Reg. No.	R	A	1	7	1	1	0	0	3	0	2	0	0	2	3

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018

3rd to 7th Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. The partial differential equation formed by eliminating arbitrary constant a, b is $z=(x+a)(y+b)$
   (A) $z=p+q$ (B) $z=p-q$
   (C) $z=p/q$ (D) $z=pq$

2. The complementary function of $(D^2+2DD'+D'^2)z=0$ is
   (A) $\phi_1(y-x)+\phi_2(y-x)$ (B) $\phi_1(y-x)+x\phi_2(y-x)$
   (C) $\phi_1(y-x)+\phi_2(y+x)$ (D) $\phi_1(y-x)+x\phi_2(y+x)$

3. The particular integral of $(D^2-2DD')z=e^{2x}$
   (A) $e^{2x}/4$ (B) $e^{2x+y}/4$
   (C) e^{2x} (D) $e^{2x}/2$

4. The complete solution of $z=px+qy+p^2q^2$ is
   (A) $z=ax+by^2+ab^2$ (B) $z=ax^2+by+ab^2$
   (C) $z=ax+by+a^2b^2$ (D) $z=ax+by+c$

5. sin$x$ is a periodic function with period
   (A) π (B) π/2
   (C) 2π (D) 4π

6. The constant a_0 of the Fourier series for the function $f(x)=k,\ 0\le x\le 2\pi$ is
   (A) k (B) 2k
   (C) 0 (D) k/2

7. The RMS value of f(x) $=x$ in $-1\le x\le 1$ is
   (A) 1 (B) 0
   (C) $1/\sqrt{3}$ (D) −1

8. Half range cosine series for f(x) is (0, π) is

(A) $\sum_{n=1}^{\infty} a_n \cos nx$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

(C) $\sum_{n=1}^{\infty} b_n \sin nx$

(D) $\frac{a_0}{2} - \sum a_n \cos nx$

9. The proper solution of the problems of vibration of string is

(A) $y(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})(ce^{\lambda at} + De^{\lambda at}$

(B) $y(x,t) = (Ax+B)(ct+1)$

(C) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$

(D) $y(x,t) = Ax + B$

10. The one dimensional wave equation is

(A) $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$

(B) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

(C) $\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$

(D) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial^2 y}{\partial t^2}$

11. One dimensional heat equation is used to find

(A) Density

(B) Temperature distribution

(C) Time

(D) Displacement

12. A rod of length l has its ends A and B kept at 0° and 100° respectively, until steady state conditions prevail. Then the initial condition is given by

(A) $u(x,0) = ax + b + 100l$

(B) $u(x,0) = \frac{100x}{l}$

(C) $u(x,0) = 100xl$

(D) $u(x,0) = (x+l)100$

13. $F\left[e^{iax} f(x)\right]$

(A) $F(s+a)$

(B) $F(s-a)$

(C) $F(sa)$

(D) $F(s/a)$

14. $F\left[xf'(x)\right] =$

(A) $\frac{dF(s)}{ds}$

(B) $i\frac{dF(s)}{ds}$

(C) $-i\frac{dF(s)}{ds}$

(D) $-\frac{dF(s)}{ds}$

15. The fourier cosine transform of $Fc\left[e^{-4x}\right]$

(A) $\sqrt{\frac{2}{\pi}} \frac{4}{16+s^2}$

(B) $\sqrt{\frac{2}{\pi}} \frac{4}{4+s^2}$

(C) $\sqrt{\frac{\pi}{2}} \frac{4}{16+s^2}$

(D) $\sqrt{\frac{\pi}{2}} \frac{4}{4+s^2}$

16. $F[f(x)*g(x)]=$

(A) $F(s)+G(s)$ (B) $F(s)-G(s)$

(C) $F(s)G(s)$ (D) $F(s)/G(s)$

17. What is Z(7)

(A) $\dfrac{z}{z-1}$ (B) $7\dfrac{z}{z-1}$

(C) $\dfrac{1}{7}\dfrac{z}{z-1}$ (D) $\dfrac{z-1}{z}$

18. What is $Z\left[na^n\right]$

(A) $\dfrac{az}{(z-a)^2}$ (B) $\dfrac{z}{(z-a)^2}$

(C) $\dfrac{a}{(z-a)^2}$ (D) $\dfrac{z}{(z-a)^3}$

19. If $z[f(t)]=F(z)$ then $\lim\limits_{z\to\infty} F(z)=$

(A) $f(0)$ (B) $f(1)$

(C) $\lim\limits_{x\to\infty} f(t)$ (D) $f(\infty)$

20. $\phi(z)=\dfrac{z^n(2z+4)}{(z-2)^3}$ has a pole of order

(A) 2 (B) 1

(C) 3 (D) 4

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

21. Form the Partial differential equation by eliminating f from $z=xy+f(x^2+y^2+z^2)$.

22. Find the half range Fourier sine series for $f(x)=x$ in $0<x<\pi$.

23. Write the one dimensional heat flow equation and all the possible solutions.

24. Find the Fourier sine transform of e^{-ax} $a>0$.

25. Find Z-transform of $r^n\cos n\theta$.

26. Find $z^{-1}\left(\dfrac{1}{(z-1)(z-2)}\right)$ by convolution.

27. Solve $p^2+q^2=x+y$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Solve (i) $z = px + qy + \sqrt{1 + p^2 + q^2}$ (ii) $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. Find also singular integral.

(OR)

b. Solve (i) $(D^2 - 2DD' + D'^2)z = \cos(x - 3y)$ (ii) $(D^2 - DD'^2)z = e^{x+2y}$.

29. a. Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π. Hence deduce $\Sigma \frac{1}{n^2} = \frac{\pi^2}{6}$.

(OR)

b. Compute the first two harmonics of the fourier series f(x) given by the following table.

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$. Find the displacement.

(OR)

b. A rod of length l has its ends A and B kept at 0°C and 100°C respectively unit steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature u(x, t).

31. a. Find the Fourier transform of f(x) if $f(x) = \begin{cases} 1 - |x| & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$.

(OR)

b. Use transform method to evaluate $\int_0^\infty \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$

32. a.i. Find $Z(a^n)$ and $Z(n^2)$.

ii. Using residues find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$.

(OR)

b. Solve the equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$ by using Z-transform.

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B.Tech. DEGREE EXAMINATION, NOVEMBER 2016
Third Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

C

1. The complete integral of $\sqrt{p}+\sqrt{q}=1$ is
   (A) $z=ax+by$
   (B) $z=a(x+y)+b$
   (C) $z=ax+by+c$
   (D) $z=ax-by+a$

2. The complete integral of $p=2qx$ is
   (A) $z=ax^2-ay+c$
   (B) $z=ax^2+ay+c$
   (C) $z=ay^2+ax+c$
   (D) $z=x^2-y+c$

3. The complementary function of $(D^2+DD'-2D'^2)z=x^2y$ is
   (A) $z=\phi_1(y-x)+\phi_2(y-2x)$
   (B) $z=\phi_1(y+x)+\phi_2(y+2x)$
   (C) $z=x\phi_1(y+x)+\phi_2(y+2x)$
   (D) $z=\phi_1(y+x)+\phi_2(y-2x)$

4. Find the particular integral of $(D^2+2DD'+D'^2)z=e^{x+2y}$
   (A) $\frac{e^{x+2y}}{9}$
   (B) $\frac{e^{x-2y}}{9}$
   (C) $\frac{e^{2x+y}}{9}$
   (D) $\frac{e^{2x-y}}{9}$

C

5. The partial differential equation $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 y}{\partial y^2}=0$ of the form
   (A) Elliptic
   (B) Parabolic
   (C) Hyperbolic
   (D) None of these

6. sinx is a periodic function with period
   (A) π
   (B) $\frac{\pi}{2}$
   (C) 2π
   (D) 4π

7. $\int_{-a}^{a} f(x)dx = 0$ if f(x) is

(A) Odd
(B) Even
(C) Periodic
(D) Neither even nor odd

8. For half range cosine series of $f(x) = \cos x$ *in* $(0, \pi)$ the value of a_0 is

(A) 4
(B) $\frac{2}{\pi}$
(C) $\frac{4}{\pi}$
(D) 0

9. The one dimensional wave equation is

(A) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$
(B) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
(C) $\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$
(D) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial y}{\partial t}$

10. How many initial and boundary conditions are required to solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$?

(A) 2
(B) 3
(C) 5
(D) 4

11. The one dimensional heat equation is

(A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
(B) $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$
(C) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$
(D) $\frac{\partial u}{\partial x} = \alpha^2 \frac{\partial^2 y}{\partial t^2}$

12. One dimensional heat equation is used to find

(A) Temperature distribution
(B) Displacement
(C) Time
(D) Mass

13. The Fourier transform of function f(x) is

(A) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ist} dt$
(B) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ixs} dx$
(C) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{isx} dx$
(D) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(s) e^{isx} dx$

14. Under Fourier cosine transform $f(x) = \frac{1}{\sqrt{x}}$ is

(A) Complex
(B) Inverse function
(C) Cosine function
(D) Self-reciprocal function

15. $F\left[e^{iax} f(x)\right]$ is

(A) $F(s+a)$
(B) $F(s-a)$
(C) $F(sa)$
(D) $F\left(\frac{s}{a}\right)$

16. $F[f(x)*g(x)]$ is

(A) F(s)+G(s)

(B) F(s) G(s)

(C) F(s)–G(s)

(D) $\frac{F(s)}{G(s)}$

17. Z(5) is

(A) $\frac{z}{z-1}$

(B) $5\frac{z}{z-1}$

(C) $\frac{1}{5}\frac{z}{z-1}$

(D) $\frac{z-1}{z}$

18. $Z\left(\frac{1}{n}\right)=$

(A) $\log\left(\frac{z}{z-1}\right)$ *if* $|z|>1$

(B) $\log\left(\frac{z}{z+1}\right)$

(C) $\log\left(\frac{z+1}{z}\right)$

(D) $\log\left(\frac{z^2}{z-1}\right)$ *if* $|z|>1$

19. Find $Z^{-1}\left(\frac{z}{(z-1)^2}\right)$

(A) n+1

(B) n

(C) n–1

(D) $\frac{1}{n}$

20. Poles of $\phi(z)=\frac{z^n}{(z-1)(z-2)}$

(A) $z=1, z=0$

(B) $z=1, z=2$

(C) $z=0, z=2$

(D) $z=0$

PART – B ($5\times4=20$ Marks)

Answer **ANY FIVE** Questions

21. Form the partial differential equation by eliminating the arbitrary function from $z=f(x^2+y^2)$.

22. Find the complete integral of $z=p^2+q^2$.

23. Find the RMS value of $f(x)=x-x^2$, *in* $-1<x<1$.

24. State any two assumptions in deriving one dimensional wave equation and write its all possible solutions.

25. Find the Fourier cosine transform of $e^{-ax}, a>0$.

26. Prove that $z(\sin n\theta)=\frac{z\sin\theta}{z^2-2z\cos\theta+1}$ *if* $|z|>1$.

27. Find $z[\{n(n-1)\}]$.

PART – C ($5\times12=60$ Marks)

Answer **ALL** Questions

28. a.i. Find the singular solution of $z=px+qy+p^2+q^2$.

ii. Find the general solution of $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$.

(OR)

b. Solve $(D^3-2D^2D')z=\sin(x+2y)+3x^2y$.

29. a. Find the Fourier series of $f(x)=x+x^2\ in(-\pi,\pi)$ of periodicity 2π. Hence deduce that $\sum\frac{1}{n^2}=\frac{\pi^2}{6}$.

(OR)

b. Compute the first two harmonic of the Fourier series of f(x) given by the following table

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x,0)=k(lx-x^2)$. If it is released from rest from this position, find the displacement y at any time and at any distance from the end x = 0.

(OR)

b. A rod 30 cm long, has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0°C and kept so. Find the resulting temperature function u(x,t).

31. a. Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and hence find the value of $\int_0^\infty\left(\frac{x\sin x-\cos x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.

(OR)

b.i. Evaluate $\int_0^\infty\frac{dx}{(a^2+x^2)(b^2+x^2)}$ using transform method. (8 Marks)

ii. Find Fourier sine transform of $\frac{1}{x}$. (4 Marks)

32. a.i. Find inverse z-transform of $\frac{(z+2)z}{z^2+2z+4}$ using long division method.

ii. Find the inverse z transform of $\frac{z}{(z-1)(z-2)}$ using residues.

(OR)

b. Solve $y_{n+2}+6y_{n+1}+9y_n=2^n$ given $y_0=y_1=0$ using Z transform method.

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B.Tech. DEGREE EXAMINATION, DECEMBER 2016
Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The complete integral of p = q is
   (A) $z = ax + by$
   (B) $z = a(x + y) + b$
   (C) $z = ax + by + c$
   (D) $z = ax - by + a$

2. The partial differential equation formed by eliminating arbitrary constant from $z = (x + a)(y + b)$
   (A) $z = p + q$
   (B) $z = p - q$
   (C) $z = \frac{p}{q}$
   (D) $z = pq$

3. Solve $(D^3 - 3D^2D') = 0$
   (A) $z = f_1(y - x) + f_2(y - 2x) + f_3(y + 2x)$
   (B) $z = f_1(y) + f_2(y) + f_3(y + 3x)$
   (C) $z = f_1(y) + xf_2(y) + f_3(y + 3x)$
   (D) $z = f_1(y) + f_2(y) + f_3(y - 3x)$

4. The partial differential equation $AU_{xx} + BU_{xy} + CU_{yy} + DU_x + EU_y + FU = G$, is elliptic if
   (A) $B^2 - 4AC > 0$
   (B) $B^2 - 4AC \geq 0$
   (C) $B^2 - 4AC \leq 0$
   (D) $B^2 - 4AC < 0$

5. The value of $\int_{-a}^{a} f(x)dx$ if f(x) is on add function
   (A) $2\int_0^a f(x)dx$
   (B) $3\int_0^a f(x)dx$
   (C) Periodic
   (D) Zero

6. The constant a_0 of the Fourier series for the function $f(x) = x$ in $0 \leq x \leq 2\pi$ is
   (A) π
   (B) 2π
   (C) 3π
   (D) 0

7. The RMS value of $f(x) = x, -1 \leq x \leq 1$ is
   (A) 1
   (B) 0
   (C) $\frac{1}{\sqrt{3}}$
   (D) -1

8. Half range sine series for $f(x)$ in $(0, \pi)$ is

(A) $\sum_{n=1}^{\infty} a_n \cos nx$

(B) $\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

(C) $\sum_{n=1}^{\infty} b_n \sin nx$

(D) $\frac{a_o}{2} - \sum_{n=1}^{\infty} a_n \cos nx$

9. In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) $\frac{T}{m}$

(B) $\frac{k}{c}$

(C) $\frac{m}{T}$

(D) $\frac{k}{m}$

10. How many initial and boundary conditions are required to solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$?

(A) 2

(B) 3

(C) 5

(D) 4

11. The steady state temperature of a rod of length l whose ends are kept at 30°C and 45°C is

(A) $u = \frac{15}{l}x + 30$

(B) $u = \frac{20}{l}x + 30$

(C) $u = \frac{10}{l}x + 20$

(D) $u = 10x/l$

12. One dimensional heat equation is used to find

(A) Density

(B) Temperature distribution

(C) Time

(D) Displacement

13. The Fourier transform of a function f(x) is

(A) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ist} dt$

(B) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

(C) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{isx} dx$

(D) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(s) e^{isx} dx$

14. The Fourier cosine transform of e^{-ax} is

(A) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + x^2}$

(B) $\sqrt{\frac{1}{\pi}} \frac{s}{s^2 + a^2}$

(C) $\sqrt{\frac{1}{\pi}} \frac{a}{s^2 + a^2}$

(D) $\sqrt{\frac{2}{\pi}} \frac{a}{s^2 + a^2}$

15. $F[f(ax)]$ is where F is Fourier transform operator

(A) $\frac{1}{s} F\left(\frac{s}{a}\right)$

(B) $\frac{1}{a} F\left(\frac{a}{s}\right)$

(C) $\frac{1}{a} F\left(\frac{s}{a}\right)$

(D) $\frac{1}{s} F(as)$

16. $F[f(x) * g(x)]$

(A) $F(s)+G(s)$

(B) $F(s)-G(s)$

(C) $F(s)G(s)$

(D) $\dfrac{F(s)}{G(s)}$

17. Find $Z(5)$

(A) $\dfrac{z}{z-1}$

(B) $5\dfrac{z}{z-1}$

(C) $\dfrac{1}{5}\dfrac{z}{z-1}$

(D) $\dfrac{z-1}{z}$

18. Find $Z\left(\dfrac{a^n}{n!}\right)$

(A) $e^{\frac{1}{z}}$

(B) $e^{\frac{a}{z}}$

(C) e^{z^2}

(D) $e^{\frac{-a}{z}}$

19. $Z\left(\sin\dfrac{n\pi}{2}\right)$ is

(A) $\dfrac{z}{z^2-1}$

(B) $\dfrac{z^2}{z+1}$

(C) $\dfrac{z}{z^2+1}$

(D) $\dfrac{z}{z+1}$

20. Find $Z\left(e^{-at}\right)$

(A) $\dfrac{z}{z-T}$

(B) $\dfrac{z}{z+e^{aT}}$

(C) $\dfrac{z}{z-e^{aT}}$

(D) $\dfrac{z}{z-e^{-aT}}$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Form the Partial Differential Equation, by eliminating arbitrary constants from $(x-a)^2+(y-b)^2+z^2=r^2$, where a and b are arbitrary constants.

22. Find the half-range sine series for $f(x)=x^2$ in $(0, \pi)$.

23. Write the one dimensional heat flow equation and all the possible solutions.

24. Find the Fourier cosine transform of $e^{-ax}, a>0$.

25. Find $z^{-1}\left\{\dfrac{3z^2-18z+26}{(z-1)(z-3)(z-4)}\right\}$ by the partial fraction method.

26. Solve $p^2+q^2=x+y$.

27. State and prove modulation theorem, for Fourier transform.

PART – C (5 × 12 = 60 Marks)

Answer ALL Questions

28. a. i. From the Partial Differential Equation by eliminating the arbitrary functions fr $z = f(x+ct)+\phi(x-ct)$. (8 Marks)

ii. Solve the equation $pq + p + q = 0$. (4 Marks)

(OR)

b.i. Solve the equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

ii. Solve $(D^2 - 2DD')z = x^3y + e^{2x}$.

29. a. Find the Fourier series expansion of period 2 for the function $f(x) = \begin{cases} \pi x \, in \, 0 \le x \le 1 \\ \pi(2-x) \, in \, 1 \le x \le 2 \end{cases}$.

(OR)

b. Compute the first two harmonics of the Fourier series f(x) given by the following table

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1

30. a. A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = k\sin^3\left(\frac{\pi x}{l}\right)$ and the releasing it from this position at time t = 0. Find the displacement of the point of the string at a distance x from one end at time t.

(OR)

b. A rod 30 cm long has its ends A and B kept 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find *u(x, t)*.

31. a. Find the Fourier transform of f(x) if $f(x) = \begin{cases} 1-|x|, for |x| < 1 \\ 0, \, for \, |x| > 1 \end{cases}$ hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$.

(OR)

b. Use transform methods to evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$.

32. a.i. Find $Z\left(\frac{1}{n}\right)$.

ii. Find $Z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$ by using residue theorem.

(OR)

b. Solve the equation $x_{n+2} - 5x_{n+1} + 6x_n = 36$, given that $x_0 = x_1 = 0$, using Z transform.

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Reg. No. | | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. The complete integral of $pq=1$ is
   (A) $az=a^2x+y+ac$ (B) $z=ax+ay+c$
   (C) $az=x+y+c$ (D) $z=x+y+c$

2. The partial differential equation formed by eliminating arbitrary function from $z=f\left(x^2+y^2\right)$ is
   (A) $xp=yq$ (B) $xy=pq$
   (C) $py=qx$ (D) $x+p=y+q$

3. Solve $\left(D^3-7DD'^2-6D'^3\right)Z=0$
   (A) $Z=f_1(y-x)+f_2(y-2x)+f_3(y+3x)$ (B) $Z=f_1(y-x)+f_2(y+2x)+f_3(y-3x)$
   (C) $Z=f_1(y+x)+f_2(y-2x)+f_3(y+3x)$ (D) $Z=f_1(y+x)+f_2(y-2x)+f_3(y+3x)$

4. The partial differential equation is parabolic if
   (A) $B^2-4AC>0$ (B) $B^2-4AC<0$
   (C) $B^2-4AC=0$ (D) $B^2-4AC\neq 0$

5. $\int_{-a}^{a} f(x)dx=2\int_{0}^{a} f(x)dx$ if f(x) is
   (A) Even (B) Odd
   (C) Neither even nor odd (D) Periodic

6. The constant a_0 of the Fourier series $f(x)=k$ in $0\le x\le 2\pi$ is
   (A) k (B) 2k
   (C) 0 (D) $\frac{k}{2}$

7. The RMS value of f(x) in $a\le x\le b$ is
   (A) 0 (B) $\sqrt{\frac{\int_a^b f(x)dx}{b-a}}$
   (C) $\sqrt{\frac{\int_a^b f(x)^2 dx}{b-a}}$ (D) $\sqrt{\frac{\int_a^b f(x)dx}{b+a}}$

8. Half range cosine series for f(x) in $(0, \pi)$ is

(A) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

(B) $\sum_{n=1}^{\infty} b_n \sin nx$

(C) $\sum_{n=1}^{\infty} a_n \cos nx$

(D) $\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty} a_n^2$

9. The one dimensional wave equation is

(A) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(B) $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial u}{\partial t}=a\frac{\partial^2 u}{\partial x^2}$

(D) $\frac{\partial^2 u}{\partial x^2}=a\frac{\partial^2 u}{\partial t^2}$

10. How many initial and boundary conditions are required to solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(A) Four

(B) Two

(C) Three

(D) Five

11. The steady state temperature of a rod of length l whose ends are kept at 30°C and 40°C is

(A) $u=\frac{10x}{l}+30$

(B) $u=\frac{20x}{l}+30$

(C) $u=\frac{10x}{l}+20$

(D) $u=\frac{10x}{l}$

12. One dimensional wave equation is used to find

(A) Temperature

(B) Displacement

(C) Time

(D) Mass

13. The Fourier inverse transform of f(x) is

(A) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

(B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-isx}dx$

(C) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$

(D) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{isx}dx$

14. The Fourier sine transform of e^{-ax} is

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$

(B) $\sqrt{\frac{2}{\pi}}\frac{s}{a^2+x^2}$

(C) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

(D) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$

15. $F\left(e^{iax}f(x)\right)$ is

(A) F(s+a)

(B) F(s−a)

(C) F(s/a)

(D) F(s a)

16. $F\left(f(x)*g(x)\right)$

(A) $F(s)G(s)$

(B) $F(s)+G(s)$

(C) $F(s)-G(s)$

(D) $F(s)/G(s)$

17. Find $z\left[(-1)^n\right]$

(A) $\frac{z+1}{z}$

(B) $\frac{z}{z-1}$

(C) $\frac{z}{z+1}$

(D) $\frac{z}{z-1}$

18. Z-transform of $\frac{1}{n!}$

(A) $e^{\frac{1}{z}}$

(B) e^{z^2}

(C) $e^{\frac{2}{z}}$

(D) e^{z^3}

19. The inverse Z-transform of f(z) can be found out by

(A) Synthetic division method

(B) Long division method

(C) Diagonalisation method

(D) Euler method

20. $Z\left(\cos\frac{n\pi}{2}\right)=$ ______

(A) $\frac{z^2-1}{z^2+1}$

(B) $\frac{z^2-+1}{z}$

(C) $\frac{z^2}{z^2+1}$

(D) $\frac{z}{z^2+1}$

PART – B (5 x 4 = 20 Marks)

Answer **ANY FIVE** Questions

21. Find the partial differential equation of all planes cutting equal intercepts form the x and y axes.

22. Find half-range Fourier sine series for $f(x)=x$ in $0<x<\pi$.

23. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in equilibrium position. If it is set to vibrate by giving each point a velocity $3x(l-x)$, write down the boundary conditions of the problem.

24. Find the Fourier transform of $f(x)=\begin{cases} x & for\,|x|<a \\ 0 & for\,|x|>a \end{cases}$

25. Find the inverse Z-transform of $\frac{1+2z^{-1}}{1-z^{-1}}$, by the long division method.

26. Solve $\sqrt{P}+\sqrt{q}=1$.

27. Find Z-transform of $r^n\cos n\theta$.

PART – C (5 x 12 = 60 Marks)

Answer **ALL** Questions

28. a.i. Form the partial differential equation by eliminating the arbitrary function from the relation $\phi\left(x^2+y^2+z^2, lx+my+nz\right)=0$.

ii. Find the singular solution of $Z = px + qy + \sqrt{1+p^2+q^2}$.

(OR)

b.i. Solve $(mz-ny)p+(nx-lz)q = ly-mx$

ii. Solve $\left(D^3-2D^2D'\right)z = \sin(x+2y)+3x^2y$

29. a. Find the Fourier series of $f(x)=x+x^2$ in $(-\pi,\pi)$ of periodicity 2π, Hence deduce $\sum\frac{1}{n^2}=\frac{\pi}{6}$

(OR)

b. The values of x and the corresponding values of f(x) over a period T are given below. Show that $f(x)=0.75+0.37\cos\theta+1.004\sin\theta$ where $\theta=\frac{2\pi x}{T}$.

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
f(x)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

30. a. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y=k\left(lx-x^2\right)$ from which it is released at time t = 0. Find the displacement y (x, t).

(OR)

b. A rod of length l has its ends A and B kept at 0°C and 100°C respectively until steady-state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature u(x, t).

31. a. Find the Fourier transform of f(x) given by $f(x)=\begin{cases}1-x^2, & for\,|x|<1\\ 0, & for\,|x|>1\end{cases}$, hence evaluate $\int_0^{\infty}\left(\frac{\sin x - x\cos x}{x^3}\right)\cos\frac{x}{2}dx$.

(OR)

b.i. Use transform method to evaluate $\int_0^{\infty}\frac{x^2}{\left(x^2+a^2\right)\left(x^2+b^2\right)}dx$.

ii. Find the Fourier transform of $f(x)=\begin{cases}1-|x| & if\,|x|<1\\ 0 & if\,|x|>1\end{cases}$ and hence evaluate $\int_0^{\infty}\frac{\sin^4 t}{t^4}dt$.

32. a.i. Use convolution theorem to find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$.

ii. Find $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$, by using residue method.

(OR)

b. Solve the equation $y_{n+2}-7y_{n+1}+12y_n=2^n$ given that $y_0=y_1=0$. Using Z-transform.

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Reg. No.										

B.Tech. DEGREE EXAMINATION, MAY 2015

Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted from the academic year 2013 – 2014 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A ($20 \times 1 = 20$ Marks)

Answer ALL Questions

1. The complete integral of the partial differential equation $p = 2qx$ is
   (A) $z = ax^2+ay+c$
   (B) $z = ax+c$
   (C) $z = ay^2+ax+c$
   (D) $z = ay+c$

2. The partial differential equation formed by eliminating arbitrary function from $z = f(xy)$ is
   (A) $xp = yq$
   (B) $xy = pq$
   (C) $xq = yp$
   (D) $x+p = y+q$

3. The partial differential equation $u_{xx}+4u_{xy}+4u_{yy}-12u_x+u_y+7u=0$ is
   (A) Parabolic
   (B) Elliptic
   (C) Hyperbolic
   (D) Conic

4. Find the complementary function of $\left(D^3-7DD'^2-6D'^3\right)z=0$.
   (A) $z=f_1(y-x)+f_2(y-2x)+f_3(y+3x)$
   (B) $z=f_1(y-x)+f_2(y+2x)+f_3(y-3x)$
   (C) $z=f_1(y+x)+f_2(y-2x)+f_3(y-3x)$
   (D) $z=f_1(y+x)+f_2(y+2x)+f_3(y-3x)$

5. If $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ is the Fourier half range sine series of f(x) in (0,l), then $\int_0^l [f(x)]^2\,dx$ is equal to
   (A) $l\sum_{n=1}^{\infty} b_n^{\,2}$
   (B) $\frac{l}{2}\left[\sum_{n=1}^{\infty} b_n^{\,2}\right]$
   (C) $2l\sum_{n=1}^{\infty} b_n^{\,2}$
   (D) $\frac{l}{2}\left[\frac{a_0^{\,2}}{2}+\sum_{n=1}^{\infty} a_n^{\,2}\right]$

6. The constant term of the Fourier series of the function $f(x)=|x|, -\pi<x<\pi$, is
   (A) $\frac{\pi}{2}$
   (B) π
   (C) 2π
   (D) 0

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7. The sum of the Fourier series of $f(x) = x+x^2$, in $-\pi < x < \pi$ at $x = \pi$ is

(A) π

(B) π^2

(C) $\frac{\pi}{2}$

(D) $\frac{\pi^2}{2}$

8. If $f(x) = x$ in $-l \le x \le l$, then a_n

(A) $\frac{-2l(-1)^n}{n\pi}$

(B) 0

(C) l

(D) $\frac{2l^2}{3}$

9. The steady state temperature of a rod of length 20 cm whose ends are kept respectively at 10°C and 30°C is

(A) $x+10$

(B) $x-10$

(C) $10x+10$

(D) $2x+50$

10. The suitable solution of heat equation is

(A) $u(x,t) = (A\cos\lambda x + B\sin\lambda x)e^{-\alpha^2\lambda^2 t}$

(B) $u(x,t) = (A\cos\lambda x + B\sin\lambda x)e^{\alpha^2\lambda^2 t}$

(C) $u(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})e^{-\alpha^2\lambda^2 t}$

(D) $u(x,t) = (Ae^{\lambda x} + Be^{-\lambda x})e^{-\alpha^2\lambda^2 t}$

11. The string is stretched between two fixed point $x = 0$ and $x = l$. The boundary conditions are (t being positive)

(A) $y(0,t) = 0,\ y(x,t) = 0$

(B) $y(x,0) = 0 \left(\frac{\partial y}{\partial t}\right)_{(x,\,0)} = 0$

(C) $y(0,t) = 0,\ y(l,t) = 0$

(D) $\left(\frac{\partial y}{\partial t}\right)_{(0,\,t)} = 0,\ \left(\frac{\partial y}{\partial t}\right)_{(l,\,t)} = 0$

12. In heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, α^2 stands for

(A) k/ρ

(B) T/m

(C) k/ρc

(D) kc/ρ

13. $F^{-1}[F(s)G(s)]$ is

(A) $f(x).g(x)$

(B) $f(x) * g(x)$

(C) $f(x)+g(x)$

(D) $f(x)-g(x)$

14. Fourier cosine transform of $\frac{1}{\sqrt{x}}$ is

(A) $\sqrt{s}$

(B) $\frac{1}{\sqrt{x}}$

(C) $\frac{1}{\sqrt{s}}$

(D) $\frac{1}{s}$

15. If $F(f(x)) = F(s)$ and $f(x) \to 0$ as $x \to \pm\infty$ then $F(f'(x))$ is

(A) $-isF(s)$

(B) $isF(s)$

(C) $sF(s)$

(D) $\frac{-F(s)}{is}$

16. $F_s(x\, f(x))$ is

(A) $-\frac{d}{ds}F_c(s)$

(B) $\frac{d}{ds}F_c(s)$

(C) $F_s(s)$

(D) $F_c(s)$

17. $Z[(-2)^n]$ is

(A) $\frac{z}{z+2}$

(B) $\frac{-z}{z+2}$

(C) $\frac{-z}{z-2}$

(D) $\frac{z}{z-2}$

18. $Z(n^2)$ is

(A) $\frac{z}{(z-1)^3}$

(B) $\frac{z(z+1)}{z^3}$

(C) $\frac{z(z+1)}{(z-1)^3}$

(D) $\frac{z+1}{(z-1)^3}$

19. $z\left(\sin\frac{n\pi}{2}\right)$ is

(A) $\frac{z^2}{z-1}$

(B) $\frac{z}{z^2+4}$

(C) $\frac{z}{z^2+1}$

(D) $\frac{z^2}{z^2+1}$

20. $Z^{-1}\left(\frac{z}{(z-a)^2}\right)$

(A) a^{n-1}

(B) $n\, a^{n+1}$

(C) $n\, a^{n-1}$

(D) a^{n+1}

PART – B ($5 \times 4 = 20$ Marks)
Answer ANY FIVE Questions

21. Form the partial differential equation by eliminating arbitrary constants 'a' and 'b' from $\log(az-1) = x+ay+b$.

22. Obtain the complete solution of the partial differential equation $p^2+q^2 = x+y$.

23. Find the RMS value of the function $f(x) = 2x-x^2$, $0 < x < 3$.

24. State any two assumptions in deriving one dimensional wave equation and write its all possible solutions.

25. If $F(f(x)) = F(s)$, then prove that $F\{f(x)\cos ax\} = \frac{1}{2}\{F(s-a)+F(s+a)\}$.

26. Prove that $z(\cos n\theta) = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$.

27. Find $Z^{-1}\left[\log\left(\frac{z}{z+1}\right)\right]$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = x^2y + \sin(x+2y)$.

(OR)

b. i. Solve: $\frac{y^2z}{x}p + xzq = y^2$

ii. Find the singular solution of $z = px + qy + \sqrt{1+p^2+q^2}$.

29. a. Expand $f(x) = x(2l-x)$ in $(0,2l)$ as a Fourier series of period $2l$. Hence deduce the sum $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$

(OR)

b. Determine the first three harmonics of the Fourier series for the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.96	1.3	1.05	1.3	– 0.88	– 0.25

30. a. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$, find the displacement.

(OR)

b. Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, subject to the conditions

(i) u is not infinite as $t \to \infty$

(ii) $u = 0$ for $x = 0$ and $x = \pi$ for all t

(iii) $u = \pi x - x^2$ for $t = 0$ in $(0, \pi)$

31. a. Find the Fourier transform of $f(x) = \begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4}\,dt$.

(OR)

b. i. Evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)}$ using transform method.

ii. State and prove Convolution theorem on Fourier transform.

32.a. Find

(i) $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$

(ii) $Z\left(\frac{1}{n!}\right)$

(OR)

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2017
Third/ Fourth/ Fifth Semester

15MA201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The partial differential equation formed by eliminating the arbitrary function from the relation $z = f\left(x^2 + y^2\right)$
   (A) $px = qy$
   (B) $py = qx$
   (C) $p = qy$
   (D) $px = q$

2. The complete solution of $z = px + qy + p^2q^2$
   (A) $z = ax + by + a^2b^2$
   (B) $z = bx + ay + ab$
   (C) $z = ax + by + \sqrt{ab}$
   (D) $z = ax + by + ab$

3. The complementary function of $\left(D^2 - 3DD' + 2D'^2\right)z = 0$
   (A) $z = \phi(y+x) - \phi_2(y-2x)$
   (B) $z = \phi(y+x) + \phi_2(y+2x)$
   (C) $z = \phi_1(y-x) + \phi_2(y+2x)$
   (D) $z = \phi_1(y+x) - \phi_2(y+2x)$

4. The particular integral of $\left(D^2 - 2DD'\right)z = e^{2x}$
   (A) $\frac{e^{-2x}}{2}$
   (B) $\frac{e^{-2x}}{4}$
   (C) $\frac{e^{2x}}{4}$
   (D) $\frac{e^{x}}{4}$

5. The constant a_0 of the Fourier series for the function $f(x) = x$ in $0 \le x \le 2\pi$
   (A) π
   (B) 2π
   (C) 3π
   (D) 0

6. The RMS value of $f(x) = x$ in $-1 \le x \le 1$ is
   (A) 1
   (B) 0
   (C) $\frac{1}{\sqrt{3}}$
   (D) -1

7. If $f(x) = x^2$ in $(-\pi, \pi)$ then the value of b_n is
   (A) 1
   (B) 0
   (C) -1
   (D) 2

8. Half – Range cosine series for f(x) in $(0,\pi)$ is

(A) $\frac{a_0^2}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

(B) $\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$

(C) $\sum_{n=1}^{\infty} b_n \sin nx$

(D) $\sum_{n=1}^{\infty} a_n \cos nx$

9. The one dimensional wave equation is

(A) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(B) $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$

(C) $\frac{\partial y}{\partial t}=\alpha\frac{\partial^2 y}{\partial x^2}$

(D) $\frac{\partial^2 u}{\partial x^2}=a\frac{\partial^2 y}{\partial t^2}$

10. How many initial and boundary condition are required to solve $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$

(A) Two
(B) Three
(C) Five
(D) Four

11. One dimensional heat equation is used to find
(A) Density
(B) Temperature distribution
(C) Time
(D) Displacement

12. The proper solution in steady state heat flow problems is

(A) $u=\left(Ae^{\lambda x}+Be^{-\lambda x}\right)e^{\alpha^2\lambda^2 t}$

(B) $u=Ax+B$

(C) $u=\left(A\cos\lambda x+B\sin\lambda x\right)e^{\alpha^2\lambda^2 t}$

(D) $u=\left(Ae^{\lambda x}+Be^{-\lambda x}\right)\left(Ce^{\lambda at}+De^{-\lambda at}\right)$

13. The Fourier transform of $f(x)$ is

(A) $\frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty} f(x)e^{ist}\,dt$

(B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}\,dx$

(C) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,dx$

(D) $\frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty} f(s)e^{isx}\,dx$

14. If $F[f(x)]=F(s)$ then $F\left[e^{iax}f(x)\right]$ is

(A) $F(as)$

(B) $F\left(\frac{s}{a}\right)$

(C) $F(s-a)$

(D) $F(s+a)$

15. $F[f(x)*g(x)]$ is

(A) $F(s)+G(s)$
(B) $F(s)-G(s)$
(C) $F(s)G(s)$
(D) $F(s)/G(s)$

16. If $F[f(x)]=F(s)$ then $\int_{-\infty}^{\infty}|f(x)|^2\,dx=$

(A) $\int_{-\infty}^{\infty}|F(s)|^2\,ds$

(B) $\int_{-\infty}^{\infty}|F(x)|^2\,dx$

(C) $\int_{0}^{\infty}|F(x)|^2\,dx$

(D) $\int_{0}^{\infty}|F(s)|^2\,ds$

17. What is $Z\left[(-2)^n\right]$

(A) $\dfrac{z}{z+2}$ (B) $\dfrac{-z}{z+2}$

(C) $\dfrac{-z}{z-2}$ (D) $\dfrac{z}{z-2}$

18. $Z\left[n\,a^n\right]=$

(A) $\dfrac{z}{(z-a)^2}$ (B) $\dfrac{a}{(z-a)^2}$

(C) $\dfrac{az}{(z-a)^2}$ (D) $\dfrac{z}{(z-a)^3}$

19. $Z\left[\cos\frac{n\pi}{2}\right]=$

(A) $\dfrac{z}{(z^2+1)}$ (B) $\dfrac{+z}{(z^2-1)}$

(C) $\dfrac{z^2}{z^2-4}$ (D) $\dfrac{z^2}{z^2+1}$

20. Poles of $\phi(z)=\dfrac{z^n}{(z-1)(z-2)}$ are

(A) $z=1,0$ (B) $z=1,2$

(C) $z=0,2$ (D) $z=0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Solve $p\tan x+q\tan y=\tan z$.

22. Find the complete integral of $p^2+q^2=x+y$.

23. Find the half – range Fourier sine series $f(x)=x$ in $0<x<\pi$.

24. State any two assumptions in deriving one dimensional wave equation and write its all possible solutions.

25. If $F\left[f(x)\right]=F(s)$ then prove that $F\left[f(ax)\right]=\dfrac{1}{|a|}F\left(\dfrac{s}{a}\right)$ where $a\neq 0$.

26. Prove that $z(\cos n\theta)=\dfrac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$ if $|z|>1$.

27. Find $Z^{-1}\left[\dfrac{1}{(z-1)(z-2)}\right]$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28.a.i Find the singular solution of $z = px + qy + p^2q^2$.

ii. Solve $x(y-z)p + y(z-x)q = z(x-y)$.

(OR)

b. Solve (i) $\left(D^2 + DD' - 6D'^2\right)z = x^2y + e^{3x+y}$ (ii) $z^2(p^2+q^2) = x^2 + y^2$.

29. a. Find the Fourier series to represent $\left(x - x^2\right)$ in the interval $[-\pi, \pi]$. Deduce the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots.$

(OR)

b. Find the Fourier series upto second harmonic from the following data:

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1	1.4	1.9	1.7	1.5	1.5	1

30. a. A tightly stretched string of length 'l' has its end fastened at $x = 0, x = l$. At $t = 0$, the string is in the form $f(x) = kx(l-x)$ and then released. Find the displacement at any point on the string at a distance x from one end and at any time t>0.

(OR)

b. A rod of length 20 cm has its end A and B kept at 30°C and 90°C respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to 0°C and maintained so, find the temperature $u(x,t)$ at a distance x from A at time t.

31. a. Find the Fourier transform of $f(x)$ given by

$$f(x) = \begin{cases} a^2 - x^2 & if\,|x| < a \\ 0 & if\,|x| > a > 0 \end{cases}$$ hence prove that $\int_0^\infty \frac{\sin x - x\cos x}{x^3}\,dx = \frac{\pi}{4}$

(OR)

b.i Find the Fourier cosine and sine transforms of $f(x) = e^{-ax}, a > 0$.

ii. Evaluate $\int_0^\infty \frac{dx}{\left(a^2+x^2\right)\left(b^2+x^2\right)}$ using transform methods.

32.a.i Find $Z^{-1}[F(z)]$ if $F(z) = \frac{z^2+2z}{z^2+2z+4}$.

ii. Find the inverse z transform of f(z) using residues $\frac{z+3}{(z+1)(z-2)} = f(z)$.

(OR)

b. Solve the equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$ given $y_0 = y_1 = 0$, using Z-transform.

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Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, JUNE 2017
Third Semester

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The complete integral of F(p, q) = 0 is
(A) 0
(B) $px + Qy + C$
(C) $z = ax + by + c$
(D) 2

2. The order and degree of a PDE $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ is
(A) 2, 1.
(B) 1, 2
(C) 2, 2
(D) 1, 1

3. If $B^2 - 4AC < 0$, then linear PDE is called
(A) Elliptic
(B) Parabolic
(C) Hyperbolic
(D) Straight line

4. The PI of $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial z}{\partial y} = 4\sin(x+y)$ is
(A) $4\sin(x+y)$
(B) $-4\cos(x+y)$
(C) $4\cos(x+y)$
(D) 0

5. The Fourier series expansion of an odd function contains ________ only.
(A) Sine terms
(B) Cosine terms
(C) Sine and cosine
(D) Cosec

6. If $f(x) = x^2$ in $(-\pi, \pi)$ then the value for b_n is
(A) 1
(B) 0
(C) –1
(D) 2

7. The process of finding the Fourier series for the function given by the numerical values is known as
(A) Complex analysis
(B) Numerical analysis
(C) Harmonic analysis
(D) Functional analysis

8. In the Fourier series expansion of $f(x) = |\sin x|$ in $(-\pi, \pi)$. What is the value of b_n?

(A) 1
(B) 0
(C) π
(D) $-\pi$

9. The wave equation is

(A) Hyperbolic
(B) Elliptic
(C) Parabolic
(D) Straight line

10. In the Laplace equation $\nabla^2 u = 0$, u is

(A) Temperature
(B) Displacement
(C) Steady state temperature
(D) Mass

11. The suitable solution of heat equation is

(A) $u(x,t) = e^{-c^2\lambda^2 t}(A\cos\lambda x + B\sin\lambda x)$
(B) $u(x,t) = e^{c^2\lambda^2 t}(A\cos\lambda x + B\sin\lambda x)$
(C) $u(x,t) = (A\cos\lambda x + B\sin\lambda x)$
(D) $u(x,t) = e^{\lambda t}(A\cos\lambda x + B\sin\lambda x)$

12. In steady state

(A) $\frac{\partial u}{\partial x} = 0$
(B) $\frac{\partial u}{\partial t} = 0$
(C) $\frac{\partial^2 u}{\partial t^2} = 0$
(D) $\frac{\partial^2 u}{\partial x^2} = 1$

13. Fourier transform pair is represented by

(A) $F(s)$ & $F^{-1}[F(s)]$
(B) $F(s)$ & $F[F(s)]$
(C) $F(x)$ & $F^{-1}[F(x)]$
(D) $F^{-1}(x)$ & $F[F(s)]$

14. The self reciprocal for $F\left[e^{-x^2/2}\right]$ is given by

(A) $e^{-s^2/2}$
(B) $e^{s^2/2}$
(C) $e^{s/2}$
(D) $e^{s/a}$

15. Fourier transform must satisfy

(A) Dirichlet's condition
(B) Absolute integrable
(C) Continuity
(D) All three

16. The inverse Fourier cosine transform f(x)

(A) $\sqrt{\frac{2}{\pi}}\int_0^\infty F_c(s)\cos x\,dx$
(B) $\sqrt{\frac{2}{\pi}}\int_0^\infty F(s)\cos sx\,ds$
(C) $\sqrt{\frac{2}{\pi}}\int_0^\infty F_c(s)\cos sx\,ds$
(D) $\sqrt{\frac{2}{\pi}}\int_{-\infty}^\infty F_c(s)\cos sx\,ds$

17. $z\left[a^n u(n)\right]$ exists only if

(A) $|z| < |a|$
(B) $|z| \le |a|$
(C) $|z| = |a|$

18. z transform of a^n is

(A) $\frac{z}{z+a}$

(B) $\frac{z}{z-a}$

(C) $\frac{z}{z \pm a}$

(D) $\frac{2z}{z+a}$

19. By initial value theorem $z[f(t)] = F(z)$ then f(0) is

(A) $f(0) = \underset{z\to\infty}{Lt} F(z)$

(B) $f(0) = \underset{z\to\infty}{Lt} z(z)$

(C) 1

(D) 0

20. Solve $y_{n+1} - 3y_n = 1$ given $y_0 = 1$.

(A) 3n

(B) 3^{n-1}

(C) 3^n

(D) 2n+1

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $z = (x^2 + a)(y^2 + b)$.

22. Find the particular integral of $\left[D^2 - 4DD' + 4D'^2\right]z = e^{2x+y}$.

23. Find the half range sine series for $f(x) = k$ in $0 < x < \pi$.

24. State any two assumptions in deriving one dimensional wave equation and write its all possible solutions.

25. Prove that $F_c[f(ax)] = \frac{1}{a}F_c\left(\frac{s}{a}\right)$.

26. If $F(f(x)) = F(s)$, then prove that $F\{f(x)\cos ax\} = \frac{1}{2}\{F(s-a) + F(s+a)\}$.

27. Prove that $z\left[\sin\frac{n\pi}{2}\right] = \frac{z}{z^2+1}$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$.

(OR)

b. Find the singular solution of $z = px + qy + \sqrt{p^2 + q^2 + 1}$.

29. a. Find the Fourier series expansion of $f(x) = (\pi - x^2)$ in $(0, 2\pi)$. Deduce the value $\frac{1}{1^2} + \frac{1}{2^2} + \ldots\ldots + \infty$.

(OR)

b. Find the Fourier series upto second harmonic for the following data.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
f(x)	1.0	1.4	1.9	1.7	1.05	1.2

30. a. A string is stretched and fastened to two points l apart. Motion is started by displacing string into the form $y = k(lx - x^2)$ from which it is released at time t = 0. Find displacement of any point of the string at a distance x from one end at any 't'.

(OR)

b. Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, subject to the conditions

(i) u is not infinite as $t \to \infty$.
(ii) $u = 0$ for x = 0 and x = π for all t.
(iii) $u = \pi x - x^2$ for t = 0 in (0, π)

31. a. Find the Fourier transform of the function f(x) is defined by

$$f(x) = \begin{cases} 1 - x^2 & if\ |x| < 1 \\ 0 & if\ |x| \geq 1 \end{cases}$$ hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$.

(OR)

b. Use Fourier transform method to evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$.

32. a. Find (i) $Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right]$ (ii) $Z(n+1)$.

(OR)

b. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transforms.

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B.Tech Degree Examination, Nov 2018

15MA201 – Transforms and Boundary value Problems

Answer key

Part A

1. D) $z = pq$
2. B) $\phi_1(y-x) + x\phi_2(y-x)$
3. A) $\dfrac{e^{2x}}{4}$
4. C) $z = ax+by+a^2b^2$
5. C) 2π
6. B) $2K$
7. C) $\dfrac{1}{\sqrt{3}}$
8. B) $\dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$
9. C) $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$
10. B) $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$
11. B) Temperature Distribution
12. B) $u(x,0) = \dfrac{100x}{l}$
13. A) $F(s+a)$
14. C) $-i\dfrac{dF(s)}{ds}$
15. A) $\sqrt{\dfrac{2}{\pi}}\ \dfrac{4}{16+s^2}$

16. C) $F(s)G(s)$

17. B) $7\dfrac{z}{z-1}$

18. A) $\dfrac{az}{(z-a)^2}$

19. A) $f(0)$

20. C) 3

Part B

21. $z = xy + f(x^2+y^2+z^2) \longrightarrow (1)$

Diff. (1) with respect to x & y

$p = y + f'(x^2+y^2+z^2)(2x+2zp) \longrightarrow (2)$ ———— (1M)

$q = x + f'(x^2+y^2+z^2)(2y+2zq) \longrightarrow (3)$ ———— (1M)

$\dfrac{p-y}{q-x} = \dfrac{x+pz}{y+zq} \longrightarrow (4)$ ———— (1M)

$(y+xz)p - (yz+x)q = y^2 - x^2$ ———— (1M)

22. Let $f(x) = \sum_{n=1}^{\infty} b_n \sin \dfrac{n\pi x}{l}$ $\quad l = \pi$ ———— (1M)

$b_n = \dfrac{2}{l}\int_0^l x \sin \dfrac{n\pi x}{l}\, dx$ ———— (1M)

$= \dfrac{2\cancel{l=\pi}}{n\pi}(-1)^{n+1}$ ———— (1M)

$\therefore f(x) = \dfrac{2\cancel{l=\pi}}{\pi} \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n} \sin \dfrac{n\pi x}{\cancel{l}\,\pi}$ ———— (1M)

23. $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ ———— (1M)

i) $u(x,t) = (A_1 e^{\lambda x} + B_1 e^{-\lambda x}) C_1 e^{\alpha^2 \lambda^2 t}$ ————

ii) $u(x,t) = (A_2 \cos\lambda x + B_2 \sin\lambda x) C_2 e^{-\alpha\lambda t}$ —— (1M)

iii) $u(x,t) = (A_3 x + B_3) C_3$ —— (1M)

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4. $F_s(e^{-ax}) = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin sx \, dx$ —— (1M)

$= \sqrt{\frac{2}{\pi}} \left\{ \frac{e^{-ax}}{a^2+s^2} (-a \sin sx - s \cos sx) \right\}_0^\infty$ —— (2M)

$= \sqrt{\frac{2}{\pi}} \frac{s}{a^2+s^2}$ —— (1M)

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25. Take $a = re^{i\theta}$

$z(a^n) = z(re^{i\theta})^n = z(r^n e^{in\theta})$ —— (1M)

$z(r^n(\cos n\theta + i \sin n\theta)) = \frac{z}{z-a} = \frac{z}{z-re^{i\theta}}$ —— (1M)

$= \frac{z}{z - r(\cos\theta + i\sin\theta)}$

$= \frac{z\{(z - r\cos\theta) + ir\sin\theta\}}{(z-r\cos\theta)^2 + r^2\sin^2\theta}$ —— (1M)

Equating the real parts on both sides

$z(r^n \cos n\theta) = \frac{z(z - r\cos\theta)}{z^2 - 2zr\cos\theta + r^2}$ —— (1M)

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26. $z^{-1}\left(\frac{1}{(z-1)(z-2)}\right) = z^{-1}\left(\frac{1}{z-1} \cdot \frac{1}{z-2}\right)$

$= z^{-1}\left(\frac{1}{z-1}\right) * z^{-1}\left(\frac{1}{z-2}\right)$ —— (1M)

$= 1 * 2^{n-1}$

$z^{-1}\left[\frac{z}{z-a}\right] = a^n$

$= \sum_{r=0}^{n} 1^r \, 2^{n-1-r} = 2^n\left[1 + \frac{1}{2} + (1/2)^2 + \cdots + \frac{1}{2^n}\right]$ —— (1M)

$= 2^n \left[\frac{1-(1/2)^{n+1}}{1-1/2}\right]$ —— (1M)

$= 2^n - 1/2$ —— (1M)

---

27. $p^2 + q^2 = x + y$

$p^2 - x = y - q^2 = a$ —— (1M)

(ix) $p^2 = x+a$ & hence $p = \sqrt{x+a}$

$y - q^2 = a$ & hence $q = \sqrt{y-a}$

But $dz = p\,dx + q\,dy$

$= (x+a)^{1/2}\,dx + (y-a)^{1/2}\,dy$ (1M)

$z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$ (1M)

28.a i) $z = px + qy + \sqrt{1+p^2+q^2} \rightarrow (1)$

$z = ax + by + \sqrt{1+a^2+b^2} \rightarrow (2)$ (1M)

$x = -\frac{a}{\sqrt{1+a^2+b^2}} \rightarrow (3)$

$y = -\frac{b}{\sqrt{1+a^2+b^2}} \rightarrow (4)$ (2M)

$1 - x^2 - y^2 = \frac{1}{1+a^2+b^2}$ (1M)

$\therefore \sqrt{1+a^2+b^2} = \frac{1}{\sqrt{1-x^2-y^2}} \rightarrow (5)$

From (3) & (4)

$a = \frac{-x}{\sqrt{1-x^2-y^2}}$ & $b = \frac{-y}{\sqrt{1-x^2-y^2}} \rightarrow (6)$ (1M)

Sub: (5) & (6) in (2), we get

$x^2 + y^2 + z^2 = 1$ (1M)

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ii) The subsidiary equations are

$\frac{dx}{x(z^2-y^2)} = \frac{dy}{y(x^2-z^2)} = \frac{dz}{z(y^2-x^2)} \rightarrow (1)$ (1M)

$x\,dx + y\,dy + z\,dz = 0$

$x^2 + y^2 + z^2 = c_1$ (2M)

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\log x + \log y + \log z = \log c_2$$

$$\log xyz = \log c_2$$

$$xyz = c_2 \quad \text{(2m)}$$

Hence the general solution is $\phi(x^2+y^2+z^2, xyz)=$ (1

28. b i) The auxiliary equation is

$$m^2 - 2m + 1 = 0 \quad \text{(2m)}$$

$$m = 1, 1$$

C.F is $\phi_1(y+x) + x\phi_2(y+x)$ (2m)

$$P.I = \frac{\cos(x-3y)}{D^2 - 2DD' + D'^2} = -\frac{1}{16}\cos(x-3y) \quad \text{(2m)}$$

$\therefore$ The complete solution is $z = \phi_1(y+x) + x\phi_2(y+x)$

$$-\frac{1}{16}\cos(x-3y) \quad \text{(2m)}$$

ii) since this is non homogeneous give marks for P.I

$$P.I = \frac{1}{D^2 - DD'^2} e^{x+2y} \quad \text{(1M)}$$

$$= \frac{1}{1-4} e^{x+2y} \quad \text{(2m)}$$

$$= -\frac{1}{3} e^{x+2y} \quad \text{(1M)}$$

29.a Let $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \rightarrow ①$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+x^2)\,dx = \frac{2}{3}\pi^2 \rightarrow ②$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left\{ (x+x^2)\frac{\sin nx}{n} - (1+2x)\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right\}_{-\pi}^{\pi}$$

$$= \frac{4}{n^2}(-1)^n \qquad (2m)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left\{ (x+x^2)\left(-\frac{\cos nx}{n}\right) - (1+2x)\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right\}_{-\pi}^{\pi}$$

$$= -\frac{2}{n}(-1)^n \qquad (2m)$$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right] \rightarrow ③ \qquad (1M)$$

put $x = \pi$ in ③ (2m)

$$\pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\therefore \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \infty = \frac{\pi^2}{6}. \qquad (1M)$$

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b. $f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + b_1\sin x + b_2\sin 2x$ (1m)

$$a_0 = \frac{2}{6}\sum f(x)$$

$$a_1 = \frac{2}{6}\sum f(x)\cos x$$

$$a_2 = \frac{2}{6}\sum f(x)\cos 2x$$

$$b_1 = \frac{2}{6}\sum f(x)\sin x$$

$$b_2 = \frac{2}{6}\sum f(x)\sin 2x$$

x	$f(x)$	$\cos x$	$y\cos x$	$\sin x$	$y\sin x$	$\cos 2x$	$y\cos 2x$	$\sin 2x$	$y\sin 2x$
0	1.0	1	1	0	0	1	1	0	0
$\frac{\pi}{3}$	1.4	0.5	0.7	0.866	1.21	−0.5	−0.7	0.866	1.21
$\frac{2\pi}{3}$	1.9	−0.5	−0.95	0.866	1.64	−0.5	−0.95	−0.866	−1.64
π	1.7	−1	−1.7	0	0	1	1.7	0	0
$\frac{4\pi}{3}$	1.5	−0.5	−0.75	−0.866	−1.3	−0.5	−0.75	0.866	1.3
$\frac{5\pi}{3}$	1.2	0.5	0.6	−0.866	−1.04	−0.5	−0.6	−0.866	−1.04
			−1.1				−0.3		−0.17

(4M)

$$\left.\begin{aligned} a_0 &= 2.9 \\ a_1 &= -0.37 \\ a_2 &= -0.1 \\ b_1 &= 0.17 \\ b_2 &= -0.06 \end{aligned}\right\}$$

(5M)

$\therefore f(x) = 1.45 - 0.33\cos x - 0.1\cos 2x + 0.17\sin x - 0.06\sin 2x$ (1M)

30.a. $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \rightarrow ①$

$y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$ ② (1M)

The boundary conditions are

$$\left.\begin{aligned} &\text{i) } y(0,t) = 0, \quad t \geq 0 \\ &\text{ii) } y(l,t) = 0, \quad t \geq 0 \\ &\text{iii) } y(x,0) = 0, \quad 0 \leq x \leq l \\ &\text{iv) } \left.\frac{\partial y}{\partial t}\right|_{t=0} = 3x(l-x), \quad 0 \leq x \leq l \end{aligned}\right\} \rightarrow ③$$

(4M)

using (i) in ②, we get $A = 0$ (1M)

using (ii) we get $p = \frac{n\pi}{l}$ (1M)

Using (iii), we get $c=0$.

$\therefore$ The most general solution is

$$y(x,t)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi a t}{l} \longrightarrow (4)$$

Using (iv) in (4), we get

$$3x(l-x)=\sum_{n=1}^{\infty}\frac{n\pi a}{l}B_n\sin\frac{n\pi x}{l} \qquad (1M)$$

$$\frac{n\pi a}{l}B_n=\frac{2}{l}\int_0^l 3x(l-x)\sin\frac{n\pi x}{l}\,dx$$

$$=\frac{6}{l}\left\{(lx-x^2)\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi}-(l-2x)\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}+(-2)\left(\cos\frac{n\pi x}{l}\right)\frac{l^3}{n^3\pi^3}\right\}_0^l$$

$$B_n\frac{n\pi a}{l}=\frac{24l^2}{n^3\pi^3}, \text{ if } n \text{ is odd}$$
$$=0, \text{ if } n \text{ is even} \qquad (1M)$$

$$\therefore B_n=\frac{24l^3}{n^4\pi^4 a}$$

$$\therefore y(x,t)=\frac{24l^3}{\pi^4 a}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^4}\sin\frac{n\pi x}{l}\sin\frac{n\pi a t}{l} \qquad (1M)$$

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b. The PDE of one dimensional heat flow is

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2} \longrightarrow (1)$$

In steady state (1) becomes

$$\frac{d^2u}{dx^2}=0 \longrightarrow (2)$$

$$u=Ax+b \longrightarrow (3) \qquad (2m)$$

$$u=\frac{100}{l}x$$

$$u(x,t)=(A\cos\lambda x+B\sin\lambda x)e^{-\alpha^2\lambda^2 t} \longrightarrow (4) \qquad (1M)$$

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(F) The boundary conditions are

i) $u(0,t) = 0$, $t \geq 0$
ii) $u(l,t) = 0$, $t \geq 0$
iii) $u(x,0) = \frac{100x}{l}$, $0 < x < l$ — 3M) (3)

1M)

Using (i) in (4), we get $A = 0$

$\therefore u(x,t) = B \sin \lambda x \, e^{-\alpha^2 \lambda^2 t} \rightarrow (5)$ — 1M)

Using (ii) in (5), we get $\lambda = \frac{n\pi}{l}$

$\therefore$ The most general solution is

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} \, e^{-\alpha^2 \frac{n^2\pi^2}{l^2} t} \rightarrow (6)$$ (1M)

Using (iii) in (6), we get

$$f(x) = \frac{100x}{l} = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} \rightarrow (7)$$

$$B_n = \frac{2}{l} \int_0^l \frac{100x}{l} \sin \frac{n\pi x}{l} \, dx$$

$$= \frac{200}{l^2} \left\{ x \left(-\cos \frac{n\pi x}{l}\right) \frac{l}{n\pi} - (1)\left(-\sin \frac{n\pi x}{l}\right) \frac{l^2}{n^2\pi^2} \right\}_0^l$$ (2M)

$$= \frac{200}{n\pi} (-1)^{n+1}$$

$$\therefore f(x) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{l} \, e^{-\alpha^2 \frac{n^2\pi^2}{l^2} t}$$ (1M)

31.a. $F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1 - |x|) e^{isx} \, dx$ — 1M)

$$= \frac{1}{\sqrt{2\pi}} \cdot 2 \int_0^{\infty} (1-x) \cos sx \, dx$$ 2M)

$$= \sqrt{\frac{2}{\pi}} \left(\frac{1 - \cos s}{s^2} \right)$$ 3M)

Using Parseval's identity,

$$\frac{2}{\pi} \int_{-\infty}^{\infty} \left(\frac{1 - \cos s}{s^2} \right)^2 ds = \int_{-1}^{1} (1 - |x|)^2 \, dx$$ (2m)

(10)

$$\frac{4}{\pi}\int_0^\infty \frac{(1-\cos s)^2}{s^4}\,ds = 2\int_0^1 (1-x)^2\,dx$$

$$\frac{16}{\pi}\int_0^\infty \frac{\sin^4 s/2}{s^4} = \frac{2}{3} \quad (3m)$$

setting $\frac{s}{2} = x$, we get

$$\int_0^\infty \frac{\sin^4 x}{x^4}\,dx = \frac{\pi}{3} \quad (2m)$$

31.b. Let $f(x) = e^{-ax}$ and $g(x) = e^{-bx}$ (1m)

$$F_s(f(x)) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx \quad (1m)$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-ax}\sin sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\left\{\frac{e^{-ax}}{a^2+s^2}\left(-a\sin sx + s\cos sx\right)\right\}_0^\infty \quad (2m)$$

$$= \sqrt{\frac{2}{\pi}}\,\frac{s}{a^2+s^2} \quad (1m)$$

$|||^{ly}$ $\quad F_s(e^{-bx}) = \sqrt{\frac{2}{\pi}}\,\frac{s}{b^2+s^2}$ (1m)

By Property

$$\int_0^\infty f(x)\,g(x)\,dx = \int_0^\infty F_s(s)\,G_s(s)\,ds \quad (1m)$$

$$\int_0^\infty e^{-(a+b)x}\,dx = \frac{2}{\pi}\int_0^\infty \frac{s^2\,ds}{(s^2+a^2)(s^2+b^2)} \quad (1m)$$

$$\left[\frac{e^{-(a+b)x}}{-(a+b)}\right]_0^\infty = \frac{2}{\pi}\int_0^\infty \frac{s^2\,ds}{(s^2+a^2)(s^2+b^2)} \quad (2m)$$

$$\frac{\pi}{2(a+b)} = \int_0^\infty \frac{s^2\,ds}{(s^2+a^2)(s^2+b^2)} \quad (1m)$$

$$\therefore \int_0^\infty \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2(a+b)} \quad (1m)$$

i) $Z(a^n) = \sum_{n=0}^{\infty} a^n z^{-n}$ —— (1m)

$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n = \frac{1}{1-a/z}$ —— (1m)

$= \frac{z}{z-a}$ —— (1m)

$Z(n^2) = Z(n \cdot n) = -z\frac{d}{dz}\{Z(n)\}$ —— (1m)

$= -z\frac{d}{dz}\left\{\frac{z}{(z-1)^2}\right\}$ —— (1m)

$= \frac{z(z+1)}{(z-1)^3}$ —— (1m)

---

ii) Let $Z^{-1}\left\{\frac{z}{(z-1)(z-2)}\right\}$ = Sum of the residue of $\{z^{n-1} F(z)\}$

= sum of the residue of $\left(z^n \cdot \frac{1}{(z-1)(z-2)}\right)$ —— (1m)

Residue at $z=1$ is $\lim_{z\to 1} (z-1)\frac{z^n}{(z-1)(z-2)} = -1$ —— (2m)

Residue at $z=2$ is $\lim_{z\to 2} (z-2)\frac{z^n}{(z-1)(z-2)} = 2^n$ —— (1m)

$Z^{-1}\left\{\frac{z}{(z-1)(z-2)}\right\} = -1 + 2^n$ —— (1m)

12.b $Z\{y_{n+2}\} + 6Z\{y_{n+1}\} + 9Z\{y_n\} = Z(2^n)$ (1M)

$$\left[z^2\bar{y}(z) - z^2 y(0) - z y(1)\right] + 6\left[z\bar{y}(z) - z y(0)\right]$$

$$+ 9\bar{y}(z) = \frac{z}{z-2}$$ (2M)

$$(z^2+6z+9)\,\bar{y}(z) = \frac{z}{z-2}$$ (1M)

$$\bar{y}(z) = \frac{z}{(z-2)(z+3)^2}$$ (1M)

By Partial fraction method

$$\frac{\bar{y}(z)}{z} = \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$ (1M)

$$A = \frac{1}{25},\ B = -\frac{1}{25} \ \& \ C = -\frac{1}{5}$$ (2M)

$$\therefore \bar{y}(z) = \frac{1}{25}\left(\frac{z}{z-2}\right) - \frac{1}{25}\left(\frac{z}{z+3}\right) - \frac{1}{5}\left(\frac{z}{(z+3)^2}\right)$$ (2M)

Taking inverse Z-transform

$$y(n) = \frac{1}{25}2^n - \frac{1}{25}(-3)^n - \frac{n}{5}(-3)^{n-1}$$ (2M)

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