# Chapter 4 Homework Answers

2023-02-01

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## Chapter 4: Longwave Radiation Spectrum

### Exercise 4.1: Methane

> Methane has a current concentration of 1.7 ppm in the atmosphere and is doubling at a faster rate than $CO_2$.

#### Part (a)

> **Would an additional 10 ppm of methane in the atmosphere have a larger or smaller impact on the outgoing IR flux than an additional 10 ppm of $CO_2$ at current concentrations?**

**Answer:** I ran MODTRAN three times.

- One run was a baseline, which used the current concentrations of $CO_2$ and $CH_4$.
- Then I ran MODTRAN with the $CO_2$ concentration increased by 10 ppm.
- Finally, I ran it a third time with the baseline value for $CO_2$, but with $CH_4$ increased by 10 ppm.

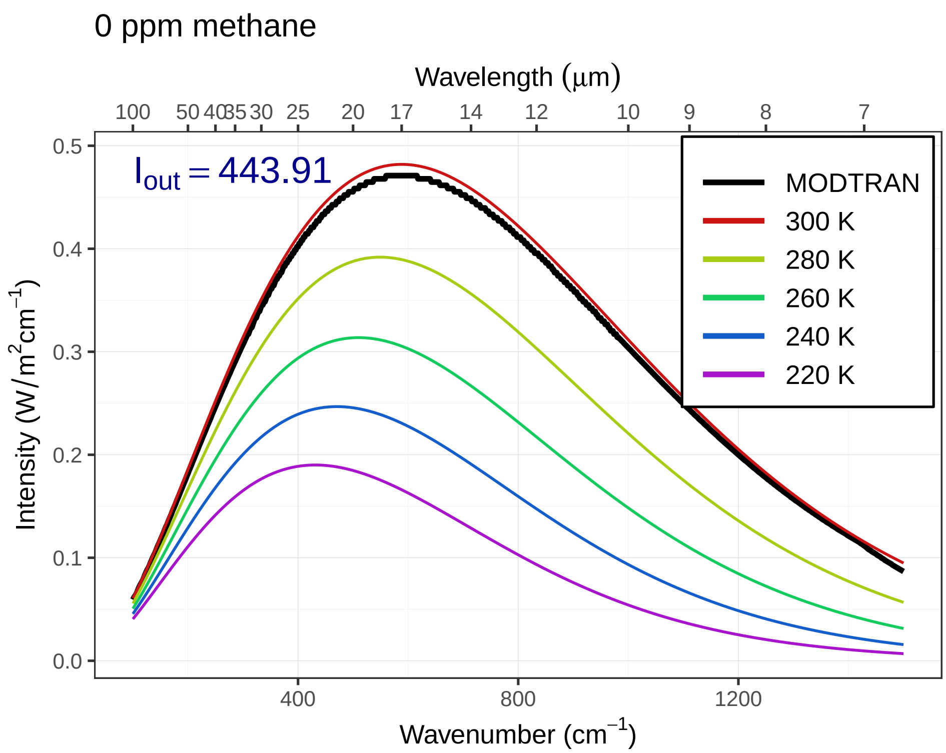
For the baseline run, the intensity of outgoing longwave light was 299. Watts per square meter. Increasing $CO_2$ by 10 ppm decreased the outgoing longwave light by 0.1 W/m$^2$, to 298. and increasing $CH_4$ by 10 ppm decreased the outgoing longwave light by 3.1 W/m$^2$ to 296.. The change in $I_{out}$ for methane is is around 25. times as much as for $CO_2$.

The difference is because absorption for $CO_2$ is strongly saturated, but the absorption for $CH_4$ is not saturated. Another way to think about this is that a 10 ppm increase in $CO_2$ increases the amount of $CO_2$ by 2.% and a 10 ppm increase in $CH_4$ increases the amount of $CH_4$ by 590.%.

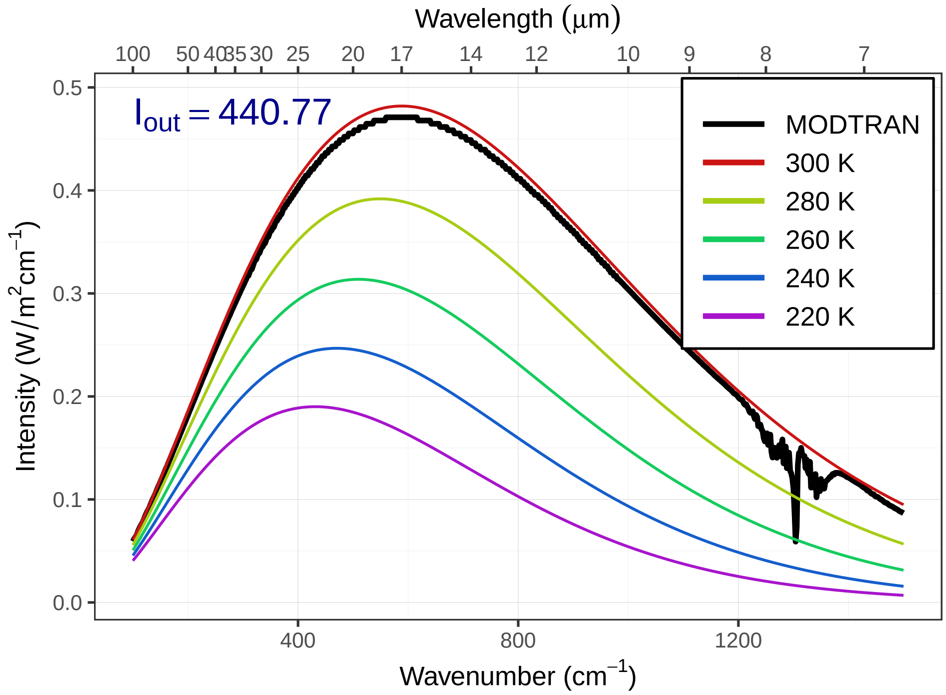
**Part (b)**

> **Where in the spectrum does methane absorb? What concentration does it take to begin to saturate the absorption in this band? Explain what you are looking at to judge when the gas is saturated.**

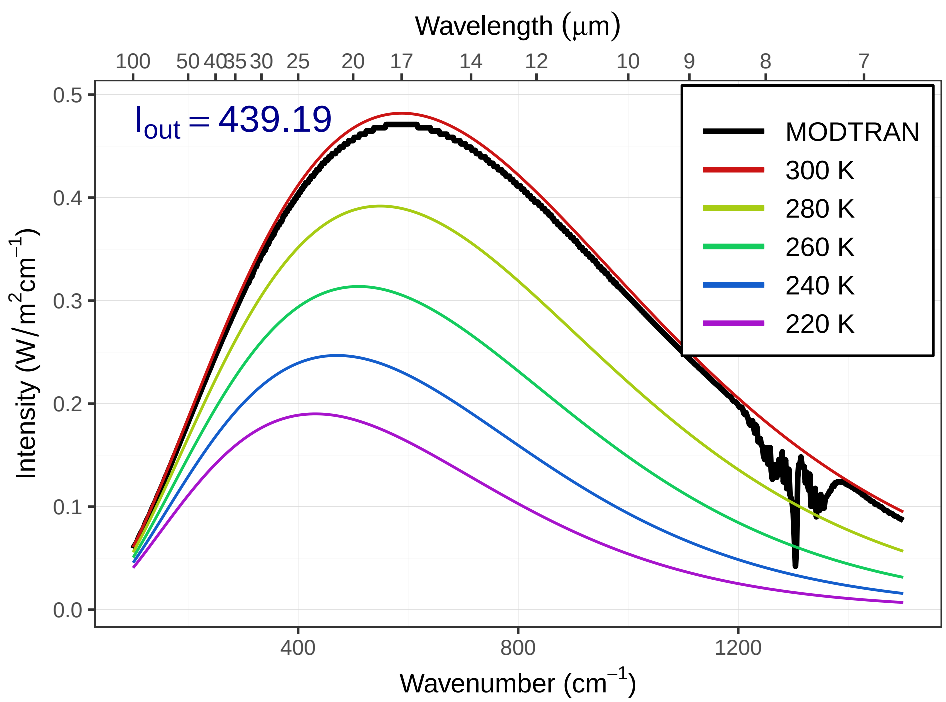
**Answer:** The plots below show the MODTRAN spectrum with all gases set to zero except methane. Methane absorbs most strongly around 1300 wavenumbers.



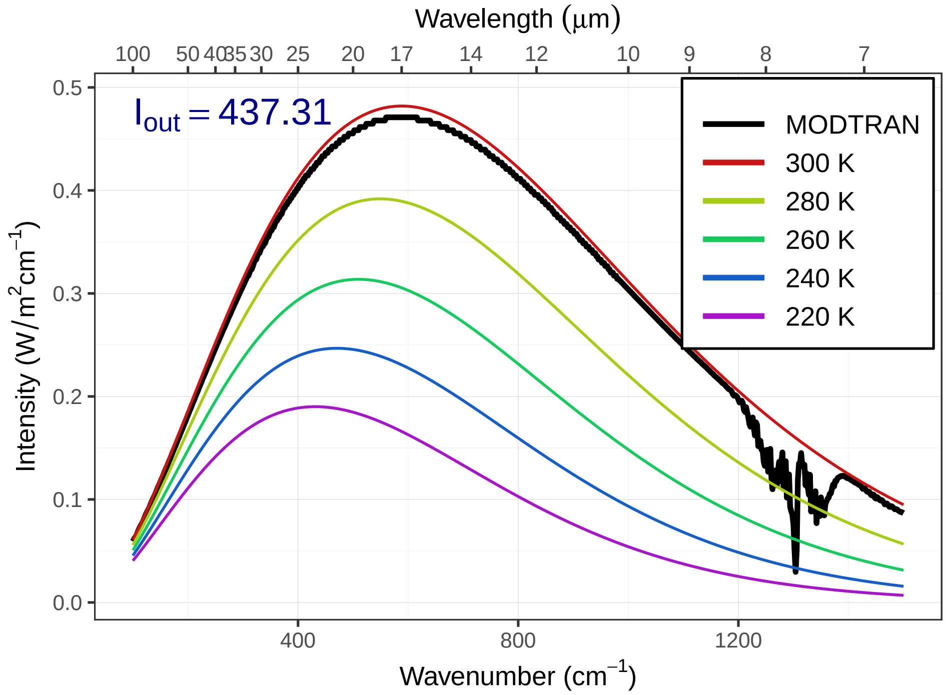
1 ppm methane



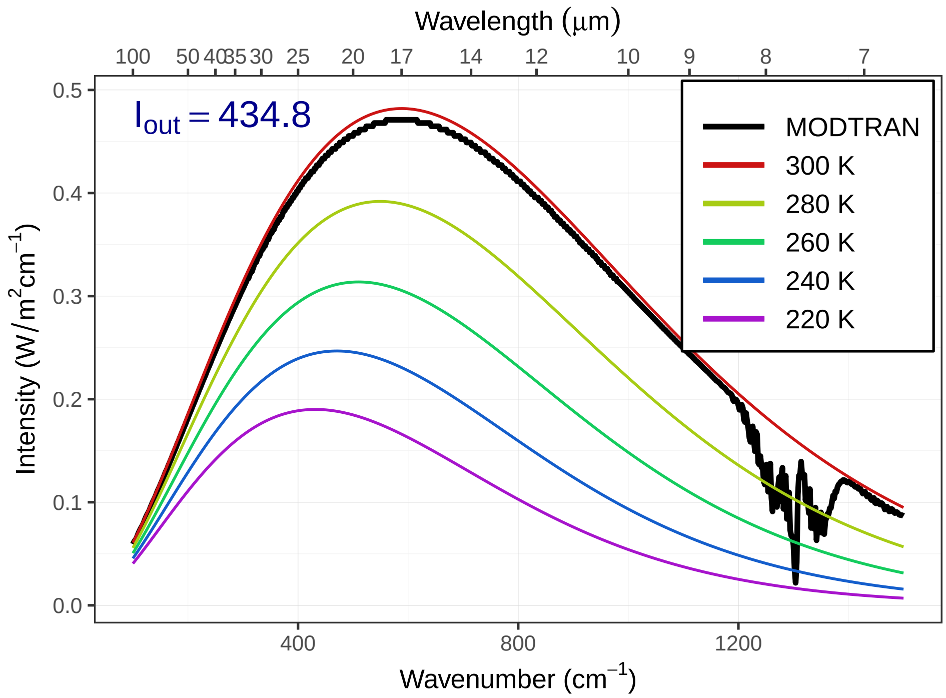

2 ppm methane



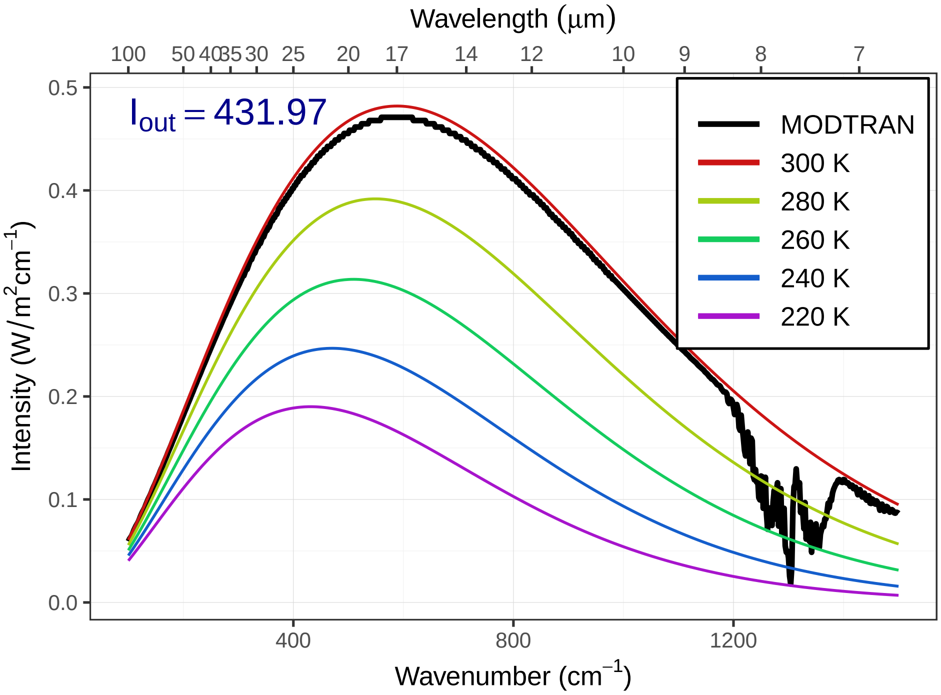
4 ppm methane



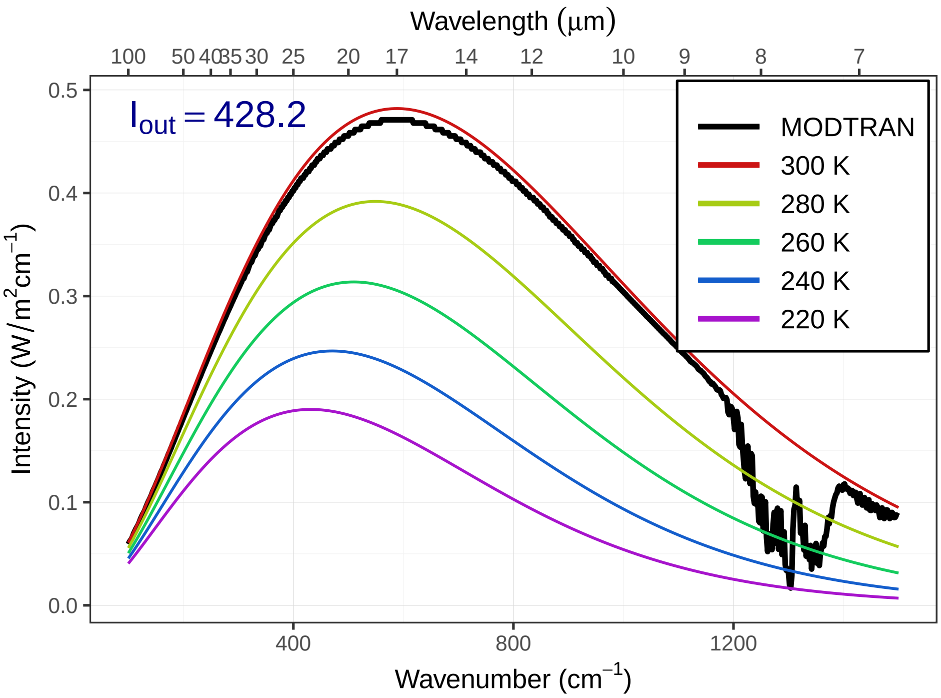

8 ppm methane



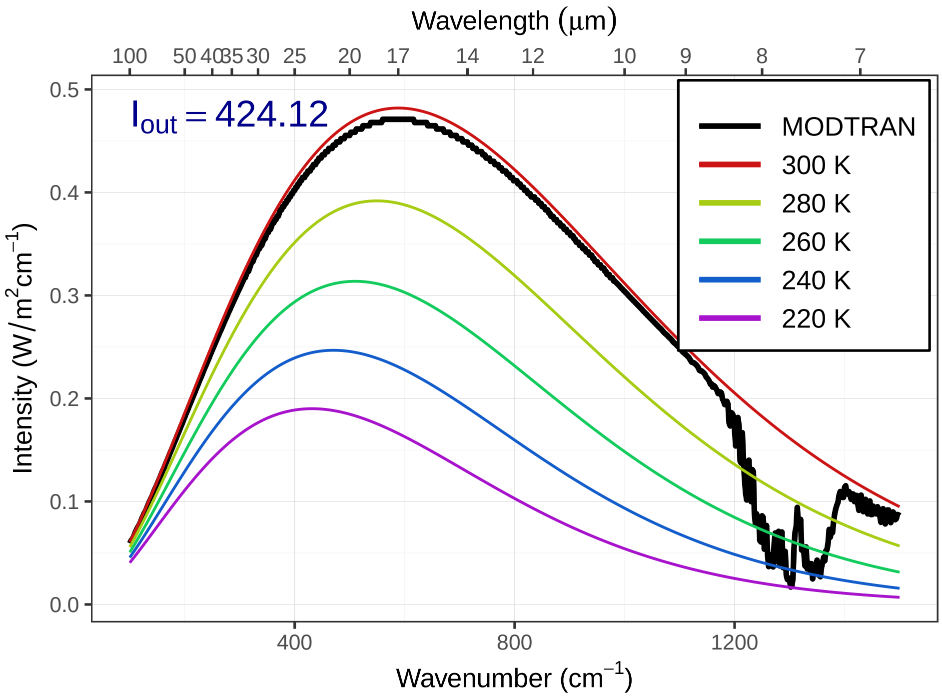
16 ppm methane



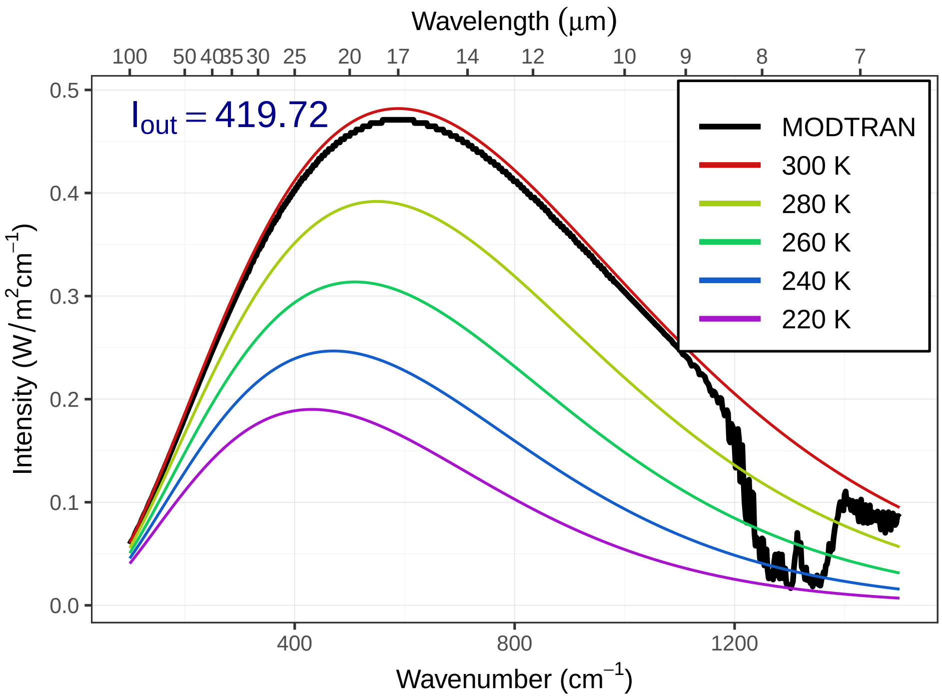

32 ppm methane



64 ppm methane




128 ppm methane



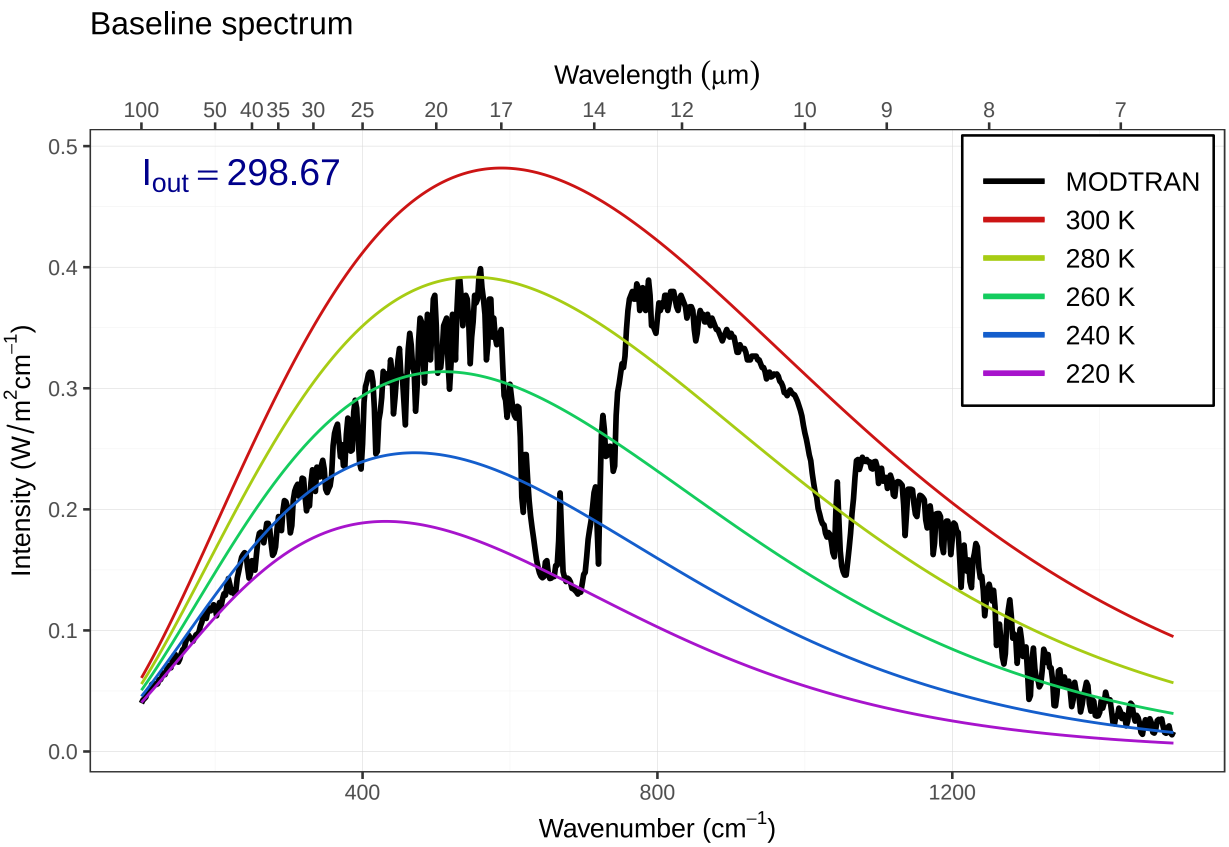

As we increase the methane concentration, the big spike around 1300 wavenumbers gets bigger until it bottoms out on the purple line. This happens somewhere around 8, 16, or 32 ppm, so any of those anwers would be correct. But the spectrum is complicated and so is its saturation, so other answers are plausible if they are supported by sound reasoning.

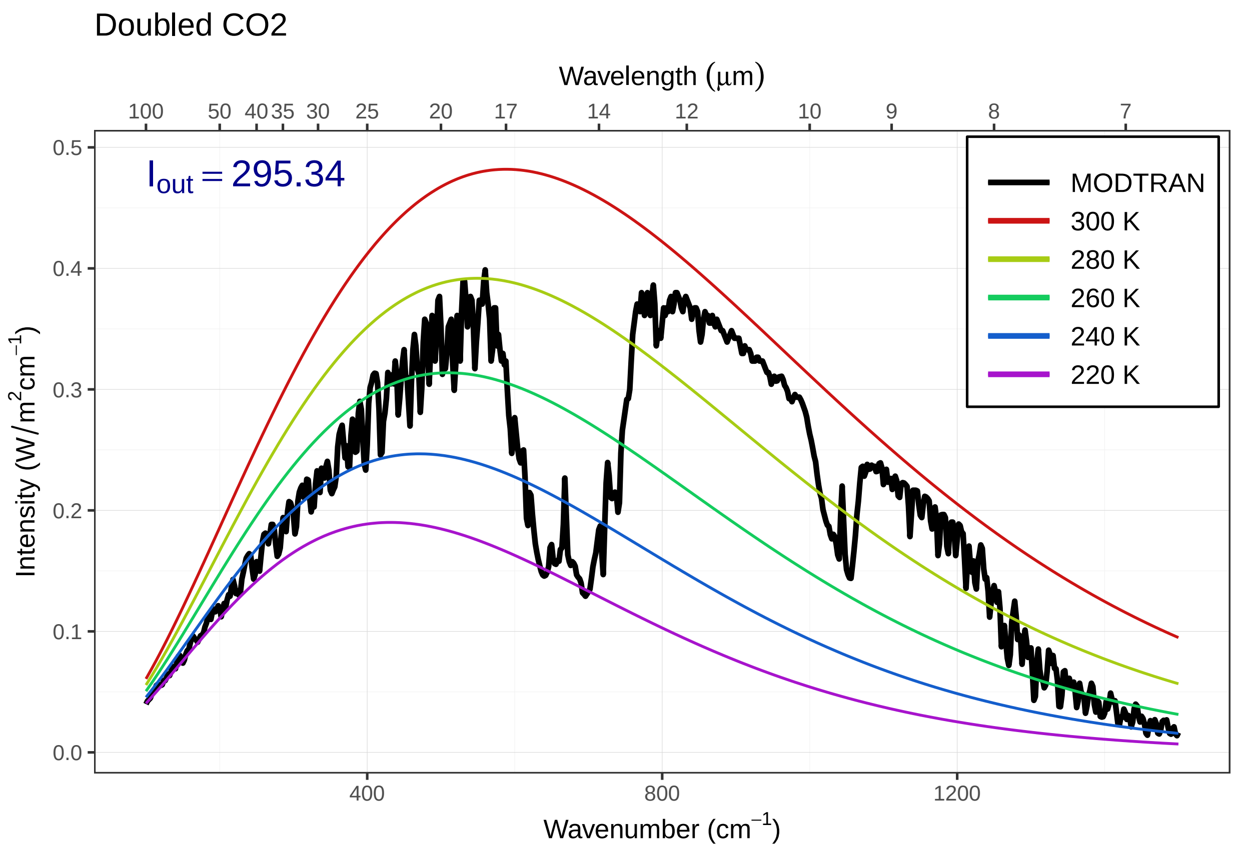
**Part (c)**

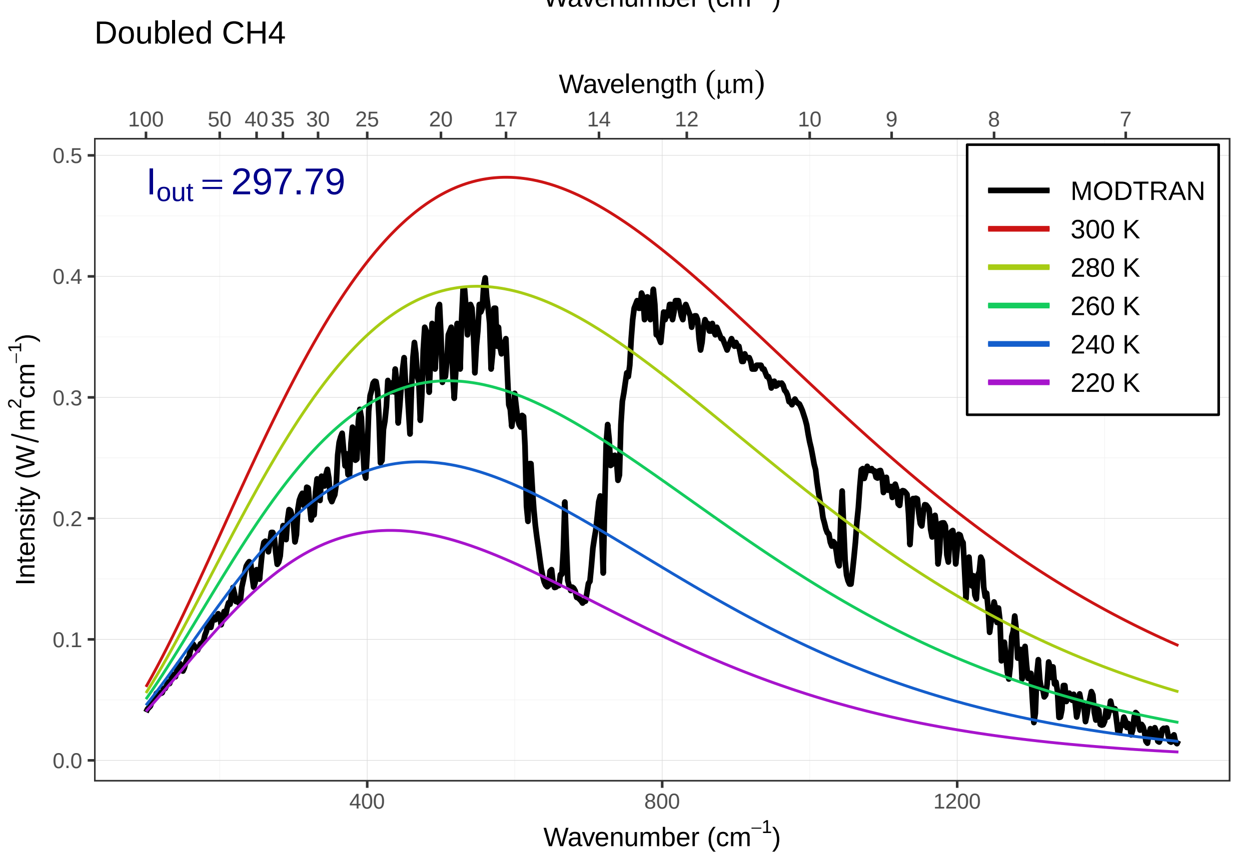
**Would a doubling of methane have as great an impact on the heat balance as a doubling of $CO_2$?**

**Answer:** The baseline value for $I_{out}$ is 299. W/m$^2$. If we double $CO_2$, it drops to 295. W/m$^2$, a decrease of 3. W/m$^2$, and if we double $CH_4$, it drops to 298. W/m$^2$, a decrease of 0.9 W/m$^2$. Doubling $CO_2$ has the larger effect because there is a lot more $CO_2$ in the atmosphere and that is more important than how saturated its absorption is.

You can see this if you look at the plots below. Notice that the effect of doubling $CO_2$ isn't to make the big $CO_2$ absorption feature get deeper, but to make it wider. You can see this if you compare the baseline to the doubled $CO_2$ spectrum where the spectrum crosses the 240 K blackbody curve near 600 cm$^{-1}$ and around 750 cm$^{-1}$. Compare this to the very small change in the methane spike near 1300 cm$^{-1}$ (you have to look very carefully at the doubled methane spectrum to notice this).




Doubled CO2
Wavelength (μm)
100
50
40
35
30
25
20
17
14
12
10
9
8
7
$I_{out} = 295.34$
MODTRAN
300 K
280 K
260 K
240 K
220 K
Intensity (W/m²cm⁻¹)
0.5
0.4
0.3
0.2
0.1
0.0
400
800
1200
Wavenumber (cm⁻¹)


Doubled CH4
Wavelength (μm)
100
50
40
35
30
25
20
17
14
12
10
9
8
7
$I_{out} = 297.79$
MODTRAN
300 K
280 K
260 K
240 K
220 K
Intensity (W/m²cm⁻¹)
0.5
0.4
0.3
0.2
0.1
0.0
400
800
1200
Wavenumber (cm⁻¹)

**Part (d)**

> **What is the "equivalent $CO_2$" of doubling atmospheric methane? That is to say, how many ppm of $CO_2$ would lead to the same change in outgoing IR radiation energy flux as doubling methane? What is the ratio of ppm $CO_2$ change to ppm methane change?**

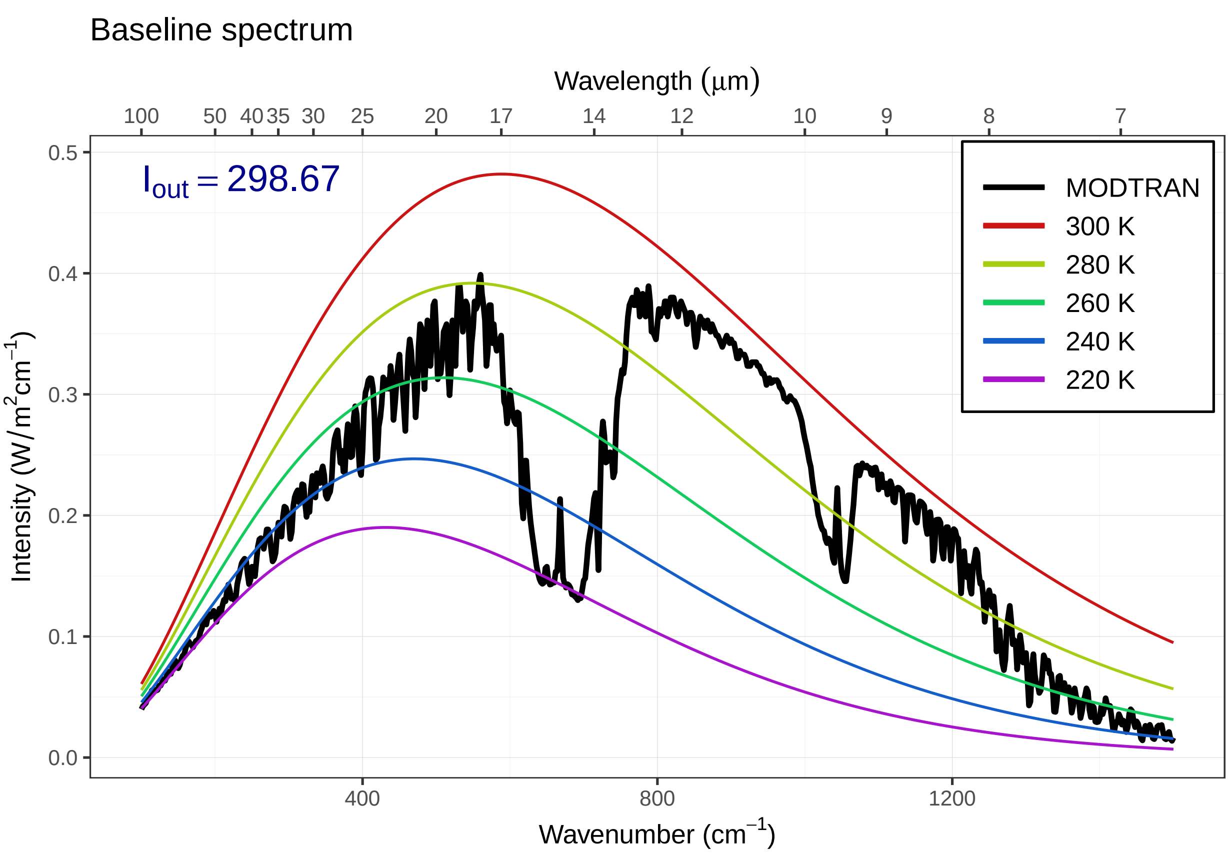
**Answer:** When we double $CH_4$, $I_{out}$ is 297.8 W/m$^2$. We need to adjust $CO_2$ to produce the same $I_{out}$ with the default value of 1.7 ppm $CH_4$. After some trial and error, this turns out to be about 480 ppm, which has $I_{out}$ = 297.8. The ratio of $\Delta CO_2$ to $\Delta CH_4$ is $80/1.7 = 47$.

## Exercise 4.3: Water vapor

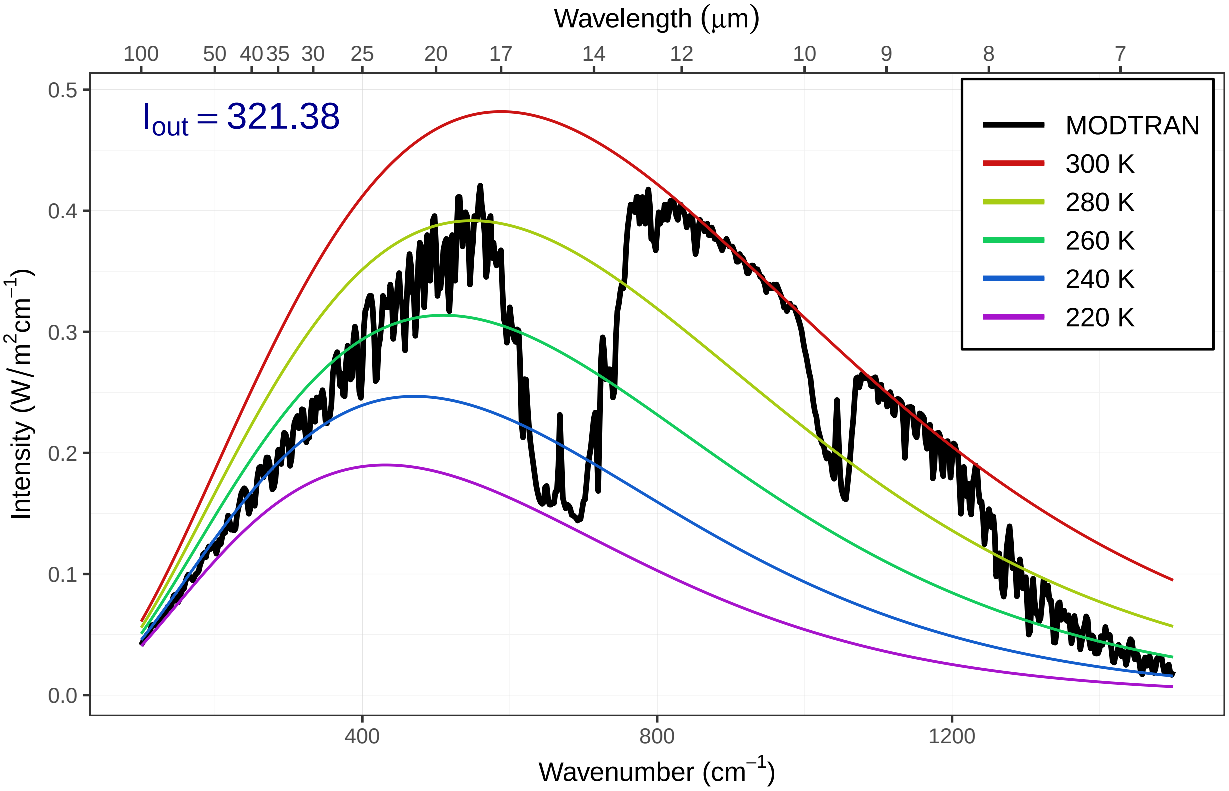
> Our theory of climate presumes that an increase in the temperature at ground level will lead to an increase in the outgoing IR energy flux at the top of the atmosphere.

**Part (a)**

> **How much extra outgoing IR would you get by raising the temperature of the ground by 5°C? What effect does the ground temperature have on the shape of the outgoing IR spectrum and why?**



Ground temperature raised 5K

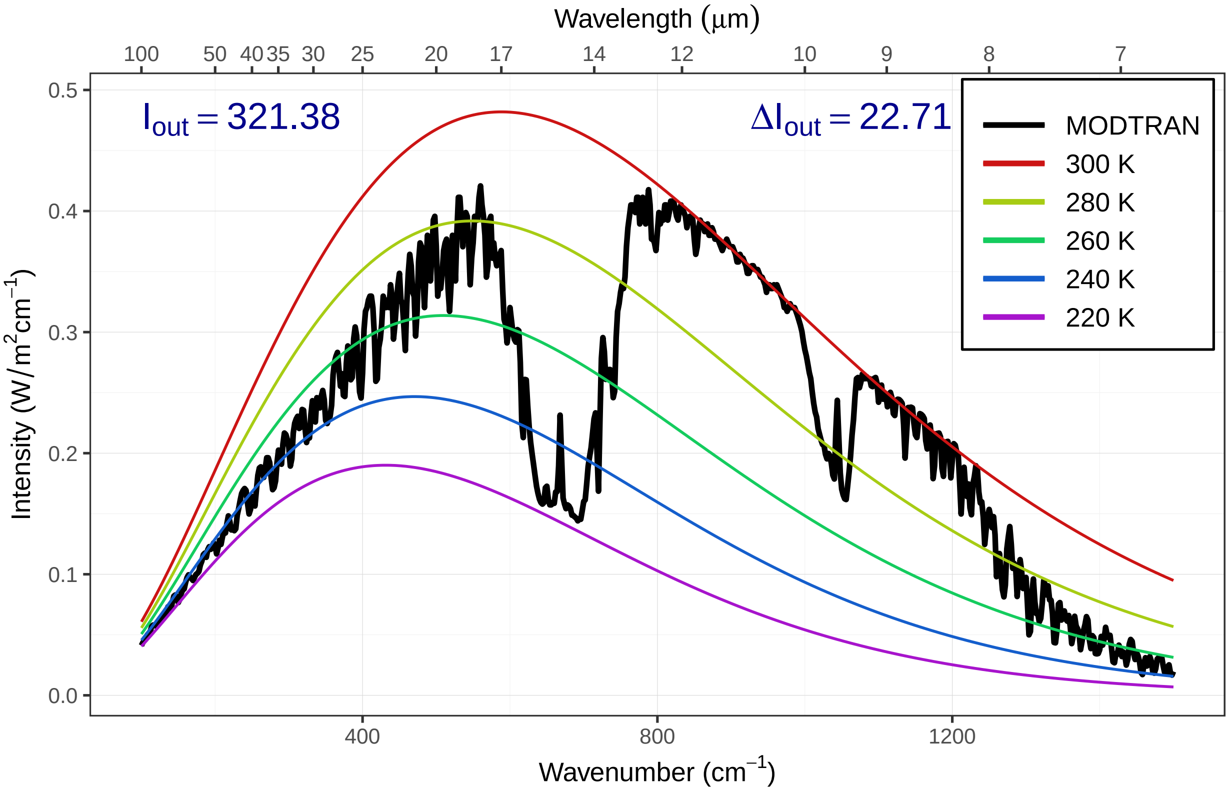



**Answer:** Raising the ground temperature raises the entire spectrum, and the intensity of outgoing longwave radiation increases from 299. W/m$^2$ to 321. W/m$^2$: a change of 23. W/m$^2$.

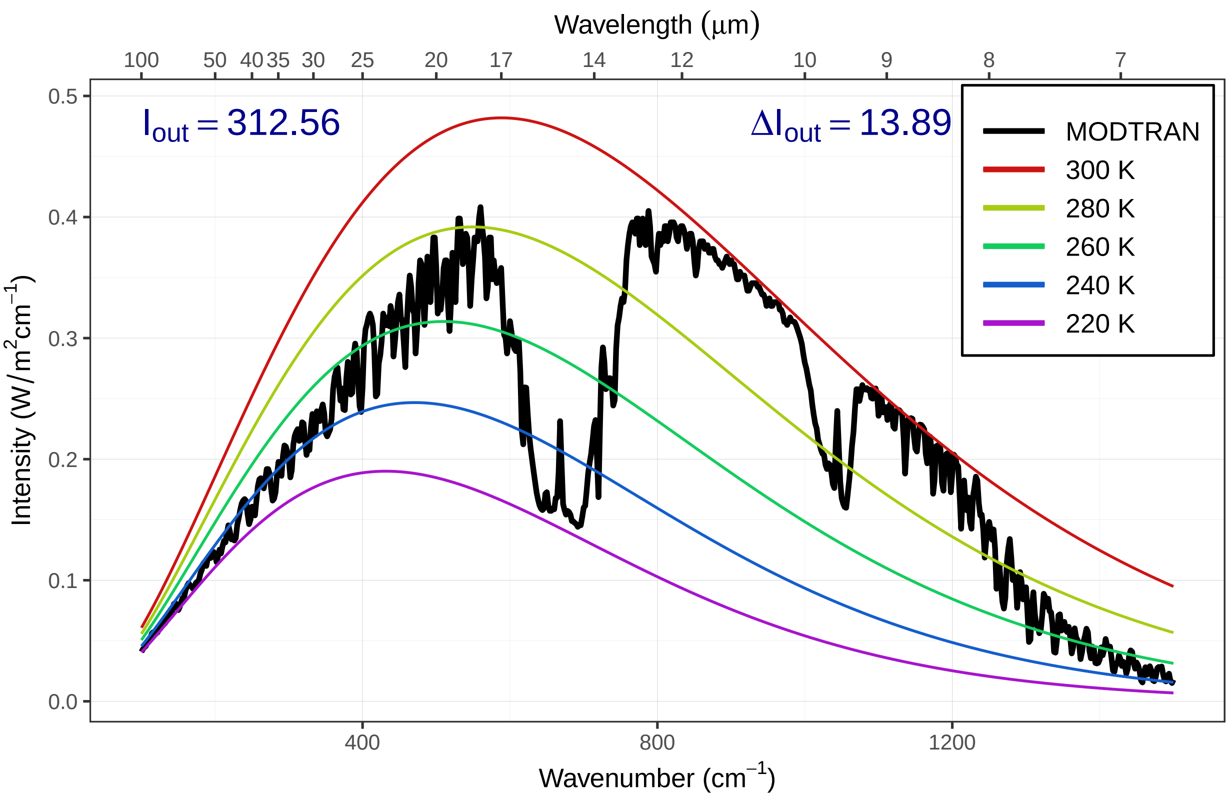
**Part (b)**

> **More water can evaporate into warm air than into cool air. Change the model settings to hold the water vapor at constant relative humidity rather than constant vapor pressure (the default), calculate the change in outgoing IR energy flux for a 5°C temperature increase. Is it higher or lower? Does water vapor make the Earth more sensitive to $CO_2$ increases or less sensitive?**

Constant vapor pressure




Constant relative humidity




**Answer:** Raising the ground temperature with constant relative humidity raises the intensity of outgoing longwave radiation from 299. W/m$^2$ to 313. W/m$^2$: a change of 14. W/m$^2$.

This is a smaller change than when we raised the surface temperature with constant vapor pressure.

This means that compensating for a change in $CO_2$ would require a bigger change in temperature with fixed relative humidity, so the climate is **more sensitive** to changes in $CO_2$ when relative humidity is fixed.

**Part (c)**

> **Now see this effect in another way.**
>
> - **Starting from the default base case, record the total outgoing IR flux.**
> - **Now double $CO_2$. The temperature in the model stays the same (that's how the model is written), but the outgoing IR flux goes down.**
> - **Using constant water vapor pressure, adjust the temperature offset until you get the original IR flux back again. Record the change in temperature.**
> - **Now repeat the exercise, but holding the relative humidity fixed instead of the water vapor pressure.**
> - **The ratio of the warming when you hold relative humidity fixed to the warming when you hold water vapor pressure fixed is the feedback factor for water vapor. What is it?**

**Answer:** In the baseline case, $I_{out}$ = 298.7 W/m$^2$. When we double $CO_2$ with constant water vapor pressure, $I_{out}$ drops to 295.3 W/m$^2$ and we have to raise the ground temperature by 0.76 K to bring $I_{out}$ back to 298.7 W/m$^2$.

When we double $CO_2$ with constant water relative humidity, $I_{out}$ drops to 295.3 W/m$^2$ and we have to raise the ground temperature by 1.2 K to bring $I_{out}$ back to 298.7 W/m$^2$.

The feedback factor is the ratio of the temperature change with relative humidity fixes to the temperature change with vapor pressure fixed: $f = 1.2\text{ K}/0.76\text{ K} = 1.6$.

Notice that there is no difference between holding vapor pressure constant and holding relative humidity constant until the temperature changes.