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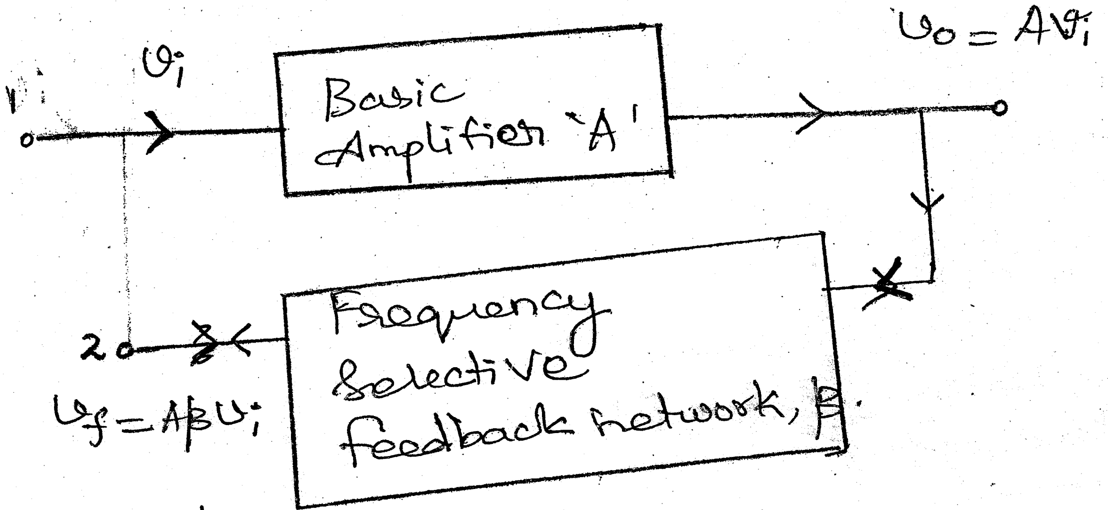
Waveform generators:

Sine Wave Generators: (oscillators)

The feedback oscillator can be used as a sine wave oscillator.
It consists of an amplifier with gain 'A' and a frequency selective feedback network with transfer ratio 'β'.

Basic structure of a feedback oscillator:

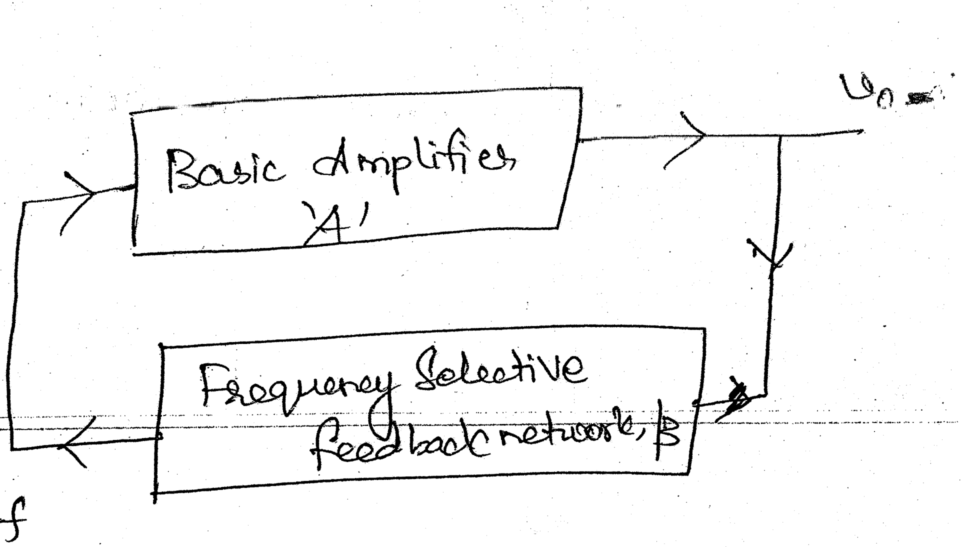


* output voltage
$\boxed{v_o = Av_i}$

* feedback voltage: $\boxed{\begin{array}{l} v_f = \beta v_o \\ v_f = A\beta v_i \end{array}} \rightarrow (1)$

* $A\beta$ = loop gain of the system.
* If $A\beta = 1$, $v_f = v_i$, from eqn (1).

∴ If we join terminals (1) & (2) and remove external signal, $\boxed{A\beta = 1}$ satisfied



$$A = \frac{v_o}{v_f}, \quad \beta = \frac{v_f}{v_o}.$$

Barkhausen criterion for oscillation:

An output signal can be continuously obtained without any input signal, if the loop gain of the system satisfies the following condition.

$$\boxed{A\beta = 1}$$

Note:

The condition $A\beta = 1$ can be satisfied only at one specific frequency f_o for the given component values.

∴ The condition for oscillation can be rewritten as

$$A(j\omega_o)\,\beta(j\omega_o) = 1 \angle 0^\circ$$

i.e. $|A\beta| = 1$

$\angle A\beta = 0^\circ$ (or) multiples of 2π.

Note:
Practically the oscillators are designed for $|A\beta| > 1$ so that oscillations grow.

Types of sine wave oscillators:

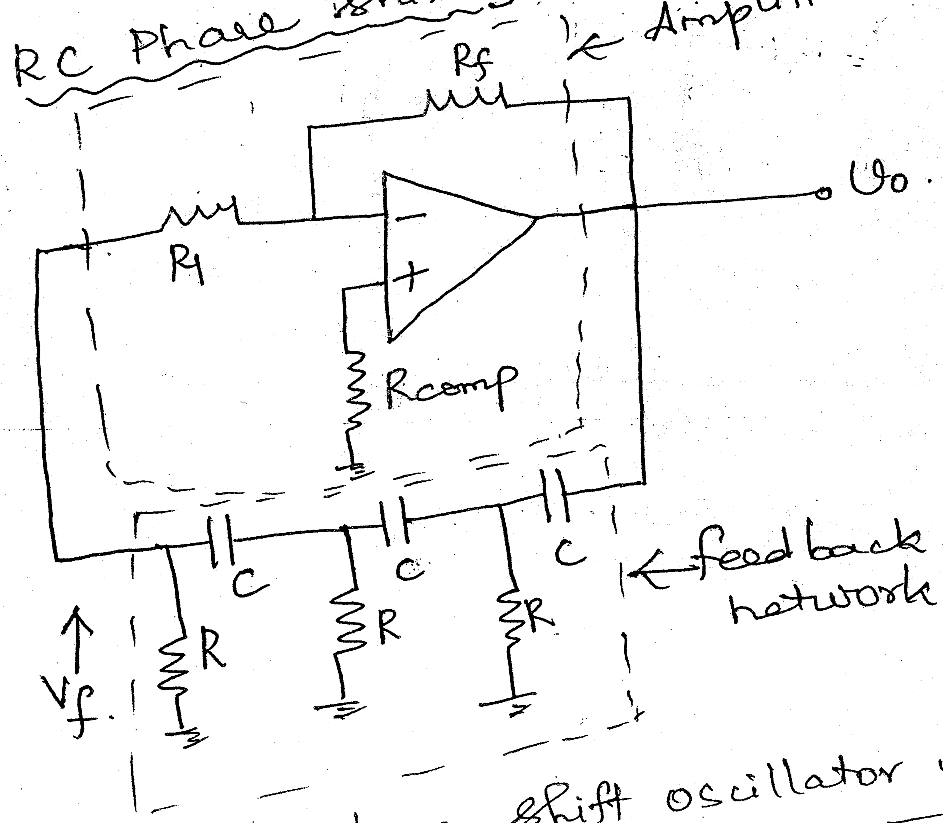
The sine wave oscillators are classified according to range of frequency.

(a) Audio frequency oscillators:
(few hertz to several 100 kHz)

(i) RC phase shift oscillator.
(ii) Wein bridge oscillator.

(b) High frequency oscillator
(i) LC oscillator.

6 (i) RC Phase shift oscillator:

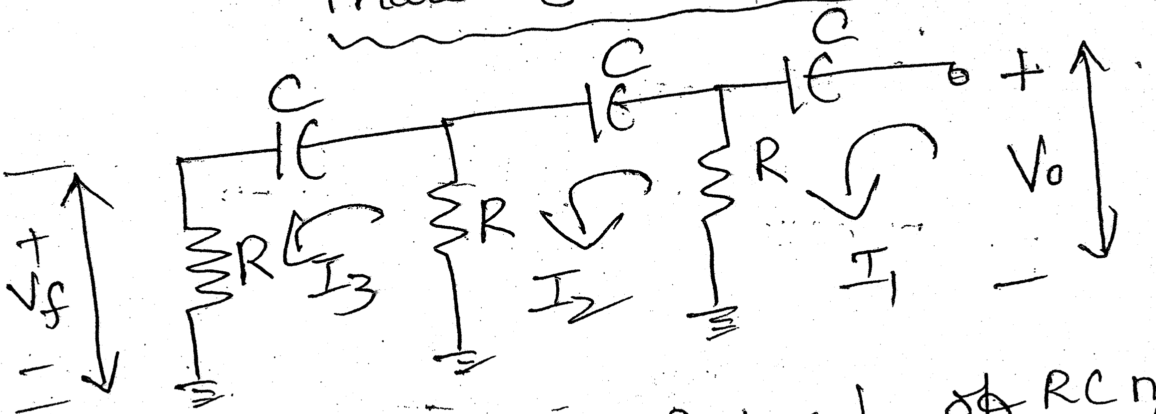


The RC phase shift oscillator uses op-amp in ~~[illegible]~~ inverting mode. Therefore

it provides a 180° phase shift. To intro[duce]
additional 180° phase shift so that tota[l]
phase shift is 360°, we use RC feedbac[k]
network.
The RC feedback network consists of
3 identical RC stages where each stage
provides a phase shift of 60°.

Analysis:

Phase shift network:



Calculating feedback factor β of RC n/w:

Applying KVL for loop 1.

$$\frac{I_1}{j\omega C} + (I_1 - I_2)R = V_o.$$

Substitute $j\omega = s$.

$$\frac{I_1}{sC} + (I_1 - I_2)R = V_o.$$

$$\boxed{I_1\left(R + \frac{1}{sC}\right) - I_2 R = V_o} \rightarrow (1)$$

Applying KVL for Loop2 :

$$I_2 \cdot \frac{1}{j\omega c} + (I_2 - I_3)R + (I_2 - I_1)R = 0.$$

sub $j\omega = s$

$$\frac{I_2}{sc} + I_2 R - I_3 R + I_2 R - I_1 R = 0.$$

$$\boxed{-I_1 R + I_2\left(2R + \frac{1}{sc}\right) - I_3 R = 0.} \rightarrow (2)$$

Applying KVL for Loop3 :

$$\frac{I_3}{j\omega c} + I_3 R + (I_3 - I_2)R = 0.$$

sub $j\omega = c$

$$\frac{I_3}{sc} + I_3 R + RI_3 - I_2 R = 0.$$

$$\boxed{0 - I_2 R + I_3\left(2R + \frac{1}{sc}\right) = 0.} \rightarrow (3)$$

$$\boxed{V_f = I_3 R} \rightarrow (4)$$

$$\begin{bmatrix} R + \frac{1}{sc} & -R & 0 \\ -R & \left(2R + \frac{1}{sc}\right) & -R \\ 0 & -R & \left(2R + \frac{1}{sc}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \left(R+\frac{1}{sC}\right)\left[\left(2R+\frac{1}{sC}\right)\left(2R+\frac{1}{sC}\right) - (-R)(-R)\right]$$

$$+R\left[-R\left(2R+\frac{1}{sC}\right) - 0\right]$$

$$= \left(R+\frac{1}{sC}\right)\left(\left(2R+\frac{1}{sC}\right)^2 - R^2\right) + R\left[-2R^2 - \frac{R}{sC}\right]$$

$$= \left(R+\frac{1}{sC}\right)\left(4R^2 + \frac{1}{s^2C^2} + \frac{4R}{sC} - R^2\right) - 2R^3 - \frac{R^2}{sC}$$

$$= \left(R+\frac{1}{sC}\right)\left(3R^2 + \frac{1}{s^2C^2} + \frac{4R}{sC}\right) - 2R^3 - \frac{R^2}{sC}$$

$$= 3R^3 + \frac{R}{s^2C^2} + \frac{4R^2}{sC} + \frac{3R^2}{sC} + \frac{1}{s^3C^3} + \frac{4R}{s^2C^2} - 2R^3 - \frac{R^2}{sC}$$

$$\boxed{\Delta = R^3 + \frac{6R^2}{sC} + \frac{5R}{s^2C^2} + \frac{1}{s^3C^3}}$$

$$\Delta I_3 = \begin{bmatrix} \left(R+\frac{1}{sC}\right) & -R & V_0 \\ -R & \left(2R+\frac{1}{sC}\right) & 0 \\ 0 & -R & 0 \end{bmatrix} = \left(R+\frac{1}{sC}\right)(0-0) + R[0-0) + V_0[R^2-0] = \boxed{V_0 R^2}$$

$$\therefore I_3 = \frac{\Delta I_3}{\Delta} = \frac{V_0 R^2}{R^3 + \frac{6R^2}{sC} + \frac{5R}{s^2C^2} + \frac{1}{s^3C^3}}$$

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$$\boxed{I_3 = \frac{V_0 R^2 s^3 C^3}{1+5RSC+6R^2S^2C^2+R^3S^3C^3}} \rightarrow (5)$$

From eqn (4) $\Rightarrow V_f = I_3 R$

Sub eqn (5)

$$\boxed{V_f = \frac{V_0 R^3 S^3 C^3}{1+5SRC+6S^2R^2C^2+R^3C^3S^3}} \rightarrow (6)$$

$$\boxed{\beta = \frac{V_f}{V_0}}$$

From eqn (6), $\beta = \frac{V_f}{V_0} = \frac{R^3 S^3 C^3}{1+5SRC+6S^2R^2C^2+S^3R^3C^3}$

Num & denominator $\div$ by $S^3C^3R^3$

$$\beta = \frac{1}{\frac{1}{S^3C^3R^3} + \frac{5}{S^2R^2C^2} + \frac{6}{SRC} + 1}$$

Sub $S = j\omega$. $\therefore S^2 = -\omega^2$, $S^3 = -j\omega^3$

$$\therefore \boxed{\beta = \frac{1}{1 + \frac{6}{j\omega RC} - \frac{5}{\omega^2R^2C^2} - \frac{1}{j\omega^3C^3R^3}}} \rightarrow (7)$$

Let cut off parameter $\alpha = \frac{1}{\omega RC}$

Sub 'α' in eqn (7).

$$\beta = \frac{1}{1 + \frac{6\alpha}{j} - 5\alpha^2 - \frac{\alpha^3}{j}}$$

$$\beta = \frac{1}{1 - j6\alpha - 5\alpha^2 + j\alpha^3}$$

$$\boxed{\beta = \frac{1}{(1-5\alpha^2) + j\alpha(\alpha^2-6)}} \rightarrow (8)$$

For $A\beta = 1$, β should be real.
i.e. imaginary term should be zero.

$$\alpha(\alpha^2-6) = 0$$

$$\alpha^2 - 6 = 0$$

$$\alpha^2 = 6$$

$$\boxed{\alpha = \sqrt{6}}$$

$$\therefore \boxed{\frac{1}{\omega RC} = \sqrt{6}}$$

$$\Rightarrow \frac{1}{2\pi f RC} = \sqrt{6} \Rightarrow f = \frac{1}{2\pi RC\sqrt{6}}$$

$\therefore$ Frequency of oscillation,

$$\boxed{f_0 = \frac{1}{2\pi RC\sqrt{6}}}$$

when $\alpha^2 = 6$

$$\beta = \frac{1}{(1-5\alpha^2)} = \frac{1}{(1-5\times 6)} = \frac{1}{(1-30)} = -\frac{1}{29}.$$

$$\boxed{\beta = -\frac{1}{29}}$$

The negative sign indicates RC feedback n/w produces a phase shift of 180°.

$$|\beta| = \frac{1}{29}$$

For sustained oscillation $|A\beta| \geq 1$

$$\therefore \left|\frac{A}{29}\right| \geq 1$$

$$\therefore \boxed{|A| \geq 29.}$$

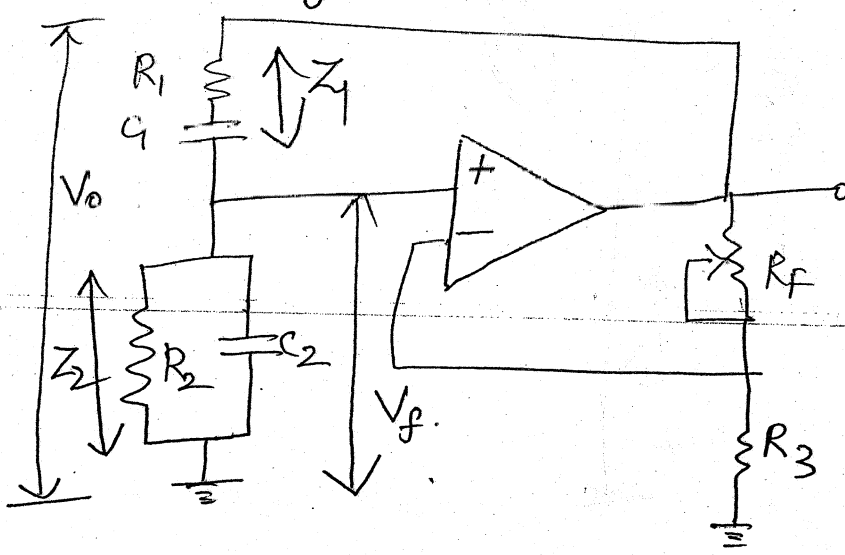
$\therefore$ The gain of inverting op-amp should be atleast 29.

Gain of inverting op-amp $A = -\frac{R_f}{R_1}$

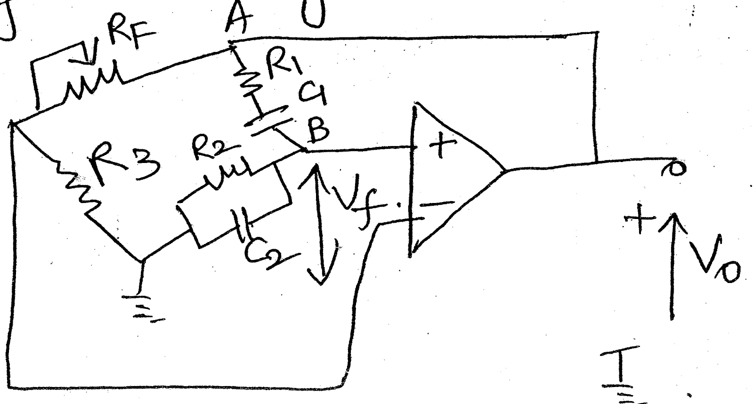
$$|A| = \left|\frac{R_f}{R_1}\right| = 29$$

$$\therefore \boxed{R_f = 29 R_1}$$

Wein bridge Oscillator:



* It is a audio frequency oscillator.
* The feedback signal is connected to the non-inverting terminal of the op-amp. ∴ The feedback network doesnot provide any phase shift.
* The circuit can be viewed as a wein bridge with series RC n/w in one arm and parallel RC n/w in adjoining arm. Resistor R_3 and R_F in other two arms.
* We achieve zero phase shift condition, by balancing the bridge.



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Analysis:

* The gain of the op-amp amplifier

$$\boxed{A = 1 + \frac{R_f}{R_3}} \rightarrow (1)$$

* $$V_f = \frac{V_0 Z_2}{Z_1 + Z_2}$$

$$\frac{V_f}{V_0} = \frac{Z_2}{Z_1 + Z_2}$$

$$\boxed{\beta = \frac{V_f}{V_0} = \frac{Z_2}{Z_1 + Z_2}} \rightarrow (2)$$

(Laplace transform)

$$Z_1 = R_1 + \frac{1}{SC_1} = \frac{SC_1R_1 + 1}{SC_1} \rightarrow (3)$$

$$Z_2 = \frac{R_2 \cdot \frac{1}{SC_2}}{R_2 + \frac{1}{SC_2}} = \frac{R_2}{1 + SC_2R_2} \rightarrow (4)$$

Sub (3) & (4) in (2)

$$\beta = \frac{R_2 / (1 + SC_2R_2)}{\frac{SC_1R_1 + 1}{SC_1} + \frac{R_2}{1 + SC_2R_2}}$$

$$= \frac{R_2 / \cancel{(1 + SC_2R_2)}}{(1 + SC_1R_1)(1 + SC_2R_2) + R_2SC_1}$$

$$= \frac{R_2 s C_1}{(1+sC_1R_1)(1+sC_2R_2) + R_2 s C_1}$$

$$= \frac{s R_2 C_1}{1 + sC_2R_2 + sR_1C_1 + s^2 R_1R_2C_1C_2 + sR_2C_1}$$

$$\boxed{\beta = \frac{s R_2 C_1}{1 + s(R_1C_1 + R_2C_2 + R_2C_1) + s^2 R_1R_2C_1C_2}}$$

Let $s = j\omega$.

$$\beta = \frac{j\omega R_2 C_1}{1 + j\omega(R_1C_1 + R_2C_2 + R_2C_1) - \omega^2 R_1R_2C_1C_2}$$

∴ The condition for oscillation is β should be real.

$$\therefore 1 - \omega^2 R_1R_2C_1C_2 = 0$$

$$\omega^2 R_1R_2C_1C_2 = 1$$

$$\omega^2 = \frac{1}{R_1R_2C_1C_2}$$

$$\omega = \frac{1}{\sqrt{R_1R_2C_1C_2}} = 2\pi f_0$$

∴ Frequency of oscillation

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}} \rightarrow (6)$$

* when oscillation condition is satisfied, (31)

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$$\beta = \frac{\cancel{j\omega} R_2 C_1}{\cancel{j\omega}(R_1 C_1 + R_2 C_2 + R_2 C_1)}$$

$$\boxed{\beta = \frac{R_2 C_1}{(R_1 C_1 + R_2 C_2 + R_2 C_1)}} \rightarrow (7).$$

* If $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$(6) \Rightarrow f_0 = \frac{1}{2\pi\sqrt{R^2C^2}} = \frac{1}{2\pi RC}.$$

$$(7) \Rightarrow \beta = \frac{RC}{(RC + RC + RC)} = \frac{RC}{3RC} = \frac{1}{3}.$$

* Note:

For sustained oscillation

$$|A\beta| \geq 1$$

$$|A/3| \geq 1$$

$$\boxed{|A| \geq 3}$$

$$A = 1 + \frac{R_F}{R_3}$$

$$1 + \frac{R_F}{R_3} = 3 \Rightarrow \frac{R_F}{R_3} = 2 \Rightarrow \boxed{R_F = 2R_3}$$

Disadvantages:

It cannot be used for generating high frequency signals as it does not provide ... phase shift.