

# Binary multiplier:

$$\begin{array}{r} A_1 \quad A_0 \quad (3) \\ A = \begin{array}{cc} 1 & 1 \\ B_1 & B_0 \end{array} \\ B = \begin{array}{cc} 1 & 0 \\ \times & \end{array} \quad (2) \end{array}$$

$$\begin{array}{l} (1) A_0 \\ (0) B_0 \end{array} \rightarrow A_0 B_0 = 0 \rightarrow P_0 = 0$$

$$\begin{array}{r} 0 \quad 0 \\ 1 \quad 1 \\ \hline 1 \quad 1 \quad 0 \quad (6) \\ \uparrow \quad \uparrow \quad \uparrow \\ P_2 \quad P_1 \quad P_0 \end{array}$$

$$\begin{array}{l} (1) A_1 \\ (0) B_0 \end{array} \rightarrow A_1 B_0 = 0$$

$$P_1 = 0 \oplus 1 = 1$$

$$\begin{array}{l} (1) A_0 \\ (1) B_1 \end{array} \rightarrow A_0 B_1 = 1$$

$$C_1 = 0 \cdot 1 = 0$$

$$\begin{array}{l} (1) A_1 \\ (1) B_1 \end{array} \rightarrow A_1 B_1 = 1$$

$$P_2 = 0 \oplus 1 = 1$$

$$C_2 = P_3 = 0$$

$$\begin{array}{r} 0000 \quad 0101 \\ 1000 \quad 0010 \\ \hline 1000 \quad 0111 \end{array}$$

$$\begin{array}{r} 12 \\ 11 \\ \hline 12 \\ 12 \\ \hline 132 \end{array}$$

## Multiplikation:

$$12 \times 11 = 132$$

$$B \Rightarrow 12 \rightarrow \begin{array}{c} B_3 B_2 B_1 B_0 \\ 1100 \end{array}$$

$$Q \Rightarrow 11 \rightarrow \begin{array}{c} Q_3 Q_2 Q_1 Q_0 \\ 1011 \end{array}$$

$$\begin{array}{c} C \quad A \quad Q \\ \begin{array}{c} 0 \rightarrow 0000 \\ 0 \rightarrow 0000 \\ 1 \rightarrow 1100 \\ 1 \rightarrow 1100 \\ 0 \rightarrow 0111 \\ 1 \rightarrow 0011 \\ 1 \rightarrow 1001 \end{array} \quad \begin{array}{c} 1011 \\ 0101 \\ 0010 \\ 0001 \\ 0000 \end{array} \quad \left. \begin{array}{l} 1 \\ 2 \end{array} \right\} \end{array}$$

$$\begin{array}{r} 128 \\ 16 \\ \hline 8 \end{array}$$



# Multiplier

①

C	A	Q	
0	0000	1011	$B \rightarrow 12 \rightarrow B_3 B_2 B_1 B_0$ $Q \rightarrow 11 \rightarrow Q_3 Q_2 Q_1 Q_0$
	1100		
$Q_0 = 1$ A+B Shift	1100	1011	} cycle 1
0	0110	0101	
<hr/>			
$Q_0 = 1$ A+B Shift	10110	0101	} Cycle 2
1	0010	0010	
<hr/>			
$Q_0 = 0$ Shift	10100	1001	} cycle 3.
0	0100	1001	
<hr/>			
$Q_0 = 1$ A+B Shift	10100	1001	} cycle 4
1	0000	0100	
<hr/>			
<p align="center">128 + 4 = 132</p>			

②

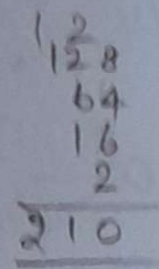
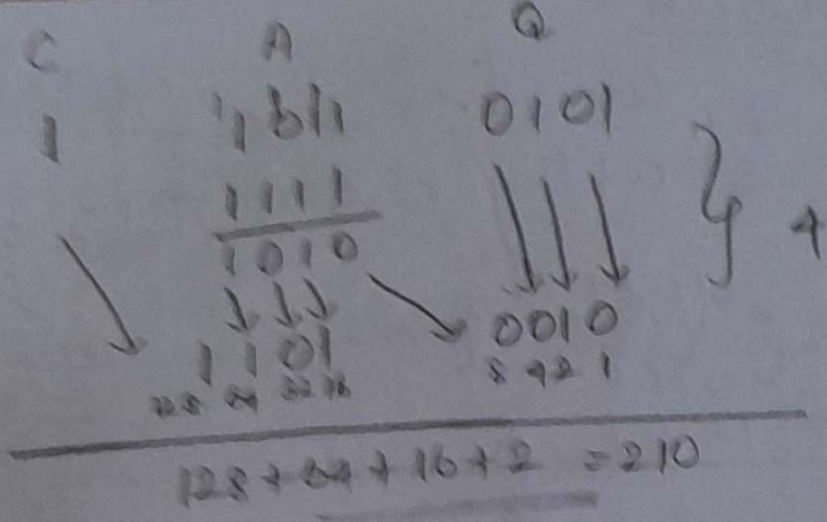
$2 \overline{) 12} \rightarrow 1111 \rightarrow B$   
 $14 \rightarrow 1110 \rightarrow Q.$

160  
15  
210

C	A	Q	
0	0000	1110	} 1
$Q_0 = 0$ Shift	0000	0111	
<hr/>			
$Q_0 = 1$ A+B Shift	0000	0111	} 2
0	1111	1011	
<hr/>			
$Q_0 = 1$ A+B Shift	10111	1011	} 3
1	0110	0101	



Q<sub>0</sub> = 1  
A + M  
Shift



Signed numbers

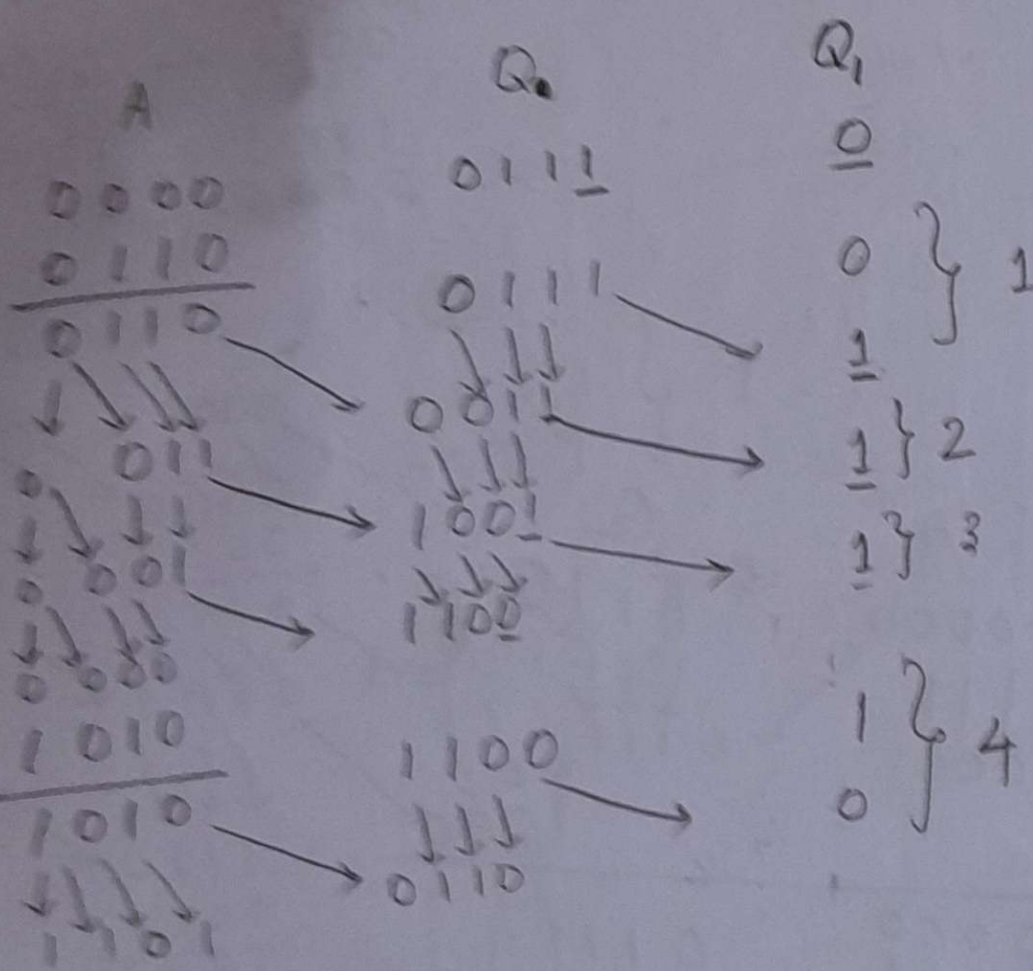
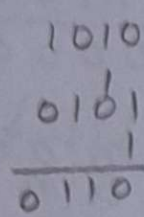
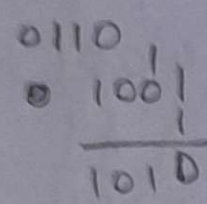
M → Multiplicand  
Q → Multiplier

$-6 \times 7$

M = -6  
Q = 7

+ M = 1010  
- M = 0110  
Q = 0111

Q<sub>0</sub> Q<sub>1</sub>  
0 0 } Shift  
1 1 } Shift  
1 0 → A - M, Shift  
0 1 → A + M, Shift



$$\begin{aligned} +M &= 0111 \\ -M &= 1001 \end{aligned}$$

$$Q \rightarrow -6 \Rightarrow 0110$$

1061

$$\underline{\underline{1010}}$$

$$+Q = 1010$$

~~-Q = 0.110~~

$+M = 0.111$

$$\begin{array}{r} 1000 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 0101 \\ \hline 0110 \end{array}$$

$$\begin{array}{r} 12 \quad 1 \quad 1 \quad 2 \\ \times \quad 2 \quad 2 \quad 0 \\ \hline 24 \quad 24 \quad 20 \quad 0 \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 1000 \\ \hline 1001 \end{array}$$

Handwritten diagram illustrating the steps of a binary subtraction algorithm (likely using a ripple-carry method) for the operation  $A - M(-)$ . The diagram shows the propagation of borrow bits (Q) through the stages of the subtraction.

The diagram is organized into four main stages, each showing the subtraction of a 4-bit value from a 4-bit value, with the result and the next stage's inputs shown below.

**Stage 1:**

- Input:  $A = 0000$ ,  $M(-) = 0111$
- Subtraction:  $0000 - 0111$
- Result:  $10000$  (with a leading 1 indicating a borrow)
- Output:  $Q_1 = 0$

**Stage 2:**

- Input:  $A = 10000$ ,  $M(-) = 0111$
- Subtraction:  $10000 - 0111$
- Result:  $0101$
- Output:  $Q_2 = 0$

**Stage 3:**

- Input:  $A = 0101$ ,  $M(-) = 0111$
- Subtraction:  $0101 - 0111$
- Result:  $1010$
- Output:  $Q_3 = 1$

**Stage 4:**

- Input:  $A = 1010$ ,  $M(-) = 0111$
- Subtraction:  $1010 - 0111$
- Result:  $1101$
- Output:  $Q_4 = 0$

The final result of the subtraction is  $1101$ .

$$\begin{array}{r}
 A \\
 0000 \\
 \downarrow \downarrow \downarrow \downarrow \\
 0000 \\
 \hline
 0111 \\
 1001 \\
 \downarrow \downarrow \downarrow \downarrow \\
 1100 \\
 0111 \\
 \hline
 10011
 \end{array}$$

$$\begin{array}{r}
 Q \\
 1010 \\
 \downarrow \downarrow \downarrow \downarrow \\
 0101 \\
 \hline
 0101 \\
 \downarrow \downarrow \downarrow \downarrow \\
 1010
 \end{array}$$

$$\begin{array}{r}
 Q_1 \\
 0 \\
 \hline
 0 \\
 \hline
 0 \\
 \hline
 1 \\
 \hline
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 1$$

$$\begin{array}{r}
 0110 \\
 1001 \\
 \hline
 1010
 \end{array}$$

$7x - 6$

$$M = 7$$
$$Q = -6$$

$$Q = 1010$$

$$+M = 0111$$

$$-M = 1001$$

$$\begin{array}{r}
 A \\
 0000 \\
 \downarrow \downarrow \downarrow \\
 0000 \\
 (+) \quad 1001 \\
 \hline
 1001 \\
 \downarrow \downarrow \downarrow \\
 1000 \\
 (+) \quad 0111 \\
 \hline
 10011
 \end{array}$$

$$\begin{array}{r}
 Q \\
 1010 \\
 \downarrow \downarrow \downarrow \\
 0101 \\
 0101 \\
 \downarrow \downarrow \downarrow \\
 1010
 \end{array}$$

$$\begin{array}{r}
 Q_1 \\
 0 \\
 \hline
 0 \\
 \hline
 \end{array}
 \left. \vphantom{\begin{array}{r} 0 \\ 0 \end{array}} \right\} 1$$

$$\begin{array}{r}
 0 \\
 \hline
 1 \\
 \hline
 \end{array}
 \left. \vphantom{\begin{array}{r} 0 \\ 1 \end{array}} \right\} 2$$



Multiplii:

$$\begin{aligned} + &\rightarrow 0 \\ - &\rightarrow 1 \end{aligned}$$

$$\begin{array}{r}
 110101 \\
 +2 \quad -1 \quad -1 \\
 \hline
 \end{array}$$

$$\begin{array}{cccccccc}
 & & & 0 & 0 & 1 & 0 & 1 & 1 \\
 & & 1 & & & & & & \\
 & 0 & 0 & 1 & 0 & 1 & 1 & x & x \\
 1 & 0 & 1 & 0 & 1 & 0 & x & x & x \\
 \hline
 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1
 \end{array}$$



16/10/2023  
Monday

# Carry Save addition

①

$$\begin{array}{r} (x) \quad 1010 \text{ (10)} \\ 0110 \text{ (6)} \\ \hline 0000 \rightarrow A \\ 1010x \rightarrow B \\ 1010xx \rightarrow C \\ 0000xxx \rightarrow D \end{array}$$

②  $A + B$

$$\begin{array}{r} 0000 \\ 10100 \\ \hline 10100 \rightarrow s_1 \\ 00000 \rightarrow c_1 \end{array}$$

④

$s_1 + c_1 + s_2$

$$\begin{array}{r} 10100 \\ 00000 \\ 0101000 \\ \hline 0111100 \rightarrow s_3 \\ 0000000 \rightarrow c_3 \end{array}$$

③  $C + D$

$$\begin{array}{r} 101000 \\ 000000 \\ \hline 0101000 \rightarrow s_2 \\ 0000000 \rightarrow c_2 \end{array}$$

$$\begin{array}{r} 2 \\ 32 \\ 16 \\ 8 \\ 4 \\ \hline 60 \end{array}$$

⑤  $s_3 + c_3 + c_2$

$$\begin{array}{r} 0111100 \\ 0000000 \\ 0000000 \\ \hline 0111100 - s_4 \\ 0000000 - c_4 \end{array}$$

$$\begin{array}{r} 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1 \\ 0111100 \end{array}$$

⑥  $0111100 \text{ (60)}$

$$\begin{array}{r} 1100 \\ 1100 \\ \hline 10100 \\ 10000 \end{array}$$

②  $12 \times 11 = 132$

②  $A + B$

①

$$\begin{array}{r} (12) \quad 1100 \\ (11) \quad 1011 \end{array}$$

$$\begin{array}{r} 1100 \\ 1100 \\ \hline 10100 \rightarrow s_1 \\ 10000 \rightarrow c_1 \end{array}$$

$$\begin{array}{r} 1100 \rightarrow A \\ 1100x \rightarrow B \\ 0000xx \rightarrow C \\ 1100xxx \rightarrow D \end{array}$$

③  $C + D$

$$\begin{array}{r} 000000 \\ 110000 \\ \hline 110000 \rightarrow s_2 \\ 000000 \rightarrow c_2 \end{array}$$



④  $S_1 + C_1 + S_2$

$$\begin{array}{r} 10100 \\ 10000 \\ \hline 1100000 \\ 1100100 \rightarrow S_3 \\ 0100000 \rightarrow C_3 \end{array}$$

⑤  $S_3 + C_3 + C_2$

$$\begin{array}{r} 1100100 \\ 0100000 \\ 0000000 \\ \hline 1000100 \rightarrow S_4 \\ 1000000 \rightarrow C_4 \\ \hline 1000001000 (132) \\ \begin{array}{r} 128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1 \\ 128 \end{array} \end{array}$$

### Division (Restoring)

- ① Left Shift
- ②  $A = A - M$
- ③ Bit = 0  $\Rightarrow Q[0] \rightarrow 1$
- ④ Bit = 1  $\rightarrow Q[0] = 0$  (Restore A)

①  $10 \div 3$

$Q \rightarrow 10 \rightarrow 1010$  (doesn't change)  
 $M \rightarrow 3 \rightarrow 00011$  (add one bit) (2)

$\boxed{-M = 11101}$

$A = 00000$

2's complement of 11

$$\begin{array}{r} 00011 \\ 11100 \\ \hline 11101 \end{array}$$

**A**

$$\begin{array}{r} 00000 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \swarrow \\ 00001 \\ \hline 11101 \\ \hline 11110 \\ 00001 \\ \hline 00010 \\ \hline 11101 \\ \hline 11111 \\ 00010 \end{array}$$

**Q**

$$\begin{array}{r} 1010 \rightarrow \text{Initial Stage} \\ \downarrow \downarrow \downarrow \downarrow \\ 010 \square \\ 010 \square \\ 100 \square \\ 1000 \end{array}$$

$$\begin{array}{r} 3 \\ 2 \overline{) 10} \\ \underline{-9} \\ 1 \end{array}$$

$$\begin{array}{r} 5 \\ 2 \overline{) 11} \\ \underline{-10} \\ 1 \end{array}$$



①  $10 \div 3$        $Q \rightarrow 10 \rightarrow 1010$   
 $3 \overline{) 10}$        $M \rightarrow 3 \rightarrow 00011$   
 $\quad \underline{9}$        $\boxed{-M = 11101}$   
 $\quad \underline{1} (R)$

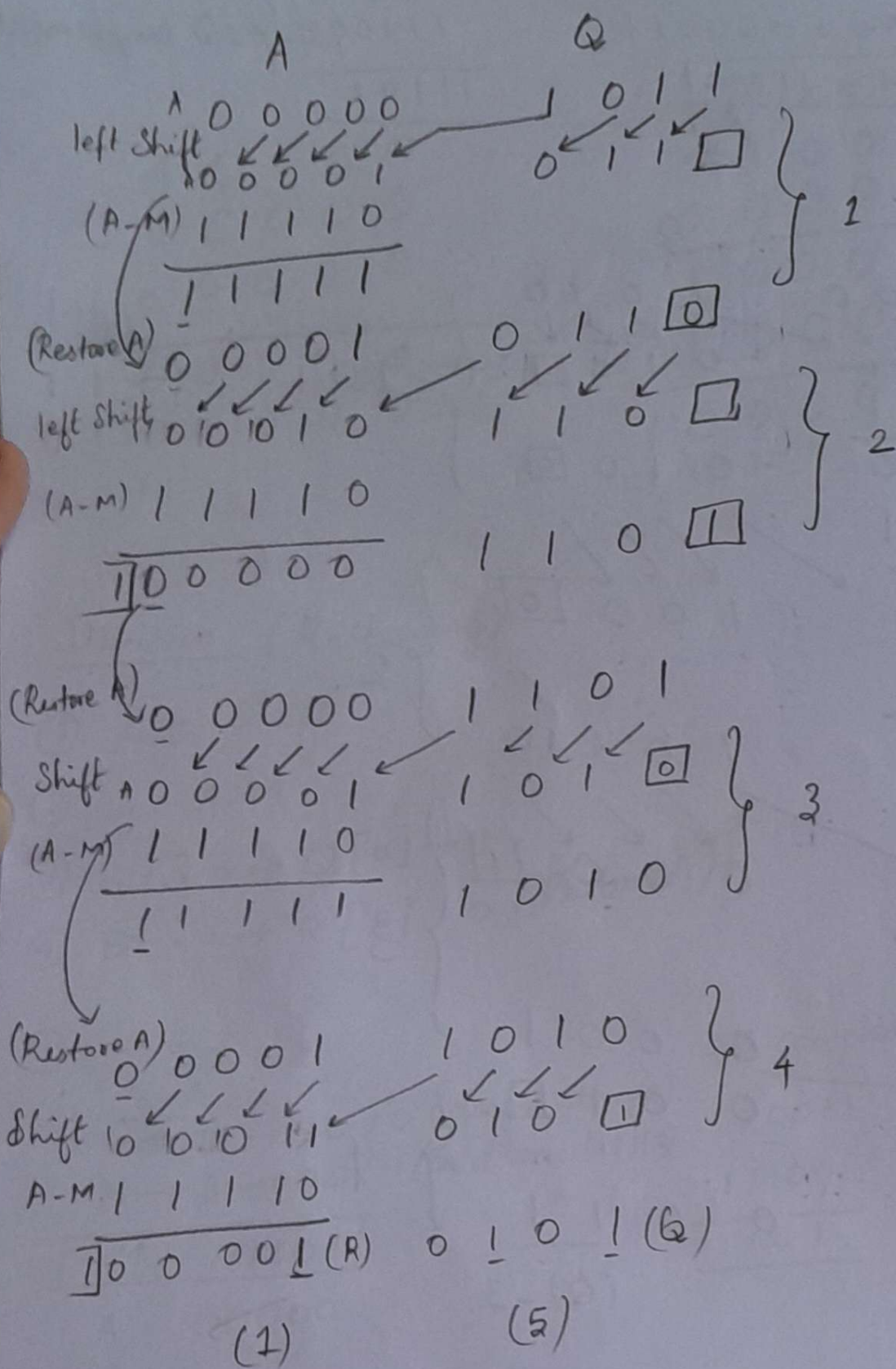
$\begin{array}{r} 00011 \\ 11100 \\ \hline 11101 \end{array}$  } 2's complement.

A	Q
$\begin{array}{r} 00000 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \\ 00001 \\ \hline 11101 \\ \hline 11110 \\ 00001 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \\ 00010 \\ \hline 11101 \\ \hline 11111 \\ 00010 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \\ 101011 \\ \hline 11101 \\ \hline 100010 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \\ 101000 \\ \hline 11101 \\ \hline 100001 \\ \hline \end{array}$	$\begin{array}{r} 1010 \\ \swarrow \swarrow \swarrow \swarrow \swarrow \\ 010\boxed{0} \\ 010\boxed{0} \\ 100\boxed{0} \\ 1000 \\ 000\boxed{1} \\ 0001 \\ 001\boxed{1} \\ 0011 \end{array}$
	} 1
	} 2
	} 3
	} 4
(R) - 1	(Q) - 3

②  $11 \div 2$        $Q \rightarrow 11 \rightarrow 1011$   
 $2 \overline{) 11}$        $M \rightarrow 2 \rightarrow 00010$   
 $\quad \underline{10}$        $\boxed{-M = 11110}$   
 $\quad \underline{1} (R)$   
 $A = 00000$

$\begin{array}{r} 00010 \\ 11101 \\ \hline 11110 \end{array}$





$$\frac{11100}{11101}$$

$$\frac{1100}{1101}$$

$$\begin{array}{r} 3 \overline{) 10} \\ \underline{-9} \\ 1 \end{array} \begin{array}{l} 3 - (Q) \\ 9 - (R) \end{array}$$

19/10/2023  
Thursday

### Non-Restoring

$$10 \div 3$$

$$Q = 10 \Rightarrow 1010$$

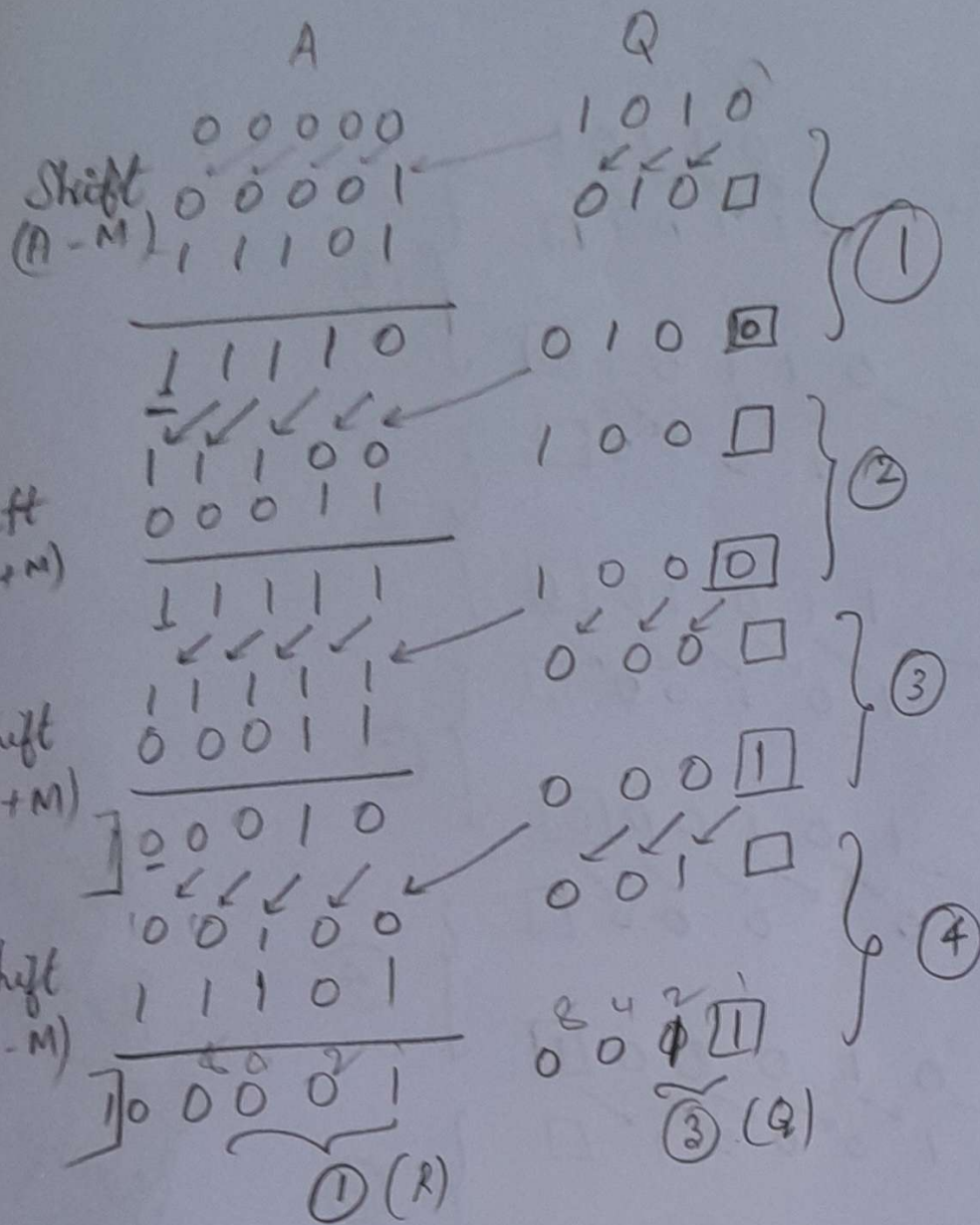
$$M = 3 \Rightarrow 00011$$

$$-M = 11101$$

### Steps:

- 1) 0  $\rightarrow$  L.S,  $A = A - M$
- 2) 1  $\rightarrow$  L.S,  $A = A + M$
- 3) 0  $\rightarrow$  Q[0] = 1
- 4) 1  $\rightarrow$  Q[0] = 0





$$\begin{array}{r} 32 \\ 16 \\ 8 \\ 4 \\ 2 \\ 1 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1 \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \end{array}$$

$$\begin{array}{r} 5 \\ 9 \overline{) 45} \\ \underline{45} \\ 0 \end{array}$$

②  $45 \div 9$  (6 cycles)

$$\begin{array}{r} 5 \\ 9 \overline{) 45} \\ \underline{45} \\ 0 \end{array}$$

$45 \rightarrow 101101 - Q$   
 $9 \rightarrow 0001001 - M$   
 $-M \rightarrow 1110111$

$$\begin{array}{r} 1110110 \\ \hline 1110111 \end{array}$$

Memory data register  
 - signal which tells the memory  
 operation is completed



