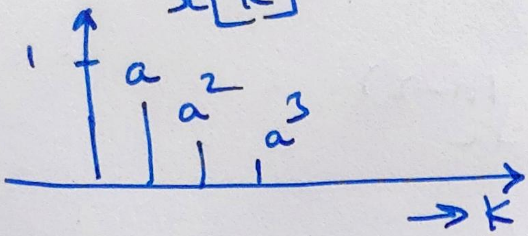
$x[n] = a^n \cdot u[n]$; $h[n] = u[n]$.

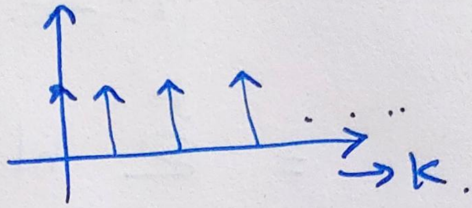
with $0 < a < 1$.

<u>Solution</u> :-

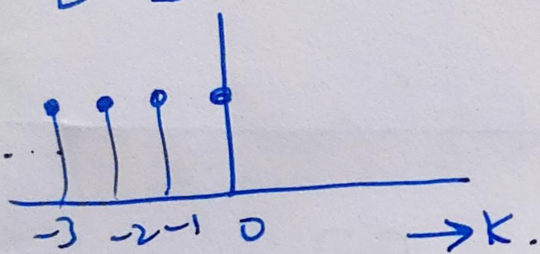
$x[k]$



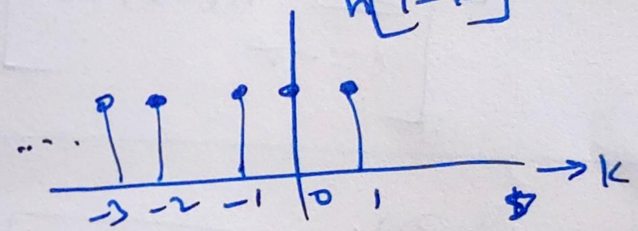
$h[k]$



$h[-k]$



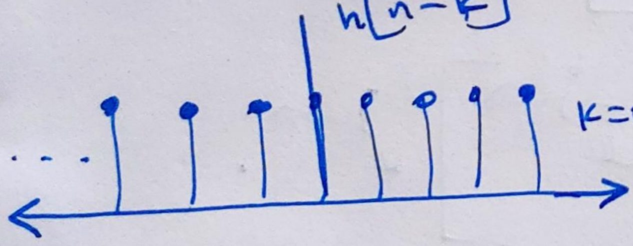
$h[1-k]$



$$y[n] = \sum_{k=0}^{n} a^k (1).$$

$k = 0$ to 1

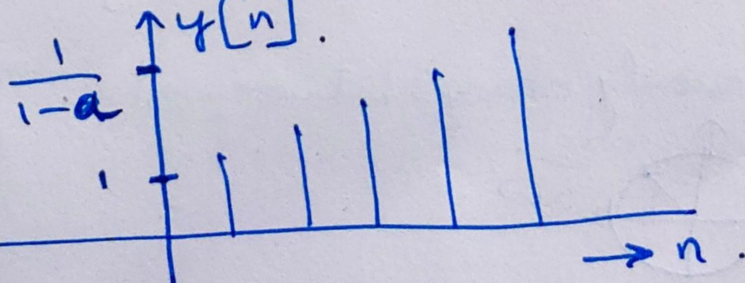
$h[n-k]$



k=0 to n

w.k.t. $\sum_{n=0}^{N} a^n = \frac{1-a^{n+1}}{1-a}$ & $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$.

$y[n] \Rightarrow$



$$a^n \longrightarrow \boxed{h[n]} \longrightarrow y[n].$$

$$y[n] = \sum_{r=-\infty}^{\infty} h(r) \cdot x(n-r).$$

$$= \sum_{r=-\infty}^{\infty} h(r) \cdot a^{(n-r)}$$

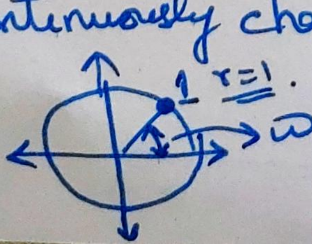
$$= \left[\sum_{r=-\infty}^{\infty} h(r) \cdot a^{-r}\right] a^n.$$

$$e^{+j\omega n.} \longrightarrow \boxed{h[n]} \longrightarrow y[n].$$

$$y[n] = \left[\sum_{r=-\infty}^{\infty} h(r) \cdot e^{-j\omega r}\right] e^{j\omega n}.$$

ω is Continuous. {Frequency domain is Continuous}.

ω Continuously changes between 0 to 2π.



$$H(e^{j\omega}) = \sum_{r=-\infty}^{\infty} h(r) \cdot e^{-j\omega r}.$$

$\rightarrow$ DTFT.

(or).

In general.,

DTFT $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}.$$

Inverse DTFT. $$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j\omega n} \, d\omega$$

$$\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} \, d\omega = \int_{-\pi}^{\pi} \sum_{r=-\infty}^{+\infty} h(r) e^{-j\omega r} \cdot e^{j\omega n} \, d\omega.$$

$$\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} \, d\omega = \int_{-\pi}^{\pi} \sum_{r=-\infty}^{\infty} h(r) \cdot e^{+j\omega(n-r)} \, d\omega.$$

(i) If $r=n$ $\Rightarrow \int_{-\pi}^{\pi} h(r) \cdot d\omega \Rightarrow h(n) \int_{-\pi}^{\pi} d\omega.$

(ii) If $r \neq n$.

$$\Rightarrow \sum_{\substack{r=-\infty \\ r \neq n}}^{\infty} h(r) \cdot \int_{-\pi}^{\pi} e^{j\omega(n-r)} \, d\omega$$

$$\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega = h(r)\int_{-\pi}^{\pi} d\omega + \sum_{\substack{r=-\infty \\ r\neq n}}^{\infty} h(r) \int_{-\pi}^{\pi} e^{j\omega(n-r)} d\omega$$

Note: $\int_{-\pi}^{\pi} e^{j\omega k} \Rightarrow \int_{-\pi}^{\pi} \cos\omega k + j\sin\omega k$

$\Rightarrow \int$ Full cycle $\Rightarrow 0.$

$$\Rightarrow \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega = h(n) \int_{-\pi}^{\pi} d\omega .$$

$$\Rightarrow \boxed{h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega}$$

IDTFT.