Lecture 5 – Op-Amp Frequency Response

Review:

- Simple RC Low-Pass Filter Response

Op-Amp Frequency Response:

- Open-Loop Frequency Response
- Gain Bandwidth Product (GBWP)
- Closed-Loop Frequency Response

Bode Plots:

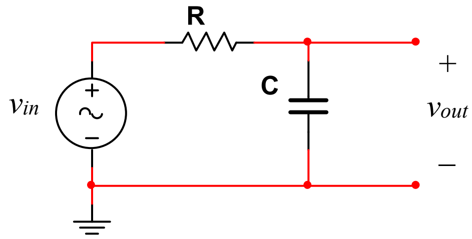
- Single-Pole RC LPF Frequency Response
- LM741 Open vs. Closed Loop Comparison

Bandwidth vs. Risetime Relationship

- $t_R BW \cong 0.35$

1. Simple RC Low-Pass Filter

Circuit:



Transfer Function:

$$H(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_0}} \qquad H(jf) = \frac{1}{1 + j\dfrac{f}{f_0}} \qquad \omega = 2\pi f,\ \omega_0 = 2\pi f_0$$

$$f_0 = \frac{1}{2\pi RC}$$

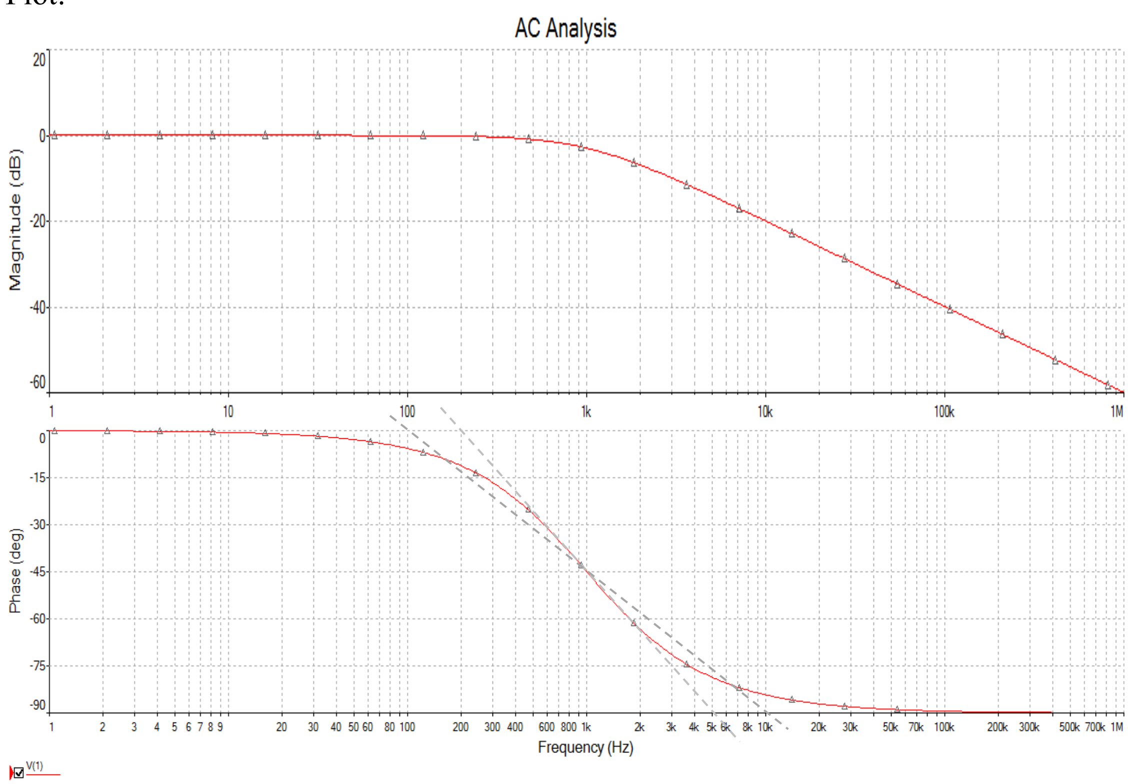
Magnitude:

$$\left| H(jf) \right| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_0} \right)^2}}$$

Phase:

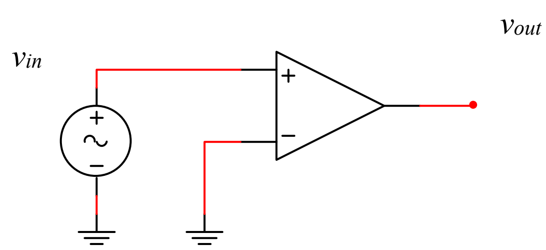
$$\angle H(jf) = -\arctan\left(\frac{f}{f_0} \right)$$

Plot:



2. Op-Amp Open Loop Frequency Response

Circuit:



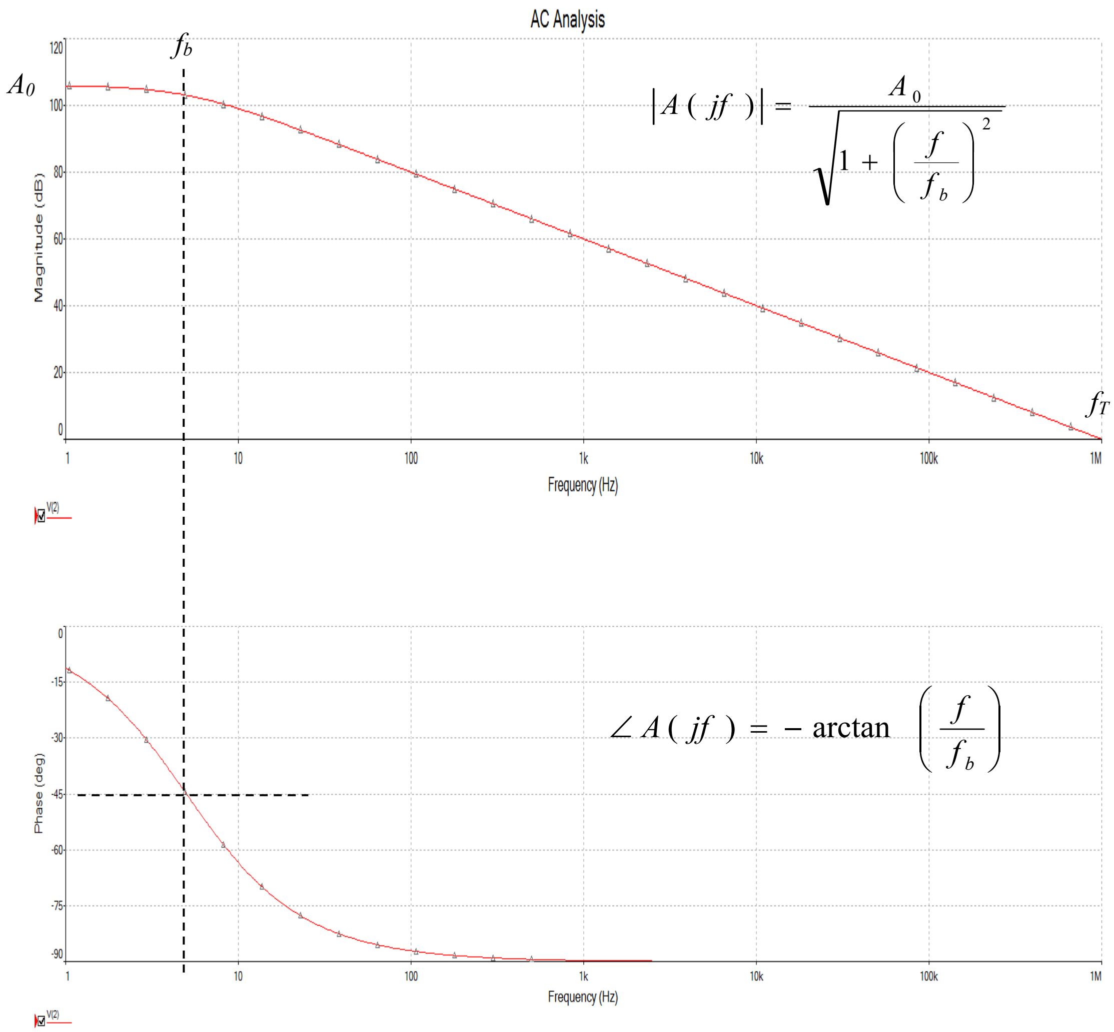
$$A(jf) = \frac{A_0}{1 + j\dfrac{f}{f_b}}$$

Where:

A_0 = Open-Loop DC Gain
f_b = Breakpoint Frequency
f_T = Unity Gain Frequency

Gain Bandwidth Product (GBWP)

$GBWP = A_0 \cdot f_b = f_T \cdot 1$

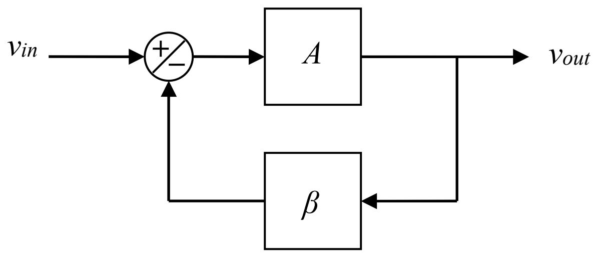


$$|A(jf)| = \frac{A_0}{\sqrt{1 + \left(\dfrac{f}{f_b}\right)^2}}$$

$$\angle A(jf) = -\arctan\left(\frac{f}{f_b}\right)$$

3. Op-Amp Closed-Loop Frequency Response

Background (from Control Theory):

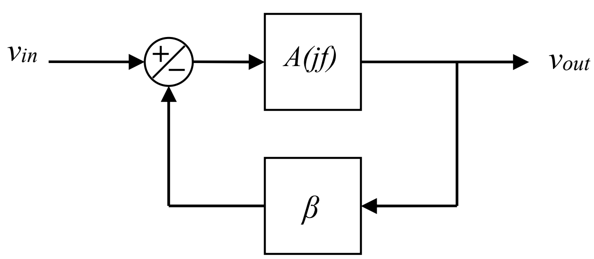


$$\frac{v_{out}}{v_{in}} \equiv A_{Closed-Loop} = \frac{A}{1+A\beta}$$

Given that the open-loop gain A is a function of frequency and exhibits a *Low-Pass Filter Response*, it can be modeled as:

$$A = A(jf) = \frac{A_0}{1 + j\frac{f}{f_b}}$$

where A_0 is the DC gain and f_b is the cutoff or breakpoint frequency of the open-loop response. Making this change in the control system yields:



$$A(jf)_{Closed-Loop} = \frac{A(jf)}{1 + A(jf)\beta}$$

Substituting the open-loop response into the closed-loop equation gives:

$$A(jf)_{Closed-Loop} = \frac{A(jf)}{1 + A(jf)\beta} = \frac{\left(\dfrac{A_0}{1 + j\dfrac{f}{f_b}}\right)}{1 + \left(\dfrac{A_0}{1 + j\dfrac{f}{f_b}}\right)\beta}$$

This equation can be rearranged into:

$$A(jf)_{Closed-Loop} = \frac{\overbrace{\frac{1}{\beta}}^{\text{Closed-Loop DC Gain}}}{1 + \underbrace{\frac{1}{A_0\beta}}_{\substack{\text{Typically small}\\ \text{enough to ignore.}}} + j\underbrace{\frac{f}{A_0 f_b \beta}}_{\text{New cut-off frequency}}}$$

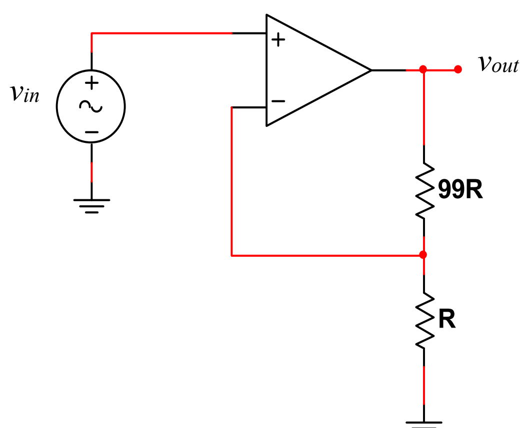
$$f_c = A_0 f_b \beta = f_T \beta$$

Final Closed-Loop Frequency Response:

$$A(jf)_{Closed-Loop} \cong \frac{\frac{1}{\beta}}{1 + j\frac{f}{f_c}}$$

Example:

Given the following op-amp circuit with f_T=1MHZ, plot the closed-loop frequency response, both magnitude and phase.

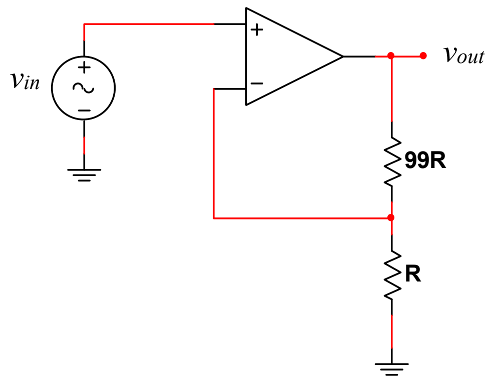


$$\beta = \frac{R}{R+99R} = \frac{1}{100}$$

$$\therefore \frac{1}{\beta} = 100$$

$$f_c = f_T\beta = 1_{MHz}\left(\frac{1}{100}\right) = 10_{kHz}$$

4. Op-Amp Closed-Loop Frequency Response EXAMPLE:



$$A(jf)_{Closed-Loop} \cong \frac{\frac{1}{\beta}}{1 + j\frac{f}{f_c}}$$

$$= \frac{100}{1 + j\frac{f}{10_{kHz}}}$$