CS181 Practice Questions: Expectation Maximization and HMMs

1. **Expectation and Maximization (Bishop 9.15)**

Show that if we maximize the expected complete-data log likelihood function (Bishop Eq. (9.55)) for a mixture of Bernoulli distributions with respect to μ_k , we obtain the M step equation (9.59).

2. **E and M (Bishop 9.20)**

Show that maximization of the expected complete-data log likelihood function (Bishop Eq. (9.62)) for the Bayesian linear regression model leads to the M step re-estimation result Eq. (9.63) for α.

3. **E and M**

Show that if we maximize the expected complete-data log-likelihood function given in eq. 9.55 for a mixture of Bernoulli's with respect to μ_k, we obtain the M-step equation 9.59.

4. **E and M**

Show that if we maximize the expected complete-data log-likelihood function given in eq. 9.55 for a mixture of Bernoulli's with respect to the mixing coefficients π_k, using a Lagrange multiplier to enforce to summation constraint (they must sum to 1), we obtain the M-step equation 9.60.

5. **Bernoulli Mixtures**

Consider the joint distribution of latent and observed variables for the Bernoulli distribution obtained by forming the product of the $p(\boldsymbol{x} \mid z, \mu)$ given by 9.52 and $p(z \mid \pi)$ given by 9.53. Show that if we marginalize this joint distribution with respect to z (i.e., sum over all possible choices for z), we obtain 9.59.

6. **When to Use HMMs (CMU)**

For each of the following scenarios, is it appropriate to use a hidden markov model to model the dataset? Why or why not.

(a) Stock market price data

(b) Recommendations on a database of movie reviews (like the book reviews from the first practical)

(c) Daily precipitation data in Boston

(d) Optical character recognition

7. **E-M For HMM's (Bishop 13.6)**

Show that if any elements of the parameters π (start probability) or A (transition probability) for a hidden Markov model are initialized to 0, then those elements will remain 0 in all subsequent updates of the EM algorithm.

8. **E-M For HMM's (Bishop 13.5)**

Verify the M-step equation 13.18 (the update rule for π_k) for the initial state probabilities of the hidden Markov model by maximization of the expected complete-data log likelihood function (given in eq. 13.17), using Lagrange multipliers to enforce the summation constraint on the components of π.