Unit 3

Session 19-21

Pulse modulation systems: Overview of PAM,PWM,PPM,
Sampling and quantization
PCM systems, Bandwidth of PCM, PCM TDM signal multiplexing, Limitations of PCM system

Pulse modulation.

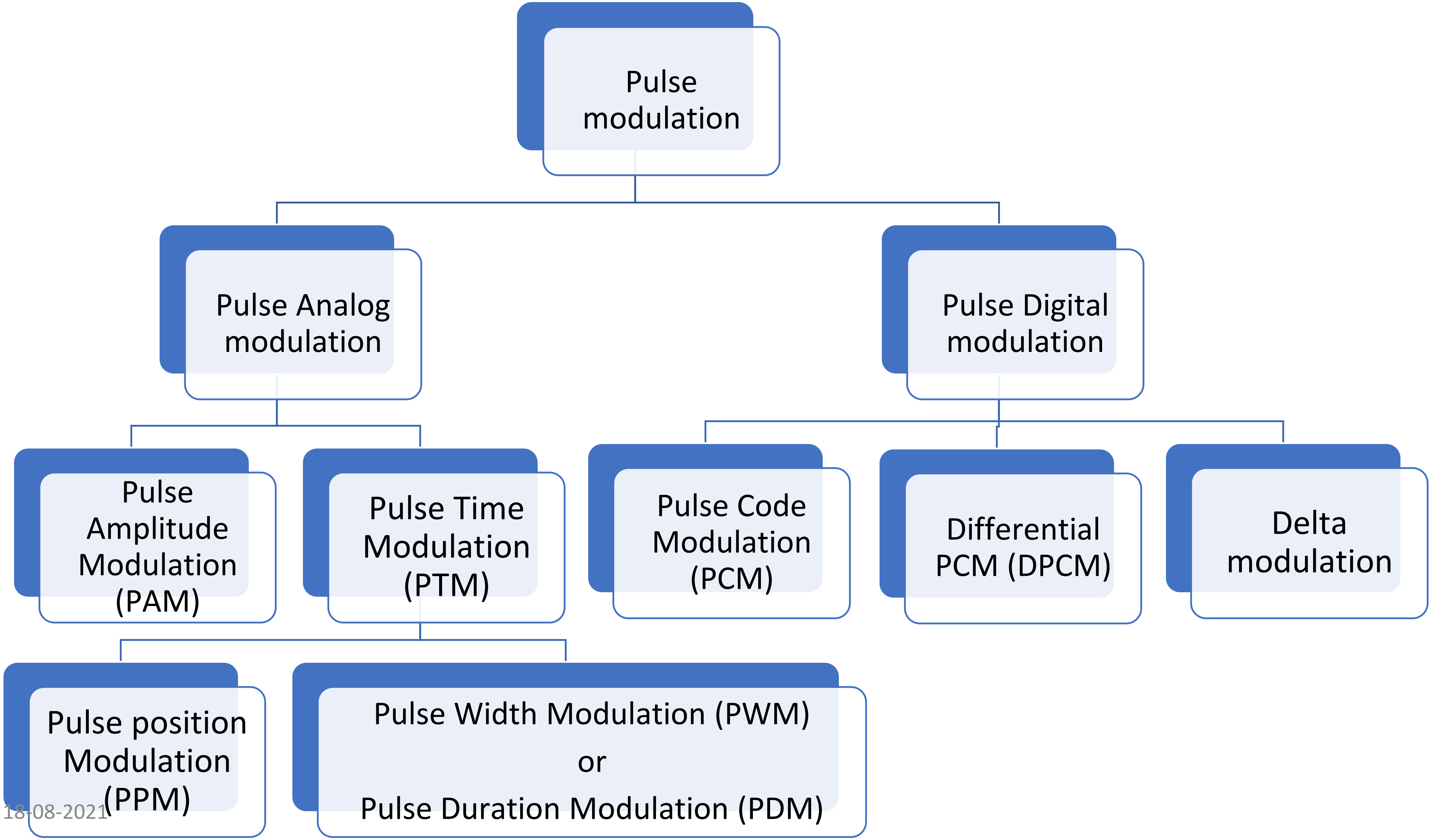
It is defined as a modulation technique in which the characteristics (amplitude / time/ position) of **pulses** (carrier signal) are varied according to instantaneous value of the modulating signal.

Advantage of Pulse Modulation

(i) Transmitted power is no longer continuous as in CW Modulation, but pulsed in nature.

(ii) The time interval between pulse can be filled with sample values from other messages, this permits the transmission of number of messages using a single channel.

Types of Pulse modulation



Pulse Amplitude Modulation

- Pulse Amplitude Modulation (PAM) is an analog modulating scheme in which the amplitude of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

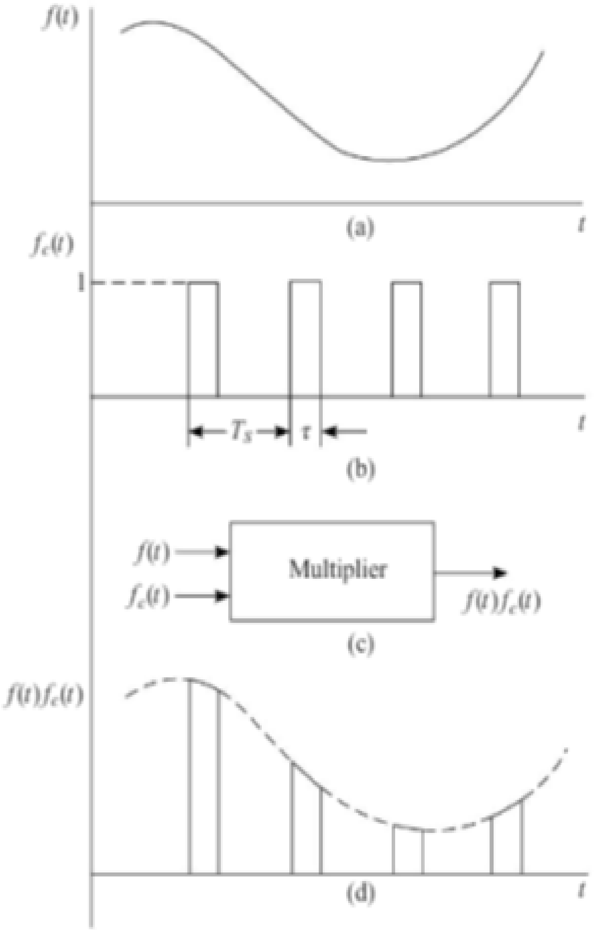
- Methods of PAM generation

➢ Naturally sampled

➢ Flat top sampled

Natural Sampling

- For a **PAM signal** produced with **natural sampling**, the **sampled signal follows the waveform of the input signal** during the time that each sample is taken.
- Naturally sampled PAM signal can be generated by multiplying Carrier pulses with message signal
- In natural PAM, a signal sampled at the Nyquist rate is reconstructed, by passing it through an efficient Low Pass Frequency (LPF) with exact cutoff frequency



(a) Natural Sampling-Baseband Signal $f(t)$, (b) Carrier Pulse Train $f_c(t)$, (c) Multiplier, (d) PAM Signal at Output of Multiplier

- Baseband signal $f(t)$
- Carrier signal $f_c(t)$

Flat Top Sampling

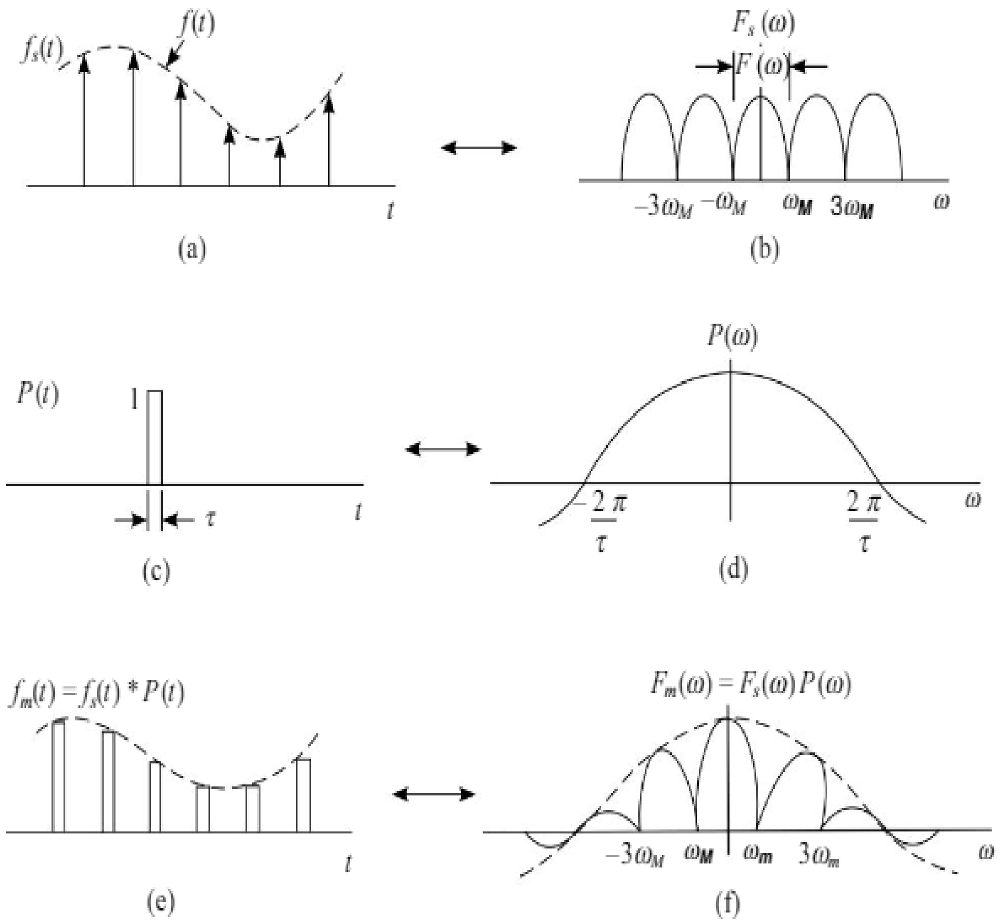
- **Flat-top sampling,** produces **pulses whose amplitude remains fixed** during the sampling time. The **amplitude value** of the pulse depends on **the amplitude of the input signal at the time of sampling.**

The flat-top sampled signal $f_m(t)$ may be considered as a convolution of the impulse sampled signal $f_s(t)$ and non-periodic pulse $p(t)$ of width τ and height 1

The spectrums of $f_s(t)$ and $p(t)$ are shown in Fig.

The spectrum of $f_m(t)$ is obtained by multiplying $F_{s(\omega)}$ with $P_{(\omega)}$.

As the $P_{(\omega)}$ value is different at different frequencies, the shape of $F_M(\omega)$ is not similar to $F_{s(\omega)}$ which shows that a distortion will be introduced if the signal is recovered by an ideal low pass filter of a cut-off frequency ω_M.



Flat Top Sampling: (a) Impulse Sampled Signal $f_s(t)$, (b) Spectrum of $f_s(t)$, (c) Non-periodic Pulse $p_c t$ of Width τ and Height 1, (d) Spectrum of $p(t)$, (e) Flat-top Sampled PAM Signal $f_m(t)$, (f) Spectrum of $f_m(t)$

Advantages and Disadvantages of Pulse Amplitude Modulation

- Advantages of Pulse Amplitude Modulation
- PAM is the simplest form of pulse modulation.
- Its implementation is quite easy.
- Disadvantages of Pulse Amplitude Modulation
- The transmission bandwidth required is very large.
- Due to the variation in amplitude, the power required by the generating unit also varies.
- Less immune to noise due to amplitude variation.
- Applications of Pulse Amplitude Modulation
- It is used in LED lighting, in microcontrollers in order to produce control signals and in the Ethernet communication system.

Bandwidth of PAM Signals

Let us assume that we have to transmit n signals, each band limited to f_M Hz. Then, for each signal, we have to take $2f_M$ samples per second. Thus, in all, we have to transmit $2nf_M$ samples per second.

Now, according to the sampling theorem, a continuous signal band limited to B Hz can be transmitted by $2B$ samples per second. Conversely, it can be stated that $2B$ samples per second define a continuous signal band limited to B Hz. Therefore, the bandwidth of the PAM system of n signals, each band limited to f_M Hz, will be nf_M Hz, because we are transmitting $2nf_M$ samples per second.

S/N Ratio of PAM System

The noise performance of PAM is identical to AM-SC signal. The figure of merit v is unity. It has already been shown that the bandwidth of PAM/AM is nf_M, where n is the number of messages multiplexed. Therefore, the bandwidth per message is f_M. Thus, the transmission of sampled signal message reproduces a bandwidth at the receiver that is equivalent to the message $f(t)$. In other words, the transmission of a sampled signal is equivalent to the direct transmission of $f(t)$. The signal and noise power at the transmitter and receiver is expected to be identical.

the power of a sampled signal and the baseband signal $f(t)$ is the same

Hence,

$$S_o = S_i = \overline{f^2(t)}$$

The bandwidth of sampled signal per message is same as that of the baseband signal $f(t)$. Hence, for white noise

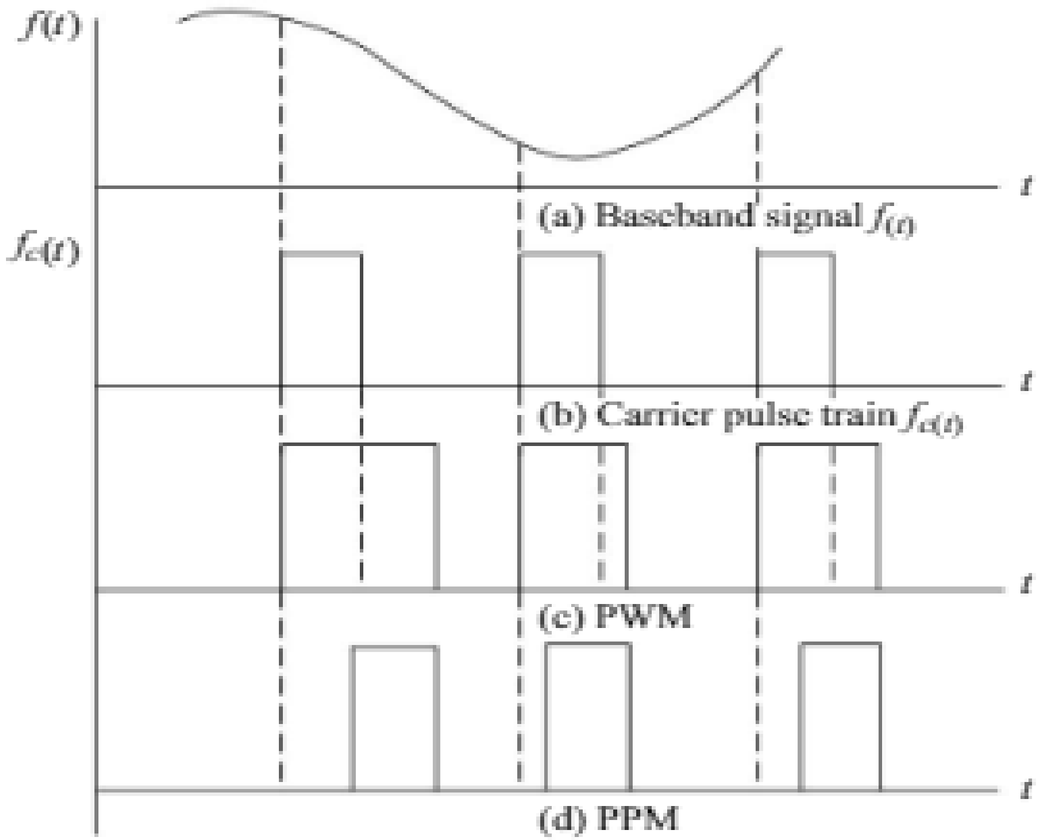
$$N_o = N_i = \eta f_M$$

where f_M is the bandwidth of $f(t)$ and $\eta/2$ is the noise power per unit bandwidth. The figure of merit is given by

$$(v)_{\text{PAM}} = \frac{(S_o/N_o)}{(S_i/N_i)} = 1$$

Pulse Time Modulation

- In PWM signal , the width of each pulse depends on the instantaneous value of the baseband signal at the sampling instant.
- In PWM , the information about the baseband signal lies in the trailing edge of the pulse, whereas in PPM , it lies in both the edges of the pulse.
- In PPM, the position of the pulses (the carrier signal) is varied in proportion to the instantaneous values of the analog signal (the message signal).



Pulse Time Modulation: (a) Baseband Signal $f(t)$, (b) Carrier Pulse Train $f_c(t)$, (c) Pulse Width Modulated Signal, (d) Pulse Position Modulated Signal

Advantages and Disadvantages of Pulse Width Modulation

- Advantages of Pulse Width Modulation

1. It is more immune to channel induced noise than PAM.
2. As noise adds to the amplitude thus the reconstruction of PWM signal from distorted PWM signal is somewhat easy.
3. The transmission and reception do not need to be synchronized.

- Disadvantages of Pulse Width Modulation

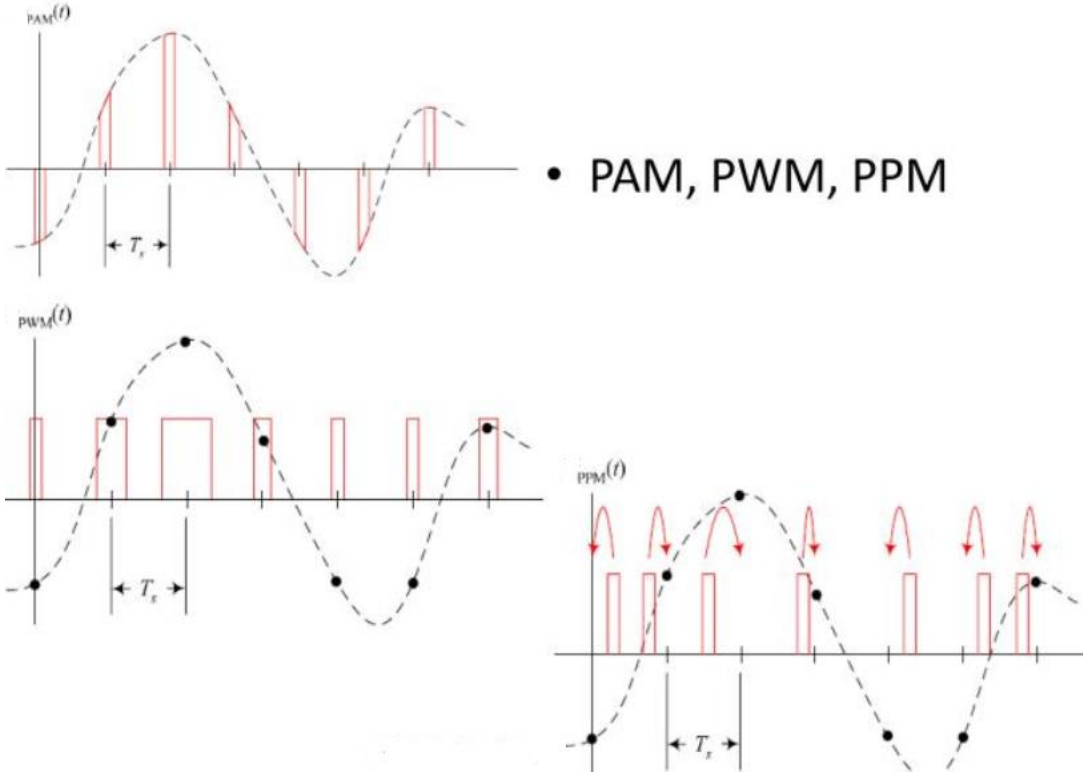
1. Due to changing width of the pulses, variation in transmission power is also noticed.(requires large power transmission compared to PPM)
2. Bandwidth requirement in case of PWM is somewhat larger than PAM.

- Applications of Pulse Width Modulation

- It is used in telecommunications, brightness controlling of light or speed controlling of fans etc.

Advantages and Disadvantages of Pulse Position Modulation

- **Advantages of Pulse Position Modulation**
- Similar to PWM, PPM also shows better noise immunity as compared to PAM. This is so because information content is present in the position of the pulses rather than amplitude.
- As the amplitude and width of the pulses remain constant. Thus the transmission power also remains constant and does not show variation.
- Recovering a PPM signal from distorted PPM is quite easy.
- Interference due to noise in more minimal than PAM and PWM.
- Disadvantages of Pulse Position Modulation
- In order to have proper detection of the signal at the receiver, transmitter and receiver must be in synchronization.
- The bandwidth requirement is large.
- Applications of Pulse Position Modulation
- The technique is used in an optical communication system, in radio control and in military applications.



- PAM, PWM, PPM

Comparison between PAM, PWM, and PPM

The comparison between the above modulation processes is presented in a single table.

PAM	PWM	PPM
Amplitude is varied	Width is varied	Position is varied
Bandwidth depends on the width of the pulse	Bandwidth depends on the rise time of the pulse	Bandwidth depends on the rise time of the pulse
Instantaneous transmitter power varies with the amplitude of the pulses	Instantaneous transmitter power varies with the amplitude and width of the pulses	Instantaneous transmitter power remains constant with the width of the pulses
System complexity is high	System complexity is low	System complexity is low
Noise interference is high	Noise interference is low	Noise interference is low
It is similar to amplitude modulation	It is similar to frequency modulation	It is similar to phase modulation

Transmission BW of PAM Signal

- Pulse duration (τ) supposed to be very small compare to the period, Ts between 2 samples
- Lets max frequency of the signal, W

$F_s >= 2W$

$T_s =< 1/2W$

$T \ll T_s =< 1/2W$

$$f\ max = \frac{1}{2\tau}$$

$\tau \ll T_s$

$f_s \geq 2fm$; $Ts \leq \frac{1}{2fm}$

$\tau \ll Ts \leq \frac{1}{2fm}$

If on and off time of PAM pulse is same **then** $f_{max} = \frac{1}{2\tau}$

$BW \geq fmax$; $BW \geq \frac{1}{2\tau}$

$BW \geq \frac{1}{2\tau} \gg fm$

Transmission BW of PDM/PPM Signal

- PPM and PDM need a sharp rise time and fall time for pulses in order to preserve the message information.
- Lets rise time, tr

$t_r \ll T_s$

$$B_T = \frac{1}{2t_r}$$

- From formula above, we know that transmission BW of PPM and PDM is higher than PAM

Example 1

- For PAM transmission of voice signal with W = 3kHz. Calculate B_T if f_s = 8 kHz and τ = 0.1 T_s

- **SOLUTION**

$$T_s = \frac{1}{f_s} = \frac{1}{8kHz} = 1.25x10^{-4}\,s$$

$$\tau = 0.1T_s = 1.25x10^{-5}\,s$$

$$\tau << \frac{1}{2W}$$

$$B_T \geq \frac{1}{2\tau} >> W$$

$$B_T \geq \frac{1}{2\tau} \geq 40kHz$$

Example 2

- For the same information as in example 1, find minimum transmission BW needed for PPM and PDM. Given tr= 1% of the width of the pulse.

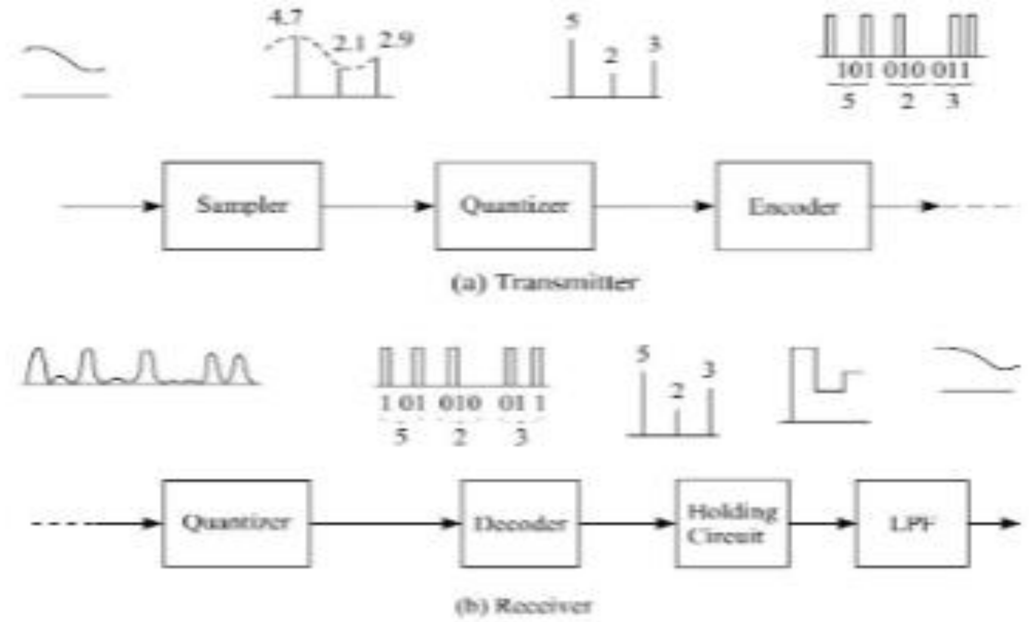
SOLUTION

$$t_r = \frac{1}{100}\tau = 1.25x10^{-7}\,s$$

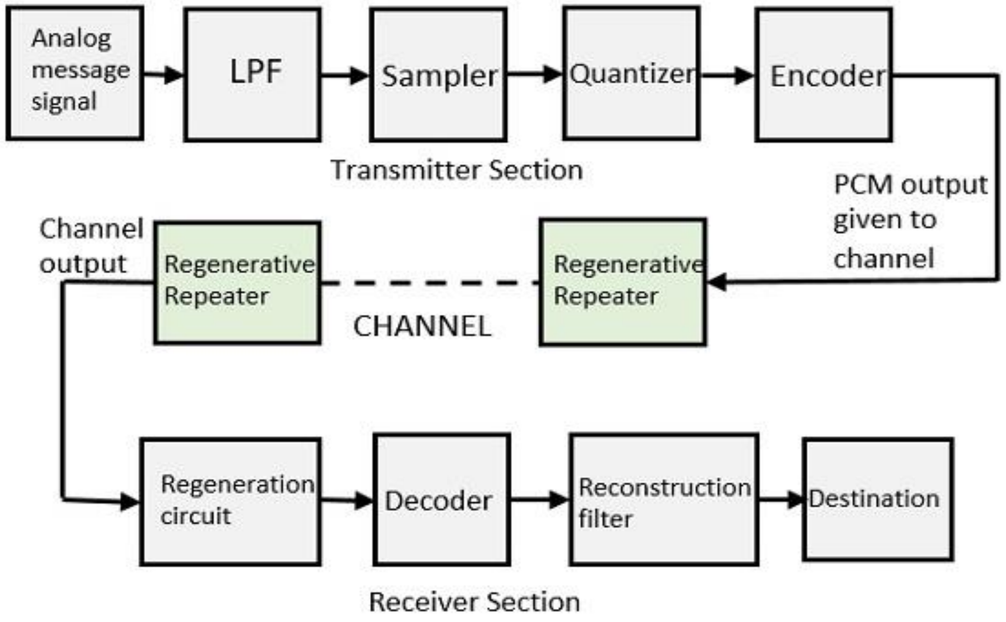
$$B_T \geq \frac{1}{2t_r}$$

$$B_T \geq 4MHz$$

Pulse code Modulation (PCM)



(a) and (b) A PCM System



Low Pass Filter (LPF)

- This filter eliminates the high frequency components present in the input analog signal which is greater than the highest frequency of the message signal, to avoid aliasing of the message signal.

Sampler

- This is the circuit which uses the technique that helps to collect the sample data at instantaneous values of the message signal, so as to reconstruct the original signal. The sampling rate must be greater than twice the highest frequency component W of the message signal, in accordance with the sampling theorem.

Quantizer

- Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer, reduces the redundant bits and compresses the value.
- The digitization of analog signals involves the rounding off of the values which are approximately equal to the analog values. The method of sampling chooses few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called as Quantization.
- The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels. Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.
- As it basically rounds off the value to a certain level this shows some variation by the actual amount. Thus we can say, quantizing a signal introduces some distortion or noise into it. This is known as **quantization error**.
- A quantizer can be of two types, **uniform and non-uniform quantizer**. In uniform quantizer, there exists a uniform spacing in between the level. As against, in non-uniform quantizer, the spacing in between the levels is not uniform. Here, we have employed a uniform quantizer.
- For a **low signal level, the quantization error is high** i.e., bad SNR. But, for a **high signal level, the quantization error is low** providing good SNR.

Error Generated by Quantization (Quantization Noise)

We start with a sampled signal {call it $m(t)$} and now we want to quantize it.

The quantized amplitude is limited to a range, say from $-m_p$ to $+m_p$.
(Note: the range of $m(t)$ may extend beyond ($-m_p$, $+m_p$) in some cases.)

Divide the range ($-m_p$, $+m_p$) into L uniformly spaced intervals. The number intervals is L and the separation between quantized levels is

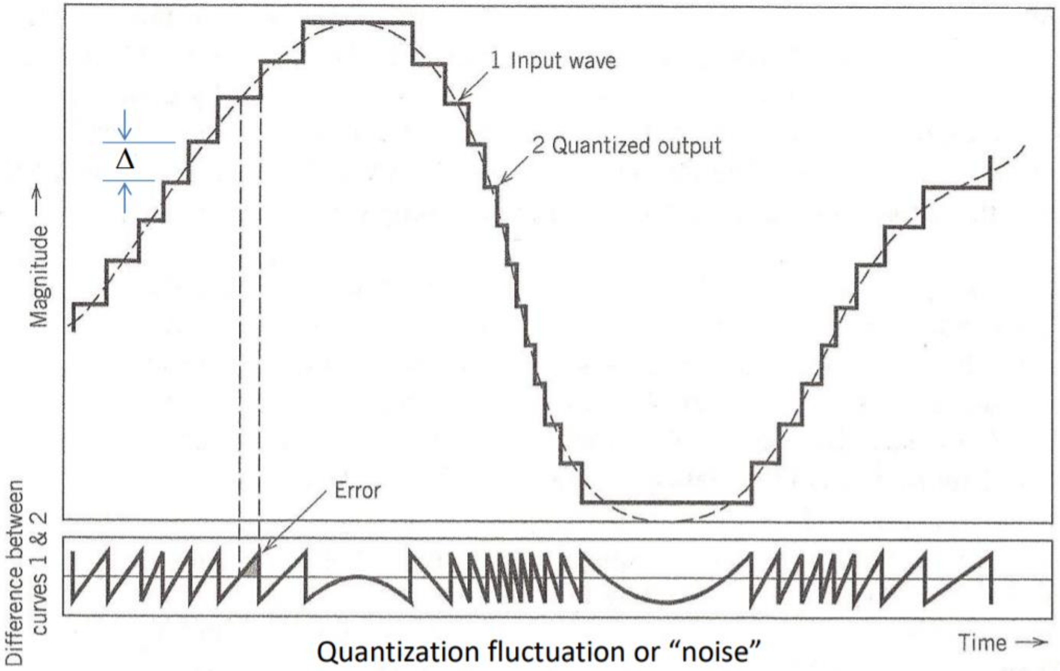
$$\Delta = \frac{2m_p}{L}$$

The k^{th} sample point of $m(t)$ is designated as $m(kT_S)$ and is assigned a value equal to the midpoint between two adjacent levels. Define:

$m(kT_S)$ = k^{th} sample's value, and
$m_q(kT_S)$ = k^{th} quantized sample's value.

Then the quantization error $q(kT_S)$ is equal to $m_q(kT_S)$ - $m(kT_S)$

Quantization noise $q(t) = m(t) - m_q(t)$



Quantization fluctuation or "noise"

Both sampling and quantization results in the loss of information, aliasing and quantization error respectively.
The quality of a Quantizer output depends upon the number of quantization levels used.
The discrete amplitudes of the quantized output are called as representation levels or reconstruction levels.
The spacing between two adjacent representation levels is called a quantum or step-size.

The quantized levels are separated by $\boxed{\Delta = \frac{2m_p}{L}}$

The maximum error for any sample point's quantized value is at most ½Δ.

The "time average" **mean-square quantization error** is

$$\overline{q^2} = \langle q^2 \rangle = \frac{m_p^2}{3L^2} = \frac{(\Delta)^2}{12}.$$

Let $N_q = \overline{q^2}$. Thus, N_q is proportional to the fluctuation of the error signal. This is usually called the **quantization noise power**.

Next we define $m(t) = m_q(t) + q(t)$.

The signal (or message) power S_0 is proportional to the mean square of $m(t)$,

$$S_0 = \sigma^2 = \langle m^2(t) \rangle; \text{ and if } m(t) \text{ is sinusoidal, } S_0 = \frac{m_p^2}{2}$$

Note: $\langle \cdots \rangle$ denotes a time average.

We want a measure of the quality of received signal (that is, the ratio of the strength of the received signal power S_0 relative to the strength of the noise power N_q due to quantization).

This is the **Signal-to-Quantization Noise Ratio** (SQNR) and is given by

$$SQNR = \frac{S_0}{N_q} = \frac{\langle m^2(t) \rangle}{\left(m_p^2 / 3L^2 \right)} = 3L^2 \left(\frac{\langle m^2(t) \rangle}{m_p^2} \right)$$

It is usually expressed in decibels,

Note: $\left(\frac{\langle m^2(t) \rangle}{m_p^2} \right) \cong \frac{1}{2}$

$$SQNR_{dB} = 10 \cdot \log_{10} \left(\frac{S_0}{N_q} \right) \cong 10 \cdot \log_{10} \left(\frac{3L^2}{2} \right)$$

Conclusion:

To reduce the quantization error relative to the message signal level, use smaller quantization steps Δ.

Encoder

- The digitization of analog signal is done by the encoder. It designates each quantized level by a binary code. The sampling done here is the sample-and-hold process. These three sections will act as an analog to the digital converter. Encoding minimizes the bandwidth used.
- To exploit the advantages of sampling and quantizing for the purpose of making the transmitted signal more robust to noise , interference and other channel impairments, we require the use of an encoding process to translate the discrete set of sample values to a more appropriate form of signal.
- Plan for representing each of this discrete set of values as a particular arrangement of discrete event is called a code. One of the discrete event in a code is called code element or symbol.
- A particular arrangement of symbols used in a code to represent a single value of a discrete set is called a code word or character.
- Ina binary code , each symbol may be either of two distinct values , as presence or absence of a pulse(denoted as 1 and 0).
- The maximum advantage over the effects of noise in a transmission medium is obtained by using a binary code , because of binary symbol withstands a relatively high level of noise and is easy to regenerate.

LINE CODES

The digital data (0's and 1's) are transmitted over the line by means of 'Line Codes' (also known as 'Data Transmission Codes' or 'Modulation Codes'). Thus, they give electrical representation of symbols 0 and 1.

Types of Line Codes

1. UNRZ (Unipolar Non-Return to Zero) Code: On off signaling

In this code, a 1 is represented by a positive pulse and a 0 is represented by no pulse. This is also known as 'On-Off Code' (Fig..1).

Fig. .1

2. BNRZ (Bipolar Non-Return to Zero) Code:

In this code, a 1 is represented by a positive pulse and a 0 is represented by a negative pulse (Fig. .2).

Fig. 2

3. URZ (Unipolar Return to Zero) Code:

In this code, a 1 is represented by a positive pulse of half symbol-width and a 0 is represented by no pulse (Fig. .3).

Fig .3

4. BRZ (Bipolar Return to Zero) Code: Alternate mark inversion

In this code, a 1 is represented by a positive pulse of half symbol-width and a 0 is represented by a negative pulse of half symbol width (Fig. 4).

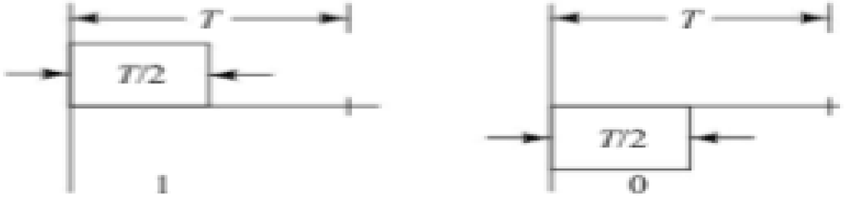


Fig .4

5. Split-Phase Code (Manchester Code):

In this code, a 1 is represented by a positive half-symbol width pulse followed by a negative half-symbol width pulse and a 0 is represented by a negative half-symbol width pulse followed by a positive half-symbol width pulse (Fig. .5). This code has a zero dc component because for both symbols, the dc component is zero. Moreover, the maximum half-width duration (positive as well as negative pulse) in this code is T (corresponding to 01 or 10), and hence, it has relatively insignificant low-frequency components. It may be noted that in other NRZ codes, this duration can be nT, resulting in a significant value of low-frequency component.

Fig 5

6. Differential Code or BRZ-AMI (Binary Return to Zero-Alternate Mark Inversion) Code:

In this code, a 1 is represented alternately by a positive pulse of half-width and a negative pulse of half-width whereas a 0 is represented by no pulse (Fig .6). Thus, the dc component of this code is zero (because of alternate positive and negative pulses).

In Telegraphy, the words 'Mark' and 'Space' are used for symbols 1 and 0 respectively. In the Differential Code, the symbol 1, i.e. 'Mark' is alternately represented by waveforms which are inverse of each other, this code includes the words 'Alternate Mark Inversion' (AMI) in its another name. Moreover, since Differential code is basically a BRZ code, another name for this code is 'BRZ-AMI Code'.

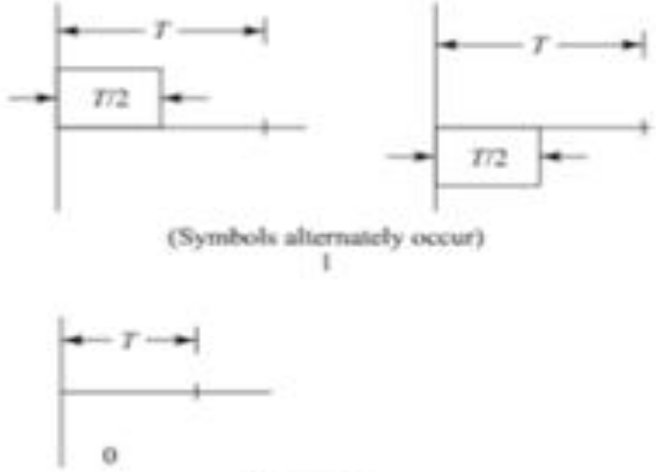


Fig. 6

Line coding

Properties of a Line Code

1. **Transmission Bandwidth:** The minimum bandwidth required depends on the highest fundamental frequency of the waveform. This should be as small as possible.
2. **Favourable Power Spectral Density:** The signal spectrum should be matched to the channel frequency response. Zero dc component is preferable.
3. **Timing (clock) Recovery:** It should be possible to extract timing or clock information from the signal.
4. **Error Detection and Correction Capability.** '0's
5. **Ease of Detection and Decoding.**
6. **Transparency:** It should be possible to correctly transmit a digital signal regardless of 0
and 1's.
7. **Power Efficiency:** For a given bandwidth and a specified error probability, the transmitt
power for a line code should be as small as possible.

Table *Properties of Line Codes*

Line Code	*Minimum Bandwidth*	*Average DC*	*Clock Recovery*	*Error Detection*
UNRZ	1/2T	V/2	Poor	No
BNRZ	1/2T	0	Poor	No
URZ	1/T	V/2	Good	No
BRZ	1/T	0	Very Good	No
Manchester	1/T	0	Best	No
BRZ-AMI	1/2T	0	Good	Yes

Reconstruction Filter

- After the digital-to-analog conversion is done by the regenerative circuit and the decoder, a low pass filter is employed, called as the reconstruction filter to get back the original signal.

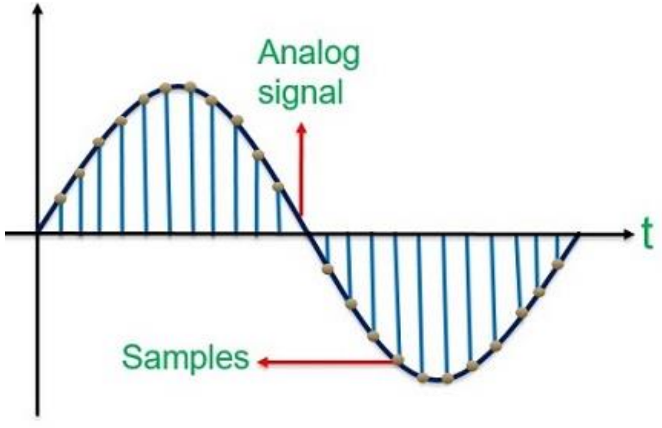
- Hence, the Pulse Code Modulator circuit digitizes the analog signal given, codes it, and samples it. It then transmits in an analog form. This whole process is repeated in a reverse pattern to obtain the original signal.

Regenerative Repeater

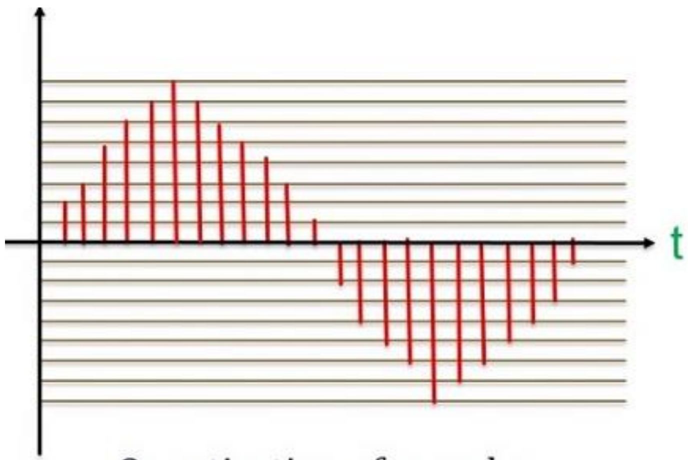
- The output of the channel has one regenerative repeater circuit to compensate the signal loss and reconstruct the signal. It also increases the strength of the signal.

Decoder

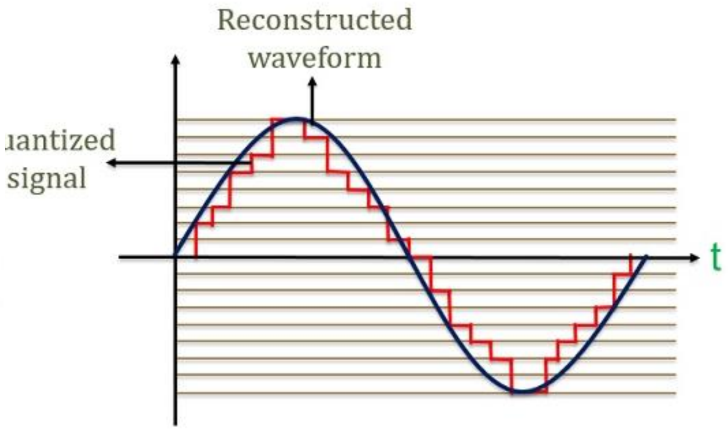
- The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator.



Sampling of analog signal



Quantization of samples



Reconstruction of analog signal at the receiver

Problems Still exist with PCM

1. **Quantization noise:**
   - The difference between the original samples to their quantized values is called Quantization noise. The noise will appear at the reconstruction of analog signal.

2. **Bandwidth:**
   - Each sample is represented by n bits , therefore the required bandwidth is multiplied a factor of at least n

3. **ISI(Inter symbol interference):**
   - Each binary representation of the samples, will be transformed at the end to some shape , usually a pulse, called a symbol. It is very likely that neighboring symbol will interfere each other, thus adding difficulties to the reconstruction of analog signal.

Companding in PCM

- The word Companding is a combination of Compressing and Expanding, which means that it does both.
- This is a non-linear technique used in PCM which compresses the data at the transmitter and expands the same data at the receiver to avoid signal distortion.
- The effects of noise and crosstalk are reduced by using this technique.
- There are two types of Companding techniques
    - A-law Companding Technique,
    - μ-law Companding Technique

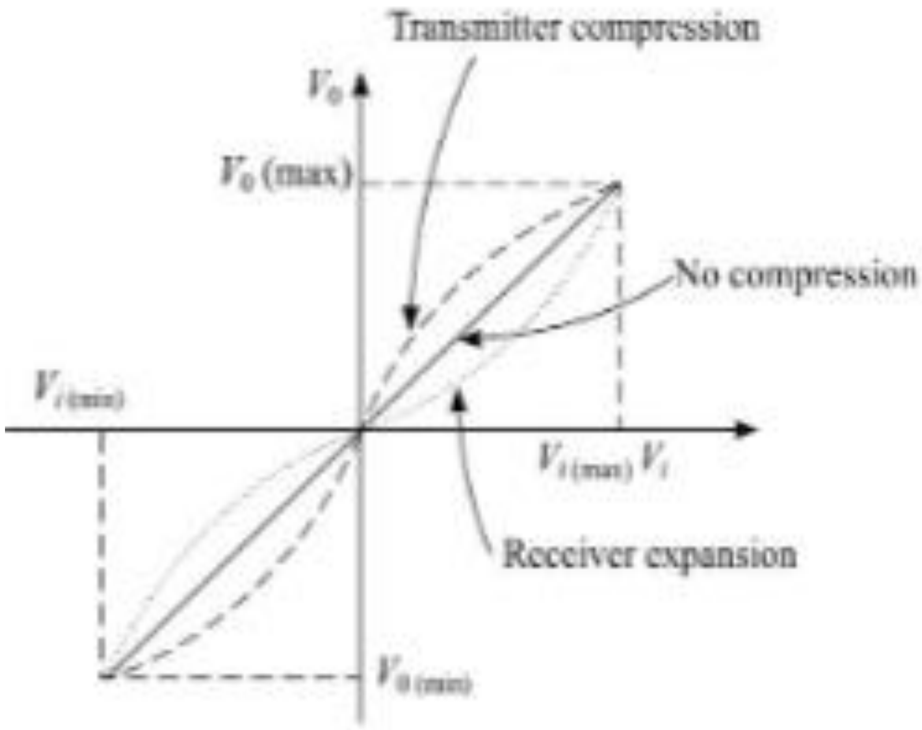


Fig. Companding

A-law Companding Technique

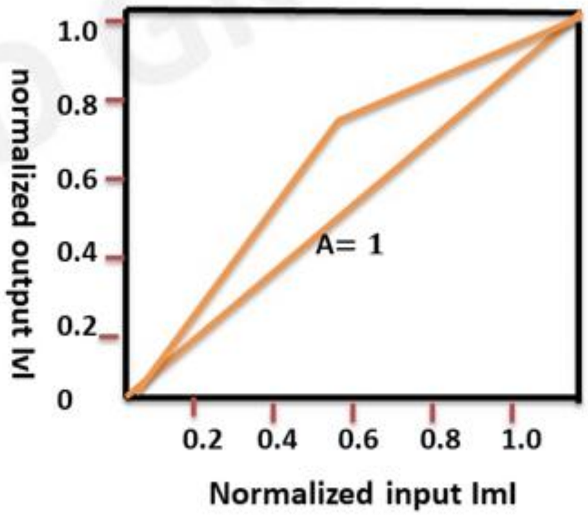
Uniform quantization is achieved at **A = 1**, where the characteristic curve is linear and there is no compression.

A-law has mid-rise at the origin. Hence, it contains a non-zero value.

A-law companding is used for PCM telephone systems.

A-law is used in many parts of the world.

$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A} & for\ 0 \le |m| \le 1/A \\ \dfrac{1+\log(A|m|)}{1+\log A} & for\ \dfrac{1}{A} \le |m| \le 1 \end{cases}$$



µ-law Companding Technique

Uniform quantization is achieved at **µ = 0,** where the characteristic curve is linear and there is no compression.

µ-law has mid-tread at the origin. Hence, it contains a zero value.

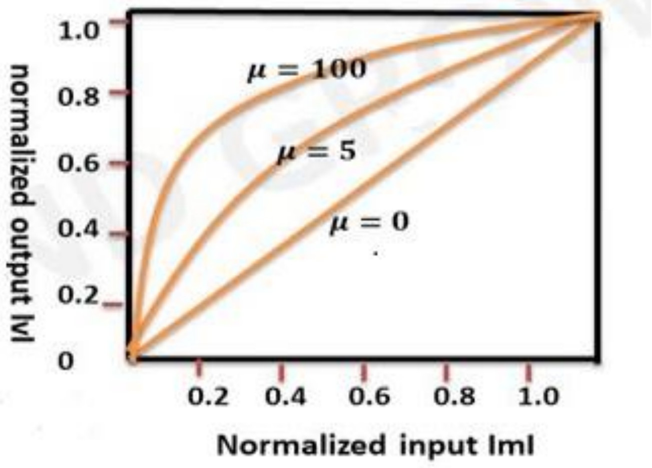
µ-law companding is used for speech and music signals.

µ-law is used in North America and Japan

$$|v| = \frac{\log(1+\mu\,|m|)}{\log(1+\mu)}$$

Drive lml respect to lvl

$$\frac{d|m|}{d|v|} = \frac{\log(1+\mu)}{\mu}(1+\mu|m|)$$



Multiplexing

- **Time-division multiplexing (TDM)** is a method of transmitting and receiving independent signals over a common signal path by means of synchronized switches at each end of the transmission line so that each signal appears on the line only a fraction of time in an alternating pattern.
- It is used when the data rate of the transmission medium exceeds that of signal to be transmitted.
- When a large number of PCM signals are to be transmitted over a common channel, multiplexing of these PCM signals is required. There are two types of Multiplexing i)Synchronous TDM ii)Asynchronous TDM.

Synchronous time division multiplexing

As in the case of PAM , TDM is possible in PCM also, Here each sample is coded into several bits. The multiplexing is possible in two ways.

i)Bits are taken , one by one from each channel sample code. After the first bits from all channel samples are taken , the commutator takes the second bits from all channel samples, and so on. This is bit interleaving.
ii)All code bits of the first channel samples are taken followed by all code bits of the second channel samples and so on. In this method, the desired commutator speed is less than the first method. This is word interleaving.

A synchronizing bit is added at the end of each frame of synchronization between the commutator and decommutator. All bits corresponding to a specific sample code from all channels constitute a frame. Thus if there are n channels and N bits per sample, then the size of a frame is nN bits. The signal that is to be time division multiplexed is bandlimited to the same frequency, resulting in the same sampling frequency for all channels and hence the Synchronous time division multiplexing.

Asynchronous time division multiplexing

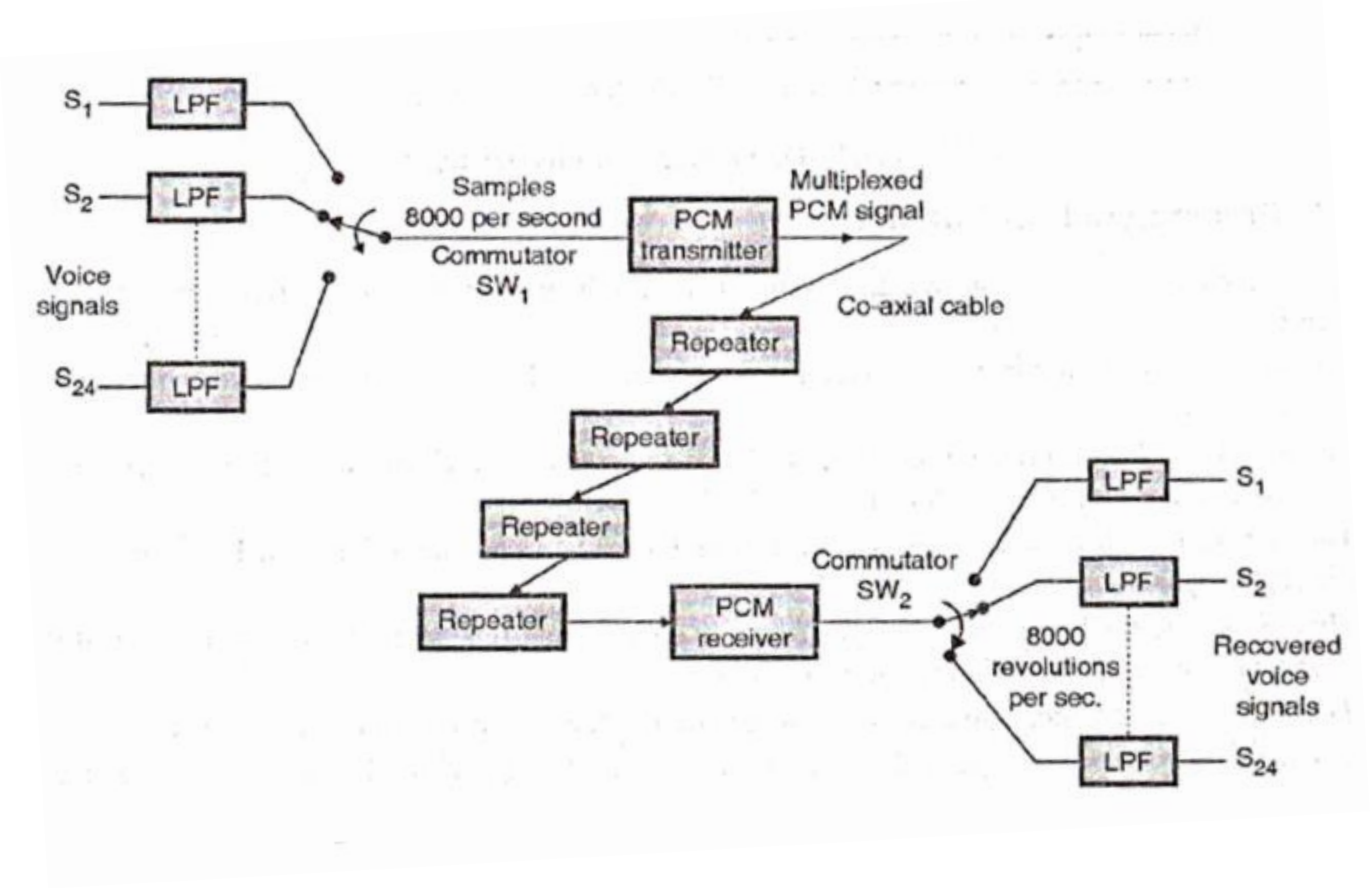
When the signal to be time division multiplexed are bandlimited to different frequency, their sampling frequencies are also different, such asynchronously samples signals are multiplexed by a technique called Pulse stuffing.
Different signals are sampled at different sampling frequencies. The samples are stored on different storage device(elastic store). While transmitting these signals , each storage device is played at different speeds in such away that the output sample rate of each device is same.

Pulse stuffing, often referred to as bit stuffing, is the practice of adding non-data bits to a binary signal before that signal is transmitted over a network. Pulse stuffing is often used as a means of controlling synchronization in systems that require both transmitter and receiver to transmit at the same bit rate.

PCM -TDM

This system is used to convey multiple signals over telephone lines using wideband coaxial cable.

- Operation of the T1 system.
- The operation of the PCM-TDM system is as follows
  - This system has been designed to accommodate 24 voice channels marked S1 to S24 .
  - Each signal is bandlimited to 3.3kHz, and the sampling is done at a standard rate of 8 kHz.
  - This is higher than the Nyquist rate.
  - The sampling is done by the commutator switch SW1 .
  - These voice signals are selected one by one and connected to a PCM transmitter by the commutator switch SW1.
  - These voice signals are selected one by one and connected to a PCM transmitter by the commutator switch SW1.
  - Each sampled signal is then applied to the PCM transmitter which converts it into a digital signal by the process of A to D conversion and companding, as explained earlier.
  - The resulting digital waveform is transmitted over a co-axial cable.
  - Periodically, after every 6000 ft, the PCM-TDM signal is regenerated by amplifiers called "repeaters".
  - They eliminate the distortion introduced by the channel and remove the superimposed noise and regenerate a clean PCM-TDM signal at their output.
  - This ensures that the received signal is free from the distortions and noise.
  - At the destination the signal is companded, decoded and demultiplexed, using a PCM receiver.
  - The PCM receiver output is connected to different low pass filters via commutator switch SW2 .
  - Synchronization between the transmitter and receiver commutators SW1 and SW2 is essential in order to ensure proper communication.



Bandwidth of a PCM

- Bandwidth of PCM signal depends on the bit rate and the pulse shape.
- Let the bit rate be R (of the PCM signal generated), then
- **R = n*fs**
- n = number of bits on the PCM word (M= 2^n …. M is no. of levels of quantization)
- fs = sampling rate to which analog signal is sampled
- For no aliasing i.e. the Nyquist rate : **fs= 2fm**,
- where **fm** is the bandwidth of the analog signal that is to be converted.
- Bandwidth of the PCM (BPCM) waveform is bounded by
- **BPCM>= R/2**
- **BPCM>= n* fm**
- (so R/2 is the minimum bandwidth of the PCM signal)
- For one using a rectangular pulse with polar NRZ (Non return to zero) line code:
- **BPCM = R = n*fs = n*2*B**
- This means the bandwidth of PCM is greater than the bandwidth of the analog signal.

Assume that there are n channels, each bandlimited to f_m, to be time division multiplexed. Let N be the length of the PCM code so that there are $2^N = M$ quantization levels. The bandwidth of the PCM system depends on the bit duration (bit time slot), which may be calculated as follows:

$$\text{sampling frequency} = 2f_m$$

and

$$\text{sampling period} = \frac{1}{2f_m}$$

As there are n channels and N bits per sample and one synchronizing bit, the total number of bits/sampling period (or frame)

$$= nN + 1$$

$$\text{Therefore, the bit duration} = \frac{\text{sampling period}}{\text{total number of bits}}$$

Hence,

$$T_b = \frac{1}{(nN+1)2f_m} \text{ sec}$$

where T_b is the bit duration.

$$BW = \frac{1}{T_b}$$

bandwidth of the PCM system becomes

$$BW = (nN+1)\,2f_m \text{ Hz}$$

Now, if $N >> 1$ and $n >> 1$ (as is the practical situation)

Then,

$$BW \approx 2nNf_m \text{ Hz}$$

Minimum Information Capacity (Bit Rate) of PCM Systems

- The information capacity is defined as the number of bits that can be transmitted per second (bit rate). Since we are using the Nyquist rate for sampling, the minimum bit rate transmitted for a binary system is

$$C_{\min} = \text{bit rate} = \frac{\text{no. of samples}}{\text{second}} \cdot \frac{\text{no. of bits}}{\text{no. of samples}}$$

$$= 2nB \text{ bits/second}$$

- *That is, the minimum bit rate is equal to double the product of the signal bandwidth and the number of binary pulses.*

Noise Considerations in PCM Systems

The performance of a PCM system is influenced by two major sources of noise:

1. *Channel noise*, which is introduced anywhere between the transmitter output and the receiver input. Channel noise is always present, once the equipment is switched on.
2. *Quantization noise*, which is introduced in the transmitter and is carried all the way along to the receiver output. Unlike channel noise, quantization noise is *signal-dependent* in the sense that it disappears when the message signal is switched off.

- **Benefits or advantages of PCM:**

  ➡It is robust against noise and interference.

  ➡Uniform transmission quality.

  ➡Efficient SNR and bandwidth trade off.

  ➡It provides secure data transmission.

  ➡It offers efficient regeneration.

  ➡It is easy to add or drop channels

➡ **The PCM signal can then be transmitted over long distance with great reliability.**

- **Limitations/drawbacks or disadvantages of PCM:**

  ➡Overload appears when modulating signal changes between samplings, by an amount greater than the size of the step.

  ➡Large bandwidth is required for transmission.

  ➡Noise and crosstalk leaves low but rises attenuation.

  ➡An IDN (Integrated Digital Network) can only be realized by gradual extension of noise.

  ➡The difference between original analog signal and translated digital signal is called quantizing error.

  ➡ **Requires synchronization between receiver and transmitter.**

Problems

- Estimate the number of levels if the number of bits per sample is as follows.
- (a) 8 (as in telephony)
- (b) 16 (as in compact disc audio systems)

Solution :

- a) The number of levels with 8 bits per sample
- $N = 2^m$,
- m= 8
- Therefore N= 256

- b) The number of levels with 16 bits per sample
- $N = 2^m$
- m= 16
- Therefore N= 65,536.

Problem: A band-limited signal $m(t)$ of 3 kHz bandwidth is sampled at rate of 33⅓ % higher than the Nyquist rate. The maximum allowable error in the sample amplitude (*i.e.*, the maximum quantization error) is 0.5% of the peak amplitude m_p. Assume binary encoding. Find the minimum bandwidth of the channel to transmit the encoded binary signal.

Solution:

The Nyquist rate is R_N = 2 x 3000 Hz = 6000 Hz (samples/second), but the actual rate is 33⅓ % higher, so the sample rate is 6000 Hz + (⅓ x 6000) = 8000 Hz.

The quantization step is Δ and the maximum quantization error is plus/minus $\Delta/2$. Hence, we can write

$$\frac{\Delta}{2} = \frac{m_p}{L} = \frac{0.5}{100} m_p \quad \Rightarrow \quad L \geq 200$$

For binary coding, L, must be a power of two; therefore, knowing that $L = 2^7$ = 128 and 2^8 = 256, we must choose n = 8 to guarantee better than a 0.5% error.

Problems

Example 24 telephone channels, each bandlimited to 3.4 kHz, are to be time division multiplexed by using PCM. Calculate the bandwidth of the PCM system for 128 quantization levels and an 8 kHz sampling frequency.

Solution

Given $n = 24$

and $M = 128$

Therefore, $2^N = 128$

or $N = \log_2 128 = 7$

By putting $2f_m = 8000$ Hz is Eq. 8.8.3, we get

$$BW = [(24 \times 7) + 1]\ 8000 \text{ Hz}$$
$$= 1.352 \text{ MHz}$$

On the other hand, the approximate value of BW, as given by Eq. 8.8.4, is

$$BW = 24 \times 7 \times 8000 \text{ Hz}$$
$$= 1.344 \text{ MHz}$$

Note: For comparison, if the same number of channels are frequency division multiplexed by using an SSB modulation, the required bandwidth, assuming 4 kHz per channel, will be

$$BW = 24 \times 4 = 96 \text{ kHz}$$

This example clearly shows the tremendous bandwidth requirement of the PCM system.

An analog voltage wave form having an absolute bandwidth of 100Hz and an amplitude range of -10v to +10v and an amplitude over a PCM system with+ or - 0.1% accuracy(full scale)

a)determine the minimum sampling rate needed

b)determine the no. of bits needed in each PCM word

c)determine the minimum bit rate required in the PCM signal

Solution

a) Given BW=100Hz, so the min.sampling rate needed = 200Hz (nyquist sampling theorem).

b) given 0.1% accuracy => (delta / 2) = (1 / 1000) => delta = (2 / 1000)

But delta is (max range - min range)/L (where L is the number of quantization levels)

c) so ([10-(-10)] / L) = 2 / 1000 => L = 10,000

The number of bits needed are log(L) (base 2) . Here log(10000) to the base 2 is 13.278 = 14 (higher number)

Therefore 14 bits are needed for each PCM word

d) Min.bit rate = (no.of bits)*(sampling rate) = 14*200 = 2800 bits/sec

Problems

A message signal of $10\cos 2\pi \times 10^4 t$ is transmitted by using 4-bit PCM system. Find all possible parameters of PCM provided binary sequence is represented with rectangular pulses.

Given – $m(t) = 10\cos 2\pi \times 10^4 t$, $n = 4$

$\therefore A_m = 10$, $f_m = 10\,KHz$

$1\,Hz = 1\,bPs$

$L = 2^n$
$= 2^4 = 16$

& $f_s \longrightarrow$ Not given

$\therefore$ by default $f_s = NR$ i.e Nyquist Rate
$= 2f_m$
$= 2 \times 10 = 20\,KHz$

$$\Delta = \frac{2A_m}{L} = \frac{2\times 10}{16} = \frac{20}{16} = \frac{5}{4} = 1.25 \text{ Volts}$$

$$Q_{e_{max}} = \frac{\pm\Delta}{2} = \pm\frac{20}{16\times 2} = 0.625$$

$BW = R_b = nf_s = 4\times 20 = 80\,KbPs$ or KHz , $T_b = 1/R_b = \frac{1}{80}$

A message signal of $10\cos \pi 10^5 t$ is transmitted by using PCM. Maximum quantization error should be almost of 0.1% of Peak amplitude of msg signal. find bit-rate R_b.

Given – $10\cos \pi 10^5 t \Longrightarrow 10\cos(2\pi \cdot 50(10^3) t)$

$\therefore A_m = 10$, $f_m = 50\,KHz$

$f_s = 2f_m = 2\times 50 = 100\,KHz$

We know $R_b = nf_s$

but $n \longrightarrow ?$

$\therefore R_b = 10 \times 100$
$= 1000\,KbPs$

$\boxed{R_b = 1\,MbPs}$

To find n

$[Q_e]_{max} \leq 0.1\%$ of A_m

$\frac{\Delta}{2} \leq 0.1\%$ of 10

$\frac{\cancel{2}A_m}{\cancel{2}\, 2^n} \leq \frac{0.1}{100} \times 10$

$\frac{10}{2^n} \leq \frac{0.1}{1000} \times 10$

$2^n \geq 1000$

$\boxed{n = 10}$

($n\uparrow$, channel Bandwidth $\uparrow$)