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Math.

→ Taylor's series :

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right]$$

$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$

This is the taylor's series expression for f(x,y) at the point (a,b)

① expand $e^x \cos y$ in powers of x and y upto 3rd term.

$$f(x,y) = e^x \cos y.$$

$$f(a,b) = 1.$$

$\partial f/\partial x \leftarrow f_x = e^x \cos y \qquad f_x(a,b) = 1$

$\frac{\partial f}{\partial y} \leftarrow f_y = -e^x \sin y \qquad f_y(a,0) = 0$

$f_{xx} = e^x \cos y \qquad f_{xx}(0,0) = 1$

$f_{yy} = -e^x \cos y \qquad f_{yy}(0,0) = -1$

$\frac{\partial^2 f}{\partial x \partial y} \leftarrow f_{xy} = -e^x \sin y \qquad f_{xy}(0,0) = 0$

$$e^x \cos y = 1 + \frac{1}{1!}\left[x(1) + y(0)\right] + \frac{1}{2!}\left(x^2(1) + 2(x)(y)(0) + \dots\right.$$

$$= 1 + \frac{1}{1!}(x) + \frac{1}{2!}(x^2 - y^2)$$

② Expand $\tan^{-1}(y/x)$ near (1,1)

$$f(x,y) = \tan^{-1}(y/x) \qquad f(1,1) = \pi/4$$

$$f_x = \frac{1}{1 + y^2/x^2}\left(-y/x^2\right) \qquad \cancel{f_x(1,1)}$$

$$= \frac{-y}{x^2+y^2} \qquad f_x(1,1) = \frac{-1}{2}$$

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$f_y = \frac{1}{1+y^2/x^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$ $\quad f_y(1,1) = \frac{1}{2}$

$f_{xx} = (-y)(x^2+y^2)^{-1} = (-y)(-1)(x^2+y^2)^{-2}(2x)$
$= 2xy(x^2+y^2)^{-2}$ $\quad f_{xx}(1,1) = 1/2$

$f_{yy} = x(x^2+y^2)^{-1} = -x(x^2+y^2)^{-2}(2y)$
$= -2xy(x^2+y^2)^{-2}$ $\quad f_{yy}(1,1) = \frac{-1}{2}$

$f_{xy} = \frac{(x^2+y^2)(-1) - (-y)(2y)}{(x^2+y^2)^2}$ $\quad f_{xy}(1,1) = 0.$

$f(x,y) = 1 + \frac{1}{1!}\left[(x-1)\left(\frac{-1}{2}\right) + (y-1)\left(\frac{1}{2}\right)\right] + \frac{1}{2!}\left[(x-1)^2\left(\frac{1}{2}\right)\right.$
$\left. + 2(x-1)(y-1)(0) + (y-1)^2\left(\frac{-1}{2}\right)\right]$

③ $f(x,y) = e^{xy}$. $f(1,1) = e$ at $(1,1)$ upto to third degree.

$f_x = e^{xy}(y)$ $\quad f_x(1,1) = e$
$f_y = e^{xy}(x)$ $\quad f_y(1,1) = e$
$f_{xx} = e^{xy}(y^2)$ $\quad f_{xx}(1,1) = e$
$f_{yy} = e^{xy}(x^2)$ $\quad f_{yy}(1,1) = e$
$f_{xy} = (e^{xy}(1) + ye^{xy}x)$ $\quad f_{xy}(1,1) = 2e$

$f_{xxx} = e^{xy}(y^3)$ $\quad f_{xxx}(1,1) = e$
$f_{yyy} = e^{xy}(x^3)$ $\quad f_{yyy}(1,1) = e$
$f_{xxy} = 2y(e^{xy}) + y^2(e^{xy})(x)$ $\quad f_{xxy}(1,1) = 3e.$
$f_{yyx} = 2xe^{xy} + x^2e^{xy}(y)$ $\quad f_{yyx}(1,1) = 3e$

→ Maxima and Minima of functions of 2 variables

let the given func be :

$f(x,y)$

$$\frac{\partial f}{\partial x} = p \qquad \frac{\partial f}{\partial y} = q$$

$$\frac{\partial^2 f}{\partial x^2} = r \qquad \frac{\partial^2 f}{\partial x\,\partial y} = s \qquad \frac{\partial^2 f}{\partial y^2} = t$$

To find the points (x_1,y_1) (x_2,y_2) .... (x_n,y_n) substitute:

→ $\frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$ find (x_1,y_1) (x_2,y_2) ...

(these are pts) ↑

To check whether the pts are max, min (or) saddle the foll.

→ ① $rt - s^2 > 0$

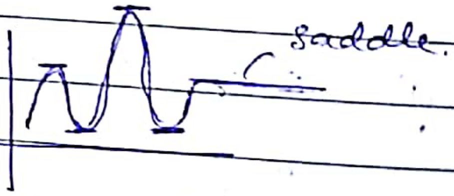
case 1 : $r > 0$ min

case 2 : $r < 0$ max

② $rt - s^2 < 0$

pt saddle.

(is max nor min)



③ $rt - s^2 = 0$

① $f(x,y) = x^3 + y^3 + 3xy$

$\frac{\partial f}{\partial x} = p = 3x^2 + 3y$ $\qquad r = 6x$

$s = 3$

$\frac{\partial f}{\partial y} = q = 3y^2 + 3x.$ $\qquad t = 6y$

$\cancel{3x^2 + 3y = 0}$

$\cancel{3(x^2 + y) = 0}$

$\cancel{x^2 = -y}$

$\cancel{3x^3 + 3y = 0}$

$\cancel{3y^3 + 3x = 0}$

$\cancel{3(x^3 - y^3) = 0}$

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$x^2+y=0$
$y^2+x=0 \Rightarrow x=-y^2$

$y^4+y=0.$
$y(y^3+1)=0$

$y=0$, $y^3+1=0$
$\boxed{-1+1=0}$ hence do synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & \cancel{0} & -1 & 1 & -1 \\ \hline & \cancel{1} & -1 & 1 & \boxed{0} \end{array}$$

$(y+1)(y^2-y+1)=0.$

$y=0$, $y=-1$
$x=0$, $x=-1$ $(0,0)$ $(-1,-1)$
$rt-s^2=6(0)(6(0)-9<0$
$(0,0)$ ← saddle.
$(-1,-1)$ $6(-1)(6(-1)-9>0$
$\cancel{rt}$ $r=6(-1)<0$ — max.
$f(-1,-1)$ $f(x)$ is max.