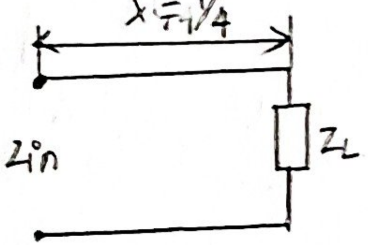
5. Derive the expression for impedance matching using quarter wave transform. (Q-5 and Q-8).



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta L}{Z_0 + jZ_L \tan\beta L}$$

$$l = \lambda/4 \; ; \; \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\beta l = \frac{\pi}{2}$$

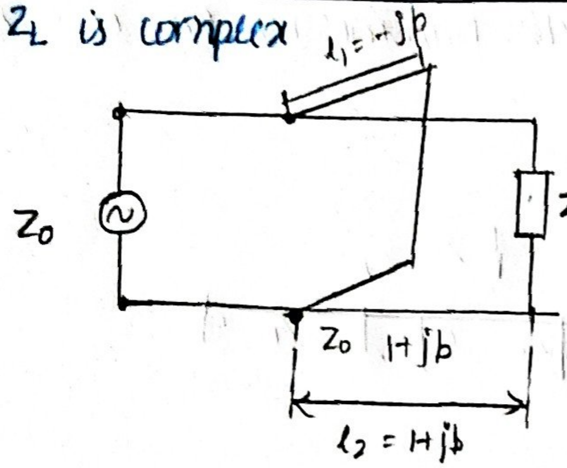
$$\tan\beta l = \tan\frac{\pi}{2} = \infty$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\pi/2)}{Z_0 + jZ_L \tan(\pi/2)} = \cancel{Z_L}\, \frac{Z_0 \tan\beta l \left\{ \frac{Z_L}{\tan\beta L} + jZ_0 \right\}}{\tan\beta L \left\{ \frac{Z_0}{\tan\beta l} + jZ_L \right\}}$$

$$Z_{in} = Z_0 \left[\frac{\frac{Z_L}{\infty} + jZ_0}{\frac{Z_0}{\infty} + jZ_L} \right] = \left(\frac{jZ_0}{jZ_L} \right) \cdot Z_0$$

$$\boxed{\therefore \; Z_{in} = \frac{Z_0^2}{Z_L}}$$

If Z_L is complex



If bothe the reflected waves are out of phase and equal in amplitude

→ Both reflected waves get cancelled

→ Reflection is zero

→ Impedance is Matched

$$Y = \frac{1}{Z_L} \Rightarrow g + jb$$

$$Y = 1 + jb$$

$$Y_{eq} = 1 + jb - j = 1$$

6. A Lossless line has a characteristics impedance of 500Ω. Determine the reflection co-efficient of the reciecving end impedance is (800+j10)Ω

$Z_R = (800 + j10)\,\Omega$; $Z_0 = 500\,\Omega$; $\Gamma_L = ?$

$$\Gamma_L = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{800 + j10 - 500}{800 + j10 + 500} = \frac{300 + j10}{1300 + j10}$$

$$= \frac{300.16\,\angle 1.90}{1300.03\,\angle 0.44}$$

$$\boxed{\Gamma_L = 0.23\,\angle 1.46}$$

11. Find the input impedance of 90Ω lossless transmission line of length 0.2λ when the load is a short.

$Z_{in} = ?$; $Z_L \simeq 0$ ($\because$ short)

$Z_0 = \sqrt{Z_S \cdot Z_L}$

$\cancel{Z_{in} = \frac{(Z_0)^2}{Z_L}} =$

$\boxed{Z_0 = 0}$ $\boxed{\because Z_L = 0}$

ụn that load is short $\boxed{Z_0 = 0}$

$\therefore \quad Z_{in} = Z_0 \tan(\beta \cdot L)$

$= Z_0 \left[\frac{2\pi}{\lambda} \cdot L\right]$

$= Z_0 \left[\frac{2\pi}{\lambda} \cdot 0.2\lambda\right]$

$= Z_0 [2\pi \cdot 0.2]$

$= 0$

$\boxed{Z_{in} = 0}$

7. A transmission line of length 0.3λ has a characteristic impedance of 50Ω and is terminated in a load impedance of $[100+j150]\Omega$. Find the

a) Voltage Reflection co-efficient (Γ_L)

b) voltage standing ~~of the line.~~ wave ratio

c) Input impedance of the line

$l = 0.3\lambda$; $Z_L = (100+j150)\Omega$; $Z_0 = 50\Omega$

a) $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{100+j150-50}{100+j150+50} = \dfrac{50+j150}{150+j150} = \dfrac{158.11\angle 71.5}{212.13\angle 45}$

$\Gamma_L = \underbrace{0.74}_{|\Gamma|}\underbrace{\angle 26.5}_{\theta}$

$\Gamma_L = 0.66 + 0.33j$

b) $\dfrac{V_{max}}{V_{min}} = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \cancel{\dfrac{1+0.66}{1-0.}}\ \dfrac{1+0.74}{1-0.7} = 6.69$

VSWR $= 6.7$

c). $Z_{in} = \dfrac{Z_0^2}{Z_L} = \dfrac{\cancel{500}(50)^2}{100+j150} = 7.69 - 11.5j$

$Z_{in} = 7.69 - 11.5j$

14. $Z_0 = 50\Omega$; $Z_L = (60+j50)\Omega$; $l = 0.4\lambda$

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{60+j50}{50} = 1.2+1j$$

$$\boxed{\bar{Z}_L = 1.2+1j}$$

iv) $\bar{Z}_{in} = 0.5+0.428j$

$$Z_{in} = \bar{Z}_{in} \cdot Z_0$$
$$= (0.5+0.428j)(50)$$
$$= 25+21.4$$

1) VSWR = 2.4 Ω

ii) $\Gamma_L = 0.42 \angle 54$

iii) $\cancel{Y_L} = \bar{Y}_L \cdot Z_0$

$$\bar{Y}_L = 0.5 - 0.42j$$

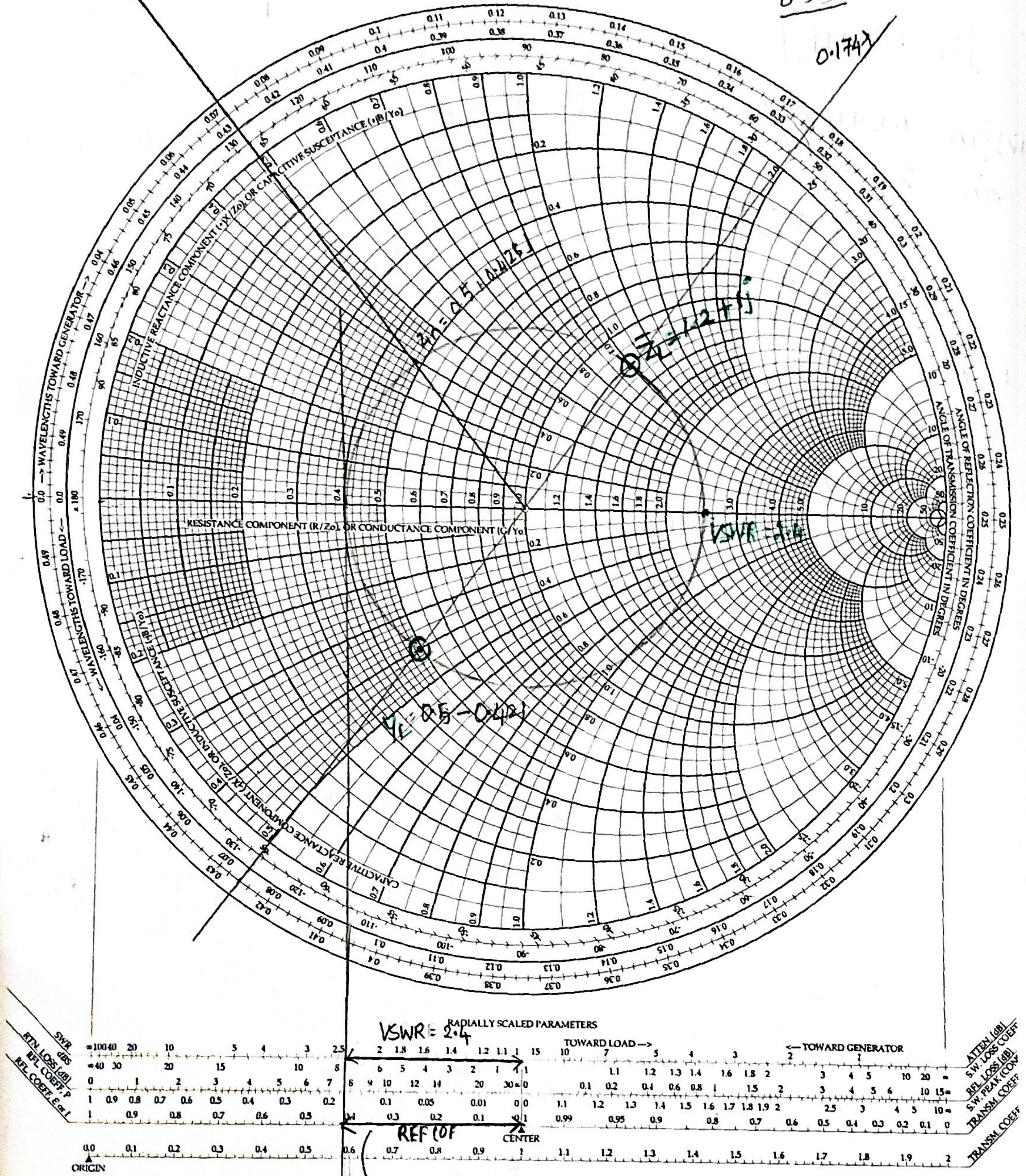
$$Y_L = \frac{1}{Z_L} = \frac{1}{(60+j50)\Omega} = (9.8 - 8.19j) \times 10^{-3}$$
$$= (9.8 - j8.2) \times 10^{-3} \mho$$

$Y_L =$

The Complete Smith Chart

Black Magic Design

0.174
0.400
0.574



$(\Gamma_L) = 0.42\angle 54°$