SRM Institute of Science and Technology
Department of Mathematics
18MAB101T-Calculus and Linear Algebra

Unit II -Tutorial I (Slot C1)

Part -B

1. If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify $f_{xy} = f_{yx}$.

2. If $z = f(x+ct) + \phi(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$.

3. (a)If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

   (b) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

4. Find $\frac{dy}{dx}$ if $x^y = y^x$.

5. If $u = x^2 + y^2, x = at^2, y = 2at$ find $\frac{du}{dt}$.

Part-C

6. If $z = f(u,v), u = x^2 - y^2, v = 2xy$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4\left(x^2+y^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$

7. If $u = \log\left(x^3+y^3+z^3-3xyz\right)$, then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

8. If $u = f(x-y, y-z, z-x)$ prove that $\sum \frac{\partial u}{\partial x} = 0$.

9. Find Taylor's expansion of x^y near (1,1) upto second degree term.

10. (a) Expand $xy^2 + 2x - 3y$ in powers of $(x+2)$ and $(y-1)$ upto third degree terms.

    (b) Find the Maclaurin's series for e^{x+y} upto second degree.