Design of FIR filters Using Fourier Series Method.

Disad :- Direct Truncation
Jump discontinuity
→ Gibbs Oscillations

The desired frequency response of FIR filter Can be represented by the Fourier Series,

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n}.$$

↳ Impulse sequence of the given filter. (with Infinite duration)

Design steps :-

(i) Find the desired impulse sequence of the filter $h_d(n)$.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\cdot e^{j\omega n}\cdot d\omega.$$

$$-\infty \le n \le +\infty.$$

(ii) Truncate $h_d(n)$ at $n = \pm\left(\frac{N-1}{2}\right)$ to get finite duration sequence, $h(n)$.

$$\Rightarrow h(n) = \begin{cases} h_d(n) & ; \ |n| \le \frac{N-1}{2} \\ 0 & ; \ \text{otherwise}. \end{cases}$$

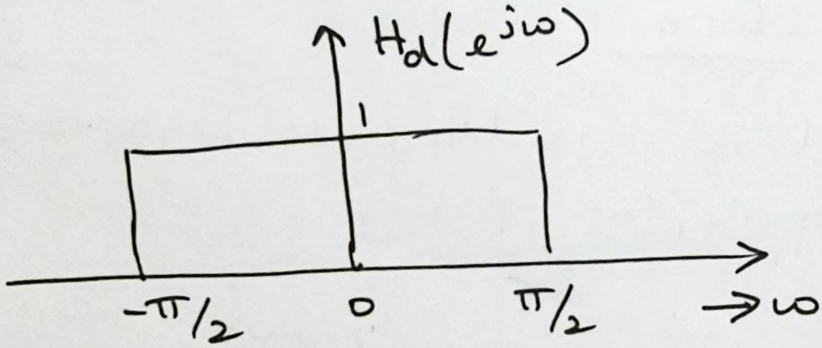
(iii) $H(z) = z^{-(N-1)/2}\left[h(0) + \sum_{n=1}^{(N-1)/2} h(n)\cdot\left[z^{n} + z^{-n}\right]\right].$

1) Design a FIR LPF with cut off frequency 1 KHz & sampling frequency of 4 KHz with 11 samples using Fourier series method. Determine frequency response & Verify the design.

Solution :-

Given $F_c = 1\,KHz$. ; $F_s = 4\,KHz$.

$$\omega_c = 2\pi \frac{F_c}{F_s} = \frac{2\pi(1\times10^3)}{4\times10^3} = 0.5\pi \ \frac{\text{rad}}{\text{sample}}.$$



(i) Desired Impulse response of the filter

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} (1)\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2} = \frac{1}{\pi n}\left[\frac{e^{j\pi/2\, n} - e^{-j\pi/2\, n}}{2j}\right]$$

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$$\boxed{h_d(n) = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right);} \quad -\infty \le n \le \infty.$$

(ii) Truncate $h_d(n)$ at $n = \pm\left(\frac{N-1}{2}\right) \Rightarrow \underline{\underline{5}}$.

$$\Rightarrow h(n) = \begin{cases} \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) & ; \ |n| \le 5 \\ 0 & ; \ \text{otherwise.} \end{cases}$$

when $n=0$; $h(0) \Rightarrow h_d(0) = \lim_{n \to 0} \frac{\sin\frac{n\pi}{2}}{n\pi}$ //

<u>L-Hospital Rule:-</u>

$$\frac{\frac{\pi}{2}\cdot\cos\frac{n\pi}{2}}{\pi} \Rightarrow \frac{\pi/2}{\pi} = 0.5 \text{ //}$$

<u>Aliter:-</u> $h(0) \Rightarrow h_d(0) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{0}\, d\omega$

$$= \frac{1}{2\pi}\left[\omega\right]_{-\pi/2}^{\pi/2} = 0.5 \text{ //}.$$

For $n=1$

$$h(1) = h(-1) = \frac{\sin\pi/2}{\pi} = 0.3183$$

For $n=2$

$$h(2) = h(-2) = \frac{\sin(2\pi/2)}{2\pi} = 0.$$

For $n=3 \Rightarrow h(3) = h(-3) = -\frac{1}{3\pi} = -0.106$.

For $n=4 \Rightarrow h(4) = h(-4) = 0$

For $n=5 \Rightarrow h(5) = h(-5) = \frac{\sin(5\pi/2)}{5\pi} = \frac{1}{5\pi} = 0.06366$.

(iii). Transfer Function, $H(z)$.

$$H(z) = z^{-\frac{(N-1)}{2}} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) \left[z^{n} + z^{-n} \right] \right]$$

$$\Rightarrow z^{-\left(\frac{11-1}{2}\right)} \left[\frac{1}{2} + \sum_{n=1}^{5} h(n) \left(z^{n} + z^{-n} \right) \right]$$

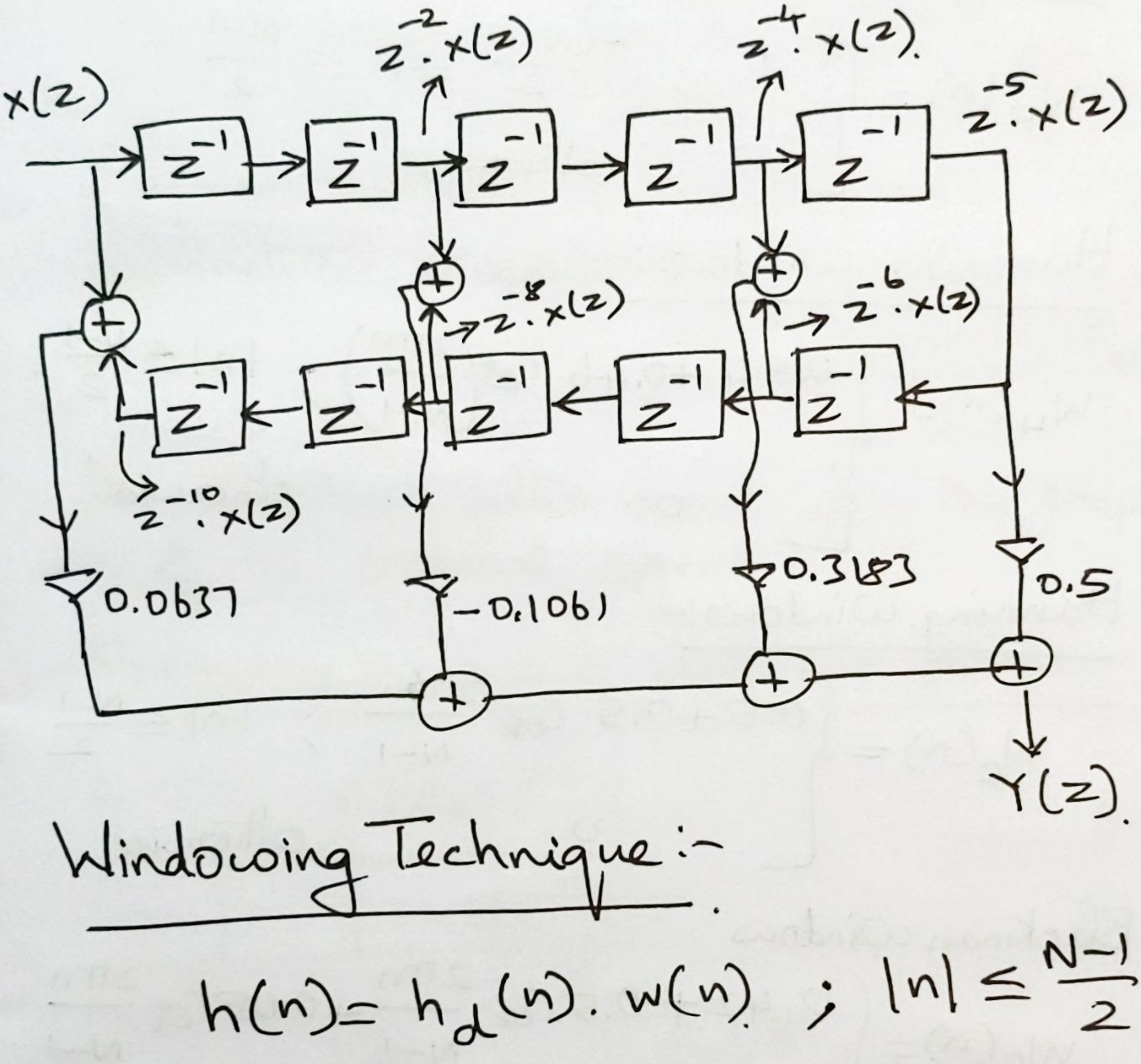
$$= z^{-5} \left\{ 0.5 + h(1)\left[z^{1} + z^{-1} \right] + h(2)\left[z^{2} + z^{-2} \right] + h(3)\left[z^{3} + z^{-3} \right] + h(4)\left[z^{4} + z^{-4} \right] + h(5)\left[z^{5} + z^{-5} \right] \right\}.$$

$$= h(0).z^{-5} + h(1)\left[z^{-4} + z^{-6} \right] + h(2)\left[z^{-3} + z^{-7} \right] + h(3)\left[z^{-2} + z^{-8} \right] + h(4)\left[z^{-1} + z^{-9} \right] + h(5)\left[1 + z^{-10} \right]$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.5\, z^{-5} + 0.3183\left[z^{-4} + z^{-6} \right] + 0 - 0.1061\left[z^{-2} + z^{-8} \right] + 0.0637\left[1 + z^{-10} \right]$$

$$Y(z) = 0.5.z^{-5}.X(z) + 0.3183\left[z^{-4} + z^{-6} \right].X(z) - 0.1061\left[z^{-2} + z^{-8} \right]X(z) + 0.0637\left[1 + z^{-10} \right].X(z).$$

Realization of FIR filter structure.



Windowing Technique:-

$$h(n) = h_d(n) \cdot w(n) \quad ; \quad |n| \leq \frac{N-1}{2}$$

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Rectangular Window:-

$$W_R(n) = \begin{cases} 1 \; ; & -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0 \; ; & \text{otherwise.} \end{cases}$$

Hamming Window:-

$$W_H(n) = \begin{cases} 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \; ; & |n| \le \frac{N-1}{2}. \\ 0 & \text{otherwise.} \end{cases}$$

Hanning Window:-

$$W_C(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \; ; & |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Blackman Window

$$W_B(n) = \begin{cases} 0.42 + 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{2\pi n}{N-1} & \text{for } |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Bartlet (or) Triangular Window:-

$$W_T(n) = \begin{cases} 1 - \frac{2|n|}{N-1} & \text{for } |n| \le \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Linear Phase Characteristics:-

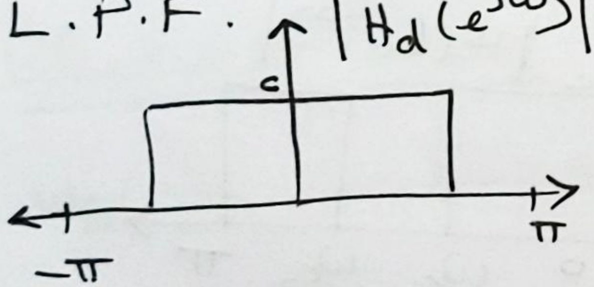
The necessary & sufficient condition for linear phase characteristics in FIR filter is Impulse response $h(n)$ of the system, with symmetrical property, $\boxed{h(n) = h(N-1-n)}$

$N \rightarrow$ Duration of the sequence

Linear phase filter donot alter the shape of the original signal.

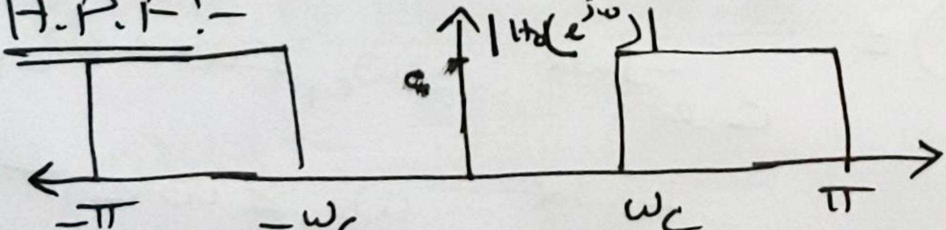
Frequency response of different filters:-

L.P.F.



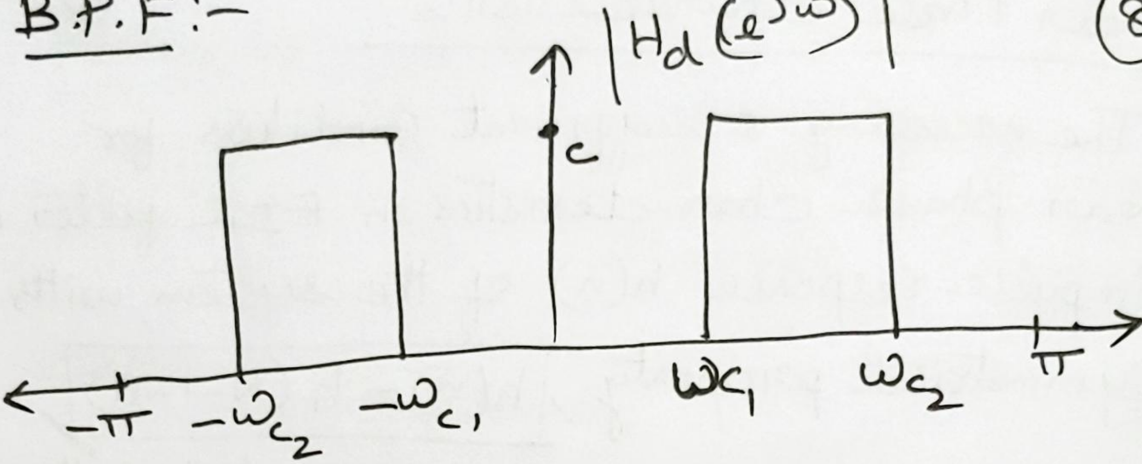
$$H_d(e^{j\omega}) = \begin{cases} c \cdot e^{-j\omega\alpha} & ; -\omega_c < \omega < \omega_c \\ 0 & \text{otherwise.} \end{cases}$$

H.P.F:-



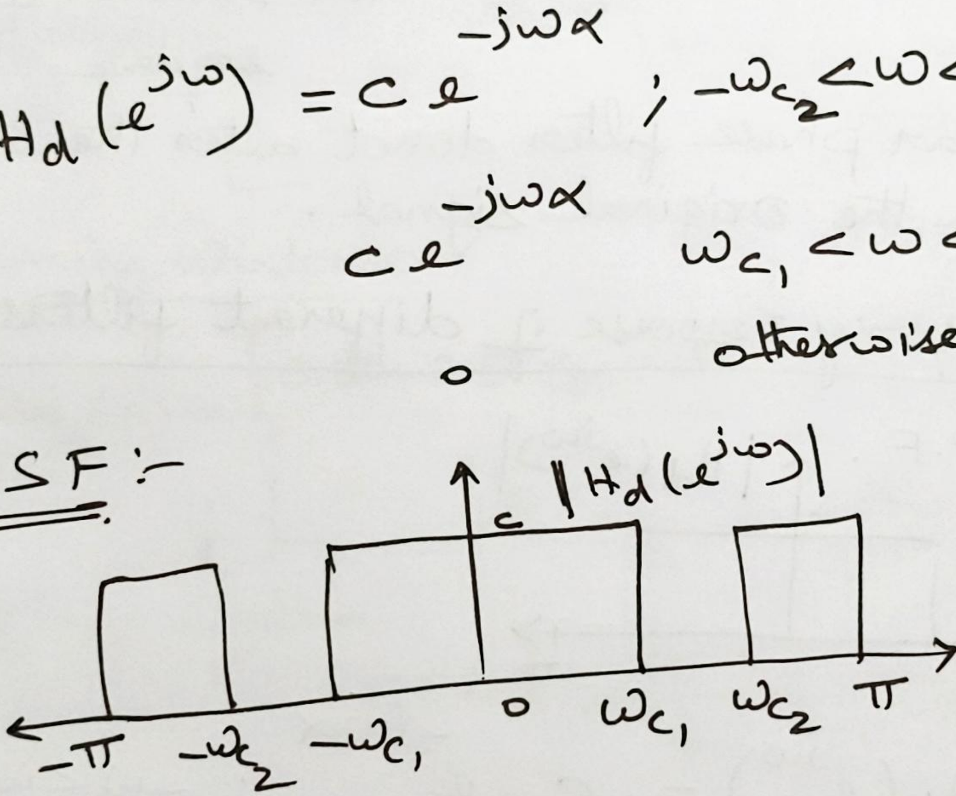
$$H_d(e^{j\omega}) = \begin{cases} c \cdot e^{-j\omega\alpha} & ; \text{for } -\pi < \omega < -\omega_c \\ c\, e^{-j\omega\alpha} & ; \text{for } \omega_c < \omega < \pi \\ 0 & \text{otherwise} \end{cases}$$

B.P.F :-



$$H_d(e^{j\omega}) = \begin{cases} c\,e^{-j\omega\alpha} & ; -\omega_{c_2} < \omega < -\omega_{c_1} \\ c\,e^{-j\omega\alpha} & \omega_{c_1} < \omega < \omega_{c_2} \\ 0 & \text{otherwise.} \end{cases}$$

BSF :-



$$H_d(e^{j\omega}) = \begin{cases} c\cdot e^{-j\omega\alpha} & -\pi < \omega < -\omega_{c_2} \\ c\,e^{-j\omega\alpha} & -\omega_{c_1} < \omega < \omega_{c_1} \\ c\,e^{-j\omega\alpha} & \omega_{c_2} < \omega < \pi \end{cases}$$

Rectangular Window

$$W_R(n) = \begin{cases} 1 & ; \text{ for } n = 0 \text{ to } N-1 \\ 0 & \text{otherwise.} \end{cases}$$

Hamming Window :-

$$W_H(n) = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & ; \text{ for } n = 0 \text{ to } N-1 \\ 0 & \text{otherwise} \end{cases}$$

Hanning Window :-

$$W_c(n) = \begin{cases} 0.5 - 0.5 \cdot \cos\left(\frac{2\pi n}{N-1}\right) & ; \text{ for } n = 0 \text{ to } N-1 \\ 0 & \text{otherwise} \end{cases}$$

Blackmann Window :-

$$W_B(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos \frac{4\pi n}{N-1} \quad \text{for } n = 0 \text{ to } N-1$$

$$0 \quad \text{otherwise.}$$

Bartlet (or) Triangular Window :-

$$W_T(n) = \begin{cases} 1 - \dfrac{2\left|n - \frac{N-1}{2}\right|}{N-1} & ; \text{ for } n = 0 \text{ to } N-1 \\ 0 & \text{otherwise.} \end{cases}$$