

B.Tech. DEGREE EXAMINATION, DECEMBER 2022
Three Semester

18MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted from the academic year 2018-2019 to 2021-2022)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
- (ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours

PART – A (25 × 1 = 25 Marks)

Answer ALL Questions

Max. Marks: 75

Marks BL CO PO

- The complete integral of $q = 2py$ is _____
 (A) $z = ax + ay^2 + b$ (B) $z = ax^2 - ay^2 + b$
 (C) $z = ax + by$ (D) $z = 2xy$
 1 2 1 2
- The solution to $pq = x$ is
 (A) $z = \frac{x^2}{2a} + ay + c$ (B) $z = \frac{y^2}{2a} + ax + c$
 (C) $z = x + y + 1$ (D) $z = x - ay$
 1 2 1 1
- The partial differential equation formed by eliminating arbitrary function in $z = f(x^2 + y^2)$ is
 (A) $xy = pq$ (B) $xp = yq$
 (C) $xq = yp$ (D) $x + p = y + q$
 1 2 1 2
- Solve $(D^3 - 3D^2D')z = 0$
 (A) $z = f_1(y-x) + f_2(y-2x) + f_3(y+2x)$
 (B) $z = f_1(y) + f_2(y) + f_3(y+3x)$
 (C) $z = f_1(y) + x f_2(y) + f_3(y+3x)$
 (D) $z = f_1(y) + f_2(y) + f_3(y-3x)$
 1 2 1 2
- The particular integral $D^2z = x^3y$ is
 (A) x^3y (B) x^4y^2
 (C) x^2y^2 (D) $\frac{x^5y}{20}$
 1 2 1 2
- The constant a_0 for the Fourier series for the function $f(x) = k, 0 \leq x \leq 2\pi$ is
 (A) k (B) $2k$
 (C) 0 (D) $k/2$
 1 1 2 1
- Which one of the following function is an even function?
 (A) $\sin x$ (B) x
 (C) e^x (D) x^2
 1 1 2 1
- The Root Mean Square value of $f(x) = x$ in $-1 \leq x \leq 1$ is
 (A) 1 (B) 0
 (C) $\frac{1}{\sqrt{3}}$ (D) -1
 1 2 2 2

9. Half range cosine series for $f(x)$ in $(0, \pi)$ is

(A) $\sum_{n=1}^{\infty} b_n \sin nx$

(C) $\frac{a_0}{2} + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2$

(B) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

(D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sin nx$

10. An example for a function which is neither even nor odd.

(A) $x \sin x$

(B) $x^2 \sin x$

(D) $x \cos x$

11. The wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, a^2 stands for

(A) $\frac{T}{m}$

(B) $\frac{k}{c}$

(C) $\frac{m}{T}$

(D) $\frac{k}{m}$

12. In heat equation $u_t = \alpha^2 u_{xx}$, α^2 stands for

(A) $\frac{k}{\rho}$

(B) $\frac{T}{m}$

(C) $\frac{k}{\rho c}$

(D) $\frac{k}{c}$

13. One dimensional heat equation is used to find

(A) Density

(B) Temperature distribution

(C) Time

(D) Displacement

14. Heat flows from _____ temperature.

(A) Higher to lower

(B) Uniform

(C) Lower to higher

(D) Stable

15. The steady state temperature of a rod of length l whose ends are kept at 30° and 40° is

(A) $u = \frac{10x}{l} + 30$

(B) $u = \frac{20x}{l} + 30$

(C) $u = \frac{10x}{l} + 20$

(D) $u = \frac{10x}{l}$

16.

The Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ is

(A) e^{-z^2}

(B) $\frac{s^2}{e^{\frac{s^2}{2}}}$

(C) $\frac{1}{e^{x^2}}$

(D) $\frac{1}{e^{-s^2/2}}$

17. $F(f(x) \cos ax) =$ _____

(A) $[F(a) + F(s-a)]/2$

(B) $[F(sa) + F(s+a)]/2$

(C) $[F(s+a) + F(s-a)]/2$

(D) $F(s/a)$

18. $F(f(x) * g(x)) =$ _____

(A) $F(s) + G(s)$

(B) $F(s) - G(s)$

(C) $F(s)G(s)$

(D) $F(s)/G(s)$

19. If $F(f(x)) = F(s)$, then $F(f(ax)) =$ _____

(A) $\frac{1}{z} F(s/a)$

(B) $\frac{1}{a} F(a/s)$

(C) $\frac{1}{a} F(s/a)$

(D) $\frac{1}{s} F(s/a)$

20. The Fourier cosine transform of e^{-4x} is

(A) $\sqrt{\frac{2}{\pi}} \frac{2}{4+s^2}$

(B) $\sqrt{\frac{2}{\pi}} \frac{4}{16-s^2}$

(C) $\sqrt{\frac{2}{\pi}} \frac{4}{16+t^2}$

(D) $\sqrt{\frac{2}{\pi}} \frac{4}{s^2+16}$

21. Find $Z\{(-1)^n\}$ is

(A) $\frac{z+1}{z}$

(B) $\frac{z}{z-1}$

(C) $\frac{z}{z+1}$

(D) $\frac{-z}{z+1}$

22. Find $Z\left\{\cos \frac{n\pi}{2}\right\}$

(A) $\frac{z}{z^2+1}$

(B) $\frac{z^2}{z^2+1}$

(C) $\frac{z}{z^2-1}$

(D) $\frac{z^2}{z^2-4}$

23. What is the value of $Z^{-1}\left\{\frac{z}{(z-1)^2}\right\}$

(A) $n+1$

(B) n

(C) $n-1$

(D) $1/n$

24. The solution of $u_n = 5 u_{n-1}, n \geq 1$ and $u_0 = 2$ is

(A) 5^n

(B) 2×5^n

(C) 2^n

(D) 5×2^n

25. Find the pole of $F(z) = \left[\frac{z^n(z+1)}{(z-1)^3} \right]$

(A) $z=1$

(B) $z=-1$

(C) $z=0$

(D) $z=3$

PART - B (5 × 10 = 50 Marks)
Answer ALL Questions

26. a. Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$.
(OR)

10 3 1 1

10 3 1 1

- b. Solve $(D^3 - 7DD' - 6D'^2)z = x^2y + \sin(x + 2y)$.

10 4 2 2

27. a. Find the half-range cosine series of $f(x) = (\pi - x)^2$ in $(0, \pi)$. Hence find the sum of series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty$.

10 3 2 2

- b. Find the first two harmonics of the Fourier series from the following table.
(OR)

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y:	1	1.4	1.9	1.7	1.5	1.2	1.0

28. a. A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string in the form $y = \lambda(lx - x^2)$ from which it is released at time $t=0$. Find the displacement $y(x, t)$.
(OR)

10 4 3 2

- b. A rod of length 'l' has its ends A and B kept at 0°C and 120°C respectively until steady state condition prevails. If the temperature at B is reduced to 0°C and kept so while that of 'A' is maintained, find the temperature distribution in the rod.

10 4 3 2

29. a. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$ hence evaluate

10 3 4 2

$$\int_0^\infty \frac{x \cos x - \sin x}{x^3} dx.$$

10 3 4

- b. Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ using transform method.
(OR)

10 3 5

30. a. Using partial fraction method, evaluate $Z^{-1} \left[\frac{z^2}{(z-1)(z-3)^2} \right]$.

(OR)

- b. Solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 1$, $y_0 = 0$ and $y_1 = 1$ using Z-transform method.

10 3 5
