Bruteforce method

Sum of all submatrices of a given matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$\boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}$ → 4 Submatrices – (1×1) → 4×1

$\begin{bmatrix}1 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\end{bmatrix}$ → 2 submatrices → (1×2) ~~(2×2)~~ $= 2\times2 = 4$

$\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$ → 2 submatrices → (2×1) $= 2\times \underbrace{2}_{\text{contributing}} = 4$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ → 1 submatrix (2×2) → $1\times4 =$

16

16 is the sum of all possible submatrices of a given matrix.

Brutforce method

Sum of all submatrices of a given matrix

1	1
1	1

$[1]\ [1]\ [1]\ [1]$ → 4 Submatrices – (1×1) → 4×1

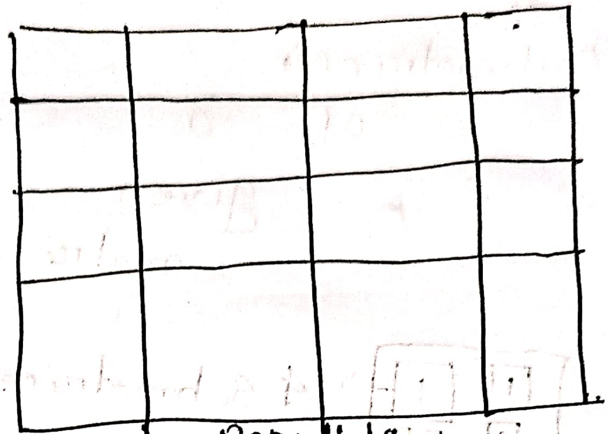
$[1\ 1]$, $[1\ 1]$ → 2 submatrices → (1×2) $\cancel{(2\times2)}$ $= 2\times2 = 4$

$\begin{bmatrix}1\\1\end{bmatrix}$, $\begin{bmatrix}1\\1\end{bmatrix}$ → 2 submatrices (2×1) $= \underbrace{2}_{\text{contributing}}\times2 = 4$

$\begin{bmatrix}1&1\\1&1\end{bmatrix}$ → 1 submatrix (2×2) → $1\times4 = 4$

16

16 is the sum of all possible submatrices of a given matrix.



Extracting all ^possible submatrices brut force

Submatrices can be considered as a rectangle

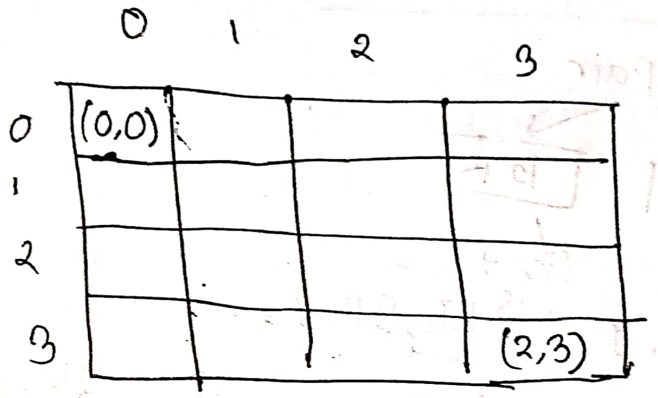
this information will help u in co-ordinate system.

→ uniquely identifying a rectangle.

↳ Top — left } corner

↳ Bottom — right } Corner

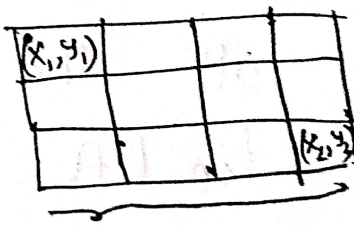
~~Top~~



Easily extract the length and breadth of the reactangle.

Suppose Top → left Co-ordinate (x_1, y_1)

Bottom-right Co-ordinate (x_2, y_2)



length is $(y_2 - y_1 + 1)$

breadth is $(x_2 - x_1 + 1)$

⇓

we get area

⇓

we can draw the reactangle.

To find a ~~rectangle~~ rectangle we Just need Top left bottom right.

All possible

Pair

$\boxed{T.L}$ $\boxed{B.R}$

(x, y) (x, y)

$\{\{LT_1, BR_1\}, \{LT_2, BR_2\}, \ldots\}$

if we define all those pairs we can easily extract submatrices.

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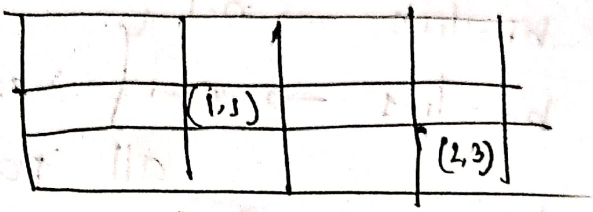
all the possible (x, y) cordinates to define all top-left &

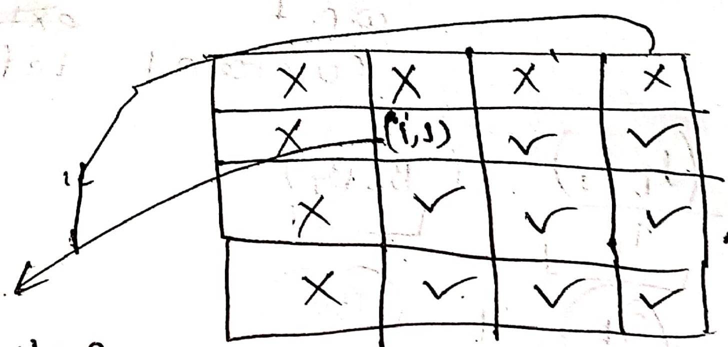
all the possible (x, y) cordinates to ~~dir~~ define bottom right

↓

(*) Top-left for Co-ordinates will be lesser than → Bottom-right Co-ordinates



(i,j) < (2,3)



→ ✓ will be bottom right cells

Bottom right never be this index

because all are smaller than top, left index

(i,j) → T.L → 0 → end
0 → end

BR → i → end
j → end

{
for li = 0 ⟶ n-1 } extract all possible top left
for lj = 0 ⟶ n-1

for bi = li+1 ⟶ n-1 }
for bj = lj+1 ⟶ n-1 } ⟶ extract all possible Bottom Right we ~~can~~ ~~extract~~ w.r.t current Left

{

(li, lj) — (Bi, Bj)

$\underbrace{(l_i, l_j)}_{TL}$ + $\underbrace{(B_i, B_j)}_{BR}$

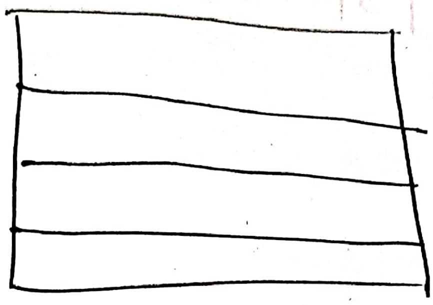
Just sumup a pairs

{
for i = li ⟶ Bi } ⟶ sum of current sub matrix
for j = lj ⟶ Bj
{
sum += arr[i][j]

}

$\boxed{\rightarrow \text{2nd method}}$

Sum



	0	1	2
0	a	b	c
1	d	e	f
2	g	h	i

~~for i -> 0 -> n-1~~

1	2	3	4	5

sum of a given sub-array

sum

prefix

1	2	3	4	5

prefix → sum

1	3	6	10	15

15 - 3 + 2

= 15 - 1

= 14

15 - 6 + 3

= 12

Prefix → sum matrix

Prefix Sum :

a	b	c
d	e	f
g	h	i

a	a+b	a+b+c
d	d+e	d+e+f
g	g+h	g+h+i

column wise

T.L

a	a+b	a+b+c
a+d	a+b+d+e	a+b+c+d+e+f
a+d+g	a+b+d+e+g+h	a+b+c+d+e+f+g+h+i

B.W

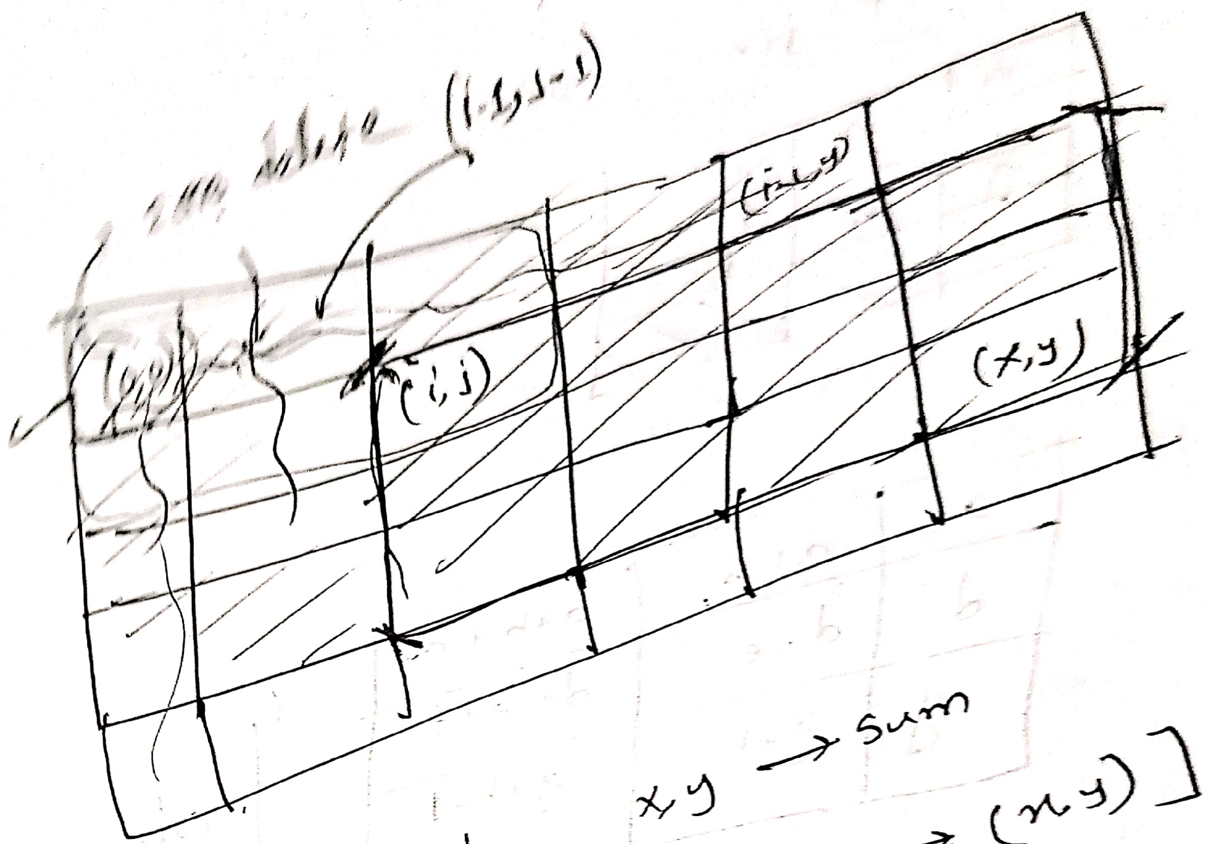
row wise

Prefix Sum matr

$TC = \cancel{d}+e+\cancel{a}+b$

$BR = \cancel{a}+\cancel{b}+c+\cancel{d}+\cancel{e}+f+g+h+i$

$BR - TL = (c+g+h+i+f)$



cell subtract twice. we need to add them

V ← value at x y → sum [0, 0 → (x, y)]

P → value at (i-1, y) → sum [(0, 0) – (i-1, y))

Q → value at (x y-1) → sum [(0, 0 → x, y-

R ← value at (i-1, j-1) → sum [0, 0 – (i-

$\boxed{V - P \to Q + R}$

G sum of all ele

TL → defi as

BR → as

$\boxed{TL}$ for $l_i = 0 \rightarrow n-1$ Prefix sum matrix

{ for $l_j = 0 \rightarrow n-1$

for $B_i = l_i \rightarrow n-1$

$\boxed{B.R}$ { for $B_j = J \rightarrow n-1$

$\rightarrow$ sum += $Pre[B_i][B_j] - Pre[l_i - 1][B_j]$

$- Pre[B_i][l_j - 1] + Pre[T_{i-1}][T_{j-1}]$

Extract sum.

$O(n^4)$

10	20	30
5	10	20
2	4	6

10	30	60
5	15	35
2	6	12

10	30	60
15	45	95
17	51	107

1	6	9	9	16	21	28	30
1	5	3	0	7	5	7	2

(0,0)

1	2	7	9	7
4	5	8	3	5
3	2	11	4	6
6	0	7	12	5

(i,j)

→ Optimistic

⇒

	0	1	2
0	a	b	c
1	d	e	f
2	g	h	i

a	b	c
d	e	f
g	h	i

$(e+f+h+i)$
$+(d+e+f)$
$+(b+e+h)$
$+ \ldots)$

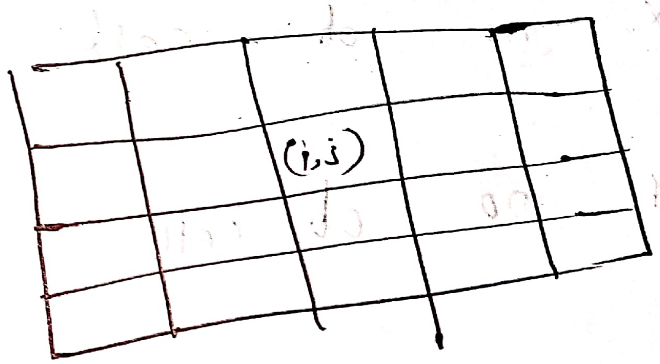
Every Element will be a part of more than 1 submatrices

→ In each submatrix, the element will be contributing for the sub

→ we just ~~find the Conti~~
Calculate the Contribution.

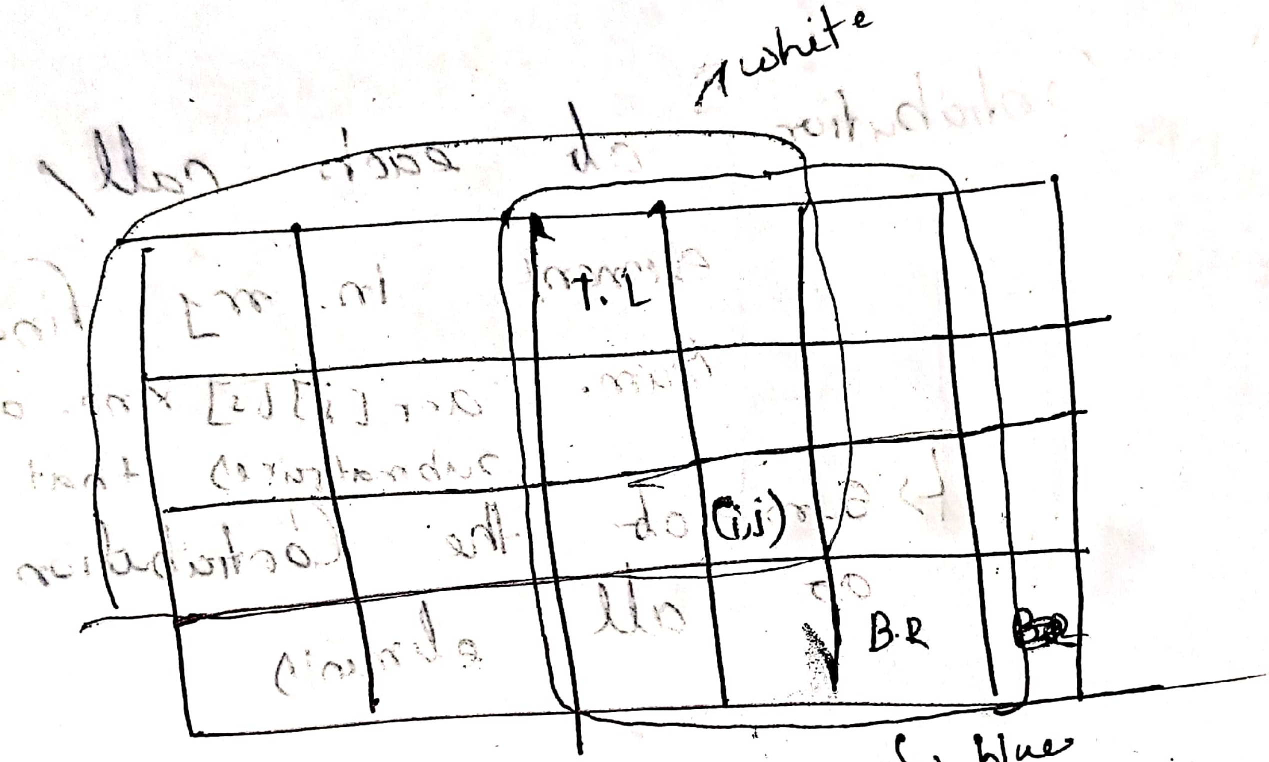
Contribution of each cell/element in my final sum = arr[i][j] × no. of submatrices that will be part of

↳ Sum of the Contribution of all elements



// all the cell when made

T.L will have (i,j)



x. no of cell in the white area.

y. no of cell in the green area

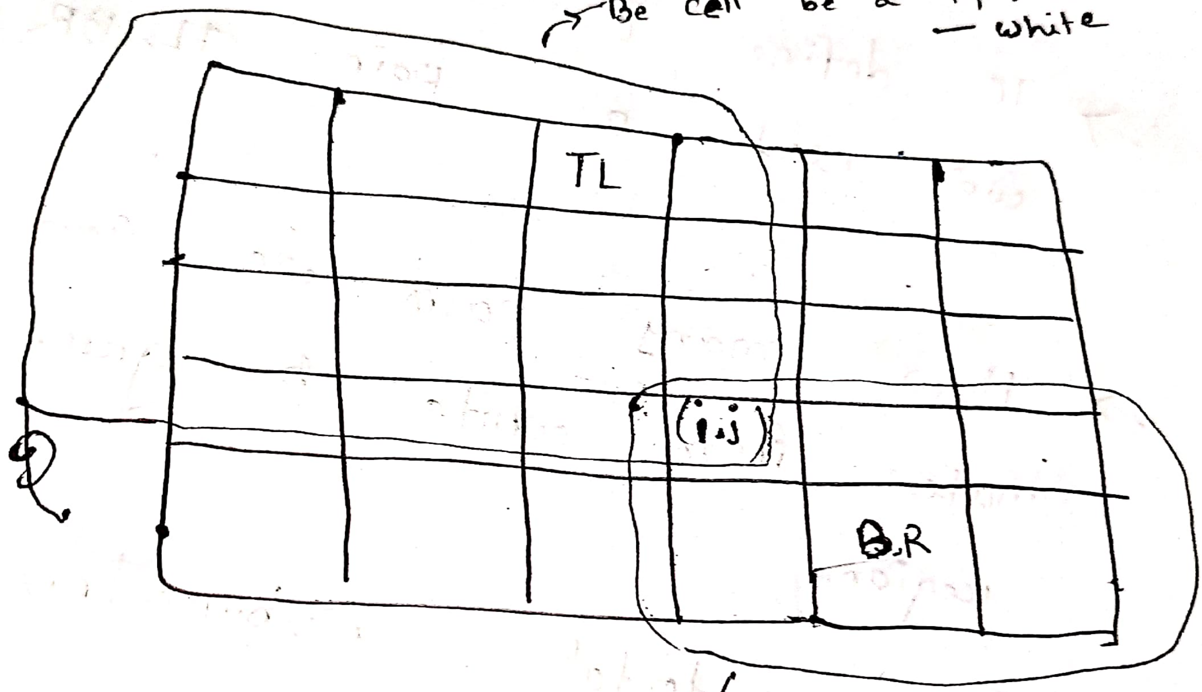
To define a submatrix we need a

pair (TL), (BR)

৫৫, ৫৭৬টাকা.

৫৫,০০০ টাকা.
২০০
৩৫০
৬০০
১৭০
৫৫৫৮০টাকা.

Be cell be a top left.
— white



Be cell be a bottom right
↳ green

all the cell, who made a BR, when have a (i,j)

X no cell in the white area

y no Cell in the green area

⇒ To define a Submatrix; we need a pair, TL, BR

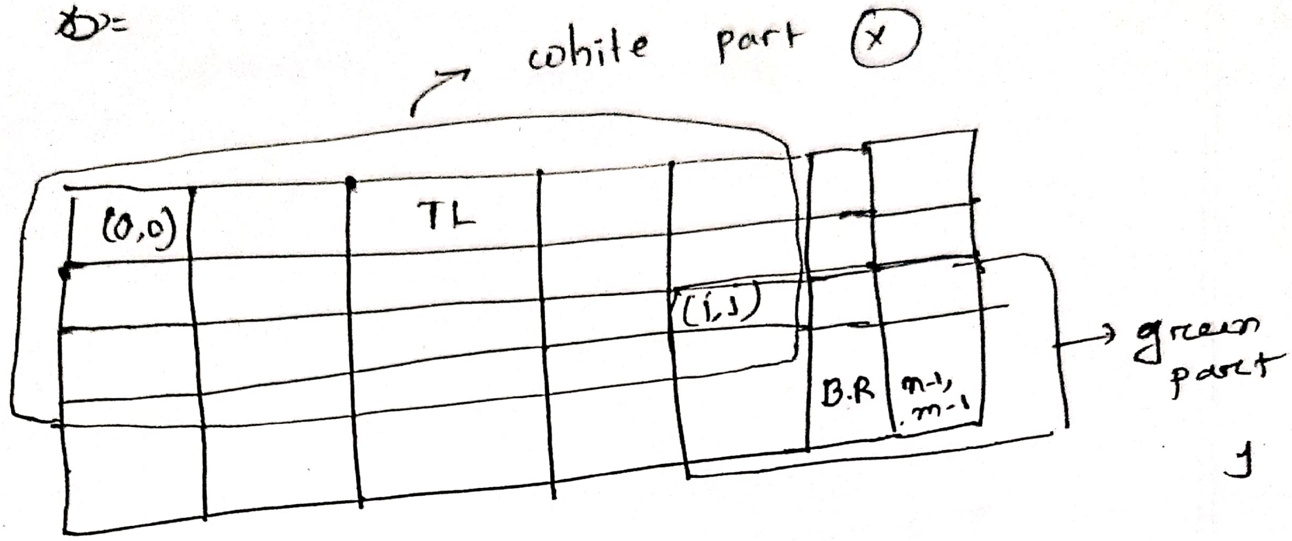
⇒ How many pairs we can make from white & green region.

⇒ The total submatrix made will be $(x * y)$

→ (i, j) is present.

-> starting indexing from (0,0)

x =



$x = (i+1) * (j+1)$
rows columns

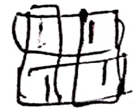
$y = (n-i) * (m-j)$ $O(n^2)$

for $i = 0$ ———— $n-1$
for $j = 0$ ———— $m-1$

sum += $\underbrace{arr[i][j]}_{\text{value of cell}}$ $*$ $[(i+1) * (j+1) * (n-i)(m-j)]$

Contribution (i,j) in my sum

no of sub matrices will be



= 2+2 part of

= 4+4+4+4