UNIT-3

TWO-DIMENSIONAL RANDOM VARIABLES

Defn:- let S be the sample space. let $X = X(S)$ & $Y = Y(S)$ be two functions each assigning a real numbers to each outcome $s \in S$ then (X, Y) is a two dimensional random variables.

Two types of two Dimensional R.V are

(i) Discrete R.V
ii) Continuous R.V

T.D Discrete Random Variable:-

If the possible values of (X,Y) are finite or countably infinite, then (X,Y) is called a two dimensional discrete random variable

TD continuous Random Variable:-

If (X, Y) can assumes all values in a specified Region R in xy-plane then (X,Y) is called a Two dimensional continuous random variable.

Joint Probability function (or) Joint Probability mass fn:-

If (X,Y) be a TDDRV such that $P(X=x_i, Y=y_j) = P(x_i, y_j) = P_{ij}$ is called the joint probability fn. (or) joint probability mass fn.

If (i) $P_{ij} \geq 0 \quad \forall i \& j$

(ii) $\sum_j \sum_i P_{ij} = 1$

Joint Probability density fn:-

If (X,Y) be a TD continuous Random Variable then $f(x,y)$ is called the joint probability density fn. of (X,Y).

If i) $f(x,y) \geq 0 \quad \forall (x,y) \in R$, R is the region.

ii) $\iint_R f(x,y)\,dx\,dy = 1$. (or) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$ (or) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$

Joint cumulative distribution fn.:-

In discrete case:-

$$F(x,y) = P[X \leq x, Y \leq y] = \sum_{j=1}^{m}\sum_{i=1}^{n} P(x_i, y_j)$$

In continuous case:-

$$F(x,y) = P[X \leq x, Y \leq y] = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\,dx\,dy.$$

Marginal Probability function (or) Marginal distribution:-

(i) In discrete case:-

* $P[X = x_i] = \sum_{j=1}^{m} P(x_i, y_j) = P_{i*} \rightarrow$ marginal probability fn. of X.

* $P[Y = y_j] = \sum_{i=1}^{n} P(x_i, y_j) = P_{*j} \rightarrow$ marginal probability fn. of Y.

(ii) In continuous case:-

* Marginal prob. density fn. of X is

$$f_X(x) = f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

* Marginal prob. density fn. of y is

$$f_Y(y) = f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx.$$

Conditional distribution:-

In discrete case:-

* Conditional probability fn. x given $Y = y_j$ is

$$P[X = x_i / Y = y_j] = \frac{P[X = x_i, Y = y_j]}{P[Y = y_j]}$$

* Conditional probability fn. Y given $X=x_i$ is

$$P[Y=y_j / X=x_i] = \frac{P[X=x_i, Y=y_j]}{P[X=x_i]}$$

In continuous case :-

* The conditional probability fn. of x given y is

$$f(X/Y) = \frac{f(x,y)}{f(y)} \quad ; \quad f(y) \geq 0$$

* The conditional probability fn. of y given x is

$$f(Y/X) = \frac{f(x,y)}{f(x)} \quad ; \quad f(x) > 0$$

Independent Condition :-

In discrete case :-

* $P[X=x_i, Y=y_j] = p(x) \cdot p(y)$ (or) $P_{ij} = P_i * P_j$

In continuous case :-

* $f(x,y) = f(x) \cdot f(y)$

Note :-

1) $P[X=x_i, Y=y_j] = P[X=x_i \cap Y=y_j]$

2) $P[a_1 \leq x \leq b_1 \cap a_2 \leq y \leq b_2] = \int_{a_2}^{b_2} \int_{a_1}^{b_1} f(x,y)\,dx\,dy$

3) $f(x,y)$ or $f_{xy}(x,y)$ are both represents Joint probability fn. .

4) $f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)$.

5) $F(\infty,\infty) = 1$; $F(-\infty, y) = F(x, -\infty) = 0$; $0 \leq F(x,y) \leq 1$.