27. Evaluate $\oint_C \frac{\left(3z^2+z\right)}{z^2-1}dz$ where C is circle $|z-1|=1$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b. Find the volume of a sphere $x^2+y^2+z^2=a^2$ by using triple integrals.

29. a. Verify Gauss-Divergence theorem for $\bar{F}=x^2\hat{i}+z\hat{j}+yz\hat{k}$ over a cube formed by $0\le x\le 1,\ 0\le y\le 1, 0\le z\le 1$.

(OR)

b. Verify Green's theorem in a plane for $\iint\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy$ where 'c' is the boundary of the region defined by $x=y^2$ and $y=x^2$.

30. a. Find the Laplace transform of periodic function

$$f(t) = \begin{cases} t, & 0<t<1 \\ 2-t, & 1<t<2 \end{cases}$$ where $f(t+2)=f(t)$ for all 't' where t is positive.

(OR)

b. Solve using Laplace transform method $y''-3y'-4y=2e^{-t}$, $y(0)=1, y'(0)=1$.

31. a. Find the Analytic function $f(z)=u+iv$, if $u+v=(x-y)\left(x^2+4xy+y^2\right)$.

(OR)

b. Find the bilinear transform which maps the points $z_1=1,\ z_2=i,\ z_3=-1$ into the points $w_1=i,\ w_2=0,\ w_3=-i$.

32. a. Expand $f(z)=\frac{1}{(z-1)(z-2)}$ as a Laurent's series valid in (i) $|z|<1$ (ii) $1<|z|<2$ and (iii) $|z|>2$.

(OR)

b. Evaluate using contour integration $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$.

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Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019
First & Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The value of the integral $\int_0^2 \int_0^1 xy\,dx\,dy$ is
(A) 1 (B) 2
(C) 3 (D) 4

2. The name of the curve $r=a(1+\cos\theta)$ is
(A) Lemniscate (B) Cycloid
(C) Cardiod (D) Hemicircle

3. Area of the double integral in Cartesian co-ordinates is
(A) $\iint dydx$ (B) $\iint rdrd\theta$
(C) $\iint xdxdy$ (D) $\iint x^2dxdy$

4. $\int_0^1\int_0^2\int_0^3 dxdydz$ is equal to
(A) 3 (B) 2
(C) 4 (D) 6

5. The relation between line integral and a surface integral is known as
(A) Green's theorem (B) Residue theorem
(C) Stokes theorem (D) Divergence theorem

6. The condition for $\bar{F}$ to be conservative is, $\bar{F}$ should be
(A) Solenoidal vector (B) Rotational vector
(C) Irrotational vector (D) Both solenoidal and irrotational

7. The unit normal vector of $\phi=xy+yz+zx$ at the point (–1, 1, 1) is
(A) $2i$ (B) i
(C) $3i$ (D) $4i$

8. Value of $\nabla\left(r^n\right)$ is

(A) $n\bar{r}$ (B) $n\bar{r}\,r^n$

(C) $n(n-1)\bar{r}$ (D) $nr^{n-2}\bar{r}$

9. An example of a function for which the laplace transform does not exist is

(A) $g(t)=t^2$ (B) $g(t)=\sin t$

(C) $g(t)=\tan t$ (D) $g(t)=e^{-at}$

10. If $L[f(t)]=F(s)$, then $L\left(e^{-at}f(t)\right)$ is

(A) $F(s-a)$ (B) $F(s+a)$

(C) $F(s)$ (D) $\frac{1}{a}F\left(\frac{s}{a}\right)$

11. $L\left(t^4\right)$

(A) $\frac{4!}{s^5}$ (B) $\frac{3!}{s^4}$

(C) $\frac{4!}{s^4}$ (D) $\frac{5!}{s^4}$

12. $L(\cos 2t)$ is

(A) $\frac{s}{s+2}$ (B) $\frac{s}{s^2+4}$

(C) $\frac{s}{s^2-4}$ (D) $\frac{2}{s^2+4}$

13. The Cauchy-Riemann equation in polar co-ordinates are

(A) $ru_r=v_\theta, u_\theta=-rv_r$ (B) $-ru_r=v_\theta,\ u_\theta=+rv_r$

(C) $u_r=rv_\theta,\ v_r=ru_\theta$ (D) $u_r=-rv_\theta,\ v_r=ru_\theta$

14. The critical point of transformation $w=z^2$ is

(A) $z=2$ (B) $z=0$

(C) $z=1$ (D) $z=-2$

15. The fixed points of the transformation $w=-\frac{(2iz-4)}{z-i}$ are

(A) $-4i,\ i$ (B) $4i,\ -i$

(C) $i,\ 2i$ (D) $-i,\ 2i$

16. If $u+iv$ is analytic, then the curves $u=c_1$ *and* $v=c_2$

(A) Cut orthogonally (B) Are parallel

(C) Coincide (D) Intersect each other

17. The annular region for the function $f(z)=\frac{1}{z(z-1))}$ is

(A) $0<|z|<1$ (B) $1<|z|<2$

(C) $1<|z|<0$ (D) $2<|z|<1$

18. If $f(z)=\frac{1}{(z-1)(z-3)^3}$ then

(A) 3 is a pole of order 3, 1 is a pole of order 2 (B) 1 is a simple pole, 3 is a pole of order 3

(C) 3 is a simple pole, 1 is a pole of order 2 (D) 1 is a pole of order 3 and 3 is a pole of order 1

19. The value of $\oint_c \frac{z}{z-2}dz$ when c is a circle $|z|=1$ is

(A) 0 (B) 2

(C) $\pi/2$ (D) π

20. If $f(z)$ is analytic inside and on C, the value of $\oint_C \frac{f(z)}{z-a}dz$, where 'C' is a simple closed curve and 'a' is any point within 'C' is

(A) $f'(a)$ (B) $2\pi i\, f(a)$

(C) $\pi i\, f(a)$ (D) 0

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$.

22. Find the angle between normals to the surface $x^2=yz$ at the points (1, 1, 1) and (2, 4, 1).

23. A fluid motion is given by $\bar{v}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$. Is this motion irrotational? If so, find the scalar potential.

24. Find $L\left[\frac{\cos 6t-\cos 4t}{t}\right]$.

25. Verify initial value theorem for $f(t)=1+e^{-t}(\sin t+\cos t)$.

26. Find the constant a, b c if $f(z)=(x+ay)+i(bx+cy)$ is analytic.