1 Proof of Theorem 1

Let $A_{\Pi^*}(i)$ and $A_{\Pi}(i)$ be the assignment of task i according to the optimal and the EFS2 algorithms. Each element of $A_{\Pi^*}(i)$ and $A_{\Pi}(i)$ is either L or R.

Let z be the task that finishes the last in EFS2. Denote $M_z \in \{L, R\}$ the most efficient device for task z, namely, $M_z = \arg\max_{M \in \{L,R\}} e_{z,M_z}$. Clearly, $e_{z,M_z} = 1$. Denote by list $\mathcal{L}$, the tasks executed on device M_z with starting times earlier than t_z^S, that is, $\mathcal{L} = \{i | A_{\Pi}(i) = M_z \text{ and } t_i^S < t_z^S\}$. All tasks in $\mathcal{L}$ must have efficiency 1 on device M. This can be proved by contradiction. Suppose there exists a task in $\mathcal{L}$ with efficiency lower than 1 on device M is scheduled earlier than task z. By EFS2, the higher efficiency task z should be scheduled earlier than this task. We have the contradiction. Formally,

$$e_{i,M_z} = 1, i \in \mathcal{L}. \tag{1}$$

Let $t_{\mathcal{L}}^C$ be the completion time when all tasks are in $\mathcal{L}$. The completion time is not larger than the starting time of task z. That is,

$$t_{\mathcal{L}}^C \geq t_z^S. \tag{2}$$

This can be proved by contraction. If $t_{\mathcal{L}}^C < t_z^S$, there must exist a task k on machine M_z starting between $[t_{\mathcal{L}}^C, t_z^S)$. On one hand, the efficiency of task k should be lower than 1, otherwise, it belongs to $\mathcal{L}$ according to (1). On the other hand, its efficiency must equal 1 or task z should be scheduled earlier than task k.

According to (2), it is easy to see,

$$\begin{aligned} t_{\Pi}^C &= \mu_{z,A_{\Pi}(z)} + t_z^S \\ &\leq \mu_{z,A_{\Pi}(z)} + t_{\mathcal{L}}^C. \end{aligned} \tag{3}$$

We next give two lemmas for the approximation analysis of each part in (3).

Lemma 1. *Let task z be the last task to be scheduled in the EFS2 algorithm. Its duration is less than or equal to $2\sqrt{2}$ times the optimal makespan.*

$$\mu_{z,A_{\Pi}(z)} \leq 2\sqrt{2} \cdot t_{\Pi^*}^C. \tag{4}$$

Proof. No matter where task i is scheduled, the optimal makespan is always larger than or equal to either $\mu_{z,M}, M \in \{L, R\}$.

$$\begin{aligned} t^C_{\Pi^*} &\geq \mu_{z,A_{\Pi^*}(z)} \\ &\geq \min_{M\in\{L,R\}} \{\mu_{z,M}\} \\ &= e_{z,A_\Pi(z)} \cdot \mu_{z,A_\Pi(z)}. \end{aligned}$$

According to the EFS2 rule, each task been scheduled satisfies,

$$e_{z,A(z)} \cdot \mu_{z,A_\Pi(z)} \geq \frac{1}{2\sqrt{2}} \cdot \mu_{z,A_\Pi(z)}.$$

The statement (4) is hence proved. □

Lemma 2. *The completion time of all tasks in $\mathcal{L}$ satisfies,*

$$t^C_{\mathcal{L}} \leq 2t^C_{\Pi^*}. \tag{5}$$

Proof. We consider two cases based on the assignment of $\mathcal{L}$.

Tasks in $\mathcal{L}$ are scheduled on the AR device Divide $\mathcal{L}$ into 2 parts, $\mathcal{L}_1$ and $\mathcal{L}_2$, where $A_\Pi(i) = A_{\Pi^*}(i) = L, \forall i \in \mathcal{L}_1$ and $A_\Pi(i) \neq A_{\Pi^*}(i) = R, \forall i \in \mathcal{L}_2$. Clearly, $t^C_{\Pi^*} \geq \sum_{i\in\mathcal{L}_1} \frac{p_i}{f_{A_{\Pi^*}(i)}}$ and $t^C_{\Pi^*} \geq \sum_{i\in\mathcal{L}_2} \frac{p_i}{f_{A_{\Pi^*}(i)}}$. Hence,

$$\begin{aligned} 2t^C_{\Pi^*} &\geq \sum_{i\in\mathcal{L}_1} \frac{p_i}{f_{A_{\Pi^*}(i)}} + \sum_{i\in\mathcal{L}_2} \frac{p_i}{f_{A_{\Pi^*}(i)}} \\ &= \sum_{i\in\mathcal{L}_1} \frac{p_i}{f_{A_\Pi(i)}} + \sum_{i\in\mathcal{L}_2} \frac{f_L}{f_R} \cdot \frac{p_i}{f_{A_\Pi(i)}} \\ &\geq \frac{f_L}{f_R} \Big(\sum_{i\in\mathcal{L}_1} \frac{p_i}{f_{A_\Pi(i)}} + \sum_{i\in\mathcal{L}_2} \frac{p_i}{f_{A_\Pi(i)}}\Big) \\ &= \frac{f_L}{f_R} \sum_{i\in\mathcal{L}} \frac{p_i}{f_{A_\Pi(i)}}. \end{aligned}$$

We next show that

$$t^C_{\mathcal{L}} = \sum_{i\in\mathcal{L}} \mu_{i,A_\Pi(i)} = \sum_{i\in\mathcal{L}} \frac{p_i}{f_{A_\Pi(i)}}, \forall i \in \mathcal{L}. \tag{6}$$

In other words, all the tasks in $\mathcal{L}$ do not incur any communication cost. To see this is true, consider that the predecessor of task i runs the EC device. From

$$\begin{cases} \mu_{i,L} = \dfrac{p_i}{f_L} + |x_i - x_j|\dfrac{c_{i,j}}{b} \\ \mu_{i,R} = \dfrac{p_i}{f_R} + |x_i - x_j|\dfrac{c_{i,j}}{b} \end{cases} \quad i \in V, \langle i,j \rangle \in A, \tag{7}$$

we have

$$\mu_{i,L} > \frac{p_i}{f_L} > \frac{p_i}{f_R} = \mu_{i,R}, i \in \mathcal{L},$$

and

$$e_{i,L} = \frac{\min\{\mu_{i,L}, \mu_{i,R}\}}{\mu_{i,L}} = \frac{\mu_{i,R}}{\mu_{i,L}} < 1, i \in \mathcal{L},$$

due to

$$\begin{cases} e_{i,L} = \dfrac{\min\{\mu_{i,L}, \mu_{i,R}\}}{\mu_{i,L}} \\ e_{i,R} = \dfrac{\min\{\mu_{i,L}, \mu_{i,R}\}}{\mu_{i,R}} \end{cases} \quad i \in V. \tag{8}$$

(1) is violated, and hence (6) holds. Therefore,

$$t^C_{\mathcal{L}} \leq \frac{2f_R}{f_L} \cdot t^C_{\Pi^*}. \tag{9}$$

Tasks in $\mathcal{L}$ are scheduled on the EC device We divide $\mathcal{L}$ into 4 sub-lists, $\mathcal{L}_{11}$, $\mathcal{L}_{12}$, $\mathcal{L}_{21}$, and $\mathcal{L}_{22}$, where $\mathcal{L}_{11} = \{i | A_\Pi(i) = A_{\Pi^*}(i), A(\Gamma_i^-) = A_\Pi(i), A(\Gamma_i^-) = A_\Pi(i)\}$, $\mathcal{L}_{12} = \{i | A_\Pi(i) = A_{\Pi^*}(i), A(\Gamma_i^-) \neq A_\Pi(i), A(\Gamma_i^-) \neq A_\Pi(i)\}$, $\mathcal{L}_{21} = \{i | A_\Pi(i) \neq A_{\Pi^*}(i) = L, A(\Gamma_i^-) = A_\Pi(i), A(\Gamma_i^-) \neq A_\Pi(i)\}$, $\mathcal{L}_{12} = \{i | A(i \neq A_{\Pi^*}(i) = L, A(\Gamma_i^-) \neq A_\Pi(i), A(\Gamma_i^-) = A_\Pi(i)\}$. The physical meanings of $\mathcal{L}_{11}$, $\mathcal{L}_{12}$, $\mathcal{L}_{21}$, and $\mathcal{L}_{22}$ are illustrated in Table 1.

Table 1: Physical meanings of each sub-list of $\mathcal{L}$

	$\mathcal{L}_{11}$	$\mathcal{L}_{12}$	$\mathcal{L}_{21}$	$\mathcal{L}_{22}$
$A_\Pi(i)$	EC w/o comm.	EC w/ comm.	EC w/o comm.	EC w/ comm.
$A_{\Pi^*}(i)$	EC w/o comm.	EC w/ comm.	AR w/ comm.	AR w/o comm.

We denote by $W_{12}^{\Pi^*} = W_{12}^{\Pi}$ the workload of tasks in $\mathcal{L}_{12}$ on the EC device; and $W_{21}^{\Pi^*}$ the workloads of tasks in $\mathcal{L}_{21}$ on the AR device in Π^*. Let W_{12}^{Π} and W_{22}^{Π} be the workloads in $\mathcal{L}_{12}$ and $\mathcal{L}_{22}$ according to Π, respectively. We have,

$$2t_{\Pi^*}^{C} \geq \sum_{i\in\mathcal{L}_{11}} \frac{p_i}{f_{A_{\Pi^*}(i)}} + W_{12}^{\Pi^*} + W_{21}^{\Pi^*} + \sum_{i\in\mathcal{L}_{22}} \mu_{i,A_{\Pi^*}(i)}. \tag{10}$$

Note that,

$$W_{21}^{\Pi^*} \geq \sum_{i\in\mathcal{L}_{21}} \frac{p_i}{f_{A_{\Pi^*}(i)}} \geq \sum_{i\in\mathcal{L}_{21}} \frac{p_i}{f_{A_{\Pi}(i)}}, \tag{11}$$

and due to (1),

$$\sum_{i\in\mathcal{L}_{22}} \mu_{i,A_{\Pi^*}(i)} \geq \sum_{i\in\mathcal{L}_{22}} \mu_{i,A_{\Pi}(i)} \geq W_{22}^{\Pi}. \tag{12}$$

Substituting terms in (10) with (11) and (12), we have

$$2t_{\Pi^*}^{C} \geq \sum_{i\in\mathcal{L}_{11}} \frac{p_i}{f_{A_{\Pi}(i)}} + W_{12}^{\Pi} + \sum_{i\in\mathcal{L}_{21}} \frac{p_i}{f_{A_{\Pi}(i)}} + W_{22}^{\Pi}. \tag{13}$$

Due to pipelining, the right hand is less than $W_{\mathcal{L}}^{\Pi}$. Therefore,

$$\begin{aligned} 2t_{\Pi^*}^{C} &\geq W_{\mathcal{L}}^{\Pi} \\ &= t_{\mathcal{L}}^{C}. \end{aligned} \tag{14}$$

We collect the two cases together and draw the conclusion,

$$\begin{aligned} t_{\mathcal{L}}^{C} &\leq \min\left\{\frac{f_R}{f_L} \cdot 2t_{\Pi^*}^{C}, 2t_{\Pi^*}^{C}\right\} \\ &= 2t_{\Pi^*}^{C}. \end{aligned} \tag{15}$$

□

Finally, combining Lemma 1 and Lemma 2, we have Theorem 1.