1. Use the backward difference for the derivative to convert the analog low-pass filter with system function $H(s) = \frac{1}{s+2}$.

Solution:-

The Mapping formula for the backward difference for the derivative is $s = \frac{1-z^{-1}}{T}$.

The system response of the digital filter is,

$$H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{T}} \Rightarrow \frac{1}{\left(\frac{1-z^{-1}}{T}\right)+2}.$$

$$= \frac{T}{1-z^{-1}+2T}. \quad \text{Assume } T = 1 \text{ sec.}$$

$$\Rightarrow \boxed{H(z) = \frac{1}{3-z^{-1}}}$$

2. Use the backward difference for the derivative and Convert the analog filter with system function, $H(s) = \frac{1}{s^2+16}$.

Solution:-

$$s = \frac{1-z^{-1}}{T}.$$

$$H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{T}} \Rightarrow \frac{1}{\left(\frac{1-z^{-1}}{T}\right)^2+16}.$$

$$H(z) = \frac{T^2}{1-2z^{-1}+z^{-2}+16T^2}$$

If $T = 1$ sec.

$$\boxed{H(z) = \frac{1}{z^{-2}-2z^{-1}+17}}$$

3 . An analog filter has the following system function. Convert this filter into a digital filter using backward difference for the derivative, $H(s) = \dfrac{1}{(s+0.1)^2+9}$.

Solution :-

$$H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{T}} = \frac{1}{\left(\frac{1-z^{-1}}{T}+0.1\right)^2+9}.$$

$$H(z) = \frac{}{z^{-2} - 2(1+0.1T)z^{-1} + (1+0.2T+9.01T^2)}$$

$$H(z) = \frac{\dfrac{T^2}{(1+0.2T+9.01T^2)}}{1 - 2\dfrac{1+0.1T}{(1+0.2T+9.01T^2)}z^{-1} + \dfrac{z^{-2}}{1+0.2T+9.01T^2}}.$$

If $T = 1$ sec.

$$\boxed{H(z) = \frac{0.0979}{1 - 0.2155z^{-1} + 0.09792z^{-2}}}$$

IIR Filter design by Impulse Invariant method.

Properties of Impulse Invariant Transformation:-

$$\frac{1}{(s+s_i)^m} = \frac{(-1)^{m-1}}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left(\frac{1}{1-e^{-sT} z^{-1}} \right) ; s \to s_i \quad \to (1)$$

$$\frac{s+a}{(s+a)^2+b^2} = \frac{1-e^{-aT} (\cos bT) z^{-1}}{1-2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}} \quad \to (2)$$

$$\frac{b}{(s+a)^2+b^2} = \frac{e^{-aT} (\sin bT) z^{-1}}{1-2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}} \quad \to (3)$$

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1. Convert the analog filter into a digital filter whose system function is $H(s) = \dfrac{s+0.2}{(s+0.2)^2+9}$.

Use the Impulse Invariant Technique.

Assume $T = 1$ sec.

Solution:-

The system function of the analog filter is of the standard form $\Rightarrow H(s) = \dfrac{s+a}{(s+a)^2+b^2}$.

where $a=0.2$ & $b=3$.

$$\therefore H(z) = \frac{1-e^{-0.2T}\cdot(\cos 3T)\,z^{-1}}{1-2e^{-0.2T}(\cos 3T)\,z^{-1}+e^{-0.4T}z^{-2}}.$$

Assume $T = 1$ sec

$$H(z) = \frac{1-(0.8187)(-0.99)\,z^{-1}}{1-2(0.8187)(-0.99)\,z^{-1}+0.6703\,z^{-2}}.$$

$$\boxed{\therefore H(z) = \frac{1+0.8105\,z^{-1}}{1+1.6210\,z^{-1}+0.6703\,z^{-2}}.}$$

2: For the analog transfer function, (b)

$$H(S) = \frac{1}{(S+1)(S+2)}$$, determine $H(z)$ using Impulse Invariant technique, Assume $T = 1$ sec.

Solution:-

Using Partial fractions, $H(s) = \frac{1}{(S+1)(S+2)} \Rightarrow \frac{A}{(S+1)} + \frac{B}{(S+2)}$

$$\Rightarrow \frac{A}{(S+1)} + \frac{B}{(S+2)} = \frac{A(S+2)+B(S+1)}{(S+1)(S+2)}.$$

$$\Rightarrow A(S+2)+B(S+1) = 1.$$

If $S=-2 \Rightarrow \underline{\underline{B=-1}}$ & If $S=-1 \Rightarrow \underline{\underline{A=1}}$

$$\therefore H(S) = \frac{1}{S+1} - \frac{1}{S+2}.$$

$$\Rightarrow H(z) = \frac{1}{1-e^{-T}z^{-1}} - \frac{1}{1-e^{-2T}z^{-1}}.$$

$$H(z) = \frac{z^{-1}\left[e^{-T} - e^{-2T}\right]}{1-\left(e^{-T}+e^{-2T}\right)z^{-1} + e^{-3T}z^{-2}}$$

$\underline{\underline{T=1\,Sec}}$

$$\boxed{\therefore H(z) = \frac{0.2326z^{-1}}{1-0.5032z^{-1}+0.0498z^{-2}}}$$

IIR Filter by Bilinear Transformation

1. Apply Bilinear Transformation to

$$H(s) = \frac{2}{(s+1)(s+3)} \; ; \text{ with } T = 0.1 \text{ sec}.$$

Solution:-

$$s \rightarrow \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{2}{T}\left(\frac{z-1}{z+1}\right).$$

$$H(z) = H(s)\Big|_{s \rightarrow \frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{2}{\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right)+1\right]\left[\frac{2}{T}\frac{z-1}{z+1}+3\right]}.$$

$$H(z) = \frac{2(z+1)^2}{(21z-19)(23z-17)}.$$

Simplying further,

$$\boxed{H(z) = \frac{0.0041\left(1+z^{-1}\right)^2}{1-1.644z^{-1}+0.668z^{-2}}}$$

2. Convert the analog filter with system function

$$H(S) = \frac{S+0.1}{(S+0.1)^2 + 9}$$ into a digital IIR

filter using bilinear transformation. The digital filter should have a resonant frequency of

$\omega_r = \frac{\pi}{4}$.

Solution :-

$$\Omega_c = \frac{2}{T} \tan\left(\frac{\omega_r}{2}\right). \Rightarrow T = \frac{2}{3} \tan\left(\frac{\pi}{8}\right)$$

$$\boxed{T = 0.276\,sec}$$

Using Bilinear Transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}} \Rightarrow \frac{\frac{2}{T}\frac{(z-1)}{(z+1)} + 0.1}{\left(\frac{2}{T}\frac{z-1}{z+1} + 0.1\right)^2 + 9}$$

$$= \frac{(2/T)(z-1)(z+1) + 0.1(z+1)^2}{\left[2/T\,(z-1) + 0.1(z+1)\right]^2 + 9(z+1)^2}$$

Substituting $T = 0.276$ sec.

$$\boxed{H(z) = \frac{1 + 0.027z^{-1} - 0.973z^{-2}}{8.572 - 11.84z^{-1} + 8.177z^{-2}}}$$

3. A digital filter with a 3dB bandwidth of 0.25π is to be designed from the analog filter whose system response is,

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$. Use bilinear Transformation & obtain $H(z)$.

<u>Solution :-</u>

w.k.t. $\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right) = \frac{2}{T}\tan(0.125) = \frac{0.828}{T}$.

The system response of the digital filter is,

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{(z-1)}{(z+1)}} \Rightarrow \frac{\Omega_c}{\frac{2}{T}\frac{(z-1)}{(z+1)} + \Omega_c}$$

$$H(z) = \frac{0.828/T}{\frac{2}{T}\frac{z-1}{z+1} + \frac{0.828}{T}} \Rightarrow \frac{0.828(z+1)}{2(z-1) + 0.828(z+1)}.$$

Simplifying further,

$$H(z) = \frac{1 + z^{-1}}{3.414 - 1.414z^{-1}}$$