Unit – I: Matrices

PART A

MULTIPLE CHOICE QUESTIONS

1. The matrix of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ is

$(a)\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ $(b)\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ $(c)\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ $(d)\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$

2. The number of positive terms in the canonical form is called

   (a) Signature (b) Index (c) Quadratic form (d) Positive definite

3. A homogeneous polynomial of second degree in any number of variables is

   (a) Canonical form (b) Quadratic form (c) Orthogonal (d) Diagonal form

4. Find the eigen values of A^2 if $A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

   (a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5

5. Find the sum and product of the eigen values of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

   (a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1

6. The eigen values of an orthogonal matrix have the absolute value _____

   (a) 0 (b) 1 (c) 2 (d) ± 1

7. All the eigen values of a symmetric matrix with real elements are

   (a) Distinct (b) Real (c) Equal (d) Conjugate complex numbers

8. Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$

   (a) Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite

9. Write the Q.F. defined by the matrix $A=\begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$

(a) $6x_1^2+2x_2^2+x_3^2+2x_1x_2-14x_1x_3$ (b) $6x_1^2+x_2^2+6x_3^2+x_1x_2-7x_1x_3$
(c) $6x_1^2+2x_2^2+x_3^2+2x_1x_2+14x_1x_3$ (d) $6x_1^2+x_2^2+6x_3^2+x_1x_2-14x_1x_3$

10. Find the eigen values of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

(a) 1, 3 (b) 3, 1 (c) 2, 1 (d) 1, 2

11. Find the eigen values of A^{10} if $A=\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

(a) $1,3^{10}$ (b) 3,1 (c) $3^2,1^{10}$ (d) $0,2$

12. If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, find the value of $|A|$

(a) 0 (b) 1 (c) -1 (d) 2

13. Find the characteristic equation of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

(*a*) $\lambda^3+\lambda^2-18\lambda-40$ (*b*) $\lambda^3-\lambda^2+18\lambda-40$
(*c*) $\lambda^3+\lambda^2+18\lambda+40$ (*d*) $\lambda^3+\lambda^2-18\lambda+40$

14. Find the nature of the quadratic form $x_1^2+2x_2^2+3x_3^2+2x_1x_2+2x_2x_3-2x_3x_1$

(a) Indefinite (b) Positive definite (c) Negative definite (d) Positive semidefinite

15. Find the eigen values of $A=\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

(*a*) $1,3,-4$ (*b*) $1,-3,-4$ (*c*) $1,-3,4$ (*d*) $-1,3,-4$

16. The matrix of the quadratic form x^2+xy is

(*a*) $\begin{pmatrix} 1 & 1/2 \\ 1/2 & 0 \end{pmatrix}$ (*b*) $\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ (*c*) $\begin{pmatrix} 0 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ (*d*) $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

17. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the third eigen value.

(a) 3 (b) b (c) 2 (d) 1

18. Two of the eigen values of 3 x 3 matrix A are 2, 1 and |A| = 12. Find the third eigen value

(a) 6 (b) 3 (c) 2 (d) 1

19. If A is an orthogonal matrix then

(a) |A| = 0 (b) A is singular (c) $A^2 = I$ (d) $A^T = A^{-1}$

20. Two eigen values of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double the third. Find them.

(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3

21. Find the inverse of the eigen values of the matrix if $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

(*a*) $-1, 1/6$ (*b*) $1, 1/6$ (*c*) $1, -1/6$ (*d*) $-1, -1/6$

22. Find rank and index of the QF whose canonical form is $3y_2^2 - 3y_3^2$

(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2

23. Find signature of the QF whose canonical form is $2y_1^2 - y_2^2 - y_3^2$,

(a) 1 (b) -1 (c) 0 (d) 6

24. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are

(a) imaginary (b) non-orthogonal (c) real (d) orthogonal

25. Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(a) $\lambda^2 - 3\lambda - 2 = 0$ (b) $\lambda^2 + 3\lambda + 2 = 0$ (c) $\lambda^2 - 3\lambda + 3 = 0$ (d) $\lambda^2 - 6\lambda + 3 = 0$