CS181 Practice Questions: POMDPs and Mixture Models

1. **Belief State Methods**

   In belief state methods, the agent has to maintain a probability distribution over the current state of the world.

   $$b(s_t) = P(s_t | o_1, \ldots, o_t,\ a_1, \ldots, a_{t-1})$$

   Express $b(s_t)$ as a function of $b(s_{t-1})$.

2. **POMDPs vs MDPs**

Why can we not use the same approaches to solve an MDP as a POMDP? That is, why is the POMDP case so much harder to compute an optimal solution for?

3. **Belief States**

Show that the set of belief states is uncountably infinite in a finite state space.

4. **Bounds (Bishop 9.17)**

Show that as a consequence of the constraint $0 \leq p(x_n|\mu_k) \leq 1$ for the discrete variable x_n, the incomplete-data log likelihood function for a mixture of Bernoulli distributions is bounded above, and hence that there are no singularities for which the likelihood goes to infinity.

5. **Gaussian (Bishop 9.3)**

Consider a Gaussian mixture model in which the marginal distribution $p(z)$ for the latent variable is given by Bishop Eq. (9.10), and the conditional distribution $p(x|z)$ for the observed variable is given by Eq. (9.11). Show that the marginal distribution $p(x)$, obtained by summing $p(z)p(x|z)$ over all possible values of z, is a Gaussian mixture of the form Eq. (9.7).

6. **EM and Mixture Models (Bishop 9.8)**

Show that if we maximize (9.40) with respect to μ_k while keeping the responsibilities $\gamma(znk)$ fixed, we obtain the closed form solution given by (9.17). (Hint use Bishop Equation 2.43)

7. Gaussian Mixture Models

Suppose we have a Gaussian mixture model where the covariance matrices Σ_k are all constrained to have a common value, call it Σ. Derive the EM condition for maximizing the likelihood function with respect to Σ under such a model.