Formation of PDE by eliminating arbitrary constants

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Formation of PDE by eliminating arbitrary constants

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2 + 1$.

Solution:

$$p = 2(x - a)$$
 \Rightarrow $x - a = \frac{p}{2}$ and

$$q = 2(y - b)$$
 \Rightarrow $y - b = \frac{q}{2}$

substituting in(1), we get
$$z=rac{p^2}{4}+rac{q^2}{4}+1$$

$$4z = p^2 + q^2 + 4$$
 which is the required pde.



Formation of PDE by eliminating arbitrary constants

2. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a^2)(y^2 + b^2)$.

Solution:

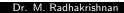
Differentiate
$$(1)$$
 partially w.r.to x and y , we get

$$p = 2x (y^2 + b^2)$$
 \Rightarrow $(y^2 + b^2) = \frac{p}{2x}$ and $q = 2y (x^2 + a^2)$ \Rightarrow $(x^2 + a^2) = \frac{q}{2y}$

$$q = 2y(x^2 + a^2)$$
 \Rightarrow $(x^2 + a^2) = \frac{q}{2y}$

substituting in(1), we get
$$z = \left(\frac{p}{2x}\right) \left(\frac{q}{2y}\right)$$

4xyz = pq which is the required pde.



Formation of PDE by eleminating arbitrary constants

3. Form the partial differential equation of all spheres whose centres lie on the 7 -axis.

Solution:

Given that the centres of the spheres lie on the Z - axis.

- \therefore Centre is (0,0,c). Let r be the radius.
- : Equation of the family of spheres is

$$x^2 + y^2 (z - c)^2 = r^2 \dots (1)$$

Differentiate (1) partially w.r.to x and y, we get

$$2x + 2(z - c)p = 0$$
 \Rightarrow $(z - c)p = -x...(2)$ and

$$2y + 2(z - c)q = 0 \Rightarrow (z - c)q = -y...(3)$$

(2) divide by (3), we get
$$\frac{(z-c)p}{(z-c)q} = \frac{-x}{-y}$$

py = qx which is the required pde.

