DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Name of the Course: Electrical and Electronics Engineering

Department of Electronics and Communication Engineering

SRM Institute of Science & Technology,

Deemed to be University, Tiruchirappalli Campus,

Trichy - 621105

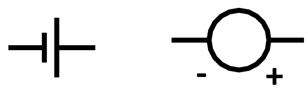
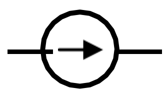
<u>AC & DC circuits</u>

Circuit parameters, Ohms law, Kirchhoff’s laws. Average and RMS values, concept of phasor representation. RLC serious circuits and series resonance. RLC parallel circuits (includes simple problems in DC & AC circuits).

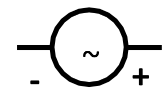
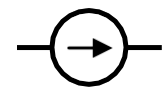
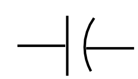
ELECTRIC CIRCUITS

Electric circuits are broadly classified as Direct Current (D.C.) circuits and Alternating Current (A.C.) circuits. The following are the various elements that form electric circuits.

D.C. Circuits

Elements	**Representation**
Voltage source	
Current source	
Resistor	

A.C. Circuits

Elements	**Representation**
Voltage source	
Current source	
Resistor	
Inductor	
Capacitor	

DC CIRCUITS

The voltage across an element is denoted as E or V. The current through the element is I.

Conductor is used to carry current. When a voltage is applied across a conductor, current flows through the conductor. If the applied voltage is increased, the current also increases. The voltage current relationship is shown in Fig. 1.

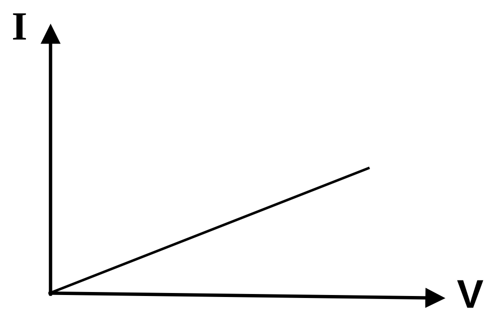


Fig. 1 Voltage – current relationship

It is seen that $I \propto V$. Thus we can write

$$I = G\,V \qquad (1)$$

where G is called the conductance of the conductor.

$$I = G\,V \qquad (1)$$

Very often we are more interested on RESISTANCE, R of the conductor, than the conductance of the conductor. Resistance is the opposing property of the conductor and it is the reciprocal of the conductance. Thus

$$R = \frac{1}{G} \quad \text{or } G = \frac{1}{R} \qquad (2)$$

Therefore $$I = \frac{V}{R} \qquad (3)$$

The above relationship is known as OHM's law. Thus Ohm law can be stated as the current flows through a conductor is the ratio of the voltage across the conductor and its resistance. Ohm's law can also be written as

$$V = R\,I \qquad (4)$$

$$R = \frac{V}{I} \qquad (5)$$

The resistance of a conductor is directly proportional to its length, inversely proportional to its area of cross section. It also depends on the material of the conductor. Thus

$$R = \rho \frac{\ell}{A} \tag{6}$$

where ρ is called the specific resistance of the material by which the conductor is made of. The unit of the resistance is Ohm and is represented as Ω. Resistance of a conductor depends on the temperature also. The power consumed by the resistor is given by

$$P = V\,I \tag{7}$$

When the voltage is in volt and the current is in ampere, power will be in watt. Alternate expression for power consumed by the resistors are given below.

$$P = R\,I \times I = I^2 R \tag{8}$$

$$P = V \times \frac{V}{R} = \frac{V^2}{R} \tag{9}$$

KIRCHHOFF's LAWS

There are two Kirchhoff's laws. The first one is called Kirchhoff's current law, KCL and the second one is Kirchhoff's voltage law, KVL.

Kirchhoff's current law deals with element currents meeting at a junction, which is a meeting point of two are more elements.

Kirchhoff's voltage law deals with element voltages in a closed loop also called as closed circuit.

Kirchhoff's current law

Kirchhoff's current law states that the algebraic sum of element currents meeting at a junction is zero.

Consider a junction P wherein four elements, carrying currents I_1, I_2, I_3 and I_4, are meeting as shown in Fig. 2.

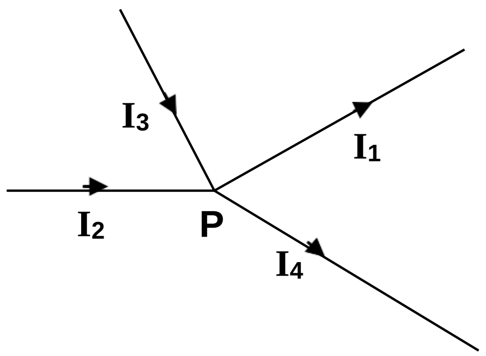


Fig. 2 Currents meeting at a junction

Note that currents I_1 and I_4 are flowing out from the junction while the currents I_2 and I_3 are flowing into the junction. According to KCL,

$$I_1 - I_2 - I_3 + I_4 = 0 \qquad (10)$$

The above equation can be rearranged as

$$I_1 + I_4 = I_2 + I_3 \qquad (11)$$

From equation (11), KCL can also stated as at a junction, the sum of element currents that flows out is equal to the sum of element currents that flows in.

Kirchhoff's voltage law

Kirchhoff's voltage law states that the algebraic sum of element voltages around a closed loop is zero.

Consider a closed loop in a circuit wherein four elements with voltages V_1, V_2, V_3 and V_4, are present as shown in Fig. 3.

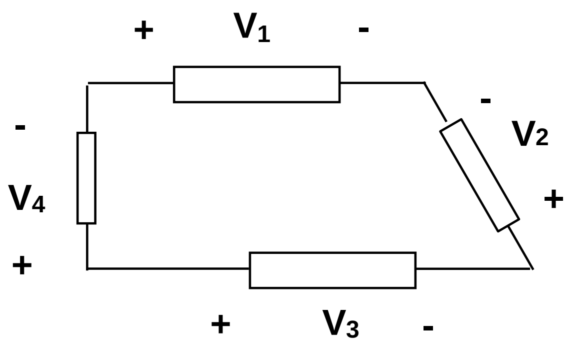


Fig. 3 Voltages in a closed loop

Assigning positive sign for voltage drop and negative sign for voltage rise, when the loop is traced in clockwise direction, according to KVL

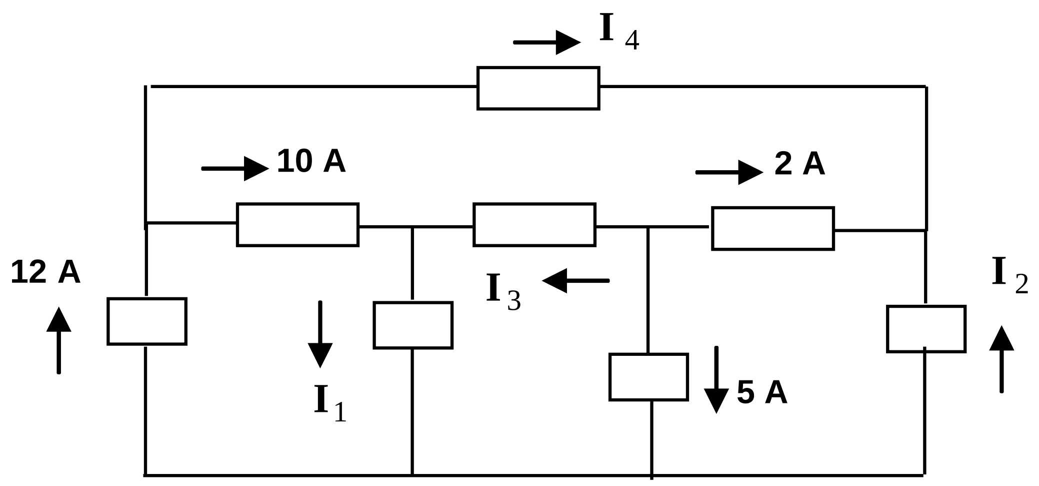
$$V_1 - V_2 - V_3 + V_4 = 0 \quad (12)$$

The above equation can be rearranged as

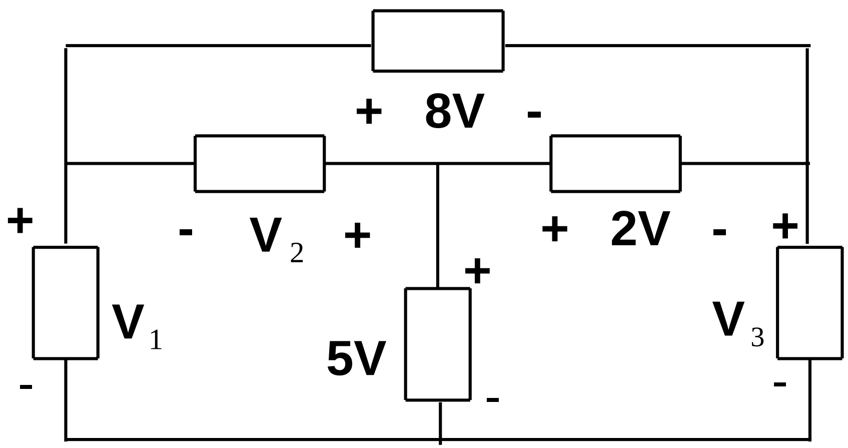
$$V_1 + V_4 = V_2 + V_3 \quad (13)$$

From equation (13), KVL can also stated as, in a closed loop, the sum of voltage drops is equal to the sum of voltage rises in that loop.

Find the currents I_1, I_2, I_3 and I_4 in the circuit shown.



Find the voltages V_1, V_2 and V_3 in the circuit shown.



Resistors connected in series

Two resistors are said to be connected in series when there is only one common point between them and no other element is connected in that common point. Resistors connected in series carry same current. Consider three resisters R_1, R_2 and R_3 connected in series as shown in Fig. 4. With the supply voltage of E, voltages across the three resistors are V_1, V_2 and V_3.

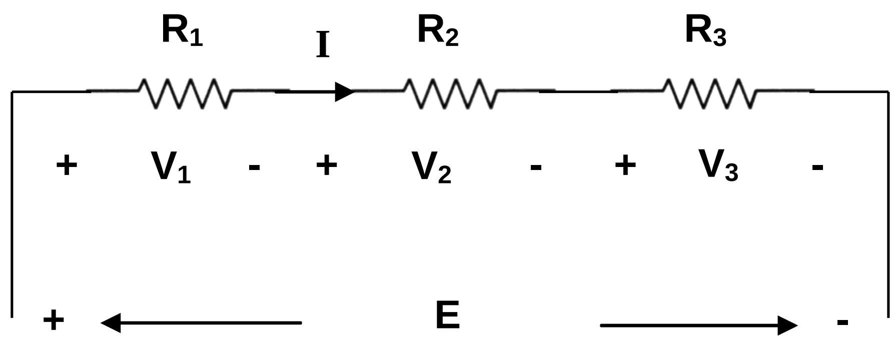


Fig. 4 Resistors connected in series

As per Ohm’s law

$$\left.\begin{aligned} V_1 &= R_1\, I \\ V_2 &= R_2\, I \\ V_3 &= R_3\, I \end{aligned}\right\} \tag{14}$$

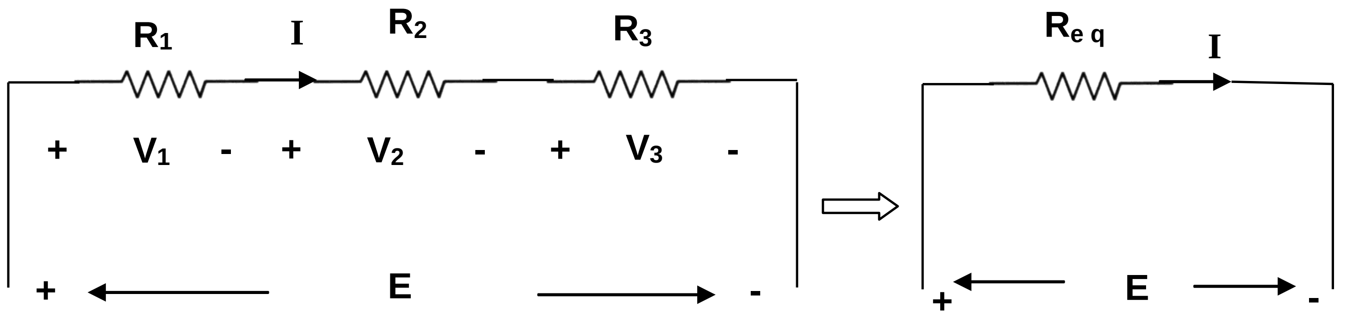


Fig. 4 Resistors connected in series

Applying KVL,

$$E = V_1 + V_2 + V_3 \quad (15)$$

$$= (R_1 + R_2 + R_3)\, I = R_{e\,q}\, I \quad (16)$$

Thus for the circuit shown in Fig. 4,

$$E = R_{e\,q}\, I \quad (17)$$

where E is the circuit voltage, I is the circuit current and $R_{e\,q}$ is the equivalent resistance. Here

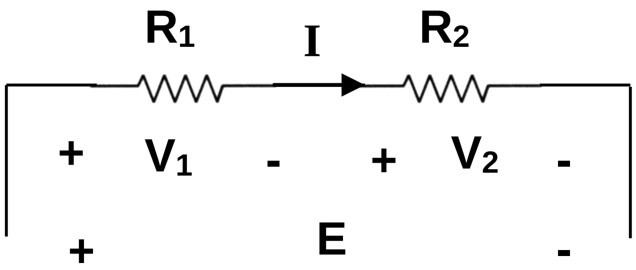
$$R_{e\,q} = R_1 + R_2 + R_3 \quad (18)$$

This is true when two are more resistors are connected in series. When n numbers of resistors are connected in series, the equivalent resistor is given by

$$R_{e\,q} = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n \quad (19)$$

Voltage division rule

Consider two resistors connected in series. Then



$V_1 = R_1 I$

$V_2 = R_2 I$

$E = (R_1 + R_2) I$ **and hence** $I = E / (R_1 + R_2)$

Total voltage of E is dropped in two resistors. Voltage across the resistors are given by

$$V_1 = \frac{R_1}{R_1 + R_2} E \quad \text{and} \tag{20}$$

$$V_2 = \frac{R_2}{R_1 + R_2} E \tag{21}$$

Resistors connected in parallel

Two resistors are said to be connected in parallel when both are connected across same pair of nodes. Voltages across resistors connected in parallel will be equal.

Consider two resistors R_1 and R_2 connected in parallel as shown in Fig. 5.

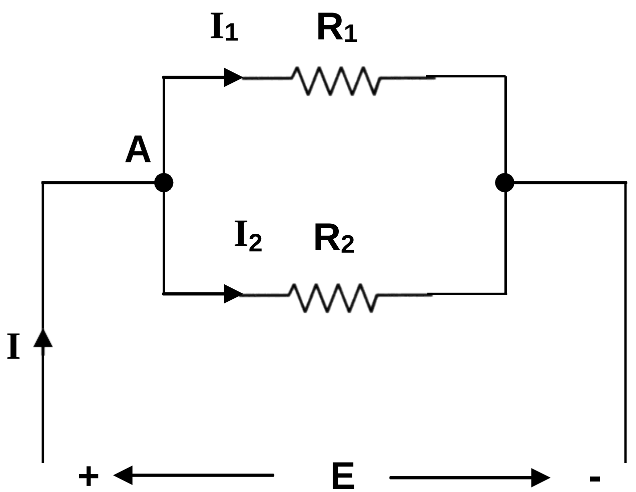
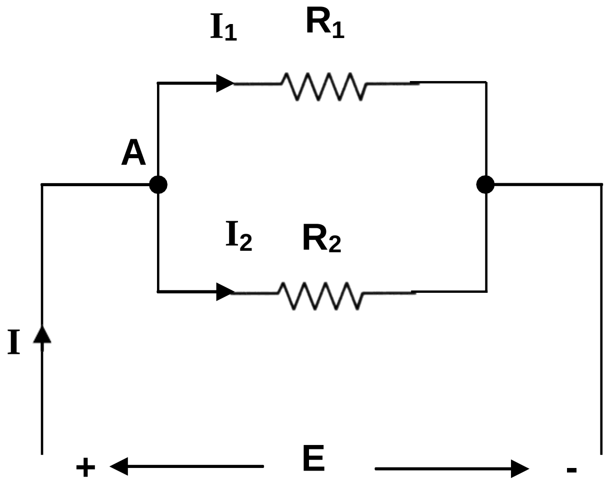


Fig. 5 Resistors connected in parallel

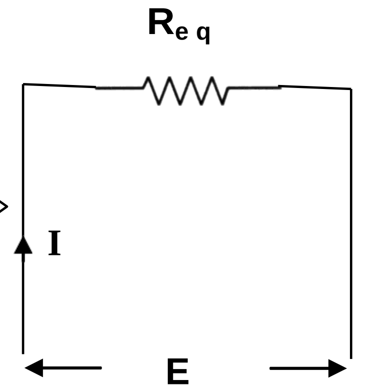
As per Ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \qquad (22)$$



$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$



Equivalent circuit

Applying KCL at node A

$$I = I_1 + I_2 = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{23}$$

From the equivalent circuit shown

$$I = \frac{E}{R_{eq}} \tag{24}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Comparing eq. (23) and (24)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

From the above $\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$

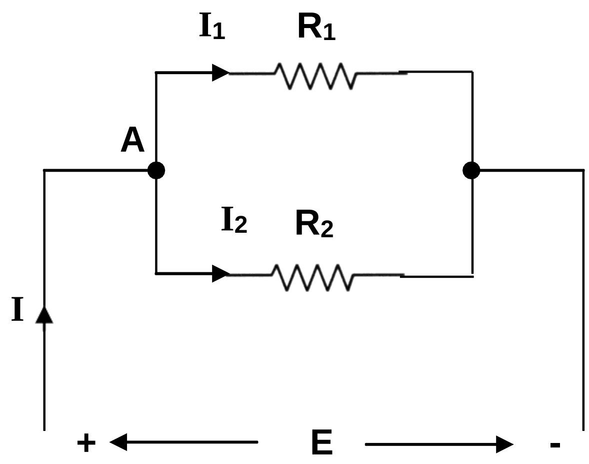
Thus $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ **(26)**

When n numbers of resistors are connected in parallel, generalizing eq. (25),

R_{eq} can be obtained from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \text{..........} \text{......} + \frac{1}{R_n} \tag{27}$$

Current division rule



$$I_1 = \frac{E}{R_1}, \quad I_2 = \frac{E}{R_2} \qquad (22)$$

Fig. 5 Resistors connected in parallel

Referring to Fig. 5, it is noticed the total current gets divided as I_1 and I_2. The branch currents are obtained as follows.

From eq. (23)

$$E = \frac{R_1 R_2}{R_1 + R_2} I \qquad (29)$$

Substituting the above in eq. (22)

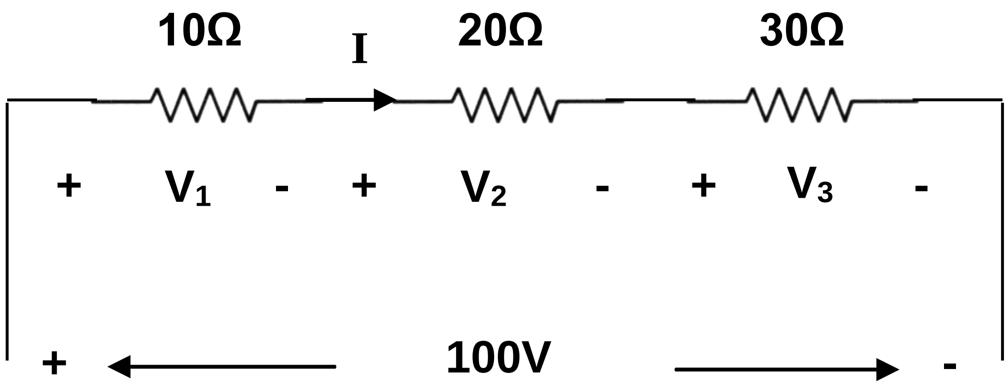
$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I \qquad (30)$$

Example 1

Three resistors 10Ω, 20Ω and 30Ω are connected in series across 100 V supply. Find the voltage across each resistor.

Solution



Current I = 100 / (10 + 20 + 30) = 1.6667 A

Voltage across 10Ω = 10 x 1.6667 = 16.67 V

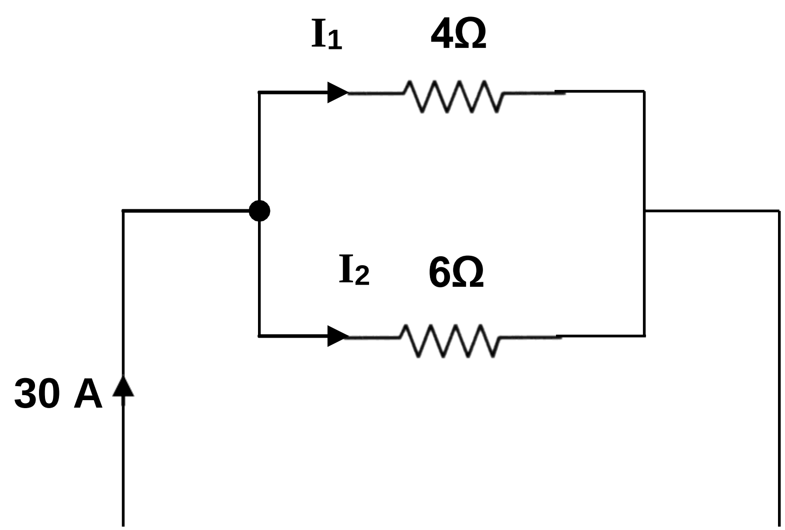
Voltage across 20Ω = 20 x 1.6667 = 33.33 V

Voltage across 30Ω = 30 x 1.6667 = 50 V

Example 2

Two resistors of 4Ω and 6Ω are connected in parallel. If the supply current is 30 A, find the current in each resistor.

Solution



Using the current division rule

Current through 4Ω $= \dfrac{6}{4+6} \times 30 = 18\,\text{A}$

Current through 6Ω $= \dfrac{4}{4+6} \times 30 = 12\,\text{A}$

Example 3

Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that the total power of 100 W is absorbed?

Solution

Let R_T be the total equivalent resistor. Then

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$$

Resistance $R_T = \frac{120}{154} = 0.7792\Omega$

Let E be the supply voltage. Then total current taken = E / 0.7792 A

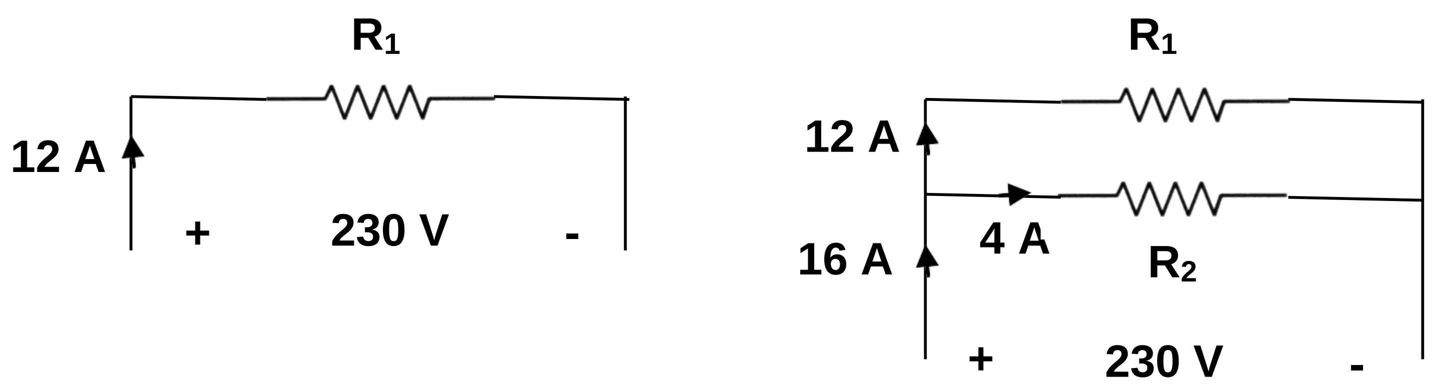
Thus $\left(\frac{E}{0.7792}\right)^2 \times 0.7792 = 100$ **and hence** $E^2 = 100 \times 0.7792 = 77.92$

Required voltage $= \sqrt{77.92} = 8.8272$ V

Example 4

When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel, to increase the load to 16 A?

Solution



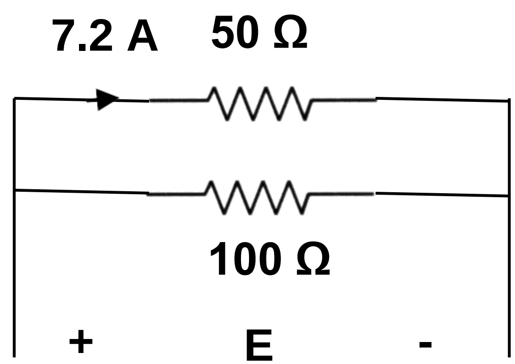
To make the load current 16 A, current through the second resistor = 16 –12 = 4 A

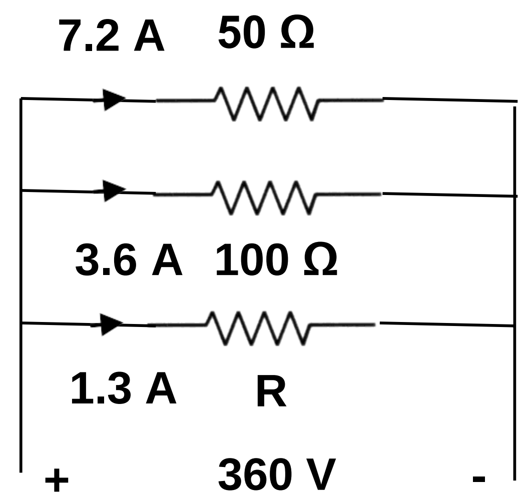
Value of second resistor R_2 = 230/4 = 57.5 Ω

<u>Example 5</u>

A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistor to be added in parallel to make the line current as 12.1A?

<u>Solution</u>





Supply voltage E = 50 x 7.2 = 360 V

Current through 100 Ω = 360/100 = 3.6 A

When the line current is 12.1 A, current through third resistor = 12.1 – (7.2 + 3.6)

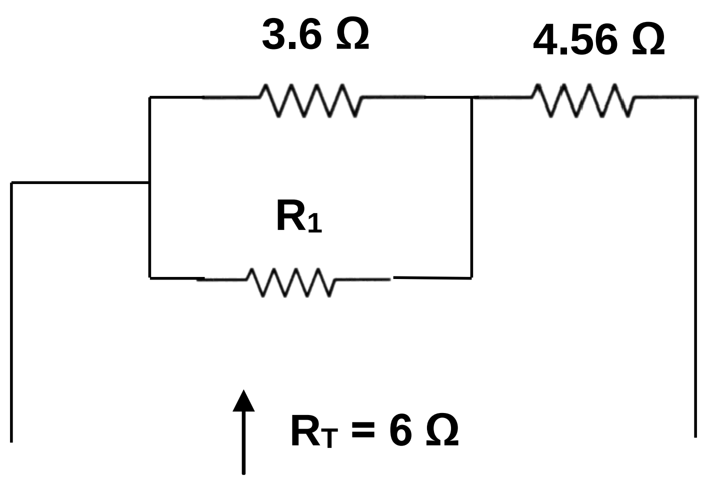
= 1.3 A

Value of third resistor = 360/1.3 = 276.9230 Ω

Example 6

A resistor of 3.6 ohms is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms, so that the total resistance of the circuit shall be 6 ohms?

Solution



$3.6 \parallel R_1 = 6 - 4.56 = 1.44\ \Omega$

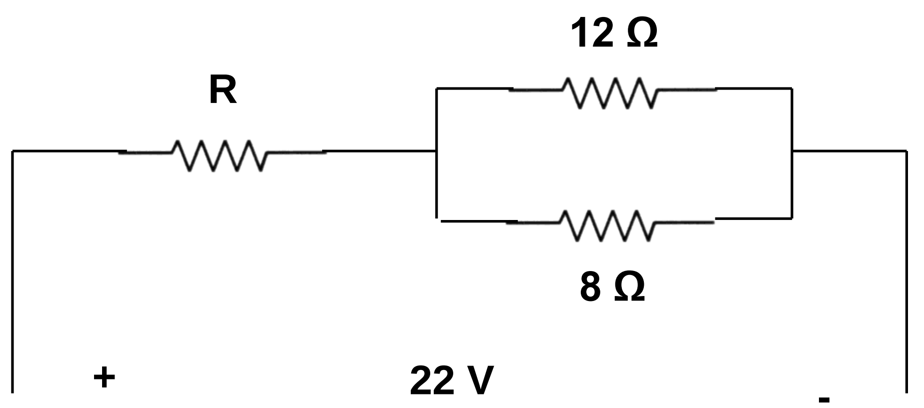
Thus $\dfrac{3.6 \times R_1}{3.6 + R_1} = 1.44$; Therefore $\dfrac{3.6 + R_1}{R_1} = \dfrac{3.6}{1.44} = 2.5$; $\dfrac{3.6}{R_1} = 1.5$

Required resistance $R_1 = 3.6/1.5 = 2.4\ \Omega$

Example 7

A resistance R is connected in series with a parallel circuit comprising two resistors 12 Ω and 8 Ω respectively. Total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Calculate the value of the resistor R.

Solution



Power dissipated = 70 W

Total current taken = 70 / 22 = 3.1818 A

Equivalent of 12 Ω ║ 8 Ω = 96/20 = 4.8 Ω

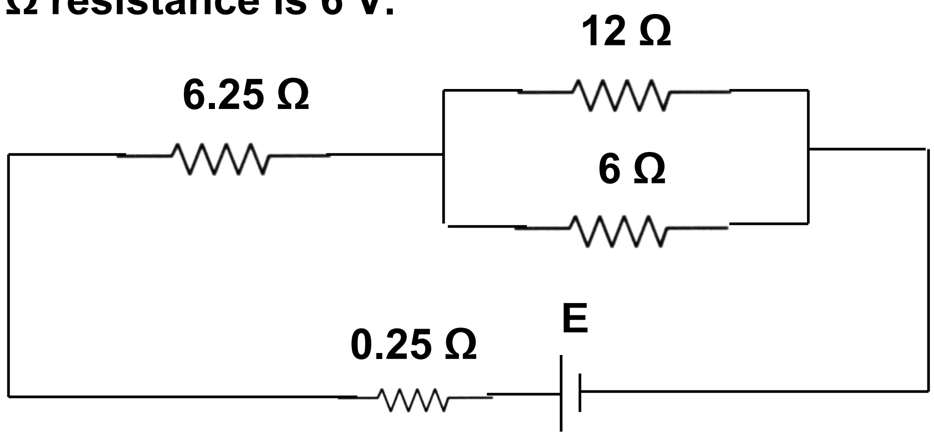
Voltage across parallel combination = 4.8 x 3.1818 = 15.2726 V

Voltage across resistor R = 22 – 15.2726 = 6.7274 V

Value of resistor R = 6.7274/3.1818 = 2.1143 Ω

Example 8

The resistors 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance and a battery which has an internal resistance of 0.25 Ω. Determine the emf of the battery if the potential difference across 6 Ω resistance is 6 V.



Voltage across 6 Ω = 6 V

Solution

Current in 6 Ω = 6/6 = 1 A

Current in 12 Ω = 6/12 = 0.5 A

Therefore current in 0.25 Ω = 1.0 + 0.5 = 1.5 A

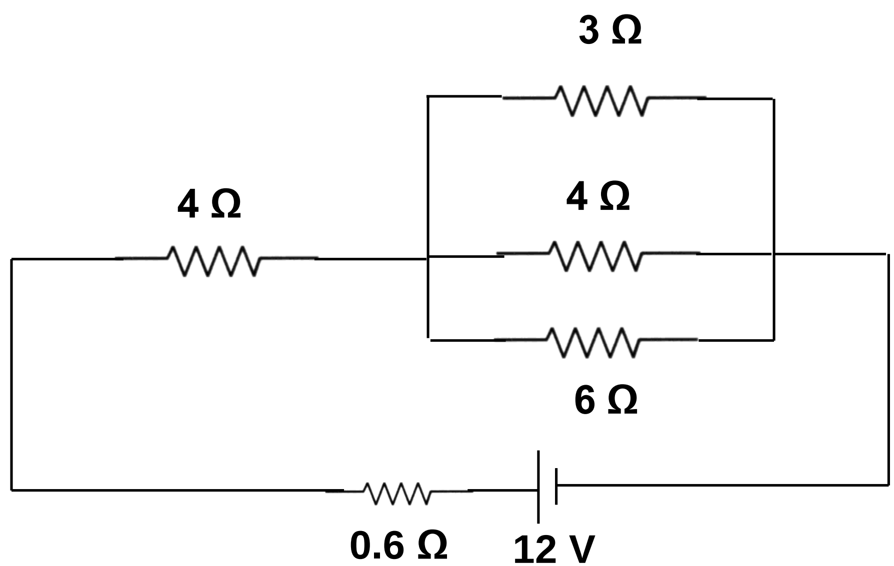
Using KVL E = (0.25 x 1.5) + (6.25 x 1.5) + 6 = 15.75 V

Therefore battery emf E = 15.75 V

Example 9

A circuit consist of three resistors 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of 12 V and an internal resistance of 0.6 Ω is connected across the circuit. Find the total current in the circuit and the terminal voltage across the battery.

Solution



4 Ω ║ 6 Ω = 24/10 = 2.4 Ω
1.4 Ω ║ 3 Ω = 7.2/5.4 = 1.3333 Ω

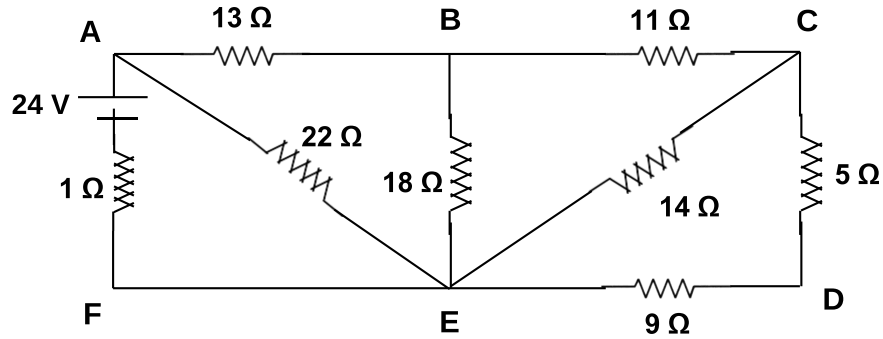
Total circuit resistance = 4 + 0.6 + 1.3333 = 5.9333 Ω

Circuit current = 12/5.9333 = 2.0225 A

Terminal voltage across the battery = 12 – (0.6 x 2.0225) = 10.7865 V

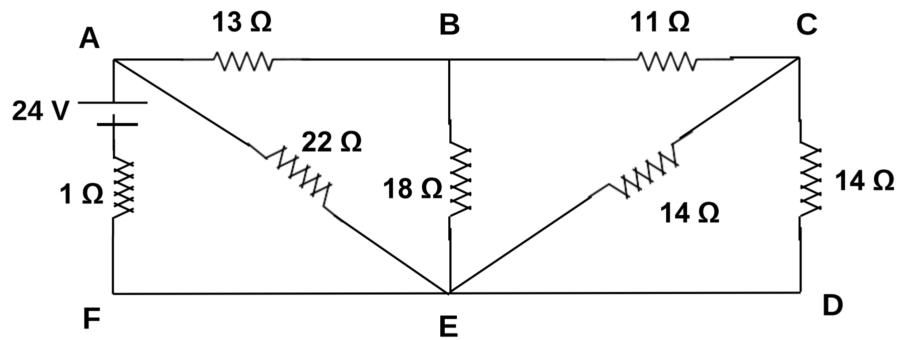
Example 10

An electrical network is arranged as shown. Find (i) the current in branch AF (ii) the power absorbed in branch BE and (iii) potential difference across the branch CD.

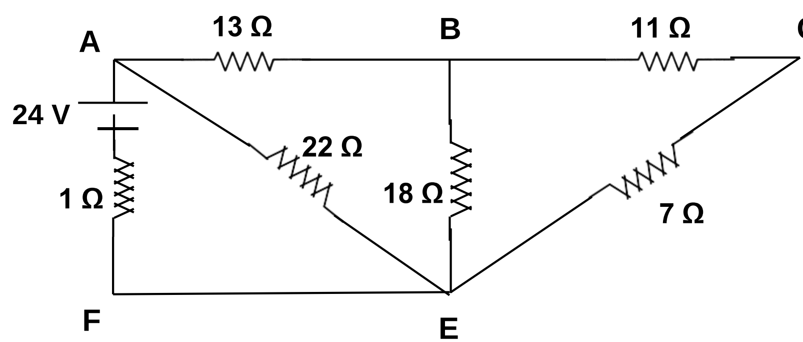


Solution

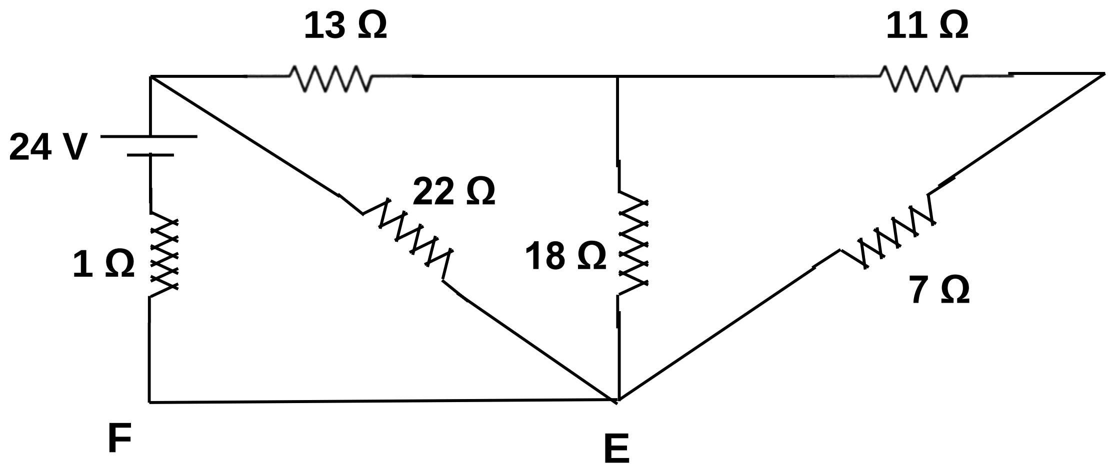
Various stages of reduction are shown.



1



2



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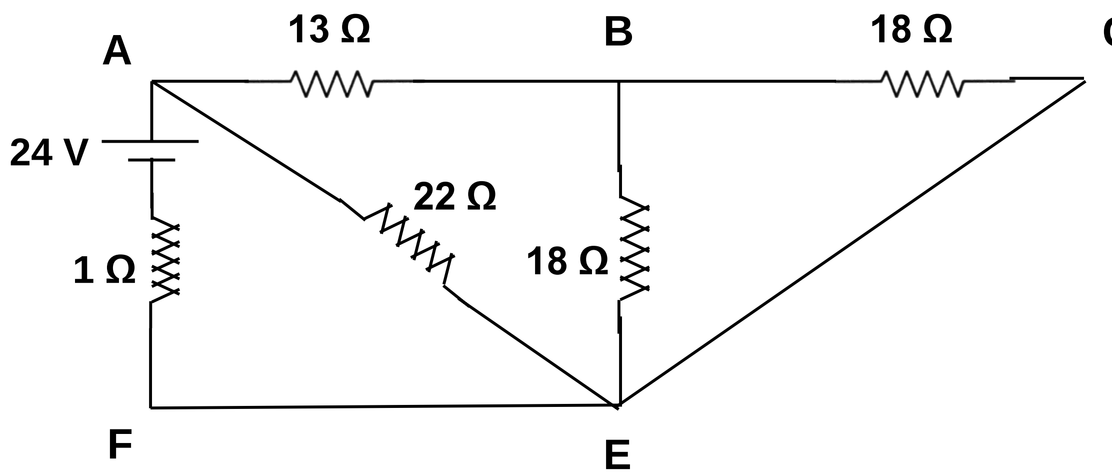
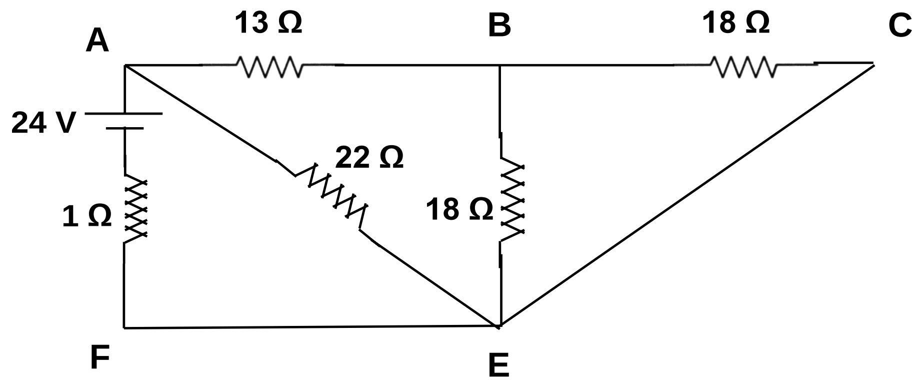
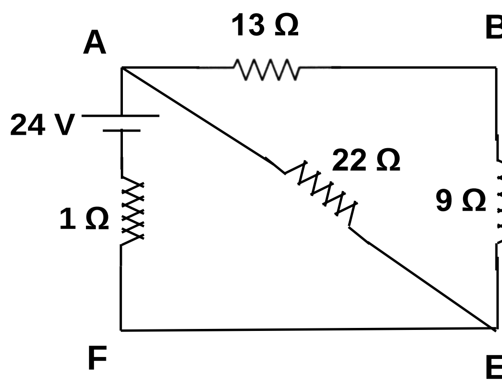


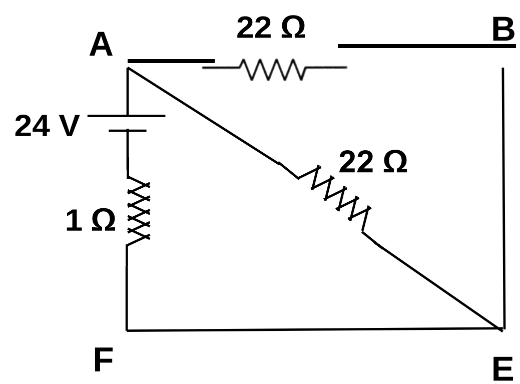
3


A
13 Ω
B
18 Ω
C
24 V
22 Ω
18 Ω
1 Ω
F
E

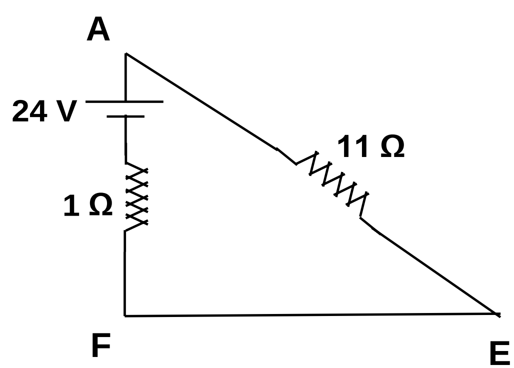
3


A
13 Ω
B
24 V
22 Ω
9 Ω
1 Ω
F
E

4


A
22 Ω
B
24 V
22 Ω
1 Ω
F
E

5


A
24 V
11 Ω
1 Ω
F
E

6

Current in branch AF = 24/12 = 2 A from F to A

Using current division rule current in 13 Ω in Fig. 4= 1 A

Referring Fig. 3, current in branch BE = 0.5 A

Power absorbed in branch BE = 0.5^2 x 18 = 4.5 W

Voltage across BE = 0.5 x 18 = 9 V

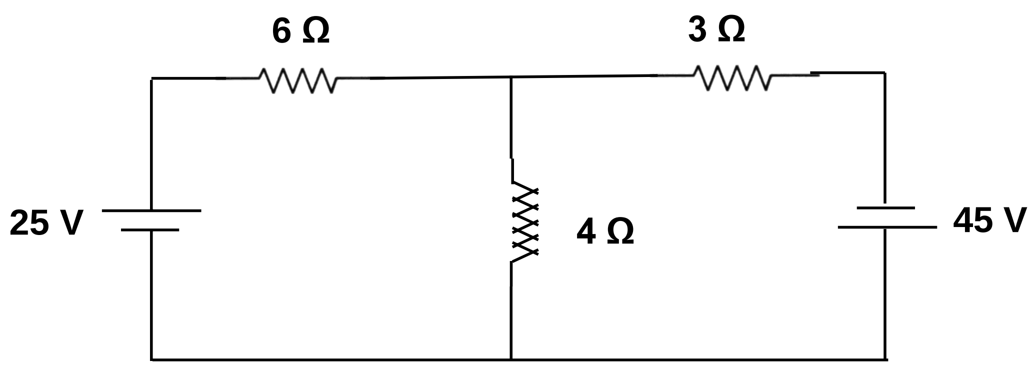
Voltage across CE in Fig. 1 $= \frac{7}{18} \times 9 = 3.5$ **V**

Referring Fig. given in the problem, using voltage division rule, voltage

across in branch CD $= \frac{5}{14} \times 3.5 = 1.25$ **V**

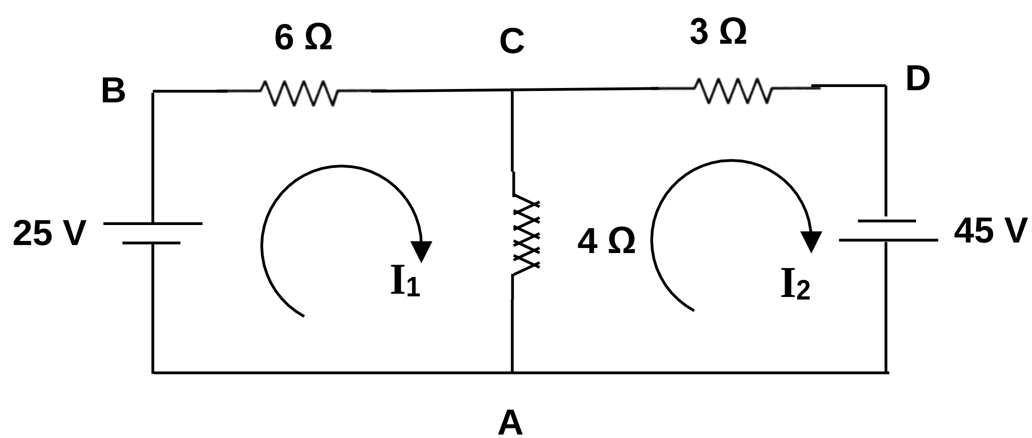
Example 11

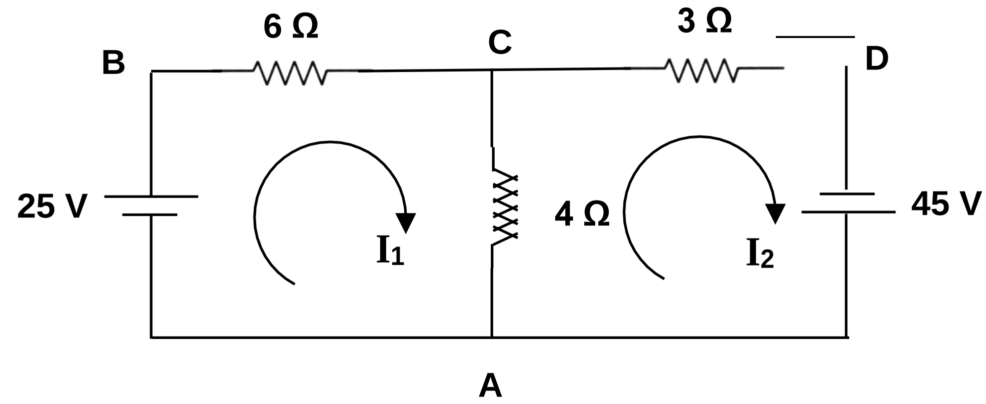
Using Kirchhoff's laws, find the current in various resistors in the circuit shown.



Solution

Let the loop current be I_1 and I_2. We can find element currents in terms of loop currents.





Considering the loop ABCA, KVL yields

$6\ I_1 + 4\ (I_1 – I_2) – 25 = 0$

For the loop CDAC, KVL yields

$3\ I_2 – 45 + 4\ (I_2 – I_1) = 0$

Thus $10\ I_1 - 4\ I_2 = 25$

$- 4\ I_1 + 7\ I_2 = 45$

On solving the above $I_1 = 6.574$ A; $I_2 = 10.1852$ A

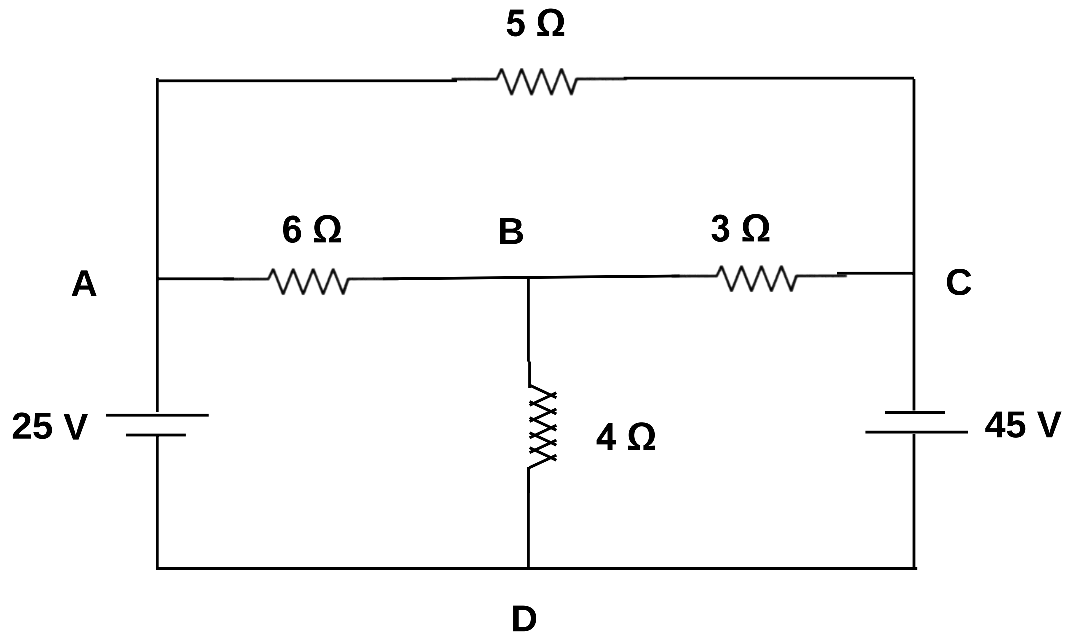
Current in 4Ω resistor = $I_1 – I_2 = 6.574 – 10.1852 = - 3.6112$ A

Thus the current in 4Ω resistor is 3.6112 A from A to C

Current in 6 Ω resistor = 6.574 A; Current in 3 Ω resistor = 10.1852 A

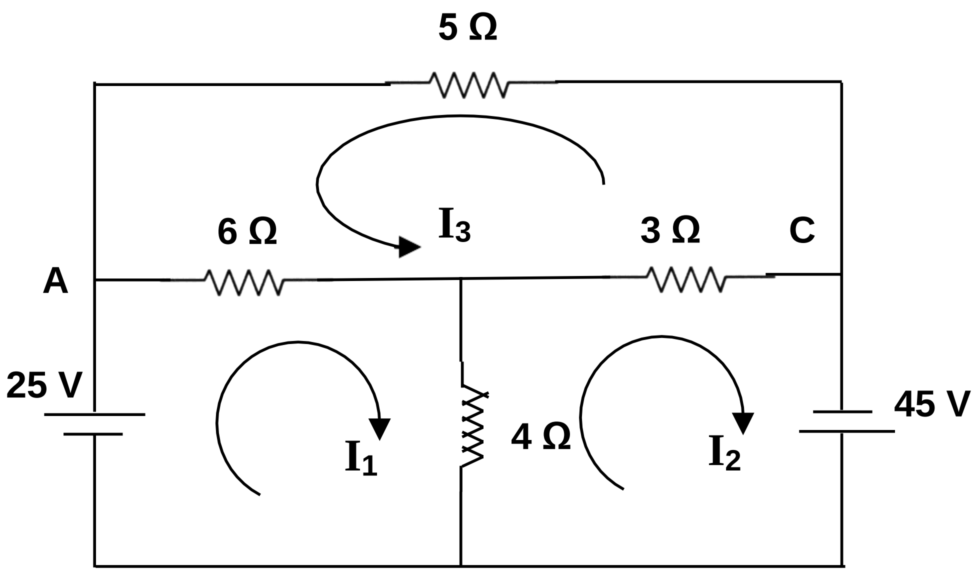
Example 12

Find the current in 5 Ω resistor in the circuit shown.



Solution

Let the loop current be I_1, I_2 and I_3.



Three loops equations are:

$$-25 + 6(I_1 + I_3) + 4(I_1 - I_2) = 0$$

$$-45 + 4(I_2 - I_1) + 3(I_2 + I_3) = 0$$

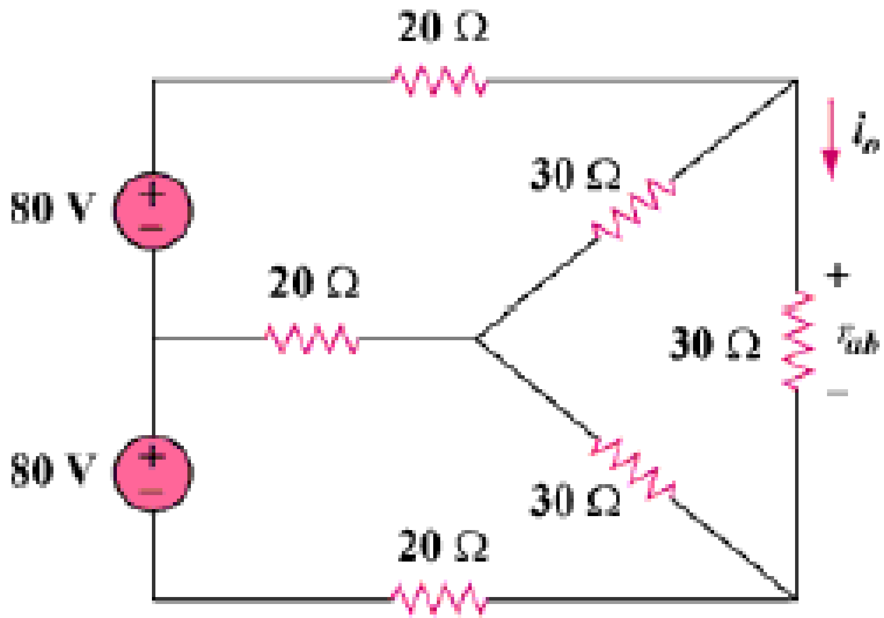
$$5 I_3 + 6(I_3 + I_1) + 3(I_2 + I_3) = 0$$

$$\begin{bmatrix} 10 & -4 & 6 \\ -4 & 7 & 3 \\ 6 & 3 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 45 \\ 0 \end{bmatrix}$$: On solving

$I_3 = -14$ A   Current in 5 Ω resistor = 14 A from A to C

Example 13

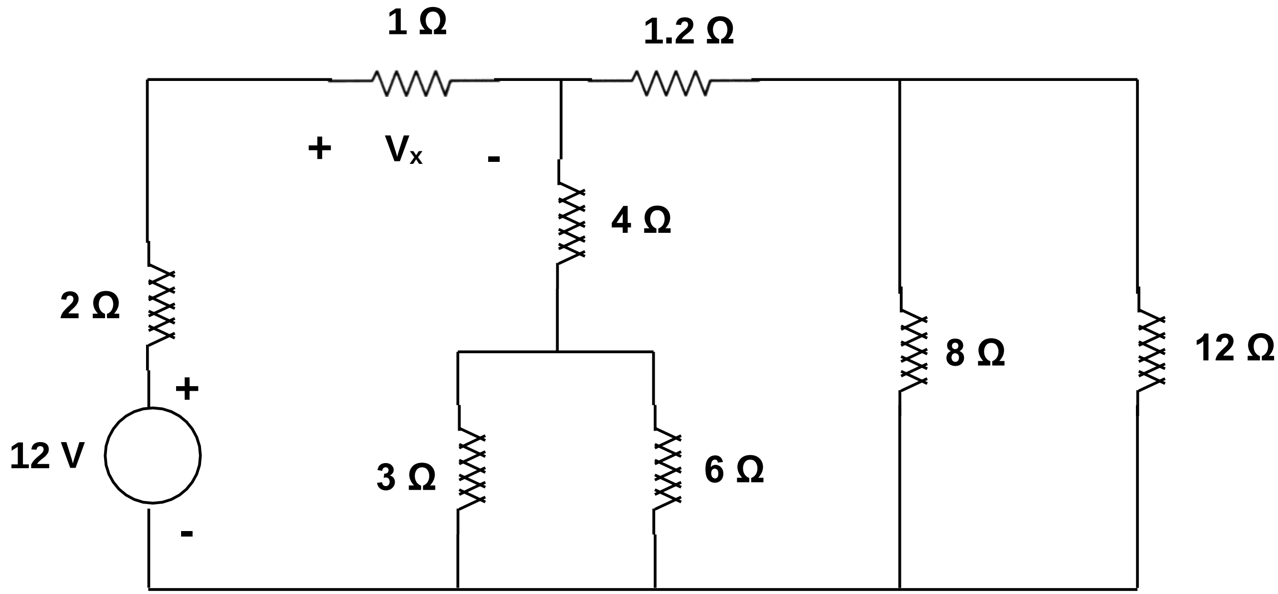
Using mesh analysis find the current i_0 and the voltage v_{ab} in the circuit shown.



Answers: **1.7777 A** **53.331 V**

Example 14

In the circuit shown, determine V_x and the power absorbed by 12 Ω resistor.

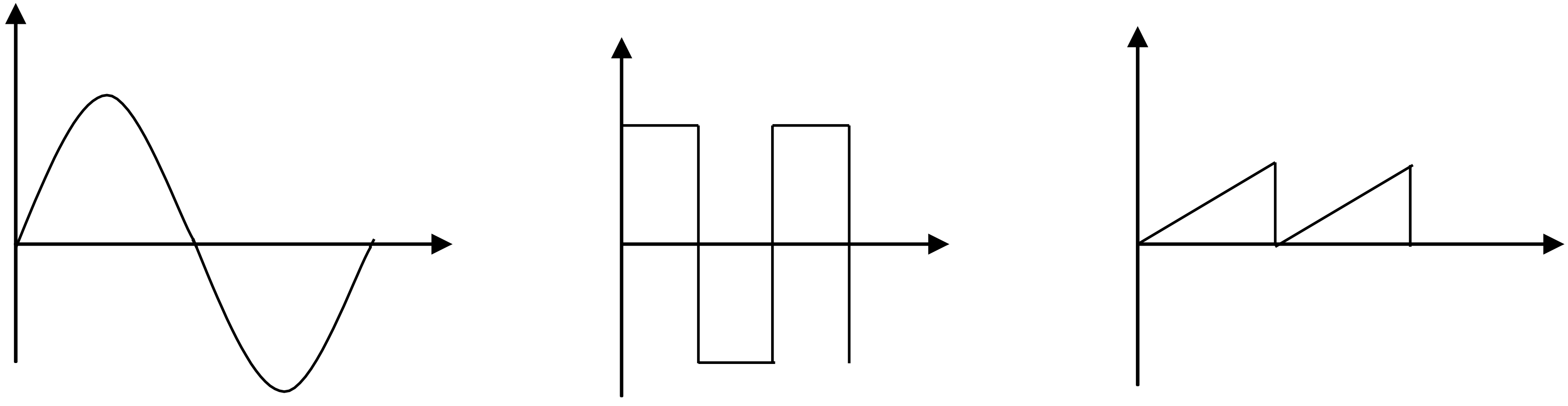


Answers: **2 V** **1.92 W**

FUNDAMENTALS OF AC

Electrical appliances such as lights, fans, air conditioners, TV, refrigerators, mixy, washing machines and industrial motors are more efficient when they operate with AC supply. The required AC voltage is generated by AC generator also called as alternator.

A __waveform__ is a graph in which the instantaneous values of any quantity are plotted against time. A periodic waveform is the one which repeats itself at regular intervals. A waveform may be sinusoidal or non sinusoidal. Examples of a few periodic waveforms are shown in Fig.1.



(a) Sinusoidal waveform **(b) Rectangular waveform** **(c) Sawtooth waveform**

Fig. 1

Alternating waveform is a waveform which reverses its direction at regular intervals. Sinusoidal and rectangular waveforms shown above are alternating waveforms. Let us see more details about sinusoidal waveform.

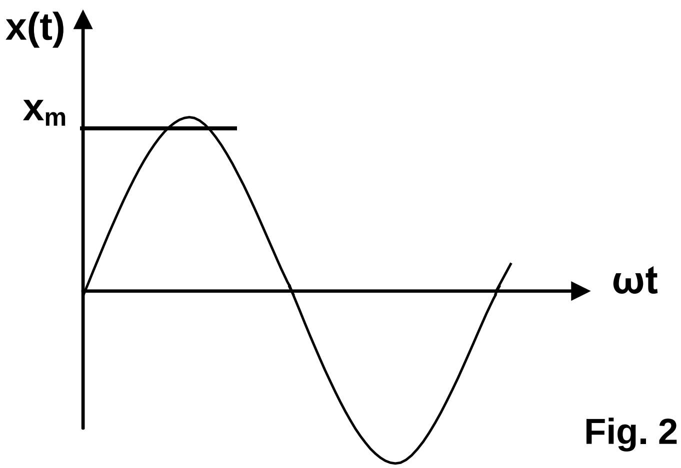


Fig. 2

Fig. 2 shows a sinusoidal waveform, which can be called as a sinusoid. It can represent a voltage or current. Its equation can be written as

$$x(t) = x_m \sin(\omega t + \varphi) \quad (1)$$

Thus a sinusoid is described in terms of

i) its maximum value

ii) its angular frequency, ω and

iii) its phase angle φ

It is evident that sinusoid repeats in a cyclic manner. The number of cycles it makes in one second is called the frequency (f). Thus the unit for frequency is cycles per second which is also commonly known as hertz (Hz). Electric supply has a frequency of 50 or 60 Hz. In communication circuit, the frequency will be in the order of Mega Hz.

The time taken by the sinusoid to complete one cycle is called the period (T) of the sinusoid. When the supply frequency is 50 Hz, the sinusoid makes 50 cycles in one second. Thus the period is 1/50 = 0.02 second. The frequency and the period are related as

$$T = \frac{1}{f} \qquad \text{or } f = \frac{1}{T} \tag{2}$$

The angular frequency of sinusoid is represented by ω and its unit is radians per second. In one cycle the angle covered is 2π radians. When the frequency is f cycles per second, the angle covered in one second will be 2πf radians. Thus

$$\omega = 2\,\pi\,f \tag{3}$$

While drawing a sinusoid, instead of ωt, time t can be taken in the x-axis.

Example 1

Consider the voltage sinusoid

$v(t) = 70 \sin(60t + 20^0)$ V

Find the amplitude, phase, angular frequency, frequency, period and the value of voltage at time t = 0.25 s.

Solution

Amplitude $v_m = 70$ V

Phase $\varphi = 20^0$

Angular frequency $\omega = 60$ rad / s

Frequency $f = \dfrac{\omega}{2\pi} = \dfrac{60}{2\pi} = 9.5511$ Hz

Period $T = \dfrac{1}{f} = \dfrac{1}{9.5511} = 0.1047$ s

Voltage value at t = 0.25 s is

$$v(0.25) = 70 \sin\left(60 \times 0.25 \times \frac{180}{\pi} + 20^0\right) = 24.59 \text{ V}$$

The two sinusoids shown in Fig. 3 are $x(t) = x_m \sin \omega t$ and $x(t) = x_m \sin(\omega t + \varphi)$

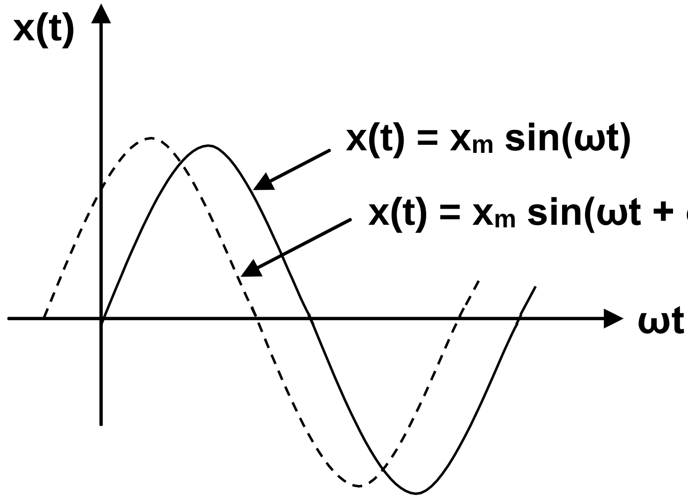


Fig. 3

The sinusoid $x(t) = x_m \sin(\omega t + \varphi)$ leads the sinusoid $x(t) = x_m \sin \omega t$ by an angle of φ. The sinusoids can also be written as

$$x(t) = x_m \sin(\theta + \varphi) \qquad (4)$$

The average value of the periodic waveform can be obtained as:

$$\text{Average value} = \frac{\text{Area under one complete cycle}}{\text{Period}} \qquad (5)$$

<u>Average value is also called as mean value.</u>

The Root Mean Square (RMS) value of periodic waveform is:

$$\text{RMS value} = \sqrt{\frac{\text{Area under squaredcurve for one cycle}}{\text{Period}}} \qquad (6)$$

Form Factor is defined as

$$\text{Form Factor} = \frac{\text{RMS Value}}{\text{Average value}} \qquad (7)$$

Peak Factor is defined as

$$\text{Peak Factor} = \frac{\text{Peak Value}}{\text{RMS value}} \qquad (8)$$

Consider a current waveform described by

$$i(t) = I_m \sin\theta \tag{9}$$

Its positive half cycle and negative half cycle of such sinusoids are negative of each other. Hence the area in one cycle is zero. For such sinusoidal wave form the average value is the average value over half cycle.

Thus

$$\text{Area of the curve} = \int_0^{\pi} I_m \sin\theta \, d\theta = I_m(-\cos\theta)\Big|_0^{\pi} = I_m(1+1) = 2\, I_m$$

$$I_{av} = \frac{2\, I_m}{\pi} = 0.6366\, I_m \tag{10}$$

When we square the waveform $i(t) = I_m \sin\theta$, the first and the second half of the cycle will be same. Therefore while computing the R M S value of $i(t) = I_m \sin\theta$ it is enough to consider only one half cycle.

$$\text{Area of square curve} = \int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2}\int_0^{\pi} (1 - \cos 2\theta)\, d\theta = \frac{I_m^2}{2}\left(\theta - \frac{\sin 2\vartheta}{2}\right)\Big|_0^{\pi} = \frac{I_m^2}{2}[\pi - 0] = \frac{\pi}{2} I_m^2$$

$$\text{Mean square value} = \frac{I_m^2}{2}$$

$$\text{RMS value} = \frac{I_m}{\sqrt{2}} \qquad (11)$$

$$= 0.7071\ I_m \qquad (12)$$

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Average value}} = \frac{0.7071\ I_m}{0.6366\ I_m} = 1.11 \qquad (13)$$

$$\text{Peak factor} = \frac{\text{Peak Value}}{\text{RMS value}} = \frac{I_m}{0.7071 I_m} = 1.414 \qquad (14)$$

We may be calculating average and RMS values of waveforms in which inclined straight line variations are present. Consider the waveform shown in Fig. 4. Its square curve is shown in Fig. 5. Area A_1 of the square curve can be calculated as follows.

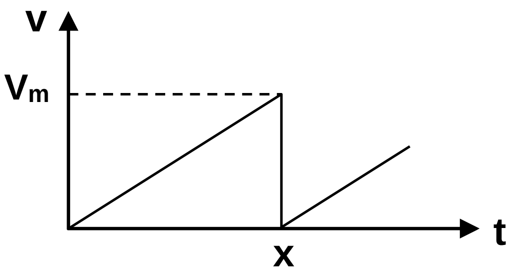


Fig. 4

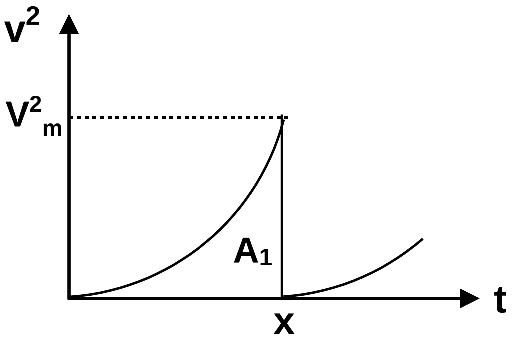


Fig. 5

Equation of the straight line is: $v = \frac{V_m}{x} t$; Then $v^2 = \frac{V_m^2}{x^2} t^2$

$$\text{Area } A_1 = \int_0^x \frac{V_m^2}{x^2} t^2 \, dt = \frac{V_m^2}{x^2} \frac{t^3}{3} \Bigg|_0^x = \frac{1}{3} V_m^2 x$$

It can be verified that the above result is true for the waveform shown in Fig. 6 also.

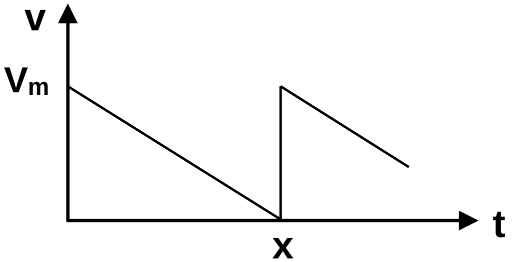


Fig. 6

Example 2

Find the average and RMS values of the waveform shown in Fig. 7

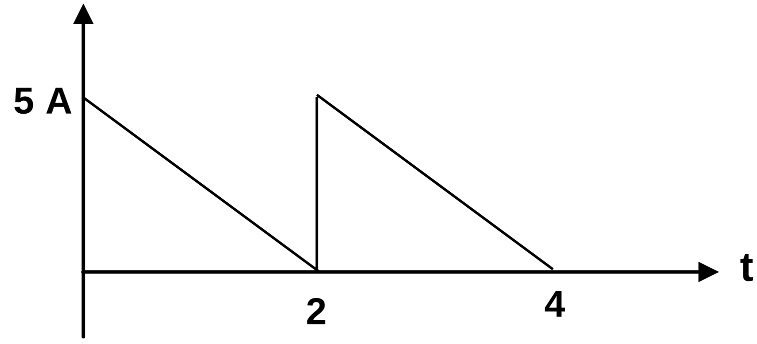


Fig. 7

Solution

$I_{av} = \frac{1}{2}$ x area of the triangle $= \frac{1}{2} \times \frac{1}{2} \times 5 \times 2 = 2.5\,A$

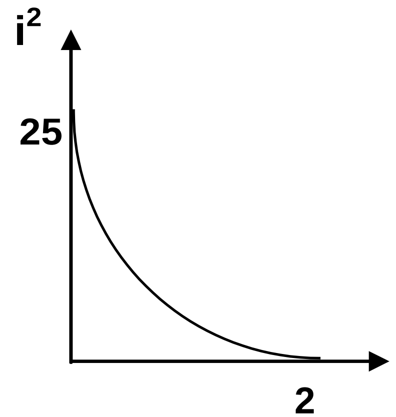


Fig. 8

The square curve is shown in Fig. 8.

Area of square curve $= \frac{1}{3} \times 25 \times 2 = 16.6663$

Mean square value $= \frac{16.6663}{2} = 8.3332$

RMS value $= \sqrt{8.3332} = 2.8867\,A$

Example 3

Find the average and RMS value of the waveform shown in Fig. 9.

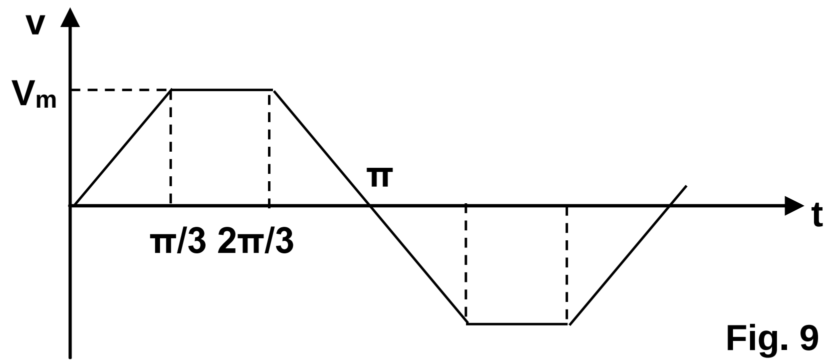


Fig. 9

Solution

Area of positive half cycle $= \frac{1}{2}\frac{\pi}{3}V_m + \frac{\pi}{3}V_m + \frac{1}{2}\frac{\pi}{3}V_m = \frac{2\pi}{3}V_m$

Average value $= \frac{2}{3}V_m = 0.6667\ V_m$

The square curve is shown in Fig. 10.

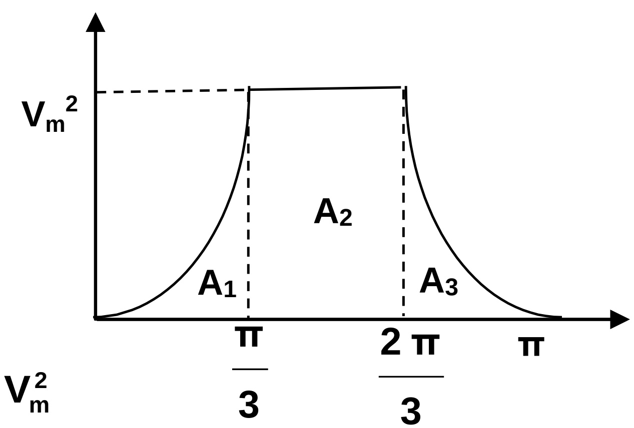


Fig. 10

Area of square curve

$$= \frac{\pi}{9}V_m^2 + \frac{\pi}{3}V_m^2 + \frac{\pi}{9}V_m^2 = \frac{5}{9}\pi\ V_m^2$$

Mean Square value $= \frac{5}{9}V_m^2$; Thus RMS value = 0.7454 V_m

Example 4

Find the average and RMS values of the half wave rectified sine wave shown in Fig. 11.

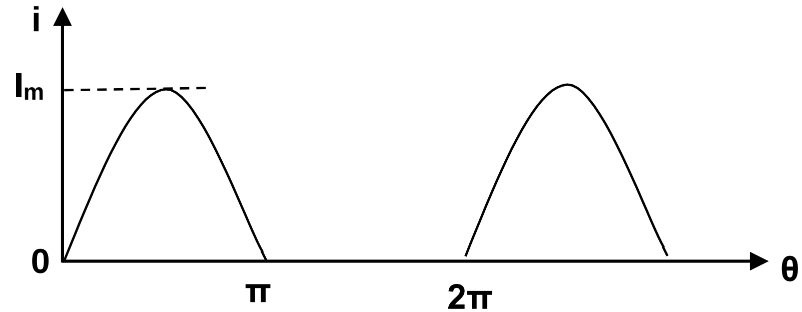


Fig. 11

Solution

As seen earlier, area of half sine wave = $2\ I_m$

Total area = $2\ I_m + 0 = 2\ I_m$

Average value $I_{av} = \dfrac{2\ I_m}{2\ \pi} = 0.3183\ I_m$

As seen earlier, area of square of half sine wave = $\dfrac{\pi}{2} I_m^2$

Total area of square curve = $\dfrac{\pi}{2} I_m^2$

Mean of square curve = $\dfrac{1}{2\pi}\ \dfrac{\pi}{2} I_m^2 = \dfrac{1}{4} I_m^2 = 0.25\ I_m^2$

RMS value $I_{RMS} = 0.5\ I_m$

Example 5

Find the average and RMS values of the full wave rectified sine wave shown in Fig. 12.

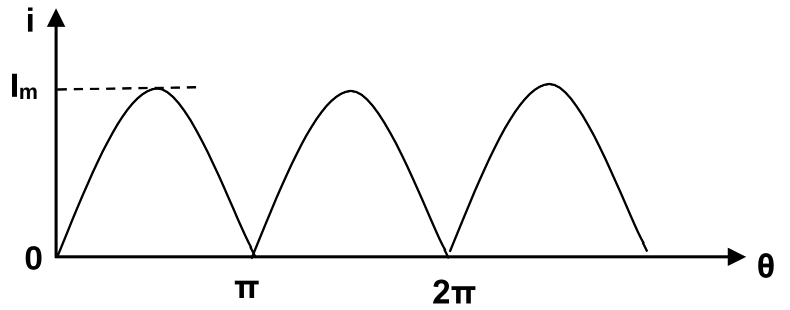


Fig. 12

Solution

As seen earlier, area of half sine wave = $2\ I_m$

Total area = $2\ I_m + 2\ I_m = 4\ I_m$

Average value $I_{av} = \dfrac{4\ I_m}{2\ \pi} = \dfrac{2}{\pi} I_m = 0.6366\ I_m$

As seen earlier, area of square of half sine wave = $\dfrac{\pi}{2} I_m^2$

Total area of square curve = $\pi\ I_m^2$

Mean of square curve = $\dfrac{1}{2\pi}\ \pi\ I_m^2 = \dfrac{1}{2} I_m^2$

RMS value $I_{RMS} = \dfrac{I_m}{\sqrt{2}} = 0.7071\ I_m$

If the waveform is the sum of several waveforms, its AVERAGE value is the sum of average values of its components and its RMS values can be obtained as follows. Let

$W = W_1 + W_2 + W_3$ and their RMS values be $W_{1\ RMS}$, $W_{2\ RMS}$ and $W_{3\ RMS}$ respectively. Then

$$W_{RMS} = \sqrt{W_{1RMS}^2 + W_{2RMS}^2 + W_{3RMS}^2}$$

<u>Example 6</u>

A conductor carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 10 A. Find the RMS value of the conductor current.

<u>Solution</u>

Conductor current $i(t) = (10 + 10 \sin \omega t)$ A

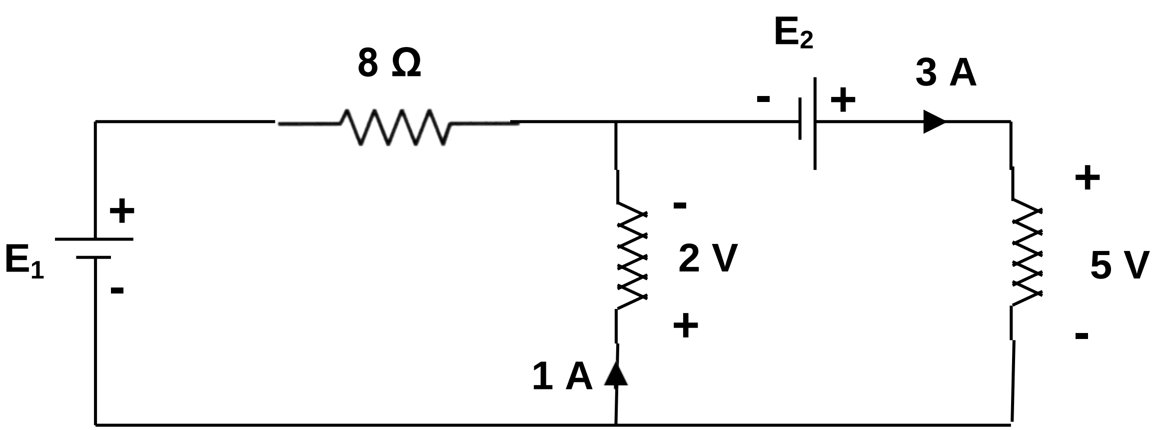
Here $W_1 = 10$ A and $W_2 = 10 \sin \omega t$ A

Therefore $W_{1\ RMS} = 10$ A; $W_{2\ RMS} = 7.071$ A

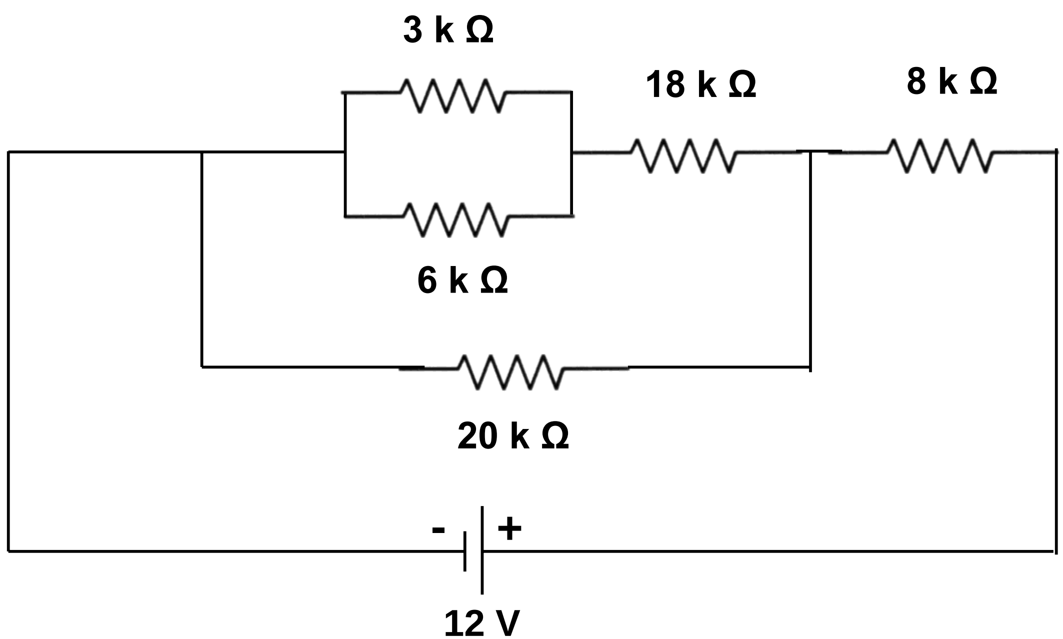
RMS value of conductor current = $\sqrt{10^2 + 7.071^2} = 12.2474A$

Descriptive Type Questions

1. Briefly explain the fringing effect.
2. Define Magnetic field intensity.
3. State and explain Kirchhoff's current law and Kirchhoff's voltage law with suitable examples.
4. Using Kirchhoff's laws, find the voltages E_1 and E_2.



5. Find the power delivered by the voltage source.



SINGLE PHASE AC CIRCUITS

PHASORS

Consider a linear ac circuit having one or more sinusoidal inputs having same frequency as shown in Fig. 1. The amplitudes and phase angles of the inputs may be different while their frequency should be same.

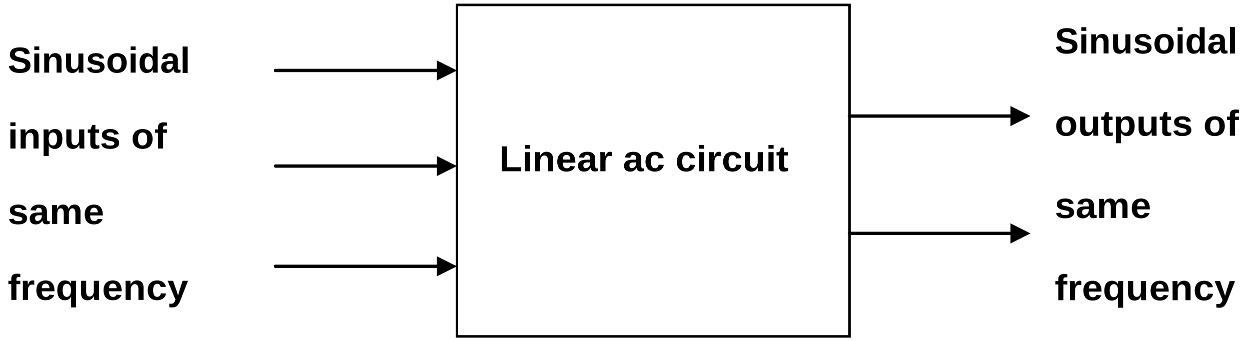


Fig. 1

The output what we may be interested may be voltage across an element or current through an element. The output waveform will be sinusoidal with the same frequency as the input signals. This could be easily verified experimentally.

<u>The steady-state analysis of such circuits can be carried out easily using phasors</u>.

A sinusoid is fully described when its maximum value, angular frequency and phase are specified.

A question may arise whether we should always deal with such sinusoidal time function to represent voltage and current in ac circuits. When all the inputs are sinusoidal time function with the same angular frequency ω, the voltage or the current in any part of the circuit will also be of sinusoidal time function with the SAME ANGULAR FREQUENCY ω. Hence it is redundant to carry information of ω, while representing voltages and currents in ac circuits. This idea gives birth to the concept of PHASORS.

The phasor corresponding to sinusoid $x(t) = x_m \cos(\omega t + \varphi)$ is $\mathbf{X} = \dfrac{x_m}{\sqrt{2}} \angle \varphi$ (1)

In case $x(t)$ is expressed as $x(t) = x_m \sin(\omega t + \varphi)$, it can be written as

$x(t) = x_m \sin(\omega t + \varphi) = x_m \cos\left(\omega t + \varphi - \dfrac{\pi}{2}\right)$ and the corresponding phasor is

$$\mathbf{X} = \frac{x_m}{\sqrt{2}} \angle \varphi - \frac{\pi}{2} \qquad (2)$$

In a similar way we can state:

If $x(t) = -x_m \cos(\omega t + \varphi)$ its phasor is $\mathbf{X} = \dfrac{x_m}{\sqrt{2}} \angle \varphi - \pi$ (3)

If $x(t) = -x_m \sin(\omega t + \varphi)$ its phasor is $\mathbf{X} = \dfrac{x_m}{\sqrt{2}} \angle \varphi + \dfrac{\pi}{2}$ (4)

Eqs. (1) to (4) are useful to find the phasor for a given sinusoid.

Fig. 1 is useful to locate the quadrant in which the phasor lies

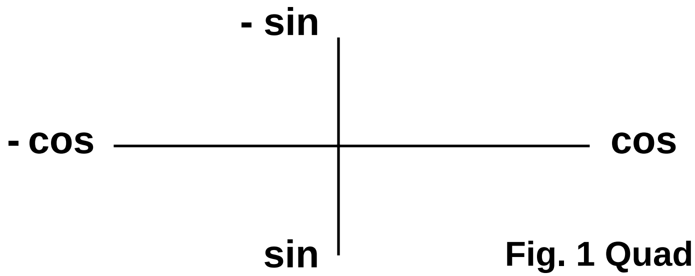


Fig. 1 Quadrants for Phasor

A few sinusoids and the corresponding phasors are;

$x_1(t) = \sqrt{2}\ 150 \cos(\omega t + 15^0)$ $\qquad X_1 = 150 \angle 15^0$

$x_2(t) = \sqrt{2}\ 150 \cos(\omega t - 75^0)$ $\qquad X_2 = 150 \angle -75^0$

$x_3(t) = \sqrt{2}\ 100 \sin \omega t$ $\qquad X_3 = 100 \angle -90^0$

$x_4(t) = \sqrt{2}\ 100 \sin(\omega t + 30^0)$ $\qquad X_4 = 100 \angle -60^0$

$x_5(t) = \sqrt{2}\ 100 \sin(\omega t - 150^0)$ $\qquad X_5 = 100 \angle -240^0$

$x_6(t) = -\sqrt{2}\ 80 \cos(\omega t + 30^0)$ $\qquad X_6 = 80 \angle 210^0$

$x_7(t) = -\sqrt{2}\ 80 \sin(\omega t - 30^0)$ $\qquad X_7 = 80 \angle 60^0$

- The above Phasors are shown in Fig. 2

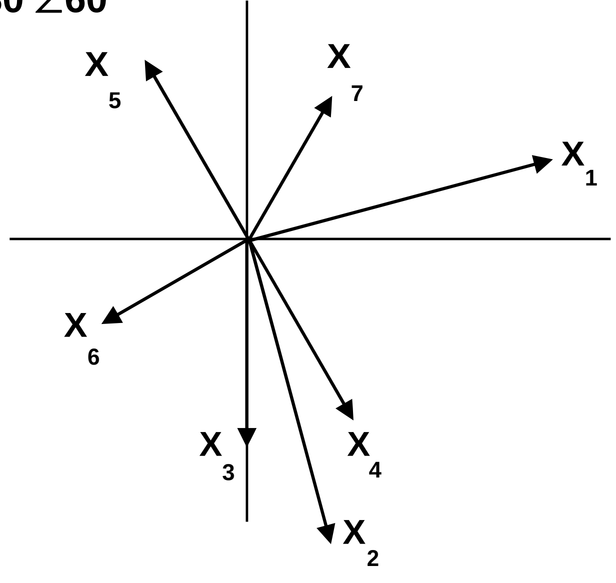


Fig. 2 Phasors of given sinusoids

The important motivation for the use of phasors is the ease with which two or more sinusoids at the same frequency can be added or subtracted. In the sinusoidal steady state, all the currents and voltages are of same frequency. Hence phasors can be used to combine currents or voltages. KCL and KVL can be easily interpreted in terms of phasor quantities.

A phasor is a transformed version of a sinusoidal voltage or current waveform and it contains the amplitude and phase angle information of the sinusoid. Phasors are complex numbers and can be depicted in a complex plane. The relationship of phasors on a complex plane is called a phasor diagram.

Example 1

Using phasor concept, find the sum of 4 voltages given by :

$v_1 = \sqrt{2}\ 50\ \sin \omega t$ $\quad$ $v_2 = \sqrt{2}\ 40 \sin(\omega t + \pi / 3)$

$v_3 = - \sqrt{2}\ 20\ \sin(\omega t - \pi / 6)$ $\quad$ $v_4 = \sqrt{2}\ 30 \sin(\omega t + 3\pi / 4)$

Solution

In phasors corresponding to the sinusoids are:

$$V_1 = 50 \angle -90° = 0.0 - j\,50.0$$

$$V_2 = 40 \angle -30° = 34.6410 - j\,20.0$$

$$V_3 = 20 \angle -120° = -10.0 - j\,17.3205$$

$$V_4 = 30 \angle 45° = 21.2132 + j21.2132$$

$$V_1 + V_2 + V_3 + V_4 = 45.8542 - j66.1073$$

$$= 80.4536 \angle -55.25^0$$

Corresponding sinusoid is obtained as

$$v_T = \sqrt{2}\ 80.4536 \cos(\omega t - 55.25^0) = 113.7786 \cos(\omega t - 55.25^0)$$

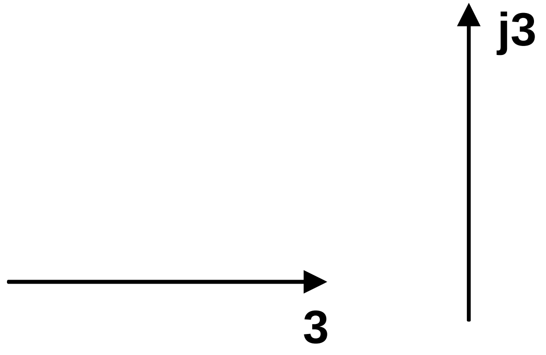
$$= 113.7786 \sin(\omega t + 34.75^0)$$

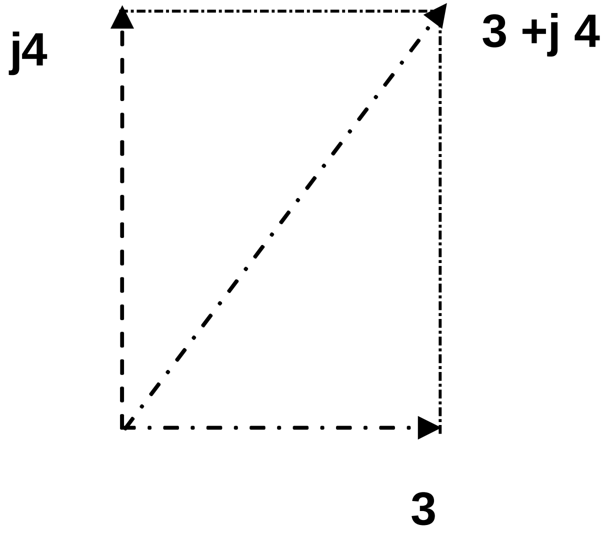
OPERATOR j

Operator j is useful in dealing with COMPLEX NUMBERS.

$$\mathbf{j} = \mathbf{1}\angle\mathbf{90^0} \qquad \mathbf{j^2} = \mathbf{1}\angle\mathbf{90^0}\ \mathbf{x}\ \mathbf{1}\angle\mathbf{90^0} = \mathbf{1}\angle\mathbf{180^0} = -\mathbf{1}$$

$$\mathbf{j^3} = -\mathbf{j} = \mathbf{1}\angle-\mathbf{90^0} \qquad \mathbf{j^4} = \mathbf{1}$$



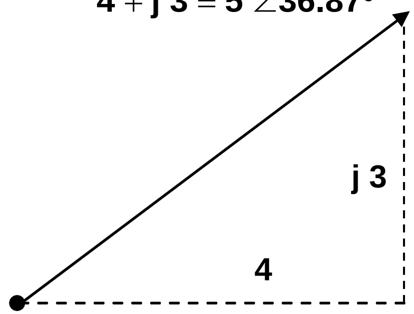


COMPLEX NUMBERS

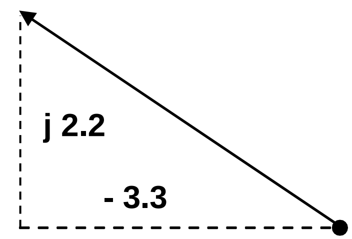
4 + j 3 - 3.3 + j 2.2 - 3 – j 4 2.4 – j 4.4 are a few complex numbers.

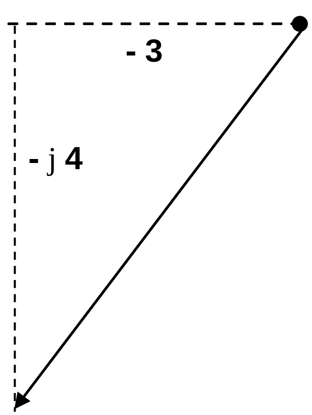
They can be represented either in RECTANGULAR FORM or POLAR FORM. The numbers shown above are in rectangular form. There is one phasor corresponding to each complex number as shown below.

$$4 + j\,3 = 5\ \angle 36.87^0$$

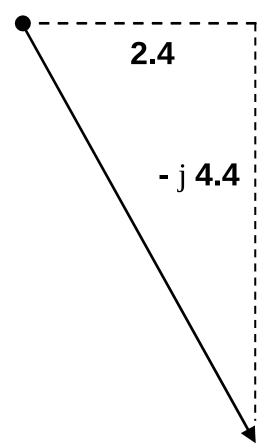


$$-3.3 + j\,2.2 = 4\ \angle 146.3^0$$





$$-3 - j4 = 5 \angle -126.9^0$$



$$2.4 - j4.4 = 5\ \angle -61.4^0$$

SINGLE ELEMENT IN STEADY STATE

Voltage-current relationship of resistor, inductor and capacitor can be obtained in phasor form. Such phasor representations are useful in solving ac circuits.

RESISTOR

Let the voltage v(t) across the resistor terminals be

$$v(t) = V_m \cos \omega t \tag{5}$$

The current through it is given by

$$i(t) = \frac{v(t)}{R} = \frac{V_m}{R} \cos \omega t \tag{6}$$

Expressing the equations (5) and (6) in phasor form we get

$$V = \frac{V_m}{\sqrt{2}} \angle 0^0 \tag{7}$$

$$I = \frac{V_m}{\sqrt{2}\,R} \angle 0^0 \tag{8}$$

The impedance of an element is defined as the ratio of the phasor voltage across it to the phasor current through it. Thus

$$Z = \frac{V}{I} \tag{9}$$

For a resistor $\qquad Z = \frac{V}{I} = R\angle 0^0$ (10)

Thus in the case of a resistor, voltage-current relationship is

$$V = R\,I \tag{11}$$

Representation of resistor in time frame and its phasor form are shown in Fig. 3.

Fig. 3 Representation of a resistor

It is to be noted that as seen by the Eqns. (7) and (8),both the voltage V and the current I have the same phase angle of 0^0. The phasor diagram showing the voltage and current in a resistor is shown in Fig. 4.

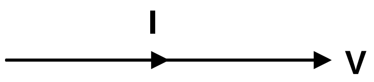


Fig. 4 Phasor diagram - resistor

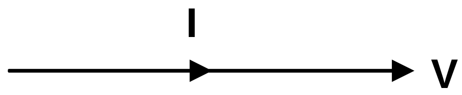


Fig. 4 Phasor diagram - resistor

In the phasor diagram shown in Fig. 4, importance must be given to the phase angles of the voltage V and the current I. The lengths of the phasors depend on their magnitude and the scale chosen. In no occasion length of a voltage phasor and the length of a current phasor can be compared since they have different units. The scale for current phasors will be like

1 cm = x Volts while the scale for the voltage phasors will be like

1 cm = y Ampere.

The impedance of the resistor is R $\angle 0°$. In a general network where R is embedded, the phasor corresponding to the voltage across R and the phasor corresponding to the current through R are always in phase.

INDUCTOR

For the inductor, the voltage-current relationship is

$$v(t) = L \frac{di(t)}{dt} \tag{12}$$

In steady state, let the current through it be

$$i(t) = I_m \cos \omega t \tag{13}$$

Then $v(t) = L \dfrac{di(t)}{dt} = - \omega L \; I_m \sin \omega t$ (14)

Expressing the above two equations in phasor form we have

$$I = \frac{I_m}{\sqrt{2}} \angle 0^0 \tag{15}$$

and $V = \omega L \dfrac{I_m}{\sqrt{2}} \angle 90^0$ (16)

The impedance of the inductor is given by

$$Z = \frac{V}{I} = \omega L \angle 90^0 = j\,\omega L = j\,X_L \tag{17}$$

$$\text{where } X_L = \omega L \tag{18}$$

Thus, the terminal relationship of an inductor in phasor form is

$$V = j\,X_L\,I \tag{19}$$

Representation of inductor in time frame and its phasor form are shown in Fig. 5.

Fig. 5 Representation of a inductor

It is to be noted that as seen by the Eqns. (15) and (16), the voltage V leads the current I by a phase angle of 90^0. The phasor diagram showing the voltage and current in an inductor is shown in Fig. 6.

Fig. 6 Phasor diagram - inductor

Fig. 6 Phasor diagram - inductor

It is to be noted that the voltage V leads the current I by 90° or we can also state that the current I lags the voltage V by 90°. The steady state impedance corresponding to the inductance L is jX_L where $X_L = \omega L$. The quantity X_L is known as the INDUCTIVE REACTANCE.

CAPACITOR

For the capacitor, the voltage-current relationship is

$$i(t) = C\frac{dv(t)}{dt} \tag{20}$$

In steady state, let the voltage across it be

$$v(t) = V_m \cos \omega t \tag{21}$$

Then $i(t) = C\dfrac{dv(t)}{dt} = -\,\omega C\ V_m \sin \omega t$ (22)

Expressing the above two equations in phasor form we have

$$V = \frac{V_m}{\sqrt{2}}\angle 0^0 \tag{23}$$

and $I = \omega C\dfrac{V_m}{\sqrt{2}}\angle 90^0$ (24)

The impedance of the capacitor is given by

$$Z = \frac{V}{I}$$

$$= \frac{1}{\omega C} \angle -90^0 = -\frac{j}{\omega C} = -j X_C \qquad (25)$$

$$\text{where } X_C = \frac{1}{\omega C} \qquad (26)$$

Thus, the terminal relationship of a capacitor in phasor form is

$$V = -j X_C \, I \qquad (27)$$

Representation of capacitor in time frame and its phasor form are shown in Fig. 7.

Fig. 7 Representation of a capacitor

It is to be noted that as seen by the Eqns. (23) and (24), the current I leads the voltage V by a phase angle of 90 ^{0} . The phasor diagram showing the voltage and current in a capacitor is shown in Fig. 8.

Fig. 8 Phasor diagram of - capacitor

It is to be noted that the current I leads the voltage V by 90° or we can also state that the voltage V lags the current I by 90°. The steady state impedance corresponding to the capacitance C is - jX_C where $X_C = \dfrac{1}{\omega C}$. The quantity X_C is known as the CAPACITIVE REACTANCE.

It is conventional to say how the current phasor is relative to voltage phasor. Thus for the resistor, the current phasor is in phase with the voltage phasor. In an inductor, the current phasor lags the voltage phasor by 90^0. In the case of a capacitor, the current phasor leads the voltage phasor by 90^0 .

<u>Example 2</u>

The voltage of $v = \sqrt{2}\ 80 \cos(100t - 55^0)$ V is applied across a resistor of 25Ω. Find the steady state current through the resistor.

<u>Solution</u>

Here $V = 80 \angle -55^0$ and $R = 25\ \Omega$

Thus, current I = $$\frac{V}{R} = \frac{80\angle -55^0}{25} = 3.2\angle -55^0 \text{ A}$$

Current $i(t) = 4.5255 \cos(100t - 55^0)$ A

Example 3

The voltage of v = $\sqrt{2}$ 20 sin (50 t - 25 ^{0}) V is applied across an inductor of 0.1 H. Find the steady state current through the inductor.

Solution

Phasor voltage V = 20 $\angle$- 25 ^{0} - 90 ^{0} = 20 $\angle$- 115 ^{0} V

Impedance Z = j ω L = j 50 x 0.1 = 5 $\angle$90 ^{0} Ω

Current I = $\dfrac{V}{Z} = \dfrac{20\angle -115^0}{5\angle 90^0} = 4 \angle - 205^0$ **A**

Converting this to the time domain

Current i(t) = 5.6569 cos (50 t - 205 ^{0}) A

= - 5.6569 cos (50 t - 25 ^{0}) A

Example 4

The voltage of v = $\sqrt{2}$ 12 cos (100 t - 25 ^{0}) V is applied across a capacitor of 50 μF. Find the steady state current through the capacitor.

Solution

Phasor voltage V = 12 $\angle$- 25 ^{0} V

Impedance $Z = -j\dfrac{1}{\omega C} = -j\dfrac{1}{100 \times 50 \times 10^{-6}} = -j\,200\Omega$

Current I = $\dfrac{V}{Z} = \dfrac{12\angle -25^0}{200\angle -90^0}$ $A = 0.06\ \angle 65^0\ A = 60\angle 65^0$ mA

Converting this to the time domain

Current i(t) = 84.8528 cos (100 t + 65 ^{0}) mA

ANALYSIS OF RLC CIRCUITS

An ac circuit generally consists of resistors, inductors and capacitors connected in series, parallel and series-parallel combinations. Often we need to simplify the circuit by finding the equivalents. Further to this, we have to make use of KVL, KCL, source transformation, voltage division and current division what we discussed in previous chapter, by replacing resistors by impedances and dc voltages and currents by voltage phasors and current phasors.

A coil used in ac circuit will have its own resistance in addition to the inductive reactance due to its inductance. One such coil is shown in Fig. 9.

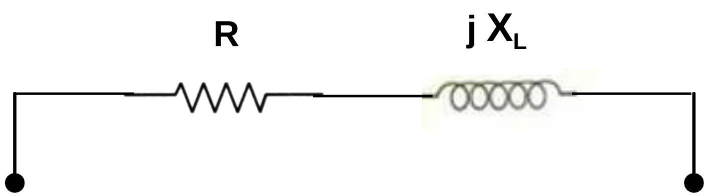


Fig. 9 A coil in an ac circuit

It is clear that the resistance R and inductive reactance j X_L are connected in series. The impedance of this coil is

$$Z = R + jX_L \tag{28}$$

Now consider a case where a resistance R and a capacitance having a capacitive reactance $- j X_C$ are connected in series as shown in Fig. 10.

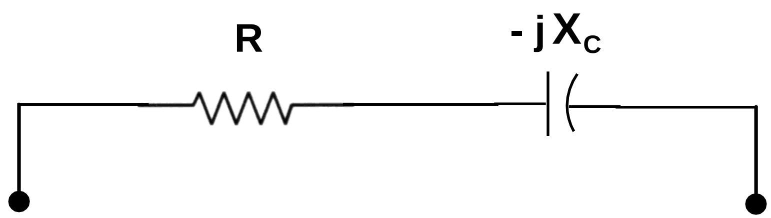


Fig. 10 A resistance and a capacitance in series

The impedance of the circuit is

$$Z = R - j X_C \tag{29}$$

IMPEDANCE AND ADMITTANCE

The steady state impedance (a complex quantity) can be written in two forms, namely Rectangular form and Polar form as

Rectangular form: $Z = R + j X$

Polar form: $Z = |Z| \angle \varphi$

If two impedances Z_1 and Z_2 are connected in series as shown in Fig. 11, then the equivalent impedance is

$$Z_{eq} = Z_1 + Z_2 \tag{30}$$

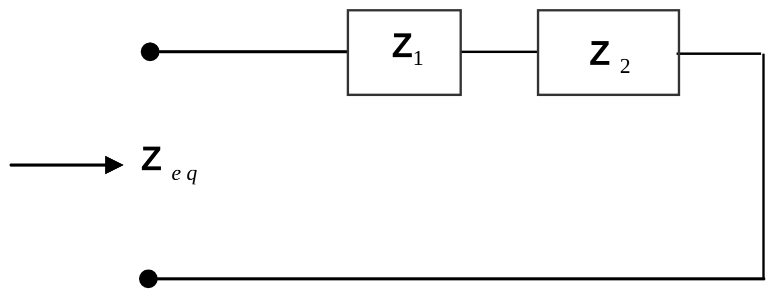


Fig. 11 Two impedances connected in series

This could be generalized to *n* number of impedances connected in series as

$$Z_{eq} = Z_1 + Z_2 + \ldots\ldots\ldots\ldots\ldots + Z_n \tag{31}$$

If *n* number of impedances Z_1, Z_2, …., Z_n are connected in parallel as shown in Fig. 12, the equivalent impedance is obtained from

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots\ldots\ldots + \frac{1}{Z_n} \tag{32}$$

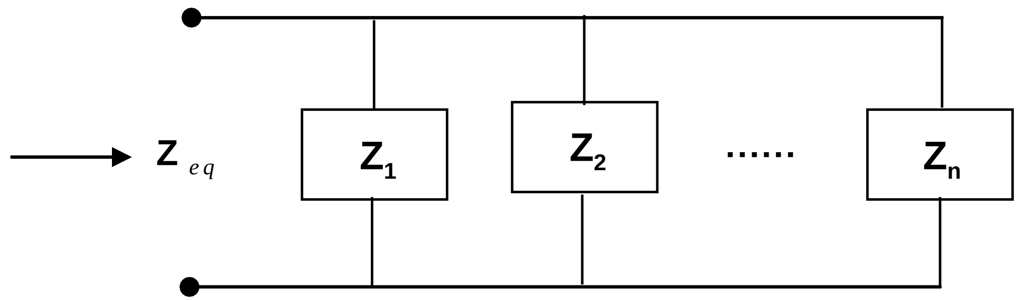


Fig. 12 n impedances connected in parallel

If two impedances Z_1 and Z_2 are connected in parallel, then the equivalent resistance is obtained from

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \tag{33}$$

$$= \frac{Z_1 + Z_2}{Z_1 Z_2}$$

Therefore

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{34}$$

While dealing with the parallel circuit, it is also useful to define another quantity called 'admittance'. ADMITTANCE is defined as the reciprocal of the impedance and it is denoted by Y. Thus

$$Y = \frac{1}{Z} \qquad (35)$$

When the admittance Y is written in rectangular form as

$$Y = G + j\,B \qquad (36)$$

G is called as 'conductance' and B is called as 'susceptance'. The unit of G, B and Y are mho or siemens and is denoted by ℧ .

When two impedances Z_1 and Z_2 are connected in parallel, referring to Eqn. (33) the equivalent admittance Y_{eq} is given by

$$Y_{eq} = Y_1 + Y_2 \qquad (37)$$

where Y_1 and Y_2 are the admittances corresponding to the impedances Z_1 and Z_2 respectively.

When n number of admittances $Y_1, Y_2, \ldots\ldots, Y_n$ are connected in parallel, Eqn. (3.44) can by generalized as

$$Y_{eq} = Y_1 + Y_2 + \ldots\ldots\ldots\ldots + Y_n \quad \textbf{(38)}$$

If there are n equal impedances Z are connected in series, then the equivalent impedance is

$$Z_{eq} = nZ \quad \textbf{(39)}$$

Similarly if there are n equal admittances Y are connected in parallel, then the equivalent admittance is

$$Y_{eq} = nY \quad \textbf{(40)}$$

RL CIRCUIT

Having studied how to combine the series and parallel impedances we shall now see how the RL, RC and RLC circuits can be analyzed.

Let us consider a simple circuit in which a resistor and an inductor are connected in series as shown in Fig. 13.

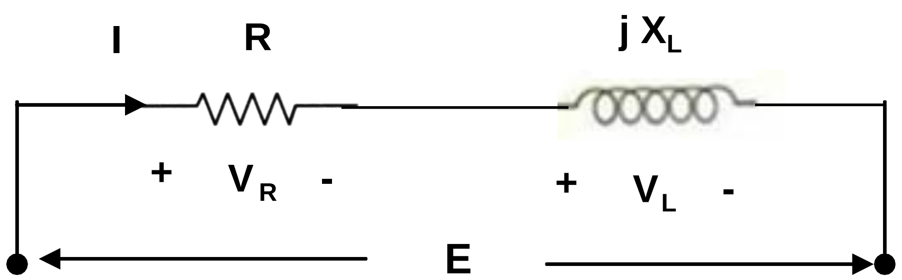


Fig. 13 RL circuit

Taking the supply voltage as reference

$$\mathbf{E} = |\mathbf{E}| \angle 0^0 \qquad (41)$$

Circuit impedance $\mathbf{Z} = \mathbf{R} + \mathbf{jX_L} = |\mathbf{Z}|\angle\boldsymbol{\theta}$ (42)

Circuit current $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}} = \dfrac{|\mathbf{E}|\angle 0^0}{|\mathbf{Z}|\angle\boldsymbol{\theta}} = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}\angle -\boldsymbol{\theta}$ (43)

$= |\mathbf{I}|\angle -\boldsymbol{\theta}$ (44)

where $|\mathbf{I}| = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}$ (45)

Further $\mathbf{V_R} = \mathbf{RI} = \mathbf{R}\ |\mathbf{I}|\angle -\boldsymbol{\theta}$ (46)

$\mathbf{V_L} = \mathbf{jX_L\,I} = \mathbf{X_L}\ |\mathbf{I}|\angle -\boldsymbol{\theta} + 90^0$ (47)

Using KVL, we get $\mathbf{V_R} + \mathbf{V_L} = \mathbf{E}$ (48)

The phasor diagram for this RL circuit can be got by drawing the phasors E, I, V_R and V_L as shown in Fig. 14.

$E = |E| \angle 0^0 \, V$

$I = |I| \angle -\theta$

$V_R = R \, |I| \angle -\theta$

$V_L = X_L \, |I| \angle -\theta + 90^0$

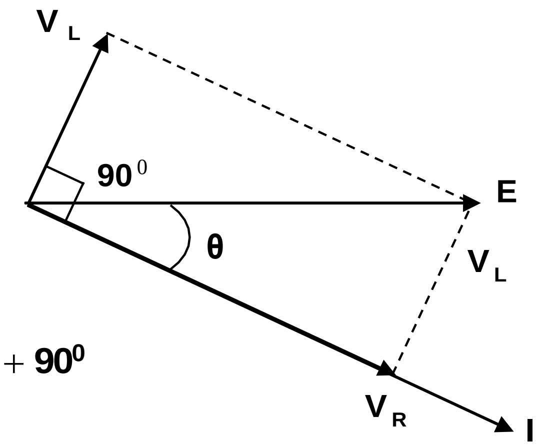
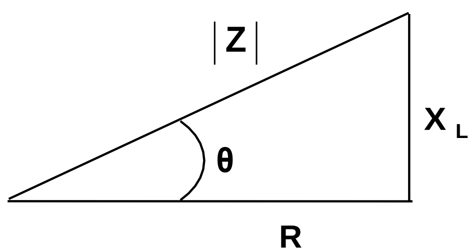


Fig.14 Phasor diagram of RL circuit

Consider the triangle formed by the phasors V_R, V_L and E. Recognizing that $|V_R| = R|I|$, $|V_L| = X_L |I|$ and $|E| = |Z||I|$ if each side of the triangle is divided by $|I|$ then R, X_L and $|Z|$ will form a triangle as shown in Fig, 15. This triangle is known as the IMPEDANCE TRIANGLE.



By knowing any two of

4 quantities, other two can be calculated

Fig. 15 Impedance diagram of RL circuit

RC CIRCUIT

Let us now consider the circuit in which a resistor and a capacitor are connected in series as shown in Fig. 16.

Fig. 16 RC circuit

Taking the supply voltage as reference

$$\mathbf{E} = |\mathbf{E}| \angle 0^0 \qquad (49)$$

Circuit impedance $\mathbf{Z} = \mathbf{R} - j\mathbf{X_C} = |\mathbf{Z}| \angle -\theta$ (50)

Circuit current $$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{|\mathbf{E}|\angle 0^0}{|\mathbf{Z}|\angle -\theta} = \frac{|\mathbf{E}|}{|\mathbf{Z}|}\angle\theta \qquad (51)$$

$$= |\mathbf{I}|\angle\theta \qquad (52)$$

where $$|\mathbf{I}| = \frac{|\mathbf{E}|}{|\mathbf{Z}|} \qquad (53)$$

Further $V_R = RI = R \; |I| \angle\theta$ (54)

$$V_C = -jX_C I = X_C \; |I| \angle\theta - 90^0 \qquad (55)$$

Using KVL, we get $V_R + V_C = E$ (56)

The phasor diagram for this RC circuit can be got by drawing the phasors V_R, V_C, E and I as shown in Fig. 17.

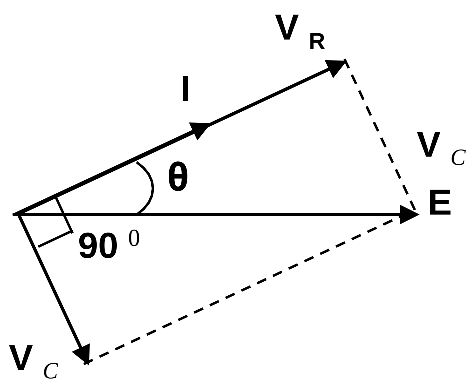


Fig. 17 Phasor diagram of RC circuit

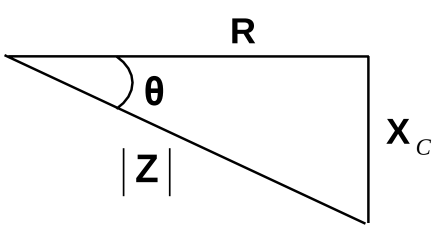


Fig. 18 Impedance triangle of RC circuit

Consider the triangle formed by the phasors V_R, V_C and E. Recognizing that $|V_R| = R|I|$, $|V_C| = X_C |I|$ and $|E| = |Z||I|$ if each side of the triangle is divided by $|I|$ then R, X_C and $|Z|$ will form a triangle as shown in Fig, 18. This triangle is known as the IMPEDANCE TRIANGLE.

RLC CIRCUITS

Analysis of RLC circuits is the series, parallel and series-parallel combination of RL and RC circuits. Equivalent of RLC circuit will be R, or RL or RC circuit as illustrated in the examples to be discussed.

POWER AND POWER FACTOR

Let $|E| \angle 0^0$ be the supply voltage in an AC circuit. The supply current may lag or lead the supply voltage. Let the supply current be $|I| \angle -\theta$. The supply current can be resolved into two components (i) A component I_p in phase with the voltage and (ii) A component I_q at right angle to the voltage as shown in Fig. 19.

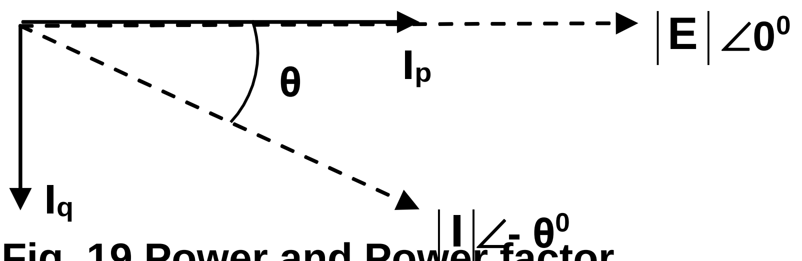


Fig. 19 Power and Power factor

Current I_p is called the active or in-phase component while I_q is known as reactive or quadrature component. As seen from Fig. 19

$$I_p = |I| \cos\theta \quad \text{and} \tag{57}$$

$$I_q = |I| \sin\theta \tag{58}$$

It is to be noted that

$|I| \cos\theta$, $|I| \sin\theta$ and $|I|$ form three sides of a right angle triangle as in Fig. 20.

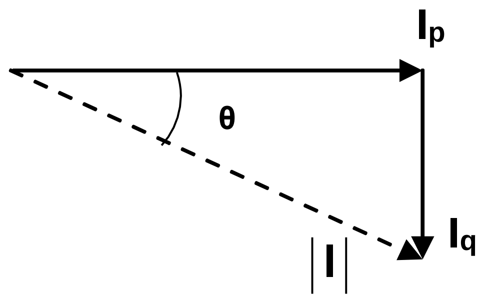


Fig. 20 Components of current

Active Power (P)

Active power is the real power consumed by the circuit. This is due to the in-phase component.

Active or real power $P = |E| \, I_p$

$$= |E| \, |I| \cos\theta \text{ Watts} \tag{59}$$

Reactive Power (Q)

The power associated with the reactive component of current I_q is known as reactive power. Its unit is Volt Ampere Reactive (VAR).

Reactive power $Q = |E| \, I_q$

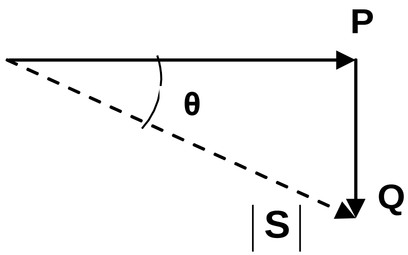
$$= |E| \, |I| \sin\theta \text{ VAR} \tag{60}$$

Apparent Power and Power Factor

The product of voltage and current, $|E|\ |I|$ is called as Apparent Power, denoted by $|S|$. Its unit is Volt Ampere (VA).

$$\text{Apparent power} \quad |S| = |E|\ |I| \quad \text{VA} \tag{61}$$

Similar to Fig. 20, real power P, reactive power Q and apparent power $|S|$ form three sides of a right angle triangle as shown in Fig. 21.



$$P = |E|\,|I| \cos\theta$$

$$Q = |E|\,|I| \sin\theta$$

$$S = |E|\,|I|$$

Fig. 21 Components of power

Power Factor (pf) is the ratio of real power to apparent power.

$$\text{Thus power factor} = \frac{|E|\,|I| \cos\theta}{|E|\,|I|}$$

$$= \cos\theta \tag{62}$$

By the above definition, it is not possible to distinguish whether the load is inductive or capacitive. If the load is inductive, the current is lagging the voltage and the nature of the power factor is LAGGING. On the other hand if the load is capacitive, the current is leading the voltage and hence the nature of the power factor is LEADING.

Whenever power factor is furnished, it must be clearly stated whether it is lagging or leading. For inductive load, the power factor is cos θ lagging; for capacitive load, the power factor is cos θ leading; for resistive load since the voltage and current are in-phase, power factor angle θ is zero and the power factor is said to be UNITY.

Power associated with R, L and C can be obtained as follows.

<u>In the case of resistor</u>, p.f. angle is zero and hence

$$\mathbf{P} = |\mathbf{E}|\ |\mathbf{I}| \cos\theta = |\mathbf{E}|\ |\mathbf{I}| = |\mathbf{Z}|\,|\mathbf{I}|\,|\mathbf{I}| = |\mathbf{I}|^2\ \mathbf{R} \qquad (63)$$

$$\mathbf{Q} = |\mathbf{E}|\ |\mathbf{I}| \sin\theta = 0 \qquad (64)$$

<u>In the case of pure inductor and pure capacitor</u>, p.f. angle = 90^0 and hence

$$\mathbf{P} = |\mathbf{E}|\ |\mathbf{I}| \cos\theta = 0 \qquad (65)$$

$$\mathbf{Q} = |\mathbf{E}|\ |\mathbf{I}| \sin\theta = |\mathbf{E}|\ |\mathbf{I}| = |\mathbf{Z}|\,|\mathbf{I}|\,|\mathbf{I}| = |\mathbf{I}|^2\ \mathbf{X} \qquad (66)$$

Example 5

In a series circuit containing pure resistance and pure inductance, the current and voltage are: $i(t) = 5 \sin(314t + \frac{2\pi}{3})$ and $v(t) = 20 \sin(314t + \frac{5\pi}{6})$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the power drawn by the circuit?

Solution Phasor current and phasor voltages are

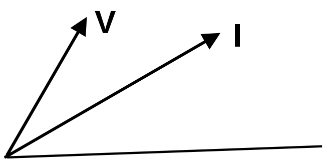
Current $I = \frac{5}{\sqrt{2}} \angle 120\text{-}90 = \frac{5}{\sqrt{2}} \angle 30^0$; Voltage $V = \frac{20}{\sqrt{2}} \angle 150\text{-}90 = \frac{20}{\sqrt{2}} \angle 60^0$

Impedance $Z = \frac{V}{I} = \frac{20\angle 60}{5\angle 30} = 4 \angle 30^0 \ \Omega = (3.4641 + j2)\Omega$

Resistance $R = 3.4641\ \Omega$

$X_L = 2\ \Omega$; $314\ L = 2$; $L = \frac{2}{314}$ H ; **Inductance** $L = 6.3694$ mH

Angle between voltage and current = 30^0



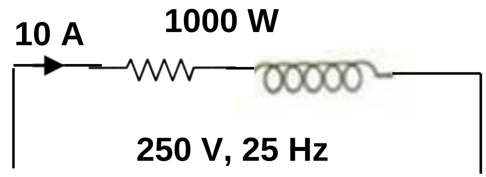
p.f. = $\cos 30^0 = 0.866$ lagging

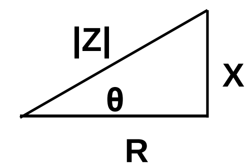
Power $P = |V||I|\cos\theta = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3$ W

Example 6

An inductive coil takes 10 A and dissipates 1000 W when connected to a supply of 250 V, 25 Hz. Calculate the impedance, resistance, reactance, inductance and the power factor.

Solution





$P = |I|^2 R$; Resistance $R = \dfrac{1000}{100} = 10\ \Omega$

$|Z| = \dfrac{250}{10} = 25\,\Omega$; From impedance triangle $X = \sqrt{25^2 - 10^2} = 22.9128\Omega$

Thus <u>impedance</u> $Z = (10 + j\,22.9128)\ \Omega = 25 \angle 66.42^0\ \Omega$

<u>Resistance</u> $R = 10\ \Omega$ <u>Reactance</u> (Inductive) $X_L = 22.9128\,\Omega$

<u>Inductance</u> $L = \dfrac{X_L}{2\pi f} = \dfrac{22.9128}{2\pi \times 25} = 0.1459\,H$

From impedance triangle, <u>power factor</u> $= \dfrac{R}{|Z|} = \dfrac{10}{25} = 0.4$ lagging

Example 7

A resistance is connected in series with a coil. With a supply of 250 V, 50 Hz, the circuit takes a current of 5 A. If the voltages across the resistance and the coil are 125 V and 200 V respectively, calculate (i) impedance, resistance and reactance of the coil (ii) power absorbed by the coil and the total power. Draw the phasor diagram.

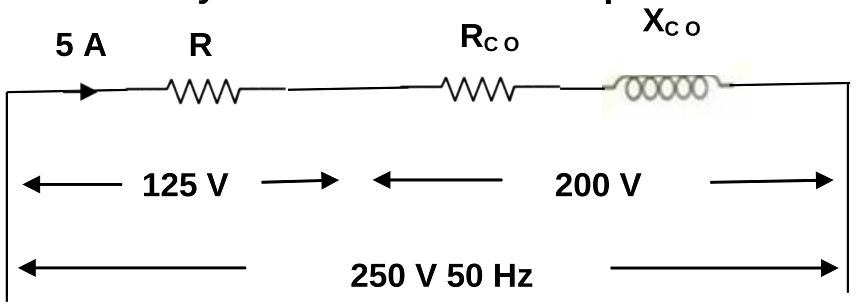


Fig. 21 Example 7

Resistance R = $\frac{125}{5} = 25\ \Omega$

$|Z_{CO}| = \frac{200}{5} = 40\ \Omega$; $\quad |Z_T| = \frac{250}{5} = 50\ \Omega$

Since $|Z_{CO}| = 40\ \Omega$, $\quad |R_{CO} + jX_{CO}| = 40$; $\quad R_{CO}^2 + X_{CO}^2 = 1600$

Since $|Z_T| = 50\ \Omega$; $\quad |(25+R_{CO}) + jX_{CO}| = 50$

$625 + 50\,R_{CO} + R_{CO}^2 + X_{CO}^2 = 2500$ **i.e.** $50\,R_{CO} = 2500 - 625 - 1600 = 275$

<u>Resistance of the coil</u> $R_{CO} = 5.5\ \Omega$ $\quad$ **Also** $X_{CO}^2 = 1600 - 5.5^2 = 1569.75$

<u>Reactance of the coil</u> $X_C = 39.62\ \Omega$

<u>Impedance of the coil</u> $Z_{CO} = (5.5 + j\,39.62) = 40\ \angle 82.1^0\ \Omega$

<u>Power absorbed by the coil</u> $P_{CO} = 5^2 \times 5.5 = 137.5\ W$

<u>Total power</u> $P_T = (5^2 \times 25) + 137.5 = 762.5\ W$

Total impedance $Z_T = (30.5 + j\,39.62) = 50\ \angle 52.41^0\Omega$

$|I|R_{CO} = 5 \times 5.5 = 27.5\ V$; $|I|X_{CO} = 5 \times 39.62 = 198.1\ V$

Phasor diagram is shown in Fig. 22.

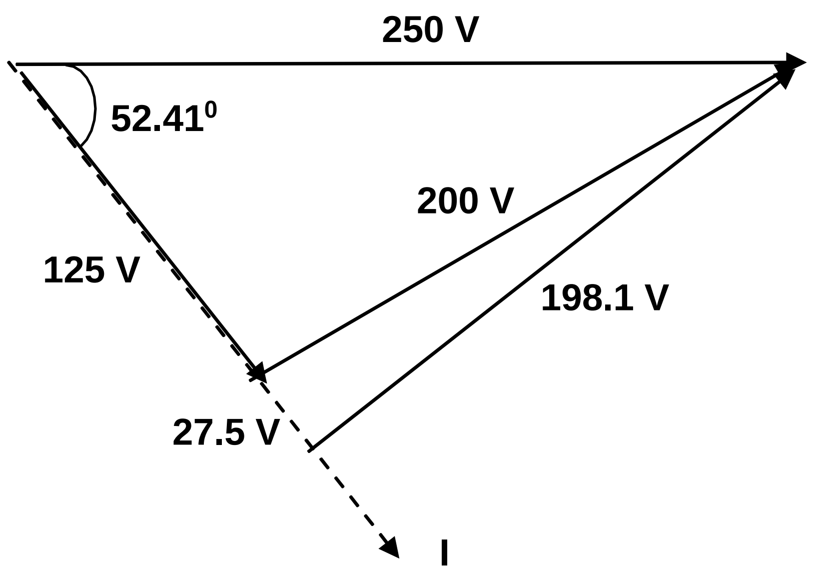


Fig. 22 Phasor diagram-Example 7

Example 8

When a resistor and a seriesly connected inductor coil, are supplied with 240 V, a current of 3 A flows lagging behind the supply voltage by 37^0. The voltage across the coil is 171 V. Find the value of the resistor, resistance and reactance of the inductor coil.

Solution

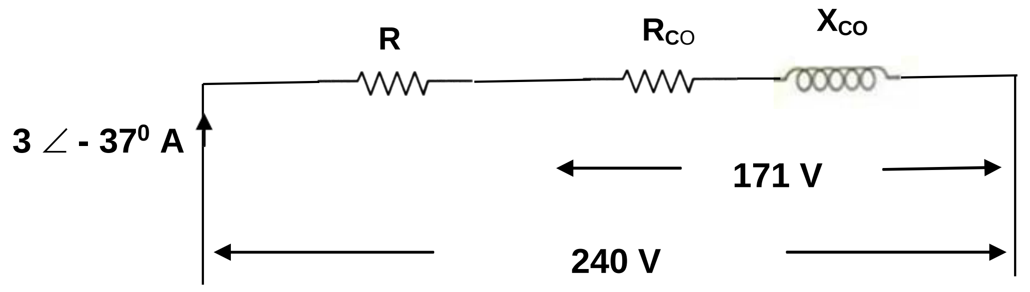


Fig. 23 Example 8

Supply voltage $E = 240 \angle 0^0$ V; Supply current $I = 3 \angle -37^0$ A

Circuit impedance $Z = \frac{E}{I} = 80 \angle 37^0\ \Omega = (63.8908 + j\,48.1452)\ \Omega$

Thus $R + R_{CO} = 63.8908\ \Omega$ and $X_{CO} = 48.1452\ \Omega$

For the coil, $|Z_{CO}| = \frac{171}{3} = 57\ \Omega$; From impedance triangle of the coil

$R_{CO} = \sqrt{57^2 - 48.1452^2} = 30.5129\ \Omega$

Value of resistor $R = 63.8908 - 30.5129 = 33.3779\ \Omega$

Resistance of coil $R_{CO} = 30.5129\ \Omega$; Reactance of coil $X_{CO} = 48.1452\ \Omega$

Example 9

When a voltage of 100 V at 50 Hz is applied to choking coil 1, the current taken is 8 A and the power is 120 W. When the same supply is applied to choking coil 2, the current is 10 A and the power is 500 W. Find the current and power when the supply is applied to two coils connected in series.

Solution

Resistance $R_1 = \dfrac{120}{8^2} = 1.875\ \Omega$

Impedance $|Z_1| = \dfrac{100}{8} = 12.5\ \Omega$; **Therefore** $X_1 = \sqrt{12.5^2 - 1.875^2} = 12.3586\ \Omega$

Resistance $R_2 = \dfrac{500}{10^2} = 5\ \Omega$

Impedance $|Z_2| = \dfrac{100}{10} = 10\ \Omega$; **Therefore** $X_2 = \sqrt{10^2 - 5^2} = 8.6603\ \Omega$

Total resistance $R_T = 6.875\ \Omega$; **Total reactance** $X_T = 21.0189\ \Omega$

Total impedance $Z_T = (6.875 + j\,21.0189) = 22.1147\ \angle 71.89^0\ \Omega$

Total current $|I_T| = \dfrac{100}{22.1147} = 4.5219$ **A** **Power** $P_T = 4.5219^2 \times 6.875 = 140.5771$ **W**

Example 10

A resistance of 100 ohm is connected in series with a 50 μF capacitor. When the supply voltage is 200 V, 50 Hz, find the (i) impedance, current and power factor (ii) the voltage across resistor and across capacitor. Draw the phasor diagram.

Solution

Resistor R = 100 Ω; Reactance of the capacitor $X_C = \dfrac{10^6}{2\pi \times 50 \times 50} = 63.662\ \Omega$

Impedance $Z = (100 - j\,63.662) = 118.5447 \angle -32.48^0$

Taking the supply voltage as reference, $E = 200 \angle 0^0$ V

Current $I = \dfrac{E}{Z} = \dfrac{200\angle 0^0}{118.5447\angle -32.48^0} = 1.6871\angle 32.48^0$ A

Power factor = cos 32.48⁰ = 0.8436 leading

Voltage across resistor $V_R = 100 \times 1.6871 \angle 32.48^0 = 168.71 \angle 32.48^0$ V

Voltage across capacitor $V_C = -j\,63.662 \times 1.6871 \angle 32.48^0 = 107.4042 \angle -57.52^0$ V

Phasor diagram is shown in Fig. 24.

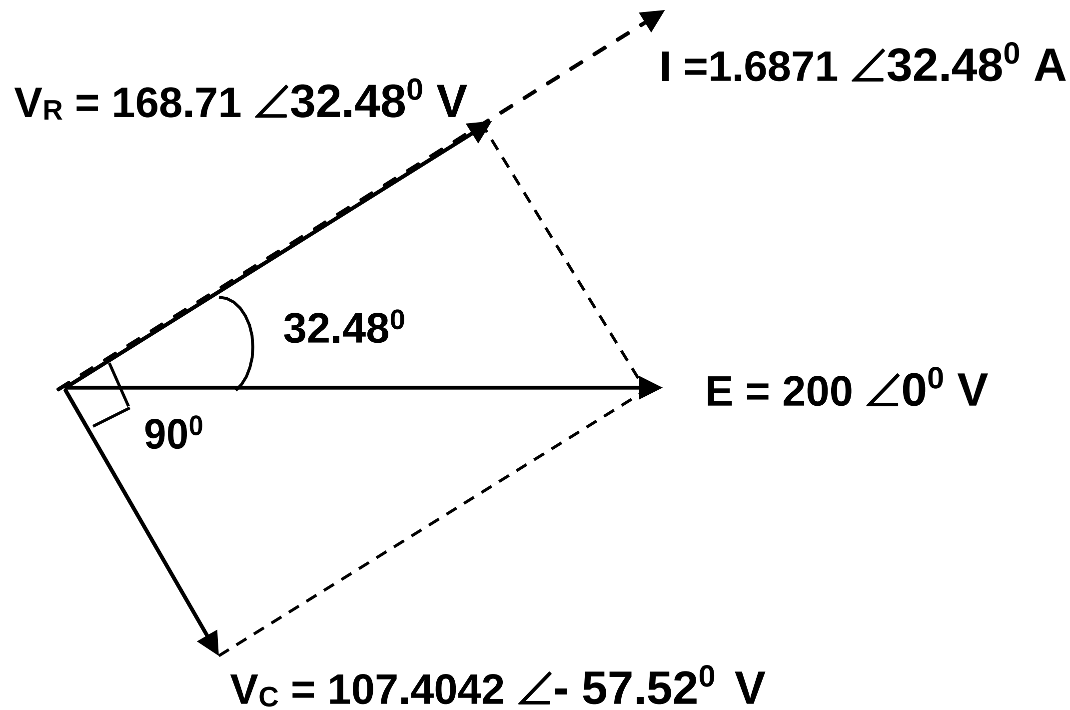


Fig. 24 Phasor diagram - Example 10

Example 11

In a circuit, the applied voltage of 150 V lags the current of 8 A by 40^0. (i) Find the power factor (ii) Is the circuit inductive or capacitive? (iii) Find the active and reactive power.

Solution

Power Factor = 0.766 leading

Circuit is capacitive.

Active Power P = 150 x 8 x 0.766 = 919.2 W

Reactive Power Q = 150 x 8 x 0.6428 = 771.36 VAR

Example 12

Find the circuit constants of a two elements series circuit which consumes 700 W with 0.707 leading power factor. The applied voltage is V = 141.4 sin 314 t volts.

Solution

$$\mathbf{P = 700W;\ |V| = \frac{141.4}{\sqrt{2}} = 99.9849\ V\ ;\ \ \cos\theta = 0.707;\ \ Since\ Power\ P = |V||I|\cos\theta}$$

$$\mathbf{|I| = \frac{700}{99.9849 \times 0.707} = 9.9025\ A\quad and\quad |Z| = \frac{99.9849}{9.9025} = 10.0969\ \Omega}$$

From the impedance triangle

Resistance $\mathbf{R = |Z|\cos\theta = 10.0969 \times 0.707 = 7.1385\,\Omega}$

Reactance $\mathbf{X_C = |Z|\sin\theta = 10.0969 \times 0.707 = 7.1385\,\Omega}$

Capacitance $\mathbf{C = \frac{1}{314 \times 7.1385} = 446.132\ \mu F}$

Example 13

A series R-C circuit consumes a power of 7000 W when connected to 200 V, 50 Hz supply. The voltage across the resistor is 130 V. Calculate (i) the resistance, impedance, capacitance, current and p.f. (ii) Write the equation for the voltage and current.

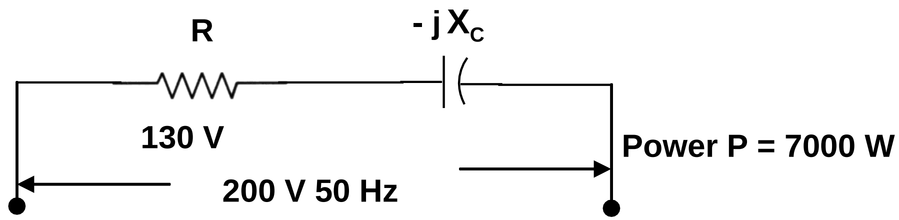


Fig. 25 Circuit – Example 13

Solution

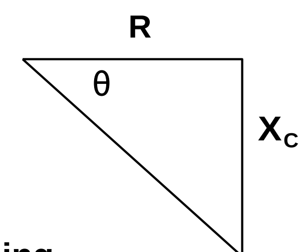
$$\left(\frac{130}{R}\right)^2 R = 7000$$

Resistance $R = \dfrac{130^2}{7000} = 2.4143\ \Omega$; Current $|I| = \dfrac{130}{2.4143} = 53.8458$ A

Since 200 x 53.8458 x cos θ = 7000, p.f. = 0.65 leading; $\theta = 49.46^0$

From impedance triangle, reactance $X_C = R \tan\theta = 2.4143 \times 1.1691 = 2.8226\ \Omega$

Impedance $Z = (2.4143 - j\,2.8226)\ \Omega = 3.7143\ \angle -49.46^0\,\Omega$



Capacitance $C = \dfrac{1}{2\,\pi \times 50 \times 2.8226} = 1127.72\ \mu F$

Current $I = 53.8458\ \angle 49.46^0$ A; **Power Factor** = 0.65 leading

Taking supply voltage as reference

$$v(t) = \sqrt{2} \times 200 \cos(2\pi \times 50t) = 282.8427 \cos 314.16t$$

$$i(t) = \sqrt{2} \times 53.8458 \cos(2\pi \times 50t + 49.46^0) = 76.15 \cos(314.16t + 49.46^0)\ A$$

Example 14

A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150 μF capacitor across 200 V, 50 Hz supply. Calculate (i) inductive reactance, capacitive reactance, impedance, current and power factor and (ii) the voltage across the coil and capacitor.

Solution

Data are marked in Fig. 26

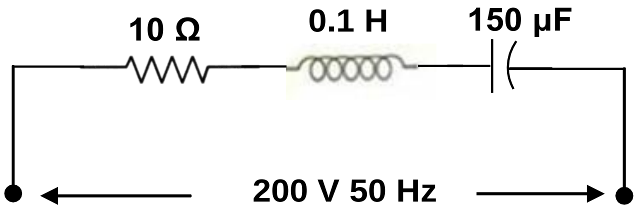


Fig. 26 Circuit – Example 14

Inductive reactance $X_L = 2\,\pi \times 50 \times 0.1 = 31.4159\ \Omega$

Capacitive reactance $X_C = \dfrac{10^6}{2\,\pi \times 50 \times 150} = 21.2207\ \Omega$

Impedance $Z = (10 + j\,31.4159 - j\,21.2207) = (10 + j\,10.1952)\ \Omega = 14.2808 \angle 45.55^0\ \Omega$

Taking supply voltage as reference

Current $I = \dfrac{200}{14.2808 \angle 45.55} = 14.0048 \angle -45.55^0\ A$ **Power factor** = 0.7003 lagging

Impedance $Z_{Coil} = (10 + j\,31.4159) = 32.9691 \angle 72.34^0\ \Omega$

Voltage $V_{Coil} = Z_{Coil} \times I = 461.7257 \angle 26.79^0\ V$

Voltage $V_{Cap} = Z_{Cap} \times I = -j\,21.2207 \times 14.0048 \angle -45.55^0 = 297.1916 \angle -135.55^0\ V$

Example 15

In the circuit shown in Fig. 27, the current leads the voltage by 50^0. Find value of R and the voltages across each circuit element. Draw the phasor diagram.

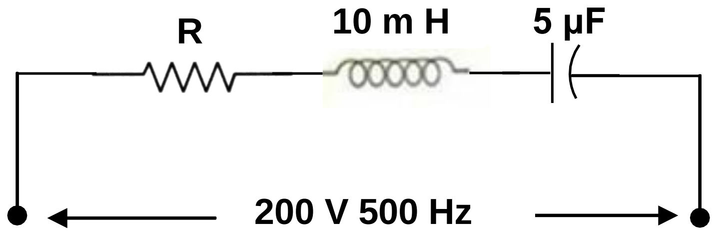


Fig. 27

Solution

Circuit is capacitive

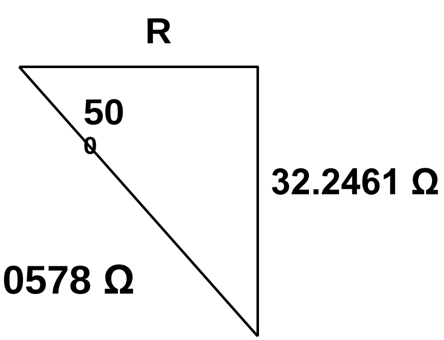
Power factor angle $\theta = 50^0$

Taking the supply voltage as reference $V = 200 \angle 0^0$ volts

$X_L = 31.4159\ \Omega$; $X_C = 63.662\ \Omega$

Impedance $Z = R + j\,31.4159 - j\,63.662 = R - j\,32.2461$

From the impedance triangle



$\tan\theta = \dfrac{32.2461}{R} = 1.19175$; Thus Resistance R = $27.0578\ \Omega$

Impedance $Z = (27.0578 - j\,32.2461) = 42.0944 \angle -50^0\ \Omega$

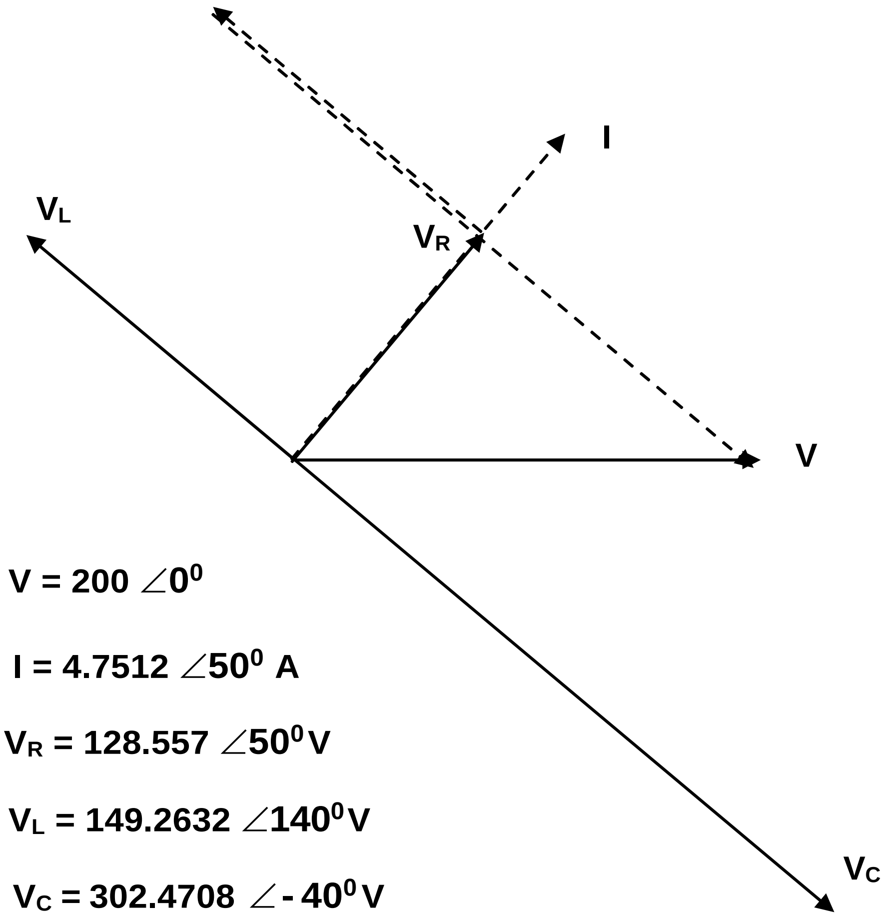
Current $I = \dfrac{V}{Z} = \dfrac{200}{42.0944\angle -50} = 4.7512\angle 50^0$ A

Voltage across R $V_R = R \times I = 128.557 \angle 50^0$ V

Voltage across L $V_L = j\,31.4159 \times 4.7512 \angle 50^0 = 149.2632 \angle 140^0$ V

Voltage across C $V_C = -j\,63.662 \times 4.7512 \angle 50^0 = 302.4708 \angle -40^0$ V

Phasor diagram is shown in Fig. 28.



$V = 200 \angle 0^0$

$I = 4.7512 \angle 50^0$ A

$V_R = 128.557 \angle 50^0$ V

$V_L = 149.2632 \angle 140^0$ V

$V_C = 302.4708 \angle -40^0$ V

Fig. 28 Phasor diagram – Example 15

Example 16

A 230 V, 50 Hz voltage is applied to a coil of L = 5 H and R = 2 Ω in series with a capacitance C. What value must C have in order that the p.d. across the coil shall be 250 V?

Solution

Refer Fig. 29

$X_L = 2\pi \times 50 \times 5 = 1570.7963\ \Omega$

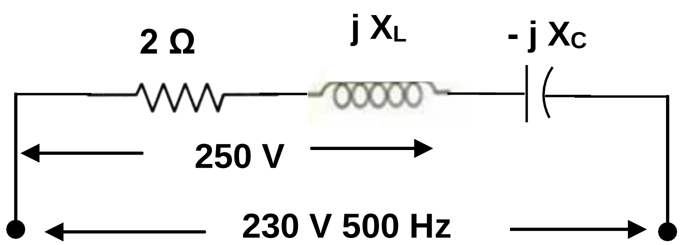


Fig. 29 Circuit – Example 16

Coil impedance $Z_{CO} = (2 + j\,1570.7963) = 1570.7975 \angle 89.93^0\ \Omega$

$$|I| = \frac{250}{1570.7975} = 0.1592\text{A};\qquad |Z_T| = \frac{230}{0.1592} = 1444.7236\Omega;$$

Therefore $2^2 + X_T^2 = 1444.7236^2$; Thus $X_T = \pm\, 1444.7\ \Omega$

i.e. $1570.8 - X_C = \pm\, 1444.7$; Thus $X_C = 126.1\ \Omega$ or $X_C = 3015.5\ \Omega$

If $X_C = 126.1\ \Omega$ circuit is INDUCTIVE; $C = \dfrac{1}{2\pi \times 50 \times 126.1} = 25.243\ \mu F$

If $X_C = 3015.5\ \Omega$ circuit is CAPACITIVE; $C = \dfrac{1}{2\pi \times 50 \times 3015.5} = 1.056\ \mu F$

Example 17

A resistance R, an inductance L = 0.5 H and a capacitance C are connected in series. When a voltage $v = 350 \cos(3000\,t - 20^0)$ volt is applied to this series combination, the current flowing is $15 \cos(3000\,t - 60^0)$ amperes. Find R and C.

Solution

Supply voltage $V = 247.4873 \angle -20^0$ Volts

Current $I = 10.6066 \angle -60^0$ A

Inductive reactance $X_L = 3000 \times 0.5 = 1500\ \Omega$

Impedance $Z = R + j\,(X_L - X_C) = \dfrac{V}{I} = \dfrac{247.4873\angle -20}{10.6066\angle -60} = 23.3333\angle 40^0\ \Omega$

$= (17.8743 + j\ 14.9984)\ \Omega$

Thus R $= 17.8743\ \Omega$

$X_L - X_C = 14.9984$;   Thus $X_C = 1500 - 14.9984 = 1485.0016\ \Omega$

Therefore C $= \dfrac{1}{3000 \times 1485.0016} = 0.2245\ \mu F$

Example 18

Calculate the admittance Y, the conductance G and the susceptance B of a circuit consisting of 10 Ω in series with an inductor of 0.1 H when the frequency is 50 Hz.

Solution

Inductive reactance $X_L = 2\pi \times 50 \times 0.1 = 31.4159\,\Omega$

Circuit impedance $Z = (10 + j\,31.4159)\,\Omega$

Circuit admittance $Y = \frac{1}{Z} = (0.0092 - j\,0.0289) = 0.03033 \angle -72.34$ mho

Thus Conductance G = 0.0092 mho; Susceptance B = 0.0289 mho Inductive

Example 19

An impedance of (7 + j 5) Ω is connected in parallel with another circuit having impedance of (10 – j 8) Ω. The supply voltage is 230 V, 50 Hz. Calculate (i) the admittance, conductance and susceptance of the combined circuit and (ii) the total current taken from the mains and the p.f.

Solution

Impedance Z_1 = (7 + j 5) Ω; Admittance Y_1 = (0.0946 – j 0.0676) mho

Impedance Z_2 = (10 - j 8) Ω; Admittance Y_2 = (0.061 – j 0.0488) mho

Total admittance $Y_T = Y_1 + Y_2$ = (0.1556 – j 0.0188) mho = 0.1567 $\angle$ - 6.89^0 mho

Conductance G = 0.1556 mho; Susceptance B = 0.0188 mho Inductive

Taking the supply voltage as reference, V = 230 $\angle$ 0^0 volts

Total current taken I = Y V = 36.041 $\angle$ - 6.89^0 A

p.f. = 0.9928 lagging

Example 25

Consider RLC series circuit with R = 100 Ω, L = 1.0 H and C = 1.0 μF. It is connected to 500 V variable frequency supply. For a range of ω = 800 to 1200 rad. per sec. in steps of 100 rad. per sec., compute the values of X_L, X_C, $|Z|$ and $|I|$ and plot them. Mark the region of leading and lagging power factor. For ω = 1000 rad. per sec., find the values of voltages across the inductance and capacitance.

Solution

The calculated values are shown in the table.

ω rad. / sec.	X_L Ω	X_C Ω	Z Ω	$\|Z\|$ Ω	$\|I\|$ A
800	800	1250	100 – j 450	461	1.08
900	900	1111	100 – j 211	233	2.15
1000	1000	1000	100	100	5.0
1100	1100	909	100 + j191	216	2.31
1200	1200	833	100 + 367	380	1.32

R = 100 Ω

L = 1.0 H

C = 1.0 μF

ω rad. / sec.	X_L Ω	X_C Ω	Z Ω	|Z| Ω	|I| A
800	800	1250	100 – j 450	461	1.08
900	900	1111	100 – j 211	233	2.15
1000	1000	1000	100	100	5.0
1100	1100	909	100 + j191	216	2.31
1200	1200	833	100 + 367	380	1.32

R = 100 Ω

L = 1.0 H

C = 1.0 μF

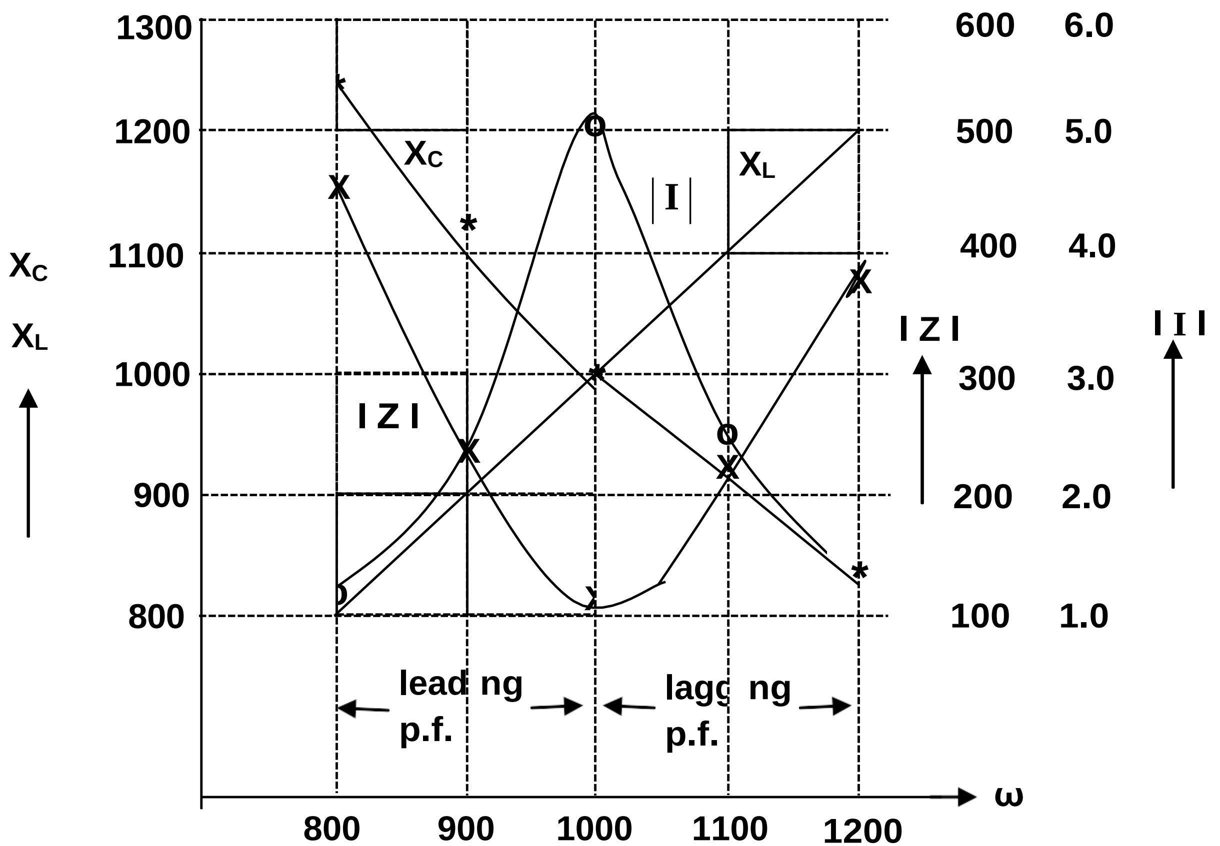


Fig. 35 Resonance charecteristics

When ω = 1000 rad. per sec., X_L = 1000 Ω and X_C = 1000 Ω. Taking the supply voltage as reference, current $I = 5 \angle 0^0$ A .

Therefore

V_L = j 1000 x 5 = j 5000 V

V_C = - j 1000 x 5 = - j 5000 V

Even though the supply voltage is 500 V, voltages across the inductance and capacitance are 5000 V.

It is to be noted that V_R = 100 x 5 = 500 V and $V_R + V_L + V_C$ = 500 V The phasor diagram is shown in Fig. 36.

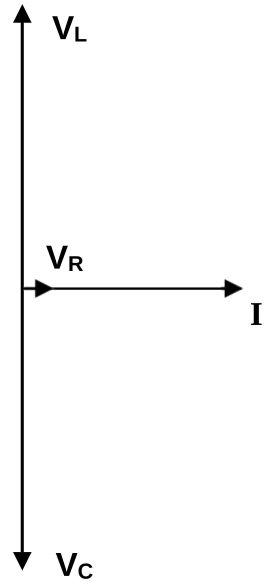
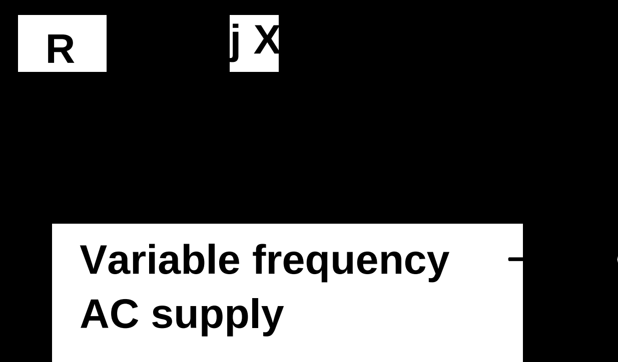


Fig. 36 Phasor diagram

An R – L – C series circuit is said to be at resonance when the applied voltage and the resulting current are in phase.

Since the supply voltage and the circuit current are in phase, the power factor of the resonant circuit is unity.



While the frequency is increased from a low value, X_L increases and X_C decreases. At one particular frequency, called resonance frequence, X_L and X_C are equal and the total impedance will be equal to R. This angular frequency is designated as ω_0. For other frequencies

$$\left|Z\right| = \sqrt{R^2 + (X_L - X_C)^2} \tag{68}$$

Thus at resonance frequency, the impedance is minimum and is equal to R and the current is at the maximum value given by $\frac{\left|E\right|}{R}$. When $\omega < \omega_0$, $X_C > X_L$ and hence p.f. is of leading nature. When $\omega > \omega_0$, $X_L > X_C$ and hence the p.f. is lagging in nature. At resonance frequency since the circuit impedance is of resistance only, the p.f is unity. Expression for angular resonance frequency ω_0 can be obtained as follows. At resonance condition, $X_L = X_C$. Therefore

$$\omega_0 L = \frac{1}{\omega_0 C}; \qquad \text{Thus } \omega_0 = \frac{1}{\sqrt{L\,C}} \text{ rad. / sec.} \tag{69}$$

$$\text{Resonance frequency } f_0 = \frac{1}{2\,\pi}\,\frac{1}{\sqrt{L\,C}} \text{ Hz} \tag{70}$$

Resonance condition can be achieved by varying the values of either L or C also.

QUALITY FACTOR

In Example 26, it was found that, at resonance condition, voltage across L or C is much larger than the supply voltage. The property of developing high voltage during resonance condition is defined by QUALITY FACTOR, also referred as Q Factor.

Q Factor, $Q = \dfrac{|V_L|}{|E|} = \dfrac{|V_C|}{|E|}$ at resonance condition. (71)

$$\text{Thus } Q = \frac{X_L\ |I|}{R\ |I|} = \frac{X_L}{R} = \frac{\omega_0 L}{R} \tag{72}$$

$$\text{Also } Q = \frac{X_C\ |I|}{R\ |I|} = \frac{X_C}{R} = \frac{1}{\omega_0 C R} \tag{73}$$

BANDWIDTH

At resonance condition, the circuit current is maximum and is given by $I_0 = \dfrac{|E|}{R}$.

The entire power input is absorbed by R and this power is given by $P_0 = I_0^2 R$. For all other frequencies around ω_0, circuit current is less than I_0 and hence the power absorbed by the circuit will be less than P_0. The power absorbed will be $\dfrac{P_0}{2}$ when the circuit current is $\dfrac{1}{\sqrt{2}} I_0$

BANDWIDTH is defined as that range of frequencies around the resonance frequency ω_0, within which the power absorbed by the circuit is greater than or equal to $\dfrac{P_0}{2}$, where P_0 is the power absorbed by the circuit at resonance condition

OR within which the circuit current is greater than or equal to $\dfrac{I_0}{\sqrt{2}}$, where I_0 is the circuit current at resonance condition.

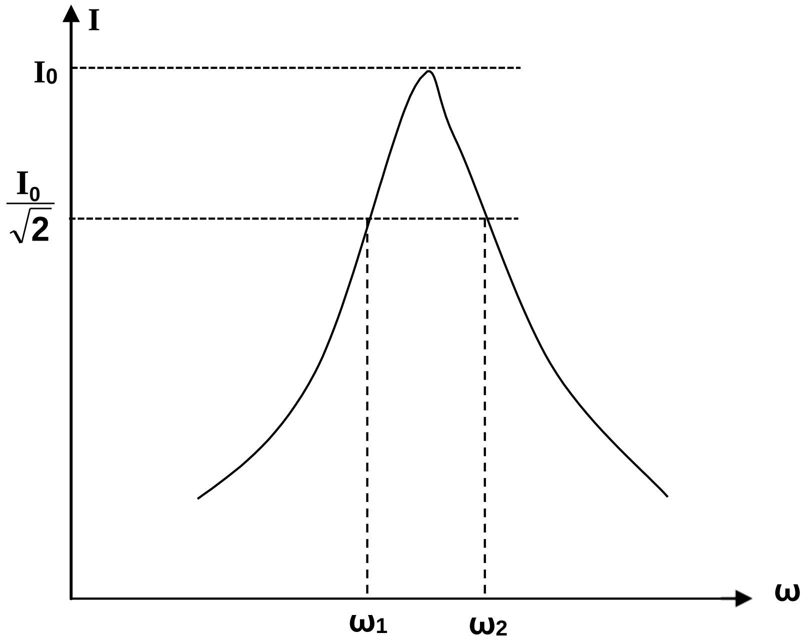


Fig. 38 Bandwidth

Referring to Fig. 38,

Bandwidth $\omega_{B\,W} = \omega_2 - \omega_1$ (74)

It can be shown that $\omega_{B\,W} = \dfrac{R}{L}$ (75)

Also from eq. (72), it can be seen that

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{\omega_{B\,W}} \tag{76}$$

Example 26

A series R-L-C circuit with R = 10 Ω, L = 10 mH and C = 1 μF is connected to 200 V variable frequency supply. Calculate the resonance frequency. Also find the circuit current and the voltages across the elements. Determine the Quality factor and Bandwidth.

Solution

$$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{0.01 \times 1} = 10^8; \quad \text{Therefore } \omega_0 = 10^4 \text{ rad. / sec.}$$

Resonant frequency $f_0 = \dfrac{\omega_0}{2\pi} = 1591.55$ **Hz.; At resonance** $X_L = X_C = \omega_0 L = 100\ \Omega$

Circuit current at resonance $I_0 = \dfrac{200}{10} = 20$ **A**

Voltage across resistor $V_R = 10 \times 20 = 200$ **V**

Voltage across L and C: $|V_L| = |V_C| = X_L I_0 = 100 \times 20 = 2000$ **V**

Quality factor $Q = \dfrac{X_L}{R} = \dfrac{100}{10} = 10$

Bandwidth $\omega_{BW} = \dfrac{R}{L} = \dfrac{10}{0.01} = 1000$ **rad. / sec. = 159.155 Hz**

Example 27

A series R-L-C circuit is connected to a 200 V, 50 Hz supply. When L is varied, the maximum current obtained is 0.4 A. At that condition, the voltage across the capacitor is 330 V. Find the circuit constants.

Solution

Resistor **R = 200 / 0.4 = 500 Ω**

$|V_L| = |V_C| = X_L I_0 = 330$; Therefore $X_L = X_C = 330 / 0.4 = 825\ \Omega$

$\omega_0 = 2\pi \times 50 = 314.1593$

Inductance $L = \dfrac{X_L}{\omega_0} = \dfrac{825}{314.1593} = 2.626\ \text{H}$

Capacitance $C = \dfrac{1}{\omega_0 X_C} = \dfrac{1}{314.1593 \times 825}\ \text{F} = 3.8583\ \mu\text{F}$

Example 20

When a 240 V, 50 Hz supply is applied to a resistor of 15 Ω which is in parallel with an inductor, total current is 22.1 A. What value must the frequency has for the total current to be 34 A?

Solution

Current in resistor and current in inductor will have a phase difference of 90^0

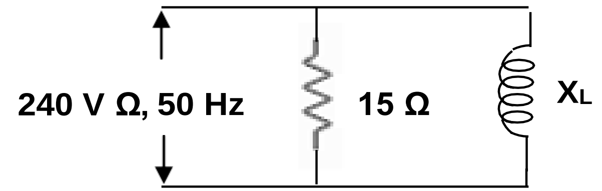
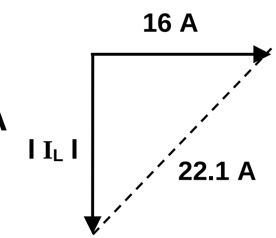


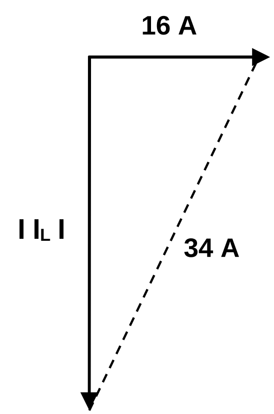
Fig. 30 – Example 20

$$\left|I_R\right| = \frac{240}{15} = 16\ \text{A}; \quad 16^2 + \left|I_L\right|^2 = 22.1^2; \quad \text{Thus } \left|I_L\right| = 15.245\ \text{A}$$



$$\text{Therefore } X_L = \frac{240}{15.245} = 15.7429\ \Omega; \quad \text{Thus } L = \frac{15.245}{2\pi \times 50} = 0.05011\text{H}$$

With new frequency



$$16^2 + \left|I_L\right|^2 = 34^2; \quad \text{Thus } \left|I_L\right| = 30\ \text{A} \quad \text{and } X_L = \frac{240}{30} = 8\ \Omega$$

New frequency $f = \dfrac{8}{2\pi \times 0.05011} = 25.4089\text{Hz}$

Example 21

A coil of resistance 15 Ω and inductance 0.05 H is connected in parallel with non-inductive resistor of 20 Ω. Find (i) the current in each branch and the total current supplied and (ii) the phase angle of the combination when a voltage of 200 V at 50 Hz is applied. Draw the phasor diagram.

Solution

$X_L = 2\pi \times 50 \times 0.05$

$= 15.708\ \Omega$

Taking supply voltage as

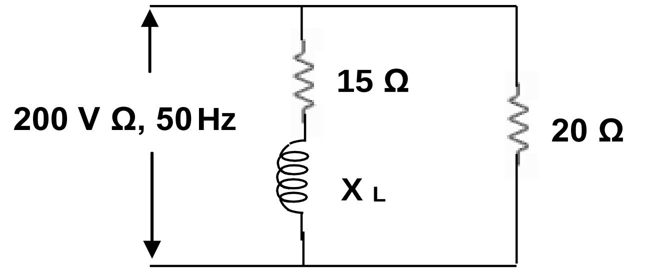


Fig. 31 – Example 21

Current in the coil $I_1 = \dfrac{200}{15 + j15.708} = 6.3594 - j\,6.6596 = 9.2083 \angle -46.32^0$ A

Current in non-inductive resistor $I_2 = 10$ A

Total current $I_T = 16.3594 - j\,6.6596 = 17.663 \angle -22.15^0$ A

Current lags the voltage by 22.15^0

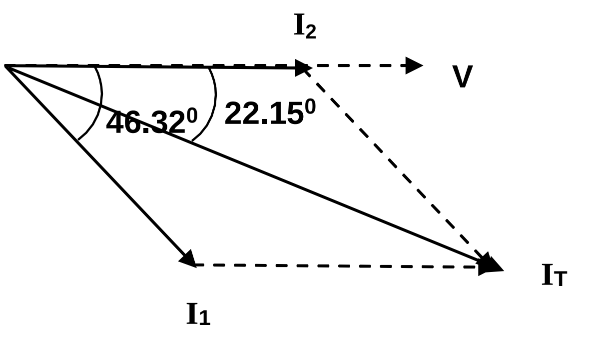


Fig. 32 – Phasor diagram - Example 22

Example 22

A coil of inductance 6 mH and resistance 40Ω is connected across a supply of 100 V, 800 Hz. Also across the supply is a circuit consisting of 4 μF in series with 50 Ω resistor. Find (i) the total current taken from the supply and (ii) the phase angle between the currents in the coil and the capacitance. Draw the phasor diagram.

Solution

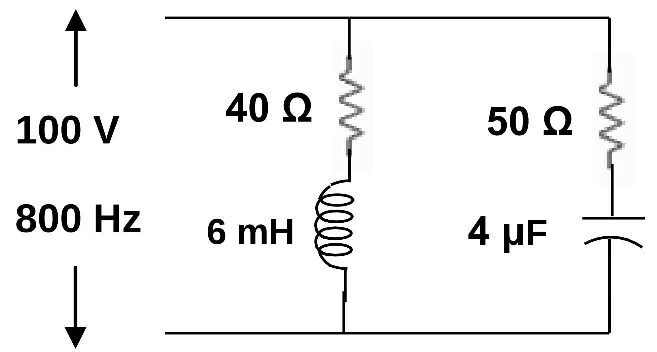


Fig. 33 – Example 22

$$X_L = 2\pi \times 800 \times 0.006$$

$$= 30.1593\ \Omega$$

$$X_C = \frac{10^6}{2\pi \times 800 \times 4} = 49.7359\Omega$$

$$Z_1 = 40 + j30.1593 = 50.0957\ \angle 37.02^0\ \Omega$$

$$Z_2 = 50 - j\,49.7359 = 70.5242\ \angle -44.85^0\ \Omega$$

$$\text{Current } I_1 = \frac{100}{40 + j30.1593} = 1.5938 - j1.2019 = 1.9962 \angle -37.02^0 \text{ A}$$

$$\text{Current } I_2 = \frac{100}{50 - j49.7359} = 1.0053 - j1.0 = 1.418 \angle 44.85^0 \text{ A}$$

<u>Total current</u> I_T = (2.5991 – j 0.2019) A = 2.6069 $\angle$ - 4.44^0 A

Angle between currents I_1 and I_2 = 81.87^0

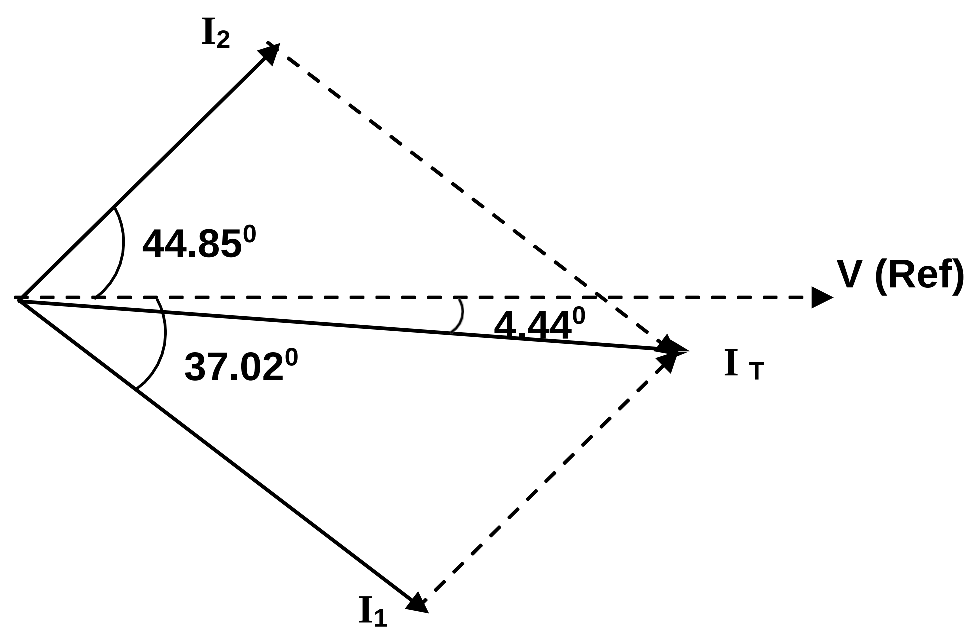


Fig. 34 – Phasor diagram - Example 22

Example 23

Two circuits, with impedances Z_1 = (10 + 15) Ω and Z_2 = (6 – j 8) Ω are connected in parallel. If the total current supplied is 15 A, find the power consumed by each impedance.

Solution

Taking the supply current as reference, $I_T = 15 \angle 0^0$ A

$$\text{Current } I_1 = \frac{6 - j8}{16 + j7} \text{ x } 15 = (1.9672 - j8.3607) = 8.589\angle -76.76^0 \text{ A}$$

$$\text{Current } I_2 = \frac{10 + j15}{16 + j7} \text{ x } 15 = (13.0328 + j8.3607) = 15.484\angle 32.68^0 \text{ A}$$

Power P_1 = $|I_1|^2 R_1 = 8.589^2 \text{ x } 10 = 737.7092$ W

Power P_2 = $|I_2|^2 R_2 = 15.484^2 \text{ x } 6 = 1438.5255$ W

Example 24

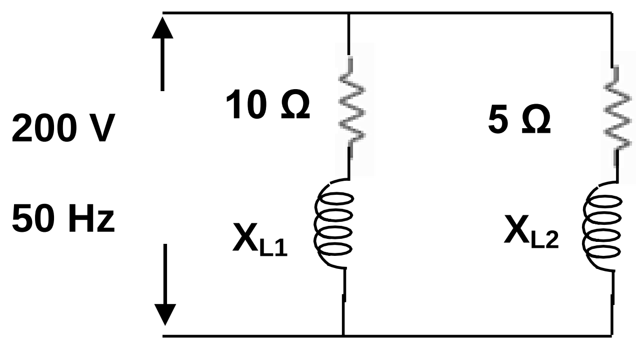
Two coils are connected in parallel across a voltage of 200 V, 50 Hz. The coils have resistances of 10 Ω and 5 Ω and inductances of 0.023 H and 0.035 H respectively. Find (i) current in each coil and total current and (ii) p.f. of the combination.

Solution

$X_{L1} = 2\,\pi \times 50 \times 0.023 = 7.2257\ \Omega$; $X_{L2} = 2\,\pi \times 50 \times 0.036 = 10.9956\ \Omega$

$Z_1 = (10 + j\,7.2257)\ \Omega = 12.3374\ \angle 35.85^0\ \Omega$

$Z_2 = (5 + j\,10.9956)\ \Omega = 12.079\ \angle 65.55^0\ \Omega$



Current I_1 $= V / Z_1 = 16.2109\ \angle -35.85^0$ A

Current I_2 $= V / Z_2 = 16.5577\ \angle -65.85^0$ A

Total current I_T $= 31.6743\ \angle -50.86^0$ A

p.f. of the combination = 0.6312 lagging