SRM Institute of Science and Technology
Department of Mathematics
21MAB203T-Probability and Stochastic Processes
2023-2024 Even
Unit III: Probability Bounds and Central Limit Theorems
Tutorial Sheet - 3

S.No.	Questions	Answers
	Part – B	
1.	A random variable has the pdf $f_X(x)= e^{-3x}$ for x ≥0. Obtain an upper bound of P(X ≥1) using Markov's inequality.	$\frac{1}{3}$
2.	A fair dice is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.	0.75
3.	A random variable X is exponentially distributed with parameter 1. Use Tchebycheff's inequality to find a lower bound of P(-1≤ X ≤ 3)	$\frac{3}{4}$
4.	If the number of items produced in a factory during a week is a random variable with mean 100 and variance 400. Use one-sided Tchebycheff's inequality to compute an upper bound on the probability that this week's production will be at least 120.	$\frac{1}{2}$
5	If $X_1, X_2, \ldots, X_n$ are Poisson variates with parameter λ=2, use the central limit theorem to estimate P(120 ≤ S_n ≤ 160), where $S_n = X_1+ X_2 +\ldots..+X_n$, and n=75.	0.7868
6	Let X be a random variable with mean 10. Use Jensen's inequality, to find a lower bound of E$\{ \frac{1}{X+1} \}$.	$\frac{1}{11}$
	Part – C	
7.	Let Z be a standard normal variate with the MGF $M_Z(t) = e^{\frac{t^2}{2}}$. Using Chernoff bounds, find an upper bound of P(Z ≥ 2). Compare the upper bound with the one-sided Tchebycheff's inequality.	0.1353 (using Chernoff bounds) 0.2 (using Tchebycheff's one-sided inequality)
8	Let X be a Poisson variate with the parameter λ=2, having the MGF $M_X(t) = e^{2(e^t-1)}$. Using Chernoff bounds, find an upper bound of P(X ≥ 3). Compare this upper bound with the Markov's inequality.	0.8054 (using Chernoff bounds) 0.6667 (using Markov's inequality)
9	Use Cauchy-Schwartz inequality to prove that for any two random variables X and Y, $\lvert\rho(X, Y)\rvert \leq 1$.	
10.	The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using Central Limit Theorem find the approximate probability that 100 of these electric bulbs will provide a total more than 6000 hours of burning time	0.0228