

Displacement Current :-

①

Displacement current is the current that flows through the capacitor when time varying voltage is given.

$$\text{In general, } I_d = \frac{dQ}{dt}$$

$$I_d = \frac{d}{dt}(C \cdot V) \Rightarrow C \frac{dV}{dt} \Rightarrow \frac{\epsilon A}{d} \frac{dV}{dt}$$

$$\text{w.k.t } V = |\vec{E}| |\vec{d}| \Rightarrow V = E \cdot d$$

$$\therefore I_d = \frac{\epsilon A}{d} \frac{d}{dt}(E \cdot d)$$

$$\frac{I_d}{A} = \frac{d}{dt} \epsilon E = \frac{d}{dt} D \Rightarrow J_d = \frac{d}{dt} D$$

$$\boxed{\vec{J}_d = \frac{\partial \vec{D}}{\partial t}}$$

According to Ampere's Circuital Law in Magnetostatics

$$\nabla \times H = J_c \quad \vec{J}_c \rightarrow \text{Conduction Current Density}$$

Taking Divergence on both sides,

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J_c \Rightarrow 0 \quad \left[\begin{array}{l} \text{Divergence of the} \\ \text{curl of any vector field} \\ \text{is zero} \end{array} \right]$$

According to Continuity equation;

(2)

$$\nabla \cdot \vec{J}_c = -\frac{\partial \rho_v}{\partial t} \neq 0.$$

→ For Time Varying field; Ampere's Circuit Law is Incompatible.

→ Maxwell modified Ampere's Circuit Law for time varying field & it is called Modified Ampere's Circuit Law.

He introduced new parameter called displacement current (Depends on changing electric field, D).

$$\Rightarrow \boxed{\nabla \times H = \vec{J}_c + \vec{J}_d} \quad \vec{J}_d \rightarrow \text{Displacement Current density.}$$

Taking Divergence on both sides;

$$\nabla \cdot (\nabla \times H) = \nabla \cdot \vec{J}_c + \nabla \cdot \vec{J}_d = 0.$$

$$\begin{aligned} \nabla \cdot \vec{J}_d &= -\nabla \cdot \vec{J}_c \\ &= -\left(-\frac{\partial \rho_v}{\partial t}\right) = \frac{\partial \rho_v}{\partial t}. \end{aligned}$$

w.k.t from Gauss law of Electric field, (3)

$$\nabla \cdot \vec{D} = \rho_v$$

$$\Rightarrow \nabla \cdot \vec{J}_d = \frac{\partial}{\partial t} \nabla \cdot \vec{D}$$

$$\Rightarrow \nabla \cdot \vec{J}_d = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

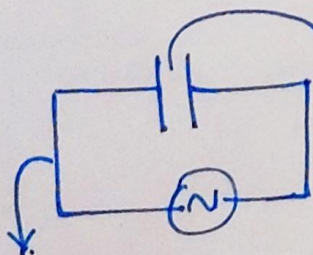
$$\boxed{\vec{J}_d = \frac{\partial \vec{D}}{\partial t}} \Rightarrow \text{Displacement Current Density.}$$

$$I_d = \int_s \vec{J}_d \cdot d\vec{s}$$

$$\boxed{\begin{matrix} \text{w.k.t.} \\ \oint_L \vec{H} d\vec{l} = I = \int_s \vec{J}_d d\vec{s} \end{matrix}}$$

$$\boxed{I_d = \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}} \Rightarrow \underline{\text{Displacement Current}}$$

$$\text{w.k.t } \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$



$$\oint_L \vec{H} d\vec{l} = \int_s \vec{J}_c \cdot d\vec{s} = I_{enc} = I$$

$\{J_d = 0\}$

$$\begin{aligned} \oint_L \vec{H} d\vec{l} &= I = \int_s \vec{J}_d \cdot d\vec{s} \\ &= \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} \\ &= \frac{\partial}{\partial t} \int_s \vec{D} \cdot d\vec{s} \Rightarrow \frac{\partial Q}{\partial t} \\ \frac{\partial Q}{\partial t} &= I \end{aligned}$$

$\{J_c = 0\}$