Frequency Sampling method :-

→ choose the ideal (desired) frequency response $H_d(e^{j\omega})$.

→ Compute $H(K)$ ⇒ Sample $H_d(e^{j\omega})$ & obtain the DFT sequence, $H(K)$.

→ $$H(K) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi K}{N}}$$

for $K = 0, 1, \ldots (N-1)$.

→ obtain Impulse response of the filter. $h(n)$.

when N is odd

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N-1}{2}} \mathrm{Re}\left[H(K)\, e^{\frac{j2\pi Kn}{N}}\right]\right]$$

when N is even

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N}{2}-1} \mathrm{Re}\left[H(K)\, e^{\frac{j2\pi Kn}{N}}\right]\right]$$

$\{H(N/2) = 0\}$

→ Determine $H(z)$. & Realize the filter design.

1. Determine the coefficients of a linear phase FIR filter of length N=15, which has symmetric unit sample response and frequency response,

$$H\left[\frac{2\pi K}{15}\right] = 1 \; ; \quad \text{for } K=0,1,2,3$$
$$= 0.4 \; ; \quad \text{for } K=4$$
$$= 0 \; ; \quad \text{for } K=5,6,7.$$

Solution:- $\alpha = \frac{N-1}{2}$; $N=15$.

1. Frequency response $\boxed{\alpha=7}$

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\omega\alpha} & K=0,1,2,3. \\ (0.4)\, e^{-j\omega\alpha} & K=4. \\ 0 & K=5,6,7. \end{cases}$$

2. Determine the sequence, H(K).

w.r.t. $H(K) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi K}{N}}$ $\;K=0,1,\ldots(N-1)$.

$$\therefore H(K) = \begin{cases} (1) \cdot e^{-j(7)\frac{2\pi K}{15}} & K=0,1,2,3 \\ (0.4)\, e^{-j(7)\frac{2\pi K}{15}} & K=4 \\ 0 & K=5,6,7 \end{cases}$$

3. obtain the impulse response of the filter,

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N = odd N = 15.

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N-1}{2}} Re\left[H(K).e^{\frac{j2\pi nK}{N}}\right]\right]$$

$$= \frac{1}{15}\left[H(0) + 2\sum_{K=1}^{7} Re\left[H(K).e^{\frac{j2\pi nK}{15}}\right]\right]$$

$$h(n) = \frac{1}{15}\left[H(0) + 2\sum_{K=1}^{3} Re\left(H(K)e^{\frac{j2\pi nK}{15}}\right) + 2 Re\, H(K)e^{\frac{j2\pi nK}{15}}\right]$$

$$h(n) = \frac{1}{15}\left[1 + 2\sum_{K=1}^{3} Re\left[e^{-j(7)\frac{2\pi K}{15}}.e^{\frac{j2\pi nK}{15}}\right] + 2 Re\left[0.4.e^{\frac{-j(7)2\pi(4)}{15}}.e^{\frac{j2\pi nK}{15}}\right]\right].$$

$$h(n) = \frac{1}{15}\left[1 + 2\sum_{K=1}^{3} Cos\frac{2\pi K(n-7)}{15} + 0.8\, Cos\frac{8\pi(n-7)}{15}\right]$$

$$h(n) = \frac{1}{15}\left[1 + 2\, Cos\frac{2\pi(n-7)}{15} + 2\, Cos\frac{4\pi(n-7)}{15} + 2\, Cos\frac{6\pi(n-7)}{15} + 0.8\, Cos\frac{8\pi(n-7)}{15}\right]$$

4. Transfer function, $H(z)$.

$$H(z) = \sum_{n=0}^{N-1} h(n).z^{-n} \Rightarrow \sum_{n=0}^{14} h(n).z^{-n}.$$

$$\Rightarrow \left[h(0).z^{0} + h(1).z^{-1} + h(2).z^{-2} + \dots h(14).z^{-14} \right].$$

w.k.t according to symmetry conditions,

$$h(n) = h(N-1-n).$$

$n=0$; $h(0) = h(14) = -0.0141$

$n=1$; $h(1) = h(13) = -0.0019$

$n=2$; $h(2) = h(12) = 0.04$

$n=3$; $h(3) = h(11) = 0.0122$

$n=4$; $h(4) = h(10) = -0.0914$

$n=5$; $h(5) = h(9) = -0.0181$.

$n=6$; $h(6) = h(8) = 0.3130$

$n=7$; $h(7) = 0.52$.

$$\therefore H(z) = -0.0141(1+z^{-14}) - 0.0019(z^{-1}+z^{-13}) + 0.04(z^{-2}+z^{-12}) + 0.0122(z^{-3}+z^{-11}) - 0.0914(z^{-4}+z^{-10}) - 0.0181(z^{-5}+z^{-9}) + 0.3130(z^{-6}+z^{-8}) + 0.52z^{-7}.$$

w.k.T. $H(z) = \frac{Y(z)}{X(z)}$.

$$Y(z) = H(z).X(z).$$

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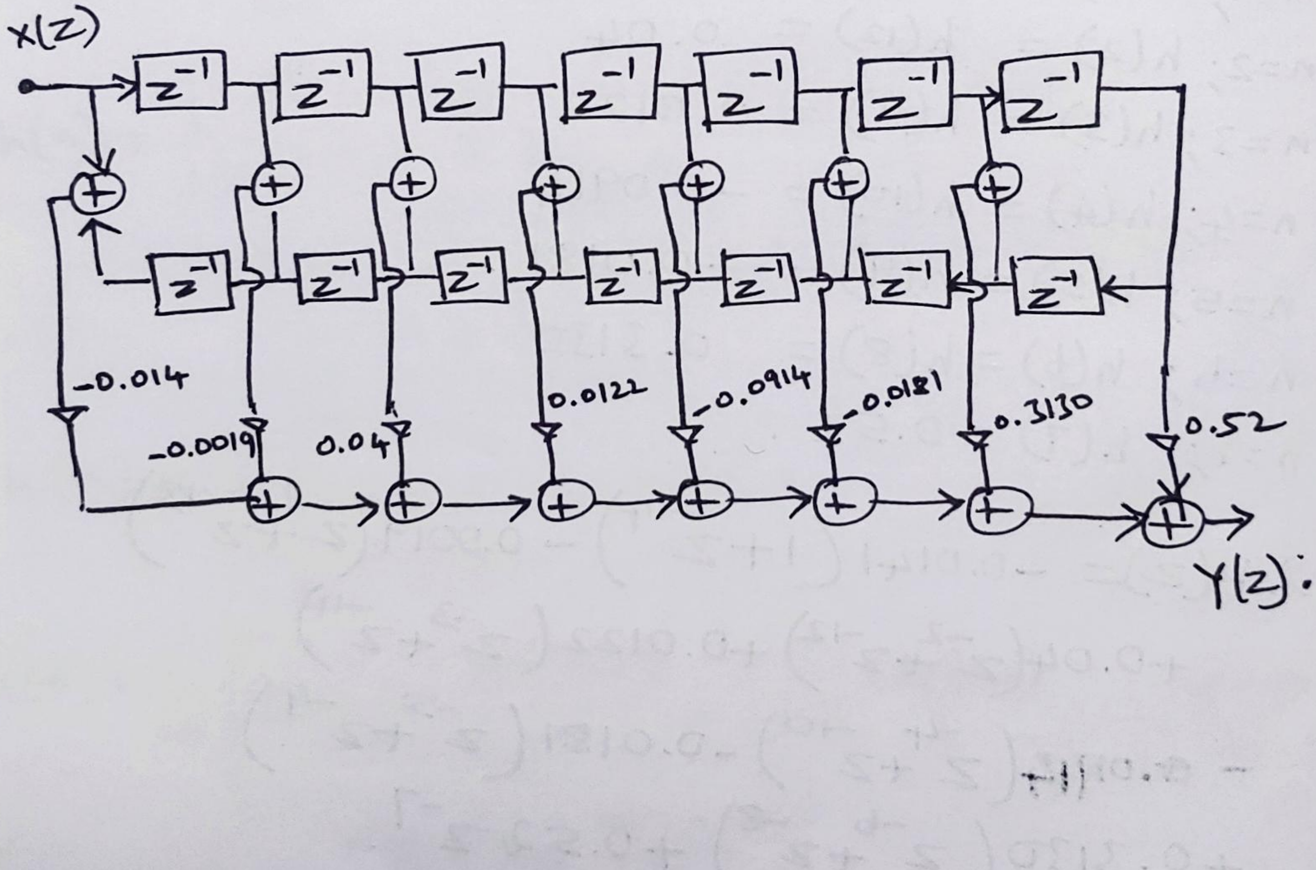
$$Y(z) = -0.0141(1+z^{-14}).X(z)$$
$$-0.0019(z^{-1}+z^{-13}).X(z)+0.04(z^{-2}+z^{-12}).X(z)$$
$$+0.0122(z^{-3}+z^{-11}).X(z)-0.0914(z^{-4}+z^{-10}).X(z)$$
$$-0.0181(z^{-5}+z^{-9}).X(z)+0.3130(z^{-6}+z^{-8}).X(z)$$
$$+0.52\,z^{-7}.X(z).$$

Realization of FIR Filter structure.



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1. Determine the sequence $x_3(n)$, corresponding to the circular convolution of the sequences, $x_1(n)$ & $x_2(n)$ by DFT & IDFT.

Solution:- $x_1(n) = (\underset{\uparrow}{2}, 1, 2, 1)$; $x_2(n) = (\underset{\uparrow}{1}, 2, 3, 4)$.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}.$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}.$$

DFT of $x_1(n)$:-

$$X_1(k) = \sum_{n=0}^{3} x_1(n) \cdot e^{\frac{-j2\pi nk}{N}} \qquad k = 0, 1, 2, 3$$

$$= 2 \cdot e^{0} + 1 e^{\frac{-j\pi k}{2}} + 2 e^{-j\pi k} + 1 e^{-j\frac{3\pi}{2}k}$$

$\Rightarrow X_1(0) = 6$; $X_1(1) = 2 + (-j) + 2(-1) + (-j(-1))$

$= 0.$

$X_1(2) = 2$; $X_1(3) = 0.$

DFT of $x_2(n)$:-

$$X_2(k) = \sum_{n=0}^{3} x_2(n) e^{\frac{-j2\pi nk}{N}} \qquad k = 0, 1, 2, 3$$

$$= 1 + 2 e^{-j\pi k/2} + 3 e^{-j\pi k} + 4 \cdot e^{-j\frac{3\pi}{2}k}$$

$X_2(0) = 10$; $X_2(1) = -2 + j2$; $X_2(2) = -2$

$X_3(2) = -2 - j2.$

$X_3(k) = X_1(k) \cdot X_2(k)$.

$X_3(0) = 60$; $X_3(1) = 0$; $X_3(2) = -4$

$X_3(3) = 0$.

IDFT of $X_3(k)$ is :

$$x_3(n) = \frac{1}{N} \sum_{k=0}^{3} X_3(k) \cdot e^{j2\pi nk/N} \qquad n = 0,1,2,3.$$

$$= \frac{1}{4}\left(60\, e^{j2\pi n(0)/4} - 4\, e^{j2\pi n(2/4)}\right)$$

$$= \frac{1}{4}\left(60 - 4\, e^{j\pi n}\right)$$

$x_3(0) = 14$; $x_3(1) = 16$; $x_3(2) = 14$

$x_3(3) = 16$.

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}.$$

① Compute Circular Convolution of two sequences (25)

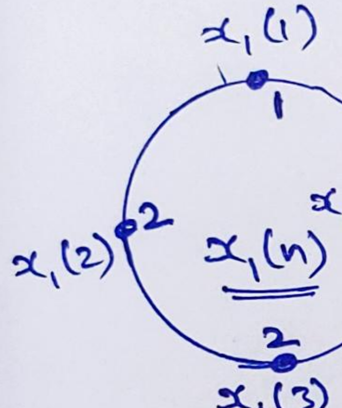
$x_1(n) = \{1, 1, 2, 2\}$ and $x_2(n) = \{1, 2, 3, 4\}$.

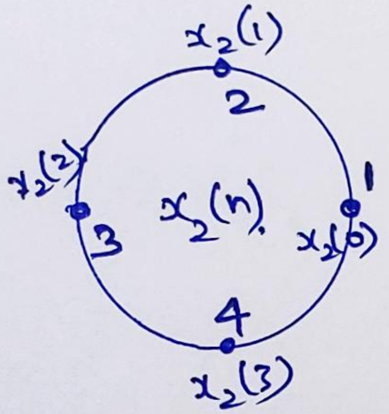
<u>Solution</u> :-

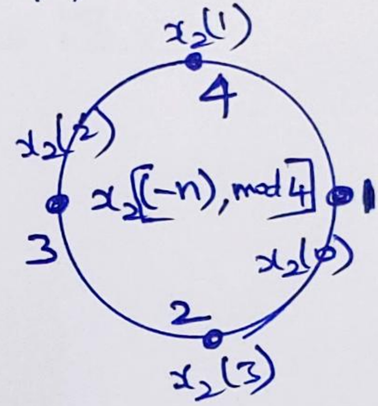
Circular Convolution of two sequences;

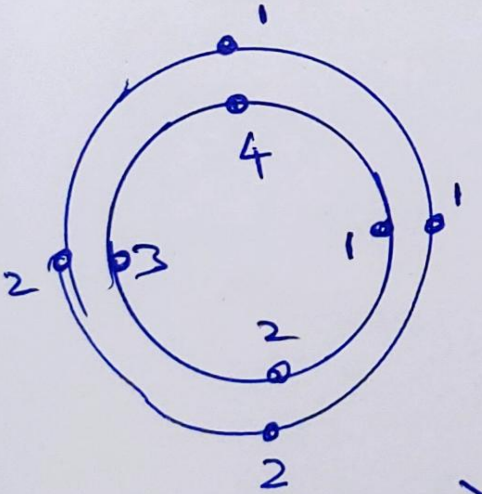
$$x_3(n) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2(m-n, (\text{mod } N)).$$

$m = 0, 1, \ldots N-1.$



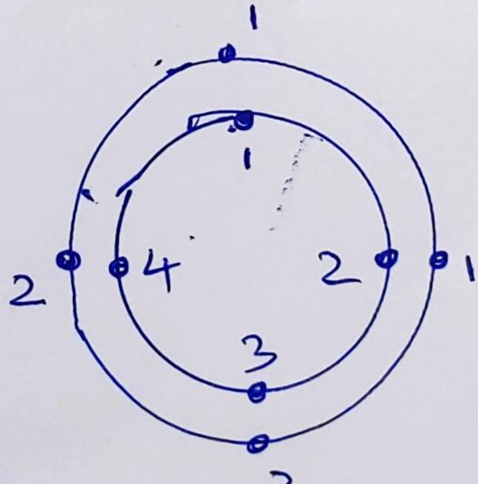






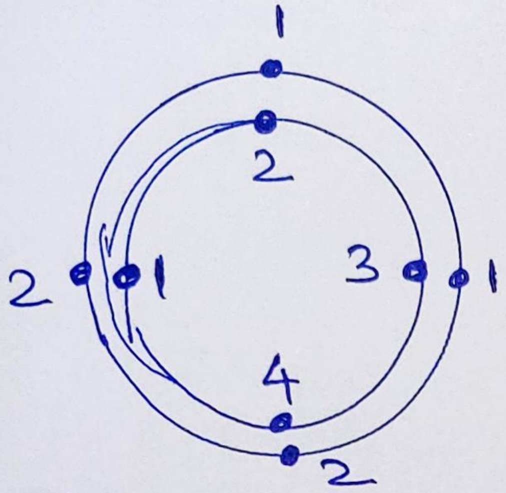
$x_1(n) \cdot x_2(-n, (\text{mod } 4))$

$= 15$



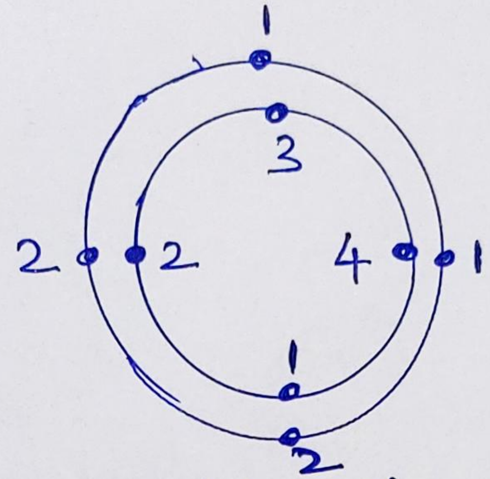
$x_1(n) \cdot x_2(1-n, (\text{mod } 4))$

$= 17$



$x_1(n) \cdot x_2(2-n, (\mathrm{mod}\,4))$

$\Rightarrow 15.$



$x_1(n) \cdot x_2(3-n, (\mathrm{mod}\,4))$

$= 13.$

$x_3(n) = \{15, 17, 15, 13\}.$

(1) Direct Form-I Realization :-

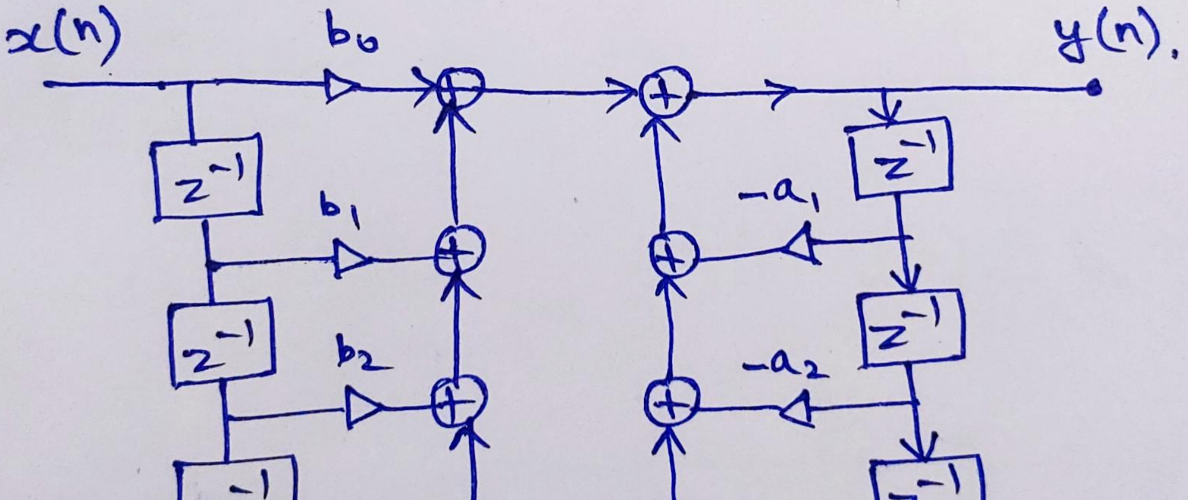
Difference equation,

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 . x(n-3) - a_1 y(n-1) - a_2 y(n-2) - a_3 . y(n-3).$$

Solution :-

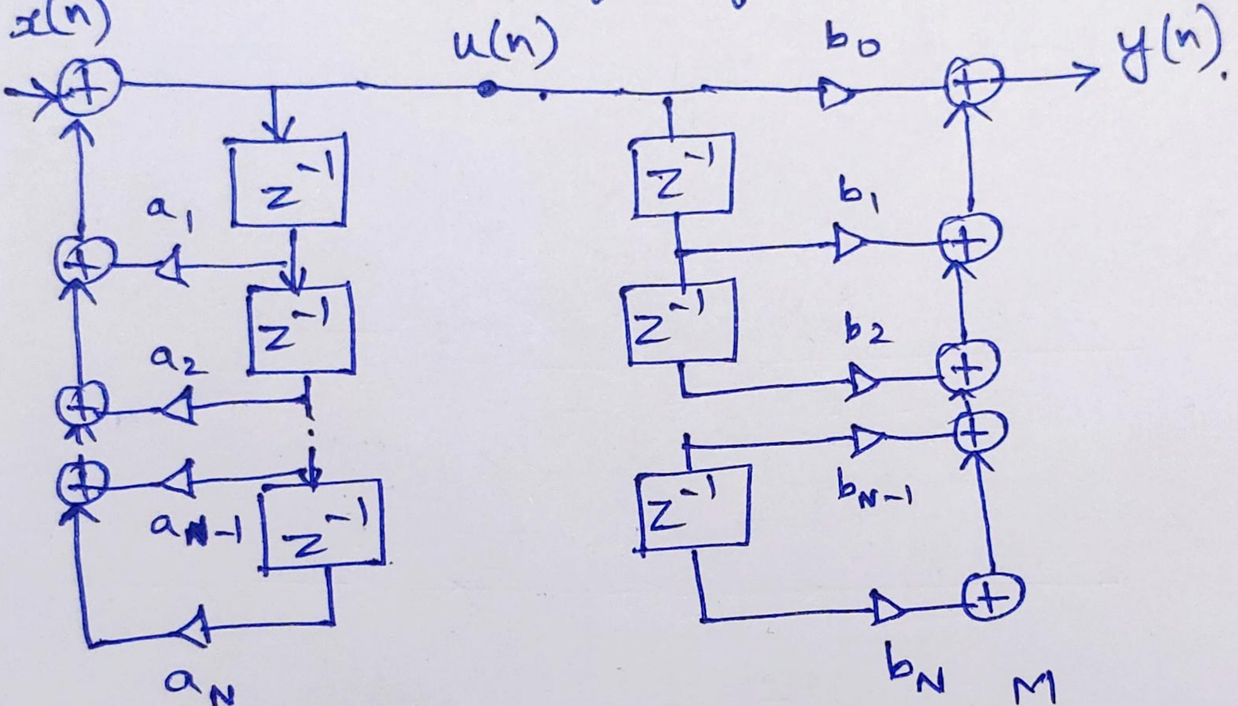
Taking z-Transform & Simplifying.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}.$$

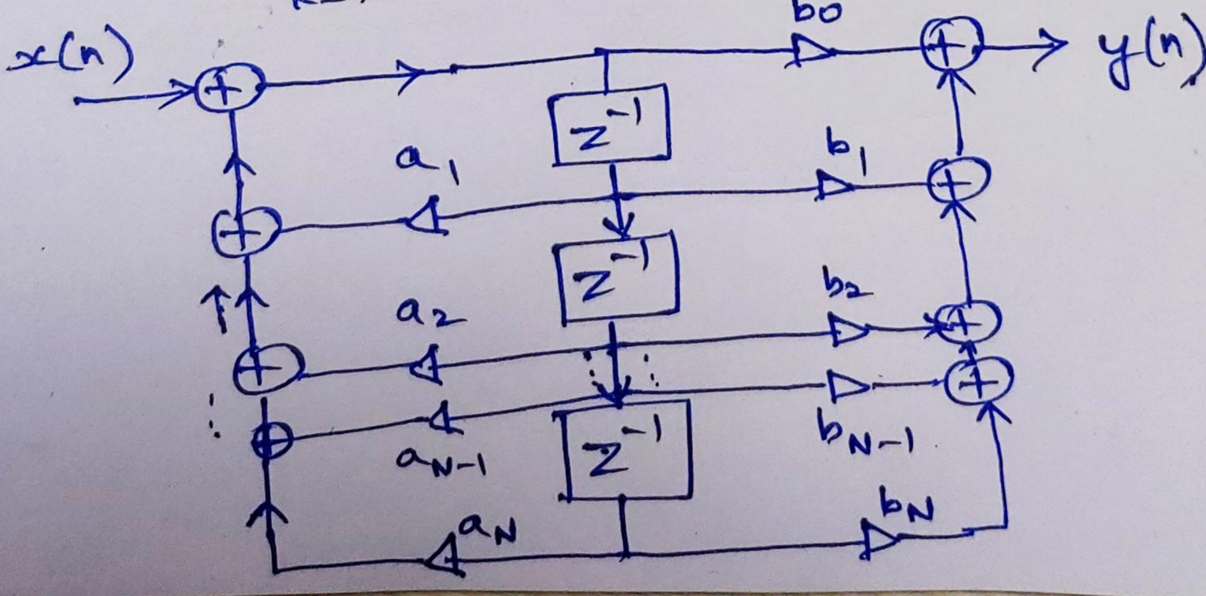


2) Direct Form II Realization :-

An Intermediate Sequence $u(n)$ is introduced to obtain the output of the filter.



$$U(z) = \frac{X(z)}{1 - \sum_{K=1}^{M} a_K \cdot z^{-K}} \; ; \quad Y(z) = U(z) \cdot \sum_{K=0}^{M} b_K \cdot z^{-K}.$$



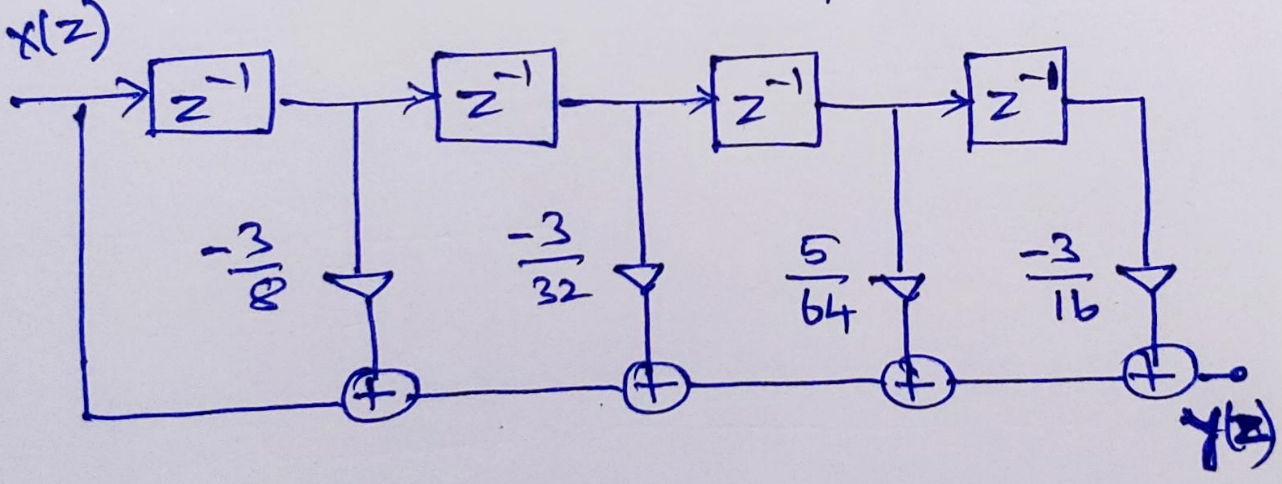
obtain direct form & Cascade form realizations for the transfer function of an FIR system given by,

$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right).$$

Solution :-

FIR Direct form Realization :-

$$H(z) = 1 - \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} + \frac{5}{64}z^{-3} - \frac{3}{16}z^{-4}.$$



Cascade Realization :-

$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$

$$\Rightarrow H_1(z) \cdot H_2(z).$$