5. Calculate the Convolution of $x(t) = e^{-t}u(t)$ & $h(t) = e^{t}u(-t)$ using Convolution property. Also verify the property by Convolution Integral. (2+4+2)=7

<u>Solution</u> :-

$x(t) = e^{-t}u(t)$; $X(j\omega) = \dfrac{1}{1+j\omega}$.

$$\boxed{\begin{aligned} &\text{w.k.t} \\ &\text{F.T. } e^{-at}u(t) = \frac{1}{a+j\omega} \\ &\text{F.T. } e^{at}u(-t) = \frac{1}{a-j\omega} \end{aligned}}$$

$h(t) = e^{t}u(-t)$; $\therefore H(j\omega) = \dfrac{1}{1-j\omega}$.

w.k.t. $x(t) * h(t)$ is IFT of $\left[X(j\omega)\cdot H(j\omega)\right]$

$$= \text{IFT}\left[\left(\frac{1}{1+j\omega}\right)\left(\frac{1}{1-j\omega}\right)\right]$$

$$= \text{IFT}\left[\frac{1}{1+\omega^2}\right]$$

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w.k.t.

$$F.T\left[e^{-a|t|}\right] \text{ for } a>0 \Rightarrow \frac{2a}{a^2+\omega^2}.$$

$$\therefore F.T\left[e^{-|t|}\right] \Rightarrow \frac{2}{1+\omega^2}.$$

Answer:

$$\therefore \text{For IFT}\left[\frac{1}{1+\omega^2}\right] = \frac{1}{2}e^{-|t|}.$$

<u>Verification by Convolution Integral.</u>

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau).h(\tau)\,d\tau.$$

$$= \int_{-\infty}^{\infty} \left\{e^{-(t-\tau)} u(t-\tau)\right\}\left\{e^{\tau}.u(-\tau)\right\}d\tau$$

$$\int_{-\infty}^{0}$$

$u(-\tau+t) \Rightarrow$ 

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$$= \int_{-\infty}^{0} e^{-(t-\tau)} \cdot u(t-\tau) \cdot e^{\tau} \, d\tau$$

$$= e^{-t} \int_{-\infty}^{t} e^{\tau} e^{\tau} \, d\tau = e^{-t} \int_{-\infty}^{t} e^{2\tau} \, d\tau$$

$$\boxed{\begin{array}{l} t \Rightarrow -ve \\ \because t < 0 \end{array}}$$

$$= e^{-t} \left[\frac{1}{2} e^{2\tau} \right]_{-\infty}^{t}$$

$$= \frac{e^{-t}}{2} \left[e^{2t} - 0 \right] = \frac{e^{t}}{2} \quad t \text{ is } -ve.$$

$$\boxed{\Rightarrow x(t) * h(t) = \frac{1}{2} e^{-|t|}}$$

$\}d\tau$