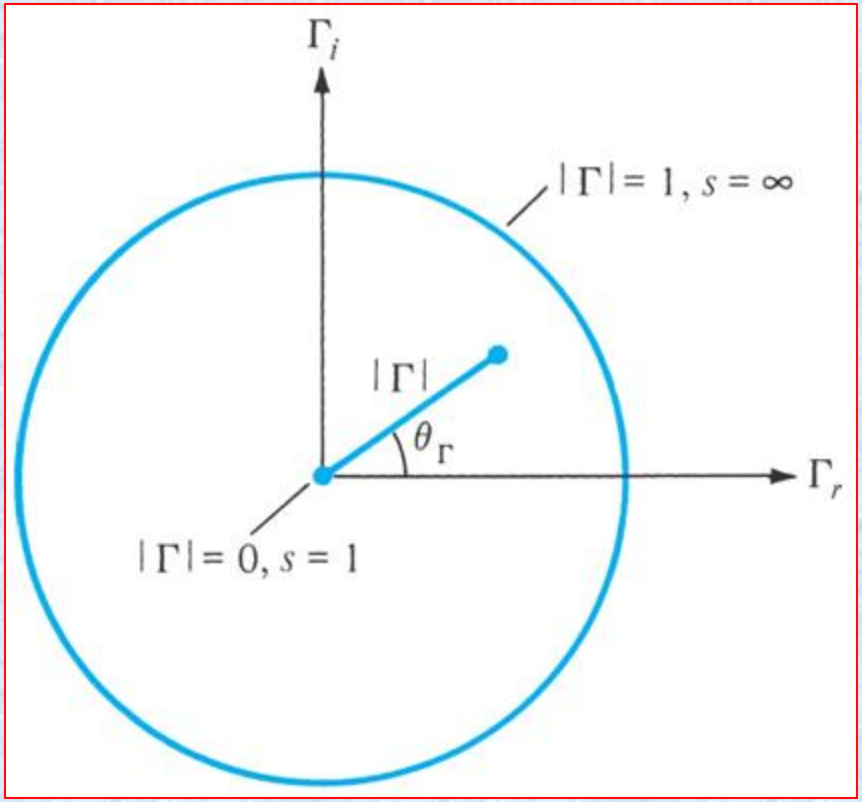
CHAPTER 11

Transmission Lines

11.5 The Smith Chart

- The Smith Chart is a graphical tool for analyzing transmission lines.
- The Smith Chart is made up of circles and arcs of circles.
- The circles are called "*constant resistance circles*"
- The arcs are "*constant reactance circles*"
- The combination of intersecting circles and arcs inside the chart allow us to locate the normalized impedance and then to find the impedance anywhere on the line.
- We will assume lossless transmission lines. ($Z_o=R_0$)

The Smith Chart



- The Smith Chart is constructed within a circle of unit radius ($|\Gamma| \leq 1$).

$$\boxed{\Gamma = |\Gamma| \angle \theta_\Gamma = \Gamma_r + j\Gamma_i}$$

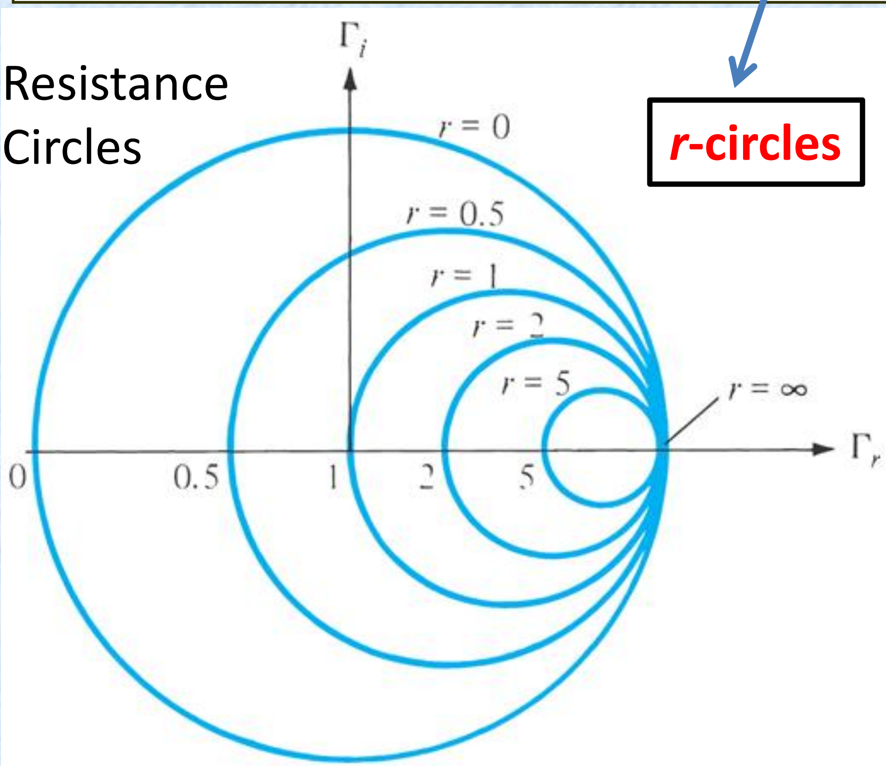
- Where Γ_r and Γ_i are the real and imaginary parts of Γ.

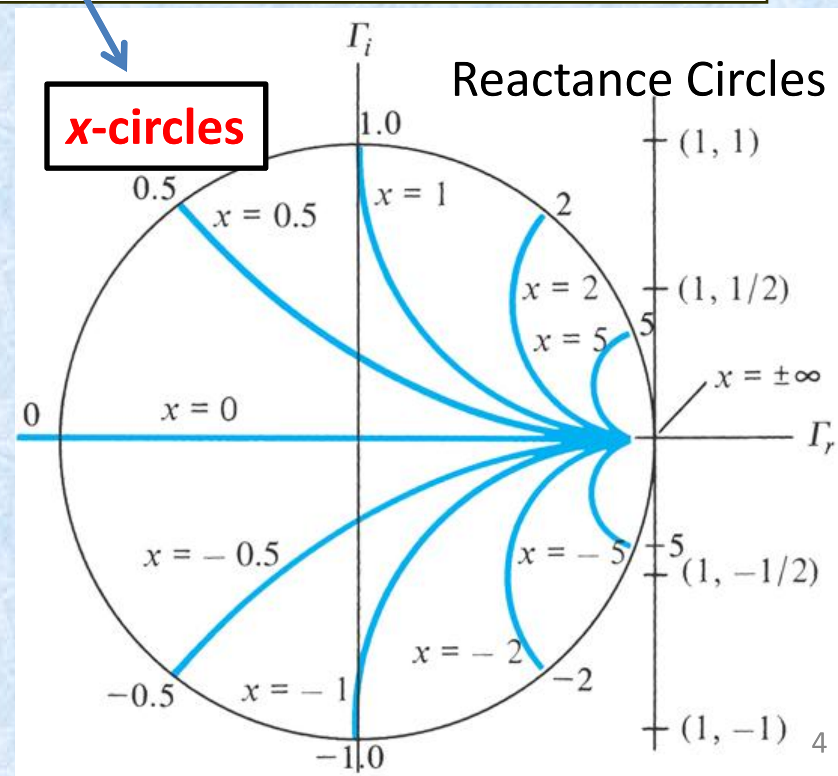
The Smith Chart

Smith Chart is normalized so that all impedances are normalized to the characteristic impedance Z_0. (Hence, it can be used for any line)

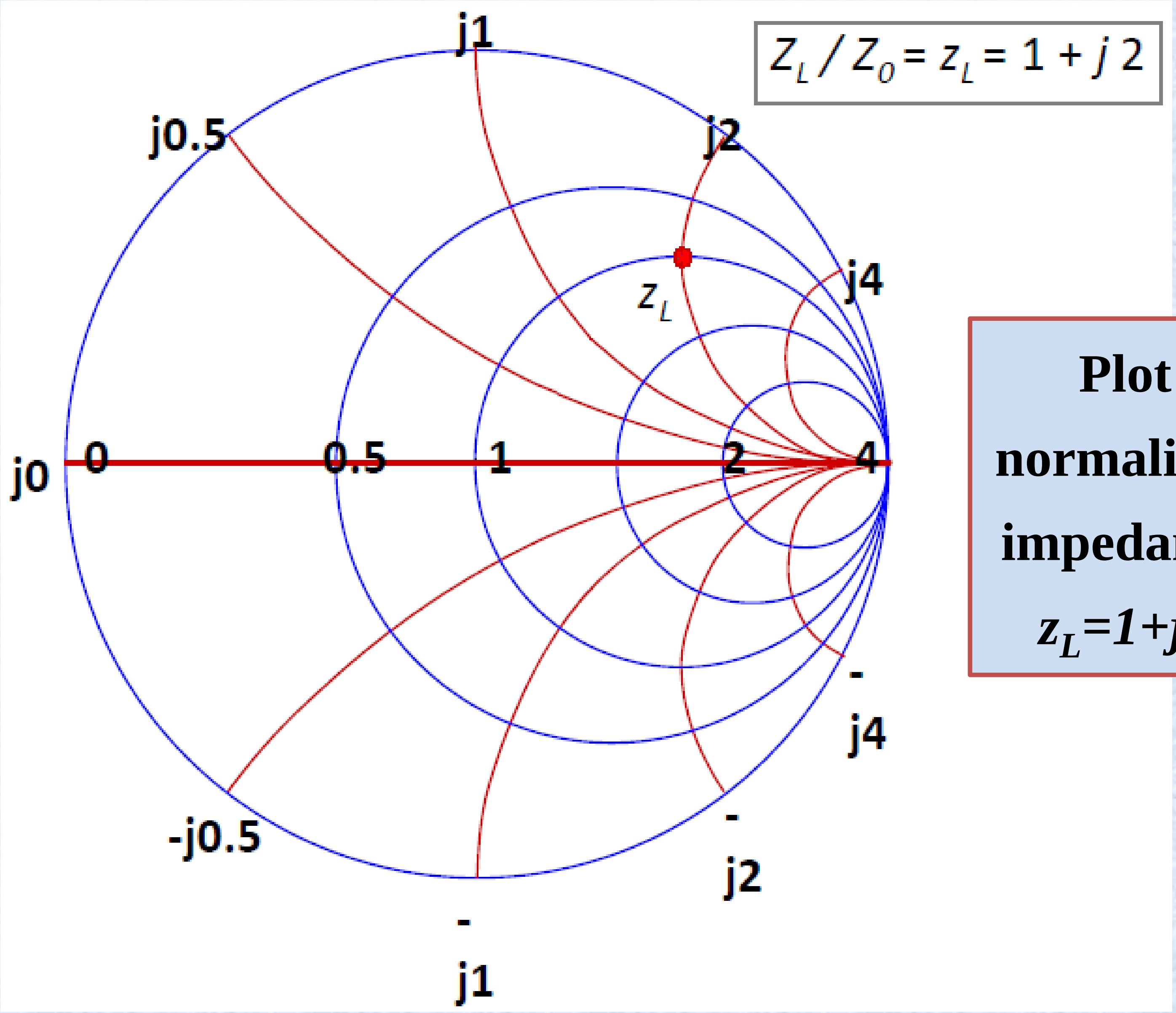
For the load impedance Z_L, the normalized impedance z_L is:

$$z_L = \frac{Z_L}{Z_0} = r + jx$$





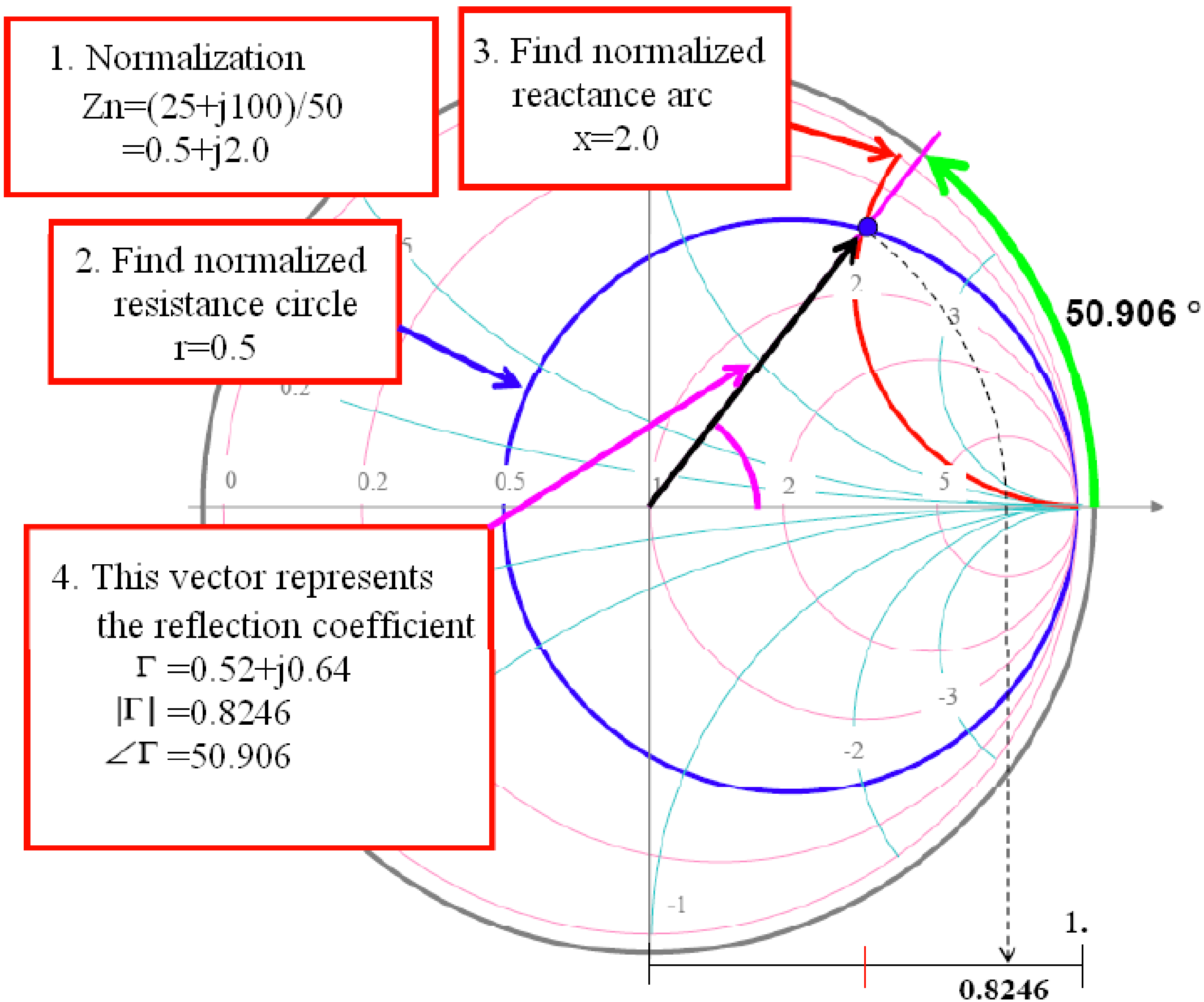
The Smith Chart



Plot normalized impedance z_L=1+j2

The Smith Chart

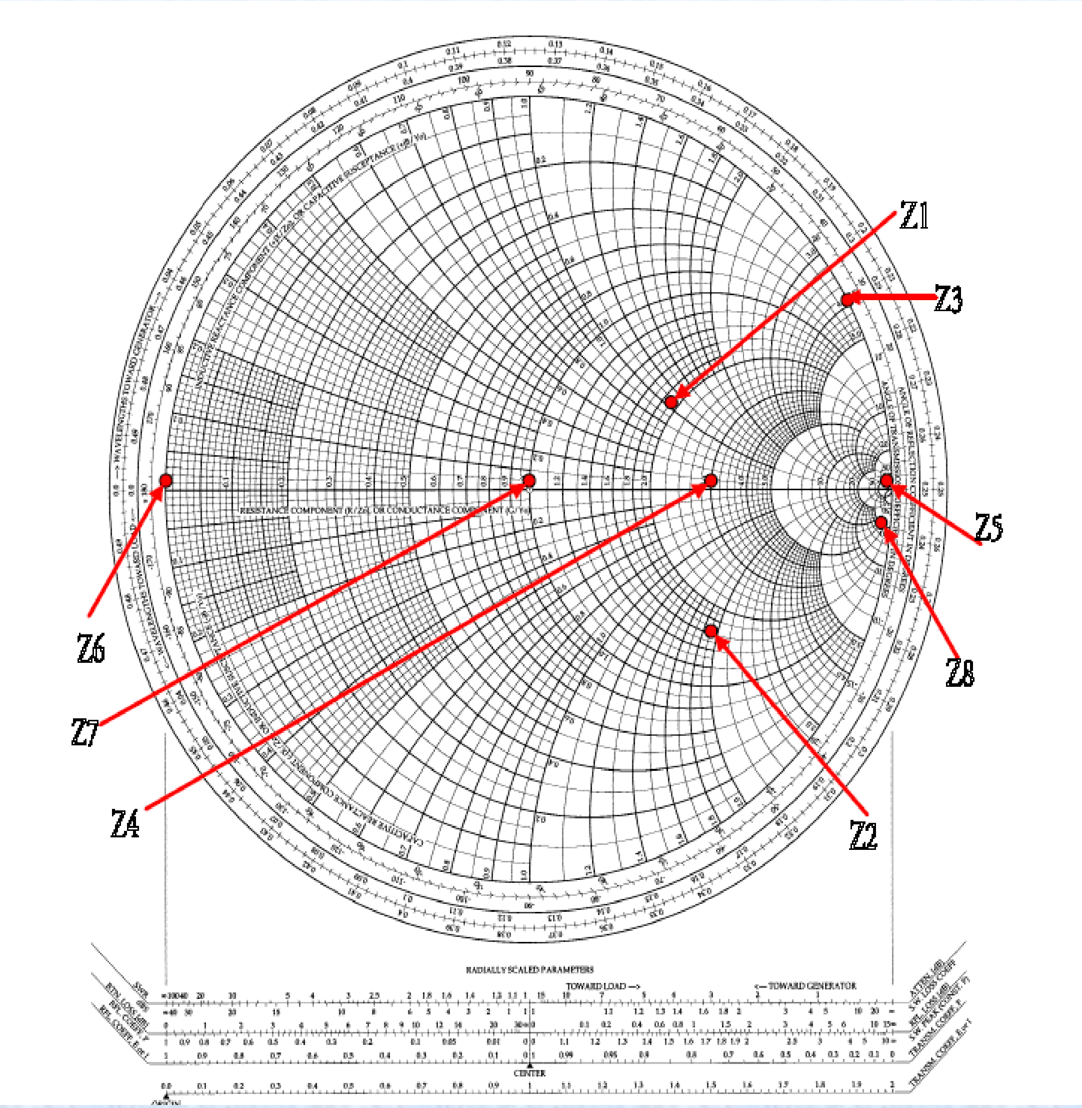
Example: Find Γ, given $Z = 25 + j\,100\,\Omega$ with $Z_0 = 50\,\Omega$



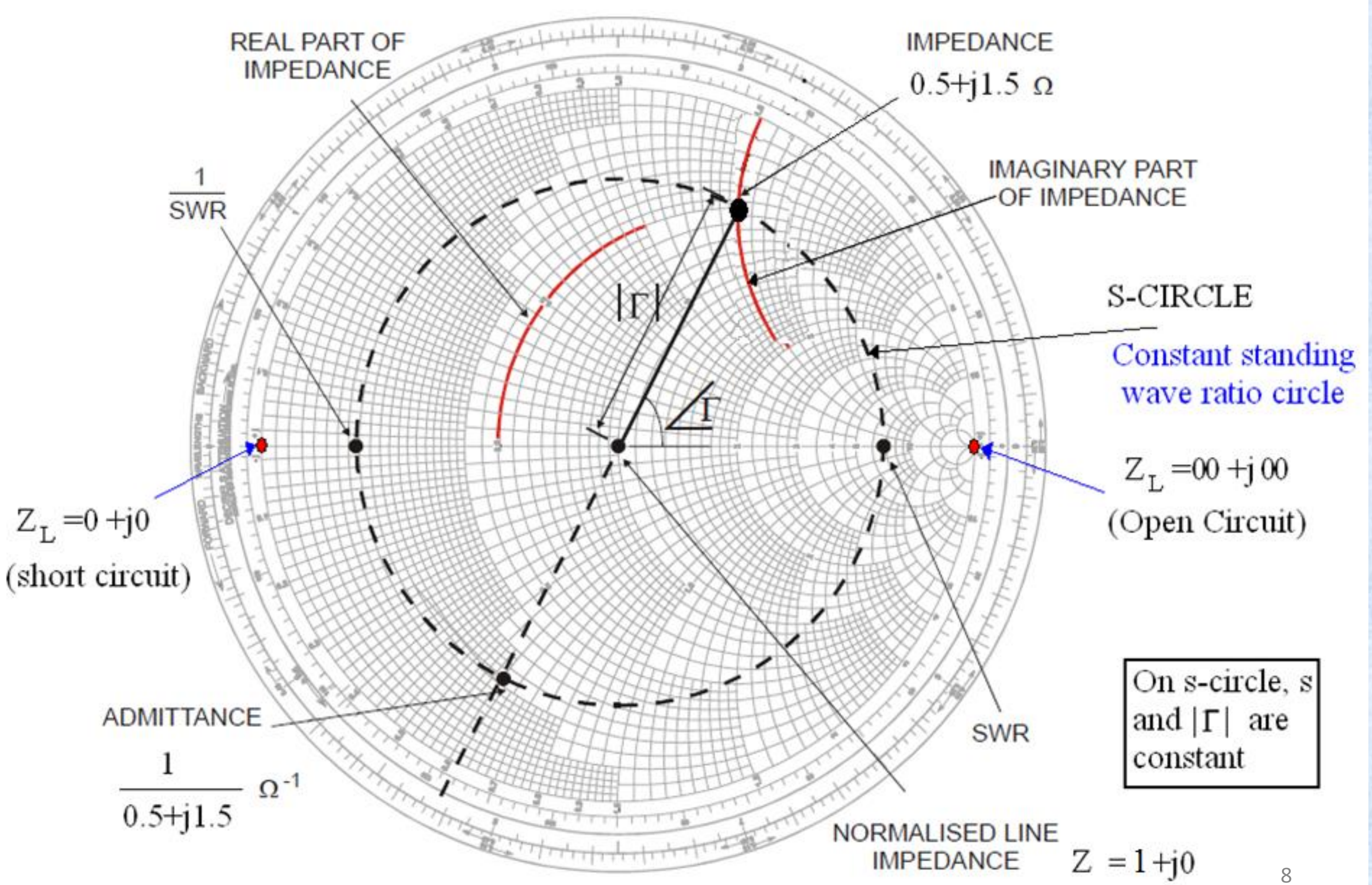
The Smith Chart

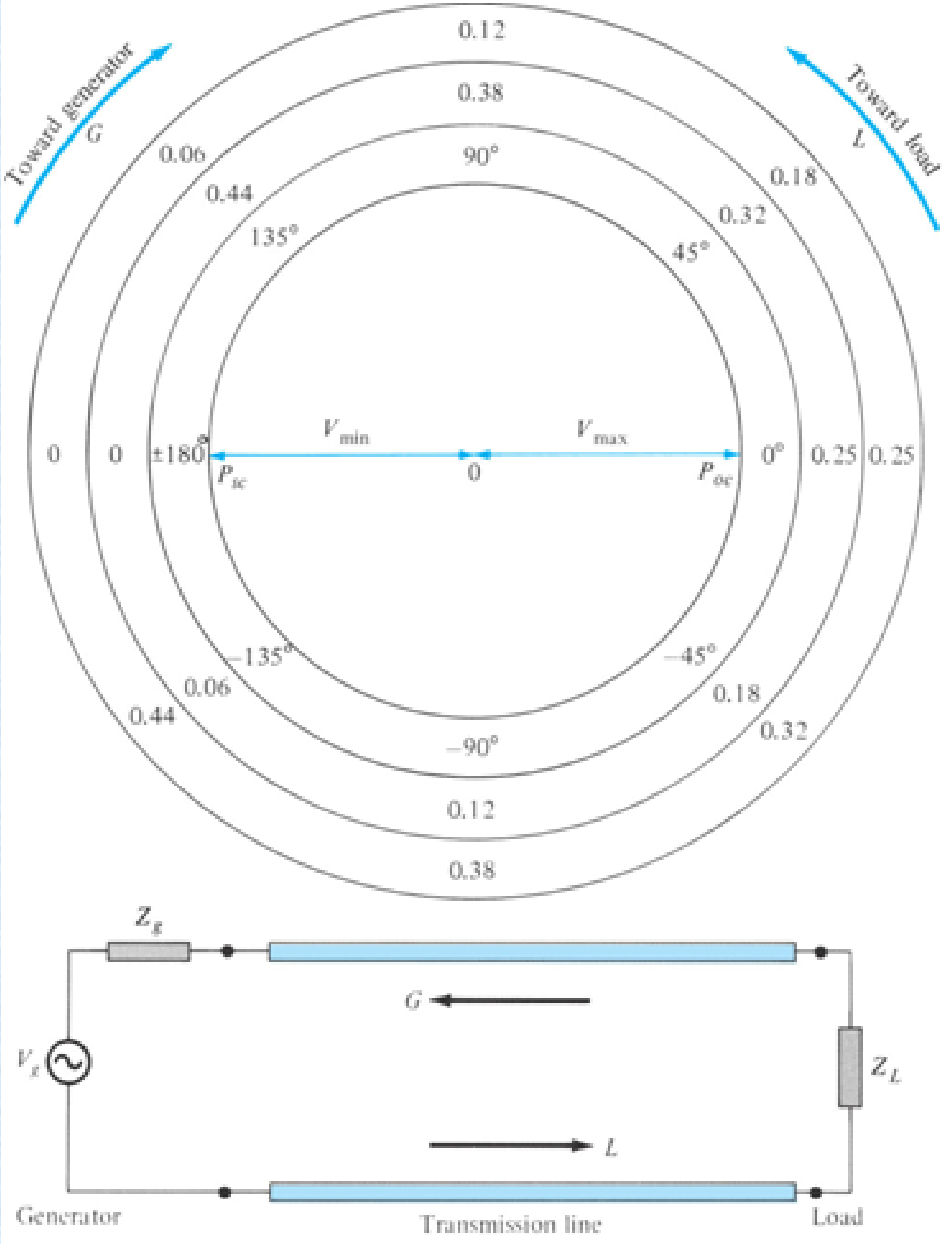
- Impedance:
  - Z1 = 100 + j50
  - Z2 = 75 -j100
  - Z3 = j200
  - Z4 = 150
  - Z5 = infinity (an open circuit)
  - Z6 = 0 (a short circuit)
  - Z7 = 50
  - Z8 = 184 -j900

- Normalized impedance (line impedance (50 Ohms) :
  - z1 = 2 + j
  - z2 = 1.5 -j2
  - z3 = j4
  - z4 = 3
  - z5 = infinity
  - z6 = 0
  - z7 = 1
  - z8 = 3.68 -j18



The Smith Chart





The Smith Chart

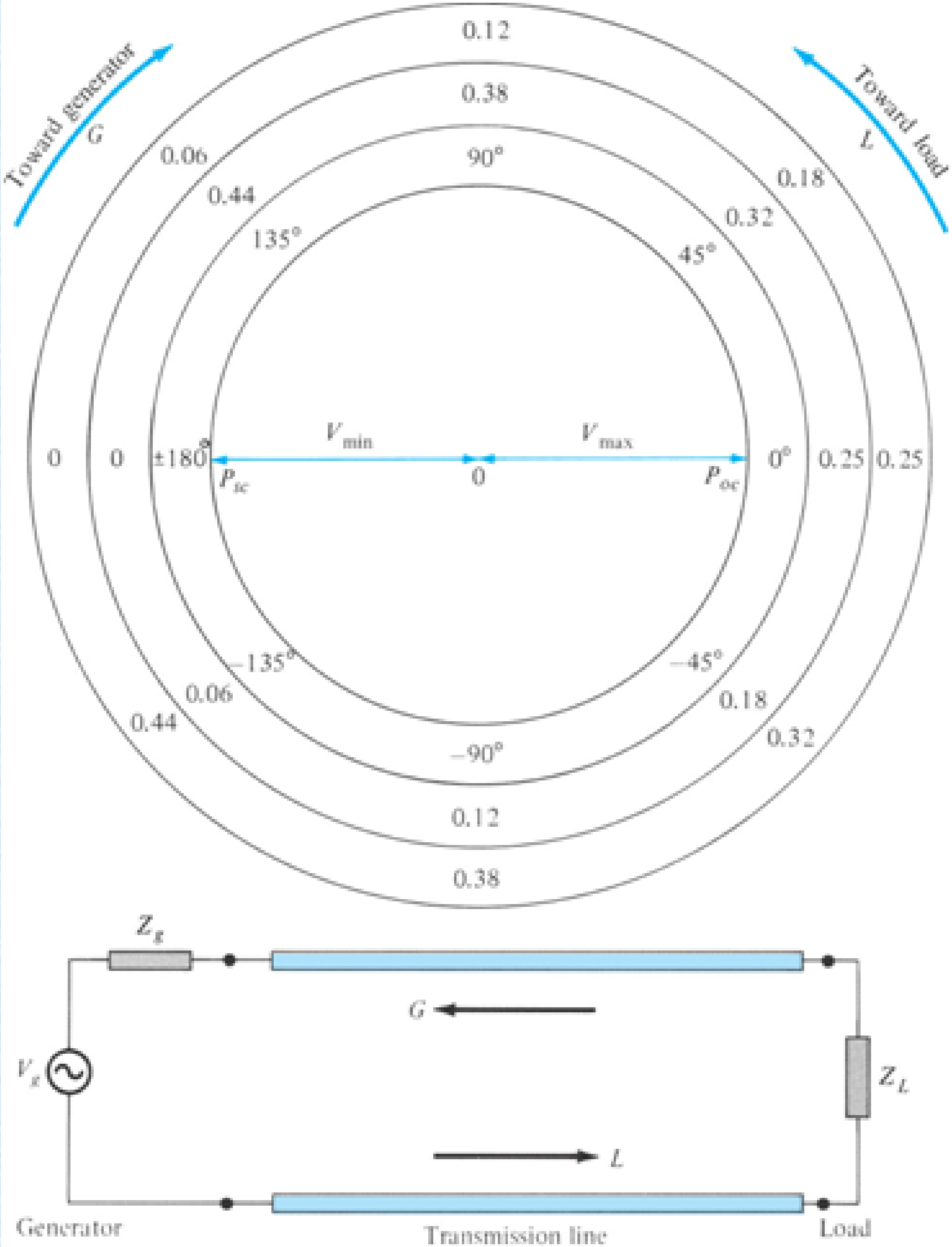
- A complete revolution (360^0) around Smith Chart represents a distance of ($\lambda/2$) on the line.

λ distance on the line corresponds to a 720^0 movement on the chart.

$\lambda \longrightarrow 720^0$

- ***Clockwise*** movement represents moving toward the **Generator**.

- ***Counter-clockwise*** movement represents moving toward the **Load**.



The Smith Chart

Three scales:

•***Outermost scale***: distance on line in terms of wavelength, moving toward Generator.

•***Middle scale***: distance on line in terms of wavelength, moving toward Load.

•***Innermost scale*** : to determine θ_Γ (in degrees)

Given Z_0, Z_L, λ and length of line, We can determine Z_{in}, Y_{in}, s, and Γ.

Example 11.4

- A lossless transmission line with Z_0=50 Ω is 30 m long and operates at 2 MHz. The line is terminated with a load Z_L=60+j40 Ω. If u=0.6c on the line, find
- (a) The reflection coefficient Γ
- (b) The standing wave ratio s
- (c) The input impedance.

Solution

Method 1 (Without the Smith Chart)

$$\text{(a) } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j40 - 50}{60 + j40 + 50} = \frac{10 + j40}{110 + j40} = 0.3523\angle 56^0$$

$$(b) \text{ s} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3523}{1 - 0.3523} = 2.088$$

Example 11.4- Solution continued

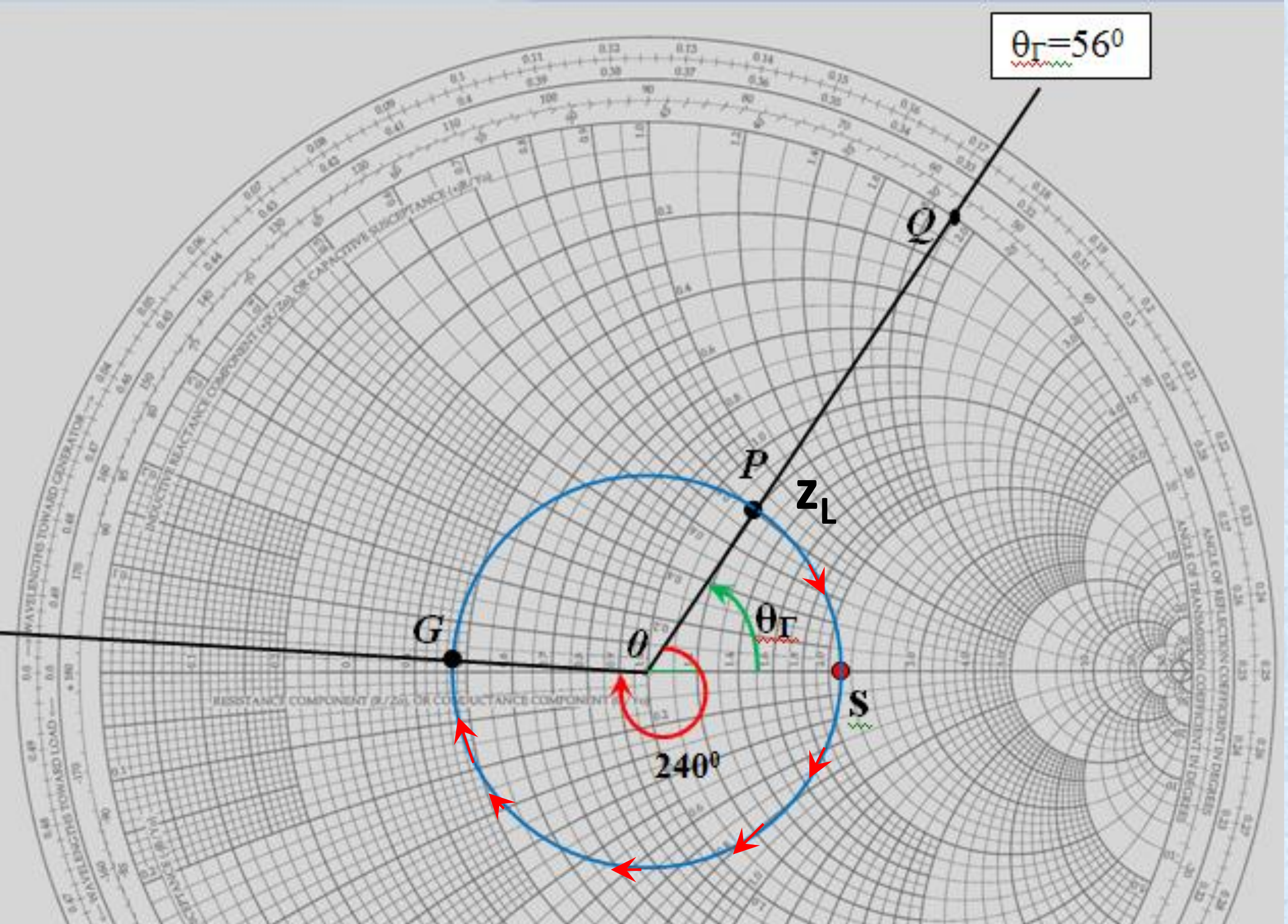
$$\text{(c) } \beta l = \frac{\omega}{u} l = \frac{2\pi(2\times10^6)}{0.6(3\times10^8)}(30) = \frac{2\pi}{3} = 120^0 \text{ (electrical length)}$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= 50\left[\frac{60 + j40 + j50\tan 120^0}{50 + j(60 + j40)\tan 120^0}\right]$$

$$= 23.97 + j1.35 = 24.01\angle 3.22^0$$

Example 11.4



(a) $Normalized: \; z_L = \dfrac{Z_L}{Z_0}$

$$= \frac{60 + j40}{50} = 1.2 + j0.8$$

Locate z_L at point P

$$|\Gamma_L| = \frac{0P}{0Q} = 0.3516$$

θ_Γ is between $0S$ and $0P$:

$\theta_\Gamma = \text{angle } P0S = 56^0$

$\rightarrow \boxed{\Gamma_L = 0.3516\angle 56^0}$

(b) To find standing wave ratio s, draw s-circle (radius 0P and center 0)

$\rightarrow$ s-circle meets Γ_r-axis at $\boxed{s = 2.1}$

(c) to find Z_{in}, express l interms of λ or in degrees.

$$\lambda = \frac{u}{f} = \frac{0.6(3\times10^8)}{2\times10^6} = 90 \text{ m}, \; l = 30 \text{ m} = \frac{30}{90}\lambda = \frac{\lambda}{3} \rightarrow \frac{720^0}{3} = 240^0$$

$\rightarrow$ Move clockwise from load toward Generator 240^0 on the s-circle from point P to point G. At G, We obtain: $z_{in} = 0.47 + j0.03$

Hence $Z_{in} = Z_0 z_{in} = 50(0.47 + j0.03) = 23.5 + j1.5\ \Omega$

Example 11.5

A load of 100+j150 Ω is connected to a 75 Ω lossless line. Find:

a) The reflection coefficient **Γ**

b) The standing wave ratio **s**

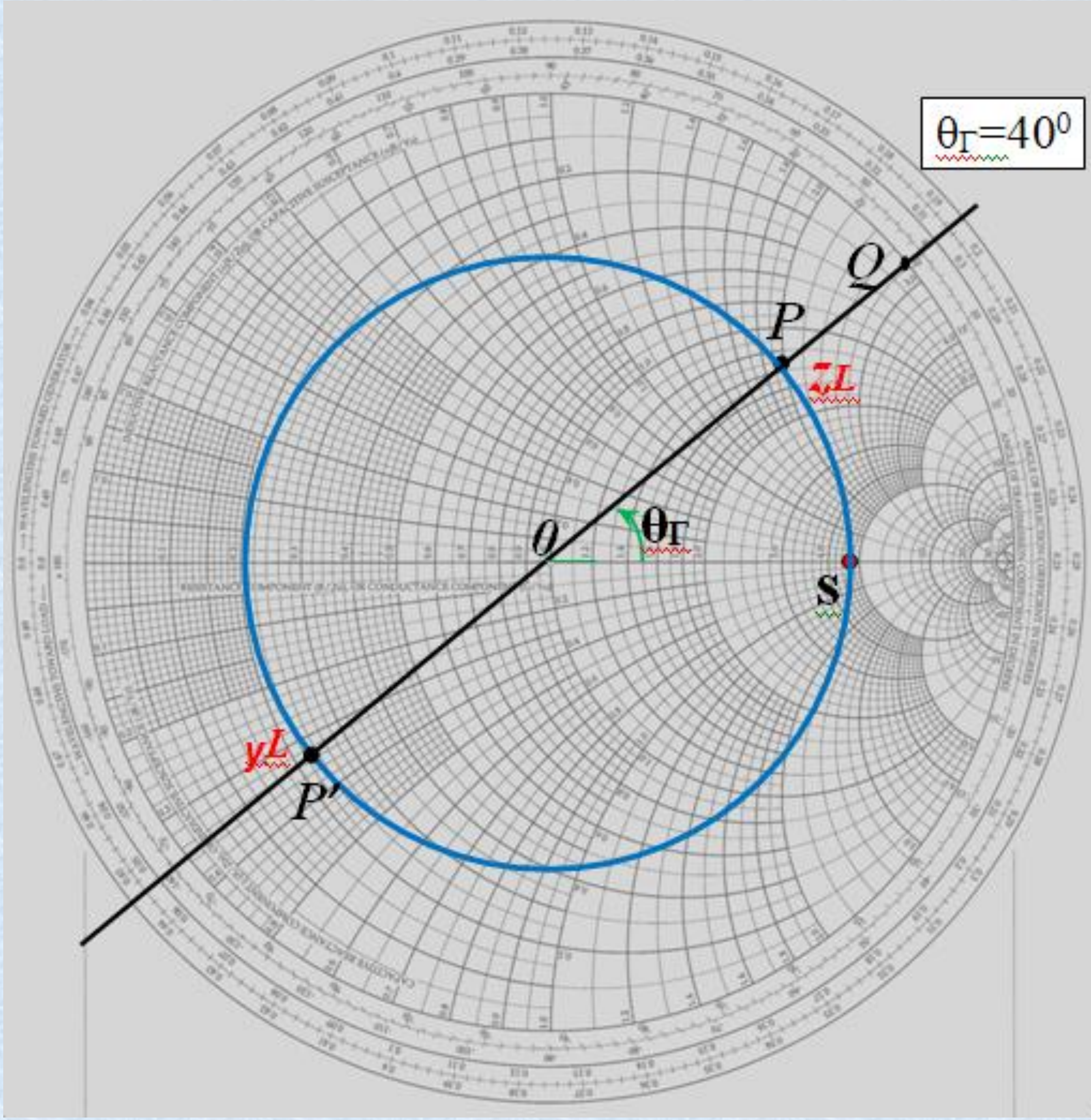
c) The load admittance $\mathbf{Y_L}$

d) Z_{in} at 0.4λ from the load.

e) The locations of $\mathbf{V_{max}}$ and $\mathbf{V_{min}}$ with respect to the load if the line is 0.6 λ long.

f) $\mathbf{Z_{in}}$ at the generator.

Example 11.5



(a) $z_L = \dfrac{Z_L}{Z_0} = \dfrac{100 + j150}{75}$

$=1.33+j2$ (point P)

$|\Gamma_L| = \dfrac{0P}{0Q} = 0.659$

$\theta_\Gamma = \text{angle } P0S = 40^0$

$\rightarrow \Gamma_L = 0.659\angle 40^0$

Check : $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

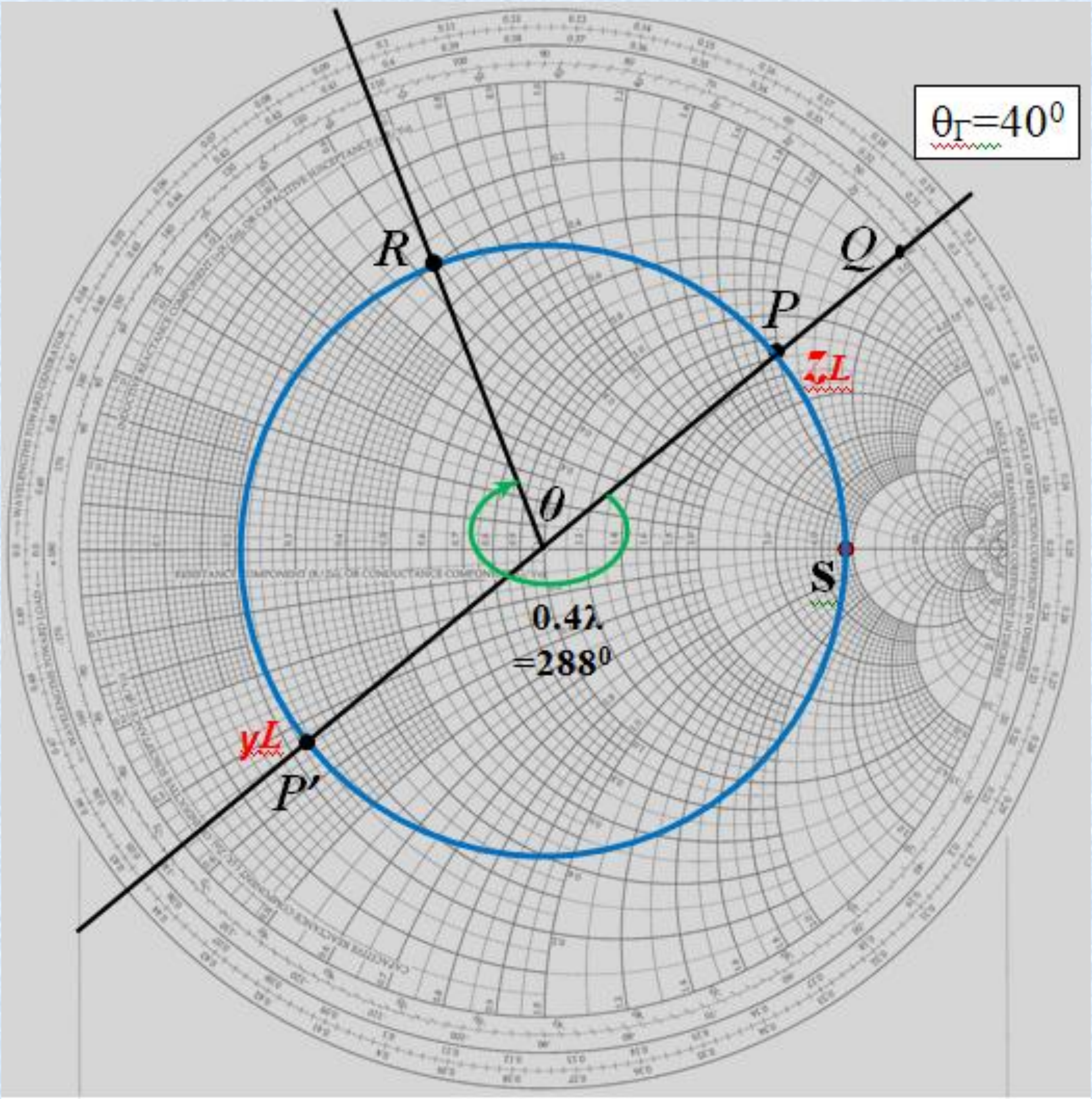
$\Gamma_L = \dfrac{100 + j150 - 75}{100 + j150 + 75}$

$\Gamma_L = 0.659\angle 40^0$

(b) Draw s-circle passing through P and obtain s=4.82

check: $s = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 4.865$

Example 11.5



(c) $y_L = 0.228 - j0.35$

$Y_L = Y_0 y_L$

$= \frac{1}{75}(0.228 - j0.35)$

$Y_L = 3.04 - j4.67\ mS$

Check :

$Y_L = \frac{1}{Z_L} = \frac{1}{100 + j150}$

$= 3.07 - j4.62\ mS$

(d) The 0.4λ corresponds

to $0.4 \times 720^0 = 288^0$

$\rightarrow$ Move 288^0 on s-circle

from P toward generator

(clockwise) to reach point R.

$z_{in} = 0.3 + j0.63$

$Z_{in} = Z_0 z_{in} = 75(0.3 + j0.63)$

$=22.5+j47.25\ \Omega$

Example 11.5 – Solution continued

(d)

$\color{red}{Check:}$

$$\beta l = \frac{2\pi}{\lambda}(0.4\lambda) = 360_0(0.4) = 144^0$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\tan\beta l}\right]$$

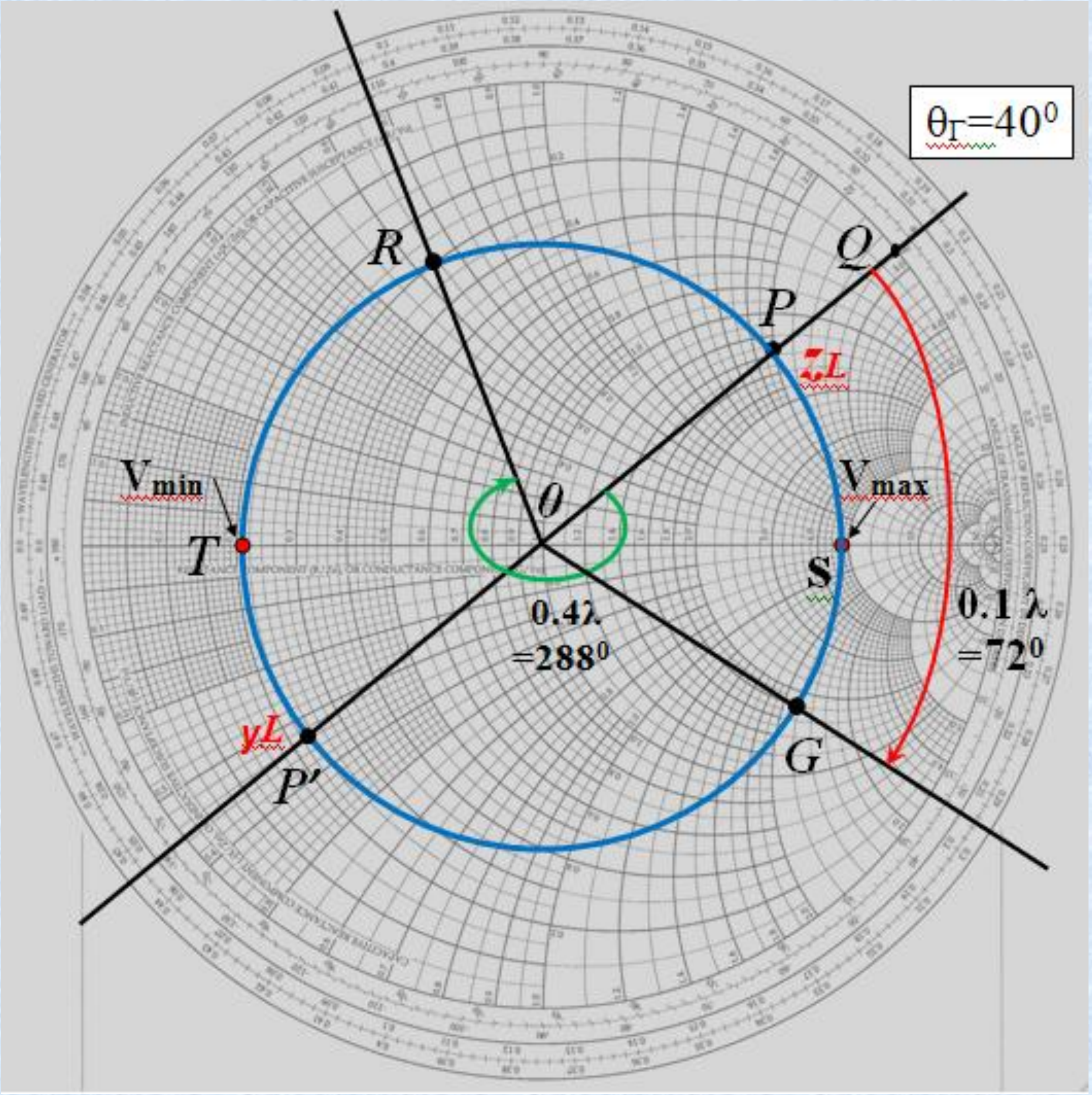
$$=75\left[\frac{100 + j150 + j75\tan 144^0}{75 + j(100 + j150)\tan\tan 144^0}\right]$$

$$=54.41\angle 65.25^0$$

or

$$Z_{in} = 21.9 + j47.6\ \Omega$$

Example 11.5



(e) $0.6\lambda \rightarrow 0.6 \times 720^0 = 432^0$

$= 1\ revolution + 72^0$

Start from P (Load), move alone s-circle 432^0, or one revolution $+72^0$, and reach Generator at point G.

$\rightarrow$ We have passed through point T (location of V_{min}) once, and point S (location of V_{max}) twice. From load:

1st V_{max} is located at :

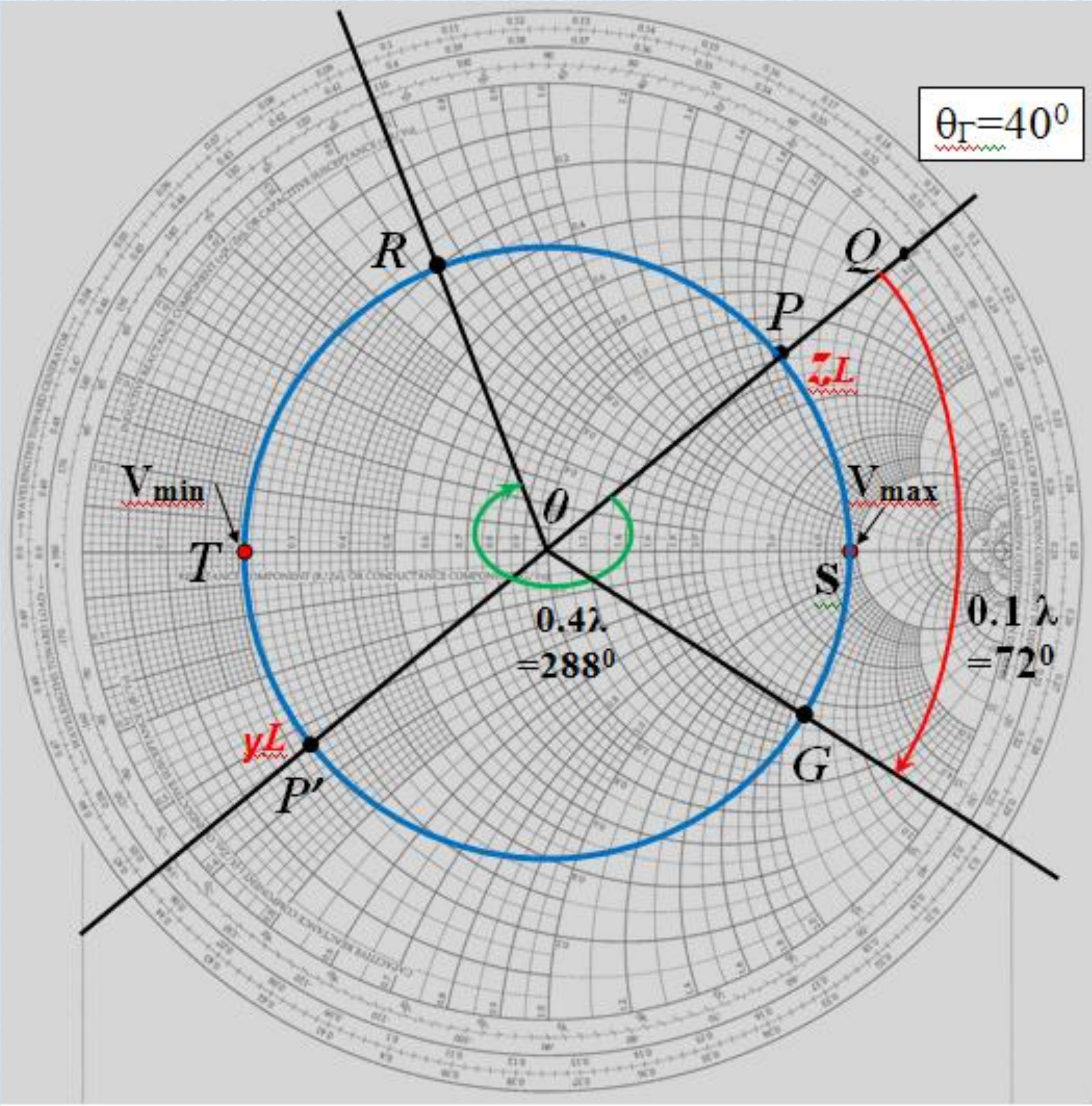
$$\frac{40^0}{720^0}\lambda = 0.055\lambda$$

2nd V_{max} is located at :

$$0.055\lambda + \frac{\lambda}{2} = 0.555\lambda$$

The only V_{min} is located at :

$$0.055\lambda + \lambda / 4 = 0.3055\lambda$$



Example 11.5

(f) At G (Generator end),

$$z_{in} = 1.8 - j2.2$$

$$Z_{in} = 75(1.8 - j2.2)$$

$$= 135 - j165$$