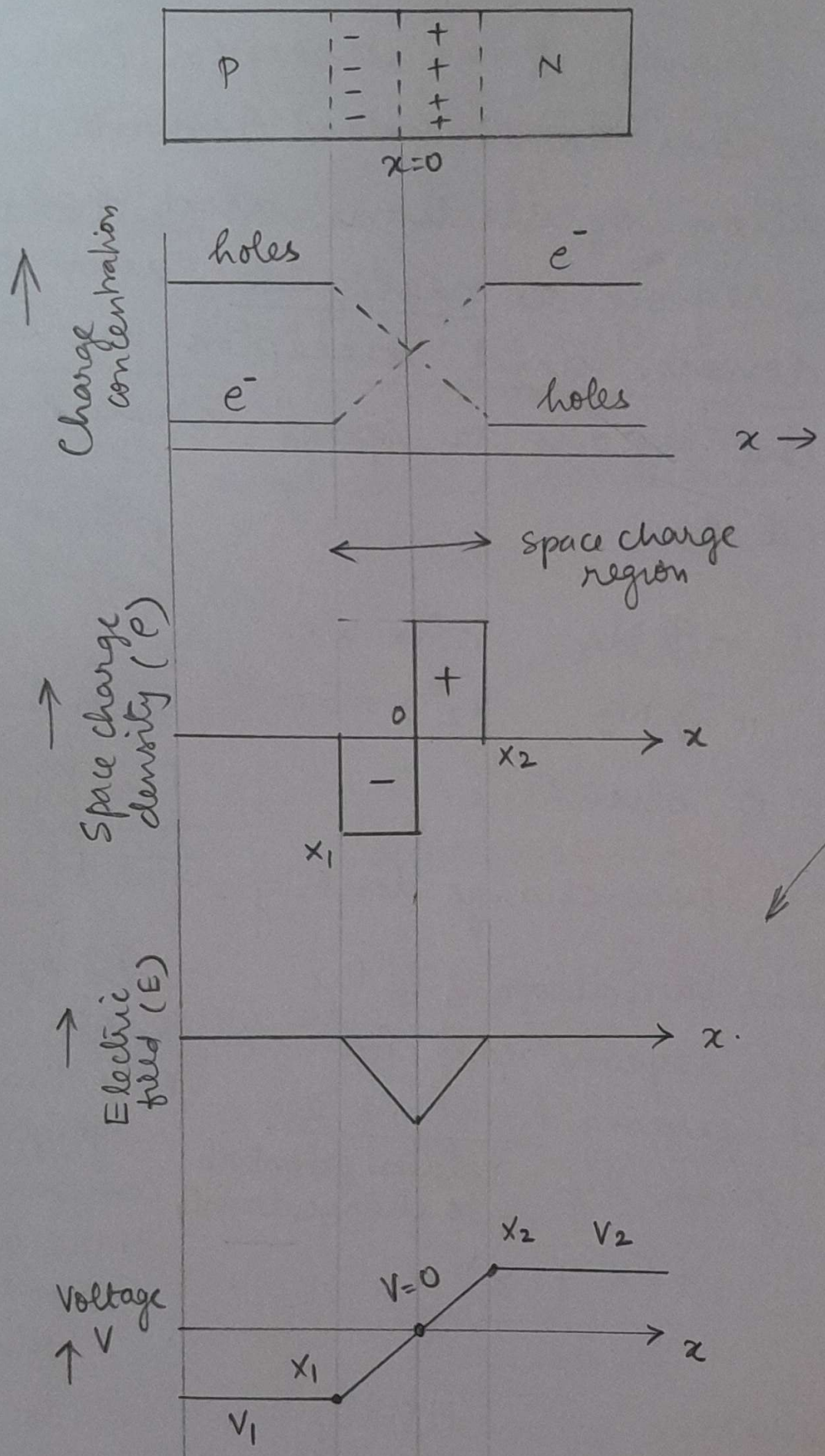


# Formation of PN Junction:-



$$E = \int \frac{\rho}{\epsilon} dx$$
 (PTD)

$$E = \frac{-\rho}{\epsilon}$$

Derivation for width of the Barrier (Depletion layer)



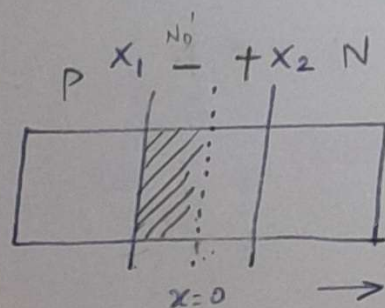
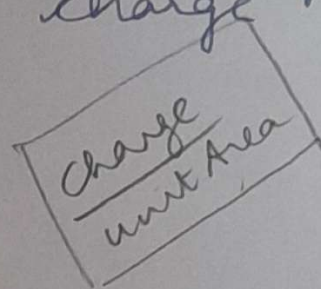
In this analysis, let us consider an alloy junction in which there is an abrupt change from acceptor ions on p-side to donor ions on N-side. Assume that the concentration of electrons and holes in the depletion region is negligible and that all of the donors and acceptors are ionized. Hence, the regions of space charge may be described as

$$\rho = -q N_A, \quad 0 > x > x_1$$

$$\rho = +q N_D, \quad x_2 > x > 0$$

$$\rho = 0 \text{ elsewhere.}$$

where  $\rho$  = space charge density.



Let  $\rho = N_D'$  for region  $x=0 < x < x_1$ . But that is negligible when compared to the total impurity concentration  $N_D$ .

$$N_D' \ll N_D$$

$$\text{So, } N_D' \approx N_D$$

$$\begin{aligned} \nabla^2 V &= -\rho/\epsilon \\ E &= -\nabla V \end{aligned}$$

$$E = -\int \frac{\rho}{\epsilon}$$

$$\int E \cdot dx = V$$

The potential variation in the space charge region can be calculated using Poisson's eqn, which is given by relates potential to charge density

$$\nabla^2 V = \frac{-\rho(x, y, z)}{\epsilon_0 \epsilon_r}$$

$\epsilon_r$  = relative permittivity

The relevant equation for the required

$$\frac{d^2 V}{dx^2} = \frac{-\rho}{\epsilon_0 \epsilon_r}$$



Applying the above equation to the P-side of the junction, we get (2)

$$\frac{d^2V}{dx^2} = \frac{qN_A}{\epsilon_0 \epsilon_r} \quad \Bigg/ \quad \frac{-qN_D}{\epsilon_0 \epsilon_r} \quad \therefore P = -qN_A$$

Integrating twice,

$$V = \frac{qN_A x^2}{2\epsilon_0 \epsilon_r} + Cx + D$$

$$E = -\left(\frac{qN_A x}{\epsilon_0 \epsilon_r}\right)$$

$$E = -\left(\frac{dV}{dx}\right)$$

where C and D are constants of integration

From the fig.:

$V = 0$  at  $x = 0$  and hence  $D = 0$  ✓

when  $x < x_1$  on the P-side, the potential is constant.

$$\left[ \therefore 0 = \frac{qN_A(0)}{2\epsilon_0 \epsilon_r} + C(0) + D \right]$$

$$\Rightarrow D = 0$$

and so,

$$\frac{dV}{dx} = 0 \text{ at } x = x_1$$

Hence,  $0 = \frac{2qN_A x_1^2}{2\epsilon_0 \epsilon_r} + Cx_1$

$$\left[ \therefore \frac{dV}{dx} = \frac{2qN_A x}{2\epsilon_0 \epsilon_r} \right]$$

$$\therefore C = -\frac{qN_A x_1}{\epsilon_0 \epsilon_r}$$

$$\therefore V = \frac{qN_A x^2}{2\epsilon_0 \epsilon_r} - \frac{qN_A x_1}{\epsilon_0 \epsilon_r} \cdot x$$

$$x_2^3 - x_1^3 = \frac{x_2^2 - x_1^2}{2}$$

$$= \frac{x_2^2 - x_1^2}{2}$$

$$x = x_2 \quad \Bigg| \quad V_2 = -\frac{qN_D x_2^2}{2\epsilon_0 \epsilon_r} + \frac{qN_D x_2}{\epsilon_0 \epsilon_r} \cdot x_2$$



As  $v = V_1$  at  $x = X_1$ , we have

$$V_1 = + \frac{q N_A}{\epsilon_0 \epsilon_r} \left[ \frac{x^2}{2} - X_1 x \right] \Big|_{\text{at } x = X_1}$$

$$= + \frac{q N_A}{\epsilon_0 \epsilon_r} \left[ \frac{X_1^2}{2} - X_1^2 \right]$$

$$= + \frac{q N_A}{\epsilon_0 \epsilon_r} \left[ - \frac{X_1^2}{2} \right]$$

$$V = - \frac{q N_D}{2 \epsilon_0 \epsilon_r} x^2 + Cx + D$$

$$\therefore V_1 = - \frac{q N_A X_1^2}{2 \epsilon_0 \epsilon_r}$$

$$\frac{dv}{dx} \Big|_{x=X_2} = 0$$

$$C = - \frac{q N_D X_2}{\epsilon_0 \epsilon_r}$$

Same procedure when applied to N-side,

$$V_2 = + \frac{q N_D X_2^2}{2 \epsilon_0 \epsilon_r}$$

$$q = N_A$$

$$x_1 A \Rightarrow \boxed{x_1 A N_A}$$

$\therefore$  The Total built in potential or contact potential is  $V_0$ , where

$$V_0 = V_2 - V_1 = \frac{q}{2 \epsilon_0 \epsilon_r} \left[ N_A X_1^2 + N_D X_2^2 \right]$$

The +ve charge on the N-side must be equal in magnitude to the negative charge on the P-side for the neutral specimen.

Hence,

$$\boxed{N_A X_1 = - N_D X_2}$$

Charge neutrality /  $\rightarrow$  all charge in a volume adds to zero.

$$N_A X_1 A = - N_D X_2 A$$



$$\frac{N_A q}{\epsilon_0 \epsilon_r}$$

above an negative

$$V = - \frac{q N_D X_2^2}{2 \epsilon_0 \epsilon_r}$$



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a substituting this relation in the above equation,

and using the fact that  $X_1$  is a negative quantity, we get

$$X_1 = - \left[ \frac{2 \epsilon_0 \epsilon_r V_0}{q N_A \left(1 + \frac{N_A}{N_D}\right)} \right]^{1/2}$$

$$\frac{q N_A N_D (N_D + N_A)}{N_D}$$

Similarly,

$$X_2 = \left[ \frac{2 \epsilon_0 \epsilon_r V_0}{q N_D \left(1 + \frac{N_D}{N_A}\right)} \right]^{1/2}$$

The total depletion width  $W = X_2 - X_1$  and hence,

$$W^2 = X_1^2 + X_2^2 - 2 X_1 X_2, \text{ and then}$$

(we take this for easy simplification)

substituting for  $X_1$  and  $X_2$  from the above equations, we find.

$$W = \left[ \frac{2 \epsilon_0 \epsilon_r V_0}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) \right]^{1/2} \checkmark$$

Inference:

The depletion width 'W' is proportional to  $(V_0)^{1/2}$ .



$$X_2 = -\frac{N_A}{N_D} \cdot X_1$$

$$\therefore V_0 = \frac{q}{2\epsilon_0\epsilon_r} \left[ N_A X_1^2 + N_D \frac{N_A^2}{N_D^2} \cdot X_1^2 \right]$$

$$\frac{2V_0\epsilon_0\epsilon_r}{q} = N_A X_1^2 + \frac{N_A^2}{N_D} X_1^2$$

$$\frac{2V_0\epsilon_0\epsilon_r}{q} = X_1^2 \left[ N_A + \frac{N_A^2}{N_D} \right]$$

$$X_1 = \left[ \frac{2V_0\epsilon_0\epsilon_r}{q N_A \left[ 1 + \frac{N_A}{N_D} \right]} \right]^{1/2}$$

As  $X_1 =$  a -ve quantity

$$X_1 = - \left\{ \frac{2V_0\epsilon_0\epsilon_r}{q N_A \left[ 1 + \frac{N_A}{N_D} \right]} \right\}^{1/2}$$

$$2 \times \left[ \frac{2\epsilon V_0}{q \frac{N_A}{N_D} (N_D + N_A)} \right]^{1/2} \times \left[ \frac{2\epsilon V_0}{q \frac{N_D}{N_A} (N_D - N_A)} \right]^{1/2}$$

$$= \frac{4\epsilon V_0}{q} \left[ \frac{1}{\sqrt{\frac{N_A}{N_D}} \sqrt{N_D + N_A} \sqrt{\frac{N_D}{N_A}} \sqrt{N_D - N_A}} \right]^{1/2}$$

$$= \frac{4\epsilon V_0}{q (N_A + N_D)}$$



$$N_A X_1 = -N_D X_2$$

$$X_2 = -\frac{N_A X_1}{N_D}$$

$$\frac{2eV_0 N_D}{q N_A (N_A + N_D)} + \frac{2eV_0 N_A}{q N_D (N_A + N_D)} + \frac{4eV_0}{q (N_A + N_D)}$$

$$(N_A X_1^2 + N_D \frac{N_A^2 X_1^2}{N_D^2})$$

$$X_1^2 \left( N_A + \frac{N_A^2}{N_D} \right)$$

$$\frac{2eV_0}{q(N_A + N_D)} \left[ \frac{N_D}{N_A} + \frac{N_A}{N_D} + 2 \right] = \frac{1}{V_0} \frac{N_D^2 + N_A^2}{N_A N_D} + 2 \frac{N_D^2 + N_A^2 + 2 N_A N_D}{N_A N_D}$$

$$\left( \frac{2V_0 \epsilon_0 \epsilon_r}{q N_A \left( 1 + \frac{N_A}{N_D} \right)} \right)$$

$$-q X_1 X_2 = + \frac{2e\epsilon_0 \epsilon_r V_0}{q \left[ (N_A N_D) \left( 1 + \frac{N_A}{N_D} \right) \left( 1 + \frac{N_D}{N_A} \right) \right]}$$

$$= - \frac{2e\epsilon_0 \epsilon_r V_0}{q (N_A + N_D)}$$

$$N_A X_1^2 + N_D$$

$$X_1^2 = \frac{2e\epsilon_0 \epsilon_r V_0}{q N_A \left( 1 + \frac{N_A}{N_D} \right)}$$

$$\int dx = x$$

$$\int x dx = \frac{x^2}{2}$$

$$X_2^2 = \frac{2e\epsilon_0 \epsilon_r V_0}{q N_D \left( 1 + \frac{N_D}{N_A} \right)}$$

$$\frac{(N_A + N_D)^2}{(N_A N_D) (N_A + N_D)}$$

$$= \frac{2eV_0}{q N_A \left( 1 + \frac{N_A}{N_D} \right)} + \frac{2eV_0}{q N_D \left( 1 + \frac{N_D}{N_A} \right)} + \frac{4eV_0}{q (N_A + N_D)}$$