B.Tech. DEGREE EXAMINATION, NOVEMBER 2019

First to Eighth Semester

15MA101 - CALCULUS AND SOLID GEOMETRY

(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:

- (i) Part A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
- (ii) Part B and Part C should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

$PART - A (20 \times 1 = 20 Marks)$

Answer ALL Questions

1. A homogeneous polynomial of second degree in any number of variables is

(A) Canonical form

(B) Quadratic form

(C) Orthogonal form

(D) Diagonal form

2. Find the Eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

(A) 1, 2, 3

(B) 1, 4, 9

(C) 2, 4, 6

(D) 0, 1, 2

3. Find the sum and product of the Eigen values of $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(A) 5, 4

(B) 4, 5

(C) 4, 4

(D) 5, 5

4. Using Cayley Hamilton, find A^{-1} if $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

 $\begin{array}{ccc}
\text{(A)} & \frac{1}{5} \begin{pmatrix} -3 & 2\\ 4 & -1 \end{pmatrix}$

(B) $\frac{1}{5}\begin{pmatrix} -3 & -2 \\ 4 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} -3 & 2 \\ 4 & -1 \end{pmatrix}$

5. If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x} =$

(A) 2y+3x

(B) 3y

(C) 2x+2y

(D) 2x

- 6. If f(x,y) is an implicit function, then $\frac{dy}{dx}$
 - (A) $-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$

(B) $(\partial f / \partial x)$

- 7. If $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, then $f_x(1,1) = \frac{1}{x}$
 - (A) $\pi/4$

(B) 1/2

(C) -1/2

- (D) 0
- 8. If $rt s^2 < 0$ at (a,b) then the point is
 - (A) Maximum point

(B) Minimum point

(C) Saddle point

- (D) Stationary point
- 9. The particular integral of $(D^2 + 6D + 9)y = 3e^{4x}$
 - (A) $\frac{3}{49}e^{4x}$

(C) $(C_1 + C_2 x)e^{-3x}$

- (D) $(C_1 + C_2 x)e^{-3x} + \frac{3}{49}e^{4x}$
- 10. The particular integral of $(D^2 + 2D + 1)y = 7$ is
 - (A) 7

(B) -7

(C) 7/2

(D) 7/3

- 11. The value of $\frac{e^{ax}}{D-a}$
 - (A) xe^{ax}

(B) e^{ax}

(C) x^2e^{ax}

- (D) $\frac{x^2}{2}e^{ax}$
- 12. If $y_1 = \cos x$, $y_2 = \sin x$, then the value of $y_1y'_2 y_2y'_1$ is
 - (A) -1

(B) 0

(C) 1

- (D) 1/2
- 13. The curvature of the straight line is
 - (A) 1

(B) 2

(C) -1

- (D) 0
- of that curve 14. Evolute of a curve is the envelope of
 - (A) Tangent

(B) Normal

(C) Parallel

(D) Locus

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- 15. What is the curvature of a circle of radius 3?
 - (A) 3

(B) -3

(C) 1/3

- (D) -1/3
- 16. The radius of curvature of $y = e^x$ at x = 0 is
 - (A) $2\sqrt{2}$

(B) 2

(C) $\sqrt{2}$

- (D) $\frac{\sqrt{2}}{\sqrt{2}}$
- 17. The number of great circles of any sphere is
 - (A) 0

(B) 1

(C) 2

- (D) Many
- 18. Section of a sphere by a plane is a
 - (A) Parabola

(B) Ellipse

(C) Circle

- (D) Straight line
- 19. The radius of a great circle of a sphere is
 - (A) Greater than the radius of the sphere
- (B) Not equal to the radius of the sphere
- (C) Less than the radius of the sphere
- (D) Equal to the radius of the sphere
- 20. The equation of the right circular cylinder whose axis is the z-axis and radius a is
 - (A) $x^2 + v^2 = a^2$

(B) $x^2 - y^2 = a^2$

(C) $x^2 + z^2 = a^2$

(D) $v^2 + z^2 = a^2$

PART – B ($5 \times 4 = 20$ Marks) Answer ANY FIVE Questions

- 21. Find the Eigen values of $A = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$.
- 22. Find the constants a and b such that the matrix $A = \begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as its Eigen values.
- 23. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\sin y)^x$.
- 24. If $x = r\cos\theta$, $y = r\sin\theta$, z = z, evaluate $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$.
- 25. Solve $(D^2 7D + 12)y = 0$.
- 26. Find the radius of curvature of the curve $r = a(1 + \cos \theta)$ at $\theta = \frac{\pi}{2}$.
- 27. Find the equation of the sphere whose centre is at (-6,1,3) and radius 4.

$PART - C (5 \times 12 = 60 Marks)$

28.a. Reduce the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ to a diagonal form using orthogonal transformation.

- b. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ and hence find A^{-1} .
- 29,a. Expand $e^x \cos y$ in powers of x and y upto terms of third degree.

(OR)

- b. Identify the saddle point and extreme points of $f(x,y) = x^3 + y^3 - 12x - 3y + 20$ Also find minimum and maximum values.
- 30.a. Solve $(D^2 + 6D + 8)y = e^{-2x} + \cos^2 x$.

(OR)

- b. Solve $(D^2 + D + 1)y = x^2 e^{-x}$.
- 31.a. Find the radius of curvature of $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = 1$.

(OR)

- b. Find the equation of the circle of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.
- 32.a. Find the equation of the sphere through the four points (0,0,0), (0,1,-1), (-1,2,0) and (1,2,3)

- b. Find the equation of the sphere through the circle
 - $x^2 + y^2 + z^2 2x + 4y + z 2 = 0, x + 3y z = 4$ and through the point (1,2,-1).