Assignment

Sub Code: 18MAB101T
Sub Name: Calculus and Linear Algebra

1. If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{\left(x+y+z\right)^2}$.
2. If $z = f\left(x+ct\right)+\phi\left(x-ct\right)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$
3. If $f\left(x, y\right) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify $f_{xy} = f_{yx}$.
4. Obtain the Maclaurin's series of $e^x \cos y$ upto second degree terms.
5. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.
6. Find the expansion for $\cos x\ \cos y$ in power of x and y up to terms of 3^{rd} degree.
7. Expand $e^x \sin y$ in power of x and y near the point $\left(-1, \frac{\pi}{4}\right)$ as far as the terms of the third degree.
8. Using Taylor's series, verify that,

$$\log(1 + x + y) = (x + y) - \frac{1}{2}(x + y)^2 + \frac{1}{3}(x + y)^3 \ldots..$$

9. Give the transformations $u = e^x cosy$ and $v = e^x siny$ and that $\emptyset$ is a function of u and v and also of x and u. Prove that $\frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial y^2} = (u^2 + v^2)\left[\frac{\partial^2\emptyset}{\partial u^2} + \frac{\partial^2\emptyset}{\partial v^2}\right]$.
10. Expand $e^x \log(1 + y)$ in power of x and y upto third degree terms by using Taylor's series.