Unit IV

FIR filters

Advantages of FIR filter over IIR:

1. FIR filters are always stable
2. FIR filters with exactly linear phase can easily be designed.
3. FIR filters are realized in both recursive & non recursive structures.
4. FIR filters are free of limit cycle oscillations. when implemented on a finite word length digital system.
5. Excellent design methods are available for various kinds of FIR filters.

Disadvantages:

1) Costly
2) Memory requirement & execution time are very high.

Linear phase FIR filter:

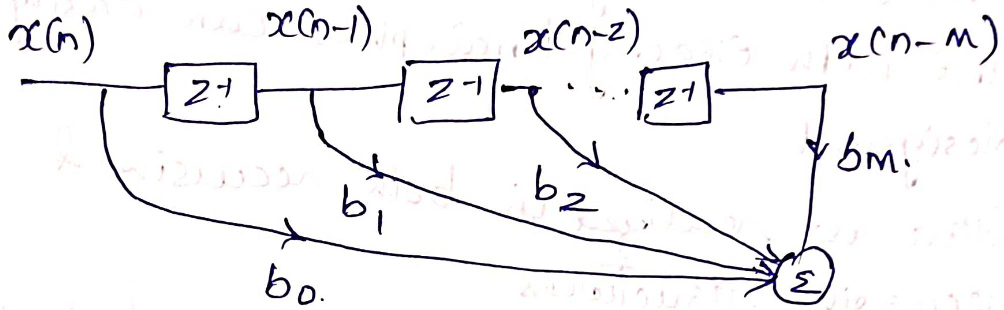
The transfer function of a FIR causal filter is given by:

$$H(Z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

where $h(n)$ is the impulse response of the filter.

Output of FIR filter

$$y(n) = \sum_{k=0}^{M} b_k \, x(n-k)$$

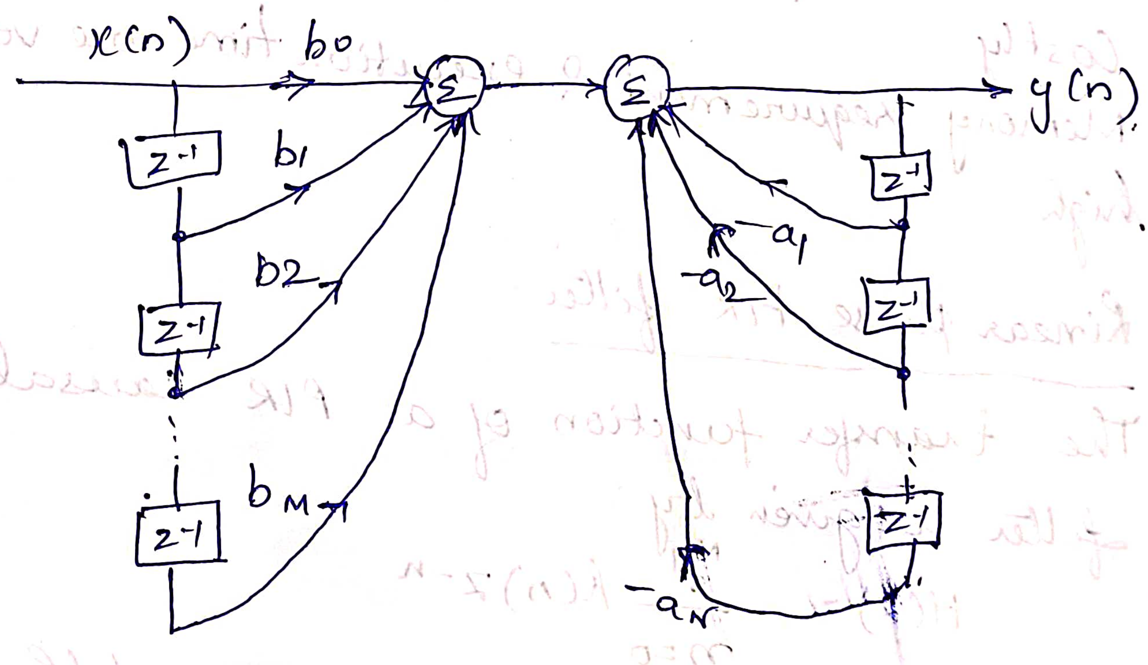


Structure of FIR filter.

IIR filter: Equation

$$y(n) = \sum_{k=0}^{M} b_k \, x(n-k) + \sum_{k=1}^{N} a_k \, y(n-k)$$

Structure:



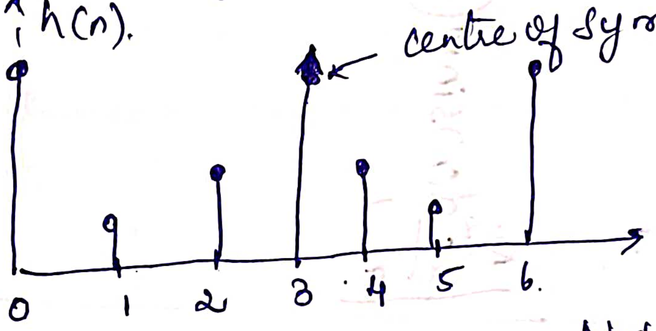
Frequency Response of Linear phase FIR filter.

Linear phase filter:-

All all the frequency component of an input signal to pass through the filter with the same delay. ∴ there is no distortion.

Frequency Response of Linear phase FIR filters:

Case 1: Symmetrical Impulse response, N odd:



$$H(e^{j\omega}) = e^{-j\omega\frac{(N-1)}{2}} \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n.$$

$$a(0) = h\left(\frac{N-1}{2}\right) \qquad a(n) = 2h\left(\frac{N-1}{2} - n\right).$$

$$|\bar{H}(e^{j\omega})| = \left| \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n \right|$$

↑
magnitude response

phase: $\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega.$

Phase response

$$\angle H(e^{j\omega}) = -\alpha\omega + \theta$$

where $\theta = 0$ for $\bar{H}(e^{j\omega}) > 0$

$\theta = \pi$ for $\bar{H}(e^{j\omega}) < 0.$

Type	Frequency response	Magnitude response	Phase response	Application
2) Symmetrical impulse response N = even	$e^{-j\omega\left(\frac{N-1}{2}\right)}\left[\sum_{n=1}^{N/2} b(n)\cos\left(n-\frac{1}{2}\right)\omega\right]$ $b(n) = 2h\left[\frac{N}{2}-n\right]$	$\left\lvert\sum_{n=1}^{N/2} b(n)\cos\left(n-\frac{1}{2}\right)\omega\right\rvert$	$-\alpha\omega+\theta$ $\theta = 0$ for $H(e^{j\omega})>0$ $\theta = \pi$ for $\bar{H}(e^{j\omega})<0$	LPF BPF
3) Antisymmetric impulse response N = odd	$e^{j\pi/2}e^{-j\omega\frac{(N-1)}{2}}\sum_{n=1}^{\frac{N-1}{2}} c(n)\sin\omega n$ $c(n) = 2h\left(\frac{N-1}{2}-n\right)$	$\left\lvert\sum_{n=1}^{\frac{N-1}{2}} c(n)\sin\omega n\right\rvert$	$-\alpha\omega+\pi/2+\theta$ $\theta = 0$ for $\bar{H}(e^{j\omega})>0$ π for $\bar{H}(e^{j\omega})<0$	Differentia for Hilbert T/f
4) Antisymmetrical impulse response N-even	$e^{j\pi/2}e^{-j\omega\frac{(N-1)}{2}}\sum_{n=1}^{N/2} d(n)\sin\omega\left(n-\frac{1}{2}\right)$ $d(n) = 2h\left(\frac{N}{2}-n\right)$	$\left\lvert\sum_{n=1}^{N/2} d(n)\sin\omega\left(n-\frac{1}{2}\right)\right\rvert$	$-\alpha\omega+\pi/2+\theta$ $\theta=0$ for $\bar{H}(e^{j\omega})>0$ $=\pi$ for $\bar{H}(e^{j\omega})<0$	,,

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The Fourier Series Method of Designing FIR filters:

The desired frequency response of an FIR filter can be represented by Fourier Series:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

Step 1:-

The Fourier coefficients $h_d(n)$ are the desired impulse response sequence of the filter;

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

Step 2:

To get an FIR filter transfer function, the series can be truncated by assigning

$$h(n) = \begin{cases} h_d(n) & \text{for } |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Then

Step 3 $$H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n) z^{-n}$$

$$\Rightarrow h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \dots + h(1) z^{-1} + h(0) + h(-1) z + h(-2) z^{2} + \dots + h\left[-\left(\frac{N-1}{2}\right)\right] z^{(N-1)/2}$$

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$$\Rightarrow h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n) z^{-n} + h(-n) z^{n}]$$

For a symmetrical impulse response having symmetry at $n=0$

$$h(-n) = h(n)$$

$$\therefore H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)[z^{n} + z^{-n}]$$

Step 4: The above transfer function is not physically realizable.

Multiply the above equation by $z^{-\left(\frac{N-1}{2}\right)}$ where $\frac{N-1}{2}$ is delay in samples:

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= z^{-\left(\frac{N-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)(z^{n} + z^{-n}) \right]$$

Step 5:- find the frequency response $H(e^{j\omega})$ from the table.

Example ①:- Design an ideal low pass filter with the frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for } -\pi/2 \le |\omega| \le \pi/2$$
$$= 0 \quad \pi/2 \le |\omega| \le \pi.$$

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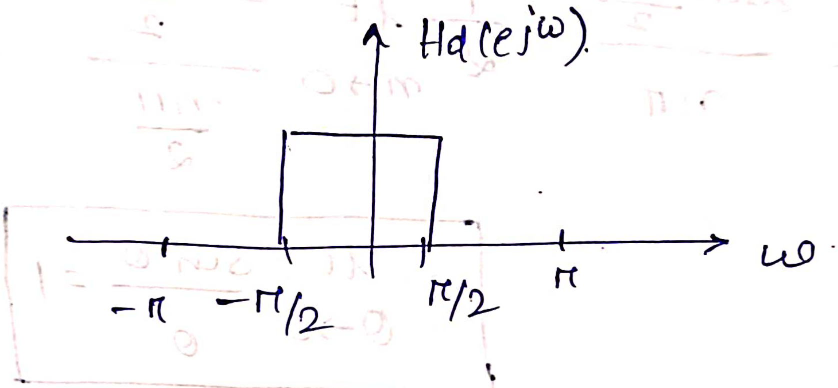
4

Find the values of $h(n)$ for $N=11$. Find $H(z)$. Plot the magnitude response.

Solution :

The frequency response of Low Pass Filter is shown in fig.

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } -\pi/2 \le |\omega| \le \pi/2 \\ 0 & \text{for } \pi/2 \le |\omega| \le \pi. \end{cases}$$



$\alpha \rightarrow$ centre of symmetry $= 0$.

$\therefore \; h_d(n) = h_d(-n)$

Step 1: $$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi j n} e^{j\omega n}\Big|_{-\pi/2}^{\pi/2} = \frac{1}{\pi n(2j)}\left[e^{\frac{j\pi n}{2}} - e^{\frac{-j\pi n}{2}}\right]$$

$$= \frac{\sin\frac{\pi n}{2}}{\pi n} \qquad -\infty \le n \le \infty$$

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Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = \begin{cases} \dfrac{\sin\frac{\pi}{2}n}{\pi n} & \text{for } |n| \le 5 \\ 0 & \text{otherwise} \end{cases} \rightarrow (1)$$

For $n=0$ eqn (1) becomes indeterminate

So,

$$h(0) = \lim_{n\to 0} \frac{\sin\frac{n\pi}{2}}{n\pi} = \frac{1}{2} \lim_{n\to 0} \frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}}$$

$$\boxed{\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

For $n=1$

$$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183$$

Similarly,

$$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = -\frac{1}{3\pi} = -0.106$$

$$h(4) = h(-4) = \frac{\sin\frac{4\pi}{2}}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.06366$$

Step 3: The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n})]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)(z^n + z^{-n})$$

$$= 0.5 + 0.3183(z^1 + z^{-1}) - 0.106(z^3 + z^{-3})$$

$$+ 0.6366(z^5 + z^{-5})$$

Step 4: The transfer function of the realizable filter is:

$$H'(z) = z^{-\frac{(N-1)}{2}} H(z)$$

$$\Rightarrow z^{-5}[0.5 + 0.3183(z + z^{-1}) - 0.106(z^3 + z^{-3})$$

$$+ 0.06366$$

$$\Rightarrow 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z^{-5}$$

$$+ 0.3183z^{-6} - 0.106z^{-8} + 0.06366z^{-10}$$

From the above eqn the filter coefficients are

$h(0) = h(10) = 0.06366$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

$h(4) = h(6) = 0.3183$

$h(5) = 0.5$

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0} a(n) \cos \omega n.$$

$$a(0) = h\left(\frac{N-1}{2}\right) \Rightarrow h(5) = 0.5$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = 0.6366$$

$$a(2) = 2h(5-2) = 2h(3) = 0$$

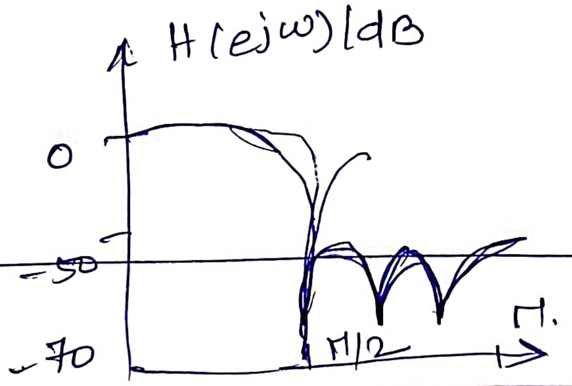
$$a(3) = 2h(5-3) = 2h(2) = -0.212$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.127.$$

$$H(e^{j\omega}) = 0.5 + 0.6366 \cos\omega - 0.212 \cos 3\omega + 0.127 \cos 5\omega$$

ω (in degrees)	0	10	20	30	40	50	60
$H(e^{j\omega})\|_{dB}$	0.4	0.21	-0.26	-0.517	-0.2	0.142	0.77
ω (in degrees)	70	80	90	120	150	180	
$H(e^{j\omega})\|_{dB}$	0.21	-1.79	-6	-20.6	-24.7	-26	



Frequency response of Low pass filter.

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