(5)

* The two detectors are coupled to form a negative feedback system in such a way to maintain the local oscillator synchronous with the carrier wave.

* Local oscillator signal is of the same phase as the carrier wave, $A_c \cos 2\pi f_c t$.
  - ⇒ I-channel output has the desired demodulated signal, $m(t)$.
  - ⇒ Q-channel output is zero

* If the L.O phase drifts from its proper value by "ϕ radians"
  - ⇒ I-channel output remains essentially unchanged
  - ⇒ There will be a small signal at the Q-channel output.

* The Q-channel output will have the same polarity as the I-channel output for one direction of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift.

$\beta \gg 1$. one radian.

1) A 20 MHz Carrier is frequency modulated by a sinusoidal signal such that the peak frequency deviation is 100 KHz. Determine the modulation index & the approximate bandwidth of the FM signal if the frequency of the modulating signal is

(i) 1 KHz.

(ii) 50 KHz.

(iii) 500 KHz.

Solution: $f_c = 20\,MHz$.

$\Delta f = 100\,KHz$.

For Tone Modulation, BW of FM ...

$$\boxed{B_T = 2(\beta+1) f_m}$$

$$\frac{\Delta f}{f_m} = \beta$$

$$\Delta f = \beta \cdot f_m.$$

$$\boxed{\beta = \frac{\Delta f}{f_m}} =$$

(i). $f_m = 1000\,Hz$.

$$\beta = \frac{100000}{1000} = \underline{\underline{100}} \rightarrow \text{WBFM}.$$

$$B_T = 2(100+1)\,1000$$

$$= \underline{\underline{202\,KHz}}.$$

(ii) $f_m = 50\,KHz$.

$\beta = 2$.

$$\frac{\Delta f}{f_m} = \frac{100000}{50000}$$

$$\beta = \underline{\underline{2}}$$

$$B_T = 2(2+1)\,\underline{\underline{50\,KHz}}$$

$$\boxed{B_T = 300\,KHz}$$

(ii) $f_m = 500\,KHz$.

$$\beta = \frac{100\,000}{500\,000}.$$

$\beta = 0.2 \Rightarrow$ Narrow Band FM.

$$B_T = 2(0.2+1)\,500\,KHz$$

$$= \underline{\underline{1\,MHz}}.$$

$$\boxed{B_T = 2 f_m} = 1\,MHz$$

1.2×2

2.4

500×2.4

2.4×5

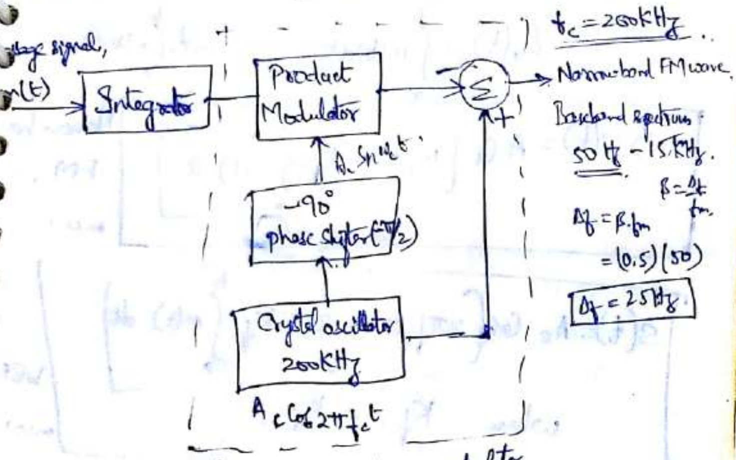
12.0

$\underline{1200\,K}$

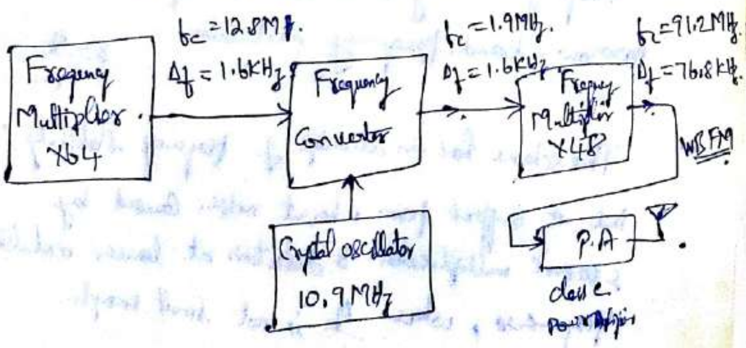
FM Generation

1

Indirect Method :- Generation of FM through phase modulation



Narrow-band phase modulator



FM. AWGN Analysis.

1

Sinusoidal modulating signal, $m(t) = A_m \cos(2\pi f_m t)$.

The instantaneous frequency of the resulting FM signal is,

$$f_i(t) = f_c + K_f \cdot A_m \cos(2\pi f_m t).$$

$$f_i(t) = f_c + \Delta f \cdot \cos(2\pi f_m t).$$

w.k.t. $f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \theta_i(t)$. (freq ⇒ rate of change of angle)

The Angle $\theta_i(t)$ of the FM signal is obtained as follows,

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau) . d\tau.$$

$$= 2\pi \int_0^t [f_c + \Delta f \cos(2\pi f_m t)]\, dt.$$

$$= 2\pi f_c t + 2\pi \Delta f \frac{\sin 2\pi f_m t}{2\pi f_m}$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$

$$\boxed{\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t}$$

The frequency modulated signal,

$$s(t) = A_c \cdot \cos(\theta_i(t)).$$

$$= A_c \cdot \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

12

The FM signal is given by,

$$S(t) = A \cdot \cos\left[2\pi f_c t + \theta_0(t)\right]$$

frequency sensitivity

$$\theta_0(t) = 2\pi K_1 \int_0^t m(t)\,dt.$$

$2\pi K_1 \int_0^t m(t)\,dt.$

Narrow band FM. $m \ll 1$

$$\boxed{\Rightarrow S(t) = A\cos\left[\omega_c t + K_1 (2\pi)\int_0^t m(t)\,dt\right]}$$

See pg. No. 4.

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t)\,dt\right] \quad \text{where } K_f = n \cdot K_1.}$$

WBFM. $m \gg 1$.

Frequency sensitivity is "n" times that of the narrow-band frequency modulator

$$\beta = \frac{\Delta f}{f_m}.$$

This scheme has an advantage of frequency stability; but it suffer from inherent noise caused by excessive multiplication & distortion at lower modulating frequencies, where $\frac{\Delta f}{f_m}$ is not small enough.

$$\therefore\ \boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]} \rightarrow A$$

$$s(t) = A_c \cos\left[2\pi f_c t + \theta_i(t)\right]$$

$\theta_i(t)$ is denoted as $\underline{\underline{\phi(t)}}$.

$$s(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right].$$

where $\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$.

The noisy signal at the Band pass filter output is :,

$$x(t) = s(t) + n(t).$$

The filtered noise $n(t)$ at the BPF o/p interms of its inphase & Quadrature Components is given by,

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) . \sin(2\pi f_c t).$$

Equivalently, the noise $n(t)$ interms of its envelope & phase is given as;

$$n(t) = r(t) . \cos\left[2\pi f_c t + \psi(t)\right].$$

where the envelope is :,

$$r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$$

and phase is

$$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right).$$

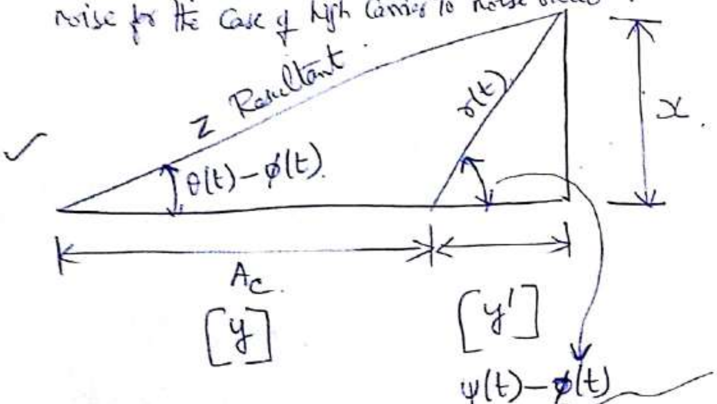
(The phase $\psi(t)$ is uniformly distributed between 0 & 2π radians)

$$\therefore\ x(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) + r(t) . \cos\left[2\pi f_c t + \psi(t)\right].$$

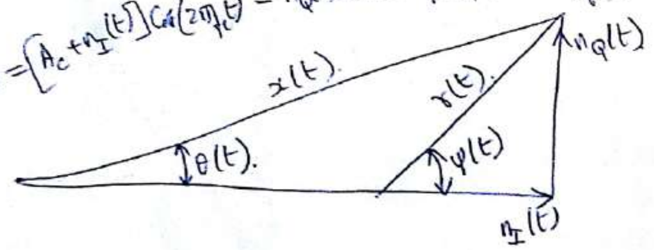
w.k.t. $x(t) = A_c \cos(2\pi f_c t + \phi(t))$
$+ r(t)\left[\cos(2\pi f_c t) + \psi(t)\right]$.

$\theta(t) \rightarrow$ Resultant phasor.

phasor diagram for FM wave plus narrowband noise for the case of high carrier to noise ratio.



For $x(t) = [A_c + n_I(t)]\cos(2\pi f_c t) - n_Q(t).\sin(2\pi f_c t)$. [For FM Threshold effect]



$\psi(t) \rightarrow$ error signal.

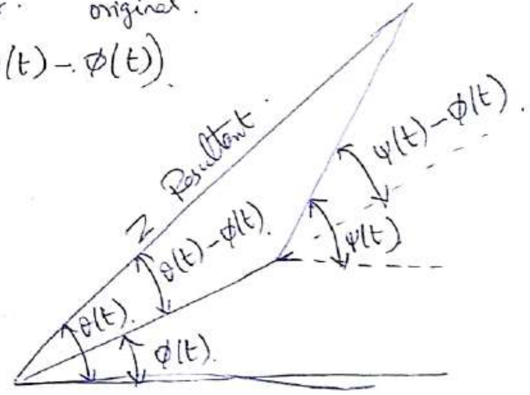
$\theta(t) \rightarrow$ resultant signal.

$\phi(t) \rightarrow$ original.

$\tan(\theta(t) - \phi(t))$

$\cos(\psi(t) - \phi(t))$
$\psi(t)$ ↓ error. $\phi(t)$ ↓ original.

$\sin(\psi(t) - \phi(t))$



$\tan(\underbrace{\theta(t)}_{\text{resultant}} - \underbrace{\phi(t)}_{\text{original}}) = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{\text{Oppo/Hypo}}{\text{Adj/Hypo}}$

$= \frac{x/z}{(y+y')/z} = \frac{x}{y+y'}$. —— I

(6)

$$\cos(\psi(t)-\phi(t)) = \frac{y'}{r(t)}$$

error. original signal.

$$y' = r(t)\cdot\cos[\psi(t)-\phi(t)] \quad \text{— (II)}$$

$$\sin(\psi(t)-\phi(t)) = \frac{x}{r(t)}$$

$$x = r(t)\cdot\sin(\psi(t)-\phi(t)) \quad \text{— (III)}$$

From (I)

$$\tan(\theta(t)-\phi(t)) = \frac{r(t)\cdot\sin(\psi(t)-\phi(t))}{A_c + r(t)\cdot\cos(\psi(t)-\phi(t))}$$

$$\theta(t)-\phi(t) = \tan^{-1}\left[\frac{r(t)\cdot\sin(\psi(t)-\phi(t))}{A_c + r(t)\cos(\psi(t)-\phi(t))}\right]$$

$$A_c \gg r(t)\cdot\cos(\psi(t)-\phi(t))$$

$\therefore \tan^{-1}(x) = x$ If "x" is small.

(7)

$$\therefore \theta(t)-\phi(t) = \tan^{-1}\left[\frac{r(t)\cdot\sin(\psi(t)-\phi(t))}{A_c}\right]$$

$$\therefore \theta(t) = \phi(t) + \frac{r(t)}{A_c}\cdot\sin(\psi(t)-\phi(t))$$

we can write as.

$$\theta(t) = 2\pi k_f\int_0^t m(t)\,dt + \frac{r(t)}{A_c}\sin(\psi(t)-\phi(t))$$

since $\phi(t) = 2\pi k_f \int_0^t m(t)\,dt$.

The discriminator output is,

$$v(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

$$\therefore v(t) = \frac{1}{2\pi}\left[2\pi k_f\, m(t) + \frac{1}{A_c}\frac{d}{dt}\left\{r(t)\cdot\sin(\psi(t)-\phi(t))\right\}\right]$$

w.k.t.

$$\Rightarrow v(t) = \frac{1}{2\pi}\left[2\pi K_f m(t) + \frac{1}{A_c}\frac{d}{dt}\left\{r(t).\sin(\psi(t)-\phi(t))\right\}\right] \quad \text{(IV)}$$

(8)

→ w.k.t. the phase "$\psi(t)$" of the narrow-band noise is uniformly distributed over 2π radians.

→ Hence $[\psi(t)-\phi(t)]$ is also uniformly distributed over 2π radians.

→ Therefore the noise $n_d(t)$ at the discriminator output would be independent of the modulating signal and depends only on the characteristics of the carrier & narrow band noise.

$$\Rightarrow n_d(t) = \frac{1}{2\pi A_c} d/dt\left[\underbrace{r(t).\sin(\psi(t))}_{n_Q(t)}\right].$$

$$\Rightarrow n_d(t) = \frac{1}{2\pi A_c} \cdot d/dt\ n_Q(t)$$

(9)

Since the differentiation of a function w.r.t time corresponds to multiplication of its Fourier Transform by $j2\pi f$.

$$\Rightarrow \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$$

Noise spectrum is,

$$S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f).$$

$$\Rightarrow \frac{N_0 \cdot f^2}{A_c^2} \qquad |f| \leq \frac{B_T}{2}.$$

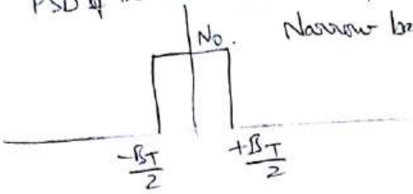
⇒ Discriminator output is.,

$$S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2} & |f| \leq \dfrac{B_T}{2}. \\ 0 & \text{otherwise}. \end{cases}$$

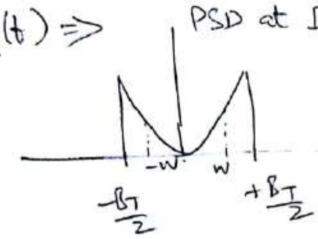
$S_N(f)$.

$S_{N_Q}(f)$ [~~O/P of discriminator~~]

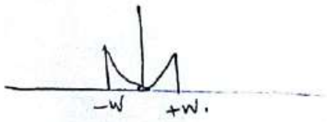
PSD of the Quadrature Component of Narrow band noise.



$S_{N_d}(f) \Rightarrow$ PSD at Discriminator O/p.



P.SD at the receiver output :-



∴ Receiver output.

$$S_{N_o}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & |f| \leq W. \\ 0 & \text{otherwise}. \end{cases}$$

∴ Average power of o/p noise is

$$= \frac{N_o}{A_c^2} \int_{-W}^{+W} f^2 \, df.$$

$$= \frac{N_o}{A_c^2} \left(\frac{f^3}{3} \right)_{-W}^{+W}$$

$$= \frac{2}{3} \frac{N_o W^3}{A_c^2}.$$

Note that the average o/p noise power is inversely proportional to the average carrier power $\left(\frac{A_c^2}{2}\right)$.

Hence, in an FM system, increasing the carrier power, has a Noise - Quieting effect.

From IV w.k.t the message is $k_f \cdot m(t)$. 12.

power $\Rightarrow k_f^2 \cdot P$.

$$\Rightarrow (SNR)_{O,FM} = \frac{k_f^2 P}{\frac{2}{3} \frac{N_0 W^3}{A_c^2}}$$

Post detection SNR.

$$\boxed{(SNR)_{O,FM} = \frac{3}{2} \cdot \frac{k_f^2 A_c^2 P}{N_0 W^3}}$$

Pre detection SNR (or) channel SNR from eqn (A).

Average power of the modulated signal, $s(t)$ $\Big\} = \frac{A_c^2}{2}$.

$$\boxed{(SNR)_{C,FM} = \frac{A_c^2}{2 \cdot W \cdot N_0}}$$

$\left(\frac{A_c}{\sqrt{2}}\right)^2 \Rightarrow \frac{A_c^2}{2}$

$\therefore$ Figure of merit, FOM is

$$\boxed{\left.\frac{(SNR)_o}{(SNR)_C}\right|_{FM} = \frac{3 k_f^2 P}{W^2}}$$

$3\left(\frac{k_f P^{1/2}}{W}\right)^2$.

$FOM = 3\left(\frac{B_T/2}{W}\right)^2$

$= \frac{3}{4}\left(B_T/W\right)^2$

From Generalized Carson's rule,

$B_T = 2(k_f P^{1/2} + W) \simeq 2 k_f P^{1/2} \Rightarrow \frac{B_T}{2} = k_f P^{1/2}$.

SSB - AWGN Analysis

13

Assume only lower side band is transmitted, then

$$s(t) = \frac{A_c}{2} m(t) \cdot \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \cdot \sin(2\pi f_c t).$$

$\rightarrow m(t)$ & $\hat{m}(t)$ are Uncorrelated. $\therefore$ their PSD's are additive.

$$\text{Average power} \Rightarrow \left(\frac{\left(\frac{A_c}{\sqrt{2}}\right)}{2}\right)^2 \cdot P + \left(\frac{\left(\frac{A_c}{\sqrt{2}}\right)}{2}\right)^2 \cdot P$$

$$= \frac{A_c^2}{8} P + \frac{A_c^2}{8} P$$

$$= \frac{A_c^2}{4} \cdot P.$$

$\therefore$ Pre detection SNR.

$$\boxed{\Rightarrow SNR^{SSB}_{Pre} = \frac{A_c^2 P}{4 \cdot N_0 W}}$$

Post detection SNR :-

(14)

$$v(t) = x(t) \cdot \cos(2\pi f_c t)$$

$$= \left\{ \left[\frac{A_c}{2} m(t) \cdot \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \cdot \sin(2\pi f_c t) \right] + n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t) \right\} \cos(2\pi f_c t)$$

$$= \left\{ \left[\frac{A_c}{2} m(t) + n_I(t) \right] \cos(2\pi f_c t) + \left(\frac{A_c}{2} \hat{m}(t) - n_Q(t) \right) \sin 2\pi f_c t \right\} \cos(2\pi f_c t)$$

$$= \left[\frac{A_c}{2} m(t) + n_I(t) \right] \cos^2(2\pi f_c t) + \left(\frac{A_c}{2} \hat{m}(t) - n_Q(t) \right) \sin(2\pi f_c t) \cdot \cos(2\pi f_c t)$$

After Low pass filtering ; $\Rightarrow$

$$y(t) = \left(\frac{A_c}{2} m(t) + n_I(t) \right) \cos^2(2\pi f_c t)$$

(15)

$$y(t) = \left[\frac{A_c}{2} m(t) + n_I(t) \right] \left[\frac{1 + \cos 4\pi f_c t}{2} \right]$$

$$= \left[\frac{A_c}{2} m(t) + n_I(t) \right] \frac{1}{2} + \text{High freq term.}$$

$$y(t) = \frac{A_c}{4} m(t) + \frac{1}{2} n_I(t)$$

signal power $\Rightarrow \frac{A_c^2}{16} P$

Noise power $\Rightarrow \frac{1}{2} \left(\frac{1}{2} N_0 W \right)$

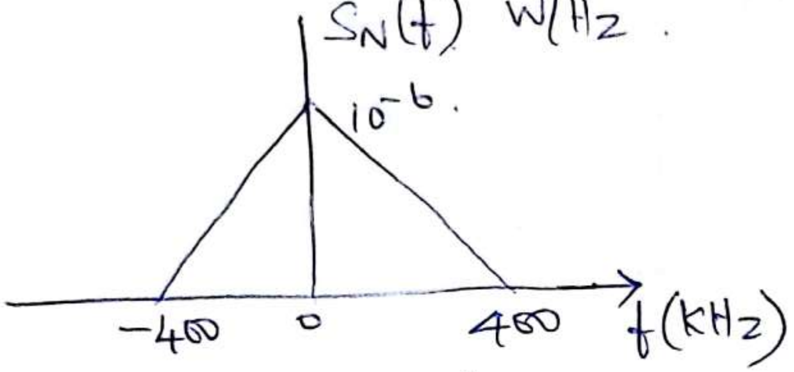
$$SNR_{Post}^{SSB} = \frac{A_c^2 P}{16} \times \frac{4}{N_0 W}$$

$$= \frac{A_c^2 P}{4 N_0 W}$$

$$\text{Figure of merit} = \frac{SNR_{Post}^{SSB}}{SNR_{Pre}^{SSB}} = \frac{\frac{A_c^2 P}{4 N_0 W}}{\frac{A_c^2 P}{4 N_0 W}}$$

$$\boxed{FOM_{SSB} = 1}$$

1. A DSB-SC modulated signal is transmitted over a noisy channel with power spectral density of noise being as shown in figure;



The message B.W. is 4KHz and the carrier frequency is 200KHz. Assuming that the average power of the modulated wave is 10 watts. Determine the output SNR of the receiver.

Solution:-

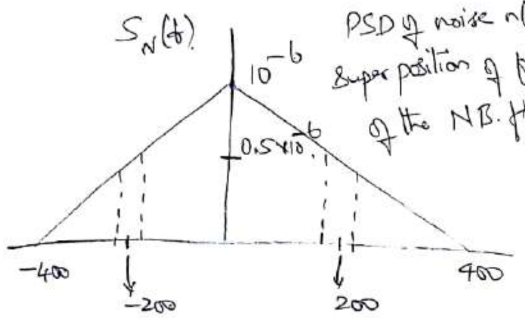
The received signal at the output of a narrow band pass filter of bandwidth 8KHz centered at the carrier frequency, $f_c = 200$KHz, we get.,

$$s(t) = A_c\, m(t) \cdot \cos 2\pi f_c t + n(t).$$

$$= \left[A_c\, m(t) + n_I(t)\right] \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t)$$

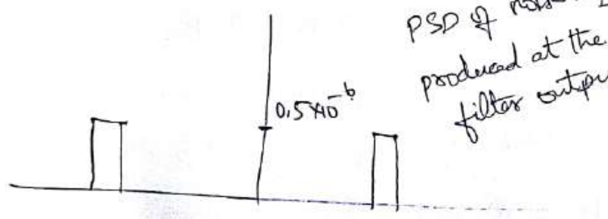
The o/p of the product modulator,

v(t) is $v(t) = A_c m(t) + n_I(t)$.



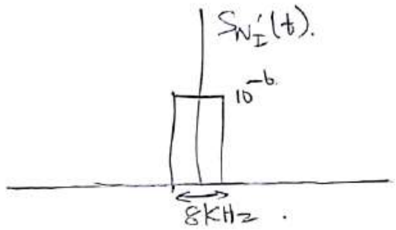
PSD of noise n(t) & Superposition of freq. response of the NB. filter.

$S_{N'}(f)$. after passing through BPF.



PSD of noise $N_I'(t)$ produced at the filter output.

PSD of Inphase component of $n_I'(t)$.



Average power of $n_I(t)$ is

$$(10^{-6}\ \text{W/Hz})(8\times10^{3}) = 0.008\ \text{watts}.$$

$$SNR = \frac{10\ \text{watts}}{0.008\ \text{watts}} = 1250.$$

$$SNR_{dB} = 10\log_{10}(1250) = \underline{31\ dB}.$$

(2) Let a message signal $m(t)$ be transmitted using single sideband modulation.

The PSD of $m(t)$ is,

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W}, & |f| \leq W, \\ 0 & \text{otherwise}, \end{cases}$$

where a & W are constants.

White Gaussian noise of zero mean & PSD, $N_0/2$ is added to the SSB modulated wave at the receiver input. Find an expression for the o/p signal to noise ratio of the receiver.

Solution :- Average signal power;

$$P = \int_{-\infty}^{\infty} S_M(f)\, df.$$

$$= \int_{-W}^{+W} a\frac{|f|}{W}\, df.$$

$$= \frac{a}{W}\left[\int_{-W}^{0} (-f)\, df + \int_{0}^{W} (+f)\, df\right].$$

$$= \frac{a}{W}\left[\left[\frac{-f^2}{2}\right]_{-W}^{0} + \left[\frac{f^2}{2}\right]_{0}^{W}\right].$$

$$= \frac{a}{W}\left[\cancel{\left(0-\frac{(-)(-)w^2}{2}\right)} + \left(\frac{w^2}{2}-0\right)\right]$$

$$= \frac{a}{W}\left[0-\frac{-(-w)^2}{2} + \left(\frac{w^2}{2}-0\right)\right]$$

$$0 - \frac{-((-w)^2)}{2}$$

$$= 0 - -w^2$$

$$= 0 + w^2.$$

$$= \frac{a}{W}\left[0-\frac{-w^2}{2} + \frac{w^2}{2}-0\right]$$

$$= \frac{a}{W}\left[\frac{w^2}{2}+\frac{w^2}{2}\right] = \frac{aw^2}{W} = \underline{a.W}.$$

W.K.T.

$$SNR^{SSB}_{o/p} = \frac{A_c^2 . P}{4.W.N_0} = \frac{A_c^2 . a.W}{4.WN_0}$$

$$\boxed{SNR^{SSB}_{o/p} = \frac{a A_c^2}{4N_0}}$$

(3) Show the improvement in post detection SNR of an FM receiver with the Pre-Emphasis & De-Emphasis.

Solution:-

The Noise power at the output is

$$N = \frac{2}{3}\,\frac{N_0 W^3}{A_c^2}.$$

This signal is now passed to De-Emphasis Circuit.

De-Emphasis is a LPF Circuit.

LPF

The T.Fn of De-emphasis is

$$H_{de}(f) = \frac{1}{1 + j\frac{f}{f_{3dB}}}.$$

$$H(\omega) = \frac{1/cS}{R + 1/cS} = \frac{1}{1+RcS}$$

$$\boxed{H(\omega) = \frac{1}{1+j\omega RC}}$$

∴ Noise PSD with de-emphasis is,

$$\Rightarrow S_{N_0}(f)\cdot\left|H_{de}(f)\right|^2$$

HPF

$$\Rightarrow \frac{R}{R+\frac{1}{cS}} \Rightarrow \frac{RcS}{RcS+1}$$

$$\boxed{\frac{j\omega RC}{1+j\omega RC}} \quad (\text{or}) \quad \frac{1}{1+\frac{1}{j\omega RC}} = \cancel{\frac{sRC}{1+sRC}}.\ \frac{s}{s+1/RC}$$

$$\therefore S_{N_0}(f)\cdot\left|H_{de}(f)\right|^2 \Rightarrow \frac{N_0 f^2}{A_c^2}\cdot\left|\frac{1}{1+j\,f/f_{3dB}}\right|^2$$

$$|f| \le W.$$

$$= \frac{N_0 f^2}{A_c^2}\cdot\frac{1}{1+\frac{f^2}{f_{3dB}^2}}.$$

$$\boxed{\begin{gathered}|A+jB| = \sqrt{A^2+B^2}\\ |A+jB|^2 = A^2+B^2\end{gathered}}$$

Noise power at the output of De-emphasis filter is,

$$N_{de} = \int_{-W}^{W} \frac{N_0 f^2/A_c^2}{1+\left(f/f_{3dB}\right)^2}\cdot df.$$

$$= \frac{N_0}{A_c^2}\int_{-W}^{+W} \frac{f^2}{1+\left(f^2/f_{3dB}^2\right)}\, df.$$

$$= \frac{N_0}{A_c^2}\int_{-W}^{+W} \frac{f^2}{\left(\frac{1}{f_{3dB}^2}\right)\left[f_{3dB}^2 + f^2\right]}\, df.$$

$$= \frac{N_0}{A_c^2}\int_{-W}^{+W} \frac{f_{3dB}^2 f^2}{\left(f_{3dB}^2+f^2\right)}\, df.$$

$$N_{de} = \frac{N_0}{A_c^2} f_{3dB}^2 \int_{-W}^{+W} \frac{f^2}{(f_{3dB}^2+f^2)} \cdot df .$$

w.k.t.

$$\int \frac{x^2}{a^2+x^2}\,dx = \int \frac{a^2+x^2-a^2}{a^2+x^2}\,dx .$$

$$= \int \left(\frac{a^2+x^2}{a^2+x^2} - \frac{a^2}{a^2+x^2} \right) dx .$$

Here $x = f$ and $a = f_{3dB}$

$$\Rightarrow \frac{N_0 f_{3dB}^2}{A_c^2} \int \left(\frac{f_{3dB}^2+f^2}{f_{3dB}^2+f^2} \right) df - \frac{N_0 f_{3dB}^2}{A_c^2} \int \frac{f_{3dB}^2}{f_{3dB}^2+f^2}\, df .$$

$$= \frac{N_0 f_{3dB}^2}{A_c^2} \left[f \right]_{-W}^{+W} - \frac{N_0 f_{3dB}^4}{A_c^2} \int \frac{1}{f_{3dB}^2+f^2}\, df .$$

$$= \frac{N_0 f_{3dB}^2}{A_c^2} [2W] - \frac{N_0 f_{3dB}^4}{A_c^2} \left[\frac{1}{f_{3dB}} \cdot \tan^{-1}\left(\frac{f}{f_{3dB}} \right) \right]_{-W}^{+W}$$

$$\boxed{\because \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}(x/a)}$$

$$\Rightarrow \frac{2 N_0 W \cdot f_{3dB}^2}{A_c^2} - \frac{N_0 f_{3dB}^4}{A_c^2 f_{3dB}} \left[2 \tan^{-1}\left(\frac{W}{f_{3dB}} \right) \right]$$

$$\boxed{N_{de} = \frac{2 N_0}{A_c^2} \left[\frac{W}{f_{3dB}} - \tan^{-1}\left(\frac{W}{f_{3dB}} \right) \right] (f_{3dB})^3}$$

For $f_{3dB} = 2.1\,KHz$ & $W = 15\,KHz$.

Find, I.

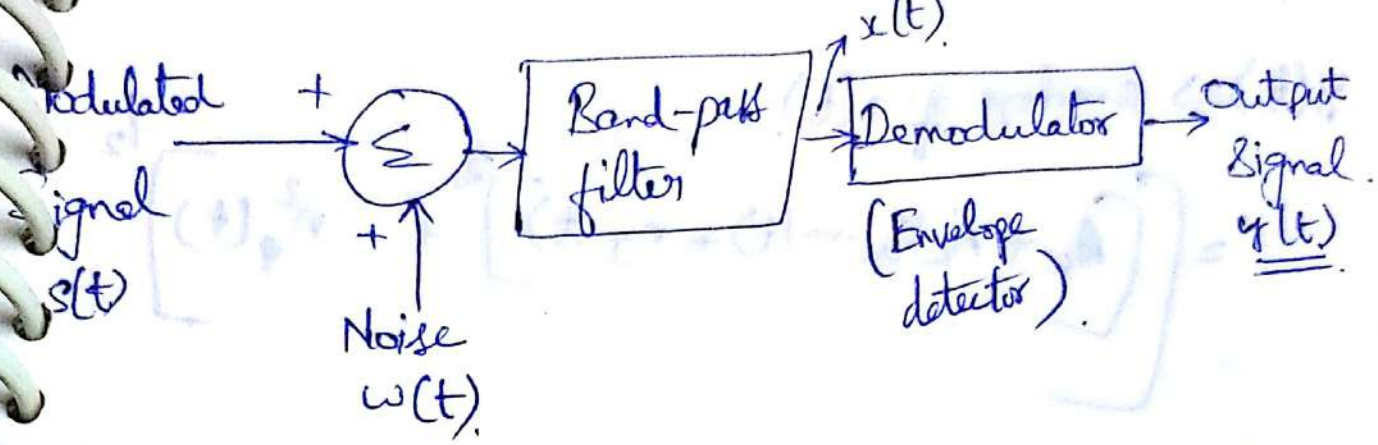
$$I = \frac{N}{N_{de}} = \frac{(W/f_{3dB})^3}{3\left[W/f_{3dB} - \tan^{-1}\left(\frac{W}{f_{3dB}} \right) \right]}$$

$$= \frac{(15/2.1)^3}{3\left[15/2.1 - \tan^{-1}(15/2.1) \right]} = \frac{(7.14)^3}{3(7.14 - 1.43)}$$

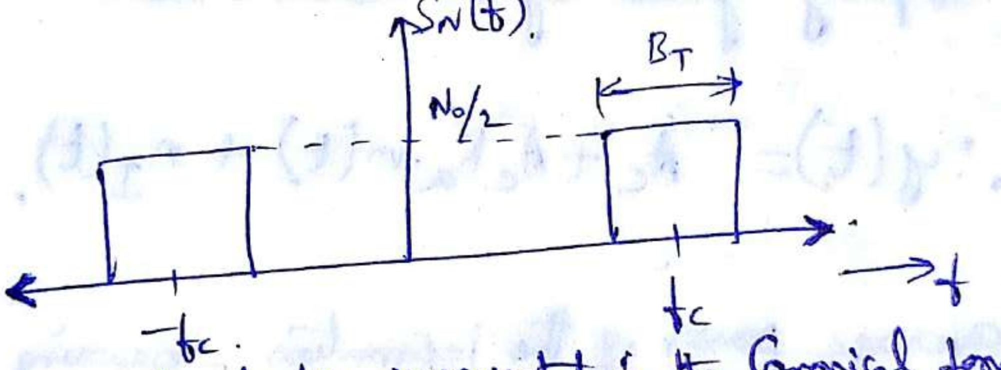
$$= 22 .$$

AWGN Analysis

Receiver model:-



Idealized characteristics of band pass filtered noise



The narrow band noise represented in the Canonical form is,

$$n(t) = n_I(t).\cos(2\pi f_c t) - n_Q(t).\sin(2\pi f_c t).$$

n_I → Inphase noise component

n_Q → Quadrature noise component.

Received Signal,

$$x(t) = S(t) + n(t).$$

$$S(t) = A_c\left(1 + ka \cdot m(t)\right).\cos 2\pi f_c t.$$

$$\therefore x(t) = \left[A_c\left(1 + K_a m(t)\right)\cos 2\pi f_c t\right] + \left[n_I(t)\cos(2\pi f_c t) - n_Q(t).\sin 2\pi f_c t\right]$$

(2)

$$\therefore x(t) = \left[A_c + A_c K_a m(t) + n_I(t)\right] \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t).$$

$y(t) \Rightarrow$ envelope of $x(t)$.

$$= \left[\left[A_c + A_c K_a \cdot m(t) + n_I(t)\right]^2 + n_Q^2(t)\right]^{1/2}$$

The Quadrature Component $n_Q(t)$ of the noise $n(t)$ is Completely rejected by the Coherent detector.

$$\therefore y(t) = A_c + A_c K_a \cdot m(t) + n_I(t).$$

The average power of the information-bearing Component $A_c \cdot K_a \cdot m(t)$ is $\frac{A_c^2 (K_a^2 \cdot P)}{2}$.

$$\left(\frac{A_c}{\sqrt{2}}\right)^2 = \frac{A_c^2}{2}$$

$$A_c^2 \left(\frac{1 + K_a^2 \cdot P}{2}\right).$$

MATLAB clc:

(3)

$$\boxed{(SNR)_{c,AM} = \frac{A_c^2 (1 + K_a^2 \cdot P)}{2 \cdot W \cdot N_0}} \Rightarrow \text{Channel SNR for A.M.}$$

O/P SNR of an AM receiver using an envelope detector.

$$(SNR)_{0,AM} = \frac{A_c^2 K_a^2 P}{2 \cdot W \cdot N_0}.$$

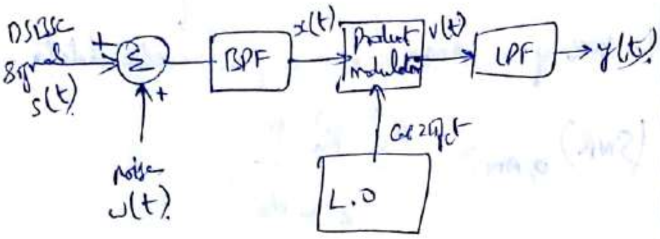
Figure of merit:

$$\left.\frac{(SNR)_0}{(SNR)_c}\right|_{AM} = \frac{A_c^2 K_a^2 P}{2 \cdot W \cdot N_0} \times \frac{2 W N_0}{A_c^2 (1 + K_a^2 P)}$$

$$\boxed{= \frac{K_a^2 \cdot P}{(1 + K_a^2 \cdot P)} < \text{unity}.}$$

$V = IR$. $\frac{V}{I} = R$. $P = VI$. $P = V \frac{V}{R}$ $\boxed{P = \frac{V^2}{R}}$ $V = IR$ $V^2 = I^2 R^2$ $= \frac{P}{R}$

DSBSC – AWGN Analysis



$$s(t) = c\, A_c \cdot \cos(2\pi f_c t) \cdot m(t)$$

$$(SNR)_{C,DSB} = \frac{c^2 \cdot A_c^2 \cdot P}{2 \cdot W \cdot N_0}$$

Received signal, $x(t) = s(t) + n(t)$

$$= c \cdot A_c \cdot \cos(2\pi f_c t) \cdot m(t) + n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$

The output of the product modulator

$$v(t) = x(t) \cdot \cos 2\pi f_c t$$

$$= c \cdot A_c \cos(2\pi f_c t) \cdot \cos(2\pi f_c t) \cdot m(t)$$
$$+ n_I(t) \cdot \cos(2\pi f_c t) \cos(2\pi f_c t)$$
$$- n_Q(t) \cdot \sin(2\pi f_c t) \cdot \cos(2\pi f_c t)$$

MATLAB clc:

$$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

$$= c \cdot A_c \frac{1}{2}(1 + \cos 4\pi f_c t) \cdot m(t)$$
$$+ n_I(t)\left[\frac{1}{2}(1 + \cos 4\pi f_c t) \cdot \cancel{m(t)})\right]$$
$$- n_Q(t)\left[\frac{1}{2}(\sin(A-B) + \sin(A+B))\right]$$

$$= \frac{1}{2} \cdot c \cdot A_c \cdot m(t) + \frac{1}{2} c A_c \cos 4\pi f_c t \cdot m(t)$$
$$+ \frac{1}{2} n_I(t) \cancel{m(t)}) + \frac{1}{2} n_I(t) \cdot \cos 4\pi f_c t \cdot \cancel{m(t)})$$
$$- n_Q(t) \cdot \frac{1}{2} \cdot \sin(4\pi f_c t)$$

$$v(t) = \frac{1}{2} c \cdot A_c m(t) + \frac{1}{2} n_I(t) + \frac{1}{2}\cos(4\pi f_c t)\left[c \cdot A_c m(t) + n_I(t)\right]$$
$$- n_Q(t) \cdot \frac{1}{2} \sin(4\pi f_c t)$$

The LPF removes high frequency components of $v(t)$

$$\Rightarrow y(t) = \frac{1}{2} c \cdot A_c m(t) + \frac{1}{2} n_I(t)$$