Reg. No.															

B.Tech. DEGREE EXAMINATION, JULY 2022
Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted from the academic year 2020 – 2021 and 2021 – 2022)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours | Max. Marks: 75

PART – A (25 × 1 = 25 Marks)
Answer **ALL** Questions

Marks BL CO PO

1. Find $\int_0^1\int_0^1 dxdy$ — 1 2 1 2
   (A) 1 (B) 2
   (C) 0 (D) 4

2. Name the curve $y^2 = 4x$ is — 1 1 1 1
   (A) Parabola (B) Hyperbola
   (C) Straight line (D) Ellipse

3. Select the formula to find area of the region R using double integral in polar co-ordinates is — 1 1 1 1
   (A) $\iint_R dr d\theta$ (B) $\iint_R r^2 dr d\theta$
   (C) $\iint_R r dr d\theta$ (D) $\iint_R (r+1) dr d\theta$

4. Identify the region of integration for the integral $\int_0^1\int_0^x f(x,y)dxdy$ — 1 1 1 2
   (A) Square (B) Triangle
   (C) Rectangle (D) Circle

5. Select the name of the curve $r = a(1+\cos\theta)$ from the options given below — 1 1 1 1
   (A) Cardioid (B) Lemniscate
   (C) Cycloid (D) Hemicircle

6. Choose the $\nabla.\vec{r}$ value, if $\vec{r}$ is the position vector of the point *(x,y,z)* with respect to the origin. — 1 2 2 1,2
   (A) 2 (B) 3
   (C) 0 (D) 1

7. Find the value of $\nabla\phi$, *if* $\phi = xyz$. — 1 2 2 1,2
   (A) $zx\vec{i} + xy\vec{j}$ (B) $zx\vec{i} + xy\vec{j} + yz\vec{k}$
   (C) $xy\vec{i} + yz\vec{j} + zx\vec{k}$ (D) $yz\vec{i} + zx\vec{j} + xy\vec{k}$

8. Find the value of curl (grad ϕ) 1 1 2 1

(A) 3 (B) 2

(C) 0 (D) 1

9. What does $\nabla \times \vec{F} = 0$ mean? 1 1 2 1

(A) Irrotational vector (B) Flux

(C) Solenoidal vector (D) Circulation

10. Name the theorem which connects the line integral and surface integral 1 1 2 1

(A) Stoke's theorem (B) Green's theorem

(C) Gauss divergence theorem. (D) Residue theorem

11. What is $L\left[e^{3t}\right]$ 1 1 3 1,2

(A) $\dfrac{1}{s+3}$ (B) $\dfrac{3}{s+3}$

(C) $\dfrac{1}{s-3}$ (D) $\dfrac{s}{s-3}$

12. Find $L[\cos 2t]$. 1 2 3 2

(A) $\dfrac{s}{s^2+2}$ (B) $\dfrac{s}{s^2+4}$

(C) $\dfrac{2}{s^2+2}$ (D) $\dfrac{4}{s^2+4}$

13. Choose the function f(t) for which the Laplace transform does not exist 1 2 3 2

(A) $f(t)=t^2$ (B) $f(t)=\sin t$

(C) $f(t)=e^{-at}$ (D) $f(t)=\tan t$

14. Interpret the value of $L\left[e^{-at}f(t)\right]$, where $L[f(t)]=F(s)$ 1 1 3 1

(A) $F(s-a)$ (B) $F(s)$

(C) $F(s+a)$ (D) $\dfrac{1}{a}F(s/a)$

15. Find the inverse Laplace transform of $\dfrac{s+3}{(s+3)^2+9}$ 1 1 3 1

(A) $e^{-3t}\cos 3t$ (B) $e^{-3t}\cos 9t$

(C) $e^{-3t}\cosh 3t$ (D) $e^{3t}\cos 3t$

16. Name the function $u(x,y)$ which satisfies $u_{xx}+u_{yy}=0$ 1 1 4 1

(A) Analytic (B) Harmonic

(C) Differential (D) Discontinuous

17. Interpret the transformation $\omega = cz$ where c is a real constant 1 2 4 1,2

(A) Rotation (B) Reflection

(C) Magnification (D) Magnification and rotation

		Marks	BL	CO	PO
18.	Find the points at which the function $f(z)=\frac{1}{z^2+1}$ fails to be analytic (A) $z=\pm 1$ (B) $z=\pm i$ (C) $z=0$ (D) $z=\pm 2$	1	2	4	1,2
19.	What is the critical point of the transformation $\omega=z^2$? (A) $z=2$ (B) $z=0$ (C) $z=1$ (D) $z=-2$	1	2	4	2
20.	State the property that "An analytic function with constant modulus is __________" (A) Zero (B) Analytic (C) Constant (D) Harmonic	1	1	4	1
21.	Name the curve which does not cross itself (A) Curve (B) Closed curve (C) Simple closed curve (D) Multiple curve	1	1	5	1
22.	What is the value of $\int_c \frac{zdz}{z-2}$ where 'c' is the circle $\|z\|=1$ (A) 0 (B) $\frac{\pi}{2}i$ (C) $\frac{\pi}{2}$ (D) 2	1	2	5	2
23.	What is the value of $\int_c \frac{f(z)}{z-a}dz$ if *f(z)* is analytic inside and on c, where c is the simple closed curve and a is any point within c. (A) $f(a)$ (B) $2\pi i\, f(a)$ (C) $\pi i\, f(a)$ (D) 0	1	1	5	1
24.	Define the annular region between two concentric circles $C_1=\|Z-a\|=R_1$ *and* $C_2=\|Z-a\|=R_2$ *where* $R_2<R_1$ (A) Within C_1 (B) Within C_2 (C) Within C_2 and outside C_1 (D) Within C_1 and outside C_2	1	2	5	1
25.	Identify the pole of *f(z)* if $f(z)=\frac{\sin z}{z}$. (A) z=0 is a simple pole (B) z=0 is a pole of order 2 (C) z=0 is a removable singularity (D) z=0 is a zero of *f(z)*	1	2	5	1,2

PART – B (5 × 10 = 50 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
26. a.	Change the order of integration and hence find the value of $I=\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$.	10	3	1	1,2

(OR)

b.	Apply triple integration to find the volume of the sphere $x^2+y^2+z^2=a^2$ without transformation.	10	4	1	1,2
27. a.i.	Compute the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point (2,–,1,2).	4	4	2	1,2
ii.	Compute the divergence and curl of the vector $\bar{V}=xyz\vec{i}+3x^2y\vec{j}+\left(xz^2-y^2z\right)\vec{k}$ at the point (2,–1,1).	6	4	2	1,2
	(OR)				
b.	Verify Gauss divergence theorem for $\vec{F}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ over the cube bounded by *x=0, x=1, y=0, y=1, z=0, z=1.*	10	4	2	1,2
28. a.	Compute Laplace transform of the square wave f(t) given by $f(t)=\begin{cases}E & for\ 0\le t\le \pi/2\\ -E & for\ \pi/2\le t\le \pi\end{cases}$ where $f(t+\pi)=f(t)$.	10	4	3	1,2
	(OR)				
b.	Solve the differential equation $\frac{dy}{dt}-y(t)=1-2t$ given that $y=-1$ when $t=0$, using Laplace transform.	10	4	3	1,2
29. a.	Construct the analytic function $f(z)=u+iv$, *if* $u=\frac{\sin 2x}{\cosh 2y+\cos 2x}$ by Milne-Thomson method.	10	4	4	1,2
	(OR)				
b.	Find the bilinear transformation which maps the points 1, i, –1 onto the points 0, 1, ∞.	10	3	4	1,2
30. a.	Apply Cauchy's integral formula to evaluate $\int_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$, where c is |z|=3.	10	4	5	1,2
	(OR)				
b.	Apply contour integration to evaluate the integral $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$.	10	4	5	1,2

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