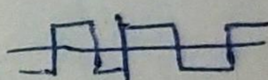
FOURIER TRANSFORM.

From Fourier series

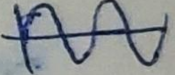
w.k.t.

Analysis eqn ; $a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$.

Synthesis eqn, $x(t) = \sum_{k=-\infty}^{+\infty} a_k\, e^{jk\omega_0 t}$.

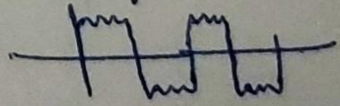


Analyzing function:- Correlating with sine and



Cosine waveforms and to analyze that the function as sine Component (or) Cosine Component.

Synthesis function:- After analyzing whether the function has sine (or) Cosine Component, the original function has to be generated by utilizing all the harmonics.



Fourier Transform :-

Continuous time Fourier Transform :-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

w.k.t.

$$\boxed{a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

w.k.t

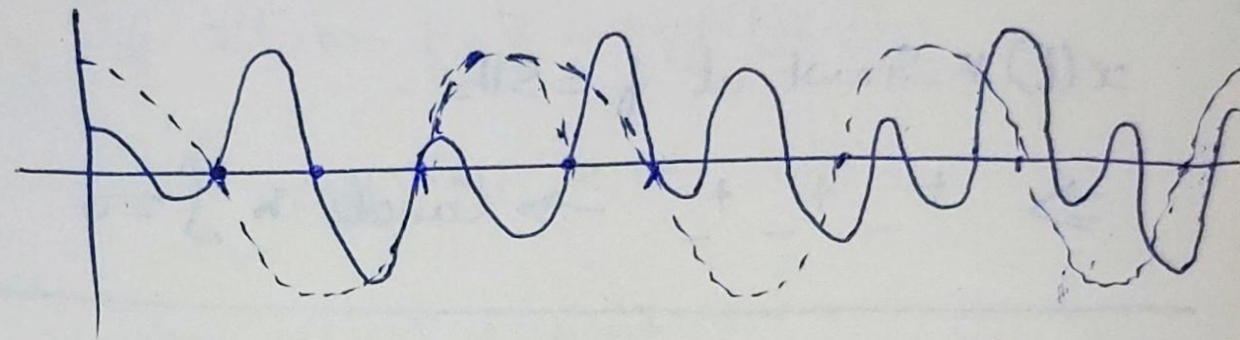
$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t}}$$

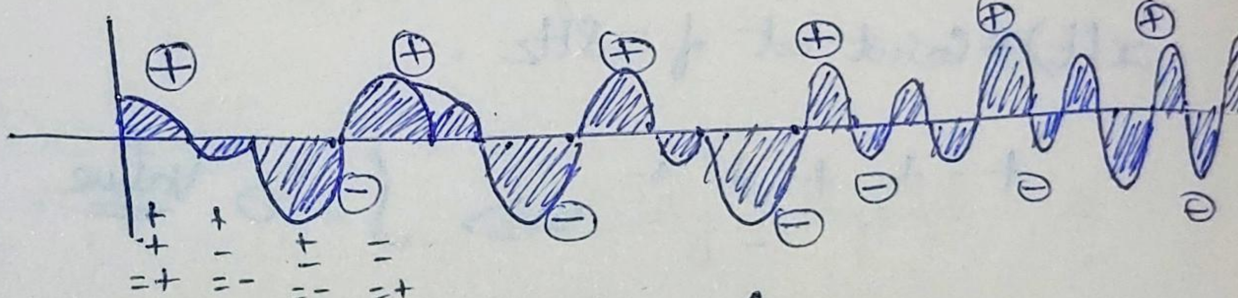
w.k.t

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cos\omega t\, dt + \int_{-\infty}^{\infty} x(t)\, j\sin\omega t\, dt.$$

Example:- $0.6 \sin(3\,Hz) + 0.8 \cos(8\,Hz)$

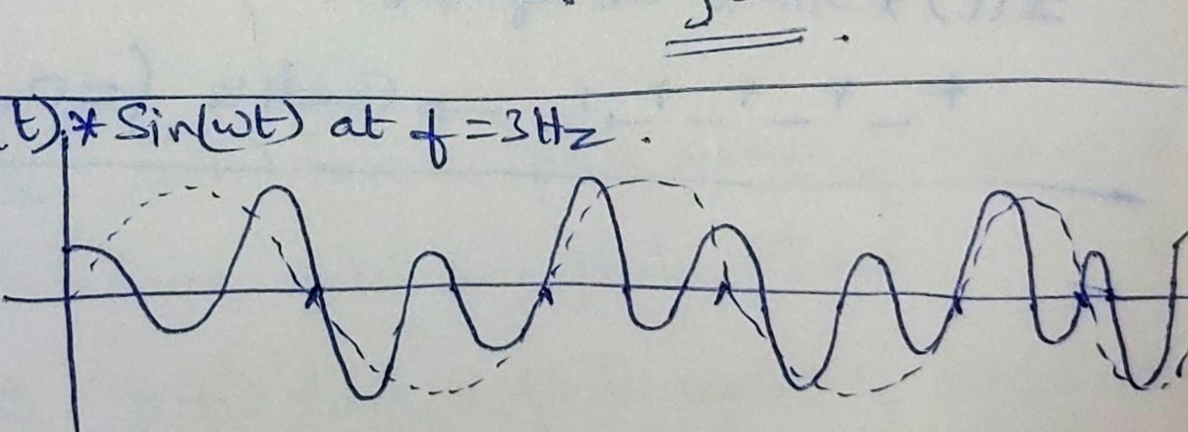
$x(t) * \cos(\omega t)$ at $f = 3Hz$.

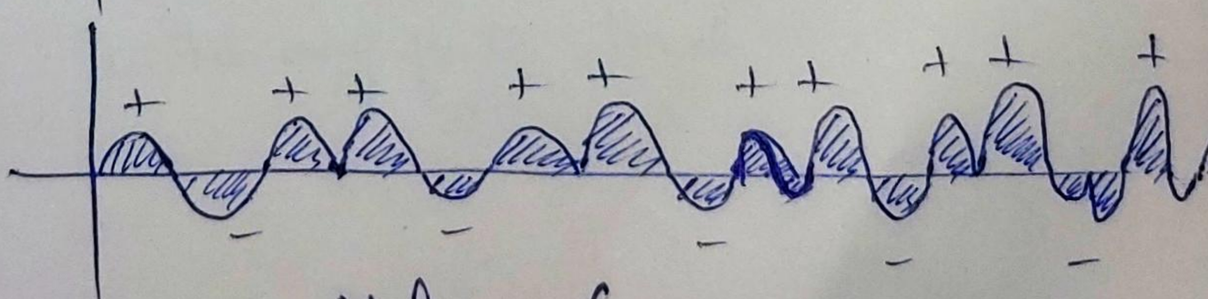




Cancels $\int = 0$.

$x(t) * \sin(\omega t)$ at $f = 3Hz$.





Value. $\int > 0$.

$x(t) * \cos\omega t$ at $f = 5\,Hz$.

$\Rightarrow + - + - + - \cdots \rightarrow$ Cancels & $\int = 0$

$x(t) * \sin\omega t$ at $f = 5\,Hz$.

$\Rightarrow + - + - + - \rightarrow$ Cancels & $\int = 0$

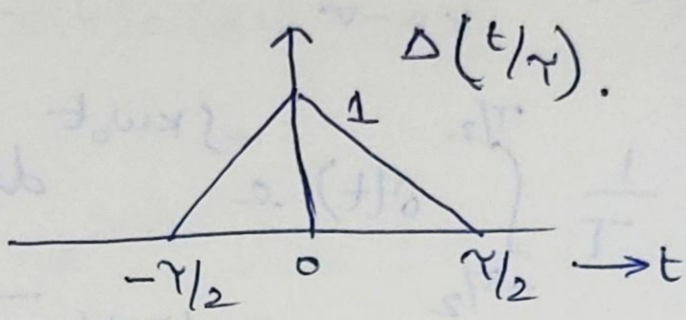
$x(t) * \cos\omega t$ at $f = 8\,Hz$.

$+ + - + + - + \Rightarrow \int > 0$ value.

$x(t) * \sin\omega t$ at $f = 8\,Hz$.

$+ - + - + - + - + - \rightarrow$ Cancels & $\int = 0$.

(4). Determine the Fourier transform of the triangular function shown below, (2b)



$(x_1, y_1) \to (-\tau/2, 0)$

$(x_2, y_2) \to (0, 1)$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x+\tau/2} = \frac{1-0}{0+\tau/2}$$

$$\frac{y}{x+\tau/2} = \frac{2}{\tau}.$$

$$y = \frac{2}{\tau}(t+\tau/2)$$

$$y = \frac{2}{\tau}\left[\frac{2t+\tau}{2}\right]$$

$$= \frac{2t+\tau}{\tau}.$$

$$\boxed{y = 1+\frac{2t}{\tau}}$$

$(x_1, y_1) \to (0, 1)$

$(x_2, y_2) \to (\tau/2, 0)$.

$$\frac{y-1}{x-0} = \frac{0-1}{\tau/2-0}.$$

$$\frac{y-1}{x} = \frac{-2}{\tau}.$$

$$(y-1) = \frac{-2t}{\tau}$$

$$y = \frac{-2t}{\tau}+1.$$

$$\boxed{y = 1-\frac{2t}{\tau}.}$$

$$x(t) = \Delta(t/\tau) = \begin{cases} \left(1+\frac{2t}{\tau}\right) & \text{for } -\frac{\tau}{2} < t < 0. \\ 1-\frac{2t}{\tau} & \text{for } 0 < t < \tau/2. \\ 0 & \text{elsewhere.} \end{cases}$$

$$X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} \Delta(t/\tau) \cdot e^{-j\omega t} \, dt.$$

$$= \int_{-\tau/2}^{0} \left(1+\frac{2t}{\tau}\right) e^{-j\omega t} \, dt + \int_{0}^{\tau/2} \left(1-\frac{2t}{\tau}\right) e^{-j\omega t} \, dt.$$

$$= \int_{0}^{\tau/2} \left(1-\frac{2t}{\tau}\right) e^{j\omega t} \, dt + \int_{0}^{\tau/2} \left(1-\frac{2t}{\tau}\right) e^{-j\omega t} \, dt.$$

$$= \int_{0}^{\tau/2} e^{j\omega t} dt - \int_{0}^{\tau/2} \frac{2t}{\tau} e^{j\omega t} dt + \int_{0}^{\tau/2} e^{-j\omega t} dt - \int_{0}^{\tau/2} \frac{2t}{\tau} e^{-j\omega t} dt$$

$$\int_{0}^{\tau/2} \left[e^{j\omega t} + e^{-j\omega t}\right] dt - \frac{2}{\tau} \int_{0}^{\tau/2} t \left[e^{j\omega t} + e^{-j\omega t}\right] dt.$$

$$= \int_{0}^{\tau/2} 2 \cos \omega t \, dt - \frac{2}{\tau} \int_{0}^{\tau/2} t . (2 \cos \omega t) \, dt.$$

$$= \int_0^{\tau/2} 2\cos\omega t\,dt - \frac{2}{\tau}\int_0^{\tau/2} 2t\cos\omega t\,dt.$$

$$= 2\left[\frac{\sin\omega t}{\omega}\right]_0^{\tau/2} - \frac{4}{\tau}\left[\left[t\left(\frac{\sin\omega t}{\omega}\right)\right]_0^{\tau/2} - \int_0^{\tau/2}\frac{\sin\omega t}{\omega}(1)\,dt\right]$$

$$= 2\left[\frac{\sin\omega t}{\omega}\right]_0^{\tau/2} - \frac{4}{\tau}\left[\left[t\left(\frac{\sin\omega t}{\omega}\right)\right]_0^{\tau/2} - \left(-\frac{\cos\omega t}{\omega^2}\right)_0^{\tau/2}\right]$$

$$= \frac{2}{\omega}\left[\sin\omega\tau/2 - \sin(0)\right] - \frac{4}{\tau}\left[\left(\frac{\tau/2\,\sin\omega\tau/2 - 0}{\omega}\right) + \left(\frac{\cos\omega\tau/2 - 1}{\omega^2}\right)\right]$$

$$= \frac{2}{\omega}\left[\cancel{\sin\omega\tau/2}\right] - \frac{4}{\omega\tau}\left(\cancel{\frac{\tau}{2}\sin\frac{\omega\tau}{2}}\right) - \frac{4}{\omega^2\tau}\left(\cos\frac{\omega\tau}{2} - 1\right)$$

$$= \frac{4}{\omega^2\tau}\left(1 - \cos\frac{\omega\tau}{2}\right)$$

$$= \frac{4}{\omega^2\tau}\left[2\sin^2\left(\frac{\omega\tau}{4}\right)\right]$$

180

$\sin^2\theta = 1-\cos 2\theta$

$\sin^2\theta/2 = 1-\cos\theta$

$\sin^2(\theta/4) = 1-\cos(\theta/2)$

$0.5 = 1$

$$= \frac{8}{\omega^2 \tau} \cdot \sin^2\left(\frac{\omega\tau}{4}\right)$$

$$= \frac{8}{\omega^2\tau}\left(\frac{\omega\tau}{4}\right)^2 \cdot \frac{\sin^2\left(\frac{\omega\tau}{4}\right)}{(\omega\tau/4)^2}.$$

$$= \frac{\tau}{2} \cdot \frac{\sin^2\left(\frac{\omega\tau}{4}\right)}{(\omega\tau/4)^2}.$$

$$= \frac{\tau}{2} \cdot \operatorname{Sinc}^2\left(\frac{\omega\tau}{4}\right).$$

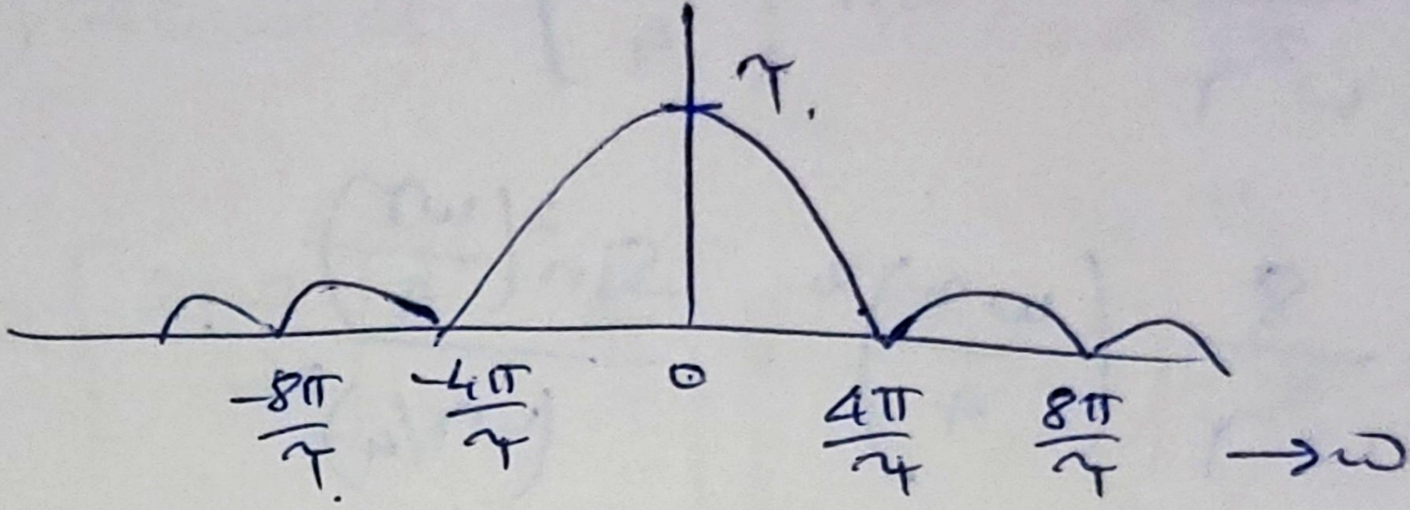
$$\Rightarrow F\left[\Delta(t/\tau)\right] = \frac{\tau}{2} \operatorname{Sinc}^2\left(\frac{\omega\tau}{4}\right).$$

$$\Rightarrow \boxed{\Delta(t/\tau) \longleftrightarrow \frac{\tau}{2} \operatorname{Sinc}^2\left(\frac{\omega\tau}{4}\right)}$$

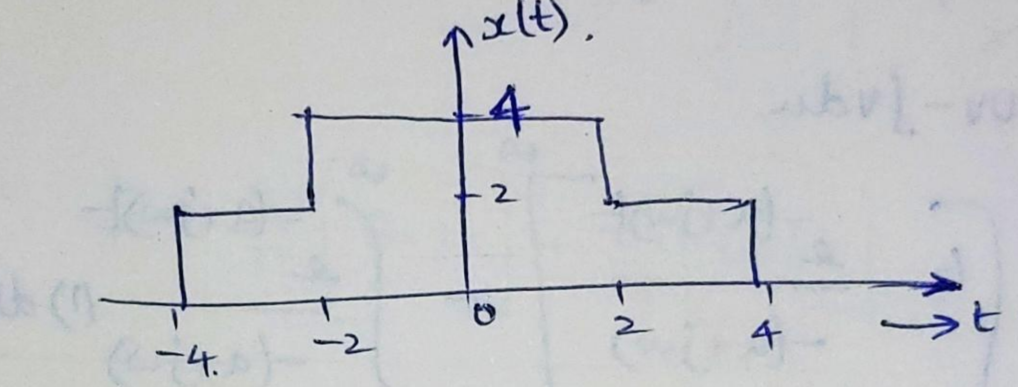
W.K.T: $F\left[\Delta(t/\tau)\right] = \frac{\tau}{2} \dfrac{\sin^2\left(\frac{\omega\tau}{4}\right)}{(\omega\tau/4)^2}.$

$$\frac{\omega\tau}{4} = \pi k.$$

$$\boxed{\omega = \frac{4\pi}{\tau} k}$$


τ.
-8π/τ
-4π/τ
0
4π/τ
8π/τ
→ ω

① . Determine the Fourier Transform of the following signal,



Solution :-

$$x(t) = \begin{cases} 2 & \text{for } -4 < t < -2 \\ 4 & \text{for } -2 < t < 2 \\ 2 & \text{for } 2 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \int_{-4}^{-2} 2e^{-j\omega t}\,dt + \int_{-2}^{2} 4e^{-j\omega t}\,dt + \int_{2}^{4} 2e^{-j\omega t}\,dt$$

$$= \left[2\frac{e^{-j\omega t}}{-j\omega}\right]_{-4}^{-2} + \left[4\frac{e^{-j\omega t}}{-j\omega}\right]_{-2}^{2} + \left[\frac{2e^{-j\omega t}}{-j\omega}\right]_{2}^{4}$$

$$= \frac{-2}{j\omega}\left[\cancel{e^{j2\omega}} - e^{j4\omega} + 2e^{-j2\omega} - \cancel{2e^{j2\omega}} + e^{-j4\omega} - \cancel{e^{-j2\omega}}\right]$$

$$= \frac{-2}{j\omega}\left[e^{j2\omega} - 2e^{j2\omega} + 2e^{-j2\omega} - e^{-j2\omega} - e^{j4\omega} + e^{-j4\omega}\right]$$

$$= \frac{-2}{j\omega}\left[-e^{j2\omega} + e^{-j2\omega} + e^{-j4\omega} - e^{j4\omega}\right]$$

$$= \frac{+2}{j\omega}\left(\frac{2}{2}\right)\left[e^{j2\omega} - e^{-j2\omega} - e^{-j4\omega} + e^{j4\omega}\right]$$

$$= \frac{4}{j\omega}\left[\left(\frac{e^{j2\omega} - e^{-j2\omega}}{2}\right) + \left(\frac{e^{j4\omega} - e^{-j4\omega}}{2}\right)\right]$$

$$= \frac{4}{\omega}\left[\sin 2\omega + \sin 4\omega\right].$$

$$= \frac{4}{\omega}\left(\frac{2\omega}{2\omega}\right)\sin(2\omega) + \frac{4}{\omega}\left(\frac{4\omega}{4\omega}\right)\sin 4\omega$$

$$= 8\,\frac{\sin 2\omega}{2\omega} + \frac{16 \sin 4\omega}{4\omega} \Rightarrow \boxed{8\ \text{Sinc}\,2\omega + 16\,\text{Sinc}\,4\omega}$$