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Reg. No. | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, MAY 2014

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Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted from the academic year 2013 – 2014 onwards)

Note:

(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. The value of $\int_0^1\int_0^x dx\,dy$ is

 (A) 1 (B) −1
 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

2. If R is the region bounded by $x = 0$, $y = 0$, $x+y = 1$ then $\iint_R dx\,dy$

 (A) 1 (B) $\frac{1}{2}$
 (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

3. Area of double integral in Cartesian co-ordinate is equal to

 (A) $\iint_R dy\,dx$ (B) $\iint_R dr\,d\theta$
 (C) $\iint_R x\,dx\,dy$ (D) $\iint_R x^2\,dx\,dy$

4. $\int_0^2\int_1^2\int_1^2 xy^2 z\,dz\,dy\,dx$ is

 (A) 24 (B) 28 Ans: 7
 (C) 20 (D) 26

5. The unit normal vector to the surface $x^2y+2xz = 4$ at the point (2, −2, 3) is

 (A) $-\frac{i}{3}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ (B) $\frac{1}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$
 (C) $-\frac{i}{3}-\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ (D) $\frac{i}{3}-\frac{2}{3}\hat{j}-\frac{2}{3}\hat{k}$

6. If $\bar{r}$ is the position vector of the point (x, y, z) with respect to the origin then $\nabla.\bar{r}$ is

 (A) 2 (B) 3
 (C) 0 (D) 1

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7. If the divergence of the vector is zero then the vector is said to be
(A) Irrotational vector
(B) Constant vector
(C) Zero vector
(D) Solenoidal vector

8. The relation between the line integral and double integral is
(A) Green's theorem
(B) Stokes theorem
(C) Surface integral
(D) Divergence theorem

9. Laplace transform of 't' is
(A) $\frac{1}{s}$
(B) $\frac{1}{s^2}$
(C) s
(D) s^2

10. L(cost) is
(A) $\frac{1}{s^2-1}$
(B) $\frac{1}{s^2+1}$
(C) $\frac{s}{s^2-1}$
(D) $\frac{s}{s^2+1}$

11. If L[f(t)] = F(s) then $L\left[\int_0^t f(u)\,du\right]$ is
(A) sF(s)
(B) $\frac{F(s)}{s}$
(C) tf(t)
(D) $\frac{f(t)}{t}$

12. The value of $L^{-1}\left(\frac{s-2}{s^2-4s+13}\right)$ is
(A) $e^{-2t}\cos 3t$
(B) $e^{-2t}\sin 3t$
(C) $e^{2t}\cos 3t$
(D) $e^{2t}\sin 3t$

13. The value of $\oint_C \frac{z}{z-2}\,dz$ where C is the circle |z| = 1 is
(A) 0
(B) $\frac{\pi}{2}i$
(C) $\frac{\pi}{2}$
(D) 2

14. If f(z) is analytic inside and on C, then the value of $\oint_C f(z)\,dz$, where C is the simp

closed curve is
(A) f(a)
(B) $2\pi i f(a)$
(C) $\pi i f(a)$
(D) 0

15. The critical point of transformation $w = z^2$ is
(A) z = 2
(B) z = 0
(C) z = 1
(D) z = −2

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16. Let $C_1 : |z-a| = R_1$ and $C_2 : |z-a| = R_2$ be two concentric circles $(R_2 < R_1)$, then annular region is defined as

(A) Within C_1
(B) Within C_2
(C) Within C_2 and outside C_1
(D) Within C_1 and outside C_2 ✓

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17. A region which is not simply connected is called

(A) Simply connected
(B) Multiply connected
(C) Simple closed curve
(D) Contour integral

18. The value of $\oint \frac{e^{-z}}{z+1} dz$ is

(A) 0
(B) $2\pi i$
(C) $-2\pi ie$
(D) $2\pi ie$

19. The point z_0 at which a function f(z) is not analytic is known as

(A) Zeros
(B) Isolated singular point
(C) Singular point
(D) Removable singular point

20. The residue of $f(z) = \frac{z}{(z-1)^2}$ is

(A) 0
(B) 1
(C) -1
(D) $2\pi i$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^a \int_y^a \frac{x\, dxdy}{x^2+y^2}$ by changing the order of integration.

22. Show that $r^n \bar{r}$ is an irrotational vector for any value of 'n' and is solenoidal only for $n = -3$.

23. Verify final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

24. Verify that the families of curves $u = c_1$ and $v = c_2$ cut each other orthogonally when $w = z^3$.

25. Find the constant a, b, c if $f(z) = x+ay+i(bx+cy)$ is analytic.

26. Evaluate $\oint_C \frac{\cos z}{z} dz$ where C is an ellipse $9x^2+4y^2 = 1$.

27. Find the angle between normals to the surface $x^2 = yz$ at the points (1,1,1) and (2,4,1).

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Find the volume of a sphere $x^2+y^2+z^2=a^2$ using triple integration.

(OR)

b. Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$.

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29.a.i Show that $\overline{F}=\left(2xy+z^3\right)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is conservative vector field and hence find the scalar potential.

ii. Evaluate $\oint_C \overline{F}\times d\bar{r}$, where $\overline{F}=xy\hat{i}+z\hat{j}+x^2\hat{k}$ where $x=t^2, y=2t, z=t^3$ from 0 to 1.

(OR)

b. Verify divergence theorem for $\overline{F}=\left(x^2-yz\right)\hat{i}+\left(y^2-xz\right)\hat{j}+\left(z^2-xy\right)\hat{k}$ taken over a rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

30. a. Solve using Laplace transform method $y''+2y'-3y=\sin t$ given $y(0)=y'(0)=0$.

(OR)

b. i. Using convolution theorem, evaluate $L^{-1}\left[\dfrac{s}{\left(s^2+a^2\right)^2}\right]$.

ii. Find the Laplace transform of $f(t)=\begin{cases} t & , 0<t\le 2 \\ 4-t & , 2\le t<4 \end{cases}$ and satisfy $f(t+4)=f(t)$.

31.a.i Find the analytic function $f(z)=u+iv$, where $u=e^x(x\sin y+y\cos y)$.

ii. Determine the region D of the w-plane into which the triangular region D enclosed by the lines $x=0$, $y=0$, $x+y=1$ is transformed under the transformation $w=2z$.

(OR)

b. Find the bilinear transformation which maps the points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=0, w_3=-i$.

32.a.i Evaluate $\oint_C \dfrac{e^{2z}}{\cos\pi z}\,dz$ where C is a circle $|z|=1$.

ii. Expand $f(z)=\dfrac{1}{(z-1)(z-2)}$ as Laurent series valid in the region

(1) $|z|<1$
(2) $1<|z|<2$
(3) $|z|>2$

(OR)

b. Evaluate $\int_0^{2\pi}\dfrac{d\theta}{5+4\cos\theta}$ by calculus of residues.

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B.Tech. DEGREE EXAMINATION, NOVEMBER 2014

Second Semester

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MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted from the academic year 2013 – 2014 onwards)

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. If $I = \int_0^1 \int_1^2 \left(x^2 + y^2\right) dx\, dy$, then the value of I is

(A) $\frac{-8}{3}$

(B) 9

✓(C) $\frac{8}{3}$

(D) 0

2. What is the value of $\int_0^1 \int_0^x dx\, dy$?

✓(A) $\frac{1}{2}$

(B) -1

(C) 1

(D) $\frac{1}{3}$

3. Change the order of integration in $\int_0^a \int_x^a f(x,y)\, dy\, dx$.

(A) $\int_0^a \int_x^a f(x,y)\, dy\, dx$

✓(B) $\int_0^a \int_0^y f(x,y)\, dy\, dx$

(C) $\int_0^a \int_0^{x^2} f(x,y)\, dx\, dy$

(D) $\int_0^x \int_0^a f(x,y)\, dx\, dy$

4. Find the value of $\int_0^{\pi} \int_0^{a\sin\theta} r\, dr\, d\theta$.

(A) πa^2

✓(B) $\frac{\pi}{4} a^2$

(C) $\frac{\pi}{4} a^3$

(D) a^2

5. If $\overline{F} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+az)\bar{k}$, is solenoidal, then the value of 'a' is

✓(A) -2

(B) 1

(C) -1

(D) 0

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6. The value of $\int_C x\,dy - y\,dx$ around the circle $x^2+y^2 = 1$ is

(A) 0
(B) π
(C) 3π
(D) 2π

7. If $\vec{u}$ and $\vec{v}$ are irrotational, then $\vec{u}\times\vec{v}$ is

(A) Solenoidal
(B) Irrotational
(C) Zero
(D) Constant

8. The value of $\iint_S \vec{r}.\vec{n}\,ds$, where S is the surface of the sphere $x^2+y^2+z^2 = a^2$ is

(A) $2\pi a^3$
(B) $3\pi a^3$
(C) $3\pi a^2$
(D) $4\pi a^3$

9. $L(2^t)$ is

(A) $\frac{1}{s-\log 2}$
(B) $\frac{1}{s+\log 2}$
(C) $\frac{1}{s-2}$
(D) $\frac{1}{s+2}$

10. $L(t^{3/2})$ is

(A) $\frac{2\sqrt{\pi}}{4s^{5/2}}$
(B) $\frac{3\sqrt{\pi}}{4s^{5/2}}$
(C) $\frac{2}{4s^{5/2}}$
(D) $\frac{3}{4s^{5/2}}$

11. The value of $L^{-1}(1)$ is

(A) $U_a(t)$
(B) 1
(C) 0
(D) $\delta(t)$

12. $L\left(f(t)*g(t)\right)$ is

(A) $F(s)*G(s)$
(B) $F(s)\div G(s)$
(C) $F(s)\,G(s)$
(D) $F(s)+G(s)$

13. If $w = f(z) = u+iv$ is an analytic function of z, then

(A) u and v are not harmonic
(B) u is harmonic, v is not harmonic
(C) Both u and v are harmonic
(D) u and v are constants

14. The fixed points of the transformation $w = \frac{z-1}{z+1}$ are

(A) $\pm i$
(B) ± 1
(C) ± 2
(D) ± 3

15. If $u = 2x(1-y)$, then harmonic conjugate of u is

(A) x^2-y^2+2y+c
(B) x^2+y^2-2y+c
(C) $-x^2+y^2+2y+c$
(D) x^2-y^2-2y+c

16. The analytic function with constant real part is

(A) Constant
(B) Function of x
(C) Function of y
(D) Function of z

17. The value of $\int_C \frac{4z^2+z+5}{z-4} dz$, where C is $9x^2+4y^2=36$ is

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(A) πi
(B) 0
(C) $-\pi i$
(D) $2\pi i$

18. If $f(z)=\frac{\sin z}{z}$, then $z=0$ is

(A) Pole
(B) Removable singularity
(C) Essential singularity
(D) Isolated singularity

19. If $I=\int_C \frac{z}{z^2-1} dz$, $C:|z|=\frac{1}{2}$, then I is

(A) 0
(B) πi
(C) -1
(D) $2\pi i$

20. If $I=\int_C e^z\, dz$, $C:|z|=1$, then I is

(A) 0
(B) πi
(C) -1
(D) $2\pi i$

PART – B ($5\times 4=20$ Marks)
Answer ANY FIVE Questions

21. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\, dx\,dy\,dz$.

22. If $\bar{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla r^n = nr^{n-2}\,\bar{r}$.

23. Find $L^{-1}\left[\cot^{-1}\left(\frac{2}{s+1}\right)\right]$.

24. Derive C-R equations in polar form.

25. Expand e^{2z} about $z=2i$ in Taylor's series.

26. Find the image of the triangular region in the z-plane bounded by the lines $x=0$, $y=0$ and $x+y=1$, under the transformation $w=2z$.

27. If $L[f(t)]=\frac{1}{s(s+1)(s+2)}$, find $\lim_{t\to 0} f(t)$ and $\lim_{t\to\infty} f(t)$.

PART – C ($5 \times 12 = 60$ Marks)

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28. a. Using triple integration, find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(OR)

b. Change the order of integration and hence evaluate $\int_0^1 \int_y^{2-y} xy\, dy\, dx$.

29. a. Show that $\bar{F}=(z^2+2x+3y)\vec{i}+(3x+2y+z)\vec{j}+(y+2zx)\vec{k}$ is irrotational, but not solenoidai. Find also its scalar potential.

(OR)

b. Verify Green's theorem in a plane for $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ where 'C' is the boundary of the region bounded by the lines $x=0$, $y=0$ and $x+y=1$.

30.a.i Find $L\left(\frac{\sin 3t \sin t}{t}\right)$.

ii. Solve: $(D^2+6D+9)x = 6t^2 e^{-2t}$, $x=0$, $Dx=0$ at $t=0$ using Laplace transform.

(OR)

b. i. Using convolution theorem, to find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

ii. Find $L(f(t))$, if $f(t)=e^t$, $0<t<2\pi$ and $f(t+2\pi)=f(t)$.

31.a.i Determine the analytic function $f(z)=u+iv$ given that $3u+2v=y^2-x^2+16xy$.

ii. Find the bilinear transformation which maps the points $z=0$, $z=1$ and $z=\infty$ into the points $w=i$, $w=1$ and $w=-i$.

(OR)

b. State and prove the two important properties of an analytic function.

32. a. Evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is a circle $|z+1+i|=2$ using Cauchy's integral formula.

(OR)

b. Evaluate $\int_0^\infty \frac{x^2\, dx}{(x^2+1)(x^2+4)}$ using contour integration technique.

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(ii) Part - B and Part - C should be answered in answer booklet.

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Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of $\int_0^{\pi/2}\int_0^{\pi/2} \sin(\theta+\phi)d\theta d\phi$ is
   (A) 2 (B) 1
   (C) 0 (D) −2

2. The value of $\int_1^a\int_1^b \frac{dxdy}{xy}$ is
   (A) $\log a + \log b$ (B) $\log a$
   (C) $\log b$ (D) $\log a \log b$

3. The value of $\int_0^1\int_0^2\int_0^3 dxdydz$ is
   (A) 3 (B) 4
   (C) 2 (D) 6

4. If R is the region bounded by $x=0, y=0, x+y=1$, then $\iint dxdy$ is
   (A) 1 (B) 1/2
   (C) 1/3 (D) 2/3

5. Curl (grad∅) is
   (A) −1 (B) 1
   (C) 0 (D) Does not exist

6. Find the constant 'a', if the vector $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal
   (A) 2 (B) −2
   (C) −1 (D) 0

7. The condition for $\vec{F}$ to be conservative is, $\vec{F}$ should be
   (A) Solenoidal vector (B) Irrotational vector
   (C) Rotational vector (D) Neither solenoidal nor irrotational

8. If $\bar{a}$ is a constant vector and $\bar{r}$ is the position vector of the point (x, y, z) with respect to the origin, then $grad(\bar{a}.\bar{r})$ is
   (A) 0 (B) 1
   (C) $\bar{a}$ (D) $2\bar{a}$

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9. $L(4^t)$ is

(A) $\frac{1}{s-4}$

(B) $\frac{s}{s^2+4}$

(C) $\frac{1}{s-\log 4}$

(D) $\frac{4}{s}$

10. $L\left(\frac{\cos at}{t}\right)$ is

(A) $\tan^{-1}(s)$

(B) $\log\sqrt{s^2+a^2}$

(C) $\cot^{-1}(s)$

(D) Does not exist

11. $L(\sin(2t+3))$ is

(A) $\frac{2}{s^2+4}$

(B) $\frac{2\cos 3}{s^2+4}+\frac{(\sin 3)s}{s^2+4}$

(C) $\frac{s}{s^2+9}$

(D) $\frac{6}{s^2+9}$

12. $L^{-1}\left(\frac{s-3}{s^2-6s+13}\right)$ is

(A) $e^{-3t}\cos 3t$

(B) $e^{2t}\cos 3t$

(C) $e^{3t}\cos 2t$

(D) $e^{-2t}\cos 2t$

13. An analytic function with constant modulus is

(A) a function of x

(B) a function of y

(C) a function of z

(D) a constant

14. The complex function $w = az$ when a is a complex constant geometrically implies

(A) Rotation

(B) Rotation and magnification

(C) Rotation and reflection

(D) Translation

15. The invariant points of the transformation $w=\frac{z-1}{z+1}$ is

(A) $\pm i$

(B) ± 1

(C) 0

(D) 2, 7

16. Cauchy Riemann equation in polar coordinates are

(A) $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial v}{\partial r}=\frac{-1}{r}\frac{\partial u}{\partial \theta}$

(B) $\frac{\partial u}{\partial \theta}=r\frac{\partial v}{\partial \theta},\ \frac{\partial v}{\partial r}=-r\frac{\partial u}{\partial \theta}$

(C) $\frac{\partial v}{\partial r}=\frac{1}{r}\frac{\partial u}{\partial \theta};\ \frac{\partial u}{\partial r}=\frac{-1}{r}\frac{\partial v}{\partial \theta}$

(D) $\frac{\partial u}{\partial r}=\frac{\partial v}{\partial \theta},\ \frac{\partial u}{\partial \theta}=-\frac{\partial v}{\partial r}$

17. If a is a simple pole, then Residue of $\left\{\frac{\phi(z)}{\psi(z)}\right\}$ at $z=a$ is

(A) $\frac{\phi'(a)}{\psi(a)}$

(B) $\frac{\phi'(a)}{\psi'(a)}$

(C) $\frac{\phi(a)}{\psi(a)}$

(D) $\frac{\phi(a)}{\psi'(a)}$

18. The value of $\int_c \frac{3z^2+7z+1}{z+1}dz,\ c:|z|=\frac{1}{2}$

(A) $2\pi i$

(B) 0

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(C) πi

(D) $\frac{\pi i}{2}$

19. If $f(z)=\frac{-1}{z-1}-2\{1+(z-1)+(z-1)^2+....\}$ then the residue of $f(z)$ at $z=1$ is

(A) 0

(B) -1

(C) -2

(D) 2

20. The singularity of $f(z)=\frac{z}{(z-2)^3}$ is

(A) Essential singularity

(B) Removable singularity

(C) Pole of order 3

(D) Pole of order 1

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Evaluate $\int_2^4\int_0^x\int_0^{\sqrt{x+y}} z\,dz\,dy\,dx$.

22. If $r=|\vec{r}|$, where $\vec{r}$ is the position vector of the point $(x, y\ z)$ with respect to the origin, prove that $\nabla f(r)=\frac{f'(r)}{r}\vec{r}$.

23. Find the work done when a force $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ displays a particle in the xy plane form (0, 0) to (1, 1) along the parabola $y^2=x$.

24. Find $L\left(\frac{e^{-t}-e^{-3t}}{t}\right)$.

25. Find $L^{-1}\left(\frac{e^{-s}}{(s+3)(s-2)}\right)$.

26. Show that $\sin z$ is an analytic function of z.

27. Evaluate $\int_c \frac{z+1}{z(z-1)}dz;\ c:|z|=2$, using Cauchy's residue theorem.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a.i. By changing into polar coordinates evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)\,dx\,dy$.

ii. Find the area of the cardiod $r=a(1+\cos\theta)$ using double integration.

(OR)

b. i. Change the order of integration and then evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$.

ii. Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$, and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=$ using triple integration.

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29. a.i. Find the angle between the surfaces $x^2-y^2-z^2=11$ and $xy+yz-zx=18$ at the poi (6, 4, 3).

ii. Show that $\overline{F}=(6xy+z^3)\overline{i}+(3x^2-z)\overline{j}+(3xz^2-y)\overline{k}$ is irrotational and find its scalar potentia

(OR)

b. Verify Green's theorem in the plane for $\int_c (3x^2-8y^2)dx+(4x-6xy)dy$ where c is t boundary of the region bounded by $x=0, y=0, x+y=1$.

30. a.i. Using convolution theorem, find $L^{-1}\left(\frac{1}{(s+3)(s-1)}\right)$.

ii. Find $L(te^{-t}\ \sin t)$.

(OR)

b. Using Laplace transform, solve $y''-3y'+2y=e^{-t}$, given $y(0)=1, y'(0)=0$.

31. a.i. An electrostatic field in the xy plane is given by the potential function $\phi=3x^2y-y^3$. Find i stream function.

ii. Determine the bilinear transformation which maps $z_1=0, z_2=1, z_3=\infty$ into $w_1=i$, $w_2=-$ $w_3=-i$ respectively.

(OR)

b. i. If $f(z)$ is an analytic function of z, show that $\nabla^2|f(z)|^2=4|f'(z)|^2$.

ii. Find the image of the circle $|z-1|=1$ under the mapping $w=\frac{1}{z}$.

32. a. Using Cauchy's integral formula evaluate $\int_c \frac{z+4}{z^2+2z+5}dz$, $c:|z+1+i|=2$.

(OR)

b. i. Evaluate $\int_0^\infty \frac{x^2dx}{(x^2+9)(x^2+4)}$ by the method of residues.

ii. Expand $f(z)=\frac{z}{(z-1)(z-3)}$ as a Laurent series in the region $0<|z-1|<2$

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QP - A

Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Second Semester

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MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

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(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. The value of the integral $\int_1^2\int_1^4 \frac{dxdy}{xy}$ is
   (A) $2(\log_e 2)^2$
   (B) $\log_e 2$
   (C) $(\log_e 2)^2$
   (D) $\log_e 4$

2. The value of the integral $\int_0^{\pi/2}\int_0^{\sin\theta} r^4\, drd\theta$ is
   (A) $\frac{6}{75}$
   (B) $-\frac{4}{75}$
   (C) $\frac{2}{75}$
   (D) $\frac{8}{75}$

3. If R is the region bounded by $x = 0, y = 0, x + y = 1$ then $\iint dxdy$ is equal to
   (A) 1
   (B) $\frac{1}{2}$
   (C) $\frac{1}{3}$
   (D) $\frac{2}{3}$

4. Area of the double integral in Cartesian co-ordinate is equal to
   (A) $\iint drd\theta$
   (B) $\iint rdrd\theta$
   (C) $\iint dxdy$
   (D) $\iint xdxdy$

5. The unit normal vector to the surface $x^2 + y^2 - z^2 = 1$ at (1, 1, 1) is
   (A) $\frac{2i+2j+2k}{\sqrt{2}}$
   (B) $\frac{2i-2j+2k}{\sqrt{2}}$
   (C) $\frac{i+j-k}{\sqrt{3}}$
   (D) $\frac{i+j+k}{\sqrt{3}}$

6. If $\phi = xyz$ then $\nabla\phi$ is
   (A) $xy\hat{i} + yz\hat{j} + zx\hat{k}$
   (B) $yz\hat{i} + zx\hat{j} + xy\hat{k}$
   (C) $zx\hat{i} + xy\hat{j} + yz\hat{k}$
   (D) 0

7. If ϕ is a scalar function, then curl(gradϕ) is

(A) Solenoidel
(B) Irrotational
(C) 0
(D) Constant vector

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8. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to origin, then $div\vec{r}$ is

(A) 1
(B) -1
(C) 2
(D) 3

9. Laplace transform of $t\cos at$ is

(A) $\frac{s^2+a^2}{(s^2-a^2)^2}$
(B) $\frac{s^2-a^2}{s^2+a^2}$
(C) $\frac{s^2-a^2}{(s^2+a^2)^2}$
(D) $\frac{s^2+a^2}{(s^2-a^2)^2}$

10. Inverse Laplace transform of $\frac{s}{(s+2)^2}$ is

(A) $e^{2t}(1-2t)$
(B) $e^{-2t}(1-2t)$
(C) $e^{-2t}(1+2t)$
(D) $e^{2t}(1+2t)$

11. $L\left(\frac{\sin 4t}{t}\right)$ is

(A) $\cot^{-1}\left(\frac{4}{s}\right)$
(B) $\tan^{-1}\left(\frac{4}{s}\right)$
(C) $\cot^{-1}\left(\frac{4}{s+1}\right)$
(D) $\tan^{-1}\left(\frac{4}{s+1}\right)$

12. $L^{-1}\left(\frac{1}{s^2+9}\right)$ is

(A) $\frac{\cos 3t}{3}$
(B) $\sin 3t$
(C) $\frac{\sin 3t}{3}$
(D) $\cos 3t$

13. Cauchy-Riemann equation in polar co-ordinates are

(A) $rU_r = V_\theta, U_\theta = -rV_r$
(B) $-rU_r = V_\theta, U_\theta = rV_r$
(C) $-rU_r = V_\theta, U_\theta = -rV_r$
(D) $U_r = rV_\theta, rU_\theta = V_r$.

14. The invariant point of the transformation $w = \frac{1}{z-2i}$ is

(A) $z = i$
(B) $z = -i$
(C) $z = 1$
(D) $z = -1$

15. An analytic junction with constant modulus

(A) Zero
(B) Analytic
(C) Harmonic
(D) constant

16. If $u + iv$ is analytic, then the curves $u = C_1$, $V = C_2$

(A) Intersect each other
(B) Cut orthogonally

17. Value of $\oint_c \frac{dz}{z-1}$ where C is $|z-1|=1$ is

(A) πi
(B) $2\pi i$
(C) 0
(D) $-\pi i$

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18. If f(z) is analytics inside and on c, the value of $\oint_c \frac{f(z)}{z-a}dz$, where c is a simple closed curve and 'a' is any point within 'c' is

(A) $f(a)$
(B) $2\pi i f(a)$
(C) $\pi i f(a)$
(D) 0

19. The point z_0 at which a function f(z) is not analytic is known as

(A) Isolated singular point
(B) Zeros
(C) Singular point
(D) Removable singular point

20. If $I=\oint_c \frac{z^2}{(z-1)^2(z+1)}dz$, where c is $|z|=\frac{1}{2}$, then I is

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) 0
(D) $\frac{1}{3}$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Show by double integration that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16a^2}{3}$

22. Show that the vector $\overline{F}$ is given by $\overline{F}=(x^2-yz)\hat{i}+(y^2-xz)\hat{j}+(z^2-xy)\hat{k}$ is irrotational. Find its scalar potential.

23. If $\bar{u}$ and $\bar{v}$ are irrotational vector, then show that $\bar{u}\times\bar{v}$ is a solenoidal vector.

24. Evaluate $L\left[\int_0^t \frac{\cos 6t-\cos 4t}{t}dt\right]$.

25. Verify final value theorem for the function $1+e^{-t}(sint+cost)$.

26. Prove that an analytic function with constant modulus is constant.

27. Evaluate $\oint_c \tan z\, dz$ where c is a circle $|z|=2$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dxdy$.

(OR)

b. Find the volume of the tetrahedron bounded by the co-ordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

29. a.i. Find the constants a and b so that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at $(1,-1,2)$.

ii. If $\overline{F}=2y\hat{i}-z\hat{j}+x\hat{k}$ evaluate $\oint_c \overline{F}\times d\overline{r}$ along the curve $x=\cos t, y=\sin t, z=2\cos t$ from $t=0$ to $\frac{\pi}{2}$.

(OR)

b. Verify Gauss-Divergence theorem for the function $F=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ taken over the cube bounded by the planes $x=0, x=1, y=0, y=1$ and $z=0, z=1$.

30. a. Solve using Laplace transform $y''+2y'-3y=\sin t$ given $y(0)=0$, $y'(0)=0$.

(OR)

b.i. Using convolution theorem, evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

ii. Find the Laplace transform of the periodic function f(t) with period '2' given by $f(t)=\begin{cases}1 & 0<t<1\\ -1 & 1\le t<2\end{cases}$

31. a.i. Find the harmonic conjugate of $u=\frac{1}{2}\log(x^2+y^2)$.

ii. Find the bilinear transformation that maps the point $z = 1, i, -1$ in the z-plane into the points $w=2, i, -2$ in the w-plane.

(OR)

b.i. Find the analytic function f(z) in terms of z if $u+v=(x-y)(x^2+4xy+y^2)$.

ii. Determine the region D of the w plane into which the triangular region D enclosed by the lines $x=0, y=0, x+y=1$ is transformed under the transformation $w=2z$.

32. a.i. Evaluate $\oint_c \frac{ze^{2z}}{(z-1)^3}dz$ by using Cauchy's integral formula where C is a circle $|z+i|=2$.

ii. Find the Laurent series expansion of $\frac{1}{(z+1)(z+3)}$ in $0<|z+1|<2$.

(OR)

b. i. Using Cauchy's residue theorem, evaluate $\oint_c \frac{z^2}{(z-1)^2(z+1)}dz$ where 'c' is $|z|=2$.

ii. Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ using Contour integration.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2016

Second Semester

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15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2015-2016)

Note:

(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. $\int_1^b\int_1^a \frac{dxdy}{xy}$ is equal to
   (A) log a +log b
   (B) log a
   (C) log b
   (D) log a log b

2. The value of $\int_0^1\int_0^2\int_0^3 dxdydz$ is
   (A) 3
   (B) 4
   (C) 2
   (D) 6

3. Area of the double integral in polar co-ordinate is equal to
   (A) $\iint_R dr d\theta$
   (B) $\iint_R r^2 dr d\theta$
   (C) $\iint_R r dr d\theta$
   (D) $\iint_R (r+1) dr d\theta$

4. What is the value of $\int_0^1\int_0^x dxdy$
   (A) $\frac{1}{2}$
   (B) -1
   (C) 1
   (D) $\frac{1}{3}$

5. Curl (grad φ) is
   (A) -1
   (B) 1
   (C) 0
   (D) φ

6. If $\varphi = xyz$ then $\nabla\phi$ is
   (A) $yz\vec{i} + zx\vec{j} + xy\vec{k}$
   (B) $xy\vec{i} + yz\vec{j} + zx\vec{k}$
   (C) 0
   (D) $zx\vec{i} + xy\vec{j} + yz\vec{k}$

7. The condition for $\vec{F}$ to be conservative is $\vec{F}$ should be
   (A) Solenoidal vector
   (B) Irrotational vector
   (C) Rotational vector
   (D) Neither solenidal nor irrotational

8. The connection between a line integral and double integral is known as
(A) Green's theorem
(B) Stoke's theorem
(C) Divergence theorem
(D) Convolution theorem

9. L (sin 3t) is equal to

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(A) $\frac{3}{s^2-3}$
(B) $\frac{s}{s^2-3}$
(C) $\frac{3}{s^2+9}$
(D) $\frac{s}{s^2+9}$

10. The value of the function $f(t) = ae^{-bt}$ using initial value theorem is
(A) a
(B) a^2
(C) ab
(D) 0

11. $L\left[\int_0^t \sin t\, dt\right] =$ ________
(A) $\frac{1}{s^2+1}$
(B) $\frac{s}{s^2+1}$
(C) $\frac{1}{s(s^2+1)}$
(D) $\frac{1}{(s^2+1)^2}$

12. L (t^4) is equal to
(A) $\frac{3!}{s^4}$
(B) $\frac{4!}{s^4}$
(C) $\frac{4!}{s^5}$
(D) $\frac{5!}{s^4}$

13. An analytic function with constant modulus is
(A) A function of x
(B) A function of y
(C) Constant
(D) A function of z

14. If u +iv is analytic, then the curves $u = c_1$ and $v = c_2$
(A) Intersect each other
(B) Cut orthogonally
(C) Are parallel
(D) Coincides

15. The function $w = \sin x \cosh y + i \cos x \sinh y$ is
(A) Continuous
(B) Differentiate at origin
(C) Analytic
(D) Need not be analytic

16. The invariant points of the transformation $W = \frac{z-1}{z+1}$ is
(A) $\pm i$
(B) ± 1
(C) 2,7
(D) 0

17. The value of $\oint_c \frac{(3z^2+7z+1)}{z+1} dz$ where C is $|z| = \frac{1}{2}$ is equal to
(A) $2\pi i$
(B) 0
(C) πi
(D) -1

18. The singularity of $f(z)=\frac{z}{(z-2)^3}$ is

(A) Essential singularity
(B) Removable singularity
(C) Pole of order 3
(D) Pole of order 1

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19. If f(z) is analytic inside and on C, the value of $\oint_c \frac{f(z)}{(z-a)^2}dz$, where C is a simple closed curve and 'a' is any point with in 'C' is

(A) $f^1(a)$
(B) 0
(C) $\pi i f^1(a)$
(D) $2\pi i f^1(a)$

20. The part $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ is consisting of negative integral powers of (z-a) is called as

(A) Analytic part of Laurent series
(B) The principal part of Laurent series
(C) Real part of Laurent series
(D) Imaginary part of Laurent series

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Using double integration, find the area of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

22. Show that $r^n\bar{r}$ is an irrotational vector for any value of 'n' and is solenoidal for $n=-3$.

23. Find $L\left[\frac{e^{-t}-e^{-3t}}{t}\right]$.

24. Verify final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

25. Find the constants a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.

26. Evaluate using divergence theorem $\iint_s (x+z)dydz+(y+z)dzdx+(x+y)dxdy$ over the sphere $x^2+y^2+z^2=4$.

27. Evaluate $\oint_C \frac{(z+1)}{z(z-1)}dz; C \text{ is } |z|=2$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

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b. Find the volume of the sphere $x^2+y^2+z^2=a^2$ by using triple integration.

29. a. Verify Stoke's theorem for $\vec{F}=(y-z+2)\vec{i}+(yz+4)\vec{j}-xy\vec{k}$ where S is an open surfa of a cube x = 0, x = 2, y = 0, y = 2 and z = 0, z = 2.

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(OR)

b.i. Show that $\vec{A}=(x^2+y^2+x)\vec{i}+(2xy+y)\vec{j}$ is irrotational and hence find the scalar potentia

ii. Evaluate $\oint_c (x^2+y^2)dx-2xydy$ taken over the rectangle bounded by the lines x = ±a, y = 0 y = b.

30. a.i. Find the Laplace transform of $f(t)=\begin{cases} t & ,0<t\le 2 \\ 4-t, & 2\le t<4 \end{cases}$ and satisfy $f(t+4)=f(t)$.

ii. Find $L[te^{-t}\sin t]$.

(OR)

b. Using Laplace transform method solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^{-t}$ given $x(0)=2$, $x'(0)=1$.

31. a. Find the analytic function $f(z)=u+iv$ where $u-v=\frac{\sin 2x}{\cosh 2y-\cos 2x}$.

(OR)

b. Find the bilinear transformation that maps the point ∞, i, 0 into 0, i, ∞ respectively.

32. a.i. Evaluate $\oint_C \frac{e^{2z}}{\cos \pi z}dz$ where C is a circle $|z|=1$.

ii. Expand $f(z)=\frac{1}{(z-1)(z-2)}$ as a Laurent's series valid in the region.

(1) $|z|<1$
(2) $1<|z|<2$
(3) $|z|>2$.

(OR)

b. Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\cos\theta}$ by contour integration.

* * * * *

29. a. Verify Green's theorem in a plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)]dy$, where C is the boundary of the region defined by the lines $x = 0$, $y = 0$, and $x + y = 1$.

(OR)

b. Verify Gauss divergence theorem for $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ where S is the surface of the cuboid bounded by the lines $x = 0, x = a, y = 0, y = b, z = 0, z = c$.

30. a.i. Prove that an analytic function with constant modulus is constant.

ii. Find the bilinear transformation that maps the points ∞, 0, i, in Z plane on the points 0, ∞, $-$i of the W-plane.

(OR)

b.i. Discuss the transformation $W = \frac{1}{Z}$.

ii. If $u + v = e^x(\cos y + \sin y)$, where $f(z) = u + iv$ is analytic, find the analytic function f(z) and hence find its derivative.

31. a.i. Verify initial value theorem and final value theorem on Laplace transform for $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

ii. Solve the integral equation using Laplace transform $y(t) = 1 + \int_0^t y(u)\sin(t - u)du$.

(OR)

b.i. Solve the differential equation using Laplace transform $y''(t) + y(t) = 2e^t$, given that $y(0) = 1, y'(0) = 2$.

ii. (1) Find $L^{-1}\left(\frac{(s-1)}{(s^2+2s+2)^2}\right)$. (3 Marks)

(2) Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$. (3 Marks)

32. a.i. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+\cos\theta}$ using contour integration.

ii. Expand $f(z) = \frac{4z+3}{(z+1)z(z+3)}$ in Laurent's series in the region given by

(1) $0 < |z+1| < 3$

(2) $1 < |z| < 3$.

(OR)

b.i. Evaluate using Cauchy's integral formula $\int_c \frac{z\,dz}{(4z+1)(z-1)(z-2)}$ where C is the circle $|z| = 3$.

ii. Expand $\frac{z}{z^2+3z+2}$ using Taylor's series in the region $1 < |z| < 2$.

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B.Tech. DEGREE EXAMINATION, JUNE 2016
Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Evaluate $\int_0^1\int_0^1 dx\,dy$

(A) 1 (B) 2
(C) 0 (D) 4

2. Area using double integral in Cartesian co-ordinate is

(A) $\iint_A dy\,dx$ (B) $\iint_R r\,dr\,d\theta$
(C) $\iint_R x\,dx\,dy$ (D) $\iint_R x^2\,dx\,dy$

3. The value of the triple integral $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz$ is

(A) 3 (B) 4
(C) 2 (D) 6

4. If R is the region bounded by $x = 0, y = 0, x + y = 1$, then $\iint_R dx\,dy =$ ________.

(A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

5. If $\bar{r}$ is the position vector of the point (x, y, z) with respect to origin then $\nabla \cdot \bar{r} =$ ________.

(A) 0 (B) 1
(C) 2 (D) 3

6. If $\bar{u}$ and $\bar{v}$ are irrotational then $\bar{u} \times \bar{v}$ is

(A) solenoidal (B) irrotational
(C) constant vector (D) zero vector

7. If $\bar{F} = \lambda y^4 z^2 \bar{i} + 4x^3 z^2 \bar{j} + 5x^2 y^2 \bar{k}$ is solenoidal, then the value of λ is ________.

(A) x (B) $-$x
(C) λ can take any real value (D) xy^2

8. Evaluate the line integral $\int_c \bar{r} \cdot d\bar{r}$ where C is the line $y = x$ in XY plane from *(1, 1)* to *(2, 2)*

(A) 0 (B) 1
(C) 2 (D) 3

9. $L^{-1}\left(\frac{1}{\sqrt{s}}\right)$

(A) $\frac{1}{\sqrt{t}}$
(B) $\frac{1}{\sqrt{\pi t}}$
(C) $t^{\frac{1}{2}}$
(D) $t^{\frac{3}{2}}$

10. $L^{-1}(1) =$ ______

(A) $\frac{1}{s}$
(B) t
(C) H(t)
(D) $\delta(t)$

11. $L\left(\frac{\cos at}{t}\right)$

(A) $\frac{s}{s^2+a^2}$
(B) $\frac{1}{s^2+a^2}$
(C) does not exist
(D) $\frac{s^2-a^2}{(s^2+a^2)^2}$

12. Evaluate using Laplace transform without integration $\int_0^\infty \left(\frac{e^{-t}-e^{-2t}}{t}\right)dt$

(A) log 2
(B) $\log\left(\frac{1}{2}\right)$
(C) $\log\left(\frac{3}{4}\right)$
(D) log 5

13. If $f(z) = u + iv$ is analytic, then the family of curves, $u=c_1$ and $v=c_2$

(A) cut orthogonally
(B) intersect each other
(C) are parallel
(D) coincides

14. If f(z) and $\overline{f(z)}$ are analytic functions of z, then f(z) is

(A) analytic function
(B) zero function
(C) constant function
(D) discontinuous function

15. The invariant point of the transformation $W = \frac{1}{Z-2i}$ is

(A) $Z = i$
(B) $Z = -i$
(C) $Z = 1$
(D) $Z = -1$

16. If a function u(x,y) satisfies the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, then u is called

(A) Analytic function
(B) harmonic function
(C) differential function
(D) continuous function

17. The residue of $f(z) = \cot z$ is

(A) π
(B) 1
(C) -1
(D) 0

18. The value of $\int_C \frac{z\,dz}{z-2}$ where C is the circle $|z| = 1$ is

(A) 0
(B) $\frac{\pi i}{2}$
(C) $\frac{\pi}{2}$
(D) 2

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19. If $f(z) = \frac{1}{(z-4)^2(z-3)^3(z-1)}$, then

(A) 4 is a simple pole, 3 is a pole of order 3 and 1 is a pole order 2
(B) 3 is a simple pole, 1 is a pole of order 3 and 4 is a pole of order 2
(C) 1 is a simple pole, 3 is a pole of order 3 and 4 is a pole of order 2
(D) 3 is a simple pole, 4 is a pole of order 1 and 1 is a pole of order 3.

20. $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ consisting of negative powers of (z - a) is called

(A) the analytic part of Laurent's series
(B) the principle part of Laurent's series
(C) the real part of Laurent's series
(D) the imaginary part of the Laurent's series

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the value of $\int_0^1\int_x^{\sqrt{x}} xy(x+y)\,dy\,dx$.

22. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin\phi\, dr\, d\phi\, d\theta$.

23. Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$.

24. Use Stoke's theorem to evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (\sin x - y)\vec{i} - \cos x\vec{j}$ and C is the boundary of the triangle whose vertices are $(0,0)$, $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$.

25. Find $L^{-1}\left(\frac{1}{s(s+1)}\right)$ using convolution theorem.

26. Show that the function $f(z) = e^z$ is analytic and find its derivative.

27. Evaluate $\int_C \frac{(4z+1)\,dz}{z(z-1)(z-3)}$ where C is the circle $|z| = 2$ using residue theorem.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a.i. Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-y-x} xyz\,dz\,dy\,dx$.

ii. Evaluate $\iint_D xy\,dx\,dy$ where D is the region of the positive quadrant bounded by the circle $x^2 + y^2 = a^2$.

(OR)

b.i. Find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

ii. Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$.

29. a.i. Find the values of the constants a,b,c so that $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ may be irrotational. For these values of a,b,c, find also the scalar potential of $\vec{F}$.

ii. Find the directional derivative of $\phi = x^2yz - 4xz^2$ at the point P(1,−2,−1)
(1) That is maximum
(2) In the direction of PQ, where Q is (3,−3,−2).

(OR)

b. Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$.

30. a.i. Verify the initial and final value theorem for the function $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

ii. Find the Laplace transform of the function f(t) defined by
$f(t) = t \text{ in } 0 \le t \le a$
$= 2a - t \text{ in } a \le t \le 2a$ and $f(t + 2a) = f(t)$

(OR)

b. Solve $y'' + 2y' - 3y = \sin t$ given $y(0) = 0, y'(0) = 0$.

31. a.i. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

ii. Show that the function $u = 2xy + 3y$ is harmonic and find the corresponding analytic function.

(OR)

b.i. Find the image of $|z - 3| = 5$ under the transformation $w = \frac{1}{z}$. Show that it can be put in the form $\left|w + \frac{3}{16}\right| = \frac{5}{16}$.

ii. Find the bilinear transformation which maps the points z = 0, z = 1 and z = ∞ into the points w = i, w = 1 and w = −i.

32. a.i. Find the Laurent's series of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ valid in the region
(1) $|z| < 2$
(2) $2 < |z| < 3$.

ii. Use Cauchy's integral formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z| = 4$.

(OR)

b. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$, using contour integration.

* * * * *

Q.P - 7

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2016

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Second Semester

15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2015-2016 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Evaluation of $\int_0^1\int_0^1 dxdy$ is
(A) 1 (B) 2
(C) 0 (D) 4

2. The area of an ellipse is
(A) πr^2 (B) $\pi a^2 b$
(C) πab^2 (D) πab

3. The name of the curve $r = a(1 + \cos\theta)$ is
(A) Lemniscate (B) Cycloid
(C) Cardioid (D) Hemicircle

4. The region of integration of the integral $\int_0^1\int_0^x f(x,y)dydx$ is
(A) Square (B) Rectangle
(C) Triangle (D) Circle

5. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, then $\nabla \cdot \vec{r}$ is
(A) 2 (B) 3
(C) 0 (D) 1

6. The condition for $\vec{F}$ to be conservative is, $\vec{F}$ should be
(A) Solenoidal vector (B) Irrotational vector
(C) Rotational vector (D) Neither solenoidal nor irrotational

7. The work done by the conservative force when it moves a particle around a closed curve is
(A) $\nabla \cdot \vec{F} = 0$ (B) $\nabla \times \vec{F} = 0$
(C) 0 (D) $\nabla \cdot (\nabla \times \vec{F}) = 0$

8. The connection between a surface integral and a volume integral is known as
(A) Green's theorem (B) Stoke's theorem
(C) Cauchy's theorem (D) Gauss divergence theorem

9. If $L(f(t)) = F(s)$, then $L\{e^{-at} f(t)\}$ is equal to

(A) $F(s+a)$ (B) $F(s-a)$

(C) $F(s)$ (D) $\frac{1}{a}F\left(\frac{s}{a}\right)$

10. $L\left(t^{-\frac{1}{2}}\right)$ is equal to

(A) $\sqrt{\frac{\pi}{s}}$ (B) $\sqrt{\frac{\pi}{2s}}$

(C) $\sqrt{\frac{1}{s}}$ (D) $\frac{1}{s}$

11. The value of $1 * e^t$ is

(A) $e^t + 1$ (B) e^t

(C) $e^t - 1$ (D) e

12. Inverse Laplace transform of $\frac{1}{s^2}$ is

(A) $\frac{t}{2}$ (B) $\frac{t^2}{2}$

(C) t^2 (D) t

13. The function $f(z) = u + iv$ is analytic if

(A) $u_x = v_y, u_y = -v_x$ (B) $u_x = -v_y, u_y = v_x$

(C) $u_x + v_y = 0$ (D) $u_y = v_y$

14. If $w = f(z)$ is analytic function of z, then

(A) $\frac{\partial w}{\partial x} = i\frac{\partial w}{\partial y}$ (B) $\frac{\partial w}{\partial x} = -i\frac{\partial w}{\partial y}$

(C) $\frac{\partial w}{\partial \bar{z}} = 0$ (D) $\frac{\partial w}{\partial z} = 0$

15. The invariant point of the transformation $w = \frac{1}{z - 2i}$ is

(A) $z = i$ (B) $z = -i$

(C) $z = 1$ (D) $z = -1$

16. An analytic function with constant modulus is

(A) Zero (B) Analytic

(C) Constant (D) Harmonic

17. The value of $\int_C \frac{z}{z-2}dz$, where C is circle $|z| = 1$ is

(A) 2 (B) $\frac{\pi}{2}$

(C) 0 (D) $2\pi i$

18. If $f(z) = \frac{\sin z}{z}$, then

(A) $z=0$ is a simple pole (B) $z=0$ is a removable singularity

(C) $z=0$ is a triple pole (D) $z=0$ is a zero pole

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19. The residue of $f(z) = \frac{e^{2z}}{(z+1)^2}$ is

(A) e^{-2} (B) $-2e^{-2}$

(C) -1 (D) $2e^{-2}$

20. The residue of $f(z) = \cot z$ is

(A) π (B) 1

(C) -1 (D) 0

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{dx\,dy}{1+x^2+y^2}$.

22. Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$.

23. Find $L(te^{-t}\sin t)$.

24. Find an analytic function whose imaginary part is $3x^2y - y^3$.

25. Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$.

26. Find the work done by the force $\vec{f} = z\vec{i} + x\vec{j} + y\vec{k}$, when it moves a particle along the arc of the curve $\vec{r} = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ from t = 0 to t = 2π.

27. Find $L^{-1}\left(\frac{1}{(s+a)(s+b)}\right)$ using convolution theorem.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Change the order of integration in $\int_0^1 \int_0^{1-y} xy\,dx\,dy$ and then evaluate it.

ii. Find the area of the cardioid $r = a(1 + \cos\theta)$ using double integration.

(OR)

b. Evaluate $\iiint_V dx\,dy\,dz$, where V is the infinite region of space (tetra-hedron) formed by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, DECEMBER 2016
Second Semester

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MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours   Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of $\int_0^2\int_0^1 4xy\,dx\,dy$ is
   (A) 4
   (B) 3
   (C) 2
   (D) 1

2. Change the order of integration in $\int_0^a\int_y^a \frac{x}{x^2+y^2}dx\,dy$
   (A) $\int_0^a\int_x^a \frac{x}{x^2+y^2}dy\,dx$
   (B) $\int_0^a\int_0^x \frac{x}{x^2+y^2}dy\,dx$
   (C) $\int_0^a\int_0^{x^2} \frac{x}{x^2+y^2}dy\,dx$
   (D) $\int_0^a\int_x^{x^2} \frac{x}{x^2+y^2}dy\,dx$

3. The value of $\int_0^\pi\int_0^{\sin\theta} r\,dr\,d\theta$ is
   (A) $\frac{\pi}{4}$
   (B) $\frac{\pi}{2}$
   (C) π
   (D) $\frac{-\pi}{2}$

4. The value of $\int_0^1\int_0^2\int_1^2 x^2yz\,dz\,dy\,dx$ is
   (A) 2
   (B) -2
   (C) 3
   (D) 1

5. Angle between two level surfaces $\varphi_1 = c$ *and* $\varphi_2 = c$ is given by
   (A) $\sin\theta = \frac{\nabla\varphi_1.\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$
   (B) $\cos\theta = \frac{\nabla\varphi_1.\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$
   (C) $\cos\theta = \frac{\nabla\varphi_1\times\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$
   (D) $\sin\theta = \frac{\nabla\varphi_1\times\nabla\varphi_2}{|\nabla\varphi_1||\nabla\varphi_2|}$

6. A vector $\overline{v}$ is said to be solenoidal if

(A) curl $\overline{v}=\overline{0}$ (B) grad $(\nabla.\overline{v})=\overline{0}$

(C) div $\overline{v}=0$ (D) div $\overline{v}\neq 0$

7. The unit normal to the surface $x^2+2y^2+z^2=7$ at the point (1, -1, 2) is

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(A) $\frac{\hat{i}-2\hat{j}-2\hat{k}}{3}$ (B) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{4}$

(C) $\frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$ (D) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

8. If $\overline{u}$ and $\overline{v}$ are irrotational, then $(\overline{u}\times\overline{v})$ is

(A) Solenoidal (B) Irrotational

(C) Rotational (D) Curvl $\overline{u}$

9. If L[f(t)=F(s), then $L\left[e^{at}f(t)\right]$ is equal to

(A) $F(s+a)$ (B) $F(s-a)$.

(C) $e^{as}F(s)$ (D) $e^{-as}F(s)$

10. $L[\cos 2t]$ is equal to

(A) $\frac{s}{s^2-4}$ (B) $\frac{s}{s^2-2}$

(C) $\frac{1}{s^2-4}$ (D) $\frac{s}{s^2+4}$

11. $L^{-1}\left[\frac{1}{s^2+9}\right]$ is equal to

(A) $\frac{\sin 3t}{3}$ (B) $\frac{\sin t}{3}$

(C) $\frac{\cos 3t}{3}$ (D) $\sin 3t$

12. $L^{-1}\left[\frac{s-2}{s^2-4s+13}\right]$ is equal to

(A) $e^{3t}\cos 2t$ (B) $e^{3t}\cos 3t$

(C) $e^{3t}\cos t$ (D) $e^{2t}\cos 3t$

13. $f(z)=\frac{1}{z^2+1}$ is analytic everywhere except at

(A) $z=2+i$ (B) $z=\pm i$

(C) i (D) 2-i

14. The invariant points of the transformation $w=\frac{2z+6}{z+7}$ are

(A) 6, -1 (B) 3, 2

(C) -3, 2 (D) -6, 1

15. The image of $|z-2i|=2$ under the transformation $w=\frac{1}{z}$ is

(A) $x^2+y^2=0$ (B) $x^2+y^2+4y=0$

(C) $x^2+y^2-4y=0$ (D) $x^2+y^2+y=0$

16. The transformation w = az (where 'a' is a real constant) represents
(A) Magnification (B) Rotation
(C) Reflection (D) Inversion

17. The value of $\int_C \frac{e^{-z}}{z+1} dz$, where C is a circle $|z| = \frac{1}{2}$ is

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(A) 1 (B) 0
(C) +1 (D) -2

18. If f(z) is analytic inside and on a simple closed curve C, and if 'a' is any point within C, then
(A) $f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$ (B) $f(a) = \int_C \frac{f(z)}{z-a} dz$
(C) $f(a) = 0$ (D) $f(a) = \frac{1}{2\pi} \int_C \frac{f(z)}{z+a} dz$

19. The poles of $f(z) = \frac{4-3z}{z(z-1)(z-2)}$ are
(A) 1, 2, -2 (B) 0, 1, 2
(C) 1, 1, -1 (D) 1, 1, 2

20. The residue of $f(z) = \frac{z}{(z^2+1)(z-2)}$ at the pole z = 2 is
(A) $\frac{1}{5}$ (B) $\frac{3}{5}$
(C) $\frac{2}{5}$ (D) $\frac{4}{5}$.

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Change the order integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and hence evaluate it.

22. Show that the vector field $\bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and hence find the scalar potential.

23. Find $L^{-1}\left[\frac{1}{s(s^2+2s+2)}\right]$.

24. Find the harmonic conjugate of $u = e^x \cos y$.

25. Evaluate $\oint_C \frac{dz}{z^2(z+4)}$ where C is a circle $|z| = 2$, using Cauchy's integral formula.

26. Verity Green's theorem for the integral $\oint_C \left[(x^2 + y)dx - xy^2 dy\right]$ taken around the boundar of the square whose vertices are (0,0), (1,0), (1,1) and (0,1).

27. Find by double integration, smaller of the areas bounded by the circle $x^2 + y^2 = 9$ an $x + y = 3$.

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PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\,dy\,dx$ and then evaluate it.

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(OR)

b. Express the volume of the sphere $x^2+y^2+z^2=a^2$ as a volume integral and hence evaluate it.

29. a. Verify stoke's theorem for $\bar{F}=(y-z+2)\hat{i}-(yz+4)\hat{j}-(xz)\hat{k}$ over the surface of a cube $x=0, y=0, z=0, x=2, y=2, z=2$ above the XOY plane.

(OR)

b. Verify Gauss divergence theorem for $\bar{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over the rectangular parallelepiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

30. a.i. Find the Laplace transform of a periodic function f(t) with period 2, given by $f(t)=\begin{cases}1, 0<t<1\\ -1, 1<t<2\end{cases}$

ii. Using convolution theorem, find $L^{-1}\left[\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

(OR)

b.i. Verify the final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

ii. Solve $y''+2y'-3y=\sin t$, given $y(0)=y'(0)=0$.

31. a.i. Show that an analytic function with constant modulus is constant.

ii. Using Milne-Thomson method, find the analytic function $f(z)=u+iv$, if $u-v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$.

(OR)

b.i. Find the image of $|z-2i|=2$ under the transformation $w=\dfrac{1}{z}$.

ii. Find the bilinear transformation which maps the points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=0, w_3=-i$ and hence find the image of $|z|<1$.

32. a. Evaluate $\int_C \dfrac{\cos \pi z^2}{(z-1)(z-2)}dz$ where C is a circle $|z|=3$, using Cauchy's integral formula.

(OR)

b.i. Find Laurent's series expansion of $\dfrac{1}{(z+1)(z+3)}$ in powers of (z +1) in the region $0<|z+1|<2$.

ii. Using Cauchy's residue theorem, evaluate $\int_C \dfrac{\sin \pi z^2+\cos \pi z^2}{z+z^2}dz$ where C is the circle $|z|=2$.

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Reg. No. | | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, MAY 2017

a.P - ⑤

First / Second Semester

15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

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Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer ALL Questions

1. The value of $\int_0^1\int_0^1 dxdy$ is

(A) 1
(B) 2
(C) 0
(D) 4

2. Change the order of integration in $\int_0^1\int_0^x dydx$ is

(A) $\int_0^1\int_1^y dxdy$
(B) $\int_0^1\int_0^x dxdy$
(C) $\int_0^1\int_0^y dydx$
(D) $\int_0^1\int_y^1 dxdy$

3. Area of double integral in polar co-ordinates is equal to

(A) $\iint_R drd\theta$
(B) $\iint_R r^2 drd\theta$
(C) $\iint_R (r+1)drd\theta$
(D) $\iint_R r\, drd\theta$

4. The name of the curve $r = a(1+\cos\theta)$ is

(A) Leminiscate
(B) Cycloid
(C) Cardioid
(D) Hemicircle

5. If $\bar{r}$ is the position vector of the point (x, y, z) with respect to the origin, the $div\bar{r}$ is

(A) 2
(B) 3
(C) 0
(D) 1

6. If $\phi = xyz$ then $\nabla\phi$ is

(A) $yz\bar{i} + zx\bar{j} + xy\bar{k}$
(B) $xy\bar{i} + yz\bar{j} + zx\bar{k}$
(C) $zx\bar{i} + xy\bar{j} + yz\bar{k}$
(D) 0

7. If the integral $\int_A^B \vec{F}.d\vec{r}$ depends only on the end points but not on the path C, then F is called

(A) Solenoidal vector
(B) Irrotiational vector
(C) Conservative vector
(D) Neither solenoidal nor irrotational

8. Using Gauss divergence theorem, find the value of $\iint_s \bar{r}.d\bar{s}$ where r is the position vector and v is the volume.

(A) 4v
(B) 0
(C) 3v
(D) v

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9. L[1] =

(A) 1/s
(B) $1/s^2$
(C) 1
(D) S

10. $L[\sinh at]$ is

(A) $\frac{s}{s^2+a^2}$ $if\ s>|a|$
(B) $\frac{a}{s^2+a^2}$ $if\ s>|a|$
(C) $\frac{a}{s^2-a^2}$ $if\ s>|a|$
(D) $\frac{s}{s^2-a^2}$ $if\ s>|a|$

11. The value of 1 * sint is

(A) 1− cost
(B) 1+ cost
(C) cost−1
(D) Sint+1

12. If $L[f(t)]=F(s)$ then $L[f'(t)]=$

(A) $L[f(t)]-f(0)$
(B) $sL[f(t)]-f'(0)$
(C) $sL[f(t)]-f(0)$
(D) $sL[f(t)]-f(0)$

13. The function f(z) = u+iv is analytic if

(A) $u_x=v_y; v_x=-u_y$
(B) $u_x=-v_y; u_y=v_x$
(C) $u_x+v_y=0; u_y-v_x=0$
(D) $u_y=v_y; u_x=v_x$

14. If a function u(x, y) satisfies $u_{xx}+u_{yy}=0$, then u is

(A) Analytic
(B) Harmonic
(C) Differentiable
(D) Continuous

15. The invariant points of the transformation $w=\frac{z-1}{z+1}$ are

(A) i, −i
(B) −1, i
(C) 1, −i
(D) 1, −1

16. An analytic function with constant modulus is

(A) Zero
(B) Constant
(C) Analytic
(D) Harmonic

17. A curve which does not cross itself is called a

(A) Curve
(B) Closed curve
(C) Simple closed curve
(D) Multiple curve

18. The value of $\int \frac{z}{z-2}dz$, where c is the circle $|z|=1$ is

(A) 0
(B) $\frac{\pi}{2}i$
(C) $\frac{\pi}{2}$
(D) 2

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19. If $f(z)=\frac{\sin z}{z}$, then

(A) z = 0 is a simple pole
(B) z = 0 is a pole of order 2
(C) z = 0 is a removable singularity
(D) z = 0 is a zero of f(z)

20. The residue of $f(z)=\frac{z}{(z-1)^2}$ at its pole is

(A) 2
(B) –1
(C) 0
(D) 1

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r dr\, d\theta$.

22. If $\phi(x,y,z)=x^2y+y^2x+z^2$, find $\nabla\phi$ at the point (1, 1, 1).

23. Find $L\left[te^{-t}\sin 3t\right]$.

24. Find the invariant points of the transformation $w=\frac{2z+4i}{1+iz}$.

25. Evaluate $\int_c \frac{(z+1)dz}{(z-1)(z-3)}$ where c is $|z|=2$ using Residue theorem.

26. Evaluate $\int_0^1\int_0^x (x+y)dydx$.

27. Prove that $\nabla(r^n)=nr^{n-2}\bar{r}$, where r is the position vector of the point (x, y z) with respect to the origin.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\, dxdy$.

(OR)

b.i. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x-y} e^{x+y+z}\,dx\,dy\,dz$.

ii. Find the area $r^2 = a^2\cos 2\theta$ using double integration.

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29. a. Prove that $\overline{A} = (2x+yz)\overline{i} + (4y+zx)\overline{j} - (6z-xy)\overline{k}$ is of solenoidal as well as irrotational. Also find the scalar potential of $\overline{A}$.

(OR)

b. Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.

30. a.i. Verify the final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

ii. Evaluate $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$.

(OR)

b. Solve $(D^2-4D+8)y = e^{2t}$, given $y(0)=2, y'(0)=-2$ using Laplace transform.

31. a. Find the analytic function $w=u+iv$ if $u=e^x(x\sin y + y\cos y)$ and hence find v.

(OR)

b. Find the bilinear transformation which maps z = 1, i, –1 onto w = 2, i, –,2.

32. a.i. Evaluate $\oint_C \frac{e^{-z}}{z+1}dz$, where C is a circle $|z|=2$ using Cauchy integral formula.

ii. Expand $f(z) = \frac{1}{z(z-1)}$ as Laurent's series in powers valid in $|z|<1$ *and* $|z|>1$.

(OR)

b. Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ using Contour integration.

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Reg. No.

Q.P. - 10

B.Tech. DEGREE EXAMINATION, JUNE 2017

Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

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Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. The value of $\int_0^1\int_0^x dxdy$ is

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{y}{2}$

(D) $\frac{y^2}{2}$

2. Area of a region in polar co-ordinate system is

(A) $\iint_R r dr d\theta$

(B) $\iint_R \theta dr d\theta$

(C) $\iint_R d\theta dr$

(D) $\iint_R dxdy$

3. $\int_0^1\int_0^1 e^{x+y}\, dxdy$ is equal to

(A) e^2

(B) $(e-1)^2$

(C) 1

(D) 0

4. Volume of a region R is given by

(A) $\iint_R dxdy$

(B) $2\iint_R dxdy$

(C) $\frac{1}{2}\iint_R dxdy$

(D) $\iiint_R dv$

5. If $\vec{F}$ is an irrotational vector, then curl $\vec{F}$ =

(A) 1

(B) 2

(C) 3

(D) 0

6. According to Green's theorem $\int_c (Pdx + Qdy) =$

(A) $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$

(B) $\iint_R \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right) dxdy$

(C) $\iint_R \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}\right) dxdy$

(D) $\iint_R \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) dxdy$

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7. If $\overline{F}$ is a conservative vector field, then

(A) $curl\ \overline{F} = 0$

(B) $grad\ \overline{F} = 0$

(C) $div\ \overline{F} = 0$

(D) Directional derivative $= 0$

8. $curl(grad\,\phi)$ is

(A) 1

(B) 0

(C) 3

(D) –1

9. $L(t^3)$ is

(A) $\frac{4}{s^3}$

(B) $\frac{2}{s^2+4}$

(C) $\frac{6}{s^4}$

(D) $\frac{s}{s^2-4}$

10. $L^{-1}\left[\frac{1}{(s-5)^2}\right]$ is

(A) $e^{5t}.t$

(B) $\frac{e^{5t}}{2^4}$

(C) $\frac{e^{5t}.t^4}{24}$

(D) $\frac{t^4}{24}$

11. If $L[f(t)] = F(s)$, then $L\left[\frac{f(t)}{t}\right]$ is if $\lim\limits_{t\to 0}\frac{f(t)}{t}$ exists

(A) $\int\limits_0^\infty F(s)ds$

(B) $\int\limits_s^\infty F(s)ds$

(C) $\int\limits_{-\infty}^\infty F(s)ds$

(D) $\int\limits_a^\infty F(s)ds$

12. $L\left[te^{2t}\right] =$

(A) $\left(\frac{1}{(s-2)^2}\right)$

(B) $-\left(\frac{1}{(s-2)^2}\right)$

(C) $\frac{1}{(s-1)^2}$

(D) $\frac{1}{s^2}$

13. Cauchy Riemann equations in polar coordinates for an analytic function f(z) are

(A) $u_r = \frac{1}{r}v_\theta,\ v_r = -\frac{1}{r}u_\theta$

(B) $u_r = v_\theta,\ v_r = -u_\theta$

(C) $u_r = \frac{1}{r}v_\theta,\ v_r = -\frac{1}{r}v_\theta$

(D) $u_r = u_\theta,\ v_r = -v_\theta$

14. $\omega = \frac{1}{z}$ is known as

(A) Translation

(B) Inversion

(C) Rotation

(D) Transition

15. The points coincide with their transformations are known as
(A) Fixed points
(B) Critical points
(C) Singular points
(D) Regular points

16. The mapping defined by an analytic function f(z) is conformal at all points z, except at points where
(A) $f'(z)=0$
(B) $f'(z)\neq 0$
(C) $f'(z)>0$
(D) $f'(z)<0$

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17. A region which is not simply connected is called a _____ region.
(A) Multiply connected
(B) Jordan connected
(C) Connected curve
(D) Multi curve

18. If f(z) is analytic and f '(z) is continuous at all points inside and on a simple closed curve c, then
(A) $\int_c f(z)dz \neq 0$
(B) $\int_c f(z)dz = 1$
(C) $\int_c f(z)dz \neq 1$
(D) $\int_c f(z)dz = 0$

19. The singular points of $f(z)=\dfrac{z+3}{(z-1)(z-2)}$ are
(A) z = 1, 3
(B) z = 1, 0
(C) z = 1, 2
(D) z = 2, 3

20. A zero of an analytic function f(z) is a value of z for which
(A) f(z) = 1
(B) f(z) ≠ 1
(C) f(z) ≠ 0
(D) f(z) = 0

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the area enclosed by the curves $y^2 = 4x$ and $x^2 = 4y$.

22. Find the angle between the normal to the surface $x^2 = yz$ at the points (1, 1, 1) and (2, 4, 1)

23. Find $L\left[\dfrac{\sin 3t \cos t}{t}\right]$.

24. Show that an analytic function with constant modulus is constant.

25. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_c \vec{F}.\vec{dr}$ where c is the arc of the parabola $y = 2x^2$ from (0, 0) to (1, 2)

26. Find the invariant points of the transformation $\omega = \dfrac{-2z+4i}{iz+1}$.

27. Find the residue at the pole of the function $f(z)=\frac{z}{z^2+1}$.

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PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b.i. Show that $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8}$.

ii. Evaluate $\int_0^{\log a} \int_0^{x} \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.

29. a.i. Prove that $\vec{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$ is irrotational and find the scalar potential.

ii. Find the directional derivative of $\phi=x^2+y^2+4xyz$ at (1, –2, 2) in the direction of $2\vec{i}-2\vec{j}+\vec{k}$.

(OR)

b. Verify Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ over the cuboid formed by $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.

30. a. Solve by Laplace transform $\frac{d^2y}{dt^2}-4\frac{dy}{dt}-5y=\sin t$, where $y(0)=1$, $y'(0)=0$.

(OR)

b. Using convolution theorem, find $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right]$.

31. a. Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic. Hence find harmonic conjugate of u.

(OR)

b. If $w=u+iv$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

32. a. i. Using Cauchy's integral formula, evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where c is $|z| = \frac{3}{2}$.

ii. Find the Laurents series of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the region $1 < |z| < 2$.

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(OR)

b. Evaluate using contour integration $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, DECEMBER 2017

Q.P – (11)

First/ Second/ Third Semester

15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

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(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of $\int_1^a \int_1^b \frac{dxdy}{xy}$
   - (A) $\log a + \log b$
   - (B) $\log a \log b$
   - (C) $\log a$
   - (D) $\log b$

2. The name of the curve $r = a(1+\cos\theta)$ is
   - (A) Cycloid
   - (B) Hypocycloid
   - (C) Cardioid
   - (D) Hemicircle

3. For the double integral the transformation from Cartesian to polar co-ordinates is
   - (A) $dxdy = drd\theta$
   - (B) $dxdy = |J| drd\theta$
   - (C) $dxdy = -Jdrd\theta$
   - (D) $dxdy = |J|^2 drd\theta$

4. The value of $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is
   - (A) 3
   - (B) 4
   - (C) 2
   - (D) 6

5. $\nabla(r^n)$ is equal to
   - (A) $n\bar{r}$
   - (B) $n\bar{r}\, r^n$
   - (C) $nr^{n-2}\bar{r}$
   - (D) nr^n

6. If $v = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+\lambda z)\hat{k}$ is solenoidal then the value of λ is
   - (A) 10
   - (B) −2
   - (C) 2
   - (D) 0

7. The condition for $\bar{F}$ to be conservative is $\bar{F}$ should be
   - (A) Solenoidal vector
   - (B) Irrotational vector
   - (C) Rotational vector
   - (D) Neither solenoidal nor irrotational

8. The workdone by the conservative force when it moves around the closed curve is

(A) 0
(B) $\nabla.\bar{F}=0$
(C) $\nabla\times\bar{F}=0$
(D) $\nabla.(\nabla\times\bar{F})=0$

9. $L(\cosh at)$ is equal to

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(A) $\dfrac{s}{s^2-a^2}$
(B) $\dfrac{a}{s^2+a^2}$
(C) $\dfrac{a}{s^2-a^2}$
(D) $\dfrac{a}{s^2+a^2}$

10. The value of $L\int_0^t \sin t\, dt$ is

(A) $\dfrac{1}{s^2+1}$
(B) $\dfrac{s}{s^2+1}$
(C) $\dfrac{1}{(s^2+1)^2}$
(D) $\dfrac{1}{s(s^2+1)}$

11. $L(t^4)$ is equal to

(A) $\dfrac{4!}{s^5}$
(B) $\dfrac{3!}{s^4}$
(C) $\dfrac{4!}{s^4}$
(D) $\dfrac{3!}{s^5}$

12. If $L[f(t)]=F(s)$, then $L\left[e^{-at}f(t)\right]$ is equal to

(A) $F(s-a)$
(B) $F(s)$
(C) $F(s+a)$
(D) $\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

13. An analytic function with constant modulus is

(A) A function of x
(B) A function of y
(C) A function of z
(D) Constant

14. Cauchy-Riemann equation in polar co-ordinates are

(A) $rU_r=V_\theta, U_\theta=-rV_r$
(B) $U_\theta=+rV_r, V_\theta=-rU_r$
(C) $-rU_r=V_\theta, U_\theta=rV_r$
(D) $U_r=rV_\theta, V_\theta=rV_\theta$

15. Critical point of transformation $w=z^2$ is

(A) z = 2
(B) z = 0
(C) z = 1
(D) z = −2

16. The function $f(z)=\log z$ is

(A) Differentiable
(B) Analytic everywhere
(C) Analytic
(D) Analytic everywhere except at the origin

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17. The annular region for the function $f(z)=\dfrac{1}{z(z-1)}$ is

(A) $0<|z|<1$ (B) $1<|z|<2$
(C) $1\le|z|<0$ (D) $|z|\ge 1$

18. The singularity of $f(z)=\dfrac{z}{(z-2)^3}$ is

(A) Essential singularity (B) Removable singularity
(C) Pole of order 3 (D) Pole of order 2

19. The value of $\oint_c \dfrac{5z^2+8z+5}{z+1}dz$, when $|z|=\dfrac{1}{2}$ is

(A) $2\pi i$ (B) 0
(C) πi (D) $\dfrac{\pi i}{2}$

Q: 20. ~~Poles of $z^2+1=0$~~ are Poles of $f(z)=\dfrac{1}{z^2+1}$ is $\pm i$

(A) 0, 1 (B) i, i
(C) i, −i (D) 1, 1

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Evaluate $\int_0^1\int_0^x (x+y)\,dy\,dx$.

22. If $r=|\bar{r}|$, where $\bar{r}$ is the position vector of the point (x, y, z) with respect to origin, prove that $f(r)\bar{r}$ is an irrotational vector.

23. Find the constants 'a' and 'b' so that the surface $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ and cut orthogonally at $(1,-1,2)$.

24. Evaluate $L\left[\dfrac{1-e^{-t}}{t}\right]$.

25. Evaluate using Laplace transform $\int_0^\infty e^{-3t}t\sin t\,dt$.

26. Show that z^n is an analytic function.

27. Evaluate $\oint_c \dfrac{(\sin\pi z^2+\cos\pi z^2)}{z+z^2}dz$, $|z|=2$ using Cauchy Residue theorem.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Change the order of integration and hence integrate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b. Find the volume bounded by the curves $y^2 = x$ and $y = x^2$ and the planes $z = 0$ and $x + y + z = 2$.

29. a. Verify Gauss Divergence theorem for $F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

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(OR)

b. Verify Stoke's theorem for $\overline{F} = (y - z + 2)\hat{i} - (yz + 4)\hat{j} - (xz)\hat{k}$ over the surface of the cube $x = 0,\ y = 0,\ z = 0, z = 2,\ y = 2, x = 2$ above the $x \circ y$ plane."

30. a. i. Using convolution theorem find the Laplace transform of $\dfrac{1}{s(s+2)^3}$.

ii. Find $Lf(t)$ if $f(t) = \begin{cases} t, & 0 < t \le \pi \\ 2\pi - t, & \pi \le t \le 2\pi \end{cases}$

given that $f(t + 2\pi) = f(t)$.

(OR)

b. Solve using Laplace transform method $(D^2 + 4)y = \cos 2t, y(0) = 3, y'(0) = 4$.

31. a.i. If $u - v = e^x(\cos y - \sin y)$, find the analytic function in terms of z.

ii. Verify that the family of curves $u = c_1$, $v = c_2$ cut orthogonally when $w = z^3$.

(OR)

b.i. Find the bilinear transformation which maps the points $z = \infty, i, 0$ into $w = 0, i, \infty$ respectively.

ii. Find the image of the rectangular region in the z plane bounded by the lines $x = 0,\ y = 0,\ x = 2\ and\ y = 1$ under the transformation $w = 2z$.

32. a.i. Evaluate $\oint_c \dfrac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz,\ |z| = 1$.

ii. Find the Laurent series expansion for $f(z) = \dfrac{z}{(z+1)(z+2)}$ over

(A) $|z| < 1$ (B) $1 < |z| < 2$ (C) $|z + 1| < 1$.

(OR)

b. Evaluate using contour intergration $\int_0^{2\pi} \dfrac{d\theta}{13 + 5\sin\theta}$.

* * * * *

b. Find the volume of a sphere $x^2+y^2+z^2=a^2$ using triple integration.

29. a. Verify Gauss Divergence theorem for $F=(x^2-yz)\bar{i}+(y^2-xz)\bar{j}+(z^2-xy)\bar{k}$ taken over a rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

(OR)

b.i. Prove that the vector $\overline{F}=(6xy+z^3)\bar{i}+(3x^2-z)\bar{j}+(3xz^2-y)\bar{k}$ is irrotational and find tis scalar potential.

ii. Show that $\overline{F}=(y^2-z^2+3yz-2x)\bar{i}+(2xy+3xz)\bar{j}+(3xy-2xz+2z)\bar{k}$ is both irrotational and solenoidal.

30. a. Find the Laplace transform of the triangular wave of period 2π define by

$$f(t)=\begin{cases} t & 0\le t\le \pi \\ 2\pi-t & \pi\le t\le 2\pi \end{cases}$$

(OR)

b. Solve $y''+4y'+3y=e^{-t}$ where $y(0)=0, y'(0)=0$ using Laplace transformation.

31. a. Find the analytic function f(z) if $u=e^x(x\sin y+y\cos y)$. Hence find V.

(OR)

b.i. Find the bilinear transformation which maps $z = 0, 1, \infty$ with points $w=i, 1, -i$ respectively.

ii. Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines in the z-plane into circles and straight lines in the w-plane.

32. a. Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ as a Laurent's series if (i) $|z|<2$ (ii) $2<|z|<3$.

(OR)

b. Using Cauchy's residue theorem, evaluate $\int_c \frac{z}{(z-1)^2(z+1)}dz$ where c is (i) $|z|=\frac{1}{2}$ (ii) $|z|=2$

* * * * *

QP – 12

Reg. No.

B.Tech. DEGREE EXAMINATION, DECEMBER 2017 Page – 43

Second Semester

MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2013-2014 and 2014-2015)

Note:

(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of $\int_0^1\int_0^1 dxdy$ is
   (A) 1 ✓ (B) 2
   (C) 0 (D) 4

2. Area of a region in Cartesian form is
   (A) $\iint y\, dxdy$ (B) $\iint x\, dxdy$
   (C) $\iint_R dxdy$ ✓ (D) $\iiint_v \iint dv$

3. $\int_0^\pi\int_0^\pi d\theta d\phi$ is
   (A) 1 (B) 0
   (C) $\pi/2$ (D) π^2 ✓

4. The value of $\int_0^1\int_0^1\int_0^1 dxdydz$ is
   (A) 1 ✓ (B) 3
   (C) 2 (D) 0

5. If $\overline{F}$ is a solenoidal vector, then
   (A) $\nabla.\overline{F}=0$ ✓ (B) $\nabla\times\overline{F}=0$
   (C) $\nabla\overline{F}=0$ (D) $\nabla.\overline{F}=1$

6. The maximum value of directional derivative is
   (A) gradient ϕ (B) curl ϕ
   (C) $|\nabla\phi|$ ✓ (D) $|\nabla.\phi|$

7. Area of a region, by using Green's theorem is
   (A) $\int_c (xdy-ydx)$ (B) $\int_c (ydx-xdy)$
   (C) $\frac{1}{2}\int_c (xdy-ydx)$ ✓ (D) $\frac{1}{2}\int_c (ydx-xdy)$

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8. By Stokes theorem $\int_c \overline{F}.\overline{dr}$ is

(A) $\iint_s \nabla\times\overline{F}ds$
(B) $\iint_s \nabla.\overline{F}ds$
(C) $\iint_s \nabla.\overline{F}.\overline{n}\,ds$
(D) $\iint_s \nabla\times\overline{F}.\hat{n}\,ds$

9. $L(t^4)=$

(A) $\frac{4!}{s^5}$
(B) $\frac{3!}{s^4}$
(C) $\frac{2!}{s^4}$
(D) $\frac{5!}{s^4}$

10. $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ is

(A) e^{at}
(B) e^{-at}
(C) te^{-at}
(D) $-te^{at}$

11. If $L[f(t)]=F(s)$, then $L[f'(t)]$ is

(A) $sL[f(t)]-f(0)$
(B) $sL[f(t)]+f(0)$
(C) $sL[f(t)]$
(D) $s^2L[f(t)]-f(0)$

12. $L(\cos t)$ is

(A) $\frac{1}{s^2-1}$
(B) $\frac{1}{s^2+1}$
(C) $\frac{s}{s^2-1}$
(D) $\frac{s}{s^2+1}$

13. Cauchy Riemann equations for an analytic function $f(z)$ is

(A) $u_x=v_y, u_x=-u_y$
(B) $u_x=v_y, v_x=u_y$
(C) $u_x=v_x, u_y=v_y$
(D) $u_x=-v_x, u_y=v_y$

14. A function u is said to be harmonic iff

(A) $u_{xx}+u_{yy}=0$
(B) $u_{xy}+u_{yx}=0$
(C) $u_x^2+u_y^2=0$
(D) $u_x+u_y=0$

15. $w=\frac{a+bz}{c+dz}$ is a bilinear transformation, when

(A) $ad-bc=0$
(B) $ad-bc\neq 0$
(C) $ab-cd\neq 0$
(D) $ac-bd\neq 0$

16. A mapping that preserves angles between oriented circles both in magnitudes and in sense is called a ________ mapping.

(A) Informal
(B) Isoganal
(C) Conformal
(D) Formal

17. The point at which a function f(z) is not analytic is known as a ____ of $f(z)$.

(A) Residue
(B) Singularity
(C) Integrals
(D) Fixed points

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18. The fixed points of the transformation $w=z^2$ are

(A) 0, 1
(B) 0, −1
(C) −1, 1
(D) −i, i

19. The poles of $f(z)=\frac{z^2+1}{1-z^2}$ is

(A) 1
(B) −1
(C) ±1
(D) 0

20. If f(z) is analytic and f '(z) is continuous at all points in the region bounded by the simple closed curves c_1, c_2 then

(A) $\int_{c_1} f(z)dz=\int_{c_2} f(z)dz$
(B) $\int_{c_1} f(z)dz\neq\int_{c_2} f(z)dz$
(C) $\int_{c_1} f'(z)dz=\int_{c_2} f'(z)dz$
(D) $\int_{c_1} f'(z)dz\neq\int_{c_2} f'(z)dz$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the area that lies outside the circle $r=2a\cos\theta$ and inside the circle $r=4a\cos\theta$.

22. Find the values of the constants a, b, c so that $\overline{F}=(axy+bz^3)\overline{i}+(3x^2-cz)\overline{j}+(3xz^2-y)\overline{k}$ may be irrotational.

23. Show that an analytic function with constant real part is constant.

24. Find the directional derivative of $\phi=2xy+z^2$ at the point (1, −1, 3) in the direction of $\overline{i}+2\overline{j}+2\overline{k}$.

25. Find $L^{-1}\left[\frac{1}{s(s^2+9)}\right]$.

26. Find the residue of $f(z)=\frac{z}{(z-1)^2}$ at its poles.

27. Find the image of $|z-2i|=2$ under the transformation $w=\frac{1}{z}$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Change the order of integration and hence evaluate $\int_0^a\int_{x^2/a}^{2a-x} xy\,dydx$.

(OR)

QP - 13

Reg. No.	R	A	1	7	1	1	0	0	3	0	2	0	4	1	9

B.Tech. DEGREE EXAMINATION, MAY 2018

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First/ Second Semester

15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2017 – 2018 only)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. $\int_0^{\pi}\int_0^{x} \sin y \, dy dx$ is equal to

   (A) $\pi/2$
   (B) $\pi/4$
   (C) 2π
   (D) π

2. $\int_0^{1/2}\int_1^{2} x dx dy$ is

   (A) 3
   (B) 3/2
   (C) 3/4
   (D) 1/2

3. $\int_2^{\pi/2}\int_0^{a} r^3 \sin 2\theta \, dr d\theta$

   (A) $\frac{a^4}{2}$
   (B) $\frac{a^4}{3}$
   (C) $\frac{a^4}{4}$
   (D) a^4

4. $\iiint_r dxdydz$ over the volume of the sphere of the radius 'a' is

   (A) $\frac{4}{3}\pi a^3$ ✓
   (B) $4\pi a^3$
   (C) $2\pi a^3$
   (D) πa^3

5. $Curl(grad\phi)$ is equal to

   ✓(A) 0
   (B) 3
   (C) 01
   (D) –1

6. The area bounded by a simple closed curve c is

   (A) $\frac{1}{2}\int_c xdy + ydx$
   (B) $\int_c xdy - ydx$
   (C) $\frac{1}{2}\int_c (xdy - ydx)$ ✓
   (D) $\int_c (xdy + ydx)$

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7. $div\bar{r}$ is
(A) r
(B) 1
(C) 0
(D) 3

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8. The maximum directional derivative of $\phi = xyz^2$ at (1, 0, 3) is
(A) 9
(B) 0
(C) −9
(D) 1

9. If $L[(t)] = F(s)$ the $L[f(at)]$ is
(A) $F\left[\frac{s}{a}\right]$
(B) $F\left[\frac{a}{s}\right]$
(C) $F[as]$
(D) $\frac{1}{a}F\left[\frac{s}{a}\right]$

10. If $L[(t)] = F(s)$ then $L[tf(t)]$ is
(A) $\frac{d}{ds}F(s)$
(B) $\frac{d}{ds}|F(s)|$
(C) $-\frac{d}{ds}F(s)$
(D) $\int_s^\infty F(s)ds$

11. If $L[f(t)] = \frac{1}{s(s+a)}$ then $\underset{t\to 0}{lt}\ f(t)$ is
(A) 0
(B) 1/a
(C) a
(D) $1/a^2$

12. The convolution of f(t) and g(t) is
(A) $\int_0^\infty f(u)g(t-u)du$
(B) $\int_0^\infty f(t-u)g(u)du$
(C) $\int_0^t f(u)g(u)du$
(D) $\int_0^t f(u)g(t-u)du$

13. If $z = u(x,y) + iv(x,y)$ is analytic then family of curves u = constant, v = constant are
(A) Parallel
(B) Not cutting
(C) Straight lines
(D) Cutting orthogonally

14. w = logz is
(A) Analytic at all points
(B) Not analytic at the origin
(C) Nowhere analytic
(D) Analytic at infinity

15. The mapping w = z+c gives
(A) Inversion only
(B) Reflection only
(C) Translation
(D) Rotation

16. In general $\omega = \dfrac{az+b}{cz+d}$ has atmost

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(A) No fixed point
(B) One fixed point
(C) Three fixed points
(D) Two fixed point

17. If a function f(z) is analytic and its derivative $f'(z)$ is continuous at all points inside and on a simple closed curve c then $\int_c f(z)dz$ is

(A) $2\pi i$
(B) $-2\pi i$
(C) 0
(D) $\dfrac{1}{2\pi i}$

18. If C is $|z|=2$ then $\int_c e^z\, dz$ is

(A) 0
(B) $2\pi i$
(C) 2
(D) $-2\pi i$

19. $f(z)=\dfrac{z+2}{(z-1)^2(z-2)}$ has

(A) Only poles
(B) A simple pole at $z=\dfrac{1}{2}$
(C) Essential singularity
(D) No poles

20. Residue of $f(z)=\dfrac{z+2}{(z-2)(z+1)^2}$ at z = 2 is

(A) $-\dfrac{9}{4}$
(B) $-\dfrac{4}{9}$
(C) $\dfrac{9}{4}$
(D) $\dfrac{4}{9}$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Find the area common to $y^2=4ax$ and $x^2=4ay$ using double integration.

22. Find a unit normal to the surface $x^2y+2xz^2=8$ at the point (1, 0, 2).

23. Find the Laplace transform of $\left(\dfrac{1-\cos t}{t}\right)$

24. Show that the function $u(x,y)=3x^2y+2x^2-y^3-2y^2$ is harmonic.

25. Expand $f(z)=\sin z$ in a Taylor's series about $z=\dfrac{\pi}{4}$.

26. Determine the poles of $f(z)=\dfrac{z^2}{(z-1)^2(z+2)}$ and the residues at one of the poles.

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27. Find $L^{-1}\left(\frac{1}{(s+1)(s+2)}\right)$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Evaluate by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$.

(OR)

b. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by triple integrals..

29. a. Determine f(r) so that $f(r)\bar{r}$ is both solenoidal and irrotiational.

(OR)

b. Verify Gauss divergence theorem, for $\bar{F}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$, taken over a cube bouonded by the planes $x=0, x=1, y=0, y=1, z=0\,and\ z=1$.

30. a. Using the Convolution theorem, find the inverse laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

(OR)

b. Solve $x''-3x'+2x=4$ given that $x(0)=2$ and $x'(0)=3$, by using Laplace transform.

31. a. If $f(z)=u+iv$ is analytic, find f(z) given that $u+v=\frac{\sin 2x}{cosh2y-\cos 2x}$.

(OR)

b. Find the bilinear map which maps the points z = 1, i, –1 onto the point w = i, 0, $-i$.

32. a. Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if (i) $|z|<2$ (ii) $|z|>3$ (iii) $2<|z|<3$.

(OR)

b. Using contour integration, Evaluate $\int_0^{2\pi} \frac{1}{5+2\cos\theta}d\theta$.

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QP-14

Reg. No. | | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, JUNE 2018

Second Semester

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MA1002 – ADVANCED CALCULUS AND COMPLEX ANALYSIS

(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. Evaluation of $\int_0^1\int_0^1 dx\,dy$ is

   (A) 1 (B) 2
   (C) 0 (D) 4

2. The value of $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin(\theta+\phi)\,d\theta\,d\phi$

   (A) 2 (B) 1
   (C) 0 (D) –2

3. The value of $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz$ is

   (A) 3 (B) 4
   (C) 2 (D) 6

4. If R is the region bounded by $x=0, y=0, x+y=1$ then $\iint_R dx\,dy$

   (A) 1 (B) 1/2
   (C) 1/3 (D) 2/3

5. Area of the double integral in polar co-ordinate is equal to

   (A) $\iint_R dr\,d\theta$ (B) $\iint_R r^2\,dr\,d\theta$
   (C) $\iint_R (r+1)\,dr\,d\theta$ (D) $\iint_R r\,dr\,d\theta$

6. Find the constant 'a', if the vector $\vec{F}=(x+3y)+(y-2z)\vec{j}+(x+az)\vec{k}$ is solenoidal.

   (A) 2 (B) –2
   (C) –1 (D) 0

7. If $\bar{r}$ is the position vector of the point (x,y,z) with respect to the origin then $\nabla . \bar{r}$ is

(A) 3
(B) 2
(C) 0
(D) 1

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8. The relation between the line integral and double integral is

(A) Stokes theorem
(B) Green's theorem
(C) Surface integral
(D) Divergence theorem

9. $L(\sin 2t)$ is

(A) $\frac{2}{s^2+4}$
(B) $\frac{2\cos 3s}{s^2+4}+\frac{\sin 3s}{s^2+4}$
(C) $\frac{s}{s^2+9}$
(D) $\frac{6}{s^2+9}$

10. $L^{-1}\left[\frac{s-3}{s^2-6s+13}\right]$ is

(A) $e^{-3t}\cos 3t$
(B) $e^{2t}\cos 3t$
(C) $e^{3t}\cos 2t$
(D) $e^{-2t}\cos 2t$

11. $L(\sin 5t)$ is

(A) $\frac{1}{s^2-1}$
(B) $\frac{5}{s^2+25}$
(C) $\frac{s}{s^2-1}$
(D) $\frac{5}{s^2+25}$

12. Inverse laplace transform of $\frac{s}{s^2-9}$ is

(A) cos 9t
(B) cos 3t
(C) cos h9t
(D) cos h3t

13. The invariant points of the transformation $w=\frac{z-1}{z+1}$ is

(A) $\pm i$
(B) ± 1
(C) 0
(D) 2, 7

14. The points at which the function $f(z)=\frac{1}{z^2+1}$ fails to be analytic at

(A) $z=\pm 1$
(B) $z=0$
(C) $z=\pm i$
(D) $z=\pm 2$

15. An analytic function with constant modulus is

(A) a function of x
(B) a function of y
(C) a function of z
(D) a constant

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16. The value of $\int_c \frac{z}{z-2} dz$ where c is the circle $|z|=1$ is

(A) 0
(B) $\frac{\pi}{2}i$
(C) $\pi/2$
(D) 2

17. The point Z_0 at which a function $f(z)$ is not analytic is known as
(A) Zeros
(B) Isolated singular point
(C) Singular point
(D) Removable singular point

18. The residue of $f(z)=\frac{z}{(z-1)^2}$ is

(A) 0
(B) 1
(C) -1
(D) $2\pi i$

19. The annular region for the function $f(z)=\frac{1}{z^2-z-6}$.
(A) $0<|z|<1$
(B) $1<|z|<2$
(C) $2<|z|<3$
(D) $|z|<3$

20. If $f(z)=\frac{\sin z}{z}$, then
(A) z=0 is a simple pole
(B) z=0 is a pole of order 2
(C) z=0 is a removable singularity
(D) z=0 is a zero of f(z)

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Evaluate $\int_0^4 \int_0^{x} \int_0^{\sqrt{x+y}} z \, dz \, dy \, dx$.

22. Show that $r^n\vec{r}$ is an irrotational vector for any value of 'n' and is solenoidal only for n= –3.

23. Find the work done when a force $F=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ displaces a particle in the xy plane from (0, 0) to (1, 1) along the parabola $y^2=x$.

24. Verify final value theorem for the function $1+e^{-t}(\cos t+\sin t)$.

25. Find the constants a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.

26. Evaluate $\int_c \frac{(z+1)}{z(z-1)} dz; c:|z|=2$, using Cauchy's residue theorem.

27. Find $L\left(\frac{e^{-2t}-e^{-3t}}{t}\right)$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

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28. a. Find the volume of the tetrahedron bounded by the planes $x=y=z=0$, and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(OR)

b. Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$.

29. a. Verify divergence theorem for $\bar{F}=(x^2-yz)\bar{i}+(y^2-xz)\bar{j}+(z^2-xy)\bar{k}$ taken over a rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

(OR)

b.i Find the angle between the surfaces $x^2-y^2-z^2=11$ and $xy+yz-zx=18$ at the point (6, 4, 3).

ii. Show that $\bar{F}=(2xy+z^3)\bar{i}+x^2\bar{j}+3xz^2\bar{k}$ is conservative vector field and hence find the scalar potential.

30. a.i Using convolution theorem, find $L^{-1}\left(\frac{1}{(s+3)(s-1)}\right)$. (8 Marks)

ii. Find $L\left(te^{-t}\sin t\right)$. (4 Marks)

(OR)

b. Solve using Laplace transform method $y''+2y'-3y=\sin t$ given $y(0)=0, y'(0)=0$.

31.a.i Determine the bilinear transformation which maps $z_1=0, z_2=1, z_3=\infty$ into $w_1=i$, $w_2=-1, w_3=-i$ respectively.

ii. Find the analytic function $f(z)=u+iv$, where $u=e^x(x\sin y+y\cos y)$.

(OR)

b.i. If $f(z)$ is an analytic function of z, show that $\nabla^2|f(z)|^2=4|f'(z)|^2$.

ii. Find the image of the circle $|z-1|=1$ under the mapping $w=1/z$.

32. a. Evaluate $\int_0^\infty \frac{x^2dx}{(x^2+9)(x^2+4)}$ by the method of residues.

(OR)

b.i. Explain $f(z)=\frac{1}{(z-1)(z-2)}$ as Laurent series valid in the region (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|Z|>2$

ii. Evaluate $\int_C \frac{e^{2z}}{\cos \pi z}dz$ where c is a circle $|z|=1$.

* * * * *

27. Evaluate $\oint_C \frac{(3z^2+z)}{z^2-1}dz$ where C is circle $|z-1|=1$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b. Find the volume of a sphere $x^2+y^2+z^2=a^2$ by using triple integrals.

29. a. Verify Gauss-Divergence theorem for $\bar{F}=x^2\hat{i}+z\hat{j}+yz\hat{k}$ over a cube formed by $0\le x\le 1, 0\le y\le 1, 0\le z\le 1$.

(OR)

b. Verify Green's theorem in a plane for $\iint (3x^2-8y^2)dx+(4y-6xy)dy$ where 'c' is the boundary of the region defined by $x=y^2$ and $y=x^2$.

30. a. Find the Laplace transform of periodic function
$f(t) = \begin{cases} t, & 0<t<1 \\ 2-t, & 1<t<2 \end{cases}$ where $f(t+2)=f(t)$ for all 't' where t is positive.

(OR)

b. Solve using Laplace transform method $y''-3y'-4y=2e^{-t}$, $y(0)=1, y'(0)=1$.

31. a. Find the Analytic function $f(z)=u+iv$, if $u+v=(x-y)(x^2+4xy+y^2)$.

(OR)

b. Find the bilinear transform which maps the points $z_1=1, z_2=i, z_3=-1$ into the points $w_1=i, w_2=0, w_3=-i$.

32. a. Expand $f(z)=\frac{1}{(z-1)(z-2)}$ as a Laurent's series valid in (i) $|z|<1$ (ii) $1<|z|<2$ and (iii) $|z|>2$.

(OR)

b. Evaluate using contour integration $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$.

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L2

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019
First & Second Semester

18MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The value of the integral $\int_0^2 \int_0^1 xy\,dx\,dy$ is
   (A) 1 (B) 2
   (C) 3 (D) 4

2. The name of the curve $r=a(1+\cos\theta)$ is
   (A) Lemniscate (B) Cycloid
   (C) Cardiod (D) Hemicircle

3. Area of the double integral in Cartesian co-ordinates is
   (A) $\iint dy\,dx$ (B) $\iint r\,dr\,d\theta$
   (C) $\iint x\,dx\,dy$ (D) $\iint x^2\,dx\,dy$

4. $\int_0^1 \int_0^2 \int_0^3 dx\,dy\,dz$ is equal to
   (A) 3 (B) 2
   (C) 4 (D) 6

5. The relation between line integral and a surface integral is known as
   (A) Green's theorem (B) Residue theorem
   (C) Stokes theorem (D) Divergence theorem

6. The condition for $\bar{F}$ to be conservative is, $\bar{F}$ should be
   (A) Solenoidal vector (B) Rotational vector
   (C) Irrotational vector (D) Both solenoidal and irrotational

7. The unit normal vector of $\phi=xy+yz+zx$ at the point $(-1, 1, 1)$ is
   (A) $2i$ (B) i
   (C) $3i$ (D) $4i$

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8. Value of $\nabla\left(r^n\right)$ is

(A) $n\bar{r}$ (B) $n\bar{r}\,r^n$

(C) $n(n-1)\bar{r}$ (D) $nr^{n-2}\bar{r}$

9. An example of a function for which the laplace transform does not exist is

(A) $g(t)=t^2$ (B) $g(t)=\sin t$

(C) $g(t)=\tan t$ (D) $g(t)=e^{-at}$

10. If $L[f(t)]=F(s)$, then $L\left(e^{-at}f(t)\right)$ is

(A) $F(s-a)$ (B) $F(s+a)$

(C) $F(s)$ (D) $\frac{1}{a}F\left(\frac{s}{a}\right)$

11. $L\left(t^4\right)$

(A) $\frac{4!}{s^5}$ (B) $\frac{3!}{s^4}$

(C) $\frac{4!}{s^4}$ (D) $\frac{5!}{s^4}$

12. $L(\cos 2t)$ is

(A) $\frac{s}{s+2}$ (B) $\frac{s}{s^2+4}$

(C) $\frac{s}{s^2-4}$ (D) $\frac{2}{s^2+4}$

13. The Cauchy-Riemann equation in polar co-ordinates are None of them is correct.

(A) $ru_r-v_\theta, u_\theta=-rv_r$ (B) $-ru_r=v_\theta, u_\theta=+rv_r$

(C) $u_r=rv_\theta, v_r=ru_\theta$ (D) $u_r=-rv_\theta, v_r=ru_\theta$

14. The critical point of transformation $w=z^2$ is

(A) z=2 (B) z=0

(C) z=1 (D) z=-2

15. The fixed points of the transformation $w=-\frac{(2iz-4)}{z-i}$ are None

(A) $-4i,\ i$ (B) $4i,\ -i$

(C) $i,\ 2i$ (D) $-i,\ 2i$

16. If $u+iv$ is analytic, then the curves $u=c_1$ *and* $v=c_2$

(A) Cut orthogonally (B) Are parallel

(C) Coincide (D) Intersect each other

17. The annular region for the function $f(z)=\frac{1}{z(z-1))}$ is

(A) $0<|z|<1$ (B) $1<|z|<2$

(C) $1<|z|<0$ (D) $2<|z|<1$

18. If $f(z)=\frac{1}{(z-1)(z-3)^3}$ then

(A) 3 is a pole of order 3, 1 is a pole of order 2 (B) 1 is a simple pole, 3 is a pole of order 3

(C) 3 is a simple pole, 1 is a pole of order 2 (D) 1 is a pole of order 3 and 3 is a pole of order 1

19. The value of $\oint_c \frac{z}{z-2}dz$ when c is a circle $|z|=1$ is

(A) 0 (B) 2

(C) $\pi/2$ (D) π

20. If $f(z)$ is analytic inside and on C, the value of $\oint_C \frac{f(z)}{z-a}dz$, where 'C' is a simple closed curve and 'a' is any point within 'C' is

(A) $f'(a)$ (B) $2\pi i\ f(a)$

(C) $\pi i\ f(a)$ (D) 0

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$.

22. Find the angle between normals to the surface $x^2=yz$ at the points (1, 1, 1) and (2, 4, 1).

23. A fluid motion is given by $\bar{v}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$. Is this motion irrotational? If so, find the scalar potential.

24. Find $L\left[\frac{\cos 6t-\cos 4t}{t}\right]$.

25. Verify initial value theorem for $f(t)=1+e^{-t}(\sin t+\cos t)$.

26. Find the constant a, b c if $f(z)=(x+ay)+i(bx+cy)$ is analytic.