18MAB102T

Advanced Calculus and Complex Analysis

Question Bank

DEPARTMENT OF MATHEMATICS
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Regulation 2018

Part B Questions

Unit I

Q.No.	Question	Bloom's Thinking Levels
1	Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$	Understand
2	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x+y)dxdy$	Understand
3	Evaluate $\int_2^4 \int_1^2 \frac{dxdy}{xy}$	Understand
4	Evaluate $\int_{-\pi/4}^{\pi/4} \int_0^{a\sqrt{\cos\theta}} \frac{rdrd\theta}{\sqrt{a^2+r^2}}$	Understand
5	Evaluate $\int_0^{\pi/2} \int_0^{2a\cos\theta} rdrd\theta$	Understand
6	Evaluate $\int_0^1 \int_0^2 \int_0^3 xyzdxdydz$	Understand
7	Find the area of $r^2 = a^2 \cos 2\theta$ by double integration	Apply
8	Find the area enclosed by $y = x$ and $y = x^2$ in the first quadrant, using double integration	Apply
9	Change the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}$	Apply
10	Find the area enclosed by the ellipse using double integration $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Apply
11	Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyzdxdydz$	Understand
12	Find $\iiint_R (x-y+z)dxdydz$, where R is given by $1 \le x \le 2, 2 \le y \le 3, 1 \le z \le 3$	Understand
13	Change into polar coordinates $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$	Apply

Q.No.	Question	Bloom's Thinking Levels
14	Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 dr d\phi d\theta$	Understand
15	Evaluate $\int_0^{\log a}\int_0^{x}\int_0^{x+y} e^{x+y+z} dzdydx$	Understand

Unit II

Q.No.	Question	Bloom's Thinking Levels
1	Find grad ϕ for the following functions. (i) $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, 1)$. (ii) $\phi = \log(x^2 + y^2 + z^2)$ at the point $(1, 2, 1)$.	Understand
2	Find the directional derivative of the following functions. (i) $\phi = x^2yz + 4xz^2$ at the point $(1, -2, 1)$ in the direction of $2\vec{i} - \vec{j} + 2\vec{k}$ (ii) $\phi = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of $2\vec{i} - \vec{j} - \vec{k}$	Understand
3	Find a unit normal vector to the following surfaces (i) $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$ (ii) $xy^2z^3 = 1$ at the point $(1, 1, 1)$	Understand
4	Find the maximum directional derivative of the following functions (i) $\phi = x^3yz$ at the point $(1, 4, 1)$ (ii) $\phi = xyz^2$ at the point $(1, 0, 3)$	Understand
5	In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2y^2z^4$maximum? Find also the magnitude of this maximum.	Apply

Q.No.	Question	Bloom's Thinking Levels
6	Find the angle between the following surfaces (i) $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point (2,-1,2) (ii) $x^2 + yz = 2$ and $x + 2y - z = 2$ at the point $(1, 1, 1)$	Apply
7	If $\vec{r}$ is the position vector of (x, y, z) w.r. to origin, then prove that (i) div $\vec{r} = 3$ (ii) curl $\vec{r} = 0$ (iii)grad$(r^n) = nr^{n-2}\vec{r}$	Understand
8	Show that $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ is a conservative field.	Apply
9	Find the value of a, if the vector $\vec{F} = (2x^2y + yz)\vec{i} + (xy^2 - xz^2)\vec{j} + (axyz - 2x^2y^2)\vec{k}$ is solenoidal.	Apply
10	Determine the constants a and b such that the curl of vector $\boldsymbol{(2xy + 3yz)\vec{i} + (x^2 + axz - 4z^2)\vec{j} - (3xy + byz)\vec{k}}$ is zero.	Apply
11	If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate the line integral $\int_c \vec{F}.d\vec{r}$ from (0,0,0) to (1,1,1) along the curve C given by $x = t, y = t^2, z = t^3$	Apply
12	If $\vec{F} = (4xy - 3x^2z^2)\vec{i} + 2x^2\vec{j} - 2x^3z\vec{k}$, then check whether the integral $\int_C \vec{F}.d\vec{r}$ is independent of the path C.	Apply
13	The scalar potential $\phi = ze^x - x^2y + y + \frac{z^2}{2} + c$ for the conservative field $\vec{F} = (e^xz - 2xy)\vec{i} - (x^2 - 1)\vec{j} + (e^x + z)\vec{k}$. Evaluate $\int_C \vec{F}.d\vec{r}$ where the end points of C are (0,1,-1) and (2,3,0).	Apply
14	If $\boldsymbol{\vec{F} = 3xy\vec{i} - y^2\vec{j}}$, evaluate $\int_C \boldsymbol{\vec{F}.d\vec{r}}$ where C is the arc of the parabola $\boldsymbol{y = 2x^2}$ from $\boldsymbol{(0, 0)}$ to $\boldsymbol{(1, 2)}$.	Apply
15	Show that $\iint_S \vec{F} \cdot \vec{n} dS = \frac{3}{2}$, where $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube bounded by planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.	Apply

Unit III

Q.No.	Question	Bloom's Thinking Levels
1	Find $L\left[t^{3/2}\right]$	Understand
2	Find $L\left[f(t)\right]$, if $f(t) = \begin{cases} (t-1)^2 & t>1 \\ 0 & t<1 \end{cases}$	Understand
3	Find $L\left[\cos 4t \sin 2t\right]$,	Understand
4	Find $L\left[\dfrac{\sin at}{t}\right]$,	Understand
5	Evaluate $\int\limits_0^\infty e^{-t}\dfrac{\sin t}{t}dt$	Understand
6	Find the inverse Laplace transform of $\dfrac{1}{(s+1)(s+2)}$	Apply
7	Verify final value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$	Justify
8	Verify initial value theorem for $f(t) = e^{-t}\sin t$	Justify
9	Find the inverse Laplace transform of $\dfrac{s+3}{s^2-4s+13}$	Apply
10	Find the inverse Laplace transform of $\dfrac{s}{s^2-4s+5}$	Apply
11	Using convolution theorem, find $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$	Apply
12	Using convolution theorem, find $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$	Apply
13	Using convolution theorem, find $L^{-1}\left[\dfrac{s^2}{(s^2+4)^2}\right]$	Apply
14	Using convolution theorem, find $L^{-1}\left[\cos t * \sin t\right]$	Apply
15	Using partial fractions, find $L^{-1}\left[\dfrac{1}{s(s+4)}\right]$	Apply

Unit IV

Q.No.	Question	Bloom's Thinking Levels
1	Test whether $f(z) = z^2$ is analytic.	Justify
2	Test whether $f(z) = \overline{z}$ is analytic.	Justify
3	Test whether $f(z) = \lvert z^2 \rvert$ is analytic.	Justify
4	Show that $u = 3x^2y - y^3$ is harmonic function.	Apply
5	Show that $u = e^x(x\cos y - y\sin y)$ is harmonic function.	Apply
6	Show that an analytic function with constant imaginary part is constant.	Apply
7	Find the fixed points of the transformation $w = \dfrac{2i - 6z}{iz - 3}$	Apply
8	Find the invariant points of the transformation $w = -\dfrac{2z + 4i}{iz + 1}$	Apply
9	Find the image of the $\lvert z + 1\rvert = 1$ where the map $w = \dfrac{1}{z}$.	Apply
10	Find the image of the $\lvert z - 2i\rvert = 2$ where the map $w = \dfrac{1}{z}$.	Apply
11	Construct the analytic function $f(z)$ for which the real part is $e^x \cos y$	Apply
12	Find a function w such that $w = u + iv$ is analytic, if $u = e^x \sin y$	Apply
13	Determine the analytic function $u + iv$ whose real part $u = x^3 - 3x^2y + 3x^2 - 3y^2 + 1$	Apply
14	Find the image of the circle $\lvert z\rvert = 3$ under the transformation $w = 2z$	Apply
15	Show that $u = \dfrac{1}{2}\log(x^2 + y^2)$ is harmonic.	Apply

Unit V

Q.No.	Question	Bloom's Thinking Levels
1	Evaluate $\oint\limits_c \frac{e^{-z}}{z+1}dz$, where c is the circle $\lvert z\rvert = 2$.	Understand
2	Evaluate $\oint\limits_c \frac{3z^2+z}{z^2-1}dz$, where c is the circle $\lvert z-1\rvert = 1$	Understand
3	Evaluate $\oint\limits_c \frac{dz}{z^3(z+4)}$ where c is the circle $\lvert z\rvert = 2$	Understand
4	Evaluate $\oint\limits_c \frac{ze^{2z}}{(z-1)^3}dz$ where c is the circle $\lvert z+i\rvert = 2$	Understand
5	Evaluate $\oint\limits_c \frac{\cos \pi z^2}{(z-1)(z-2)}dz$ where c is the circle $\lvert z\rvert = 3$	Understand
6	Evaluate $\oint\limits_c \frac{e^{3z}}{(z+i\pi)^7}dz$ where c is the circle $\lvert z\rvert = 4$	Understand
7	Expand $\frac{\sin z}{z-\pi}$ about $z = \pi$	Apply
8	Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$	Apply
9	Find the poles and its residues for $f(z) = \frac{z^2}{(z-1)^2(z+2)}$	Understand
10	Evaluate $\int\limits_C \frac{z+1}{(z-1)(z-3)dz}$ where c is —z—=2 using residue theorem.	Understand
11	Expand $z^2e^{1/z}$ in Laurent series about $z = 0$	Apply
12	Expand $\frac{e^{2z}}{(z-1)^3}$ in Laurent series about $z = 1$	Apply
13	Determine the nature of singularities of $\frac{\sin z - z}{z^3}$	Justify
14	Determine the nature of singularities of $\frac{e^{1/z}}{(z-a)^2}$	Justify
15	Find the residue at $z = 0$ for $\frac{1}{z^2e^z}$	Understand

Part C Questions

Unit I

Q.No.	Question	Bloom's Thinking Levels
1	Change the order of integration and evaluate $\int_{0}^{a}\int_{x^2/a}^{2a-x} \boldsymbol{xydxdy.}$	Apply
2	Change the order of integration in I=$\int_{0}^{4a}\int_{x^2/4a}^{2\sqrt{ax}} \boldsymbol{dydx}$and hence evaluate.	Apply
3	Change the order of integration and evaluate $\int_{0}^{1}\int_{x^2}^{2-x} \boldsymbol{xydxdy.}$	Apply
4	Find the area enclosed by the curves $\boldsymbol{y^2 = 4ax}$ and $\boldsymbol{x^2 = 4ay}$.	Apply
5	Transform into polar coordinate and evaluate $\int_{0}^{2}\int_{0}^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}dydx.$	Apply
6	Show that $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8}$	Apply
7	Show that $\int_{0}^{a}\int_{0}^{\sqrt{a^2-x^2}}\int_{0}^{\sqrt{a^2-x^2-y^2}} \frac{dzdydx}{\sqrt{a^2-x^2-y^2-z^2}} = \frac{\pi^2 a^2}{8}$	Apply
8	Find the volume of the sphere $x^2+y^2+z^2 = a^2$, using triple integral.	Apply
9	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ by triple integral.	Apply
10	Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ and the coordinate planes.	Apply
11	Find the volume of the sphere $x^2+y^2+z^2 = a^2$ in the positive octant.	Apply
12	Find the smaller of the areas bounded by $\boldsymbol{y = 2 - x}$ **and** $\boldsymbol{x^2 + y^2} = 4$ using double integral.	Apply
13	By transforming into polar coordinates evaluate $\iint \frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circles $x^2+y^2 = 4$ and $x^2+y^2 = 16$.	Apply

Q.No.	Question	Bloom's Thinking Levels
14	Find the area of the cardioid $r = a(1 + \cos\theta)$, using double integral.	Apply
15	Evaluate $\iint_A r^3 dr d\theta$, where A is the area between circles $r = 2\sin\theta$ and $r = 4\sin\theta$	Apply

Unit II

Q.No.	Question	Bloom's Thinking Levels
1	Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$	Apply
2	Find the constants a, b and c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ may be irrotational. Also find the scalar potential?	Apply
3	Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at the point $(-1, 2, 1)$.	Apply
4	Find the directional derivative of $\phi = xy + yz + zx$ at the point $(3, 1, 2)$ in the direction of the vector $2\vec{i} + 3\vec{j} + 6\vec{k}$.	Apply
5	Evaluate $\iint_S \vec{F}.\vec{n}dS$ if $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.	Apply
6	Evaluate $\iint_S \vec{F}.\vec{n}dS$, where $\vec{F} = z\vec{i} + x\vec{j} - y^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between z=0 and z=2.	Apply
7	Evaluate $\iint_S \vec{F}.\vec{n}dS$, where $\vec{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is part of the plane $2x + 3y + 6z = 12$ which is in the first octant.	Apply

Q.No.	Question	Bloom's Thinking Levels
8	Verify Green's theorem in the plane for $\oint\limits_c (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$	Justify
9	Verify Green's theorem in the plane for $\int\limits_C [(3x^2 - 8y^2)\, dx - (4y - 6xy)dy]$, where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.	Justify
10	Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c,$	Justify
11	Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ over the cube bounded by $x = \pm 1, y = \pm 1, z = \pm 1$	Justify
12	Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	Justify
13	Verify Stoke's theorem for $\vec{F} = y^2z\vec{i} + z^2x\vec{j} + x^2y\vec{k}$ where S is the open surface of the cube formed by the planes $x = -a, x = a, y = -a, y = a, z = -a, z = a$ in which $z = -a$ is cut open.	Justify
14	Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the open surface of the cube formed by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$ above the $xy-$plane.	Justify
15	Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative field. Find the scalar potential and work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$.	Apply

Unit III

Q.No.	Question	Bloom's Thinking Levels
1	Verify initial and final value theorem for $f(t) = 3e^{-2t}$	Justify
2	Find $L\left[\dfrac{\cos at - \cos bt}{t}\right]$	Apply
3	Find $L\left[te^{-2t}\sin t\right]$	Apply
4	Find $L\left[t\sin 3t\cos 2t\right]$	Apply
5	Verify initial and final value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$	Justify
6	Verify initial and final value theorem for $f(t) = e^{-t}(t+2)^2$	Justify
7	Find the Laplace transform of the function $f(t) = \begin{cases} \sin\omega t & 0 < t \le \dfrac{\pi}{\omega} \\ 0 & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases}$ with period $\dfrac{2\pi}{\omega}$.	Apply
8	Find the Laplace transform of the function $f(t) = \begin{cases} t & 0 \le t \le a \\ 2a - t & a < t \le 2a \end{cases}$ and $f(t+2a) = f(t)$.	Apply
9	Find the Laplace transform of the function $f(t) = \begin{cases} -1 & 0 < t \le \dfrac{a}{2} \\ 1 & \dfrac{a}{2} < t < a \end{cases}$ and $f(t+a) = f(t)$.	Apply
10	Find $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ using convolution theorem.	Apply
11	Find $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$ using convolution theorem.	Apply
12	Using partial fraction method, find $L^{-1}\left[\dfrac{s}{(s+1)(s^2+1)}\right]$	Apply

Q.No.	Question	Bloom's Thinking Levels
13	Using partial fraction method, find $L^{-1}\left[\dfrac{1-s}{(s+1)(s^2+4s+13)}\right]$	Apply
14	Solve $(D^2-2D+1)y=e^t$, given $y(0)=2, y'(0)=1$	Apply
15	Solve, using Laplace transform $\dfrac{d^2y}{dt^2}-4\dfrac{dy}{dt}+3y=e^{-t}$, given $y(0)=1, y'(0)=0$.	Apply

Unit IV

Q.No.	Question	Bloom's Thinking Levels
1	Show that an analytic function with (i) constant real part is constant (ii) constant modulus is constant.	Remember
2	If $f(z)=u+iv$ is an analytic function of z, show that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2=4\lvert f'(z)\rvert^2$	Justify
3	If $f(z)=u+iv$ is an analytic function of z, show that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)\log\lvert f(z)\rvert=0$	Justify
4	Show that the function $u=e^x\cos y$ is harmonic and find the harmonic conjugate of u.	Apply
5	Find the analytic function $f(z)=u+iv$ if $u-v=e^x(\cos y-\sin y)$	Apply
6	Find the analytic function $f(z)=u+iv$ if $u-v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$	Apply
7	Find an analytic function $f(z)=u+iv$, given that $2u+3v=\dfrac{\sin 2x}{\cosh 2y-\cos x}$	Apply

Q.No.	Question	Bloom's Thinking Levels
8	Under the transformation $w = \frac{1}{z}$, find the image of the region (i) $x > c$, where $c > 0$ (ii) $y > c$, where $c > 0$.	Remember
9	Find the bilinear mapping which maps $-1, 0, 1$ of the z-plane onto $i, 1, -i$ of the w-plane.	Remember
10	Find the bilinear mapping which maps $0, 1, \infty$ of the z-plane onto $-1, -i, 1$ of the w-plane.	Remember
11	Find the bilinear map which maps $z = 1, i, -1$ onto the points $w = i, 0, -i$.	Remember
12	If $u = \frac{\sin 2x}{\cosh 2y + \cos x}$, find the corresponding analytic function $f(z) = u + iv$	Apply
13	Find the image of the rectangular region bounded by $x = 0, y = 0, x = 1, y = 2$ under the map $w = (1 + i)z + 2$.	Apply
14	Show that the transformation $w = \frac{1}{z}$ maps a circle in the z-plane into a circle in the w-plane or to a straight line in the w-plane.	Apply
15	Prove that $u = e^x(x \cos y - y \sin y)$ is harmonic and find the corresponding analytic function $f(z) = u + iv$.	Apply

Unit V

Q.No.	Question	Bloom's Thinking Levels
1	Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$, where C is the circle (i) $\lvert z+1+i\rvert = 2$ (ii) $\lvert z+1-i\rvert = 2$.	Apply
2	Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$, where C is $\lvert z\rvert = 3$.	Apply
3	Find the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in $\lvert z\rvert < 2$.	Remember
4	Expand $\frac{1}{z^2-3z+2}$ in the region (i) $1 < \lvert z\rvert < 2$ (ii) $0 < \lvert z-1\rvert < 2$ (iii) $\lvert z\rvert > 2$	Remember
5	Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ as a Laurent's series if (i)$2 < \lvert z\rvert < 3$ (ii) —z—¿3.	Remember
6	Find the Laurent's series of $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ in $1 < \lvert z-1\rvert < 3$.	Remember
7	Using residue theorem, find $\int_C \frac{dz}{(z^2+4)^2}$, where C is the circle $\lvert z-i\rvert = 2$.	Apply
8	Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$, where C is $\lvert z-i\rvert = 2$, using residue theorem.	Apply
9	Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$, where C is $\lvert z\rvert = 3$, using residue theorem.	Apply

Q.No.	Question	Bloom's Thinking Levels
10	Evaluate $\int_C \frac{z-1}{(z+1)^2(z+2)} dz$, where C is $\lvert z-i\rvert = 2$, using residue theorem.	Apply
11	Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ by using contour integration.	Apply
12	Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2}$, $\lvert p\rvert < 1$, using contour integration.	Apply
13	Evaluate $\int_0^{2\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$, using contour integration.	Apply
14	Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}, a > 0, b > 0.$	Apply
15	Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$, using contour integration.	Apply