| Reg. No. | | | | | | | | | | | | | | |
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# B.Tech. DEGREE EXAMINATION, NOVEMBER 2023
Fourth, Fifth and Sixth Semester

## 18MAB302T – DISCRETE MATHEMATICS FOR ENGINEERS
*(For the candidates admitted from the academic year 2020-2021 & 2021-2022)*

**Note:**
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part - B & Part - C** should be answered in answer booklet.

Time: 3 hours | Max. Marks: 100

## PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

| | Marks | BL | CO |
|---|---|---|---|
| 1. The cardinality of the power set of the set {0, 1, 2} is<br>(A) 8 (B) 6<br>(C) 7 (D) 9 | 1 | 1 | 1 |
| 2. If A is a non-empty set with n elements, then the number of possible relations on the set A is<br>(A) $2^n$ (B) $2^{n-1}$<br>(C) $2^{n^2}$ (D) $2^{n+1}$ | 1 | 1 | 1 |
| 3. A relation R on set A is defined as if $x\mathbb{R}y, y\mathbb{R}x$, then $x = y \ \forall x, y \in A$, then $\mathbb{R}$ is _____<br>(A) Symmetric (B) Asymmetric<br>(C) Antisymmetric (D) Transitive | 1 | 2 | 1 |
| 4. A function f: A→B is said to be __________ if for every y∈B there exist at least one element x∈A such that f(x)=y.<br>(A) Surjective (B) Bijective<br>(C) Injective (D) Automorphism | 1 | 2 | 1 |
| 5. If the object A is chosen in M ways and B in N ways then either A or B is chosen in _________ ways.<br>(A) M/N (B) MN<br>(C) M+N (D) M–N | 1 | 1 | 2 |
| 6. Assuming that repetitions are not permitted, how many four-digit numbers are less than 4000, can be formed from the six digits 1, 2, 3, 5, 7, 8?<br>(A) 125 (B) 124<br>(C) 63 (D) 180 | 1 | 2 | 2 |
| 7. Every integer n>1 can be represented uniquely as a product of<br>(A) Prime members (B) Composite numbers<br>(C) Even numbers (D) Odd numbers | 1 | 1 | 2 |

8. For any positive integers a and 3 there exists unique integers q and r such that a=3q+r where r must satisfy 1 2 2

(A) $1 < r < 3$ (B) $0 < r < 3$

(C) $0 \leq r < 3$ (D) $0 < r \leq 3$

9. The biconditional is conjunction of two ________ statement. 1 1 3

(A) Negation (B) Compound

(C) Connective (D) Conditional

10. Negation of $P \rightarrow (P \vee \neg Q)$ is

(A) $\neg P \rightarrow (\neg P \vee Q)$ (B) $P \wedge (\neg P \wedge Q)$

(C) $\neg P \vee (\neg P \vee \neg Q)$ (D) $\neg P \rightarrow (\neg P \rightarrow Q)$

11. P→Q is logically equivalent to ________. 1 2 3

(A) $\neg P \vee \neg Q$ (B) $P \vee \neg Q$

(C) $\neg P \vee Q$ (D) $\neg P \wedge Q$

12. A premises may be introduced at any point in the derivation is called 1 1 3

(A) Rule T (B) Rule US

(C) CP rule (D) Rule P

13. Fourth root of unity namely 1, –1, i,–i form a group with respect to 1 2 4

(A) Addition (B) Subtraction

(C) Multiplication (D) Division

14. If G is a finite group and order of group is m, then for all $a \in G$ 1 1 4

(A) $a^m \neq e$, an identity (B) $a^m = e$, an identity

(C) $a^m = a$ (D) $a^m = a^{-1}$

15. A finite integral domain is a ________. 1 1 4

(A) Subfield (B) Vector

(C) Field (D) Ring

16. If in a ring R, the exist an elements a, b such that a*b =0 implies either a = 0 or b = 0 or both a = 0 and b = 0 then R is 1 2 4

(A) Ring with unit element (B) Ring with zero divisor

(C) Ring without zero divisor (D) Boolean ring

17. A graph G is said to be a simple graph 1 1 5

(A) G has no loops (B) There is exactly one edge between any given pair of vertices

(C) Both (A) and (B) (D) It contains only parallel edges

18. If G is an undirected graph with 12 edges. Also, it is given that two vertices are of degree 2, two are of degree 3, and one of degree 4 and remaining are of degree 5. How many total vertices are there in G? 1 2 5

(A) 8 (B) 7

(C) 9 (D) 10

| | | | | |
|---|---|---|---|---|
| 19. A path in a connected graph G=V, E) is called Hamilton path if<br>(A) It includes every edge exactly once (B) It includes every vertex exactly once<br>(C) It includes every edge exactly twice (D) It includes every vertex exactly twice | | 1 | 1 | 5 |
| 20. A degree of pendent vertex is<br>(A) 0 (B) 1<br>(C) 2 (D) 3 | | 1 | 2 | 5 |

## PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

| | Marks | BL | CO |
|---|---|---|---|
| 21. Prove that $(A-C)\cap(C-B)=\varnothing$ analytically where A, B and C are sets. | 4 | 3 | 1 |
| 22. Draw the Hasse diagram for (D12, \|) where $D_{12}$ is the set of positive integers divisor of 12. | 4 | 4 | 1 |
| 23. Find the number of ways of preparing a garland with 3 yellow, 4 pink and 2 red roses of different sizes such that the two red roses come together. | 4 | 3 | 2 |
| 24. Construct the truth table for the following $(P\rightarrow Q)\leftrightarrow(\neg Q\rightarrow\neg P)$ | 4 | 4 | 3 |
| 25. Prove the following implication without using truth table $P\Rightarrow(Q\rightarrow P)$. | 4 | 3 | 3 |
| 26. If α, β are elements of the symmetric group $S_4$ given by $\alpha=\begin{pmatrix}1&2&3&4\\3&4&2&1\end{pmatrix}$ and $\beta=\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}$ Find $\alpha\beta$, $\beta\alpha$, $\alpha^2$ and $\alpha^{-1}$. | 4 | 3 | 4 |
| 27. Prove that the number of edges in a bipartite graph with n vertices is atmost $n^2/4$. | 4 | 4 | 5 |

## PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

| | Marks | BL | CO |
|---|---|---|---|
| 28. a.i. R is the relation on the set of integers such that (a, b) ∈R if 3a+4b=7n for some integer n, prove R is equivalence relation? | 12 | 3 | 1 |
| ii. If A={1, 2, 3, 4, 5}, B={1, 2, 3, 8, 9} and f:A→B and g:A→A are defined by f={(1, 8), (3,9), (4,3), (2, 1), (5,2)} and g={(1, 2), (3, 1), (2, 2), (4, 3), (5, 2)}. Find $f\circ g, g\circ f, f\circ f, g\circ g$ if they exist. | 12 | 3 | 1 |
| **(OR)** | | | |
| b. Let $A=\{1,2,3,4\}$ and $R=\{(1,2),(2,3),(2,1),(3,4)\}$ using Warshall's algorithm, find the transitive closure of R. | 12 | 3 | 1 |
| 29. a.i. If there are 5 points inside a square of side length 2. Prove that two of the points are within a distance of $\sqrt{2}$ of each other | 12 | 4 | 2 |
| ii. In a group of 72 students, 47 have background in electronics, 59 have background in mathematics and 42 have background in both the subject, how many students do not have background in any of the subjects? | 12 | 4 | 2 |

**(OR)**

b. Use the Euclidean algorithm to find gcd(1819, 3587) and also express linear combination of the given number. 12 4 2

30. a.i. Prove the following by using direct method 12 4 3

$P\vee Q, Q\rightarrow R, P\rightarrow S, \neg S \Rightarrow R\wedge(P\vee Q)$

ii. Show that $P\rightarrow Q, P\rightarrow R, Q\rightarrow \neg R$ and $P$ are inconsistent. 12 4 3

**(OR)**

b.i. Use indirect method of proof to show that 12 4 3

$R\rightarrow\neg Q, R\vee S, S\rightarrow\neg Q, P\rightarrow Q \Rightarrow \neg P$

ii. Use mathematical induction to show that $n!\geq 2^{n-1}\ \forall n\geq 1$. 12 4 3

31. a.i Let $Q^{+}=\{\text{set of all postive rational number}\}$, let $*$ be defined on $Q^{+}$ by $a*b=\dfrac{ab}{3}$ $a,b\in Q^{+}$, prove that $(Q^{+},*)$ is an abelian group. 12 4 4

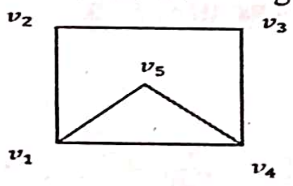
ii. Prove that the intersection of two subgroups of a group is also a subgroup of a group. 12 4 4

**(OR)**

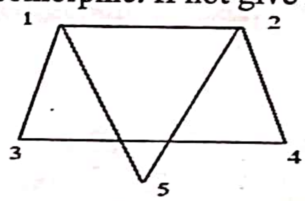
b. Find the code words generated by the parity check matrix 12 4 4

$$H=\begin{bmatrix}1&1&1\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$ when the encoding function is $e:B^{3}\rightarrow B^{6}$.

32. a.i. Check whether the following graphs are isomorphic. If not give reason. 12 3 3




$G_1(V_1,E_1)$

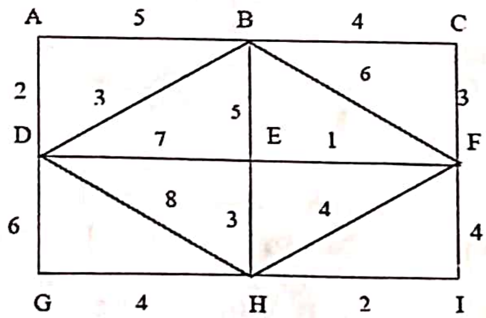



$G_2(V_2,E_2)$

ii. Prove that a tree with n vertices has n-1 edges. 12 3 3

**(OR)**

b. Use Kuskal's algorithm to find a minimum spanning tree for the weighted graph. 12 4 4

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# B. Tech Degree Examination, Nov 2023

## 18MAB302T - Discrete Mathematics for Engineers.

## Answer Key

### PART - A (20 x 1 = 20 Marks)

1. A. 8
2. C. $2^{n^2}$
3. C. Antisymmetric
4. A. Surjective
5. C. M+N
6. D. 180
7. A. Prime members
8. C. $0 \le r < 3$
9. D. Conditional
10. B. $P \wedge (\neg P \wedge Q)$
11. C. $\neg P \vee Q$
12. D. Rule P
13. C. Multiplication
14. B. $a^m = e$, an identity
15. C. Field
16. C. Ring without zero divisor
17. C. Both (A) and (B)
18. B. 7
19. B. It includes every vertex exactly once
20. B. 1

### PART - B (5x4 = 20 Marks)

Answer any five Questions

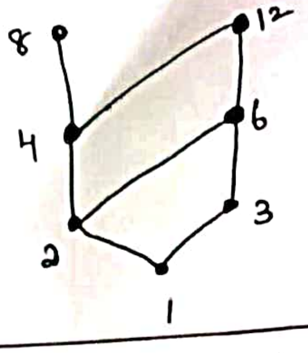
21. $(A-C) \cap (C-B) = \{x \mid x \in A \text{ and } x \notin C \text{ and } x \in C \text{ and } x \notin B\}$

$= \{x / x \in A \text{ and } (x \in C \text{ and } x \in \bar{C}) \text{ and } x \notin B\}$

$= \{x / (x \in A \text{ and } x \in \phi) \text{ and } x \in \bar{B}\}$

$= \{x / x \in A \cap \phi \text{ and } x \in \bar{B}\}$ (4m)

$= \{x / x \in \phi \cap \bar{B}\} = \{x / x \in \phi\} = \phi.$

22. 


(4m)

23. The number of ways of preparing a garland with 3 yellow, 4 pink and 2 red roses of different sizes such that the two red roses come together is $\frac{7! \times 2!}{2} = 5040$. (4m)

24.

| P | Q | ¬P | ¬Q | P→Q | ¬Q→¬P | (P→Q)↔(¬Q→¬P) |
|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

(4m)

25. TPT : $P \to (Q \to P)$ is a tautology

$$
\begin{aligned}
P \to (Q \to P) &\equiv P \to (\neg Q \vee P) \\
&\equiv \neg P \vee \neg Q \vee P \\
&\equiv \neg P \vee P \vee \neg Q \\
&\equiv T \vee \neg Q \\
&\equiv T
\end{aligned}
$$

(4m)

26. $\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ (1m) $\beta\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ (1m) $\alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ (1m) $\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ (1m)

27. The vertex set $V_1$ contains $x$ vertices and $V_2$ contains $(n-x)$ vertices.
The largest number of ~~ver~~ edges, $f(x) = x(n-x)$

$f'(x) = n - 2x$ and $f''(x) = -2$ (2m)

$f'(x) = 0$ when $x = \frac{n}{2}$ and $f''\left(\frac{n}{2}\right) < 0$. Hence $f(x)$ is maximum when $x = \frac{n}{2}$. Therefore maximum number of edges required (2m)

$= f\left(\frac{n}{2}\right) = \frac{n^2}{4}$.

## Part – C (5 × 12 = 60 Marks)

### Answer all questions.

28)

a.i) Reflexive: $3a + 4a = 7a$ when $a$ is an integer (2m)

Symmetry: $3b + 4a = 7a + 7b - (3a + 4b)$

$= 7(a+b) - 7n = 7(a+b-n)$

where $a+b-n$ is an integer. $\therefore (b,a) \in R$ when $(a,b) \in R$ (2m)

Transitive: let $(a,b)$ and $(b,c) \in R$

$3a + 4b = 7m$ — ① and $3b + 4c = 7n$ — ②

① + ② $\Rightarrow 3a + 7b + 4c = 7m + 7n$

$\Rightarrow 3a + 4c = 7(m+n-b)$ where $m+n-b$ is an integer (2m)

$\therefore (a,c) \in R$

$\Rightarrow R$ is an equivalence relation.

a. ii) $f \circ g = \{(1,1), (2,1), (3,8), (4,9), (5,1)\}$ (2m)

$g \circ f$ is not defined. [ range $(f) \notin$ dom $(g)$ ] (1m)

$f \circ f$ is not defined [ range $(f) \notin$ dom $(f)$ ] (1m)

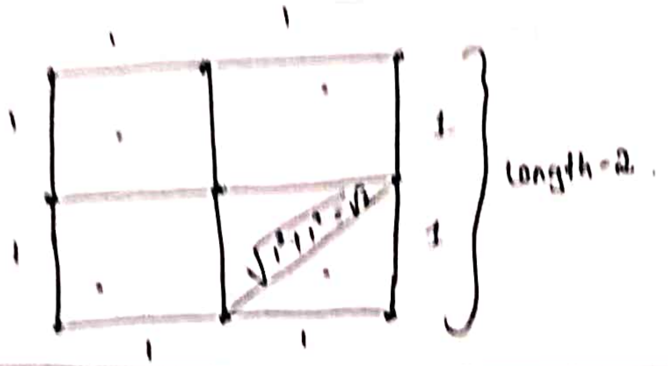
$g \circ g = \{(1,2), (2,2), (3,2), (4,1), (5,2)\}$ (2m)

28) b) $R = \{(1,2), (2,3), (2,1), (3,4)\}$  $W_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (2m)

| $k$ | pos. of 1's in col. $k$ | pos. of 1's in row $k$ | New pos of 1's in $W_K$ | $W_K$. | |
|---|---|---|---|---|---|
| 1 | 2 | 2 | (2,2) | $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | (2m) |
| 2 | 1, 2 | 1, 2, 3 | (1,1) (1,2), (1,3), (2,1), (2,2), (2,3) | $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | (2m) |
| 3 | 1, 2 | 4 | (1,4), (2,4) | $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | (2m) |
| 4 | 1, 2, 3 | – | – | $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | (2m) |

Transitive closure of $R = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\}$ (2m)

27) a i)




Points - Pigeons

Subsquares - pigeonholes.

(6m)

a ii) $|E| = 47$ ; $|H| = 59$ ; $|E \cap H| = 42$

$|E \cup H| = |E| + |H| - |E \cap H|$ (4m)

$= 47 + 59 - 42$

$= 64.$

No. of Students do not have background in any of the subjects (2m)

$= 72 - 64 = 8.$

29) b)

$3587 = 1 \cdot 1819 + 1768$

$1819 = 1 \cdot 1768 + 51$

$1768 = 34 \cdot 51 + 34$

$51 = 1 \cdot 34 + 17$

$34 = 2 \cdot 17 + 0$ (6m)

$\gcd(1819, 3587) = 17$

$17 = 51 - 1 \cdot 34$

$= 51 - 1 \cdot (1768 - 34 \cdot 51)$

$= 35 \cdot 51 - 1 \cdot 1768$

$= 35 \cdot (1819 - 1 \cdot 1768) - 1 \cdot 1768$

$= 35 \cdot 1819 - 36 \cdot 1768$

$= 35 \cdot 1819 - 36\,(3587 - 1 \cdot 1819)$

$= 71 \cdot 1819 - 36 \cdot 3587.$ (6m)

30) a i)

| Step No | Statement | Reason | |
|---|---|---|---|
| 1. | $P \vee Q$ | P | (1m) |
| 2. | $Q \to R$ | P | |
| 3. | $P \to S$ | P | |
| 4. | $\neg S$ | P | |
| 5. | $\neg P$ | Modus tollen [3 & 4] | |
| 6. | $\neg P \to Q$ | from ①, $p \to q \equiv \neg p \vee q$ | (2m) |
| 7. | $Q$ | Modus ponen 5 & 6 | (2m) |
| 8. | $R$ | Modus ponen 2 & 7 | |
| 9. | $R \wedge (P \vee Q)$ | conjunction of 1 and 8. | (1m) |

a ii) Step N

10 a ii)

| Step No | Statement | Reason | |
|---|---|---|---|
| 1. | $P \to Q$ | P | |
| 2. | $P \to R$ | P | (1m) |
| 3. | $Q \to \neg R$ | P | |
| 4. | $P$ | P | |
| 5. | $Q$ | 1,4, Modus ponen | |
| 6. | $\neg R$ | 3,5, Modus ponen | (2m) |
| 7. | $P \to \neg R$ | 1,3, Hypothetical Syllogism | |
| 8. | $\neg P$ | 6,7, Modus tollen | |
| 9. | $P \wedge \neg P$ | 4&8 - conjunction | (2m) |
| 10. | $F$ | 9, Negation law | |
| 11. | $F \wedge (\neg P \vee R)$ | conjunction of 2 and 10. | (1m) |
| 12. | $F$ | 12, Dominant law | |

30. bi)

| Step No | Statement | Reason | |
|---|---|---|---|
| 1. | $R \to \neg Q$ | P | |
| 2. | $R \vee S$ | P | (1m) |
| 3. | $S \to \neg Q$ | P | |
| 4. | $P \to Q$ | P | |
| 5. | $P$ | AP | |
| 6. | $Q$ | 4 & 5, Modus ponen | |
| 7. | $\neg R \to S$ | from ② | (2m) |
| 8. | $\neg R \to \neg Q$ | 3 & 7, Hypothetical Syllogism | |
| 9. | $Q \to R$ | contrapositive of 8 | |
| 10. | $Q \to \neg Q$ | 1 & 9, Hypothetical Syllogism | (2m) |
| 11. | $\neg Q$ | from 10. | |
| 12. | $Q \wedge \neg Q$ | 6 & 11, conjunction | (1m) |
| 13. | $F$ | 12, Negation law. | |

③

30. b. ii) $S_n : n! \geq 2^{n-1}$

$S_1 : 1! \geq 2^0$ which is true (1m)

Let $S_k$ be true i.e $k! \geq 2^{k-1}$ — ①

Now $(k+1)! = (k+1)\,k!$

$\geq (k+1) \cdot 2^{k-1}$ [from ①]

$\geq 2 \cdot 2^{k-1}$ Since $k+1 \geq 2$ (1m)

$= 2^k$

$\Rightarrow S_{k+1}$ is also true

$\Rightarrow S_n$ is true for $n = 1, 2, 3, \ldots$ (1m)

31. a i) $a * b = \frac{ab}{3}$ where $a, b \in Q^+$

When $a, b \in Q^+$, $\frac{ab}{3} \in Q^+ \Rightarrow Q^+$ is closed under the operation $*$

Now $(a*b)*c = a*(b*c)$ (2m)

$\Rightarrow \frac{ab}{3} * c = a * \frac{bc}{3} \Rightarrow \frac{abc}{9} = \frac{abc}{9}$

$\Rightarrow$ Associative holds good.

Identity : $a * e = a$, $a \in Q^+$

$\frac{ae}{3} = a \Rightarrow ae = 3a \Rightarrow ae - 3a = 0$

$a(e-3) = 0$ (3m)

$\Rightarrow e = 3$

Inverse : $a * a^{-1} = e$

$\frac{aa^{-1}}{3} = 3 \Rightarrow aa^{-1} = 9 \Rightarrow a^{-1} = \frac{9}{a}$.

Commutative : $a * b = b * a$

$\Rightarrow \frac{ab}{3} = \frac{ba}{3}$. (1m)

$\Rightarrow (Q^+, *)$ forms an abelian group.

a. ii) Let $a \in H_1 \cap H_2$ Then $a \in H_1$ and $a \in H_2$

$b \in H_1 \cap H_2$ Then $b \in H_1$ and $b \in H_2$

$H_1$ is a sbgp of $G$ : $H_2$ is a sbgp of $G$

$a * b^{-1} \in H_1$ (2m) : $a * b^{-1} \in H_2$ — (2m)

$\Rightarrow a * b^{-1} \in H_1 \cap H_2$, $\because a, b \in H_1 \cap H_2$ — (2m)

$\Rightarrow H_1 \cap H_2$ is a subgroup of $G$.

(4)

31. b

$$H = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] = [A^T \mid I_{n-m}]$$ (2m)

$$G = [I_m \mid A] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$ (2m)

$B^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}$

$e(000) = [000000]$

$e(001) = [001011]$

$e(010) = [010101]$

$e(100) = [100111]$ (8m)

$e(011) = [011110]$

$e(101) = [101100]$

$e(110) = [110010]$

$e(111) = [111001]$

The codewords generated are 000000, 001011, 010101, 100111, 011110, 101100, 110010 and 111001.

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32. a. i)

$G_1$ : $|V_1| = 5$, $|E_1| = 6$

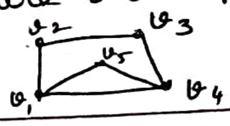
$G_2$ : $|V_2| = 5$, $|E_2| = 6$ (2m)

Degree sequence : 3, 2, 2, 3, 2.

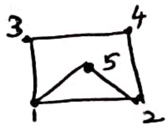
Degree sequence : 3, 3, 2, 2, 2

The graphs $G_1$ and $G_2$ are isomorphic. (4m)

$G_1$ :




$G_2$ :

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a ii) The property is true for $n = 1, 2, 3$ (1m)

Let us assume it is true for all trees with less than $n$ vertices. Now consider a tree $T$ with $n$ vertices. Let $e_k$ be the edge connecting the vertices $v_i$ and $v_j$ of $T$. Delete the edge $e_k$, $T$ becomes disconnected. (3m)

$T_1 \rightarrow$ $r$ vertices, $r-1$ edges

$T_2 \rightarrow$ $n-r$ vertices, $n-r-1$ edges.

Induction hypothesis

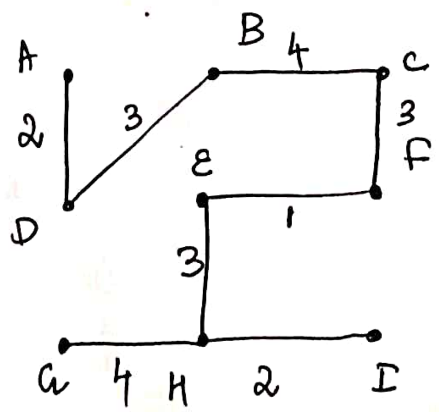
$\therefore$ T has $(r-1) + (n-r-1) + 1 = n-1$ edges (2m)

Thus a tree with $n$ vertices has $(n-1)$ edges.

32) b.

| Edge | Weight | Included in the spanning tree or not | If not included circuit formed |
|---|---|---|---|
| EF | 1 | Yes | – |
| AD | 2 | Yes | – |
| HI | 2 | yes | – |
| BD | 3 | yes | – |
| CF | 3 | yes | – |
| EH | 3 | Yes | – |
| BC | 4 | yes | – |
| FH | 4 | NO | E-F-H-E |
| FI | 4 | NO | E-F-I-H-E |
| GH | 4 | Yes | – |
| AB | 5 | – | – |
| BE | 5 | – | – |
| BF | 6 | – | – |
| DG | 6 | – | – |
| DE | 7 | – | – |
| DH | 8 | – | – |

(8m)




The total length of the minimum spanning tree = 22.

(2m)