Electromagnetic waves in Lossless Dielectrics

Maxwell's First equation,

$$\nabla \times H = \sigma \vec{E} + j\omega \varepsilon_e E.$$
$$= \vec{J} + \frac{\sigma \vec{D}}{\partial t}.$$

Dissipation Factor, D_f :- The ratio of Conduction Current density to displacement Current density in the medium is.,

$$\frac{\sigma \vec{E}}{\omega \varepsilon_e \vec{E}} = \frac{\sigma}{\omega \varepsilon} = 1.$$

For Good Conductor., $\frac{\sigma}{\omega \varepsilon_e} >> 1.$

For Good Dielectric., $\frac{\sigma}{\omega \varepsilon_e} << 1.$

power Factor in terms of dissipation factor is.,

$$\text{power Factor} = \sin\left(\tan^{-1}(D_f)\right).$$

In a lossless Dielectric medium;

$$\frac{\sigma}{\omega \varepsilon} << 1 \Rightarrow \sigma << \omega \varepsilon.$$

$$\Rightarrow \sigma = 0 ; \quad \varepsilon = \varepsilon_0 \varepsilon_r ; \quad \mu = \mu_0 \mu_r$$

General wave equation.,

$$\nabla^2 E = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \&$$

$$\nabla^2 H = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \varepsilon \frac{\partial^2 H}{\partial t^2}.$$

Since, $\sigma = 0$;

$$\nabla^2 E = \mu \varepsilon \frac{\partial^2 E}{\partial t^2} ; \quad \nabla^2 H = \mu \varepsilon \frac{\partial^2 H}{\partial t^2}.$$

w.k.t. Attenuation Constant, α is

$$\alpha = \omega \sqrt{\frac{\mu \varepsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon^2}} - 1\right]} \rightarrow (3).$$

Condition for Good dielectric is $\frac{\sigma}{\omega \varepsilon} << 1 \rightarrow (1)$

$$\frac{\sigma^2}{\omega^2 \varepsilon^2} << 1 ; \quad 1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \simeq 1 \rightarrow (2)$$

③

w.k.t. phase constant; β is

$$\beta = \omega \sqrt{\frac{\mu\varepsilon_e}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_e^2}}+1\right)} \rightarrow (4)$$

Substituting (2) in (3) & (4) we get.,

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon_e}$$

$$\boxed{\text{wave length, } \lambda = \frac{2\pi}{\beta}\ \text{mts}}$$

Velocity, $V = \dfrac{\omega}{\beta}$

$$V = \frac{\omega}{\omega\sqrt{\mu\varepsilon_e}} = \frac{1}{\sqrt{\mu\varepsilon_e}} = 3\times10^8\ \text{m/sec}$$

$\Rightarrow c$.

Characteristic Impedance (or)

Intrinsic Impedance :-

It is the ratio of the square root of permeability to the dielectric constant of the medium & it is denoted by η.

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon_e}}.$$

Intrinsic Impedance → General Formula ⇒

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon_e}}.$$

w.k.t for a good dielectric, $\sigma = 0$.

$$\eta = \sqrt{\frac{j\omega\mu}{0 + j\omega\varepsilon_e}} = \sqrt{\mu/\varepsilon_e}.$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon_e}}$$

$$= \sqrt{\frac{j\omega\mu}{j\omega\varepsilon_e\left(1 + \frac{\sigma}{j\omega\varepsilon_e}\right)}} = \sqrt{\frac{\mu}{\varepsilon_e}\left(1 + \frac{\sigma}{j\omega\varepsilon_e}\right)^{-1}}.$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon_e}\left(1 - \frac{\sigma}{j\omega\varepsilon_e}\right)}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon_e}\left(1 + \frac{j\omega\varepsilon_e\sigma}{\omega^2\varepsilon_e^2}\right)}$$

$$\boxed{\frac{\sigma}{j\omega\varepsilon_e} \times \frac{-j\omega\varepsilon_e}{-j\omega\varepsilon_e} = \frac{-j\omega\varepsilon_e\sigma}{\omega^2\varepsilon_e^2}}$$

w.k.t.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$\therefore \left(1+\frac{j\sigma}{2\omega\varepsilon_e}\right)^2 = 1 + (2)\left(\frac{j\sigma}{2\omega\varepsilon_e}\right) + \frac{2(2-1)}{2}\left(\frac{j\sigma}{2\omega\varepsilon_e}\right)^2 + \ldots$$

$$= 1 + \frac{j\sigma}{\omega\varepsilon_e} + \ldots$$

$$\sqrt{1+\frac{j\sigma}{\omega\varepsilon_e}} = \sqrt{\left(1+\frac{j\sigma}{2\omega\varepsilon_e}\right)^2} \Rightarrow 1 + \frac{j\sigma}{2\omega\varepsilon_e}$$

$$\boxed{\therefore \eta = \sqrt{\frac{\mu}{\varepsilon_e}}\left(1+\frac{j\sigma}{2\omega\varepsilon_e}\right)}$$

$$\eta = \sqrt{\mu/\varepsilon_e} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\boxed{\eta = 377\sqrt{\frac{\mu_r}{\varepsilon_{e_r}}}}$$

$\eta_0 = 377\,\Omega$

Electromagnetic Plane Wave in Good Conductor:-

(6)

For a Good Conductor, Conductivity is 10^7 ℧/m (is in the order of).

$$\frac{\sigma}{\omega \varepsilon_e} >> 1.$$

$$\frac{\sigma^2}{\omega^2 \varepsilon_e^2} >> 1 \quad ; \quad 1 + \frac{\sigma^2}{\omega^2 \varepsilon_e^2} \simeq \frac{\sigma^2}{\omega^2 \varepsilon_e^2}.$$

General wave equation ;

$$\nabla^2 E = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \varepsilon_e \frac{\partial^2 E}{\partial t^2}.$$

$$\nabla^2 H = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \varepsilon_e \frac{\partial^2 H}{\partial t^2}.$$

In phasor form ;

$$\nabla^2 E = \left[j\omega\mu (\sigma + j\omega \varepsilon_e) \right] \vec{E}.$$

$$\nabla^2 H = \left[j\omega\mu (\sigma + j\omega \varepsilon_e) \right] \vec{H}.$$

w.k.t. propagation Constant, $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$.

$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$ $\quad \gamma = \sqrt{j\omega\mu(\sigma + j\omega)}$.

For a good Conductor, $\sigma >> \omega\varepsilon$.

$\Rightarrow \gamma = \sqrt{j\omega\mu(\sigma)}$.

$\gamma = \sqrt{\omega\mu\sigma}\,(\sqrt{j})$.

In general, $j = \cos 90 + j\sin 90^\circ$.

$(j)^{1/2} = (\cos 90 + j\sin 90)^{1/2}$ $\quad \boxed{(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta}$

$= \left[\cos(1/2)90^\circ + j\sin(1/2)90^\circ\right]$

$= \cos 45 + j\sin 45$.

$= \cos \pi/4 + j\sin \pi/4$

$$\boxed{\sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \Rightarrow 1\angle 45^\circ}$$

$\therefore \gamma = \sqrt{\omega\mu\sigma}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$.

$\gamma = \frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}} + j\frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}}$.

w.k.t $\gamma = \alpha + j\beta$.

Aliter :-

w.k.t. $\alpha = \omega\sqrt{\dfrac{\mu\varepsilon_e}{2}\left[\sqrt{1+\dfrac{\sigma^2}{\omega^2\varepsilon_e^2}}-1\right]}$.

$= \omega\sqrt{\dfrac{\mu\varepsilon_e}{2}\left[\sqrt{\dfrac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$.

$= \omega\sqrt{\dfrac{\mu\varepsilon_e}{2}\left(\dfrac{\sigma}{\omega\varepsilon_e}-1\right)} = \omega\sqrt{\dfrac{\mu\varepsilon_e}{2}\left(\dfrac{\sigma}{\omega\varepsilon_e}\right)}$

$= \omega\sqrt{\dfrac{\mu\sigma}{2\omega}} = \sqrt{\dfrac{\mu\sigma\omega^2}{2\omega}} = \boxed{\sqrt{\dfrac{\omega\mu\sigma}{2}} \Rightarrow \alpha}$

w.k.t. $\beta = \omega\sqrt{\dfrac{\mu\varepsilon_e}{2}\left(\sqrt{1+\dfrac{\sigma^2}{\omega^2\varepsilon_e^2}}+1\right)}$

$\Rightarrow \boxed{\beta = \sqrt{\dfrac{\omega\mu\sigma}{2}}}$

Intrinsic Impedance, $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma+j\omega\varepsilon_e}}$.

$\eta = \sqrt{\dfrac{j\omega\mu}{j\omega\varepsilon_e\left(1+\dfrac{\sigma}{j\omega\varepsilon_e}\right)}}$

w.k.t $\dfrac{\sigma}{j\omega\varepsilon_e} >> 1$

$\sigma >> j\omega\varepsilon$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(\frac{\sigma}{j\omega\varepsilon}\right)}} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}} \cdot \angle 45^\circ}$$

Velocity :-

Velocity of the wave in the Conductor,

$$v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \omega\sqrt{\frac{2}{\omega\mu\sigma}}$$

$$= \sqrt{\frac{2\omega^2}{\omega\mu\sigma}} = \boxed{\sqrt{\frac{2\omega}{\mu\sigma}} \Rightarrow v}$$

1. A 300 MHz Uniform plane wave propogates through fresh water for which, $\sigma=0, \mu_r=1$, and $\varepsilon_r=78$. Calculate : (i) Attenuation Constant, (ii) phase Constant, (iii) Wavelength, (iv) Intrinsic Impedance.

<u>Solution :-</u>

(i). For the given medium, ie., Fresh water, the Conductivity, $\sigma=0$. Assuming medium, to be a lossless medium, the attenuation Constant, $\alpha=0$.

(ii) <u>phase Constant :-</u> $\beta = \omega\sqrt{\mu\varepsilon}$.

$$= \left[2\pi(300\times10^{6})\right]\sqrt{(4\pi\times10^{-7})(1)(8.854\times10^{-12})(78)}.$$

$\beta = 55.529$ rad/m.

(iii). wavelength, $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{55.529} = 0.1131$ m.

(iv). Intrinsic Impedance, $\eta = \sqrt{\mu/\varepsilon} = \sqrt{\dfrac{(4\pi\times10^{-7})(1)}{(8.854\times10^{-12})78}}$.

$\boxed{\eta = 42.636\,\Omega}$

(2). A 9375 MHz Uniform plane wave is propagating in polystyrene. If the amplitude of the E-Field intensity is 20 V/m & the material is assumed to be lossless, find: (i) Attenuation Constant, (ii) Phase Constant, (iii) Wavelength in Polystyrene (iv) Velocity of propagation, (v) Intrinsic Impedance (vi) propagation Constant, (vii) Amplitude of the magnetic field intensity.

For polystyrene, $\mu_r = 1$, $\varepsilon_r = 2.56$;

<u>Solution</u> :- For a lossless medium, $\sigma = 0$, then

(i) Attenuation Constant, $\alpha = 0$

(ii) Phase Constant, $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}$.

$$= 2\pi(9375\times10^{6})\sqrt{(4\pi\times10^{-7})(1)(8.854\times10^{-12})(256)}$$

$$= 314.37 \text{ rad/m}.$$

(iii) Wavelength, $\lambda = \frac{2\pi}{\beta} = 0.01998 \text{ m}$.

(iv) Velocity of propagation, $v = f.\lambda. = 1.873\times10^{8} \text{ m/s}$

(v) Intrinsic Impedance, $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = 235.45\Omega$

(vi) Propagation Constant, $\gamma = \alpha + j\beta = (0 + j\,314.37)\text{m}^{-1}$.

$$\boxed{\gamma = j\,314.37 \text{ m}^{-1}}$$

(vii) The amplitude of electric field intensity is given as 20 V/m.

$E_x = 20$ V/m.

w.k.t, $\eta = \frac{E_x}{H_y} \Rightarrow 235.45 = \frac{20}{H_y}$.

∴ The amplitude of Magnetic field Intensity, $\boxed{H_y = \frac{E_x}{\eta} = 0.08494 \, A/m}$

③. A plane wave propagation through a medium with $\varepsilon_r = 8$, $\mu_r = 2$ has $E = 0.5 \sin(10^8 t - \beta z)\bar{a}_x$ V/m.

Determine (i) β (ii) Wave Impedance, (iii) Wave Velocity (iv) $\bar{H}$ field.

<u>Solution</u> :-

$$\bar{E} = 0.5 . \sin(10^8 t - \beta z)\bar{a}_z \text{ V/m}.$$

$$E_m = 0.5 \; ; \; f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = 15.9155 \text{ MHz}.$$

(i) Velocity of propagation, $v = \frac{\omega}{\beta} \Rightarrow \frac{1}{\sqrt{\mu \varepsilon}}$.

$$\underline{\underline{V = 0.7495 \times 10^8 \text{ m/sec}}}.$$

$$\beta = \frac{\omega}{v} = 1.3342 \text{ rad/m}$$

(iii) Intrinsic Impedance, $\underline{\underline{\eta}} = \sqrt{\frac{\mu}{\varepsilon_e}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_{e0} \varepsilon_{er}}}$

$= 188.36\,\Omega$.

(iv) The amplitude of $\bar{H}$-field is $H_m = \frac{E_m}{\eta}$

$= \frac{0.5}{188.36} = 2.655 \times 10^{-3}\ A/m$.

$\therefore$ The Magnetic Field, $\vec{H}$ is given by;

$$\bar{H} = H_m \sin(\omega t - \beta z)\, \bar{a}_y.$$

$$\boxed{\bar{H} = 2.655 \times 10^{-3} \cdot \sin(10^8 t - 1.3342)\, \bar{a}_y\ A/m}$$