

5. Calculate the Convolution of 2412 7

$x(t) = e^{-t} u(t)$ & $h(t) = e^t u(-t)$ using Convolution property. Also verify the property by Convolution Integral.

Solution:-

$$x(t) = e^{-t} u(t) ; \quad x(j\omega) = \frac{1}{1+j\omega}.$$

w.k.t.

$$\text{F.T. } e^{-at} u(t) = \frac{1}{a+j\omega}$$

$$\text{F.T. } e^{at} u(-t) = \frac{1}{a-j\omega}$$

$$h(t) = e^t u(-t) ; \quad \therefore H(j\omega) = \frac{1}{1-j\omega}.$$

w.k.t.

$$x(t) * h(t) \stackrel{\text{is}}{=} \text{IFT of } [x(j\omega) \cdot H(j\omega)]$$

$$= \text{IFT} \left[\left(\frac{1}{1+j\omega} \right) \left(\frac{1}{1-j\omega} \right) \right]$$

$$= \text{IFT} \left[\frac{1}{1+\omega^2} \right]$$

W.K.T.

$$\text{F.T.} \left[e^{-a|t|} \right] \text{ for } a \geq 0 \Rightarrow \frac{2a}{a^2 + \omega^2}$$

$$\therefore \text{F.T.} \left[e^{-|t|} \right] \Rightarrow \frac{2}{1 + \omega^2}$$

Answer

$$\therefore \text{For IFT} \left[\frac{1}{1 + \omega^2} \right] = \frac{1}{2} e^{-|t|}$$

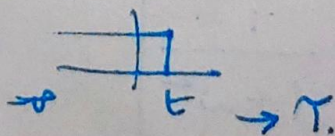
Verification by Convolution Integral

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau) \cdot h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \left[e^{-(t-\tau)} u(t-\tau) \right] \left[e^{\tau} u(-\tau) \right] d\tau$$

$$u(-\tau + t)$$

\Rightarrow



$$\int_{-\infty}^{\infty}$$

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$$= \int_{-\infty}^0 e^{-(t-\tau)} \cdot u(t-\tau) \cdot e^{\tau} d\tau$$

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$$= e^{-t} \int_{-\infty}^t e^{\tau} e^{\tau} d\tau = e^{-t} \int_{-\infty}^t e^{2\tau} d\tau.$$

$$= e^{-t} \left[\frac{1}{2} e^{2\tau} \right]_{-\infty}^t$$

↓

$t \Rightarrow -\infty$
 $\therefore t < 0$

$$= \frac{e^{-t}}{2} \left[e^{2t} - 0 \right] = \frac{e^{-t}}{2} \quad t \Rightarrow -\infty.$$

$\Rightarrow x(t) * h(t) = \frac{1}{2} e^{-|t|}.$

} d\tau