①. A rectangular waveguide with dimensions $a = 2.5$ cm, $b = 1$ cm is to operate below 15.1 GHz. How many TE & TM modes can the waveguide transmit if the guide is filled with a medium characterized by $\sigma = 0$, $\varepsilon = 4 \cdot \varepsilon_0$; $\mu_r = 1$? Calculate the cut off frequencies of the modes.

Solution:-

w.k.t. Cut off frequency, $f_c = \frac{u'}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$

w.k.t. $u' = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{(1)(4)}} = \frac{c}{2}$.

$$f_c = \frac{c}{4}\sqrt{\frac{1}{a^2}\left(m^2 + \frac{a^2}{b^2}n^2\right)} \Rightarrow \frac{c}{4a}\sqrt{m^2 + \frac{a^2}{b^2}\cdot n^2}.$$

$$= \frac{3\times10^8}{4(2.5\times10^{-2})}\sqrt{m^2 + \frac{(2.5\times10^{-2})^2}{(1\times10^{-2})^2}\cdot n^2}$$

$$f_c = 3\sqrt{m^2 + 6.25n^2}\ \text{GHz}.$$

For different values of "m" & "n" find $f_c$ value below 15.1 GHz.

If "m" is fixed & increasing "n" will quickly reach $f_c > 15.1$ GHz

$TE_{01} \Rightarrow 3\sqrt{m^2 + 6.25n^2} = 3\sqrt{0 + 6.25}$

$\{m=0, n=1\}$

$= 3(2.5) = 7.5\,GHz.$

$TE_{02} \Rightarrow 3\sqrt{0 + (6.25)(4)} = 15\,GHz.$

$TE_{03} \Rightarrow f_c = 22.5\,GHz \Rightarrow f_c > 15.1\,GHz.$

Therefore, the maximum value of $\underline{\underline{n=2}}$.

For $TE_{10}$ mode $\Rightarrow f_c = 3\,GHz.$

$TE_{20}$ mode $\Rightarrow f_c = 6\,GHz.$

$TE_{30}$ mode $\Rightarrow f_c = 9\,GHz.$

$TE_{40}$ mode $\Rightarrow f_c = 12\,GHz.$

$TE_{50}$ mode $\Rightarrow f_c = 15\,GHz \Rightarrow$ Same as for $TE_{02}$ mode.

Therefore, the maximum value of $m=5$.

(because for $TE_{60}$ mode, $f_c = 18\,GHz \Rightarrow f_c > 15.1\,GHz$)

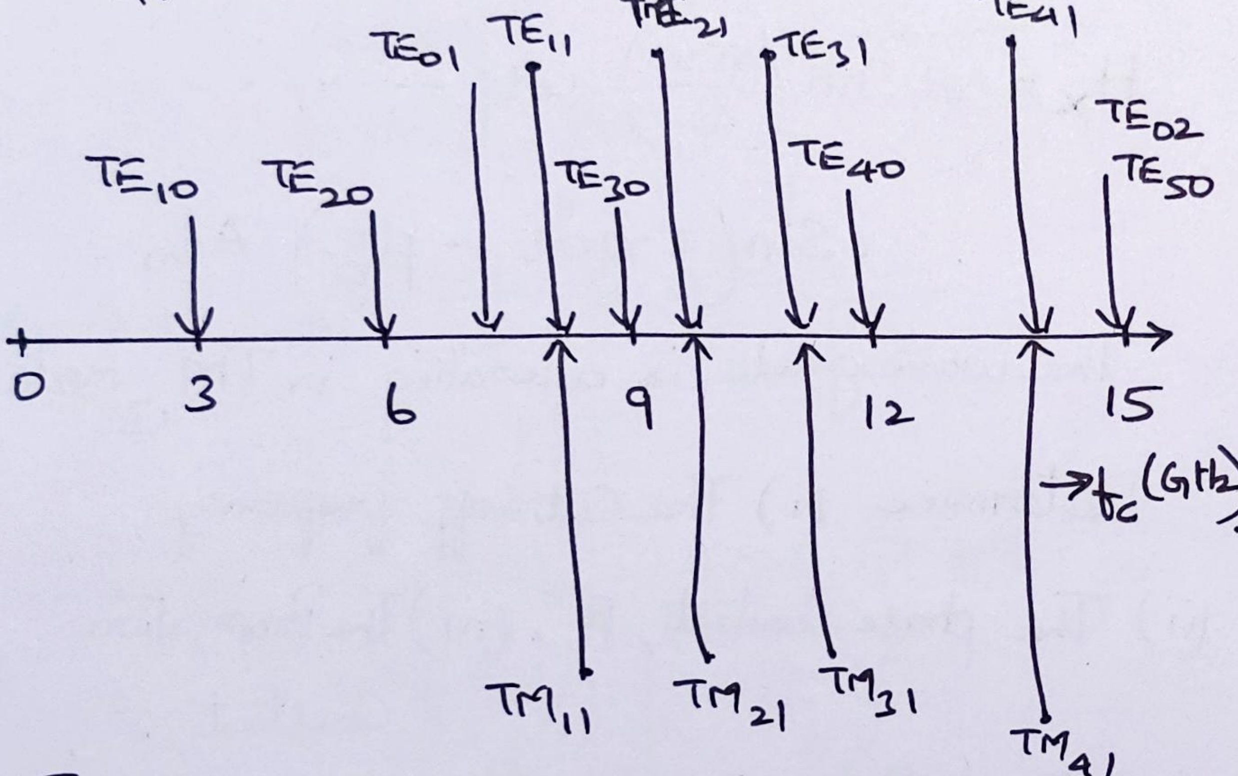
For $TE_{11}$, $TM_{11}$ (degenerate modes),

$f_c = 3\sqrt{7.25} = 8.078\,GHz.$

$TE_{21}, TM_{21} \Rightarrow f_c = 3\sqrt{10.25} = 9.6\,GHz.$

$TE_{31}, TM_{31} \Rightarrow f_c = 11.72\,GHz.$

$TE_{41}, TM_{41} \Rightarrow f_c = 14.14\,GHz.$




The waveguide can transmit the 15 modes with the cut off frequency of $f_c = 15\,GHz$ and is illustrated in the above figure.

(2) In a rectangular wave guide for which $a = 1.5\,cm$, $b = 0.8\,cm$, $\sigma = 0$ and $\varepsilon_e = 4 \cdot \varepsilon_0$.

$$H_x = 2 \sin\left(\frac{\pi x}{a}\right) \cdot \cos\left(\frac{3\pi y}{b}\right) \cdot \sin\left(\pi \times 10^{11} t - \beta z\right) \, A/m.$$

The wave guide is operating in $TM_{13}$ mode.

Determine (i) The Cut-off frequency, (ii) The phase Constant, $\beta$, (iii) The Propagation Constant, $\gamma$, (iv) The Intrinsic wave Impedance, $\eta$.

Solution:-

(i) Cut off frequency, $f_{c_{m,n}} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$.

w.k.t, $u' = \frac{1}{\sqrt{\mu \varepsilon_e}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0} \cdot \sqrt{\mu_r \varepsilon_{er}}} = \frac{c}{\sqrt{(1)(4)}}$

$$\boxed{u' = \frac{c}{2}}$$

$$\therefore f_{c13} = \frac{3\times10^8}{2\times2}\sqrt{\frac{1}{(1.5\times10^{-2})^2} + \frac{9}{(0.8\times10^{-2})^2}}$$

$$\boxed{f_{c13} = 28.57\,GHz}$$

(ii) phase Constant, $\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

w.k.t. $\beta' = \frac{\omega}{u'}$ & $u' = \frac{1}{\sqrt{\mu\varepsilon_e}}$

$$\therefore \beta' = \omega\sqrt{\mu\varepsilon_e}$$

w.k.t $\omega = 2\pi f = \pi\times10^{11} \Rightarrow f = 50\,GHz.$

$$\therefore \beta = 2\pi f\left(\frac{1}{u'}\right)\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= 2\pi f\left(\frac{2}{c}\right)\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{\pi\times10^{11}(2)}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta = \frac{\pi\times10^{11}.(2)}{3\times10^8}\sqrt{1-\left(\frac{28.57\times10^9}{50\times10^9}\right)^2} \Rightarrow 171.81\,\frac{rad}{m}.$$

(iii) propagation Constant, $\gamma = j\beta.$

$$\Rightarrow \boxed{\gamma = j171.81\,m^{-1}}$$

(iv) Intrinsic Impedance, $\eta_{TM_{13}} = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$$\eta' = \sqrt{\frac{\mu}{\varepsilon_e}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{1}{4}}$$

$$= 377 \sqrt{\frac{1}{4}} \Rightarrow \frac{377}{\sqrt{\varepsilon_r}}.$$

$$\eta' = \frac{377}{\sqrt{4}} \sqrt{1 - \left(\frac{28.57 \times 10^9}{50 \times 10^9}\right)^2}$$

$$\boxed{\eta_{TM13} = 154.7\,\Omega}$$