SLOT: C1

Reg. No.:																A

S.R.M.Institute of Science and Technology

(Under section 3 of UGC Act 1956)

DEPARTMENT OF MATHEMATICS

18MAB101T – Calculus and Linear Algebra

Cycle Test # 02

Date: **19.08.2018** Duration: **2** periods Max.:**50** Marks

Part – A: Answer ALL Questions (5 x 4 = 20 Marks)

1. Using Total differentiation, calculate $\frac{dz}{dt}$, where $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$.

2. If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, determine $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$.

3. Examine $f(x,y) = 3x^2 - y^2 + x^3$ for maximum and minimum values.

4. Solve: $\left(D^2 + 4D + 4\right)y = e^{-2x}$.

5. Get the particular integral of $\left(D^2 + 2D + 1\right)y = x^3$.

Part – B: Answer ANY THREE Questions (3 x 10 = 30 Marks)

6. Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ up to the third degree terms.

7. Compute the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

8. Solve: $(x^2D^2 + xD + 1)\ y = 4\sin(\log x)$.

9. By method of variation of parameters, solve $\left(D^2 + 1\right)y = \sec x$.

18MAB101T – Calculus & Linear Algebra

C_1 slot | **Set (A)**

CLAT # 02 — Page 1

PART – A : Answer ALL Questions : 5 × 4 = 20 marks

01. $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt}$ → (1 mark)

$$\begin{array}{ccc} & z & \\ x & & y \\ | & & | \\ t & & t \end{array}$$

$= (y^2+2xy)(2at) + (2xy+x^2)(2a)$ → (1 mark)

$\dfrac{dz}{dt} = [(2at)^2 + 2(at^2)(2at)]\,2at + (2(at^2)(2at) + (at^2)^2)(2a)$ → (1 mark)

$\boxed{\dfrac{dz}{dt} = 2a^3t^3\,[8+5t]}$. → (1 mark)

02. $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial z} & \frac{\partial y}{\partial z} & \frac{\partial z}{\partial z} \end{vmatrix}$ → (1 mark)

$= \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$ → (2 marks)

$= r\,(\cos^2\theta + \sin^2\theta) = r.$ → (1 mark)

03. $f_x = 6x + 3x^2 \qquad f_y = -2y$ → (1 mark)

Stationary points are occurs only at
$f_x = 6x + 3x^2 = 0\;;\; f_y = -2y = 0.$
Solving, we get the stationary points are $(0,0)$, $(-2,0)$. → (1 mark)

$A = f_{xx} = 6+6x \quad B = f_{xy} = 0\;;\; C = f_{yy} = -2$
$\Delta = AC - B^2 = -12(1+x) \qquad (0,0) \to$ Saddle point → (1 mark)

The point $(-2,0)$ have MAXIMUM, ∴ Max. Value is 4. → (1 mark)

Page 2

04. Auxillary Equation is $m^2 + 4m + 4 = 0$.
Solving we get $\boxed{m = -2,-2}$ (Real, equal) → (1 mark)

$\boxed{C.F. = (Ax+B)e^{-2x}}$ → (1 mark)

$PI = \dfrac{1}{D^2+4D+4}e^{-2x} = \dfrac{1}{0}e^{-2x}$
$= x\dfrac{1}{2D+4}e^{-2x} = x\dfrac{1}{0}e^{-2x}$ → (1 mark)

$\boxed{PI = \dfrac{x^2}{2}e^{-2x}}$ Hence the Complete Soln is $y = CF + PI$. → (1 mark)

05. $PI = \dfrac{1}{(D+1)^2}x^3$
$= (1+D)^{-2}(x^3)$
$= (1 - 2D + 3D^2 - 4D^3)(x^3)$
$\boxed{PI = x^3 - 6x^2 + 18x - 24.}$

(OR) $= \dfrac{1}{D^2+2D+1}(x^3)$
$= [1+(2D+D^2)]^{-1}(x^3)$ → (1 mark)
$= [1-(2D+D^2)+(2D+D^2)^2](x^3)$ → (1 mark)
$= (1 - 2D + 3D^2 - 4D^3)(x^3)$ → (1 mark)
$= x^3 - 6x^2 + 18x - 24.$ → (1 mark)

PART – B – Answer ANY THREE Questions (3 × 10 = 30 marks)

06. $f(x,y) = x^2y + 3y - 2 \qquad (a,b) = (1,-2) \qquad \boxed{f(1,-2) = -10}$

$f_x = 2xy$ $\boxed{f_x(1,-2) = -4}$	$f_y = x^2+3$ $\boxed{f_y(1,-2) = 4}$	$f_{xy} = 2x$ $\boxed{f_{xy}(1,-2) = 2}$
$f_{xx} = 2y$ $\boxed{f_{xx}(1,-2) = -4}$	$f_{yy} = 0$ $\boxed{f_{yy}(1,-2) = 0}$	$f_{xxy} = 2$ $\boxed{f_{xxy}(1,-2) = 2}$
$f_{xxx} = 0$ $\boxed{f_{xxx}(1,-2) = 0}$	$f_{yyy} = 0$ $\boxed{f_{yyy}(1,-2) = 0}$	$f_{xyy} = 0$ $\boxed{f_{xyy}(1,-2) = 0}$

2+2+2 (6 marks)

$f(x,y) = f(1,-2) + \frac{1}{1!}[(x-1)f_x(1,-2) + (y+2)f_y(1,-2)]$
$+ \frac{1}{2!}[(x-1)^2 f_{xx}(1,-2) + 2(x-1)(y+2)f_{xy}(1,-2) + (y+2)^2 f_{yy}(1,-2)]$
$+ \frac{1}{3!}[(x-1)^3 f_{xxx}(1,-2) + 3(x-1)^2(y+2)f_{xxy}(1,-2) + 3(x-1)(y+2)^2 f_{xyy}(1,-2) + (y+2)^3 f_{yyy}(1,-2)]$ (2 marks)

$f(x,y) = -10 - 4(x-1) + 4(y+2) - 2(x-1)^2 + (x-1)(y+2) + (x-1)^2(y+2).$ → (2 marks)

07. Let $f(x,y,z) = d^2 = (x-1)^2+(y-2)^2+(z+1)^2 \longrightarrow (1)$ (1 mark)

and $g(x,y,z) = x^2+y^2+z^2 = 24 \longrightarrow (2)$ (1 mark)

By Lagrange's multiplier method,

$f_x + \lambda g_x = 0 \Rightarrow 2(x-1)+(2x)\lambda = 0 \Rightarrow (1+\lambda)x = 1 \rightarrow (3)$

$f_y + \lambda g_y = 0 \Rightarrow 2(y-2)+(2y)\lambda = 0 \Rightarrow (1+\lambda)y = 2 \rightarrow (4)$

$f_z + \lambda g_z = 0 \Rightarrow 2(z+1)+(2z)\lambda = 0 \Rightarrow (1+\lambda)z = -1 \rightarrow (5)$

3x1 = (3 marks)

Eliminate $(1+\lambda)$ from (3),(4), & (5),

$\frac{(3)}{(4)} \Rightarrow \frac{x}{y} = \frac{1}{2} \Rightarrow \boxed{2x=y} \longrightarrow$ (1 mark)

$\frac{(4)}{(5)} \Rightarrow \frac{y}{z} = \frac{2}{-1} \Rightarrow \boxed{y=-2z} \longrightarrow$ (1 mark)

$\therefore \boxed{2x = y = -2z}$

Sub $y=2x$ and $z=-x$ in (2) we get $\boxed{x=\pm 2}$ $\boxed{y=\pm 4}$ $\boxed{z=\mp 2}$

Hence the Stationary points are $(2,4,-2)$ and $(-2,-4,+2)$. (1 mark)

Shortest distance $= f(2,4,-2) = \sqrt{6} \longrightarrow$ (1 mark)

Longest distance $= f(-2,-4,2) = \sqrt{54} = 3\sqrt{6}. \longrightarrow$ (1 mark)

08. Euler's Type: Take $\boxed{x=e^z}$ $\boxed{z=\log x}$

Then $xD \equiv D'$ and $x^2D^2 = D'(D'-1)$. $\longrightarrow$ (2 marks)

$\therefore$ Given Variable Diff Eqn can be reduced into Constant Coeffts. $[D'^2+1]y = 4\sin z \longrightarrow (1)$ $\longrightarrow$ (2 marks)

Auxillary Eqn. $m^2+1=0 \Rightarrow m=\pm i$

$\boxed{C.F = A\cos z + B\sin z = A\cos(\log x) + B\sin(\log x)}$ (1 mark)

$P.I. = \frac{1}{D'^2+1} 4\sin z \quad D'^2 \rightarrow -1.$

$= \frac{1}{0} 4\sin z.$ $\longrightarrow$ (1 mark)

$PI = z.\frac{1}{2D'}(4\sin z) = 2z\left[\int \sin z\, dz\right] \longrightarrow$ (1 mark)

$= -2z\cos z. \longrightarrow$ (1 mark)

$\boxed{PI = -2(\log x)\cos(\log x)} \longrightarrow$ (1 mark)

Hence the Complete soln is $y = CF+PI$

$y = A\cos(\log x) + B\sin(\log x) - 2(\log x)\cos(\log x)$. $\longrightarrow$ (1 mark)

09. Method of Variation of Parameters: $X = \sec x$.

Given D.E. be $(D^2+1)y = \sec x. \longrightarrow$ (1 mark)

Auxillary Eqn be $m^2+1=0 \Rightarrow m=\pm i \longrightarrow$ (1 mark)

$\boxed{C.F = A\cos x + B\sin x} \longrightarrow$ (1 mark)

$f_1(x) = \cos x \quad f_2 = \sin x$

$f_1 f_2' - f_2 f_1' = 1 \longrightarrow$ (1 mark)

$PI = P f_1(x) + Q f_2(x)$

$P = -\int \frac{f_2(x).X}{f_1 f_2' - f_2 f_1'} dx = -\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} dx = \log\cos x.$ (2 marks)

$Q = \int \frac{f_1(x)\, X}{f_1 f_2' - f_2 f_1'} dx = \int \cos x \cdot \frac{1}{\cos x} dx = \int dx = x.$ (2 marks)

$\boxed{PI = \cos x \log(\cos x) + x\sin x} \longrightarrow$ (1 mark)

Hence the Complete soln is $y = CF+PI$

$\boxed{y = A\cos x + B\sin x + \cos x \log(\cos x) + x\sin x}$ (1 mark)

SLOT-C_1 - SET-A

Reg. No.																

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS **SET-B**

CYCLE TEST-II

Sub. Code: 18MAB101T	Duration : 2 Periods
Title : Calculus and Linear Algebra	Max. Marks : 50
Slot : C1	Date : 19/09/2018

Part-A (5 x 4 = 20marks)
(Answer all questions)

1. Express $e^{x}cosy$ in powers of x and y as far as the terms of the second degree.
2. If $u = 2xy, v = x^2 - y^2, x = r\cos\theta$ and $y = r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.
3. If $u = xy + yz + zx$, where $x = e^t, y = e^t$ and $z = \frac{1}{t}$, find $\frac{du}{dt}$.
4. Find the particular integral of $(D^2 - 4D + 3)y = \sin 3x$.
5. Convert the equation $(x^2D^2 - xD + 4)y = x^2\sin(\log x)$ as a linear equation with constant coefficients.

Part-B (3 x 10 = 30marks)
(Answer any three questions)

6. If $z = f(u,v)$, where $u = x^2 - y^2$ and $v = 2xy$, show that
$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2 + y^2)(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2})$.
7. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
8. Solve $(D^2 + 5D + 4)y = e^{-x}\sin 2x$.
9. Solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ by method of variation of parameters.

****

18MAB101T – CALCULUS AND LINEAR ALGEBRA

SET – B

CYCLE TEST – II

SLOT : C1 SET – B

ANSWER KEY

PART – A

(1) $$f(x,y) = f(0,0) + \frac{1}{1!}\left[x f_x(0,0) + y f_y(0,0)\right] + \frac{1}{2!}\left[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$ — (1M)

$$\left.\begin{array}{l|l} f(0,0) = 1 & f_{xx}(0,0) = 1 \\ f_x(0,0) = 1 & f_{xy}(0,0) = 0 \\ f_y(0,0) = 0 & f_{yy}(0,0) = -1 \end{array}\right\}$$ — (2M)

$$\therefore \boxed{e^x \cos y = 1 + x + \frac{1}{2}(x^2 - y^2)}$$ — (1M)

(2) $$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(r,\theta)}$$ — (1M)

$$= -4(x^2+y^2) \times r(\cos^2\theta + \sin^2\theta)$$ — (2M)

$$= -4r^3$$ — (1M)

(3) $$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt}$$ — (1M)

$$= (y+z)\, e^t + (z+x)(-e^{-t}) + (x+y)\left(-\frac{1}{t^2}\right)$$ — (2M)

$$\left.\begin{array}{l} = 1 + \frac{1}{t}e^t - \frac{1}{t}e^{-t} - 1 - \frac{1}{t^2}e^t - \frac{1}{t^2}e^{-t} \\ = \frac{2}{t}\sinh t - \frac{2}{t^2}\cosh t \end{array}\right\}$$ — (1M)

(4) $P.I = \frac{1}{D^2 - 4D + 3} \sin 3x$ — (1M)

$= \frac{1}{30}(2\cos 3x - \sin 3x)$ — (3M)

(5) putting $x = e^z$ or $z = \log x$ — (1M)

Replace $x^2 D^2 = \theta(\theta - 1)$
$xD = \theta$ where $\theta = \frac{d}{dz}$ } — (1M)

$\therefore (\theta^2 - 2\theta + 4)y = e^{2z} \sin z$ — (2M)

PART-B

(6) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$ — (2M)

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$ — (2M)

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$ — (2M)

$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$ — (2M)

$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$ — (2M)

(8) $m^2 + 5m + 4 = 0$ — (1M)

$m = -1, -4$ — (1M)

$C.F = Ae^{-x} + Be^{-4x}$ — (2M)

$P.I = \frac{1}{D^2 + 5D + 4} e^{-x} \sin 2x$ — (1M)

$= -\frac{1}{26} e^{-x}(3\cos 2x + 2\sin 2x)$ — (4M)

$y = C.F + P.I$ — (1M)

(7) Volume $V = 8xyz = f(x, y, z)$

subject to: $g(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$

Now $F = f + \lambda g$

$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 0 \quad \& \quad \frac{\partial F}{\partial z} = 0$

$\Rightarrow x = \frac{a}{\sqrt{3}}, \quad y = \frac{b}{\sqrt{3}}, \quad z = \frac{c}{\sqrt{3}}$

$\therefore$ Maximum Volume $= \frac{8abc}{3\sqrt{3}}$

(9) $m = \pm 2i$ — (1M)

$C.F = A\cos 2x + B\sin 2x$
$= Af_1 + Bf_2$ — (2M)

Wronskian $W = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$
$= 2$ — (2M)

$P.I = Pf_1 + Qf_2$ — (1M)

$P = -\int \frac{f_2 X}{W} dx$ — (2M)
$= \sin 2x + \log(\sec 2x + \tan 2x)$

$Q = \int \frac{f_1 X}{W} dx$ — (2M)
$= -\cos 2x$

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

SET-C

CYCLE TEST-I

Sub. Code: 18MAB101T
Title: Calculus and Linear Algebra
Slot : C1

Duration : 2 Periods
Max. Marks : 50 Marks
Date :19/09/2018

(Answer all questions)

Part-A (5x 4 = 20 marks)

1. If $u = \sin(x/y)$, $x = e^t$, $y = t^2$, find $\frac{du}{dt}$ using total derivative.
2. Examine for extreme values of $x^2 + y^2 + 6x + 12$.
3. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.
4. Solve $(D^2 + 4)y = \sin 2x$.
5. Obtain the particular integral of $(D^2 - 2D + 1)y = e^x(3x^2 - 2)$.

Part-B (3X10= 30marks), (Answer any three questions)

6. The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.
7. Find the Taylor series expansion of $e^x \sin y$ near the point $(-1, \frac{\pi}{4})$ upto third degree term.
8. Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by using method of variation of parameter.
9. Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$.

[set-C]

18MAB101t - Calculus & Linear Algebra.

PART-A

1. $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$ —— (1M)

$= \cos(x/y)\left(\frac{1}{y}\right)e^t + \cos(x/y)\left(\frac{-x}{y^2}\right)(2t)$ —— (1M)

$= \frac{e^t}{t^2}\left[\cos\left(\frac{e^t}{t^2}\right) - \frac{2}{t}\right]$ —— (2M)

2. Let $f(x,y) = x^2+y^2+6x+12$

For maximum or minimum, $f_x = f_y = 0$ —— (1M)

$\Rightarrow 2x+6=0 \Rightarrow x=-3, \quad 2y=0 \Rightarrow y=0$ —— ~~(1M)~~

$\therefore (-3,0)$ is a stationary pt. —— (1M)

$r = f_{xx} = 2, \; s = f_{xy} = 0, \; t = f_{yy} = 2$ at $(-3,0)$

$\Rightarrow rt-s^2 > 0, \; r>0$ (1M)

$\therefore (-3,0)$ is minimum pt & minimum value $= 3$ (1M)

3. $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix} = \begin{vmatrix} -\frac{yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ z/y & -\frac{xz}{y^2} & \frac{x}{y} \\ \frac{y}{z} & \frac{x}{z} & -\frac{xy}{z^2} \end{vmatrix} = 4.$ —— (1M)

(1M) (2M)

4. A.E: $m^2+4=0 \Rightarrow m = \pm 2i$

C.F $= C_1\cos 2x + C_2\sin 2x$ —— (1M)

$P.I = \frac{1}{D^2+4}\sin 2x$

$= \frac{x}{2D}\sin 2x$

$= -\frac{x}{4}\cos 2x.$ —— (2M)

The complete sol. is $y = C_1\cos 2x + C_2\sin 2x - \frac{x}{4}\cos 2x.$ —— (1M)

5. $P.I = \frac{1}{(D-1)^2} e^x(3x^2-2)$

$= e^x \frac{1}{D^2}(3x^2-2)$ — (2M)

$= e^x\left(\frac{x^4}{4} - x^2\right)$. — (2M)

PART-B

6. Consider the Lagrange's function

$F(x,y,z) = 400xyz^2 + \lambda(x^2+y^2+z^2-1)$ — (1) — (2M)

At the critical points, we have

$400yz^2 + 2\lambda x = 0$ — (2), $400xz^2 + 2\lambda y = 0$ — (3), $800xyz + 2\lambda z = 0$ — (4) — (3M)

(2) $\times x$ + (3) $\times y$ + (4) $\times z \Rightarrow \lambda = -800xyz^2$ — (1M)

From (2), $400yz^2 - 1600x^2yz^2 = 0$

$\Rightarrow x = \pm\frac{1}{2},\ y = \pm\frac{1}{2}$

From (4), $z = \pm\frac{1}{\sqrt{2}}$ — (3M)

$\therefore T = 400(1/2)(1/2)(1/2) = 50$ — (1M)

7. $f(x,y) = e^x \sin y$, $f(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_x = e^x \sin y$, $f_x(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_{xx} = e^x \sin y$, $f_{xx}(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_{xxx} = e^x \sin y$, $f_{xxx}(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_y = e^x \cos y$, $f_y(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_{yy} = -e^x \sin y$, $f_{yy}(-1, \pi/4) = -\frac{1}{e\sqrt{2}}$

$f_{yyy} = -e^x \cos y$, $f_{yyy}(-1, \pi/4) = -\frac{1}{e\sqrt{2}}$

$f_{xxy} = e^x \cos y$, $f_{xxy}(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_{xy}(-1, \pi/4) = \frac{1}{e\sqrt{2}}$

$f_{xyy}(-1, \pi/4) = -\frac{1}{e\sqrt{2}}$

— (6M)

$$e^x \sin y = \frac{1}{e\sqrt{2}}\Big\{1 + \frac{1}{1!}\left[(x+1) + (y-\tfrac{\pi}{4})\right] + \frac{1}{2!}\left[(x+1)^2 + 2(x+1)(y-\tfrac{\pi}{4}) - (y-\tfrac{\pi}{4})^2\right] + \frac{1}{3!}\left[(x+1)^3 + 3(x+1)^2(y-\tfrac{\pi}{4}) - 3(x+1)(y-\tfrac{\pi}{4})^2 - (y-\tfrac{\pi}{4})^3\right]\Big\} + \cdots$$

— (4M)

8. $(D^2+1)y = \operatorname{cosec} x$.

A.E: $m^2+1=0$

$m = \pm i$

$C.F = c_1\cos x + c_2 \sin x = c_1 f_1 + c_2 f_2$ — (2M)

$\therefore f_1 f_2' - f_2 f_1' = 1$ — (1M)

$$P = -\int \frac{f_2}{f_1f_2' - f_2f_1'} F(x)\,dx = -\int dx = -x. \quad \text{— (2M)}$$

$$Q = \int \frac{f_1}{f_1f_2' - f_2f_1'} F(x)\,dx = \int \cos x \operatorname{cosec} x\,dx = \log(\sin x) \quad \text{— (2M)}$$

$P.I = Pf_1 + Qf_2 = -x\cos x + \sin x \log(\sin x)$ — (1M)

$\therefore$ The complete sol. is

$y = c_1\cos x + c_2 \sin x - x\cos x + \sin x \log(\sin x)$. — (2M)

9. Put $x = e^z \Rightarrow z = \log x$

$(D^2 - 2D + 1)y = z^2 e^{-2z}$ — (2M)

A.E: $m = 1, 1$

$C.F = (c_1 + c_2 z)e^z$ — (1M)

$$P.I = \frac{1}{(D-1)^2} e^{-2z} z^2$$

$$= e^{-2z} \frac{1}{(D-3)^2} z^2$$

$$= \frac{e^{-2z}}{9}\left(z^2 + \frac{4z}{3} + \frac{2}{3}\right)$$

— (4M)

Hence the complete sol.

$$y = (c_1 + c_2 z)e^z + \frac{e^{-2z}}{9}\left(z^2 + \frac{4z}{3} + \frac{2}{3}\right)$$

— (2M)

i.e., $$y = (c_1 + c_2 \log x)x + \frac{1}{9x^2}\left((\log x)^2 + \frac{4}{3}\log x + \frac{2}{3}\right)$$

— (1M)