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Institute of Science and Technology

DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
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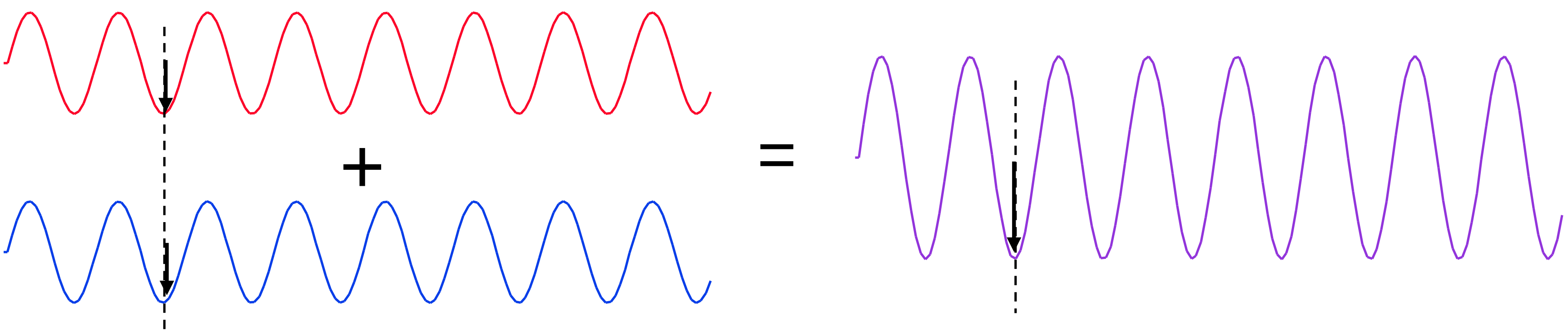
18PYB101J - Electromagnetic Theory, Quantum Mechanics, Waves and Optics
Module-IV (Waves and Optics) Lecture-1-8

Interference and Diffraction

Superposition of Waves

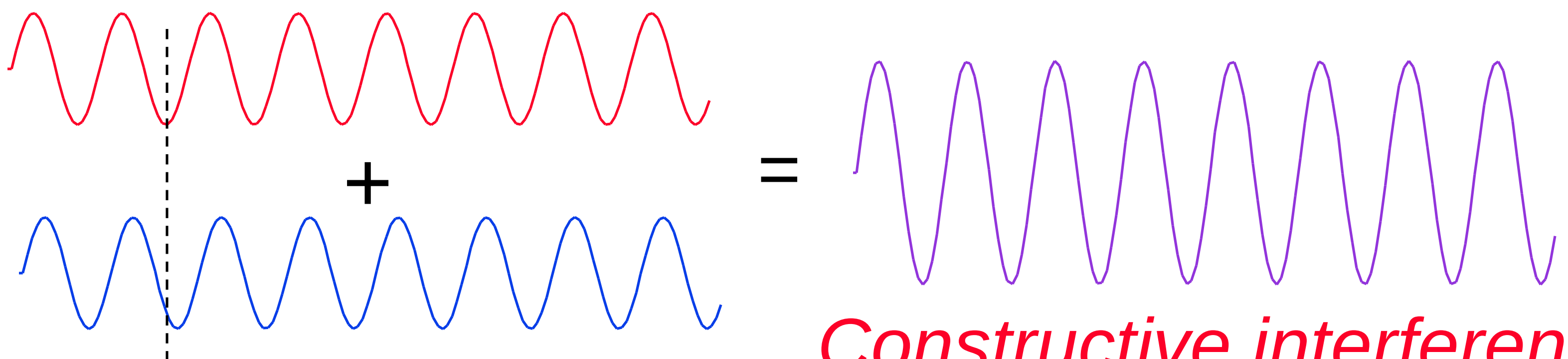
In general, when we combine two waves to form a composite wave, the composite wave is the algebraic sum of the two original waves, point by point in space [Superposition Principle].

When we add the two waves we need to take into account their:
Direction
Amplitude
Phase



Superposition of Waves

The combining of two waves to form a composite wave is called: Interference



(Waves almost in phase)

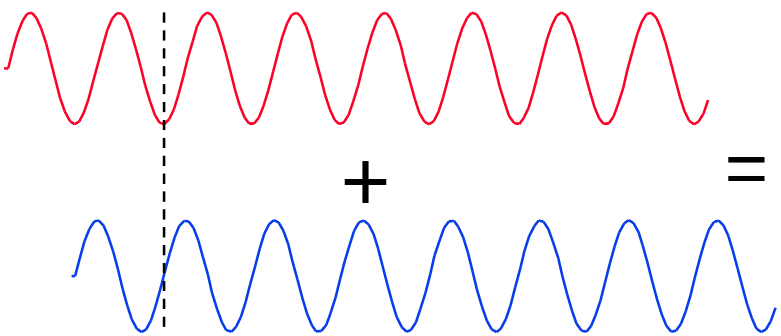
Constructive interference

The interference is constructive, if the waves reinforce each other.

Superposition of Waves

The combining of two waves to form a composite wave is called:

Interference



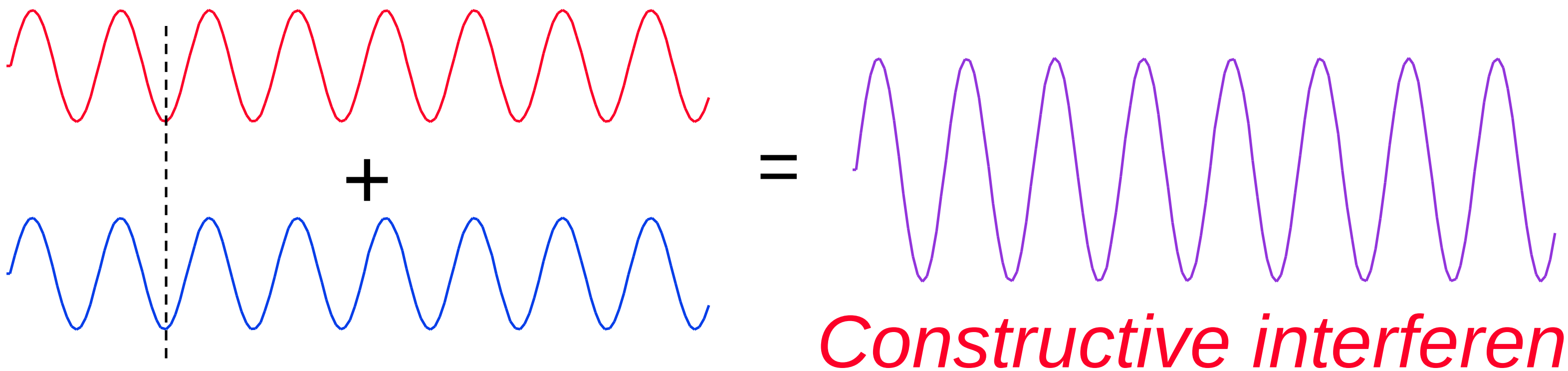
(Close to π out of phase)

(Waves almost cancel.)

Destructive interference

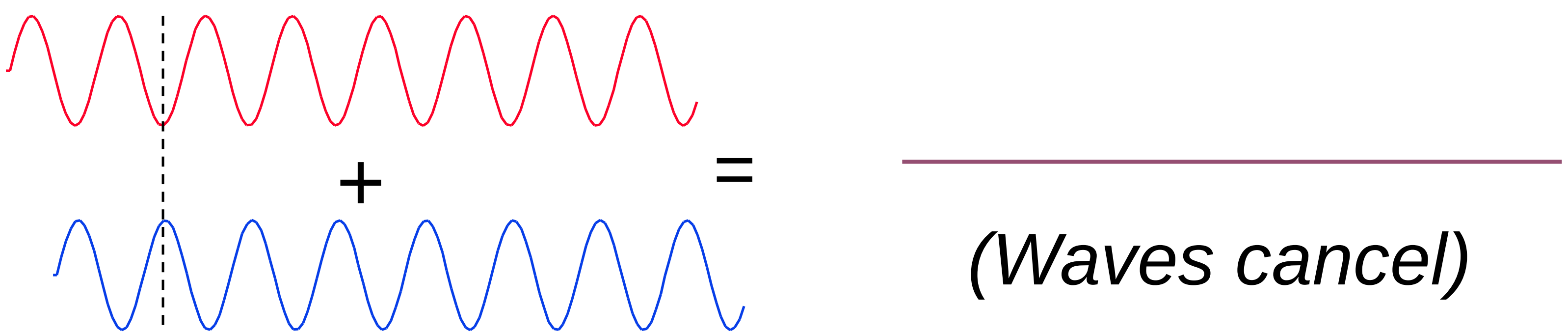
The interference is destructive
if the waves tend to cancel each other.

Interference of Waves



(In phase)

Constructive interference



(Waves cancel)

(π out of phase)

Destructive interference

Conditions for interference

When waves come together they can interfere constructively or destructively. To set up a stable and clear interference pattern, two conditions must be met:

- The sources of the waves must be coherent, which means they emit identical waves with a constant phase difference.

- The waves should be monochromatic - they should be of a single wavelength.

Conditions for interference

Let's say we have two sources sending out identical waves in phase. Whether constructive or destructive interference occurs at a point near the sources depends on the path-length difference, d, which is the distance from the point to one source minus the distance from the point to the other source.

- Condition for constructive interference:

(path-length difference) $d = 2n\lambda/2$, where n is any integer.

- Condition for destructive interference:

(path-length difference) $d = (2n+1)\lambda/2$, where n is any integer.

THEORY OF INTERFERENCE PATTERN

Let S_1 and S_2 represent the two pinholes of the Young's interference experiment. We would determine the positions of maxima and of minima on the line LL' which is parallel to the y-axis and lies in the plane containing the points S, S_1 and S_2 (see Fig).

We will show that the interference pattern (around the point 0) consists of a series of dark and bright lines perpendicular to the plane of Fig.; 0 being the foot of the perpendicular from the point S on the screen. For an arbitrary point *P* (on the line *LL')* to correspond to a maximum we must have

$$S_2P - S_1P = n\lambda \,;\, n = 0,1,2,\ldots$$

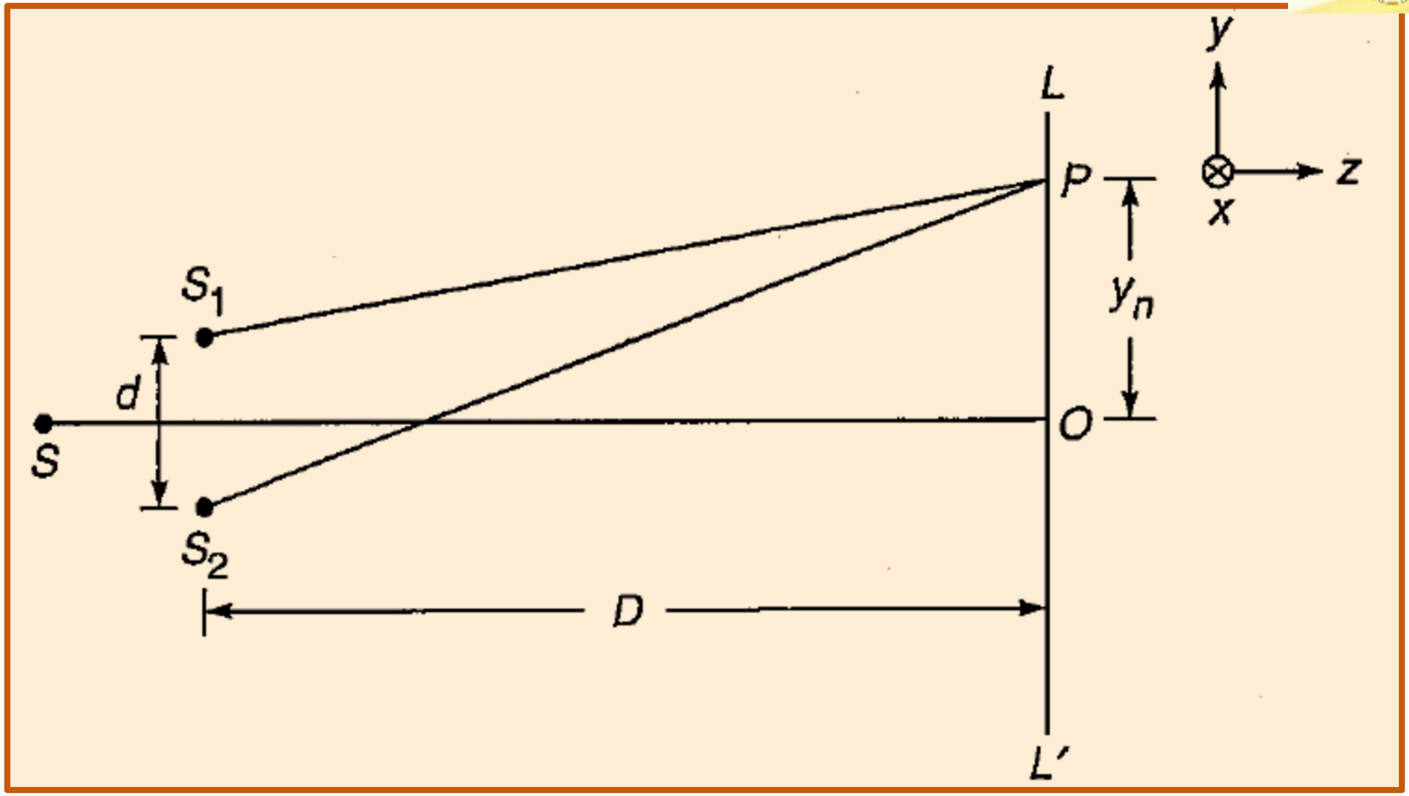


Fig.: Arrangement for producing Young's interference pattern.

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$$Now, (S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \tfrac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \tfrac{d}{2}\right)^2\right]$$

$$= 2y_n d$$

Where

S_1S_2 = d and OP = y_n

Thus

$$S_2P - S_1P = \frac{2y_n d}{S_2P + S_1P}$$

If y_n, *d* << *D* then negligible error will be introduced if $S_2P + S_1P$ is replaced by *2D.* Thus if we replace $S_2P + S_1P$ by 2D, the error involved is about 0.005%. In this approximation

$$S_2P - S_1P \approx \frac{y_n d}{D}$$

Thus we obtain $$y_n = \frac{\mathrm{n}\lambda D}{d}$$

Thus the dark and bright fringes are equally spaced and the distance between two consecutive dark (or bright) fringes is given by

$$\beta = y_{n+1} - y_{n+1} = \frac{(\mathrm{n+1})\lambda D}{d} - \frac{\mathrm{n}\lambda D}{d}$$

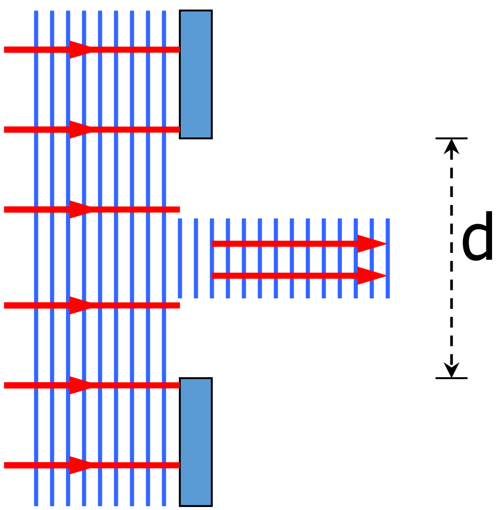
or

$$\beta = \frac{\lambda D}{d}$$

which is the expression for the fringe width.

Diffraction

Light is an electromagnetic wave, and like all waves, "bends" around obstacles.



$\lambda << d$

most noticeable when the dimension of the obstacle is close to the wavelength of the light

Diffraction

- Diffraction of light is the phenomenon of bending of light waves around the corners and their spreading into the geometrical shadows.

- Fresnel explained that the diffraction phenomenon was the result of mutual interference between the secondary wavelets from the same wave front.

Types of Diffraction

The diffraction phenomenon are usually divided into two classes

i) Fresnel class of diffraction phenomenon where the source of light and screen are in general at a finite distance from the diffracting aperture

ii) Fruanhofer class of diffraction phenomenon where the source and the screen are at infinite distance from the aperture, this is easily achieved by placing the source on the focal plane of a convex lens and placing screen on focal plane of another convex lens. This class of diffraction is simple to treat and easy to observe in practice

Difference between Fresnel and Fraunhofer's Diffractions

Fresnel Diffraction:

1. Point source of light or an illuminated narrow slit is used as light source
2. Light incident on the aperture or obstacle is a spherical or cylindrical wave front
3. The source and screen are at finite distance from the aperture or obstacle producing diffraction
4. Lenses are not used to focus the rays

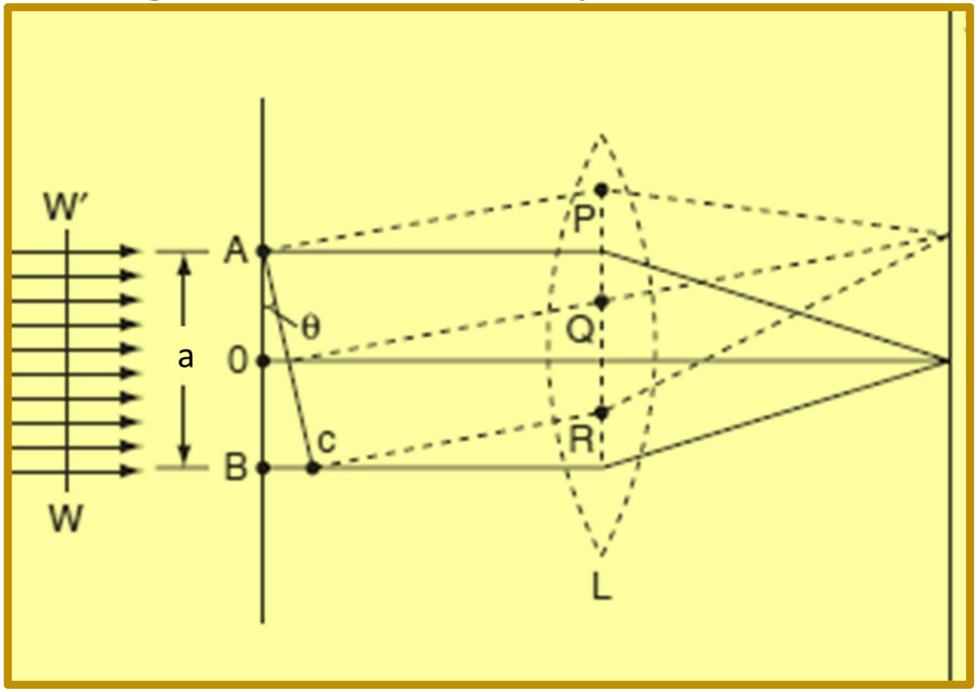
Fraunhofer diffraction:

1. Extended source of light at infinite distance is used as light source
2. Light incident on the aperture or obstacle is a plane wave front
3. The source and screen are at infinite distance from the aperture or obstacle producing diffraction
4. Converging lens is used to focus the rays

Fraunhofer Single Slit Diffraction

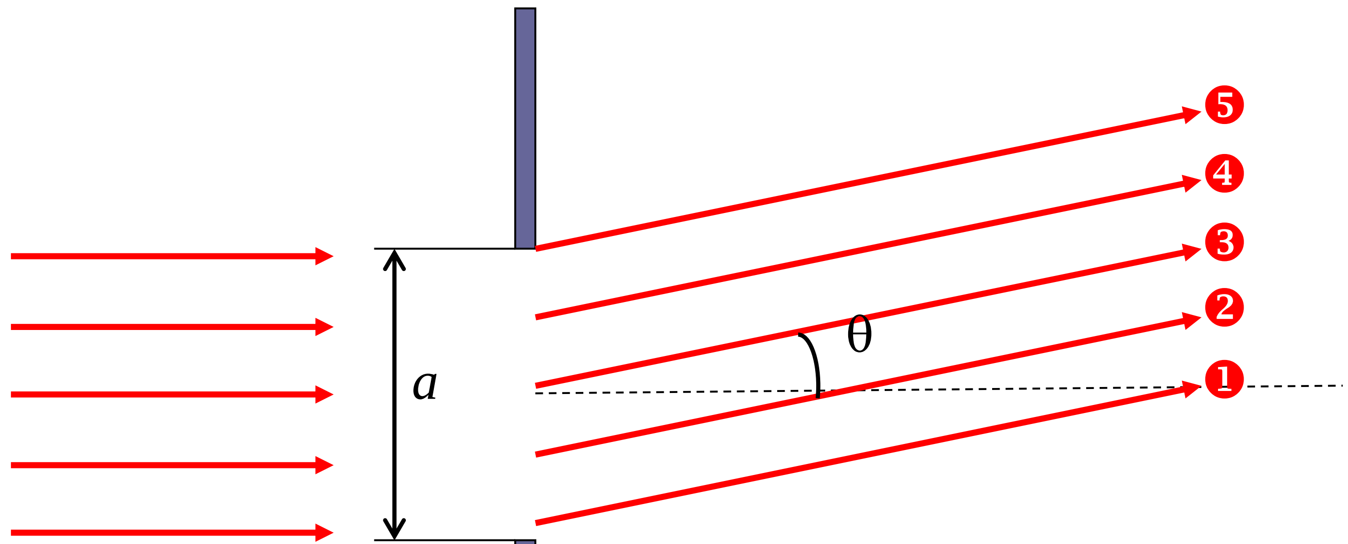
Description:

- The adjacent figure represents a narrow slit *AB* of width *'a'*.
- Let a plane wave front of monochromatic light of wavelength 'λ' is incident on the slit.
- Let the diffracted light be focused by means of a convex lens on a screen



Fraunhofer Single Slit Diffraction

Consider the effect of finite slit width

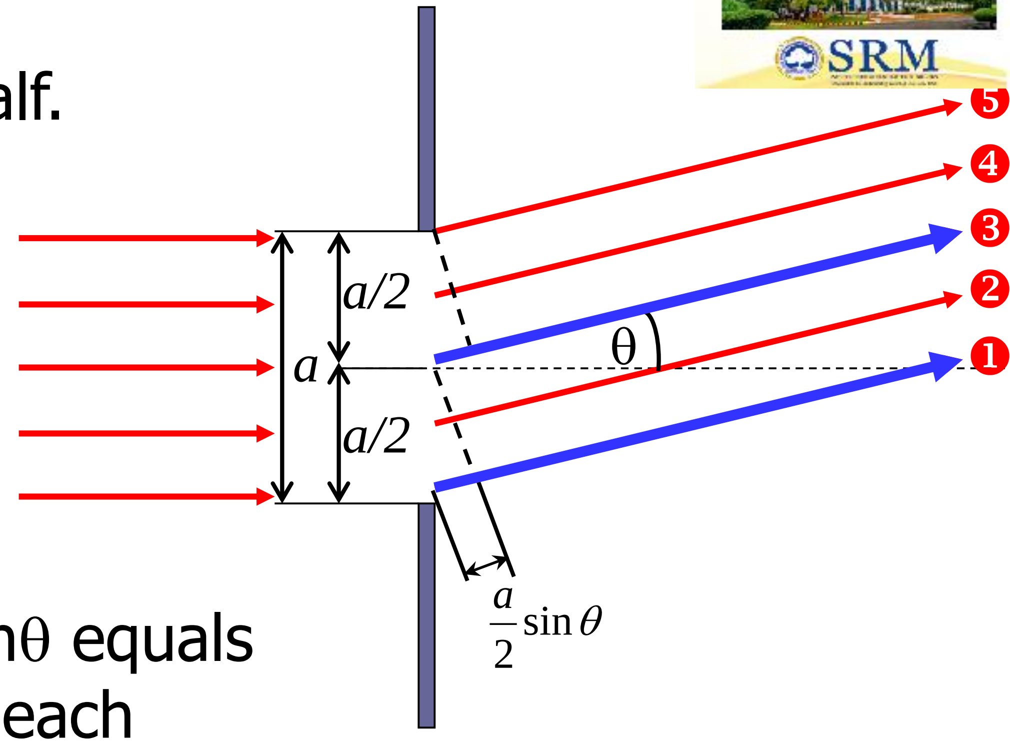


Single slit:

- each point in slit acts as source of light waves
- these different light waves interfere.

Imagine dividing the slit in half.

Wave ❶ travels farther* than wave ❸ by (a/2)sinθ. Same for waves ❷ and ❹.



If the path difference (a/2)sinθ equals λ/2, these wave pairs cancel each other → destructive interference

Destructive interference: $$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

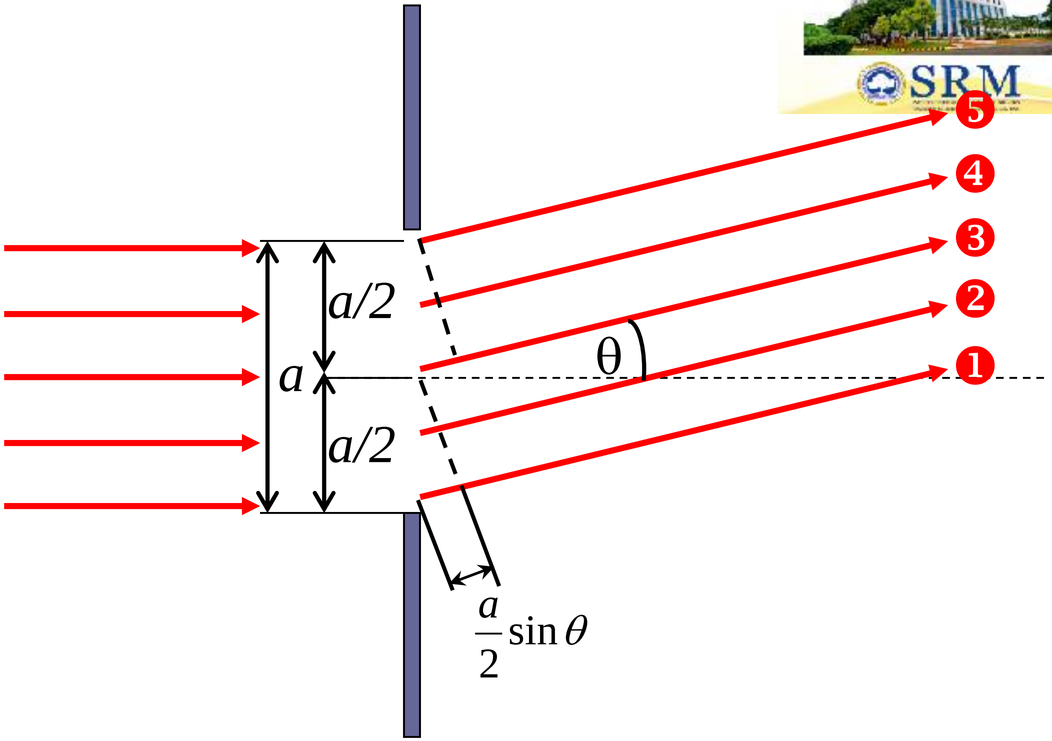
*All rays from the slit are converging at a point P very far to the right and out of the picture.

Destructive interference:

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a}$$



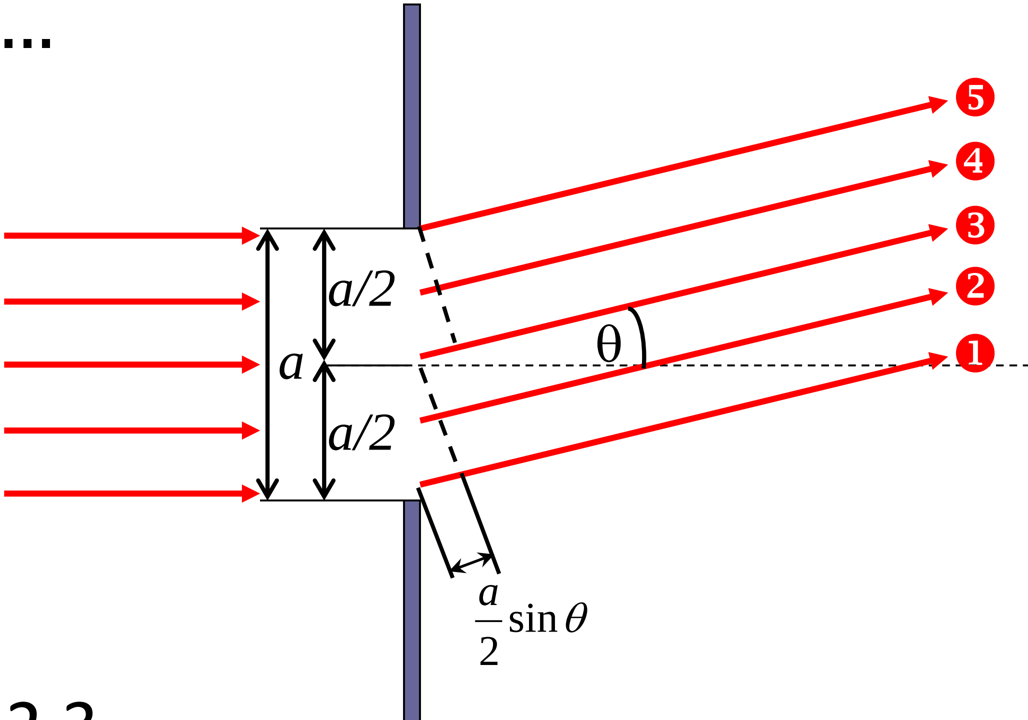
If you divide the slit into 4 equal parts, destructive interference occurs when $\sin\theta = \frac{2\lambda}{a}$.

If you divide the slit into 6 equal parts, destructive interference occurs when $\sin\theta = \frac{3\lambda}{a}$.

In general, destructive interference occurs when

$$\boxed{a \sin\theta = m\lambda} \qquad m = 1, 2, 3, \ldots$$

- gives positions of dark fringes
- **no** dark fringe for m=0



In general, constructive interference occurs when

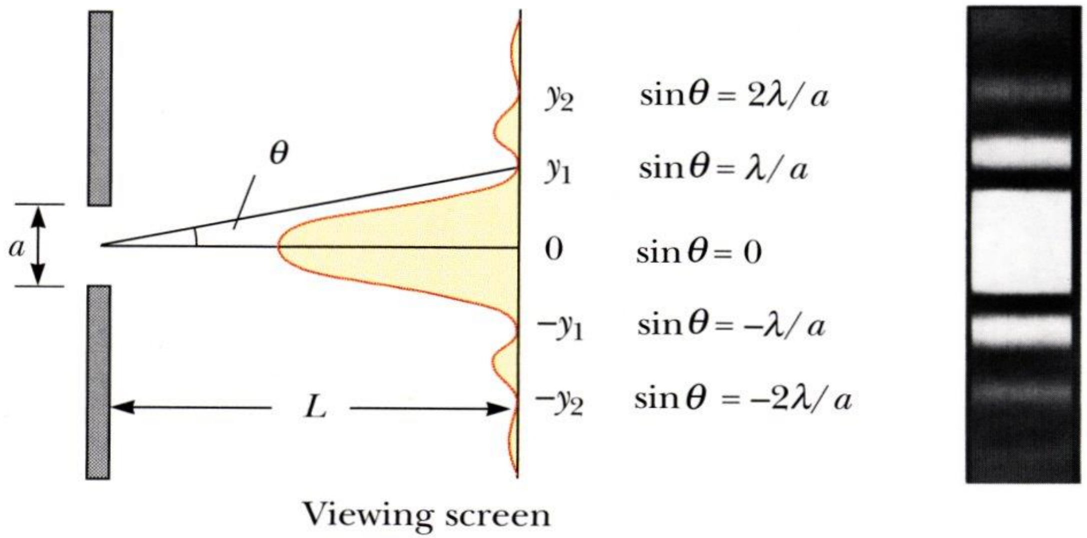
$$\boxed{a \sin\theta = (2m+1)\lambda} \qquad m = 1,2,3,\ldots$$

•gives positions of bright fringes

The bright fringes are approximately halfway in between.

Single Slit Diffraction Intensity

The general features of that distribution are shown below.



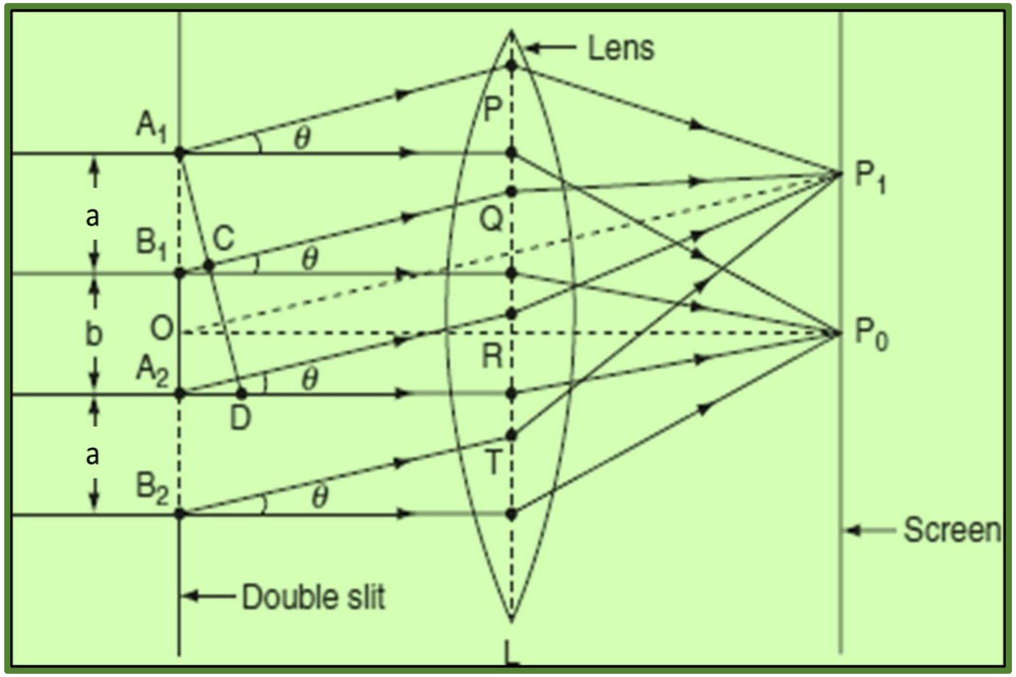
Most of the intensity is in the central maximum. It is twice the width of the other (secondary) maxima.

Intensity :

$$I^2 = R^2 = A^2 \frac{\sin^2\alpha}{\alpha^2}$$

Fraunhofer Double Slit Diffraction

- The double slits have been represented as A_1B_1 and A_2B_2 in Fig.
- The slits are narrow and rectangular in shape.
- Let the width of both the slits be equal and it is 'a' and they are separated by opaque length 'b'.
- A monochromatic plane wave front of wave length 'λ' is incident normally on both the slits.
- The secondary wavelets travelling in the direction of OP_0 are brought to focus at P_0 on the screen SS' by using a converging lens L.



- P_0 corresponds to the position of the central bright maximum. The intensity distribution on the screen is the combined effect of interference of diffracted secondary waves from the slits.

- Draw a normal from A_1 to B_1Q. Now, B_1C is the path difference between the diffracted waves at an angle 'θ' at the slit A_1B_1.

From the triangle A_1B_1C

$$sin\theta = \frac{B_1C}{A_1B_1} = \frac{B_1C}{a} \quad or \quad B_1C = a\,sin\theta$$

The corresponding phase difference

$$\delta = \frac{2\pi}{\lambda} a \sin\theta$$

- The diffracted wave at the two slits combine to produce interference.

- The path difference between the rays coming from corresponding points in the slits A_1B_1 and A_2B_2 can be found by drawing a normal from A_1 to A_2R.

- A_2D is the path difference between the waves from corresponding points of the slits.

The intensity at P_1 is

$$I^2 = R^2 = 4A^2 \frac{\boldsymbol{sin^2\alpha}}{\boldsymbol{\alpha^2}} \boldsymbol{cos^2\beta}$$

$$= 4I_0 \frac{\boldsymbol{sin^2\alpha}}{\boldsymbol{\alpha^2}} \boldsymbol{cos^2\beta}$$

Where $I_0 = A^2$

- The above represents the intensity distribution on the screen. The intensity at any point on the screen depends on α, β, and the intensity of central maximum is $4I_o$.

- The term $\cos^2 \beta$ corresponds to interference and $\sin^2\alpha$ corresponds to diffraction.

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The conditions for interference and diffraction maxima and minima:

Interference:

Maxima

- If the path difference $A_2D = (a + b)\sin\theta_n = \pm n\lambda$ where $n = 1, 2, 3\ldots$, then ‘θ_n’ gives the directions of the maxima due to interference of light waves coming from the two slits.

- The $\pm$ sign indicates maxima on both sides with respect to the central maximum.

The conditions for interference and diffraction maxima and minima:

Interference:

Minima:

- If the path difference $A_2D = (a + b)\sin\theta_n = \pm(2n+1)\lambda/2$ where $n = 1, 2, 3\ldots$, then 'θ_n' gives the directions of minima due to interference of the secondary waves from the two slits on both sides with respect to central maximum.

The conditions for interference and diffraction maxima and minima:

Diffraction:

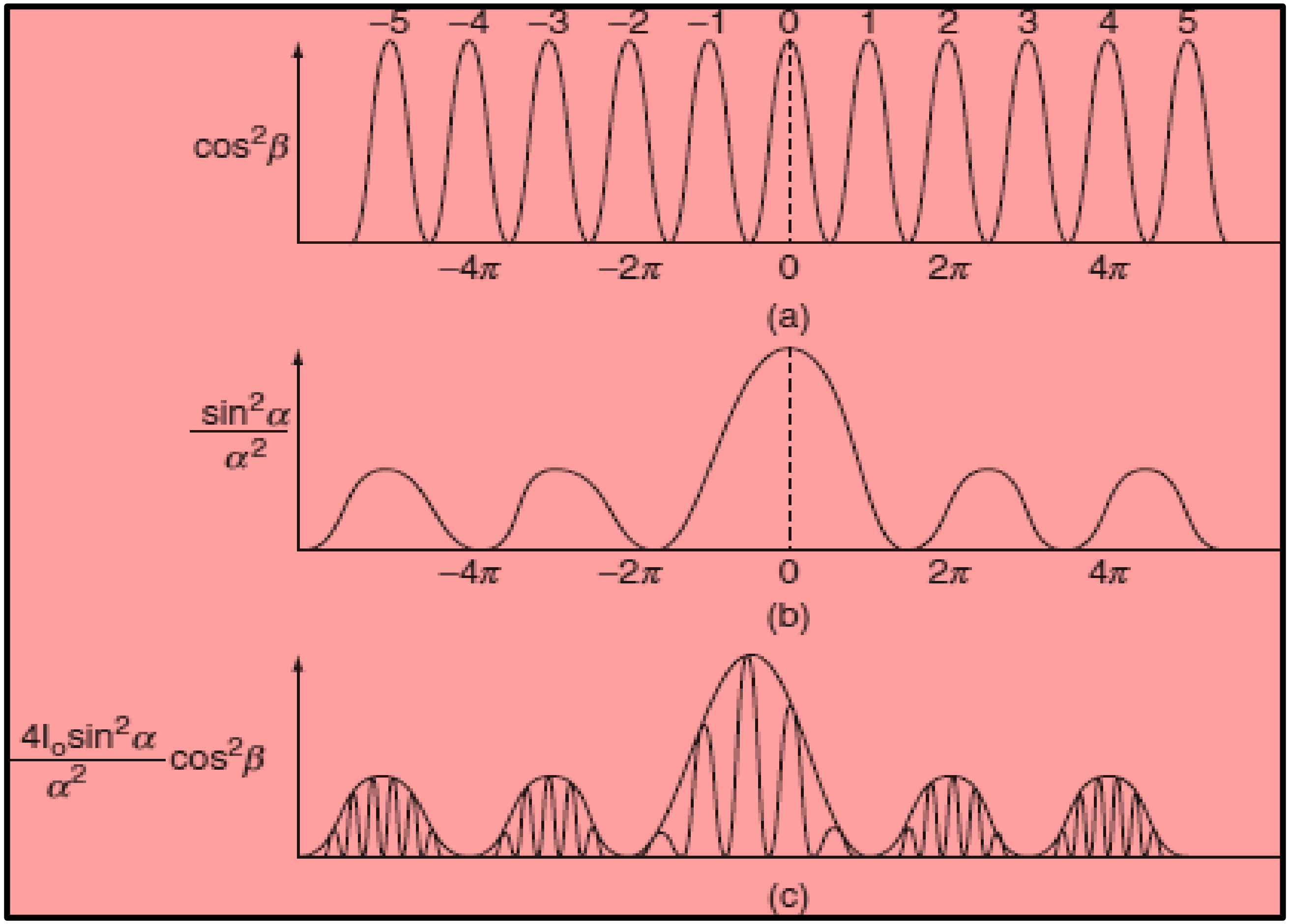
Minima

- If the path difference $B_1C = a\sin\theta_n = \pm\, n\lambda$, where n = 1, 2, 3… then θ_n gives the directions of diffraction minima. The ± sign indicates minima on both sides with respect to central maximum.

Maxima

- If the path difference $B_1C = a\sin\theta_n = \pm\,(2n+1)\lambda/2$, where n = 1, 2, 3… then θ_n gives the directions of diffraction maxima.

The intensity distribution on the screen due to double slit diffraction is shown in Fig.



Fraunhofer Diffraction due to N-Slits (Grating)

- An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as Diffraction grating

- Gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass, with a fine diamond point.

- The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit.

- This is known as plane transmission grating.

- When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.

Theory:

- A section of a *plane transmission grating MN* placed perpendicular to the plane of the paper is as shown in the figure.

- Let *'a'* be the width of each slit and *'b'* the width of each opaque space.

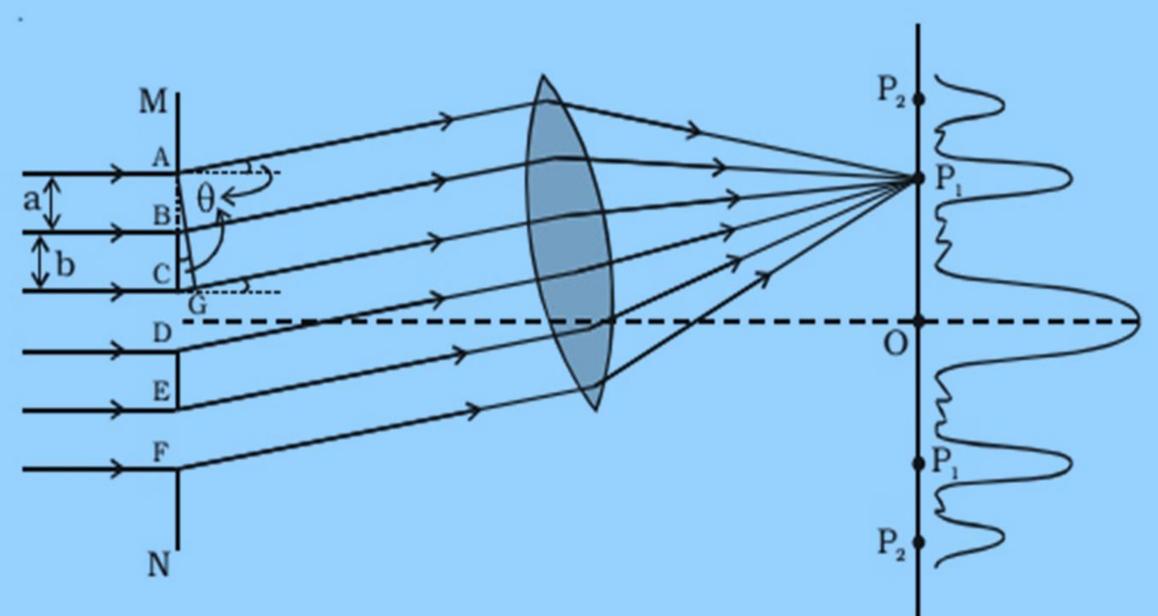


Figure: *Diffraction grating*

- MN represents the section of a plane transmission grating. AB, CD, EF … are the successive slits of equal width *a* and BC, DE … be the rulings of equal width b (Fig). Let e = $a + b$.

- Let a plane wave front of monochromatic light of wave length λ be incident normally on the grating.

- According to Huygen’s principle, the points in the slit AB, CD… etc act as a source of secondary wavelets which spread in all directions on the other side of the grating.

- Let us consider the secondary diffracted wavelets, which makes an angle θ with the normal to the grating.

- The path difference between the wavelets from one pair of corresponding points A and C is CG = (a + b) sin θ. It will be seen that the path difference between waves from any pair of corresponding points is also (a + b) sin θ

- The point P1 will be bright, when

$$(a + b) \sin\theta = m\lambda \text{ where } m = 0, 1, 2, 3$$

- (a + b) sin θ = 0, satisfies the condition for brightness for m = 0. Hence the wavelets proceeding in the direction of the incident rays will produce maximum intensity at the centre O of the screen. This is called zero order maximum or central maximum.

- Similarly, for second order maximum, $(a + b) \sin \theta_2 = 2\lambda$

- On either side of central maxima different orders of secondary maxima are formed at the point P1, P2.

- In general, $(a + b) \sin \theta = m \lambda$ is the condition for maximum intensity, where m is an integer, the order of the maximum intensity.

$$Sin\theta = \frac{m\lambda}{a+b} \quad or \quad Sin\theta = Nm\lambda$$

where N = 1/a+b , gives the number of grating element or number of lines per unit width of the grating

Difference between Diffraction and Interference

S.No	*Interference*	*Diffraction*
1.	Interference may be defined as waves emerging from two different sources, producing different wave fronts.	Diffraction on the other hand can be termed as secondary waves that emerge from the different parts of the same wave.
2.	In interference the intensity of all the positions on maxima are of similar intensity in interference.	In diffraction, there is a variance of the intensity of positions.
3.	The width of the fringes in interference is equal in interference.	The width of the fringes is not equal in interference.
4.	It is absolutely dark in the region of minimum intensity, in the case of interference.	In the case of diffraction, there is a variance in the intensity of interference.
5.	If the number of sources are few such as two sources, then they are referred to as interference sources.	If the number of sources are many, that is more than two then it is referred to as diffraction sources