PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dydx$ by change of order of integration.

(OR)

b. Find the volume of the sphere $x^2+y^2+z^2=a^2$ by triple integral.

29. a. Show that the vector $\vec{F}=\left(6xy+z^3\right)\vec{i}+\left(3x^2-z\right)\vec{j}+\left(3xz^2-y\right)\vec{k}$ is irrotational find the scalar potential.

(OR)

b. Verify Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ where 'S' is the surface of the cuboid formed by the planes $x=0, x=a, y=0, y=b, z=0$ *and* $z=c$.

30. a. Solve using Laplace transform method $\left(D^2+4\right)y=\cos 2t, y(0)=3$ and $y'(0)=4$.

(OR)

b.i. Using convolution theorem find $L^{-1}\left(\frac{1}{s^2(s+1)^2}\right)$. (8 Marks)

ii. Verify the final value theorem for the function $1+e^t(\sin t+\cos t)$. (4 Marks)

31. a. Find the bilinear transformation which maps the points $z=0$, $z=1$ and $z=\infty$ into the point $w=i$, $w=1$ and $w=-1$.

(OR)

b. Find the analytic function $w=u+iv$ where $u=e^x(x\sin y+y\cos y)$ and hence find the harmonic conjugate v.

32. a. Find the Laurent series expansion for $f(z)=\frac{1}{(z+1)(z+2)}$ over (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

(OR)

b. Evaluate using Contour integration $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. $\int_1^b \int_1^a \frac{dxdy}{xy}$ is equal to
(A) $\log a+\log b$ (B) $\log b-\log a$
(C) $\log a\times\log b$ (D) $\log(a-b)$

2. The change of order of integration of $\int_0^1 \int_0^x dydx$ is
(A) $\int_0^1 \int_y^1 dxdy$ (B) $\int_0^1 \int_0^y dydx$
(C) $\int_0^1 \int_1^y dxdy$ (D) $\int_0^1 \int_0^x dxdy$

3. The name of the curve $r=a(1+\cos\theta)$ is
(A) Ellipse (B) Cardoid
(C) Lemniscate (D) Hemisphere

4. Evaluate $\int_0^1 \int_0^1 dxdy$
(A) 4 (B) 2
(C) 0 (D) 1

5. The work done by the conservative force when it moves a particle around a closed curve is
(A) $\nabla\times\vec{F}=\vec{0}$ (B) $\nabla.\vec{F}=0$
(C) 0 (D) $\nabla.(\nabla\times\vec{F})=0$

6. The connection between a line integral and the double integral is known as
(A) Stoke's theorem (B) Green's theorem
(C) Divergence theorem (D) Convolution theorem

7. The value of $\nabla\phi$ for $\phi = xy + yz + zx$ at the point (–1, 1, 1) is
   (A) $2\vec{i}$ (B) $3\vec{i}$
   (C) $4\vec{j}$ (D) $3\vec{k}$

8. $Curl(grad\,\phi)$ is
   (A) ϕ (B) 1
   (C) –1 (D) 0

9. Laplace transform of $\sin 3t$ is
   (A) $\frac{3}{s^2-9}$ (B) $\frac{3}{s^2+9}$
   (C) $\frac{s}{s^2+9}$ (D) $\frac{s}{s^2-9}$

10. If $L(f(t)) = F(s)$ then $L(f'(t))$
    (A) $sL(f(t)) - f(0)$ (B) $L(f(t)) - f(0)$
    (C) $sL(f(t)) + f(0)$ (D) $L(f(t)) + f(0)$

11. The inverse laplace transform of $\frac{1}{s^2-a^2}$ is
    (A) *sinhat* (B) *sinat*
    (C) $\frac{sinat}{a}$ (D) $\frac{sinhat}{a}$

12. The inverse laplace transform of $\frac{2}{s-b}$ is
    (A) $2t\,e^{-bt}$ (B) $2e^{-bt}$
    (C) $2e^{bt}$ (D) $2te^{bt}$

13. The function $f(z) = u + iv$ is analytic if
    (A) $u_x = v_y$ *and* $u_y = -v_x$ (B) $u_x + v_y = 0$ *and* $u_y - v_x = 0$
    (C) $u_x + v_y = 0$ *and* $u_y + v_x = 0$ (D) $u_y = v_x$ *and* $u_x = v_y$

14. If a function $u(x,y)$ satisfies $u_{xx} + u_{yy} = 0$ then u is
    (A) Differentiable (B) Continuous
    (C) Harmonic (D) Analytic

15. The invariant points of the transformation $w = \frac{z-1}{z+1}$ are
    (A) -1 *and* i (B) i *and* 1
    (C) 1 *and* -1 (D) i *and* $-i$

16. Any analytic function with constant modulus is
    (A) Zero (B) Constants
    (C) Harmonic (D) Analytic

17. A curve which does not cross itself is called a
    (A) Not a curve (B) Closed curve
    (C) Multiple curve (D) Simple curve

18. If $f(z) = \frac{\sin z}{z}$ then
    (A) z = 0 is a removable singularity (B) z = 0 is a simple pole
    (C) z = 0 is an essential singularity (D) z = 0 is a zero of f(z)

19. The value of $\oint_c \frac{5z^2+8z+5}{z+1}dz$ where $c: |z| = 1/2$
    (A) $-2\pi i$ (B) $2\pi i$
    (C) 0 (D) πi

20. The value of $\int_c \frac{z}{z-2}dz$ where c is the circle $|z| = 1$ is
    (A) $i\pi/2$ (B) 0
    (C) 2 (D) $\pi/2$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^{\pi} \int_0^{a\sin\theta} r\,dr d\theta$.

22. Find a unit normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2).

23. Find Laplace transform of $e^t \sin^2 t$.

24. Show that the function $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic

25. Find the poles and its residues for $f(z) = \frac{z^2}{(z-1)^2(z+2)}$.

26. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$.

27. Find the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$.