Fast Fourier Transform (FFT) :-

FFT is an algorithm that efficiently Computes DFT.

DFT → N^2 Computations.

FFT → $N.\log(N)$ Computations.

The DFT of a sequence $x(n)$ of length, N is given by a Complex Valued sequence $[x(k)]$.

$$x(k) = \sum_{n=0}^{N-1} x(n) . e^{-j2\pi nk/N} \quad ; \; 0 \leq k \leq N-1$$

$$\boxed{W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}}$$

Let $W_N = e^{-j2\pi/N}$.

$$\therefore x(k) = \sum_{n=0}^{N-1} x(n) . W_N^{-nk} \quad ; \; 0 \leq k \leq N-1.$$

Similarly, IDFT becomes ;

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \, W_N^{-nk} \quad ; \; 0 \leq n \leq N-1.$$

FFT → N Multiplications. N−1 Additions.

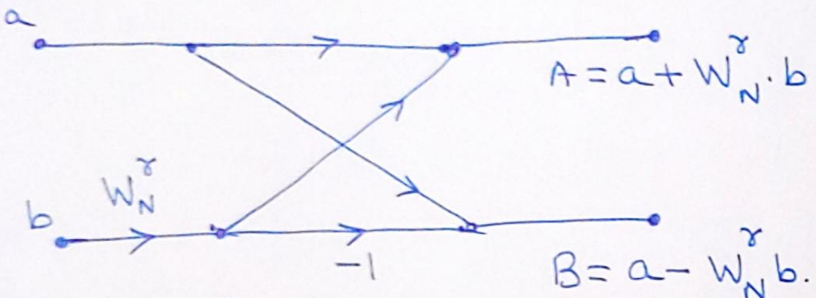
DFT → N^2 Multiplications. $N(N-1)$ Additions.

Symmetry Property :- $W_N^{k+N/2} = -W_N^k$.

Periodicity Property :- $W_N^{k+N} = W_N^k$.

DIT-FFT:-

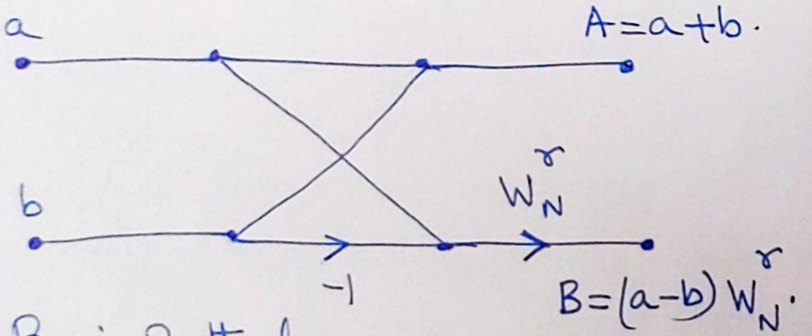
Input data is shuffled in bit reversed order



Basic Butterfly Flow Graph — DIT FFT Algorithm

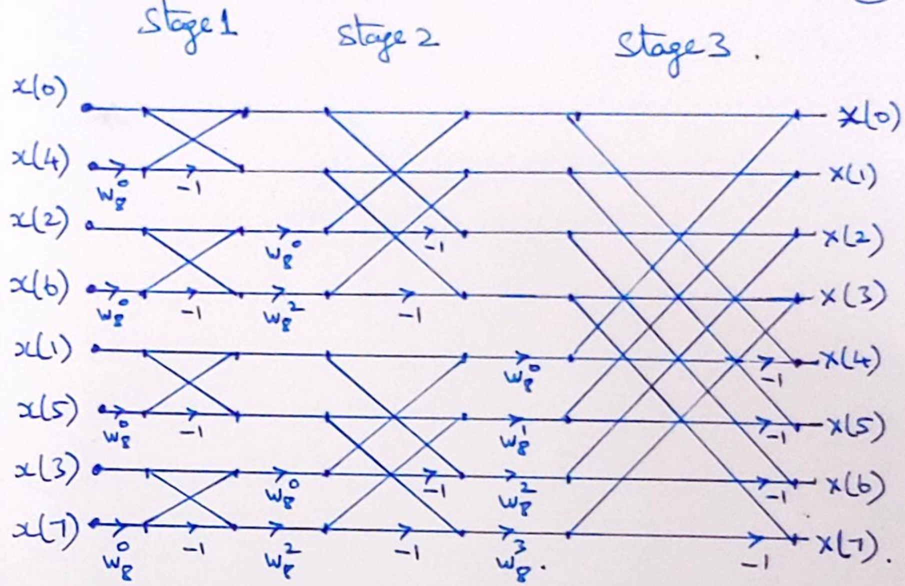
DIF-FFT:-

Input sequence $x(n)$ appears in natural order, while output $X(k)$ appears in the bit reversed order.

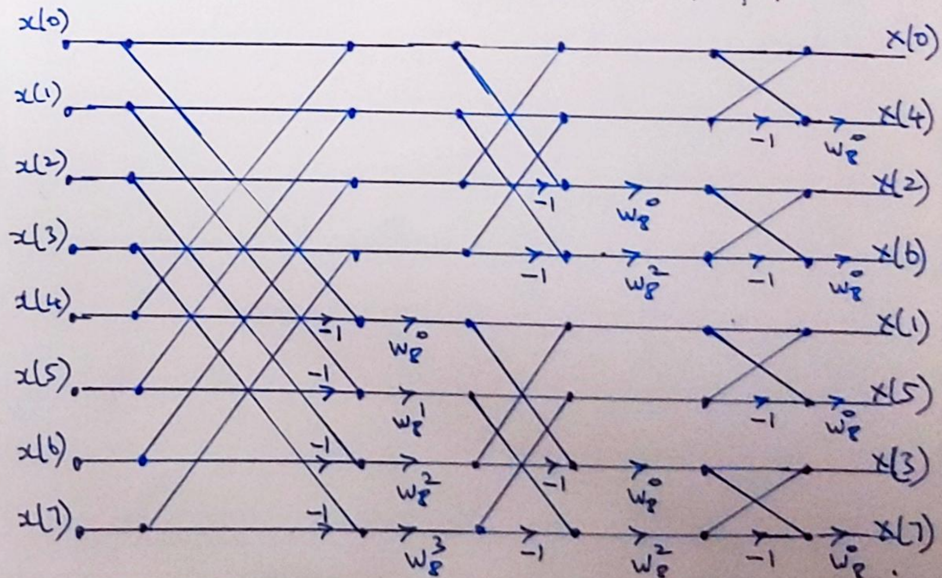


Basic Butterfly for DIF-FFT.

8-POINT DIT FFT.



8-POINT DIF FFT.



1. Given $x(n) = 2^n$ and $N = 8$, Find $X(k)$ Using DIT FFT algorithm.

Solution :-

Given $x(n) = 2^n$ and $N = 8$.

$x(0) = 1$; $x(1) = 2$; $x(2) = 4$; $x(3) = 8$;

$x(4) = 16$; $x(5) = 32$; $x(6) = 64$; $x(7) = 128$.

$\Rightarrow x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$.

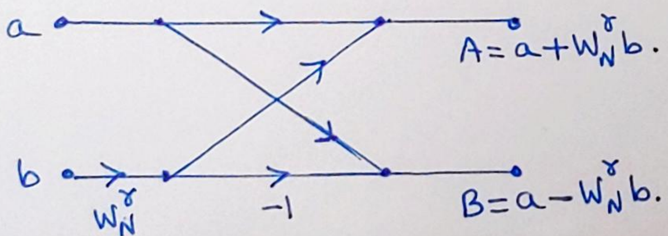
$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}.$$

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)0} = 1$$

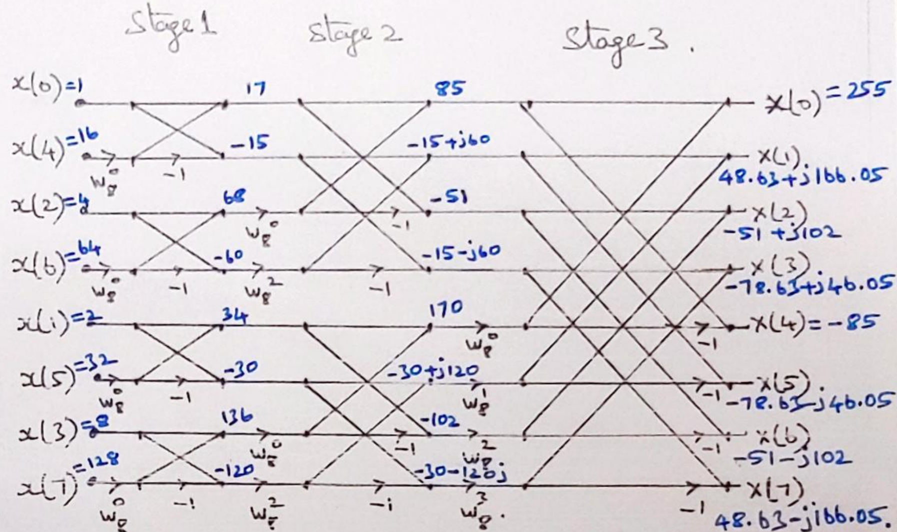
$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)1} = e^{-j\pi/4} = \cos\pi/4 - j\sin\pi/4 = 0.707 - j0.707.$$

$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)2} = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = 0 - j = -j.$$

$$W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)3} = e^{-j(3\pi/4)} = \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4} = -0.707 - j0.707.$$



8-POINT DIT FFT.



Computing for X(1) :- (stage 3)

$$(-30+j120)[0.707-j0.707] + [-15+j60].$$

$$= -21.21 + j21.21 + j84.84 + 84.84 - 15 + j60$$

$$= 48.63 + j166.05.$$

Computing for X(7):- (stage 3)

$$[(-30-120j)(-0.707-j0.707)(-1)] + [-15-j60]$$

$$= -[21.21 + j21.21 + j84.84 - 84.84] - 15 - j60$$

$$= 63.63 - j106.05 - 15 - j60 \Rightarrow 48.63 - j166.05.$$

$\Rightarrow X[k] = \{255, 48.63+j166.05, -51+j102, -78.63+j46.05, -85, -78.63-j46.05, -51-j102, 48.63-j166.05\}$.

2. Given $x(n) = 2^n$ and $N = 8$; Find $X(k)$ using DIF FFT algorithm.

Solution:-

Given $x(n) = 2^n$ and $N = 8$.

$x(0) = 1$; $x(1) = 2$; $x(2) = 4$; $x(3) = 8$;

$x(4) = 16$; $x(5) = 32$; $x(6) = 64$; $x(7) = 128$.

$x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$.

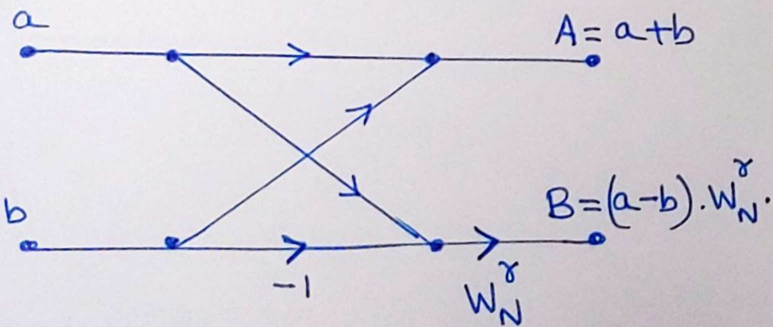
$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)0} = 1.$$

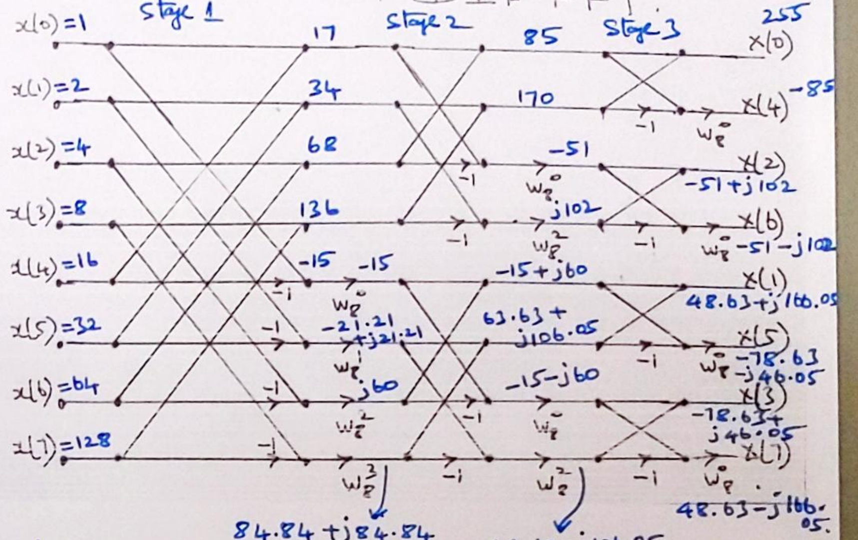
$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)1} = e^{-j\pi/4} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = 0.707 - j0.707.$$

$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)2} = e^{-j\pi/2} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = 0 - j = -j$$

$$W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)3} = e^{-j(3\pi/4)} = \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4} = -0.707 - j0.707.$$



8-POINT DIF FFT



Computing for X(1) :- stage 3.

$(-15 + j60) + (63.63 + j106.05) \Rightarrow 48.63 + j166.05.$

Computing for X(5) :- stage 3.

$[(63.63 + j106.05)(-1) + (-15 + j60)] \cdot W_8^0.$

$\Rightarrow -78.63 - j46.05.$

$\Rightarrow X[k] = \{255, 48.63 + j166.05, -51 + j102, -78.63 + j46.05, -85, -78.63 - j46.05, -51 - j102, 48.63 - j166.05\}.$