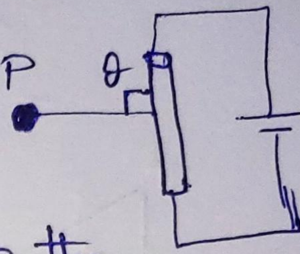
Magnetic Field $\vec{H}$ due to straight Conductor.

Magnetic field at Point, P is directly proportional to the Current, I and the ~~distance~~ length l, of the Conductor, & inversely proportional to the the distance between the Conductor and the magnetic field at Point, P.



$$dB \propto \frac{I\,dl \cdot \sin\theta}{r^2}$$

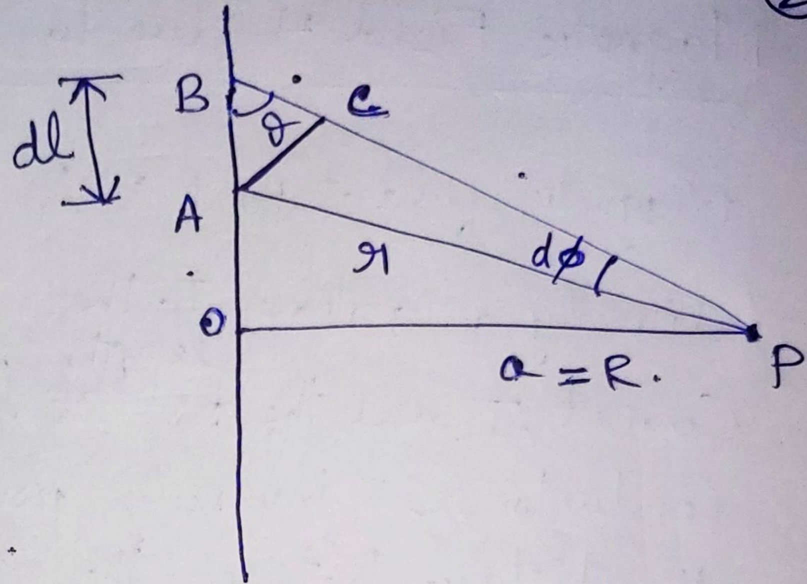
$$\vec{dB} = K \frac{I \vec{dl} \sin\theta}{r^2} \hat{a}_r \cdot \quad \left(\hat{a}_r = \frac{\vec{R}}{|\vec{R}|} \right)$$

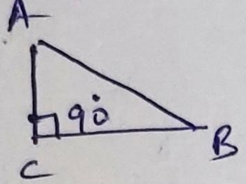
$$K = \frac{\mu_0}{4\pi}$$

$$\vec{B} = \int \vec{dB} = \int K \cdot \frac{I \vec{dl} \sin\theta}{r^2} \hat{a}_r .$$

$$\vec{B} = K \cdot I \int \frac{\vec{dl} \sin\theta}{r^2} \hat{a}_r$$

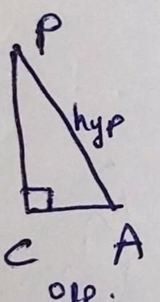
$$\boxed{\vec{dB} = \frac{\mu_0 \cdot I dl \cdot \sin\theta}{4\pi r^2} \hat{a}_r \text{ A/m}}$$





$$\sin\theta = \frac{AC}{AB}.$$

$$\boxed{AC = dl \sin\theta}$$

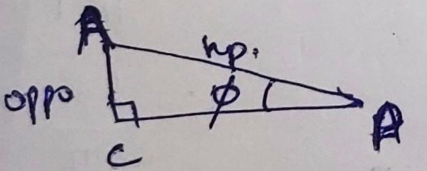


$$\sin\phi = \frac{AC}{AP}.$$

ϕ is very small

$\Rightarrow \sin d\phi = d\phi.$

$$\sin d\phi = \frac{AC}{AP} = \frac{AC}{r}.$$



$$d\phi = \frac{AC}{r}.$$

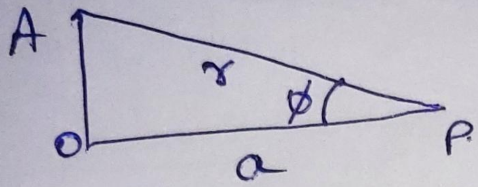
$$\boxed{AC = r\,d\phi}$$

$$\Rightarrow \boxed{r\,d\phi = dl \sin\theta = AC}.$$

w.k.t.

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}\,\hat{a}_r$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \, r \, d\phi}{r^2} \hat{a}_r = \frac{\mu_0}{4\pi} \frac{I}{r} d\phi \, \hat{a}_r.$$



$$\cos\phi = \frac{OP}{AP} = \frac{a}{r} \Rightarrow \boxed{r = \frac{a}{\cos\phi}}$$

$$\Rightarrow \vec{dB} = \frac{\mu_0}{4\pi} \frac{I \, d\phi}{a/\cos\phi} . \hat{a}_r$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I . \cos\phi \, d\phi}{a} \hat{a}_r$$

$$\int \overline{dB} = \frac{\mu_0}{4\pi} \int_{-\phi_1}^{\phi_2} \frac{I \cos\phi \, d\phi}{a} \hat{a}_r .$$

$$\overline{B} = \frac{\mu_0}{4\pi} \frac{I}{a} \hat{a}_r \int_{-\phi_1}^{\phi_2} \cos\phi \, d\phi$$

$$= \frac{\mu_0 I}{4\pi a} \Big[\sin\phi \Big]_{-\phi_1}^{\phi_2} \hat{a}_r = \frac{\mu_0 I}{4\pi a} \Big[\sin\phi_2 + \sin\phi_1 \Big] \hat{a}_r$$

If $\phi_1 = \phi_2 = 90^\circ$.

$$\vec{B} = \frac{\mu_0 I}{2\pi a} . \hat{a}_r .$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R} . \hat{a}_r}$$

Amperes Law :-

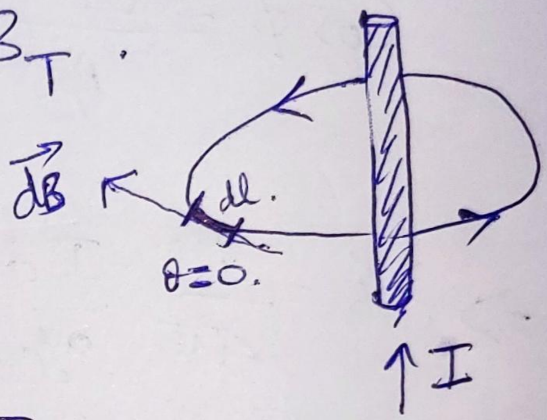
$\mu_0 \rightarrow$ permeability.

$$B_T = \mu_0 I.$$

$$B = \frac{B_T}{\text{Total length}}$$

$$= (B)(\text{Total length}) = B_T.$$

$$\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I.}$$



$$\oint B.dl \cos\theta = \mu_0 I.$$

$$\Rightarrow \oint B.dl = \mu_0 I.$$

$$B \oint dl = \mu_0 I.$$

$$B \oint_0^{2\pi r} dl = \mu_0 I.$$

$$B.2\pi r = \mu_0 I.$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \Rightarrow \boxed{\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{a}_r.}$$

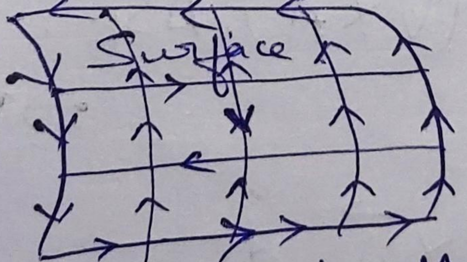
Stoke's Theorem

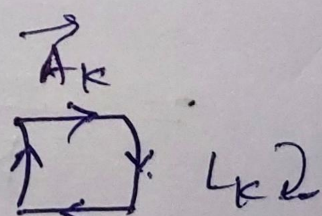
It relates line integral & surface Integral.

Definition:-

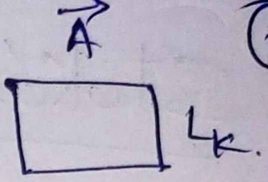
The line integral of a vector around a closed path is equal to the surface Integral of the normal component of its curl over any closed surface.

$$\oint_l H \cdot dl = \iint_S (\nabla \times H)\, ds.$$





The line integral is directly proportional to the curl of the field.



∴ For Total closed Path

$$\oint_L \vec{A}\,\vec{dl} = \sum_K \oint_{L_K} \vec{A}.\vec{dl}$$

$$= \sum_K \frac{\oint_{L_K} \vec{A}.dl}{\Delta S_K} \times \Delta S_K.$$

The Circulation per unit area is the Curl of that particular field.

$$= \sum_K (\nabla \times \vec{A}).\Delta S_K.$$

⇒ For only one cell

$$= \oiint \nabla \times \vec{A}\,.\,ds$$

⇒ For a Complete Surface.

$$\boxed{\int_l H\,dl \Rightarrow \oiint \nabla \times H\;ds.}$$

1. Find the incremental field strength at P_2 due to Current element of $2\pi \hat{a}_z \, \mu$ A/m at P_1. The Co-ordinates of P_1 & P_2 are $(4,0,0)$ & $(0,3,0)$ respectively.

Solution:-

w.k.t. $d\vec{H} = \dfrac{I\,dl}{4\pi r^2} \hat{a}_r$.

$$\hat{a}_r = \frac{\vec{R}}{|\vec{R}|} = \frac{(0-4)\hat{a}_x + (3-0)\hat{a}_y + (0-0)\hat{a}_z}{\sqrt{(4)^2+(3)^2+0}}$$

$$\hat{a}_r = \frac{-4}{5}\hat{a}_x + \frac{3}{5}\hat{a}_y .$$

$$I\,dl \cdot \hat{a}_r \Rightarrow \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & 2\pi \\ -4/5 & 3/5 & 0 \end{bmatrix}$$

$$= \frac{-2\pi}{5}(3\hat{a}_x + 4\hat{a}_y)$$

$$d\vec{H} = \frac{\frac{-2\pi}{5}(3\hat{a}_x + 4\hat{a}_y)}{4\pi(5)^2} = -4\times10^{-3}(3\hat{a}_x + 4\hat{a}_y)\,\mu A/m$$

$$\boxed{d\vec{H} = [-12\hat{a}_x - 16\hat{a}_y]\ \mu.A/m}$$

(2). Find the magnetic field Intensity at the origin due to a Current element $I\vec{dL} = 3\pi(\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z)\ \mu A/m$, at the point, $P(3,4,5)$ in free space. (8).

Solution :-

w.k.t $\vec{dH} = \frac{I\,dl}{4\pi r^2}\hat{a}_r$.

$$\hat{a}_r = \frac{\vec{R}}{|\vec{R}|} = \frac{-3\hat{a}_x - 4\hat{a}_y - 5\hat{a}_z}{\sqrt{3^2+4^2+5^2}}$$

$$= -0.42\hat{a}_x - 0.56\hat{a}_y - 0.707\hat{a}_z .$$

$$I.\vec{dL} \times \hat{a}_R = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 3\pi & 6\pi & 9\pi \\ -0.42 & -0.56 & -0.707 \end{vmatrix}$$

$$= 2.672\hat{a}_x - 5.336\hat{a}_y + 2.67\hat{a}_z$$

$$\vec{dH} = \frac{I\vec{dL} \times \vec{a}_R}{4\pi(\sqrt{50})^2} \Rightarrow \boxed{4.2\hat{a}_x - 8.49\hat{a}_y + 4.25\hat{a}_z \ \ nA/m}$$