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$f(x, y)$

$p = 0 \quad q = 0 \quad , r, t$

$rt - s^2 > 0$

$r < 0$ (max) $\quad r > 0$ (min)

$rt - s^2 < 0 \longrightarrow$ saddle point

* find max & min

$$f(x, y) = x^3 + y^3 - 3axy$$

$$p = \frac{\partial f}{\partial x} = 3x^2 - 3ay$$

$$q = \frac{\partial f}{\partial y} = 3y^2 - 3ay$$

$$r = \frac{\partial^2 f}{\partial x^2} = 6x \qquad s = \frac{\partial^2 f}{\partial x \partial y} = -3a$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6y$$

$p = 0 \qquad q = 0$

$$x^2 - ay = 0 \quad + \quad y^2 - ax = 0$$

$$x^3 - y^3 = 0$$

$(x+y)(x^2+xy+y^2)=0$

$x=+y$

$y^2-ay=0$

$y(y-a)=0$

$y=0 \qquad y=0$

$(0,0) \qquad (a,a)$

$rt-s^2 = -9a^2 < 0$

Hence $(0,0)$ is saddle point

$r=6a \qquad t=6a \qquad s^2=+9a^2$

$36a^2-9a^2 = 27a^2$

$r=6a \qquad a>0 \longrightarrow$ minimum

$a<0 \longrightarrow$ maxima

+ find the max and min values for the function $f(x,y) = x^3+3xy^2-15x^2-15y^2+72x$

$p = 3x^2+3y^2-30x+72$

$q = 6xy-30y$

$r = 6x-30$

$s = 6y \qquad t = 6x-30$

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$p = 0$

$3x^2 + 3y^2 - 30x - 72 = 0$

when $y = 0$

$3x^2 - 30x - 72 = 0$

$x^2 - 10x - 24 = 0$

$x = 4, 6$

when $x = 5$

$y = \pm 1$

$q = 0$

$6xy - 30y = 0$

$6xy = 30y$

$6y(x - 5) = 0$

$x = 5$

$y = 0$

$(4,0)$	$(6,0)$	$(5,1)$	$(5,-1)$
$36 > 0$	$36 > 0$	$[6(5)-30]$	
$r < 0$	$r > 0$	< 0	< 0
maximum	minimum	saddle	saddle

* find min and max value for

$\sin x + \sin y + \sin(x+y)$

$p = \cos x + \cos(x+y)$

$q = \cos y + \cos(x+y)$

$r = -\sin x - \sin(x+y)$

$$s = -\sin(x+y)$$

$$t = -\sin y - \sin(x+y)$$

$p = 0$

$$\cos x + \cos(x+y) = 0$$

$$\cos x = -\cos(x+y)$$

$$\cos x = \cos(\pi - (x+y))$$

$$x = \pi - (x+y)$$

$q = 0$

$$\cos y + \cos(x+y) = 0$$

$$\cos y = -\cos(x+y)$$

$$\cos y = \cos(\pi - (x+y))$$

$$y = \pi - (x+y)$$

$x = y$

$$(\pi/3, \pi/3)$$

$$[-\sin \pi/3 - \sin 2\pi/3][-\sin \pi/3 - \sin 2\pi/3]$$

$$- \sin^2(x+y)$$

$$= 3\sin \pi/3 = \frac{3 \times \sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2}$$

Method of Lagrangian Multiplyer (λ)

$f(x, y, z) \rightarrow$ function needed to be maximised / minimized

$g(x, y, z) \rightarrow$ constraint

$$f + \lambda g = 0$$

Suppose we require to find the max/min values of $f(x, y, z)$ where (x, y, z) are subject to constraint $g(x, y, z) = 0$.

we define a function $F = f + \lambda g$ where λ is called the Lagrangian multiplyer, which is independent of x, y, z

the necessary condition for maximum and minimum is given by

$$\frac{\partial F}{\partial x} = 0 \qquad \frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} = 0$$

Solving these equations for the unknowns λ, x, y, z we get the point (x, y, z)

* Find the minimum value of $x^2+y^2+z^2$ subject to $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

Auxilary equation

$$F = f + \lambda g = 0$$

$$(x^2+y^2+z^2) + \lambda\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right] = 0$$

$$\frac{\partial F}{\partial x} = 2x + \lambda\left[-\frac{1}{x^2}\right] = 0 \qquad 2x = \lambda/x^2$$

$$\frac{\partial F}{\partial y} = 2y + \lambda\left[-\frac{1}{y^2}\right] = 0 \qquad 2y = \lambda/y^2$$

$$\frac{\partial F}{\partial z} = 2z + \lambda\left[-\frac{1}{z^2}\right] = 0 \qquad 2z = \lambda/z^2$$

$$\frac{\lambda}{2} = x^3 = y^3 = z^3 \qquad \underline{x = y = z}$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1 \qquad \frac{3}{x} = 1$$

$$\frac{1}{x}+\frac{1}{x}+\frac{1}{x} = 1 \qquad x = 3$$

$$\underline{(3, 3, 3)}$$

* Find the volume of the largest rectangular parallelopoid that can be inscribed in the ellipsoid.

f = volume

$g \rightarrow$ equation of elliposid

The volume of parallelopoid $f = 8xyz$

The equation of elliposed

$$g = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$$

$$F = f + \lambda g$$

$$F = 8xyz + \lambda\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right]$$

$$\frac{\partial F}{\partial x} = 8yz + \frac{2x\lambda}{a^2} = 0$$

$$\frac{\partial F}{\partial y} = 8xz + \frac{2y\lambda}{b^2} = 0$$

$$\frac{\partial F}{\partial z} = 8xy + \frac{2z\lambda}{c^2} = 0$$

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$$8yz = -\frac{2x\lambda}{a^2}$$

$$-\lambda = \frac{8yza^2}{2x}$$

$$-\lambda = \frac{4yza^2}{x}$$

$$\frac{4yza^2}{x} = \frac{4xzb^2}{y} = \frac{4xyc^2}{z}$$

$$\frac{4yza^2}{x} = \frac{4xzb^2}{y}$$

$$y^2\cancel{z}\,a^2 = x^2\cancel{z}\,b^2$$

$$y^2a^2 = x^2b^2$$

$$\frac{y^2}{x^2} = \frac{b^2}{a^2}$$

In constraint

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{y^2}{b^2} + \frac{y^2}{b^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = \frac{b^2}{3} \qquad y = \pm\frac{b}{\sqrt{3}}$$

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