18ECC205J- ANALOG AND DIGITAL COMMUNICATION

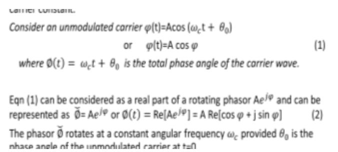
UNIT-1

S-7 Frequency modulation, Types of FM-Narrow Band FM, Wide Band FM, Phase modulation

S-8 Generation of Narrowband FM

S-9 Demodulation of FM : Foster seely discriminator

Angle modulation


Consider an unmodulated carrier φ(t)=Acos (ω_ct + θ_0)
or φ(t)=A cos φ (1)
where Ø(t) = $\omega_c t + \theta_0$ is the total phase angle of the carrier wave.
Eqn (1) can be considered as a real part of a rotating phasor A$e^{j\varphi}$ and can be represented as Ø̆= A$e^{j\varphi}$ or Ø(t) = Re[A$e^{j\varphi}$] = A Re[cos φ + j sin φ] (2)
The phasor Ø̆ rotates at a constant angular frequency ω_c provided θ_0 is the

- Process of varying the total phase angle of a carrier wave in accordance with the instantaneous value of the modulating signal, keeping the amplitude of the carrier constant.
- *Consider an unmodulated carrier* φ(t)=Acos (ω_ct + θ_0)

or φ(t)=A cos ψ(t) (1)

ψ(t) $= \omega_c t + \theta_0$ *is the total phase angle of the carrier wave.*

- Eqn (1) can be considered as a real part of a rotating phasor A$e^{j\varphi}$ and can be represented as $\breve{\emptyset}$= A$e^{j\varphi}$ or $\emptyset(t) =$ Re[A$e^{j\varphi}$] = A Re[cos φ + j sin φ] (2)
- The phasor $\breve{\emptyset}$ rotates at a constant angular frequency ω_c provided θ_0 is the phase angle of the unmodulated carrier at t=0.

Instantaneous frequency

- The constant angular frequency ω_c of the phasor is related to its total phase angle φ(t). $\varphi = \omega_c t + \theta_0$ (3)
- Differentiating (3) we get d φ/dt = ω_c [θ_0 is independent of time] (4)
- This derivative varies with time and hence the angular frequency of the phasor $\check{\varphi}$ will also change with time
- The time dependent angular frequency is called as instantaneous angular frequency and is denoted as d φ/dt = ω_i (5)
- ω_i is time dependent

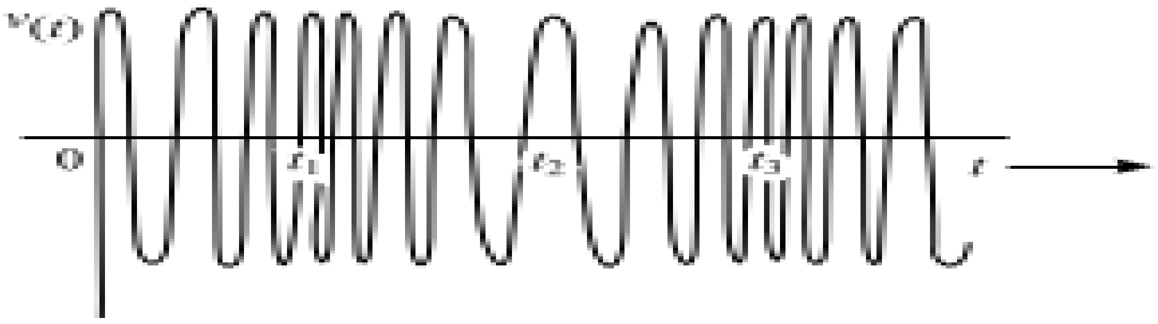


Fig: Waveform of a carrier wave with varying frequency

Types of Angle modulation

- Two types of angle modulation
  - Frequency modulation
  - Phase modulation

- Phase modulation – The phase angle ψ(t) is varied linearly with the modulating signal f(t) about an unmodulated phase angle ω_ct
- Frequency modulation – The instantaneous frequency ω_i varies linearly with a modulating signal f(t) about an unmodulated frequency ω_c.

Representation of Frequency modulated signal

- The instantaneous value of the angular frequency ω_i is equal to the frequency ω_c of the unmodulated carrier plus a time varying component proportional to f(t) .
- Mathematically ,$\omega_i = \omega_c + K_f$ f(t) (6)
  - *Where K_f is the frequency sensitivity (Hz/V)*
- The total phase angle of the FM wave can be obtained by integrating (5)

$$\varphi_i = \int \omega_i dt = \int [\, \omega_c + K_f f(t)] = \omega_c t + K_f \int f(t) dt \qquad (7)$$

- The corresponding FM wave can be given by $\emptyset_{FM}(t) = A\cos \varphi_i$ (8)

- Sub (7) in (8) we get $\emptyset_{FM}(t) = A\cos[\omega_c t + K_f \int f(t) dt]$ (9)

Representation of Frequency modulated signal- Contd

- We know f(t) =$E_m$cos ω_mt (10)
- Sub (10) in (9) $\emptyset_{FM}$(t) = A cos[ω_ct + K_f $\frac{E_m}{\omega_m}$sin ω_mt] (11)
- Let $\Delta\omega$= $K_f E_m$. Hence $\emptyset_{FM}$(t) = Acos[ω_ct + $\frac{\Delta\omega}{\omega_m}$ sin ω_mt] (12)

  Where $\boldsymbol{\Delta\omega}$ is the **frequency deviation**

- Let m_f= $\frac{\Delta\omega}{\omega_m}$ Then $\emptyset_{FM}$(t) = A cos [ω_ct + m_f sin ω_mt] (13)

  Where $\boldsymbol{m_f}$ is the **modulation index** - Ratio of frequency deviation to the modulating frequency

Representation of Phase modulated signal

- The total phase angle of the carrier wave is given by φ_i(t) = ω_ct $+\ \theta_0$
- For a phase modulated signal, the phase angle is varied linearly with the modulating signal.

Hence $\theta \,\alpha$ f(t)

θ= K_p f(t)

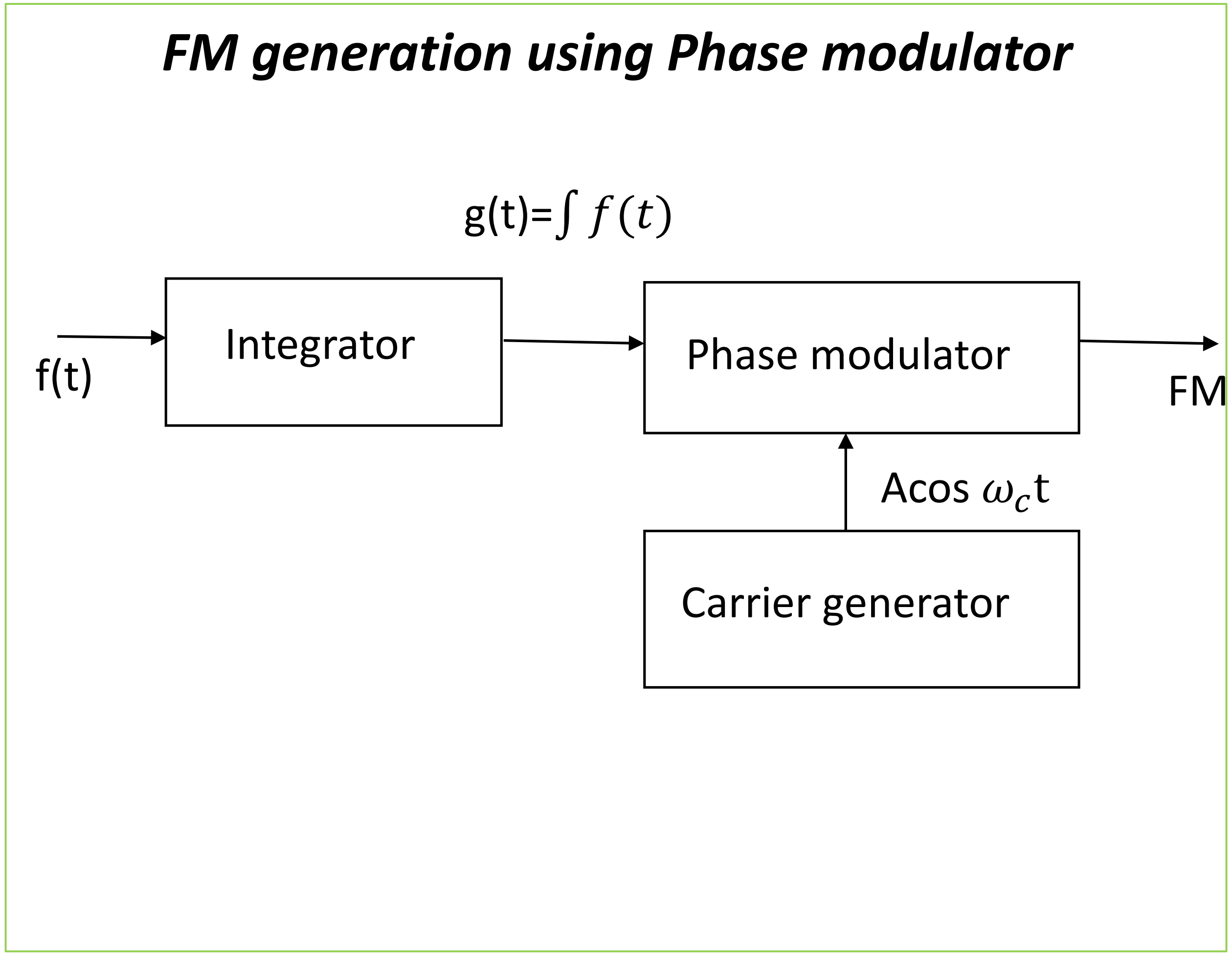
- The phase modulated signal $\emptyset_{PM}$(t) = A cosφ_i(t)

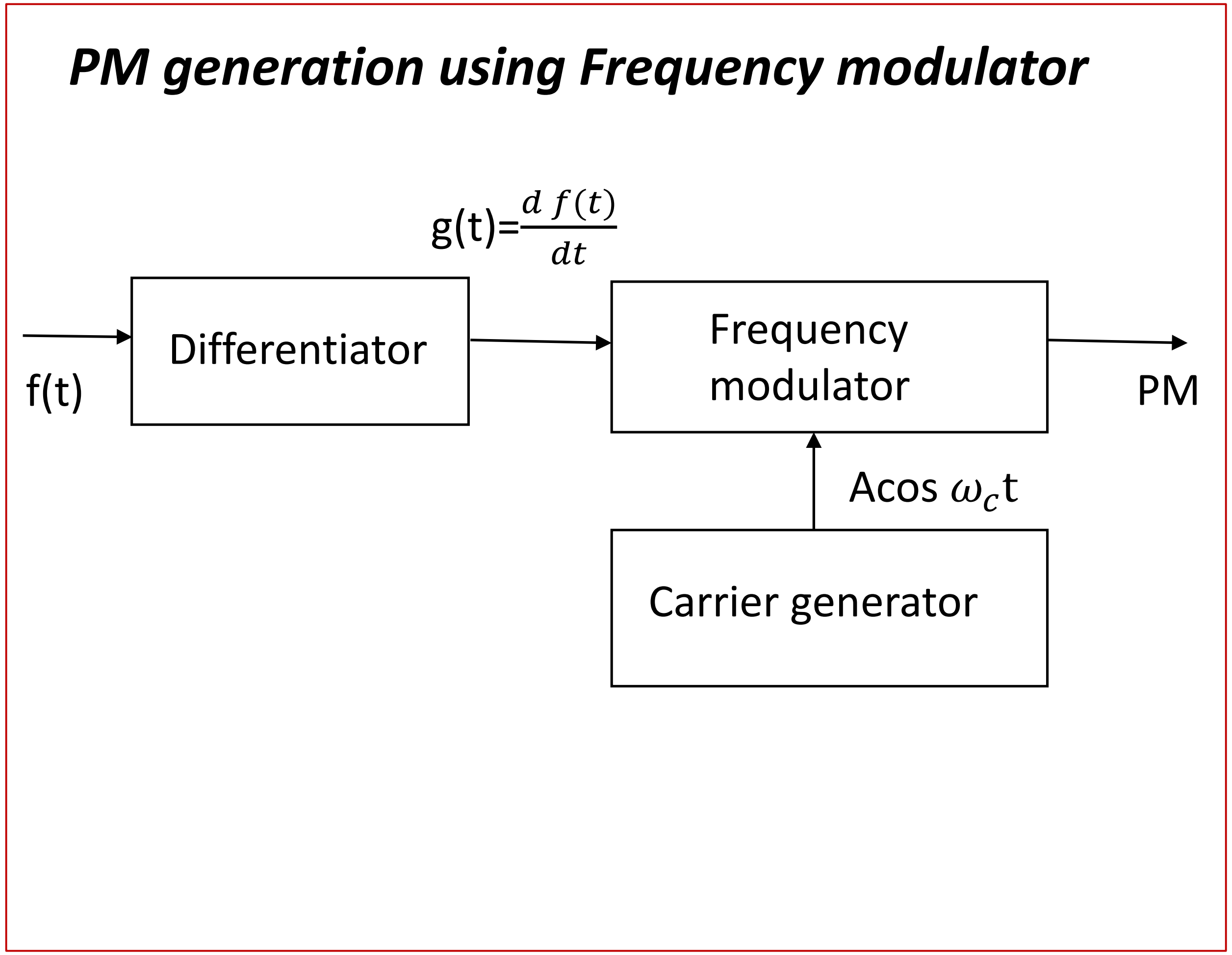
$\emptyset_{PM}$(t) = A cos[ω_ct +K_p f(t)]

The maximum change in total phase angle from the centre phase is known as phase deviation (Δθ)

$$\Delta\theta = m_f = \frac{\Delta\omega}{\omega_m}$$

Relationship between PM and FM





Transmission bandwidth of FM signal

- Bandwidth=2nω_m

  where n is the number of sidebands

  n≈m_f
- BW=2$m_f\,\omega_m$=2$\Delta\omega$=2Δf

Bandwidth using Carson's rule

$$\boldsymbol{BW=2(\Delta\omega+\omega_m)=2(\Delta f+f_m)}$$

Depending upon the value of Δω, FM is classified as narrowband FM (NBFM) and wideband FM (WBFM)

Bandwidth of PM signal

BW(PM) ≈ 2Δω

= $2K_pE_m\omega_m$

Modulation index of PM signal

$m_p = K_pE_m = \theta_d$

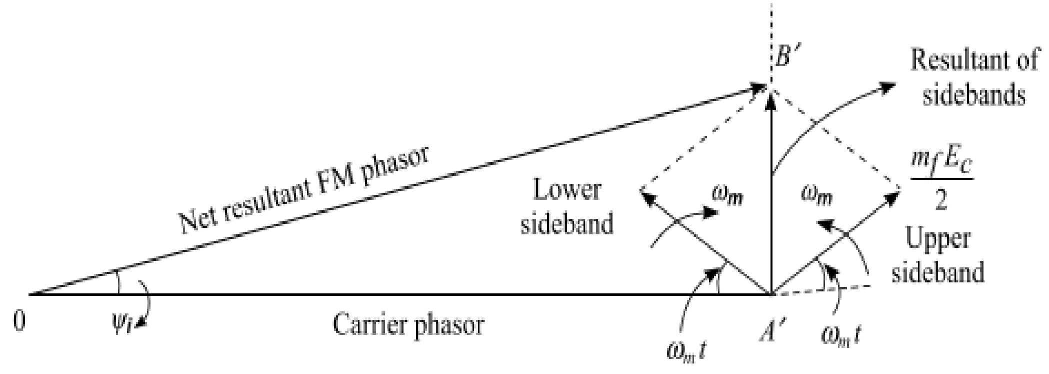


Figure: Phasor diagram of FM

Comparison between NBFM and WBFM

NBFM

- Frequency deviation is very small
- $BW = 2\omega_m$
- K_f is very small
- BW is narrow
- m_f is very small
- Only two sidebands

WBFM

- Frequency deviation is very large
- $BW = 2\Delta\omega$
- K_f is very large
- BW is wide
- m_f is very large
- ‘n’ number of sidebands

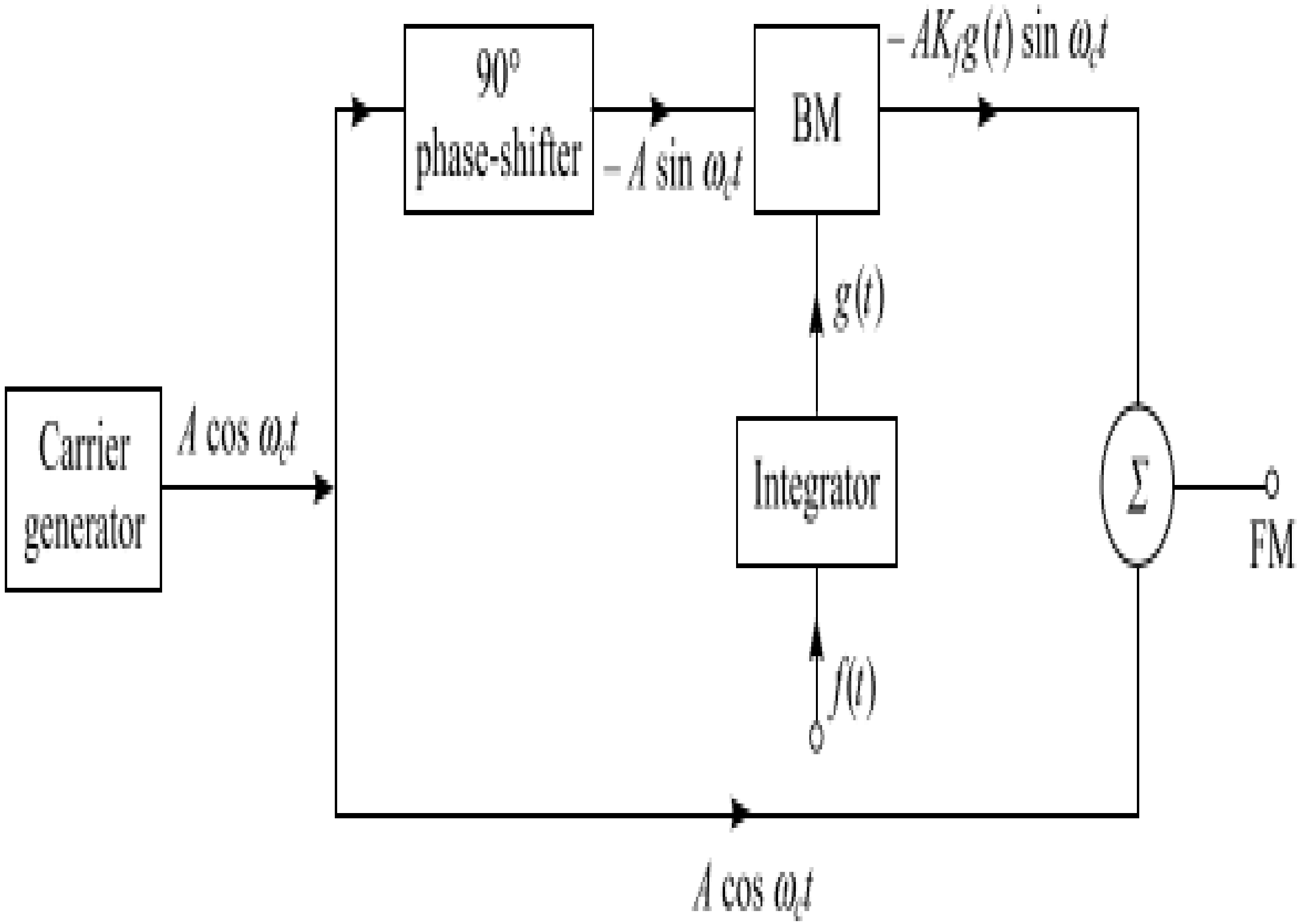
International regulation for FM signal

- The following values are prescribed by CCIR (Consultative Committee for International Radio) for commercial FM broadcast stations.

  - Maximum frequency deviation $\pm 75 KHz$.
  - Frequency stability of the carrier $\pm 2 KHz$.
  - Allowable bandwidth per channel = 200KHz.

Power content in FM signal $\frac{A^2}{2}$

Generation of Narrowband FM



Carrier signal A$\cos\omega_c$t

Phase shifted carrier - $A\sin\omega_c$t

Message signal f(t)= $E_m\cos\omega_m$t

g(t)= $\int f(t)$ = $\int f(t)$ = $\int E_m\cos\omega_m$t

$=\frac{E_m}{\omega_m}\sin\omega_m$t

Output of balanced modulator is

- $A\sin\omega_c$t * $\frac{E_m}{\omega_m}\sin\omega_m$t

$\emptyset_{NBFM}$(t) = A$\cos\omega_c$t - KA$\frac{E_m}{\omega_m}\sin\omega_m$t sin ω_ct

Let K$\frac{E_m}{\omega_m}$ = m_f

$\emptyset_{NBFM}$(t) = A$\cos\omega_c$t - A$m_f\sin\omega_m$t sin ω_ct

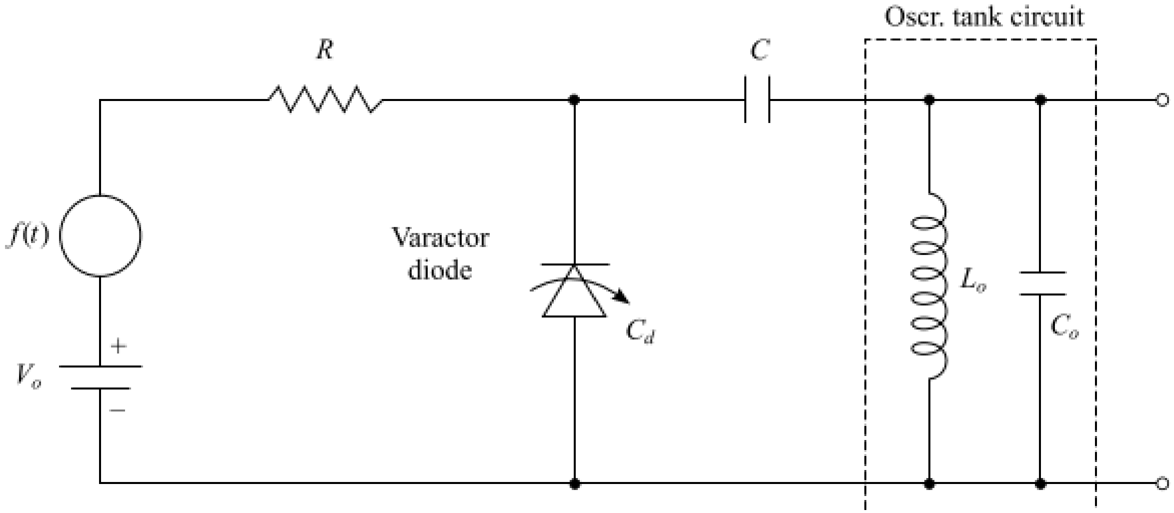
Varactor diode FM modulation

Principle of Operation:

- Modulating signal directly modulates the carrier that is generated by an electronic circuit.
- The oscillator circuit involves a parallel circuit.

- Frequency of oscillation of the carrier generator is

$$\boxed{\omega_c = \frac{1}{\sqrt{LC}}}$$



- The Carrier frequency ω_c can be made to vary according to the modulating signal f(t), if L or C is varied according to f(t).

Varactor diode FM modulation Contd..

Operation:

- Varactor diode is a semiconductor diode whose junction capacitance changes with the applied d.c bias voltage.
- The varactor diode is shunted with the oscillator tank circuit.
- $C < C_d$ to keep the r.f voltage from the oscillator across the diode small as compared to V_o, the polarizing voltage.
- X_c at highest modulating frequency is kept large as compared to R.
- V_o is reverse bias voltage across the varactor diode.

Varactor diode FM modulation Contd..

- The capacitance C_d of the diode is given by C_d= K$\sqrt{V_D}$ (1)

  Where V_D is the total instantaneous voltage across the diode

  K the proportionality constant.

$$V = V_o + f(t) \quad (2)$$

- The total capacitance of the oscillator tank circuit is $(C_o + C_d)$
- The instantaneous frequency of oscillation

$$\omega_i = {}^{1}\!\Big/_{\sqrt{L_o(C_o + C_d)}} \quad (3)$$

- Sub (1) in (3), we get $\omega_i = {}^{1}\!\Big/_{\sqrt{L_o(C_o + K\sqrt{V_D})}}$ (4)
- ω_i is dependent on V_D which in turn depends on the modulating signal f(t).

Varactor diode FM modulation Contd..

Distortion due to non-linearity:

- From (4) it is understood that ω_i does not change linearly with V_D.
- This non-linearity produces distortion due to the frequency variations caused by the higher harmonics of the modulating frequency.
- Assume that the oscillator tank circuit comprises only the diode capacitance C_d and C_o is absent.

$$\omega_i = {}^{1}\Big/\sqrt{L_o K \sqrt{V_D}} = \frac{V_D^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} \qquad (5)$$

The R.H.S of the above equation can be represented by a Taylor series about the polarizing voltage V_o as given below.

$$\frac{V_D^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} = \frac{V_o^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} + \frac{(V_D - V_o)}{4(L_o K V_o^{\frac{3}{2}})^{\frac{1}{2}}} - \frac{3(V_D - V_o)^2}{16(L_o K V_o^{\frac{7}{2}})^{\frac{1}{2}}} \qquad (6)$$

Varactor diode FM modulation Contd..

- The higher order terms can be neglected if $(V_D - V_o)$ is small.
- Let $(V_D - V_o) = \Delta V = f(t) = V_m \sin\omega_m t$ (7)

$$(V_D - V_o)^2 = V_m^2 sin^2 \omega_m t = \frac{V_m^2}{2}(1 - \cos 2\omega_m t) \quad (8)$$

- Sub (7) and (8) in (6)

$$\omega_i = \frac{V_D^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} = \frac{V_o^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} + \frac{V_m \sin\omega_m t}{4(L_o K V_o^{\frac{3}{2}})^{\frac{1}{2}}} - \frac{3V_m^2}{(32 L_o K V_o^{\frac{7}{2}})^{\frac{1}{2}}} + \frac{3V_m^2 cos 2\omega_m t}{32(L_o K V_o^{\frac{7}{2}})^{\frac{1}{2}}} \quad (9)$$

Varactor diode FM modulation Contd..

- ***% second harmonic distortion*** is the ratio of amplitude of the $cos2\omega_m$ term and the fundamental term

$$\boxed{\%\ \text{second harmonic distortion} = \frac{3V_m}{8V_o} \times 100}$$

By adjusting proper ratio of V_m and V_o second harmonic distortion may be reduced.

Ignoring the effect of second harmonic of f(t)

$$\omega_i = \frac{V_o^{\frac{1}{4}}}{(L_o K)^{\frac{1}{2}}} + \frac{V_m \sin\omega_m t}{4(L_o K V_o^{\frac{3}{2}})^{\frac{1}{2}}} = \omega_c + (\Delta\omega)\sin\omega_m t$$

$$\text{Modulation index } m_f = \frac{\Delta\omega}{\omega_m} = \frac{V_m}{4\omega_m(L_o K V_o^{\frac{3}{2}})^{\frac{1}{2}}}$$

Thus the modulation index not only depends on the modulating voltage V_m but also on the polarizing voltage V_o.

Demodulation of FM signals

- The process of recovering the modulating signal from a modulated carrier is known as modulation.

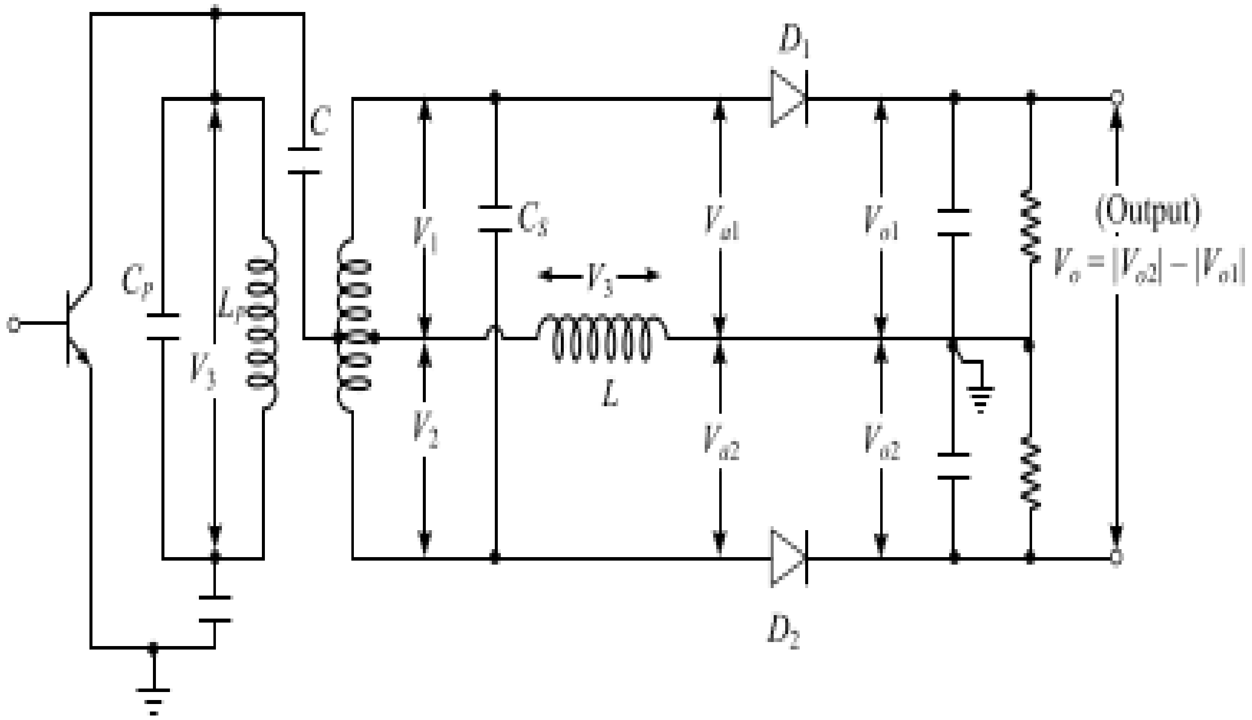
- *The detector performs the modulation in two steps.*
  - FM signal is converted to its corresponding AM signal using frequency dependent circuits (frequency discriminators)
  - The original message signal is recovered from this AM signal by using linear diode detector.

Types of FM discriminators

1)Slope detector
- Simple slope detector or single-tuned discriminator circuit
- Balanced slope detector or stagger tuned discriminator circuit

2)Phase Difference discriminator
- Foster-Seeley discriminator
- Ratio detector

Foster-Seeley discriminator



Operation:

- The circuit has inductively coupled doubled-tuned circuit
- The primary and secondary are tuned to the same frequency (f_{if})
- Centre of secondary is connected to the collector end of primary through a capacitor C.

Functions of capacitor C

  - Blocks d.c from primary to secondary
  - Couples the primary signal frequency to center-tapping of secondary.

- The primary voltage V_3 appears across the inductance L.
- The center-tapping of the transformer has equal and opposite winding.
- Hence V_1 and V_2 are equal in magnitude but opposite in phase.

Foster-Seeley discriminator Contd..

- The radio frequency voltages V_{a1} and V_{a2} applied to the diodes D_1 and D_2 are

$$V_{a1} = V_3 + V_1 \text{ and } V_{a2} = V_3 - V_2$$

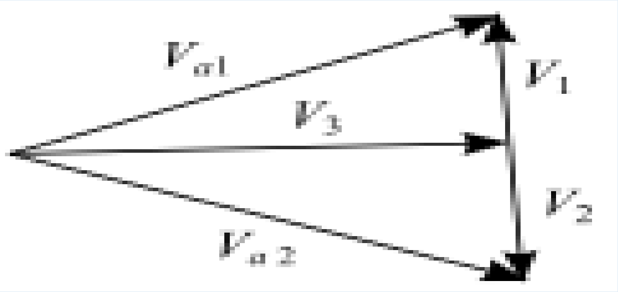
- Voltages V_{a1} and V_{a2} depend on the phasor relation between V_1, V_2, V_3.
- The phasor position of V_1 and V_2 are always equal and are in phase opposition.
- The phase position of V_1 and V_2 relative to V_3 will depend on the tuned secondary at the resonance or off resonance.

Foster-Seeley discriminator Contd..

At resonance

$f_{input} = f_{if}$

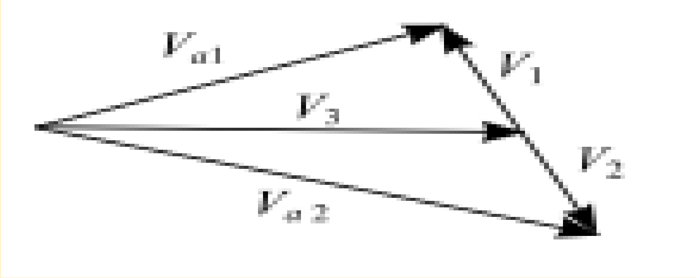
- V_3 in phase quadrature with V_1 and V_2.
- The resultant voltages V_{a1} and V_{a2} are equal in magnitude.



Off resonance

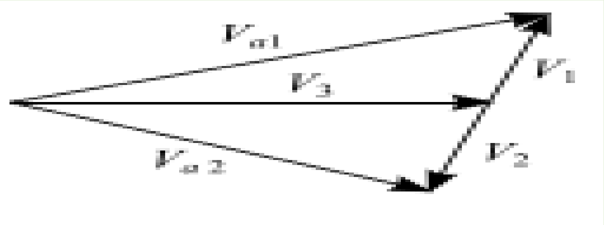
$f_{input} > f_{if}$ by an amount $\frac{f_{if}}{2Q_s}$

- Phase difference between V_3 and V_1 is 45 degrees.
- Since V_2 is in phase opposition with V_1, phase difference between V_3 and V_2 is 135 degrees.
- The magnitude of V_{a1} is reduced whereas V_{a2} is increased.



$f_{input} < f_{if}$ by an amount $\frac{f_{if}}{2Q_s}$

- Phase difference between V_3 and V_2 is 45 degrees.
- Since V_2 is in phase opposition with V_1, phase difference between V_3 and V_1 is 135 degrees.
- The magnitude of V_{a1} is increased whereas V_{a2} is decreased.



Foster-Seeley discriminator Contd..

- Thus the amplitude of V_{a1} and V_{a2} will vary with the instantaneous frequency f as shown in figure 1 (a).
- The RF voltage V_{a1} and V_{a2} are separately rectified by the diodes D_1 and D_2 to produce voltages V_{o1} and V_{o2} that represent the amplitude variations of V_{a1} and V_{a2}.
- The output voltage is given by $V_0 = |V_{o2}| - |V_{o1}|$.
- The discriminator characteristics is zero at resonance, positive above resonance and negative below resonance.
- It is linear for the region between the peaks of V_{a1} and V_{a2} and this range is the peak separation region which should be more than twice the frequency deviation .

Disadvantage

- Any variation in amplitude of the input FM signal due to noise modifies the discriminator characteristics as shown in figure 2 (b).
- The undesired frequency components corresponding to amplitude variations lead to distorted output.
- Distortions can be reduced by using a limiter in FM receiver.

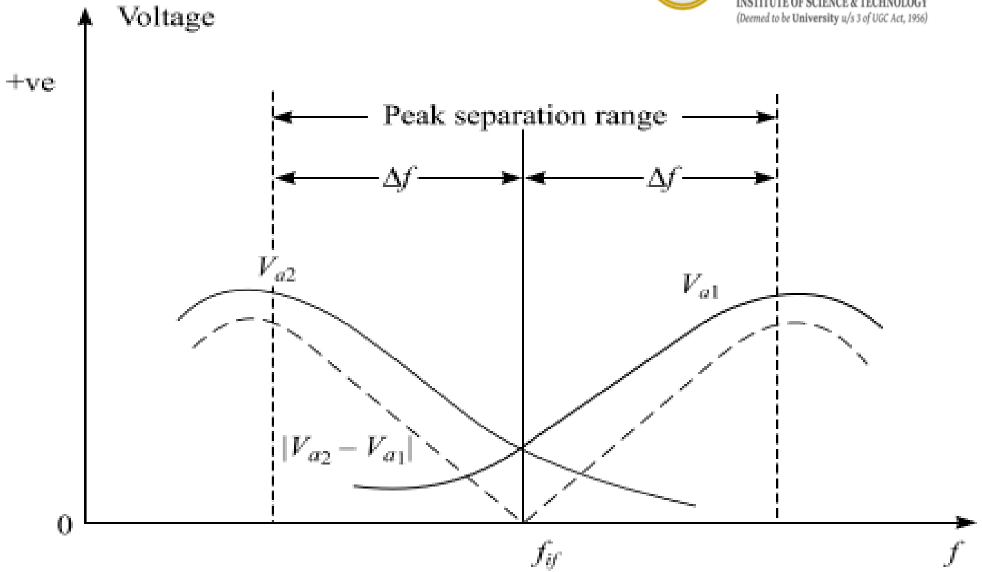


Figure 1 (a): Discriminator Characteristics

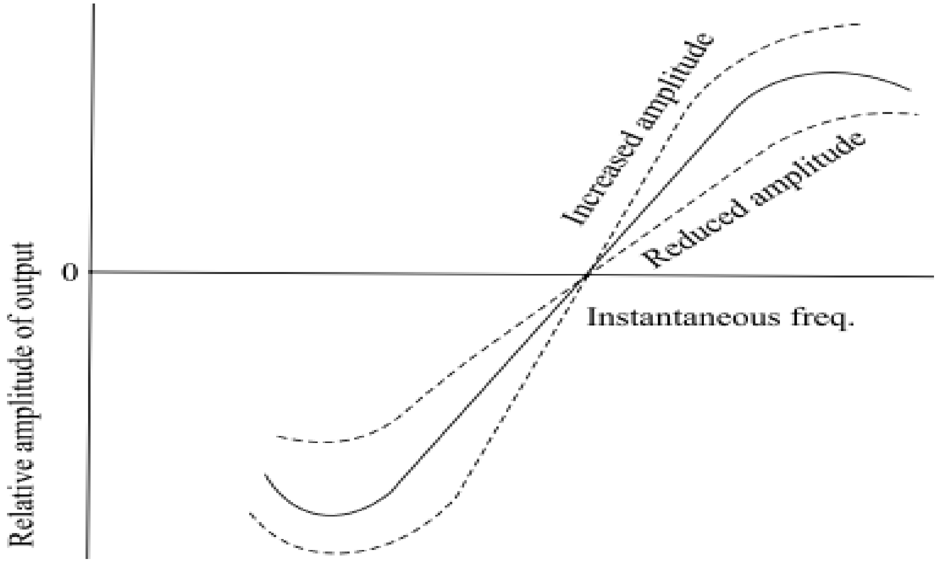


Figure 1 (b): Discriminator Characteristics

1) A single-tone modulating signal $\cos(15\pi 10^3 t)$, frequency modulates a carrier of 10 MHz and produces a frequency deviation of 75 KHz. Find i) the modulation index ii) Phase deviation produced in the FM wave.
iii) If another modulating signal produces a modulation index of 100 while maintaining the same deviation, find the frequency and amplitude of the modulating signal, assuming $k_f = 15$ kHz per volt

Solution:

i) FM modulation index

$$m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m} = \frac{75 \times 10^3}{7.5 \times 10^3}$$

$$\boxed{m_f = 10}$$

ii) Phase deviation

$$\boxed{\Delta\theta = m_f = 10 \text{ rad}}$$

iii) gn: $m_f = 100$, $\Delta f = 75$ KHz

$$f_m = \frac{\Delta f}{m_f} = \frac{75 \times 10^3}{100}$$

$$\boxed{f_m = 750 \text{ Hz}}$$

$$E_m = \frac{\Delta f}{K_f} = \frac{75}{15} = 5\text{V} \qquad \boxed{E_m = 5\text{V}}$$

2) The maximum deviation allowed in an FM broadcast system is 75 KHz. If the modulating signal is a single-tone sinusoid of 10 KHz, find the bandwidth of the FM signal. What will be the change in the bandwidth, if the modulating frequency is doubled? Determine the bandwidth when modulating signal's amplitude is also doubled.

Solution:

Given $\Delta f = 75$ KHz, $f_m = 10$ KHz

$BW = 2(\Delta f + f_m) = 2(75+10) \times 10^3$

$\boxed{BW = 170 \text{ KHz}}$

When modulating frequency is doubled, $f_m = 20$ KHz

$BW = 2(75+20) \times 10^3$

$\boxed{BW = 190 \text{ KHz}}$

When modulating signal's amplitude is doubled, frequency deviation becomes $2 \times 75 = 150$ KHz

$[\because f_m = \Delta f / K_f]$

$BW = 2 \times 150 + 2 \times 20$

$\boxed{BW = 340 \text{ KHz}}$

3) A modulating signal $5\cos 2\pi\, 15\times10^3 t$, angle modulates a carrier $A\cos\omega_c t$.

i) Find the modulation index and the bandwidth for a) the FM system b) the PM system

ii) Determine the change in the bandwidth and the modulation index for both FM and PM, if f_m is reduced to 5KHz. Assume $K_p = K_f = 15$ KHz/V.

Solution:

i) Given $E_m = 5V$, $f_m = 15KHz$

a) FM system:

$$\Delta f = K_f E_m = 15\times10^3 \times 5$$

$$\Delta f = 75\,KHz$$

$$m_f = \frac{75}{15}$$

$$\boxed{m_f = 5}$$

$$BW = 2(\Delta f + f_m) = 2\times 90$$

$$\boxed{BW = 180\,KHz}$$

b) PM System.

Frequency deviation $\Delta f = K_p E_m f_m$

$$\boxed{\Delta f = 1125\,MHz}$$

$$BW = 2(\Delta f + f_m) \simeq 2\Delta f$$

$$\boxed{BW = 2250\,MHz}$$

$m_p = k_p E_m = 15 \times 10^3 \times 5$

$\boxed{m_p = 75 \text{ KHz}}$

(ii) Now $f_m = 5$ KHz

For FM:

$m_f = \frac{\Delta f}{f_m} = \frac{75}{5}$

$\boxed{m_f = 15}$

$BW = 2(\Delta f + f_m) = 2(75+5)$

$\boxed{BW = 160 \text{ KHz}}$

FM modulation index changes considerably with a change in the modulating frequency, but the bandwidth changes only slightly.

For PM system:

$\Delta f = k_p E_m f_m = 15 \times 10^3 \times 5 \times 5 \times 10^3$

$\boxed{\Delta f = 375 \text{ MHz}}$

$BW = 2(\Delta f + f_m) \simeq 2\Delta f$

$\boxed{BW = 750 \text{ MHz}}$

$\boxed{m_p = k_p E_m = 75 \text{ KHz}}$

In PM, the BW changes considerably with change in f_m but m_p remains unchanged.

4) A semiconductor junction diode is used to modulate the frequency of an oscillator. The junction capacitance is the total tuning capacitance of the oscillator tank circuit. When a d.c bias voltage of 15 V is applied to the diode, the oscillator frequency generated is 5 MHz. If a single-tone modulating voltage 4 sin12560 t modulates the carrier: find a) the percentage second harmonic distortion and b) the frequency modulation index.

Solution:

Gn. The polarizing voltage $V_0 = 15\,V$

$f_c = 5\,MHz$

$\omega_c = 10\pi \cdot 10^6$ rad/s

$$(L_0 k)^{1/2} = \frac{V_0^{1/4}}{\omega_c} = \frac{15^{1/4}}{10\pi \cdot 10^6}$$

$\omega_m = 12560$ rad/sec, $V_m = 4\,V$.

a) % 2^{nd} harmonic distortion $= \frac{3 V_m}{8 V_0} \times 100$

$\boxed{\text{Percentage distortion} = 10\%.}$

b) modulation index

$$m_f = \frac{V_m}{4\omega_m (L_0 k V_0^{3/2})^{1/2}} = \frac{V_m}{4\omega_m (L_0 k)^{1/2} \cdot V_0^{3/4}}$$

$$m_f = \frac{4 \times 10\pi \times 10^6}{4 \times 12560 \times (15)^{1/4} \times (15)^{1/4}}$$

$\boxed{m_f = 166.6}$