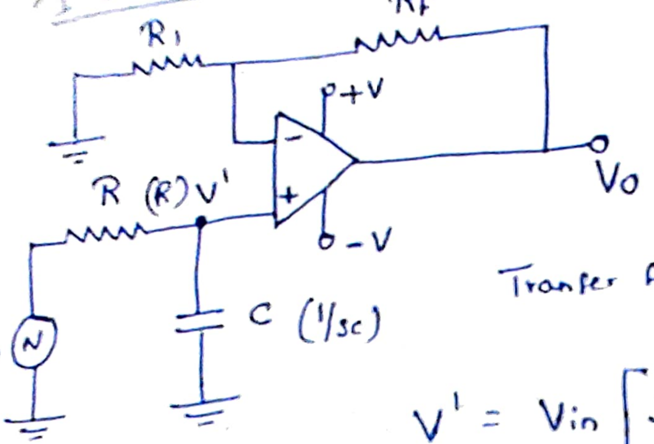
1st order Butterworth Low-Pass Filter



- First order has one pole

Negative Feed back, i/p to Non-inv. terminal.

Transfer function $H(s) = \dfrac{V_o(s)}{V_i(s)}$

$$V' = V_{in}\left[\frac{1/sC}{R + 1/sC}\right] = V_{in}\left[\frac{1}{1+RCs}\right]$$

$$V_{out} = V'\left[1 + \frac{R_f}{R_1}\right]$$

$$V_{out} = \underbrace{\left[1 + \frac{R_f}{R_1}\right]}_{A}\left[\frac{1}{1+RCs}\right] \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{A}{1+RCs}$$

This system has one pole

Root of this system $1 + RCs = 0$

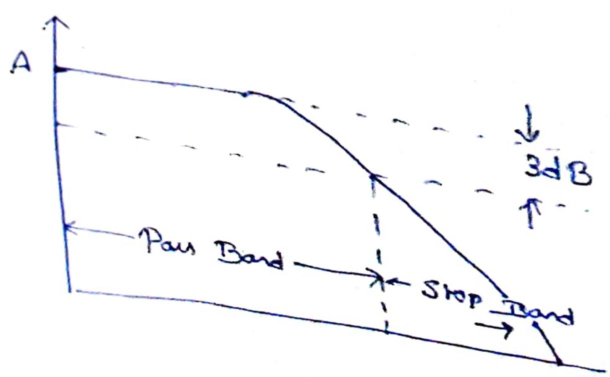
$$\boxed{s = -1/RC}$$

$s = j\omega$; $\omega_H = \dfrac{1}{RC}$

$$\boxed{H(j\omega) = \frac{A}{1 + j\left(\frac{\omega}{\omega_H}\right)} = \frac{A}{1 + j\left(f/f_H\right)}}$$

Magnitude. $|H(j\omega)| = \dfrac{A}{\sqrt{1 + (f/f_H)^2}}$.

$$\boxed{A = 1 + R_f/R_1}$$



$f < f_H$, $|H(j\omega)| \simeq A$

$f = f_H$, $|H(j\omega)| = A/\sqrt{2}$

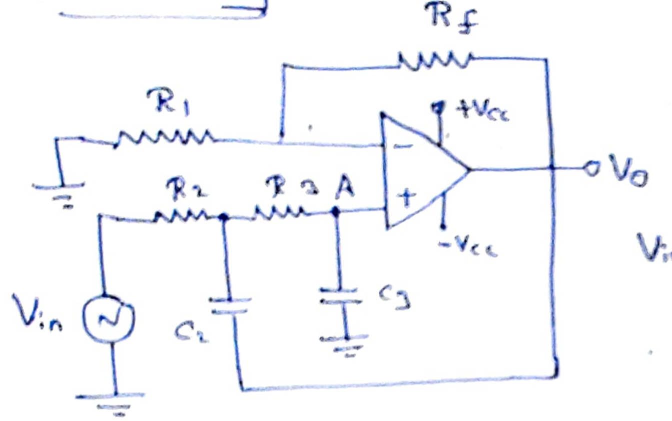
$= 0.707\,A$

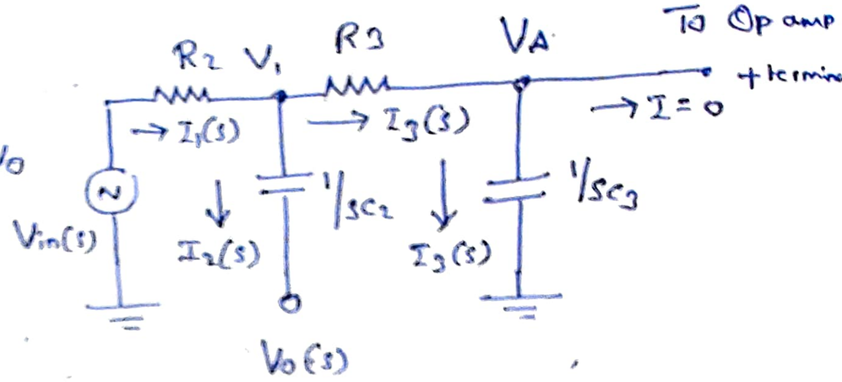
Second order Low-Pass Filter

Constant gain A_f from 0Hz to high cut-off freq f_h

* Frequency response of Filter must be very close to an ideal
* Non-inverting Opamp Configuration, Two RC-Networks

Circuit Diagram:





RC N/w in Laplace Domain

$$I_1 = I_2 + I_3$$

$$\frac{V_{in} - V_1}{R_2} = \frac{V_1 - V_o}{(1/sC_2)} + \frac{V_1 - V_A}{R_3} \rightarrow (1)$$

$$V_A = V_1\left[\frac{1/sC_3}{R_3 + 1/sC_3}\right]$$

$$V_A = \frac{V_1}{1 + sR_3C_3}$$

$$V_1 = V_A(1 + sR_3C_3) \rightarrow (2)$$

Sub (2) in (1)

$$\frac{V_{in} - V_A[1 + sR_3C_3]}{R_2} = \frac{V_A[1+sR_3C_3] - V_o}{(1/sC_2)} + \frac{V_A[1+sR_3C_3] - V_A}{R_3}$$

$$\frac{V_{in}}{R_2} + V_o(sC_2) = V_A\left[\frac{(1+sR_3C_3)}{R_2} + \frac{sC_2[1+sR_3C_3] + R_2[1+sR_3C_3] - R_2}{R_2R_3}\right]$$

$$\therefore (R_3V_{in} + V_o s R_2R_3C_2) = V_A\left[(1+sR_3C_3)(R_3 + R_2R_3sC_3 + R_2) - R_2\right]$$

$$V_A = \frac{R_3V_{in} + V_o s R_2R_3C_2}{\left[(1+sR_3C_3)(R_3 + R_2R_3C_2s + R_2) - R_2\right]} \rightarrow (3)$$

From Non-inverting mode

$$V_o = A_F V_A$$

$$A_F = 1 + \frac{R_f}{R_1} \quad ; \quad V_A - \text{the voltage at the non-inverting terminal.}$$

$$V_o = \cancel{V_A}\, A_F \left[\frac{R_3 V_{in} + V_o S R_2 R_3 C_2}{(1+SR_3C_3)(R_3+R_2R_3C_2S+R_2) - R_2} \right]$$

$$\frac{A_F R_3 V_{in}}{(1+SR_3C_3)(R_3+R_2R_3C_2S+R_2) - R_2} = V_o \left[1 - \frac{SR_2R_3C_2}{(1+SR_3C_3)(R_3+R_2R_3C_2S+R_2) - R_2} \right]$$

$$\therefore A_F R_3 V_{in} = V_o \left[(1+SR_3C_3)(R_3+R_2R_3C_2S+R_2) - R_2 - SR_2R_3C_2 \right]$$

$$\frac{V_o}{V_{in}} = \frac{A_F}{S^2 + \frac{(R_3C_3 + R_2C_3 + R_2C_2 - A_F R_2 C_2)S}{R_2R_3C_2C_3} + \frac{1}{R_2R_3C_2C_3}} \rightarrow (4)$$

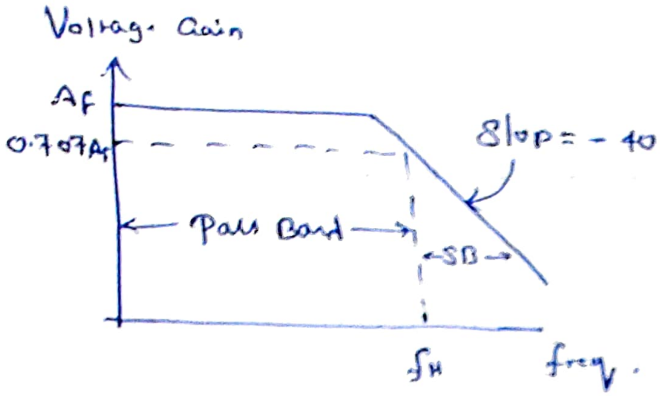
$$\frac{V_o(s)}{V_{in}(s)} = \frac{A}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

A - overall gain , ξ - damping of system ; ω_n = natural freq. of oscillation

$$\omega_n^2 = \frac{1}{R_2R_3C_2C_3} \quad ; \quad \cancel{\omega_H^2} =$$

$$(2\pi f_H)^2 = \frac{1}{R_2R_3C_2C_3}$$

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$



$$\frac{V_o}{V_{in}} = \left|\frac{V_o}{V_{in}}\right| \angle\phi$$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_f}{\sqrt{1+(f/f_H)^4}}$$

A_f = Gain of filter in pass Band

f = Input freq. in Hz

f_H = high cut-off freq. in Hz.

For $0 < f < f_H$, the gain is almost Constant.

At $f = f_H$, gain reduces to $0.707\,A_f$, 3dB down from A_f

At $f > f_H$. the gain decreases at a rate of 40dB/dec.