Formation of PDE by eliminating arbitrary constants

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1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2 + 1$.

Solution:

Given $z = (x-a)^2 + (y-b)^2 + 1 - - - - - - - - - - - -(1)$
Differentiate (1) partially w.r.to x and y , we get

$p = 2\,(x-a) \quad \Rightarrow \quad x - a = \dfrac{p}{2}$ and

$q = 2\,(y-b) \quad \Rightarrow \quad y - b = \dfrac{q}{2}$

substituting in(1), we get $z = \dfrac{p^2}{4} + \dfrac{q^2}{4} + 1$

$$4z = p^2 + q^2 + 4 \quad \text{which is the required pde.}$$

2. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = \left(x^2 + a^2\right)\left(y^2 + b^2\right)$.

Solution:

Given $z = \left(x^2 + a^2\right)\left(y^2 + b^2\right) - - - - - - - - - - -(1)$

Differentiate (1) partially w.r.to x and y , we get

$p = 2x\left(y^2 + b^2\right) \quad \Rightarrow \quad \left(y^2 + b^2\right) = \frac{p}{2x}$ and

$q = 2y\left(x^2 + a^2\right) \quad \Rightarrow \quad \left(x^2 + a^2\right) = \frac{q}{2y}$

substituting in(1), we get $z = \left(\frac{p}{2x}\right)\left(\frac{q}{2y}\right)$

$4xyz = pq$ which is the required pde.

3. Form the partial differential equation of all spheres whose centres lie on the Z -axis.

Solution:

Given that the centres of the spheres lie on the Z - axis.

$\therefore$ Centre is $(0, 0, c)$. Let r be the radius.

$\therefore$ Equation of the family of spheres is

$$x^2 + y^2 (z - c)^2 = r^2 \ldots (1)$$

Differentiate (1) partially w.r.to x and y , we get

$2x + 2(z - c)\, p = 0 \quad \Rightarrow \quad (z - c)\, p = \; - x \ldots (2)$ and

$2y + 2(z - c)\, q = 0 \quad \Rightarrow \quad (z - c)\, q = \; - y \ldots (3)$

(2) divide by (3), we get $\dfrac{(z - c)\, p}{(z - c)\, q} = \dfrac{-x}{-y}$

$$py = qx \quad \text{which is the required pde.}$$