* Del Operator:

The gradient has the formal appearance of a vector ∇, "multiplying" a scalar T:

$$\nabla T = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right) T$$

The term in parantheses is called del:

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Del is not a vector, in the usual sense, Indeed, it doesn't mean much until we provide it with a function to act upon. To be precise, then we say that ∇ is a vector operator that acts upon T, not a vector that multiplies T.

With this qualification, though, ∇ mimics the behaviour of an ordinary vector in virtually every way; almost anything that can be done with other vectors can also be done with ∇, if we merely translate "multiply" by "act upon". So by all means take the vector appearance of ∇ seriously: it is a marvelous piece of notational simplification.

There are three ways the operator ∇ can act :

1. On a scalar function T : ∇T (the gradient)
2. On a vector function v, via the dot product : $\nabla \cdot v$ (the divergence)
3. On a vector function v, via the cross product : $\nabla \times v$ (the curl)

* The Divergence :

From the definition of ∇ we construct divergence

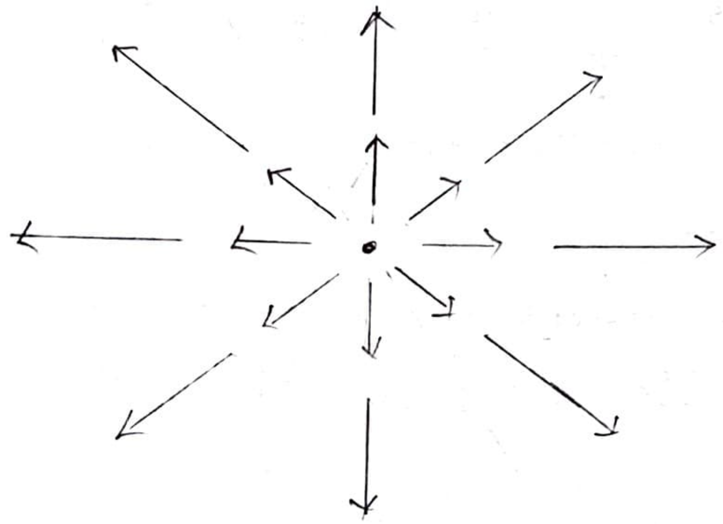
$$\nabla \cdot v = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(v_x \hat{x} + v_y \hat{y} + v_z \hat{z}\right)$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Geometrical Interpretation : The name divergence is well chosen, for $\nabla \cdot v$ is a measure of how much the vector v spreads out (diverges) from the point. If the material spreads out then you dropped it at a point of positive divergence; if it collects together, you it at a point of negative divergence

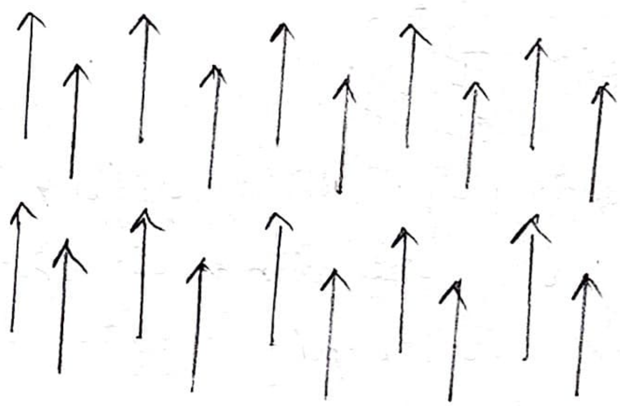
For example:

a)



The vector functions in this figure has a positive divergence

b)



The function in this figure has zero divergence

* The curl:

From the definition of ∇ we construct the curl:

$$\nabla \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$

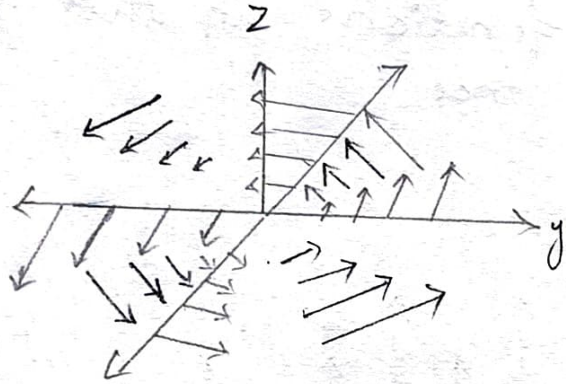
$$= \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Geometrical Interpretation :

The name curl is also well chosen for $\nabla \times v$ is a measure of how much the vector v swirls around the point.

Example :

a)



The functions in this figure have a substantial curl, pointing in the z direction, as the natural right hand rule would suggest

* The Gradient :

A derivative is supposed to tell us how fast the function varies, if we move a little distance

Geometrical Interpretation of the gradient:

Like any vector, the gradient has magnitude and direction.

The dot product,

$$dT = \nabla T \cdot d\mathbf{l}$$

$$= |\nabla T|\,|d\mathbf{l}| \cos\theta .$$

where θ is the angle between ∇T and $d\mathbf{l}$.

Now, if we fix the magnitude $|d\mathbf{l}|$ and search around in various directions, the maximum change in T evidentally occurs when $\theta = 0$. That is, for a fixed distance $|d\mathbf{l}|$, dT is greatest when it moves in the same direction as ∇T.

The gradient ∇T points in the direction of maximum increase of the function T.

The magnitude $|\nabla T|$ gives the slope (rate of increase) along this maximal direction.

If we want to locate the extrema of a function of three variables, set its gradient equal to zero.