PART – C (5 × 12 = 60 Marks)

28. a. Using triple integration, find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(OR)

b. Change the order of integration and hence evaluate $\int_0^1 \int_y^{2-y} xy\, dy\, dx$.

29. a. Show that $\vec{F}=\left(z^2+2x+3y\right)\vec{i}+\left(3x+2y+z\right)\vec{j}+\left(y+2zx\right)\vec{k}$ is irrotational, but not solenoidal. Find also its scalar potential.

(OR)

b. Verify Green's theorem in a plane for $\int_C \left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy$ where 'C' is the boundary of the region bounded by the lines $x = 0$, y = 0 and x+y = 1.

30.a.i Find $L\left(\frac{\sin 3t\, \sin t}{t}\right)$.

ii. Solve: $(D^2+6D+9)x = 6t^2 e^{-2t}$, $x = 0$, $Dx = 0$ at t = 0 using Laplace transform.

(OR)

b. i. Using convolution theorem, to find $L^{-1}\left[\frac{s}{\left(s^2+a^2\right)^2}\right]$.

ii. Find $L\,(f(t))$, if $f(t) = e^t$, $0< t<2\pi$ and $f\,(t+2\pi) = f(t)$.

31.a.i Determine the analytic function $f(z) = u+iv$ given that $3u+2v = y^2-x^2+16xy$.

ii. Find the bilinear transformation which maps the points z = 0, z = 1 and z = ∞ into the points w = i, w = 1 and w = $-i$.

(OR)

b. State and prove the two important properties of an analytic function.

32. a. Evaluate $\int_C \frac{z+4}{z^2+2z+5}\, dz$ where C is a circle $|\,z+1+i\,| = 2$ using Cauchy's integral formula.

(OR)

b. Evaluate $\int_0^\infty \frac{x^2\, dx}{(x^2+1)\,(x^2+4)}$ using contour integration technique.

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Reg. No.										

B.Tech. DEGREE EXAMINATION, NOVEMBER 2014
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted from the academic year 2013 – 2014 onwards)

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If $I=\int_0^1\int_1^2 \left(x^2+y^2\right) dx\, dy$, then the value of I is

(A) $\frac{-8}{3}$ (B) 9

(C) $\frac{8}{3}$ (D) 0

2. What is the value of $\int_0^1\int_0^x dx\, dy$?

(A) $\frac{1}{2}$ (B) -1

(C) 1 (D) $\frac{1}{3}$

3. Change the order of integration in $\int_0^a\int_x^a f(x,y)\, dy\, dx$.

(A) $\int_0^a\int_x^a f(x,y)\, dy\, dx$ (B) $\int_0^a\int_0^y f(x,y)\, dy\, dx$

(C) $\int_0^a\int_0^{x^2} f(x,y)\, dx\, dy$ (D) $\int_0^x\int_0^a f(x,y)\, dx\, dy$

4. Find the value of $\int_0^\pi \int_0^{a\sin\theta} r\, dr\, d\theta$.

(A) πa^2 (B) $\frac{\pi}{4}a^2$

(C) $\frac{\pi}{4}a^3$ (D) a^2

5. If $\vec{F}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+az)\vec{k}$, is solenoidal, then the value of 'a' is

(A) -2 (B) 1

(C) -1 (D) 0

6. The value of $\int_C x\,dy - y\,dx$ around the circle $x^2+y^2 = 1$ is

(A) 0 (B) π
(C) 3π (D) 2π

7. If $\vec{u}$ and $\vec{v}$ are irrotational, then $\vec{u}\times\vec{v}$ is

(A) Solenoidal (B) Irrotational
(C) Zero (D) Constant

8. The value of $\iint_S \vec{r}.\vec{n}\,ds$, where S is the surface of the sphere $x^2+y^2+z^2 = a^2$ is

(A) $2\pi a^3$ (B) $3\pi a^3$
(C) $3\pi a^2$ (D) $4\pi a^3$

9. $L(2^t)$ is

(A) $\frac{1}{s-\log 2}$ (B) $\frac{1}{s+\log 2}$
(C) $\frac{1}{s-2}$ (D) $\frac{1}{s+2}$

10. $L(t^{3/2})$ is

(A) $\frac{2\sqrt{\pi}}{4s^{5/2}}$ (B) $\frac{3\sqrt{\pi}}{4s^{5/2}}$
(C) $\frac{2}{4s^{5/2}}$ (D) $\frac{3}{4s^{5/2}}$

11. The value of $L^{-1}(1)$ is

(A) $U_a(t)$ (B) 1
(C) 0 (D) $\delta(t)$

12. $L\left(f(t)*g(t)\right)$ is

(A) $F(s)*G(s)$ (B) $F(s)\div G(s)$
(C) $F(s)\,G(s)$ (D) $F(s)+G(s)$

13. If $w = f(z) = u+iv$ is an analytic function of z, then

(A) u and v are not harmonic (B) u is harmonic, v is not harmonic
(C) Both u and v are harmonic (D) u and v are constants

14. The fixed points of the transformation $w = \frac{z-1}{z+1}$ are

(A) $\pm i$ (B) ± 1
(C) ± 2 (D) ± 3

15. If $u = 2x(1-y)$, then harmonic conjugate of u is

(A) x^2-y^2+2y+c (B) x^2+y^2-2y+c
(C) $-x^2+y^2+2y+c$ (D) x^2-y^2-2y+c

16. The analytic function with constant real part is

(A) Constant (B) Function of x
(C) Function of y (D) Function of z

17. The value of $\int_C \frac{4z^2+z+5}{z-4}\,dz$, where C is $9x^2+4y^2 = 36$ is

(A) πi (B) 0
(C) $-\pi i$ (D) $2\pi i$

18. If $f(z) = \frac{\sin z}{z}$, then z = 0 is

(A) Pole (B) Removable singularity
(C) Essential singularity (D) Isolated singularity

19. If $I = \int_C \frac{z}{z^2-1}\,dz$, $C: |z| = \frac{1}{2}$, then I is

(A) 0 (B) πi
(C) -1 (D) $2\pi i$

20. If $I = \int_C e^z\,dz$, C: | z | = 1, then I is

(A) 0 (B) πi
(C) -1 (D) $2\pi i$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$.

22. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla r^n = nr^{n-2}\,\vec{r}$.

23. Find $L^{-1}\left[\cot^{-1}\left(\frac{2}{s+1}\right)\right]$.

24. Derive C-R equations in polar form.

25. Expand e^{2z} about $z = 2i$ in Taylor's series.

26. Find the image of the triangular region in the z-plane bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$, under the transformation $w = 2z$.

27. If $L[f(t)] = \frac{1}{s(s+1)(s+2)}$, find $\lim_{t\to 0} f(t)$ and $\lim_{t\to\infty} f(t)$.

b. i. Change the order of integration and then evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.

ii. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$, and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ using triple integration.

29. a.i. Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3).

ii. Show that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find its scalar potential.

(OR)

b. Verify Green's theorem in the plane for $\int_c (3x^2 - 8y^2)dx + (4x - 6xy)dy$ where c is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.

30. a.i. Using convolution theorem, find $L^{-1}\left(\frac{1}{(s+3)(s-1)}\right)$.

ii. Find $L(te^{-t} \sin t)$.

(OR)

b. Using Laplace transform, solve $y'' - 3y' + 2y = e^{-t}$, given $y(0) = 1$, $y'(0) = 0$.

31. a.i. An electrostatic field in the xy plane is given by the potential function $\phi = 3x^2y - y^3$. Find its stream function.

ii. Determine the bilinear transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = i$, $w_2 = -1$, $w_3 = -i$ respectively.

(OR)

b. i. If $f(z)$ is an analytic function of z, show that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$.

ii. Find the image of the circle $|z-1| = 1$ under the mapping $w = \frac{1}{z}$.

32. a. Using Cauchy's integral formula evaluate $\int_c \frac{z+4}{z^2+2z+5} dz$, $c: |z+1+i| = 2$.

(OR)

b. i. Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+9)(x^2+4)}$ by the method of residues.

ii. Expand $f(z) = \frac{z}{(z-1)(z-3)}$ as a Laurent series in the region $0 < |z-1| < 2$

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2015
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted from the academic year 2013 – 2014 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The value of $\int_0^{\pi/2} \int_0^{\pi/2} \sin(\theta+\phi) d\theta d\phi$ is
(A) 2 (B) 1
(C) 0 (D) −2

2. The value of $\int_1^a \int_1^b \frac{dxdy}{xy}$ is
(A) $\log a + \log b$ (B) $\log a$
(C) $\log b$ (D) $\log a \log b$

3. The value of $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is
(A) 3 (B) 4
(C) 2 (D) 6

4. If R is the region bounded by $x = 0, y = 0, x + y = 1$, then $\iint dxdy$ is
(A) 1 (B) 1/2
(C) 1/3 (D) 2/3

5. Curl (grad$\emptyset$) is
(A) −1 (B) 1
(C) 0 (D) Does not exist

6. Find the constant 'a', if the vector $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal
(A) 2 (B) −2
(C) −1 (D) 0

7. The condition for $\vec{F}$ to be conservative is, $\vec{F}$ should be
(A) Solenoidal vector (B) Irrotational vector
(C) Rotational vector (D) Neither solenoidal nor irrotational

8. If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, then $grad(\vec{a}.\vec{r})$ is
(A) 0 (B) 1
(C) $\vec{a}$ (D) $2\vec{a}$

9. $L(4^t)$ is

(A) $\frac{1}{s-4}$ (B) $\frac{s}{s^2+4}$

(C) $\frac{1}{s-log4}$ (D) $\frac{4}{s}$

10. $L\left(\frac{\cos at}{t}\right)$ is

(A) $\tan^{-1}(s)$ (B) $\log\sqrt{s^2+a^2}$

(C) $\cot^{-1}(s)$ (D) Does not exist

11. $L(\sin(2t+3))$ is

(A) $\frac{2}{s^2+4}$ (B) $\frac{2\cos 3}{s^2+4}+\frac{(\sin 3)s}{s^2+4}$

(C) $\frac{s}{s^2+9}$ (D) $\frac{6}{s^2+9}$

12. $L^{-1}\left(\frac{s-3}{s^2-6s+13}\right)$ is

(A) $e^{-3t}\cos 3t$ (B) $e^{2t}\cos 3t$

(C) $e^{3t}\cos 2t$ (D) $e^{-2t}\cos 2t$

13. An analytic function with constant modulus is

(A) a function of x (B) a function of y

(C) a function of z (D) a constant

14. The complex function $w = az$ when a is a complex constant geometrically implies

(A) Rotation (B) Rotation and magnification

(C) Rotation and reflection (D) Translation

15. The invariant points of the transformation $w=\frac{z-1}{z+1}$ is

(A) $\pm i$ (B) ± 1

(C) 0 (D) 2, 7

16. Cauchy Riemann equation in polar coordinates are

(A) $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r}=\frac{-1}{r}\frac{\partial u}{\partial \theta}$ (B) $\frac{\partial u}{\partial \theta}=r\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r}=-r\frac{\partial u}{\partial \theta}$

(C) $\frac{\partial v}{\partial r}=\frac{1}{r}\frac{\partial u}{\partial \theta}; \frac{\partial u}{\partial r}=\frac{-1}{r}\frac{\partial v}{\partial \theta}$ (D) $\frac{\partial u}{\partial r}=\frac{\partial v}{\partial \theta}, \frac{\partial u}{\partial \theta}=-\frac{\partial v}{\partial r}$

17. If a is a simple pole, then Residue of $\left\{\frac{\phi(z)}{\psi(z)}\right\}$ at $z=a$ is

(A) $\frac{\phi'(a)}{\psi(a)}$ (B) $\frac{\phi'(a)}{\psi'(a)}$

(C) $\frac{\phi(a)}{\psi(a)}$ (D) $\frac{\phi(a)}{\psi'(a)}$

18. The value of $\int_c \frac{3z^2+7z+1}{z+1}dz,\ c:|z|=\frac{1}{2}$

(A) $2\pi i$ (B) 0

(C) πi (D) $\frac{\pi i}{2}$

19. If $f(z)=\frac{-1}{z-1}-2\{1+(z-1)+(z-1)^2+....\}$ then the residue of $f(z)$ at $z=1$ is

(A) 0 (B) -1

(C) -2 (D) 2

20. The singularity of $f(z)=\frac{z}{(z-2)^3}$ is

(A) Essential singularity (B) Removable singularity

(C) Pole of order 3 (D) Pole of order 1

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_2^4\int_0^x\int_0^{\sqrt{x+y}} z\,dz\,dy\,dx$.

22. If $r=|\vec{r}|$, where $\vec{r}$ is the position vector of the point (x, y z) with respect to the origin, prove that $\nabla f(r)=\frac{f'(r)}{r}\vec{r}$.

23. Find the work done when a force $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ displays a particle in the xy plane form (0, 0) to (1, 1) along the parabola $y^2=x$.

24. Find $L\left(\frac{e^{-t}-e^{-3t}}{t}\right)$.

25. Find $L^{-1}\left(\frac{e^{-s}}{(s+3)(s-2)}\right)$.

26. Show that $sinz$ is an analytic function of z.

27. Evaluate $\int_c \frac{z+1}{z(z-1)}dz;\ c:|z|=2$, using Cauchy's residue theorem.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. By changing into polar coordinates evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)dxdy$.

ii. Find the area of the cardiod $r=a(1+\cos\theta)$ using double integration.

(OR)

b. Find the volume of the tetrahedron bounded by the co-ordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

29. a.i. Find the constants a and b so that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at (1, −1, 2).

ii. If $\overline{F}=2y\hat{i}-z\hat{j}+x\hat{k}$ evaluate $\oint_c \overline{F}\times d\overline{r}$ along the curve $x=\cos t, y=\sin t, z=2\cos t$ from $t=0$ to $\frac{\pi}{2}$.

(OR)

b. Verify Gauss-Divergence theorem for the function $F=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ taken over the cube bounded by the planes $x=0, x=1, y=0, y=1$ and $z=0, z=1$.

30. a. Solve using Laplace transform $y''+2y'-3y=\sin t$ given $y(0)=0, y'(0)=0$.

(OR)

b.i. Using convolution theorem, evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

ii. Find the Laplace transform of the periodic function f(t) with period '2' given by $f(t)=\begin{cases}1 & 0<t<1\\ -1 & 1\le t<2\end{cases}$

31. a.i. Find the harmonic conjugate of $u=\frac{1}{2}\log(x^2+y^2)$.

ii. Find the bilinear transformation that maps the point $z = 1, i, -1$ in the z-plane into the points $w=2, i, -2$ in the w-plane.

(OR)

b.i. Find the analytic function f(z) in terms of z if $u + v = (x-y)(x^2+4xy+y^2)$.

ii. Determine the region D of the w plane into which the triangular region D enclosed by the lines $x=0, y=0, x+y=1$ is transformed under the transformation $w=2z$.

32. a.i. Evaluate $\oint_c \frac{ze^{2z}}{(z-1)^3}dz$ by using Cauchy's integral formula where C is a circle $|z+i|=2$.

ii. Find the Laurent series expansion of $\frac{1}{(z+1)(z+3)}$ $in\ 0<|z+1|<2$.

(OR)

b. i. Using Cauchy's residue theorem, evaluate $\oint_c \frac{z^2}{(z-1)^2(z+1)}dz$ where 'c' is $|z|=2$.

ii. Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ using Contour integration.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2015
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The value of the integral $\int_1^2\int_2^4 \frac{dxdy}{xy}$ is
(A) $2(\log_e 2)^2$ (B) $\log_e 2$
(C) $(\log_e 2)^2$ (D) $\log_e 4$

2. The value of the integral $\int_0^{\pi/2}\int_0^{\sin\theta} r^4\, drd\theta$ is
(A) $\frac{6}{75}$ (B) $\frac{4}{75}$
(C) $\frac{2}{75}$ (D) $\frac{8}{75}$

3. If R is the region bounded by $x=0, y=0, x+y=1$ then $\iint dxdy$ is equal to
(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

4. Area of the double integral in Cartesian co-ordinate is equal to
(A) $\iint drd\theta$ (B) $\iint rdrd\theta$
(C) $\iint dxdy$ (D) $\iint xdxdy$

5. The unit normal vector to the surface $x^2+y^2-z^2=1$ at (1, 1, 1) is
(A) $\frac{2i+2j+2k}{\sqrt{2}}$ (B) $\frac{2i-2j+2k}{\sqrt{2}}$
(C) $\frac{i+j-k}{\sqrt{3}}$ (D) $\frac{i+j+k}{\sqrt{3}}$

6. If $\phi=xyz$ then $\nabla\phi$ is
(A) $xy\hat{i}+yz\hat{j}+zx\hat{k}$ (B) $yz\hat{i}+zx\hat{j}+xy\hat{k}$
(C) $zx\hat{i}+xy\hat{j}+yz\hat{k}$ (D) 0

7. If $\emptyset$ is a scalar function, then curl(grad$\emptyset$) is
(A) Solenoidel (B) Irrotational
(C) 0 (D) Constant vector

8. If $\bar{r}$ is the position vector of the point (x, y, z) with respect to origin, then $div\bar{r}$ is
(A) 1 (B) -1
(C) 2 (D) 3

9. Laplace transform of $tcosat$ is
(A) $\frac{s^2+a^2}{(s^2-a^2)^2}$ (B) $\frac{s^2-a^2}{s^2+a^2}$
(C) $\frac{s^2-a^2}{(s^2+a^2)^2}$ (D) $\frac{s^2+a^2}{(s^2-a^2)^2}$

10. Inverse Laplace transform of $\frac{s}{(s+2)^2}$ is
(A) $e^{2t}(1-2t)$ (B) $e^{-2t}(1-2t)$
(C) $e^{-2t}(1+2t)$ (D) $e^{2t}(1+2t)$

11. $L\left(\frac{\sin 4t}{t}\right)$ is
(A) $\cot^{-1}\left(\frac{4}{s}\right)$ (B) $\tan^{-1}\left(\frac{4}{s}\right)$
(C) $\cot^{-1}\left(\frac{4}{s+1}\right)$ (D) $\tan^{-1}\left(\frac{4}{s+1}\right)$

12. $L^{-1}\left(\frac{1}{s^2+9}\right)$ is
(A) $\frac{\cos 3t}{3}$ (B) $\sin 3t$
(C) $\frac{\sin 3t}{3}$ (D) $\cos 3t$

13. Cauchy-Riemann equation in polar co-ordinates are
(A) $rU_r = V_\theta, U_\theta = -rV_r$ (B) $-rU_r = V_\theta, U_\theta = rV_r$
(C) $-rU_r = V_\theta, U_\theta = -rV_r$ (D) $U_r = rV_\theta, rU_\theta = V_r$

14. The invariant point of the transformation $w = \frac{1}{z-2i}$ is
(A) $z = i$ (B) $z = -i$
(C) $z = 1$ (D) $z = -1$

15. An analytic junction with constant modulus
(A) Zero (B) Analytic
(C) Harmonic (D) constant

16. If $u + iv$ is analytic, then the curves u = C_1, V=C_2
(A) Intersect each other (B) Cut orthogonally
(C) Are parallel (D) Are perpendicular

17. Value of $\oint_c \frac{dz}{z-1}$ where C is $|z-1| = 1$ is
(A) πi (B) $2\pi i$
(C) 0 (D) $-\pi i$

18. If f(z) is analytics inside and on c, the value of $\oint_c \frac{f(z)}{z-a}dz$, where c is a simple closed curve and 'a' is any point within 'c' is
(A) $f(a)$ (B) $2\pi i f(a)$
(C) $\pi i f(a)$ (D) 0

19. The point z_0 at which a function f(z) is not analytic is known as
(A) Isolated singular point (B) Zeros
(C) Singular point (D) Removable singular point

20. If $I = \oint_c \frac{z^2}{(z-1)^2(z+1)}dz$, where c is $|z| = \frac{1}{2}$, then I is
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) 0 (D) $\frac{1}{3}$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Show by double integration that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$

22. Show that the vector $\overline{F}$ is given by $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Find its scalar potential.

23. If $\bar{u}$ and $\bar{v}$ are irrotational vector, then show that $\bar{u} \times \bar{v}$ is a solenoidal vector.

24. Evaluate $L\left[\int_0^t \frac{\cos 6t - \cos 4t}{t} dt\right]$.

25. Verify final value theorem for the function $1+e^{-t}(sint+cost)$.

26. Prove that an analytic function with constant modulus is constant.

27. Evaluate $\oint_c \tan z\, dz$ where c is a circle $|z| = 2$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dxdy$.

(OR)

29. a. Verify Green's theorem in a plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)]dy$, where C is the boundary of the region defined by the lines $x = 0$, $y = 0$, and $x + y = 1$.

(OR)

b. Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ where S is the surface of the cuboid bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$, $z = c$.

30. a.i. Prove that an analytic function with constant modulus is constant.

ii. Find the bilinear transformation that maps the points ∞, 0, i, in Z plane on the points 0, ∞, $-i$ of the W-plane.

(OR)

b.i. Discuss the transformation $W = \frac{1}{Z}$.

ii. If $u + v = e^x(\cos y + \sin y)$, where $f(z) = u + iv$ is analytic, find the analytic function f(z) and hence find its derivative.

31. a.i. Verify initial value theorem and final value theorem on Laplace transform for $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

ii. Solve the integral equation using Laplace transform $y(t) = 1 + \int_0^t y(u)\sin(t - u)du$.

(OR)

b.i. Solve the differential equation using Laplace transform $y''(t) + y(t) = 2e^t$, given that $y(0) = 1, y'(0) = 2$.

ii. (1) Find $L^{-1}\left(\frac{(s-1)}{(s^2 + 2s + 2)^2}\right)$. (3 Marks)

(2) Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$. (3 Marks)

32. a.i. Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + \cos\theta}$ using contour integration.

ii. Expand $f(z) = \frac{4z + 3}{(z+1)z(z+3)}$ in Laurent's series in the region given by

(1) $0 < |z + 1| < 3$

(2) $1 < |z| < 3$.

(OR)

b.i. Evaluate using Cauchy's integral formula $\int_c \frac{z\,dz}{(4z+1)(z-1)(z-2)}$ where C is the circle $|z| = 3$.

ii. Expand $\frac{z}{z^2 + 3z + 2}$ using Taylor's series in the region $1 < |z| < 2$.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, JUNE 2016
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. Evaluate $\int_0^1\int_0^1 dxdy$

(A) 1 (B) 2
(C) 0 (D) 4

2. Area using double integral in Cartesian co-ordinate is

(A) $\iint_A dydx$ (B) $\iint_R rdrd\theta$
(C) $\iint_R xdxdy$ (D) $\iint_R x^2dxdy$

3. The value of the triple integral $\int_0^1\int_0^2\int_0^3 dxdydz$ is

(A) 3 (B) 4
(C) 2 (D) 6

4. If R is the region bounded by $x = 0, y = 0, x + y = 1$, then $\iint_R dxdy =$ ________.

(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

5. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to origin then $\nabla.\vec{r} =$ ________.

(A) 0 (B) 1
(C) 2 (D) 3

6. If $\vec{u}$ and $\vec{v}$ are irrotational then $\vec{u} \times \vec{v}$ is

(A) solenoidal (B) irrotational
(C) constant vector (D) zero vector

7. If $\vec{F} = \lambda y^4z^2\vec{i} + 4x^3z^2\vec{j} + 5x^2y^2\vec{k}$ is solenoidal, then the value of λ is ________.

(A) x (B) −x
(C) λ can take any real value (D) xy^2

8. Evaluate the line integral $\int_c \vec{r}.d\vec{r}$ where C is the line $y = x$ in XY plane from (1, 1) to (2, 2)

(A) 0 (B) 1
(C) 2 (D) 3

9. $L^{-1}\left(\frac{1}{\sqrt{S}}\right)$

(A) $\frac{1}{\sqrt{t}}$

(B) $\frac{1}{\sqrt{\pi t}}$

(C) $\frac{1}{t^2}$

(D) $\frac{3}{t^2}$

10. $L^{-1}(1) =$ ______.

(A) $\frac{1}{s}$

(B) t

(C) H(t)

(D) $\delta(t)$

11. $L\left(\frac{\cos at}{t}\right)$

(A) $\frac{s}{s^2+a^2}$

(B) $\frac{1}{s^2+a^2}$

(C) does not exist

(D) $\frac{s^2-a^2}{(s^2+a^2)^2}$

12. Evaluate using Laplace transform without integration $\int_0^\infty \left(\frac{e^{-t}-e^{-2t}}{t}\right)dt$

(A) log 2

(B) $\log\left(\frac{1}{2}\right)$

(C) $\log\left(\frac{3}{4}\right)$

(D) log 5

13. If $f(z) = u + iv$ is analytic, then the family of curves, $u=c_1$ and $v= c_2$.

(A) cut orthogonally

(B) intersect each other

(C) are parallel

(D) coincides

14. If f(z) and $\overline{f(z)}$ are analytic functions of z, then $f(z)$ is

(A) analytic function

(B) zero function

(C) constant function

(D) discontinuous function

15. The invariant point of the transformation $W = \frac{1}{Z-2i}$ is

(A) Z = i

(B) Z = −i

(C) Z = 1

(D) Z = −1

16. If a function u(x,y) satisfies the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, then u is called

(A) Analytic function

(B) harmonic function

(C) differential function

(D) continuous function

17. The residue of $f(z) = \cot z$ is

(A) π

(B) 1

(C) −1

(D) 0

18. The value of $\int_c \frac{zdz}{z-2}$ where C is the circle $|z|=1$ is

(A) 0

(B) $\frac{\pi i}{2}$

(C) $\frac{\pi}{2}$

(D) 2

19. If $f(z)=\frac{1}{(z-4)^2(z-3)^3(z-1)}$, then

(A) 4 is a simple pole, 3 is a pole of order 3 and 1 is a pole order 2

(B) 3 is a simple pole, 1 is a pole of order 3 and 4 is a pole of order 2

(C) 1 is a simple pole, 3 is a pole of order 3 and 4 is a pole of order 2

(D) 3 is a simple pole, 4 is a pole of order 1 and 1 is a pole of order 3.

20. $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ consisting of negative powers of (z - a) is called

(A) the analytic part of Laurent's series

(B) the principle part of Laurent's series

(C) the real part of Laurent's series

(D) the imaginary part of the Laurent's series

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the value of $\int_0^1 \int_x^{\sqrt{x}} xy(x+y)dydx$.

22. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin\phi\, dr d\phi d\theta$.

23. Find the angle between the normals to the surface $xy = z^2$ at the points (−2, −2, 2) and (1, 9, −3).

24. Use Stoke's theorem to evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F}=(\sin x - y)\vec{i} - \cos x\vec{j}$ and C is the boundary of the triangle whose vertices are $(0,0),\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$.

25. Find $L^{-1}\left(\frac{1}{s(s+1)}\right)$ using convolution theorem.

26. Show that the function $f(z)=e^z$ is analytic and find its derivative.

27. Evaluate $\int_c \frac{(4z+1)dz}{z(z-1)(z-3)}$ where C is the circle $|z|=2$ using residue theorem.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Evaluate $\int_0^1\int_0^{1-z}\int_0^{1-y-z} xyz\, dxdydz$.

ii. Evaluate $\iint_D xydxdy$ where D is the region of the positive quadrant bounded by the circle $x^2+y^2=a^2$.

(OR)

b.i. Find the area bounded by the parabolas $y^2=4ax$ and $x^2=4ay$.

ii. Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\, dydx$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Change the order of integration in $\int_{0}^{a}\int_{x^2/a}^{2a-x} xy\,dy\,dx$ and then evaluate it.

(OR)

b. Express the volume of the sphere $x^2+y^2+z^2=a^2$ as a volume integral and hence evaluate it.

29. a. Verify stoke's theorem for $\bar{F}=(y-z+2)\hat{i}-(yz+4)\hat{j}-(xz)\hat{k}$ over the surface of a cube x = 0, y = 0, z = 0, x = 2, y = 2, z = 2 above the XOY plane.

(OR)

b. Verify Gauss divergence theorem for $\bar{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over the rectangular parallelepiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

30. a.i. Find the Laplace transform of a periodic function f(t) with period 2, given by $f(t)=\begin{cases}1, 0<t<1\\ -1, 1<t<2\end{cases}$

ii. Using convolution theorem, find $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

(OR)

b.i. Verify the final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

ii. Solve $y''+2y'-3y=\sin t$, given $y(0)=y'(0)=0$.

31. a.i. Show that an analytic function with constant modulus is constant.

ii. Using Milne-Thomson method, find the analytic function $f(z)=u+iv$, if $u-v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$.

(OR)

b.i. Find the image of $|z-2i|=2$ under the transformation $w=\dfrac{1}{z}$.

ii. Find the bilinear transformation which maps the points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=0, w_3=-i$ and hence find the image of $|z|<1$.

32. a. Evaluate $\int_C \dfrac{\cos\pi z^2}{(z-1)(z-2)}dz$ where C is a circle $|z|=3$, using Cauchy's integral formula.

(OR)

b.i. Find Laurent's series expansion of $\dfrac{1}{(z+1)(z+3)}$ in powers of (z +1) in the region $0<|z+1|<2$.

ii. Using Cauchy's residue theorem, evaluate $\int_C \dfrac{\sin\pi z^2+\cos\pi z^2}{z+z^2}dz$ where C is the circle $|z|=2$.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, DECEMBER 2016
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
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(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of $\int_0^2\int_0^1 4xy\,dx\,dy$ is
(A) 4 (B) 3
(C) 2 (D) 1

2. Change the order of integration in $\int_0^a\int_y^a \dfrac{x}{x^2+y^2}dx\,dy$
(A) $\int_0^a\int_x^a \dfrac{x}{x^2+y^2}dy\,dx$ (B) $\int_0^a\int_0^x \dfrac{x}{x^2+y^2}dy\,dx$
(C) $\int_0^a\int_0^{x^2} \dfrac{x}{x^2+y^2}dy\,dx$ (D) $\int_0^a\int_x^{x^2} \dfrac{x}{x^2+y^2}dy\,dx$

3. The value of $\int_0^\pi\int_0^{\sin\theta} r\,dr\,d\theta$ is
(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$
(C) π (D) $\dfrac{-\pi}{2}$

4. The value of $\int_0^1\int_0^2\int_1^2 x^2yz\,dz\,dy\,dx$ is
(A) 2 (B) -2
(C) 3 (D) 1

5. Angle between two level surfaces $\varphi_1=c$ and $\varphi_2=c$ is given by
(A) $\sin\theta=\dfrac{\nabla\varphi_1.\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$ (B) $\cos\theta=\dfrac{\nabla\varphi_1.\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$
(C) $\cos\theta=\dfrac{\nabla\varphi_1\times\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$ (D) $\sin\theta=\dfrac{\nabla\varphi_1\times\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$

6. A vector $\overline{v}$ is said to be solenoidal if
(A) curl $\overline{v} = \overline{0}$
(B) grad $(\nabla.\overline{v}) = \overline{0}$
(C) div $\overline{v} = 0$
(D) div $\overline{v} \neq 0$

7. The unit normal to the surface $x^2 + 2y^2 + z^2 = 7$ at the point (1, -1, 2) is
(A) $\frac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$
(B) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{4}$
(C) $\frac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}$
(D) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

8. If $\overline{u}$ *and* $\overline{v}$ are irrotational, then $(\overline{u} \times \overline{v})$ is
(A) Solenoidal
(B) Irrotational
(C) Rotational
(D) Curvl $\overline{u}$

9. If L[f(t)=F(s), then $L\left[e^{at} f(t)\right]$ is equal to
(A) $F(s+a)$
(B) $F(s-a)$
(C) $e^{as}F(s)$
(D) $e^{-as}F(s)$

10. $L[\cos 2t]$ is equal to
(A) $\frac{s}{s^2-4}$
(B) $\frac{s}{s^2-2}$
(C) $\frac{1}{s^2-4}$
(D) $\frac{s}{s^2+4}$

11. $L^{-1}\left[\frac{1}{s^2+9}\right]$ is equal to
(A) $\frac{\sin 3t}{3}$
(B) $\frac{\sin t}{3}$
(C) $\frac{\cos 3t}{3}$
(D) $\sin 3t$

12. $L^{-1}\left[\frac{s-2}{s^2-4s+13}\right]$ is equal to
(A) $e^{3t}\cos 2t$
(B) $e^{3t}\cos 3t$
(C) $e^{3t}\cos t$
(D) $e^{2t}\cos 3t$

13. $f(z) = \frac{1}{z^2+1}$ is analytic everywhere except at
(A) $z = 2 + i$
(B) $z = \pm i$
(C) i
(D) 2-i

14. The invariant points of the transformation $w = \frac{2z+6}{z+7}$ are
(A) 6, -1
(B) 3, 2
(C) -3, 2
(D) -6, 1

15. The image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$ is
(A) $x^2 + y^2 = 0$
(B) $x^2 + y^2 + 4y = 0$
(C) $x^2 + y^2 - 4y = 0$
(D) $x^2 + y^2 + y = 0$

16. The transformation w = az (where 'a' is a real constant) represents
(A) Magnification
(B) Rotation
(C) Reflection
(D) Inversion

17. The value of $\int_C \frac{e^{-z}}{z+1} dz$, where C is a circle $|z| = \frac{1}{2}$ is
(A) 1
(B) 0
(C) +1
(D) -2

18. If f(z) is analytic inside and on a simple closed curve C, and if 'a' is any point within C, then
(A) $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a} dz$
(B) $f(a) = \int_C \frac{f(z)}{z-a} dz$
(C) $f(a) = 0$
(D) $f(a) = \frac{1}{2\pi}\int_C \frac{f(z)}{z+a} dz$

19. The poles of $f(z) = \frac{4-3z}{z(z-1)(z-2)}$ are
(A) 1, 2, -2
(B) 0, 1, 2
(C) 1, 1, -1
(D) 1, 1, 2

20. The residue of $f(z) = \frac{z}{(z^2+1)(z-2)}$ at the pole z = 2 is
(A) $\frac{1}{5}$
(B) $\frac{3}{5}$
(C) $\frac{2}{5}$
(D) $\frac{4}{5}$

PART – B (5 x 4 = 20 Marks)

Answer **ANY FIVE** Questions

21. Change the order integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and hence evaluate it.

22. Show that the vector field $\overline{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and hence find the scalar potential.

23. Find $L^{-1}\left[\frac{1}{s(s^2+2s+2)}\right]$.

24. Find the harmonic conjugate of $u = e^x \cos y$.

25. Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where C is a circle $|z| = 2$, using Cauchy's integral formula.

26. Verity Green's theorem for the integral $\oint_C \left[(x^2 + y)dx - xy^2 dy\right]$ taken around the boundary of the square whose vertices are (0,0), (1,0), (1,1) and (0,1).

27. Find by double integration, smaller of the areas bounded by the circle $x^2 + y^2 = 9$ and $x + y = 3$.

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B.Tech. DEGREE EXAMINATION, JUNE 2017
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The value of $\int_0^1 \int_0^x dxdy$ is
   (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$
   (C) $\frac{y}{2}$ (D) $\frac{y^2}{2}$

2. Area of a region in polar co-ordinate system is
   (A) $\iint_R r dr d\theta$ (B) $\iint_R \theta dr d\theta$
   (C) $\iint_R d\theta dr$ (D) $\iint_R dxdy$

3. $\int_0^1 \int_0^1 e^{x+y}\, dxdy$ is equal to
   (A) e^2 (B) $(e-1)^2$
   (C) 1 (D) 0

4. Volume of a region R is given by
   (A) $\iint_R dxdy$ (B) $2\iint_R dxdy$
   (C) $\frac{1}{2}\iint_R dxdy$ (D) $\iiint_R dv$

5. If $\vec{F}$ is an irrotational vector, then curl $\vec{F}$ =
   (A) 1 (B) 2
   (C) 3 (D) 0

6. According to Green's theorem $\int_c (Pdx+Qdy) =$
   (A) $\iint_R \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy$ (B) $\iint_R \left(\frac{\partial Q}{\partial y}-\frac{\partial P}{\partial x}\right)dxdy$
   (C) $\iint_R \left(\frac{\partial P}{\partial x}-\frac{\partial Q}{\partial y}\right)dxdy$ (D) $\iint_R \left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)dxdy$

7. If $\overline{F}$ is a conservative vector field, then

(A) $curl\ \overline{F}=0$

(B) $grad\ \overline{F}=0$

(C) $div\ \overline{F}=0$

(D) Directional derivative = 0

8. $curl(grad\,\phi)$ is

(A) 1

(B) 0

(C) 3

(D) −1

9. $L(t^3)$ is

(A) $\frac{4}{s^3}$

(B) $\frac{2}{s^2+4}$

(C) $\frac{6}{s^4}$

(D) $\frac{s}{s^2-4}$

10. $L^{-1}\left[\frac{1}{(s-5)^2}\right]$ is

(A) $e^{5t}.t^4$

(B) $\frac{e^{5t}}{2^4}$

(C) $\frac{e^{5t}.t^4}{24}$

(D) $\frac{t^4}{24}$

11. If $L[f(t)]=F(s)$, then $L\left[\frac{f(t)}{t}\right]$ is if $\lim_{t\to 0}\frac{f(t)}{t}$ exists

(A) $\int_0^{\infty} F(s)ds$

(B) $\int_s^{\infty} F(s)ds$

(C) $\int_{-\infty}^{\infty} F(s)ds$

(D) $\int_a^{\infty} F(s)ds$

12. $L\left[te^{2t}\right]=$

(A) $\left(\frac{1}{(s-2)^2}\right)$

(B) $-\left(\frac{1}{(s-2)^2}\right)$

(C) $\frac{1}{(s-1)^2}$

(D) $\frac{1}{s^2}$

13. Cauchy Riemann equations in polar coordinates for an analytic function f(z) are

(A) $u_r=\frac{1}{r}v_\theta,\ v_r=-\frac{1}{r}u_\theta$

(B) $u_r=v_\theta,\ v_r=-u_\theta$

(C) $u_r=\frac{1}{r}v_\theta,\ v_r=-\frac{1}{r}v_\theta$

(D) $u_r=u_\theta,\ v_r=-v_\theta$

14. $\omega=\frac{1}{z}$ is known as

(A) Translation

(B) Inversion

(C) Rotation

(D) Transition

15. The points coincide with their transformations are known as

(A) Fixed points (B) Critical points
(C) Singular points (D) Regular points

16. The mapping defined by an analytic function f(z) is conformal at all points z, except at points where

(A) $f'(z)=0$ (B) $f'(z)\neq 0$
(C) $f'(z)>0$ (D) $f'(z)<0$

17. A region which is not simply connected is called a _____region.

(A) Multiply connected (B) Jordan connected
(C) Connected curve (D) Multi curve

18. If f(z) is analytic and f '(z) is continuous at all points inside and on a simple closed curve c, then

(A) $\int_c f(z)dz \neq 0$ (B) $\int_c f(z)dz = 1$

(C) $\int_c f(z)dz \neq 1$ (D) $\int_c f(z)dz = 0$

19. The singular points of $f(z)=\dfrac{z+3}{(z-1)(z-2)}$ are

(A) z = 1, 3 (B) z = 1, 0
(C) z = 1, 2 (D) z = 2, 3

20. A zero of an analytic function f(z) is a value of z for which

(A) f(z) = 1 (B) f(z) ≠ 1
(C) f(z) ≠ 0 (D) f(z) = 0

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the area enclosed by the curves $y^2 = 4x$ and $x^2 = 4y$.

22. Find the angle between the normal to the surface $x^2 = yz$ at the points (1, 1, 1) and (2, 4, 1)

23. Find $L\left[\dfrac{\sin 3t\ \cos t}{t}\right]$.

24. Show that an analytic function with constant modulus is constant.

25. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_c \vec{F}.\vec{dr}$ where c is the arc of the parabola $y = 2x^2$ from (0, 0) to (1, 2)

26. Find the invariant points of the transformation $\omega = \dfrac{-2z+4i}{iz+1}$.

27. Find the residue at the pole of the function $f(z)=\dfrac{z}{z^2+1}$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b.i. Show that $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \dfrac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}} = \dfrac{\pi^2}{8}$.

ii. Evaluate $\int_0^{\log a} \int_0^{x} \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

29. a.i. Prove that $\vec{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$ is irrotational and find the scalar potential.

ii. Find the directional derivative of $\phi=x^2+y^2+4xyz$ at (1, −2, 2) in the direction of $2\vec{i}-2\vec{j}+\vec{k}$.

(OR)

b. Verify Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ over the cuboid formed by $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.

30. a. Solve by Laplace transform $\dfrac{d^2y}{dt^2}-4\dfrac{dy}{dt}-5y=\sin t$, where $y(0)=1$, $y'(0)=0$.

(OR)

b. Using convolution theorem, find $L^{-1}\left[\dfrac{1}{s^2(s+1)^2}\right]$.

31. a. Show that the function $u=\dfrac{1}{2}\log(x^2+y^2)$ is harmonic. Hence find harmonic conjugate of u.

(OR)

b. If $w=u+iv$ is analytic, prove that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

32. a. i. Using Cauchy's integral formula, evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where c is $|z| = \frac{3}{2}$.

ii. Find the Laurents series of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the region $1 < |z| < 2$.

(OR)

b. Evaluate using contour integration $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$.

* * * * *

b. Find the volume of a sphere $x^2+y^2+z^2=a^2$ using triple integration.

29. a. Verify Gauss Divergence theorem for $F=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$ taken over a rectangular parallelopiped $0\le x\le a,\ 0\le y\le b,\ 0\le z\le c$.

(OR)

b.i. Prove that the vector $\vec{F}=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$ is irrotational and find tis scalar potential.

ii. Show that $\vec{F}=(y^2-z^2+3yz-2x)\vec{i}+(2xy+3xz)\vec{j}+(3xy-2xz+2z)\vec{k}$ is both irrotational and solenoidal.

30. a. Find the Laplace transform of the triangular wave of period 2π define by

$$f(t)=\begin{cases} t & 0\le t\le \pi \\ 2\pi-t & \pi\le t\le 2\pi \end{cases}$$

(OR)

b. Solve $y''+4y'+3y=e^{-t}$ where $y(0)=0,\ y'(0)=0$ using Laplace transformation.

31. a. Find the analytic function f(z) if $u=e^x(x\sin y+y\cos y)$. Hence find V.

(OR)

b.i. Find the bilinear transformation which maps z = 0, 1, ∞ with points $w=i,1,-i$ respectively.

ii. Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines in the z–plane into circles and straight lines in the w-plane.

32. a. Expand $f(z)=\dfrac{z^2-1}{(z+2)(z+3)}$ as a Laurent's series if (i) $|z|<2$ (ii) $2<|z|<3$.

(OR)

b. Using Cauchy's residue theorem, evaluate $\int_c \dfrac{z}{(z-1)^2(z+1)}dz$ where c is (i) $|z|=\frac{1}{2}$ (ii) $|z|=2$

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Reg. No.														

B.Tech. DEGREE EXAMINATION, DECEMBER 2017
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013-2014 and 2014-2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The value of $\int_0^1\int_0^1 dxdy$ is
(A) 1 (B) 2
(C) 0 (D) 4

2. Area of a region in Cartesian form is
(A) $\iint y\,dxdy$ (B) $\iint x\,dxdy$
(C) $\iint_R dxdy$ (D) $\iiint\iint_v dv$

3. $\int_0^\pi\int_0^\pi d\theta d\phi$ is
(A) 1 (B) 0
(C) $\pi/2$ (D) π^2

4. The value of $\int_0^1\int_0^1\int_0^1 dxdydz$ is
(A) 1 (B) 3
(C) 2 (D) 0

5. If $\vec{F}$ is a solenoidal vector, then
(A) $\nabla.\vec{F}=0$ (B) $\nabla\times\vec{F}=0$
(C) $\nabla\vec{F}=0$ (D) $\nabla.\vec{F}=1$

6. The maximum value of directional derivative is
(A) gradient ϕ (B) curl ϕ
(C) $|\nabla\phi|$ (D) $|\nabla.\phi|$

7. Area of a region, by using Green's theorem is
(A) $\int_c(xdy-ydx)$ (B) $\int_c(ydx-xdy)$
(C) $\frac{1}{2}\int_c(xdy-ydx)$ (D) $\frac{1}{2}\int_c(ydx-xdy)$

8. By Stokes theorem $\int_c \vec{F}.\vec{dr}$ is

(A) $\iint_s \nabla\times\vec{F}ds$ (B) $\iint_s \nabla.\vec{F}ds$

(C) $\iint_s \nabla.\vec{F}.\vec{n}\,ds$ (D) $\iint_s \nabla\times\vec{F}.\hat{n}\,ds$

9. $L(t^4) =$

(A) $\frac{4!}{s^5}$ (B) $\frac{3!}{s^4}$

(C) $\frac{2!}{s^4}$ (D) $\frac{5!}{s^4}$

10. $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ is

(A) e^{at} (B) e^{-at}

(C) te^{-at} (D) $-te^{at}$

11. If $L[f(t)] = F(s)$, then $L[f'(t)]$ is

(A) $sL[f(t)] - f(0)$ (B) $sL[f(t)] + f(0)$

(C) $sL[f(t)]$ (D) $s^2L[f(t)] - f(0)$

12. $L(\cos t)$ is

(A) $\frac{1}{s^2-1}$ (B) $\frac{1}{s^2+1}$

(C) $\frac{s}{s^2-1}$ (D) $\frac{s}{s^2+1}$

13. Cauchy Riemann equations for an analytic function $f(z)$ is

(A) $u_x = v_y, u_x = -u_y$ (B) $u_x = v_y, v_x = u_y$

(C) $u_x = v_x, u_y = v_y$ (D) $u_x = -v_x, u_y = v_y$

14. A function u is said to be harmonic iff

(A) $u_{xx} + u_{yy} = 0$ (B) $u_{xy} + u_{yx} = 0$

(C) $u_x^2 + u_y^2 = 0$ (D) $u_x + u_y = 0$

15. $w = \frac{a+bz}{c+dz}$ is a bilinear transformation, when

(A) $ad - bc = 0$ (B) $ad - bc \neq 0$

(C) $ab - cd \neq 0$ (D) $ac - bd \neq 0$

16. A mapping that preserves angles between oriented circles both in magnitudes and in sense is called a ________mapping.

(A) Informal (B) Isoganal

(C) Conformal (D) Formal

17. The point at which a function f(z) is not analytic is known as a ____ of $f(z)$.

(A) Residue (B) Singularity

(C) Integrals (D) Fixed points

18. The fixed points of the transformation $w = z^2$ are

(A) 0, 1 (B) 0, –1

(C) –1, 1 (D) –i, i

19. The poles of $f(z) = \frac{z^2+1}{1-z^2}$ is

(A) 1 (B) –1

(C) ±1 (D) 0

20. If f(z) is analytic and f '(z) is continuous at all points in the region bounded by the simple closed curves c_1, c_2 then

(A) $\int_{c_1} f(z)dz = \int_{c_2} f(z)dz$ (B) $\int_{c_1} f(z)dz \neq \int_{c_2} f(z)dz$

(C) $\int_{c_1} f'(z)dz = \int_{c_2} f'(z)dz$ (D) $\int_{c_1} f'(z)dz \neq \int_{c_2} f'(z)dz$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the area that lies outside the circle $r = 2a\cos\theta$ and inside the circle $r = 4a\cos\theta$.

22. Find the values of the constants a, b, c so that $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ may be irrotational.

23. Show that an analytic function with constant real part is constant.

24. Find the directional derivative of $\phi = 2xy + z^2$ at the point (1, –1, 3) in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$.

25. Find $L^{-1}\left[\frac{1}{s(s^2+9)}\right]$.

26. Find the residue of $f(z) = \frac{z}{(z-1)^2}$ at its poles.

27. Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\,dydx$.

(OR)

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Find the volume of the tetrahedron bounded by the planes $x=y=z=0$, and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(OR)

b. Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx$.

29. a. Verify divergence theorem for $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-xz\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ taken over a rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

(OR)

b.i Find the angle between the surfaces $x^2-y^2-z^2=11$ and $xy+yz-zx=18$ at the point (6, 4, 3).

ii. Show that $\vec{F}=\left(2xy+z^3\right)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ is conservative vector field and hence find the scalar potential.

30. a.i Using convolution theorem, find $L^{-1}\left(\frac{1}{(s+3)(s-1)}\right)$. (8 Marks)

ii. Find $L\left(te^{-t}\sin t\right)$. (4 Marks)

(OR)

b. Solve using Laplace transform method $y''+2y'-3y=\sin t$ given $y(0)=0,\ y'(0)=0$.

31.a.i Determine the bilinear transformation which maps $z_1=0, z_2=1, z_3=\infty$ into $w_1=i$, $w_2=-1$, $w_3=-i$ respectively.

ii. Find the analytic function $f(z)=u+iv$, where $u=e^x(x\sin y+y\cos y)$.

(OR)

b.i. If $f(z)$ is an analytic function of z, show that $\nabla^2|f(z)|^2=4|f'(z)|^2$.

ii. Find the image of the circle $|z-1|=1$ under the mapping $w=1/z$.

32. a. Evaluate $\int_0^\infty \frac{x^2 dx}{\left(x^2+9\right)\left(x^2+4\right)}$ by the method of residues.

(OR)

b.i. Explain $f(z)=\frac{1}{(z-1)(z-2)}$ as Laurent series valid in the region (*i*) $|z|<1$ (*ii*) $1<|z|<2$ (*iii*) $|Z|>2$.

ii. Evaluate $\int_C \frac{e^{2z}}{\cos \pi z} dz$ where c is a circle $|z|=1$.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, JUNE 2018
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. Evaluation of $\int_0^1\int_0^1 dx\, dy$ is
(A) 1 (B) 2
(C) 0 (D) 4

2. The value of $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin(\theta+\phi)\, d\theta\, d\phi$
(A) 2 (B) 1
(C) 0 (D) –2

3. The value of $\int_0^1\int_0^2\int_0^3 dx\, dy\, dz$ is
(A) 3 (B) 4
(C) 2 (D) 6

4. If R is the region bounded by $x=0, y=0, x+y=1$ then $\iint_R dx\, dy$
(A) 1 (B) 1/2
(C) 1/3 (D) 2/3

5. Area of the double integral in polar co-ordinate is equal to
(A) $\iint_R dr d\theta$ (B) $\iint_R r^2\, dr\, d\theta$
(C) $\iint_R (r+1)\, dr\, d\theta$ (D) $\iint_R r\, dr\, d\theta$

6. Find the constant 'a', if the vector $\vec{F}=(x+3y)+(y-2z)\vec{j}+(x+az)\vec{k}$ is solenoidal.
(A) 2 (B) –2
(C) –1 (D) 0

7. If $\vec{r}$ is the position vector of the point (x,y,z) with respect to the origin then $\nabla . \vec{r}$ is
(A) 3 (B) 2
(C) 0 (D) 1

8. The relation between the line integral and double integral is
(A) Stokes theorem (B) Green's theorem
(C) Surface integral (D) Divergence theorem

9. $L(\sin 2t)$ is
(A) $\frac{2}{s^2+4}$ (B) $\frac{2\cos 3s}{s^2+4}+\frac{\sin 3s}{s^2+4}$
(C) $\frac{s}{s^2+9}$ (D) $\frac{6}{s^2+9}$

10. $L^{-1}\left[\frac{s-3}{s^2-6s+13}\right]$ is
(A) $e^{-3t}\cos 3t$ (B) $e^{2t}\cos 3t$
(C) $e^{3t}\cos 2t$ (D) $e^{-2t}\cos 2t$

11. $L(\sin 5t)$ is
(A) $\frac{1}{s^2-1}$ (B) $\frac{5}{s^2+25}$
(C) $\frac{s}{s^2-1}$ (D) $\frac{5}{s^2+25}$

12. Inverse laplace transform of $\frac{s}{s^2-9}$ is
(A) cos 9t (B) cos 3t
(C) cos h9t (D) cos h3t

13. The invariant points of the transformation $w=\frac{z-1}{z+1}$ is
(A) $\pm i$ (B) ± 1
(C) 0 (D) 2, 7

14. The points at which the function $f(z)=\frac{1}{z^2+1}$ fails to be analytic at
(A) $z=\pm 1$ (B) $z=0$
(C) $z=\pm i$ (D) $z=\pm 2$

15. An analytic function with constant modulus is
(A) a function of x (B) a function of y
(C) a function of z (D) a constant

16. The value of $\int_c \frac{z}{z-2}dz$ where c is the circle $|z|=1$ is
(A) 0 (B) $\frac{\pi}{2}i$
(C) $\pi/2$ (D) 2

17. The point Z_0 at which a function $f(z)$ is not analytic is known as
(A) Zeros (B) Isolated singular point
(C) Singular point (D) Removable singular point

18. The residue of $f(z)=\frac{z}{(z-1)^2}$ is
(A) 0 (B) 1
(C) -1 (D) $2\pi i$

19. The annular region for the function $f(z)=\frac{1}{z^2-z-6}$.
(A) $0<|z|<1$ (B) $1<|z|<2$
(C) $2<|z|<3$ (D) $|z|<3$

20. If $f(z)=\frac{\sin z}{z}$, then
(A) z =0 is a simple pole (B) z=0 is a pole of order 2
(C) z=0 is a removable singularity (D) z=0 is a zero of f(z)

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^4\int_0^{x}\int_0^{\sqrt{x+y}} z\,dz\,dy\,dx$.

22. Show that $r^n\vec{r}$ is an irrotational vector for any value of 'n' and is solenoidal only for n= –3.

23. Find the work done when a force $F=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ displaces a particle in the xy plane from (0, 0) to (1, 1) along the parabola $y^2=x$.

24. Verify final value theorem for the function $1+e^{-t}(\cos t+\sin t)$.

25. Find the constants a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.

26. Evaluate $\int_c \frac{(z+1)}{z(z-1)}dz;\ c:|z|=2,$ using Cauchy's residue theorem.

27. Find $L\left(\frac{e^{-2t}-e^{-3t}}{t}\right)$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Change the order of integration and evaluate $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dydx$.

(OR)

b. Find the volume of a sphere $x^2+y^2+z^2=a^2$ using triple integration.

29. a. Show that $\vec{F}=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ is conservative vector field and hence find the scalar potential.

(OR)

b. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$, taken round the rectangle bounded by $x=\pm a, y=0, y=b$.

30. a. Find the Laplace transform of $f(t)=\begin{cases} t, & 0<t\le 2a \\ 4a-t, & 2a\le t<4 \end{cases}$ and satisfy $f(t+4a)=f(t)$.

(OR)

b. Solve $y''+2y'-3y=\sin t$ given $y(0)=y'(0)=0$.

31. a. Verify whether the function $\frac{1}{2}\log(x^2+y^2)$ is harmonic. Find the harmonic conjugate. Also find f(z).

(OR)

b.i. Find the images of the infinite strips (A) $1/4<y<1/2$ (B) $0<y<1/2$ under the transformation $w=1/z$.

ii. Find the bilinear transformation which maps the point $z_1=\infty, z_2=i, z_3=0$ onto the points $w_1=0, w_2=i, w_3=\infty$ respectively.

32. a. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$ by calculus of residues.

(OR)

b. Find the Laurent expansion to represent the function $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ where (i) $|z|<2$, (ii) $2<|z|<3$ and (iii) $|z|>3$.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, DECEMBER 2018
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. $\int_0^1\int_0^x dxdy$ is equal to
(A) 1 (B) ½
(C) 2 (D) 3

2. Change the order of integration in $\int_0^a\int_0^x dxdy$ is
(A) $\int_0^a\int_0^x dxdy$ (B) $\int_0^a\int_0^x xdydx$
(C) $\int_0^a\int_y^a dxdy$ (D) $\int_0^a\int_0^y dxdy$

3. $\int_0^2\int_1^2\int_1^2 xy^2z\,dzdydx$ is
(A) 7 (B) 28
(C) 20 (D) 26

4. Area of the double integral in polar co-ordinate is equal to
(A) $\iint_R drd\theta$ (B) $\iint_R r^2drd\theta$
(C) $\iint_R (r+1)drd\theta$ (D) $\iint_R rdrd\theta$

5. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, then $\nabla.\vec{r}$ is
(A) 2 (B) 3
(C) 0 (D) 1

6. If $\vec{F}=\lambda y^4z^2\vec{i}+4x^3z^2\vec{j}+5x^2y^2\vec{k}$ is a solenoidal, then the value of λ is
(A) x (B) $-x$
(C) Any value (D) 0

7. If the value of $\int_A^B \vec{F}.d\vec{r}$ does not depend on the curve c, but only on the terminal points A and V then $\vec{F}$ is called

(A) Solenoidal vector (B) Consenvative vector
(C) Irrotational vector (D) Rotational vector

8. The relation between the line integral and double integral is

(A) Green's theorem (B) Stokes theorem
(C) Surface integral (D) Divergence theorem

9. $L(\sin \omega t)$

(A) $\frac{s}{s^2+\omega^2}$ (B) $\frac{\omega}{s^2+\omega^2}$
(C) $\frac{s}{s^2-\omega^2}$ (D) $\frac{\omega}{s^2-\omega^2}$

10. $L(te^t)$

(A) $\frac{1}{(s+1)^2}$ (B) $\frac{1}{(s+1)}$
(C) $\frac{1}{(s-1)}$ (D) $\frac{1}{(s-1)^2}$

11. $L\left(t^{-1/2}\right)$

(A) $\sqrt{\pi/s}$ (B) $\sqrt{\pi/2s}$
(C) $\sqrt{1/s}$ (D) $1/s$

12. $L^{-1}\left(\frac{b}{s+a}\right)$

(A) ae^{-bt} (B) be^{-at}
(C) ae^{bt} (D) be^{at}

13. The function $f(z)=u+iv$ is analytic if

(A) $u_x=v_y, u_y=-v_x$ (B) $u_x=-v_y, u_y=v_x$
(C) $u_x=v_y, u_y=v_x$ (D) $u_x=-v_y, u_y=-v_x$

14. If a function $u(x,y)$ satisfies $u_{xx}+u_{yy}=0$, then u is

(A) Analytic (B) Harmonic
(C) Differentiable (D) Continuous

15. If $u+iv$ is analytic, then the curves $u=c_1$ and $v=c_2$

(A) Cut orthogonally (B) Intersect each other
(C) Are parallel (D) Coincides

16. The value of $\int_c \frac{zdz}{z-2}$ where c is the circle $|z|=1$ is

(A) $z=2$ (B) $z=0$
(C) $z=1$ (D) $z=-2$

17. The critical point of transformation $\omega=z^2$ is

(A) $z=2$ (B) $z=1$
(C) $z=0$ (D) $z=-2$

18. The residue of $f(z)=\frac{1-e^{2z}}{z^3}$ is

(A) 0 (B) 2
(C) −2 (D) 1

19. If f(z) is analytic inside and on c, the value of $\int_c f(z)dz$, where c is the simple closed curve is

(A) f(a) (B) $2\pi/f(a)$
(C) $\pi i/f(a)$ (D) 0

20. The residue of $f(z)=\frac{z}{(z-1)^2}$ is

(A) 0 (B) 1
(C) −1 (D) $2\pi i$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the area between $x^2+y^2=a^2$ and $x+y=a$.

22. Evaluate $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$ by changing the order of integration.

23. Find the angle between the surfaces $x^2+yz=2$ and $x+2y-z=2$ at (1, 1, 1).

24. Find the directional derivative of $f(x,y,z)=x^2yz+4xz^2$ at the point (1, −2, −1) in the direction of the vector $2\vec{i}-\vec{j}-2\vec{k}$.

25. Verify final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

26. Find the constant a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.

27. Evaluate $\int_c \frac{\cos z}{z}dz$ where c is an ellipse $9x^2+4y^2=1$.