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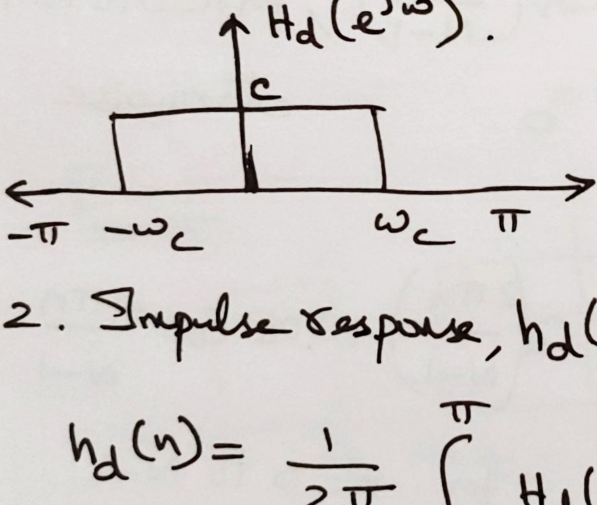
② Design a linear phase FIR LPF, Using Rectangular window by taking 7 samples of window sequence with a cut-off frequency $\omega_c = 0.2\pi$ rad/sample.

Solution:-

Given $\rightarrow$ LPF, Rectanguler window, $N=7$ & $\omega_c = 0.2\pi$ rad/sample.

1. Frequency response of LPF



$$H_d(e^{j\omega}) = e^{-j\omega\alpha} \quad ; \quad -\omega_c < \omega < \omega_c$$

$$\qquad 0 \quad \text{otherwise.}$$

2. Impulse response, $h_d(n)$ of LPF,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega \Rightarrow \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j(n-\alpha)}\left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right].$$

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$$h_d(n) = \frac{1}{\pi(n-\alpha)} \cdot \sin \omega_c (n-\alpha) \, ; \text{ for } n \neq \alpha$$

$n = 0, 1, 2, 3, 4, 5, 6.$

when $n = \alpha$

$$h_d(n) \Rightarrow \lim_{(n-\alpha) \to 0} \frac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)}.$$

$$= \frac{1}{\pi} \lim_{(n-\alpha)\to 0} \frac{\sin \omega_c (n-\alpha)}{(n-\alpha)}.$$

Applying L'Hospitals Rule, $\lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A.$

$$\Rightarrow \frac{1}{\pi} \cdot \omega_c \text{ for } n = 3; \; n = \alpha.$$

$$\boxed{\begin{aligned} &\left\{ \frac{1}{\pi} \frac{(-\alpha\omega_c) \cos(\omega_c(n-\alpha))}{(-\alpha)} \right. \\ &\Rightarrow \frac{1}{\pi} \omega_c \cos(0) \\ &= \omega_c / \pi . \end{aligned}}$$

$$\therefore h_d(n) = \begin{cases} \dfrac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)} & \text{for } n \neq \alpha \\ & n = 0,1,2,3,4,5,6 \\ \dfrac{\omega_c}{\pi} & \text{for } n = \alpha \\ & n = 3. \end{cases}$$

3. Impulse response $h(n)$ of the filter,

$$h(n) = h_d(n) \cdot \times [W_R(n)]$$

w.k.t. $W_R(n) = 1$; for $n = 0$ to $N-1 \Rightarrow 0$ to 6
$= 0$; otherwise.

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$$\therefore h(n) = \begin{cases} \dfrac{\sin \omega_c (n-\alpha)}{\pi (n-\alpha)} \times 1 & ; \text{ for } n = 0,1,2,4,5,6. \\[2ex] \dfrac{\omega_c}{\pi} \times 1 & ; \text{ for } n = 3. \end{cases}$$

$$\alpha = \frac{N-1}{2}. \quad \alpha = 3.$$

4. Transfer function $H(z)$ of the filter.

$$H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n}.$$

$$= \sum_{n=0}^{6} h(n) \cdot z^{-n}$$

$$= h(0) \cdot z^{0} + h(1) \cdot z^{-1} + h(2) \cdot z^{-2} + h(3) \cdot z^{-3} + h(4) \cdot z^{-4} + h(5) z^{-5} + h(6) \cdot z^{-6}.$$

According to Symmetry Condition, $h(n) = h(N-1-n)$

$\Rightarrow$ when $n=0 \rightarrow h(0) = h(6)$

$n=1 \rightarrow h(1) = h(5)$

$n=2 \rightarrow h(2) = h(4)$

$n=3 \rightarrow h(3) = h(3)$.

$$n=0; \quad h(0) = \frac{\sin(0.2\pi(0-\alpha))}{\pi(0-\alpha)} = \frac{\sin 0.2\pi(0-3)}{\pi(0-3)}$$

$$= 0.1009.$$

$$n=1;\quad h(1)=\frac{\sin(0.2\pi(1-3))}{\pi(1-\alpha)}=0.1514$$

$\alpha=3$

$$n=2;\quad h(2)=\frac{\sin(0.2\pi(2-\alpha))}{\pi(2-\alpha)}=0.1871$$

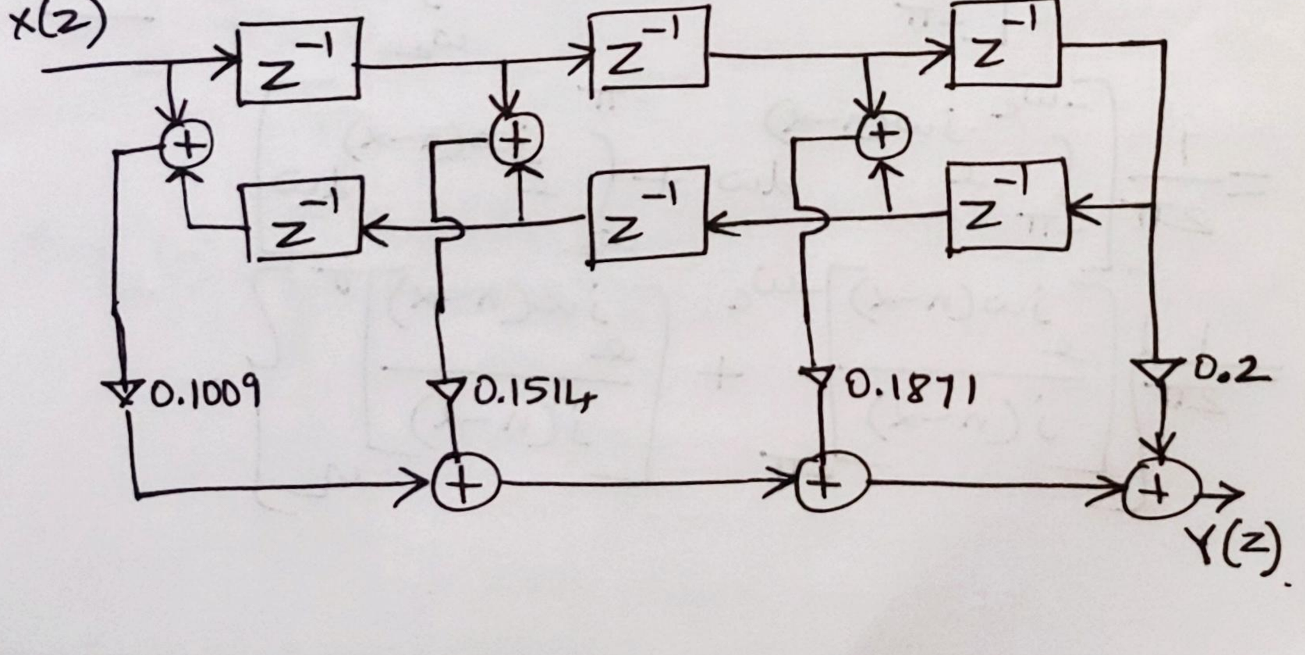
$$n=3;\quad h(3)=\frac{\omega_c}{\pi}=\frac{0.2\pi}{\pi}=0.2$$

$$\therefore H(z)=0.1009z^{0}+0.1514(z^{-1}+z^{-5})+0.1871(z^{-2}+z^{-4})$$
$$+0.2\cdot z^{-3}+0.1009z^{-6}.$$

$$\frac{Y(z)}{X(z)}=0.1009(1+z^{-6})+0.1514(z^{-1}+z^{-5})+0.1871(z^{-2}+z^{-4})+0.2z^{-3}.$$

$$Y(z)=0.1009(1+z^{-6})\cdot X(z)+0.1514(z^{-1}+z^{-5})\cdot X(z)$$
$$+0.1871(z^{-2}+z^{-4})\cdot X(z)+0.2z^{-3}$$

Realization of FIR Filter Structure.

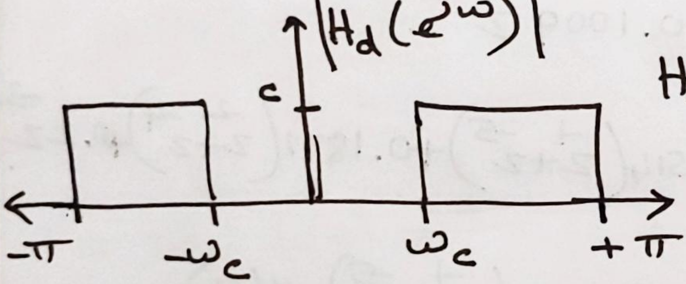


3. Design a Linear phase FIR high pass filter, Using Hamming window, with cut off frequency, $\omega_c = 0.8\pi$ rad/samples and $N=7$.

Solution :-

Given → HPF, Hamming window, $N=7$
& $\omega_c = 0.8\pi$ rad/samples.

1. Frequency response of H.P.F.



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; -\pi < \omega < -\omega_c, \ \omega_c < \omega < \pi \\ 0 & ; \text{otherwise.} \end{cases}$$

2. Impulse response $h_d(n)$ of HPF.

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{-j\omega\alpha} \cdot e^{j\omega n}\, d\omega\right].$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-\omega_c} + \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega_c}^{\pi}\right\}.$$

$$= \frac{1}{2j\pi(n-\alpha)}\left[e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\left(\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j}\right) - \left(\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right)\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)\right] \quad \text{for } n \neq \alpha,\ n = 0, 1, \dots 6.$$

when $\underline{\underline{n = \alpha}}$.

$$h_d(n) = \lim_{(n-\alpha)\to 0} \frac{\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)}$$

$$h_d(n) = \frac{1}{\pi}\left[\lim_{(n-\alpha)\to 0} \frac{\sin\pi(n-\alpha)}{(n-\alpha)} - \lim_{(n-\alpha)\to 0} \frac{\sin\omega_c(n-\alpha)}{(n-\alpha)}\right]$$

$$= \frac{\pi - \omega_c}{\pi} \Rightarrow 1 - \frac{\omega_c}{\pi}.$$

$$\Rightarrow h_d(n) = \begin{cases} \dfrac{\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)} & \text{for } n \neq \alpha,\ n = 0, 1, \dots 6 \\[2ex] 1 - \dfrac{\omega_c}{\pi} & \text{for } n = \alpha,\ n = 3. \end{cases}$$

3. Impulse response, $h(n)$ of the filter.

$$h(n) = h_d(n) \times \left[W_H(n) \right].$$

w.k.t.

$$W_H(n) = 0.54 - 0.46 \cos\frac{2\pi n}{N-1} \quad ; \text{ for } n = 0 \text{ to } N-1$$

$$0 \quad \text{otherwise.}$$

$$h(n) = \begin{cases} \left(\dfrac{\sin \pi(n-\alpha) - \sin w_c(n-\alpha)}{\pi(n-\alpha)}\right)\left(0.54 - 0.46 \cdot \cos\dfrac{2\pi n}{N-1}\right) & \text{for } n = 0, 1, 2, 4, 6. \\ \left(1 - \dfrac{w_c}{\pi}\right)\left(0.54 - 0.46 \cdot \cos\dfrac{2\pi n}{N-1}\right) & \text{for } n = 3 \end{cases}$$

$$\alpha = \frac{N-1}{2}$$

4. Transfer function $H(z)$ of the filter. $\alpha = 3$.

$$H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n} \Rightarrow \sum_{n=0}^{6} h(n) \cdot z^{-n}.$$

$$H(z) = h(0) \cdot z^{0} + h(1) \cdot z^{-1} + h(2) z^{-2} + h(3) \cdot z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) \cdot z^{-6}.$$

According to symmetry condition, $h(n) = h(N-1-n)$.

$\Rightarrow$ when $n = 0 \rightarrow h(0) = h(6)$

$n = 1 \rightarrow h(1) = h(5)$

$n = 2 \rightarrow h(2) = h(4)$.

$n = 3 \rightarrow h(3) = h(3)$.

when $n = 0$

$$h(0) = \left(\frac{\sin\pi(0-3) - \sin 0.8\pi(0-3)}{\pi(0-3)}\right)\left(0.54 - 0.46\cos\left(\frac{2\pi(0)}{6}\right)\right)$$

$h(0) = -0.0081 \Rightarrow h(6)$.

when $n=1$; $h(1) = h(5) = 0.0469$.

$n=2$; $h(2) = h(4) = -0.1441$.

$n=3$; $h(3) = 0.2$

$(1-0.8)(1) = 0.2$

$$\therefore H(z) = -0.0081z^{0} + 0.0469z^{-1} - 0.1441z^{-2} + 0.2z^{-3} - 0.1441z^{-4} + 0.0469z^{-5} - 0.0081z^{-6}.$$

$$\frac{Y(z)}{X(z)} = -0.0081(1+z^{-6}) + 0.0469(z^{-1}+z^{-5}) - 0.1441(z^{-2}+z^{-4}) + 0.2z^{-3}$$

$$Y(z) = -0.0081(1+z^{-6})X(z) + 0.0469(z^{-1}+z^{-5})X(z) - 0.1441(z^{-2}+z^{-4}).X(z) + 0.2z^{-3}.X(z).$$

Realization of FIR Filter Structure.