Electric Circuits I

Mesh Analysis

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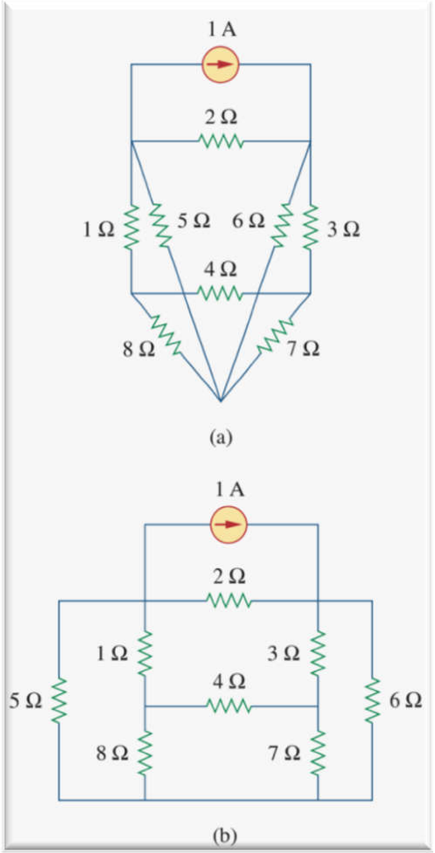
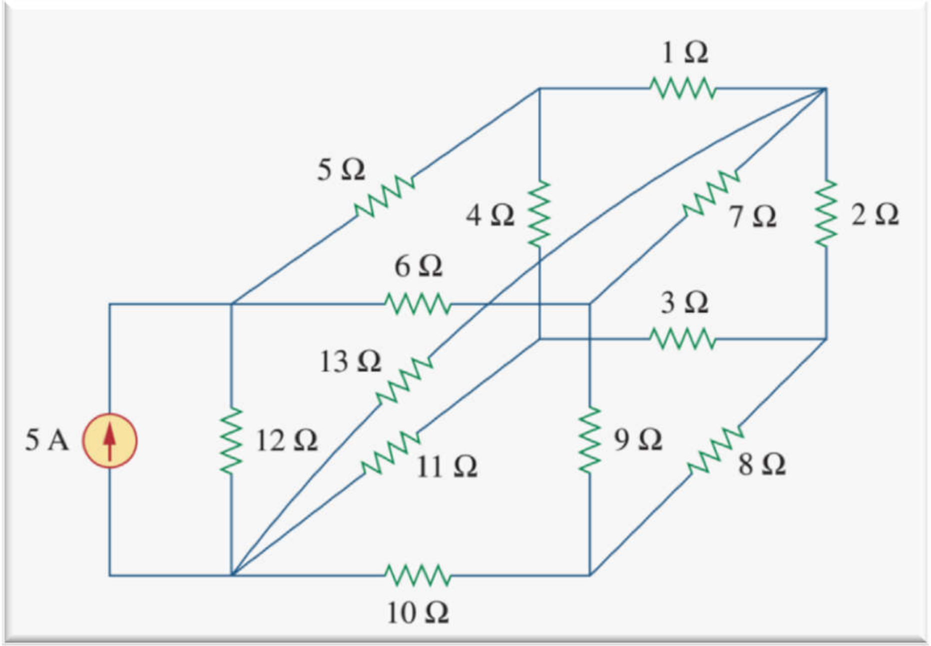
Mesh Analysis

- **Mesh analysis is also known as loop analysis or the mesh-current method.**
- **Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.**
- **The current through a mesh is known as mesh current.**
- **Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.**
- **A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches.**

Mesh Analysis



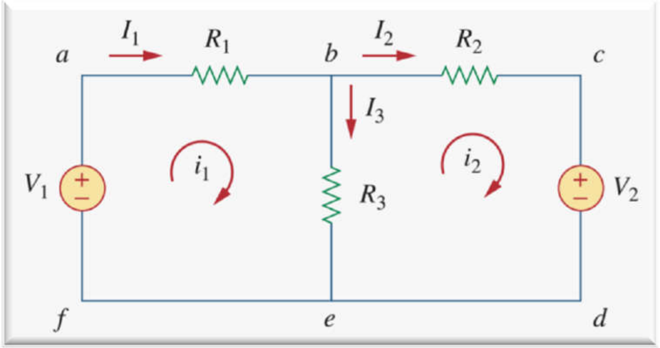


(a) A planar circuit with crossing branches,
(b) The same circuit redrawn with no crossing branches.

A nonplanar circuit.

Mesh Analysis

- **A mesh is a loop which does not contain any other loops within it.**
- **paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh.**



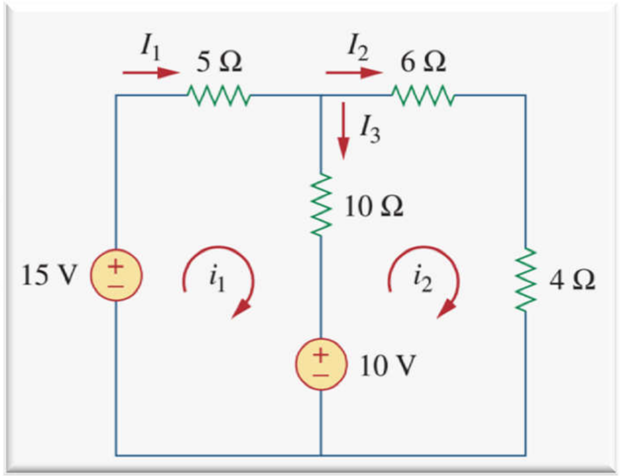
Steps to Determine Mesh Currents:

1. **Make a clear diagram.**
2. **Assign mesh currents $i_1, i_2, i_3, \ldots, i_n$ to the n meshes.**
3. **Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.**
4. **Solve the resulting *n* simultaneous equations to get the mesh currents.**

- ***The direction of the mesh current is arbitrary (clockwise or counterclockwise).***

Mesh Analysis

Example 1: For the shown circuit, find the branch currents and using mesh analysis.



We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2)+10=0$$

$$3i_1 - 2i_2=1 \qquad (1)$$

For mesh 2

$$6i_2 + 4i_2 + 10(i_2 - i_1)-10=0$$

$$i_1 - 2i_2 =-1 \qquad (2)$$

Solve equations 1 and 2 to get i_1 and i_2.

$$3i_1 - 2i_2=1 \qquad (1)$$

$$i_1 - 2i_2 =-1 \qquad (2)$$

$$2i_1=2$$

$$\boxed{i_1=1\text{A}}$$

Substitute i_1 in eq.1 or eq.2 to get i_2

$$3 \times 1 - 2i_2=1$$

$$-2i_2=1 -3 =-2$$

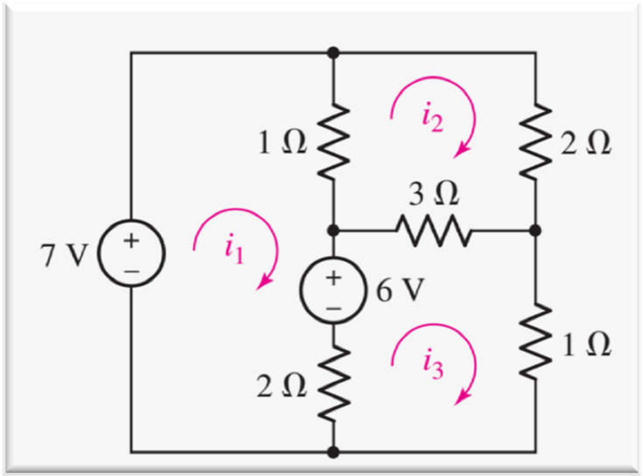
$$\boxed{i_2=1\text{A}}$$

$$\boxed{I_1 = i_1=1\text{A}} \quad \boxed{I_2 = i_2=1\text{A}}$$

$$\boxed{I_1 = i_1 - i_2=1-1=0\text{A}}$$

Mesh Analysis

Example 2: For the shown circuit, find the mesh currents using mesh analysis.



$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

Simplifying,

$$3i_1 - i_2 - 2i_3 = 1 \qquad (1)$$
$$-i_1 + 6i_2 - 3i_3 = 0 \qquad (2)$$
$$-2i_1 - 3i_2 + 6i_3 = 6 \qquad (3)$$

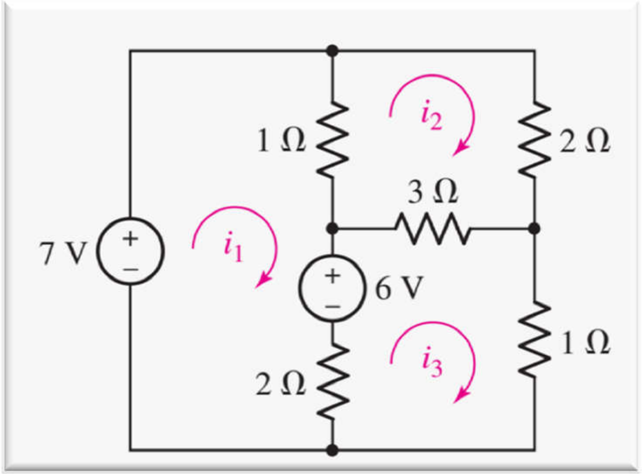
Multiply eq. 2 by 3 and then add the resulting eq. from eq.1.

$$3\boldsymbol{i_1} - \boldsymbol{i_2} - \boldsymbol{2i_3} = 1 \qquad \textbf{(1)}$$
$$-3\boldsymbol{i_1} + \boldsymbol{18i_2} - \boldsymbol{9i_3} = 0$$

$$\boldsymbol{17i_2} - \boldsymbol{11i_3} = 1 \qquad \textbf{(4)}$$

Mesh Analysis

Example 2: For the shown circuit, find the mesh currents using mesh analysis.



Multiply eq. 2 by -2 and then add the resulting eq. from eq.3.

$$-2i_1 - 3i_2 + 6i_3 = 6 \qquad (3)$$

$$2i_1 - 12i_2 + 6i_3 = 0$$

$$-15i_2 + 12i_3 = 6 \qquad (5)$$

Multiply eq. 4 by 12/11 and then add the resulting eq. from eq.5.

$$-15i_2 + 12i_3 = 6 \qquad (5)$$

$$18.54i_2 - 12i_3 = 12/11$$

$$3.54i_2 = 7.09$$

$$\boxed{i_2 = 2\text{A}}$$

Substitute i_2 in eq.4 or eq.5 to get i_3

$$-15 \times 2 + 12i_3 = 6$$

$$\boxed{i_3 = \frac{6+30}{12} = 3A}$$

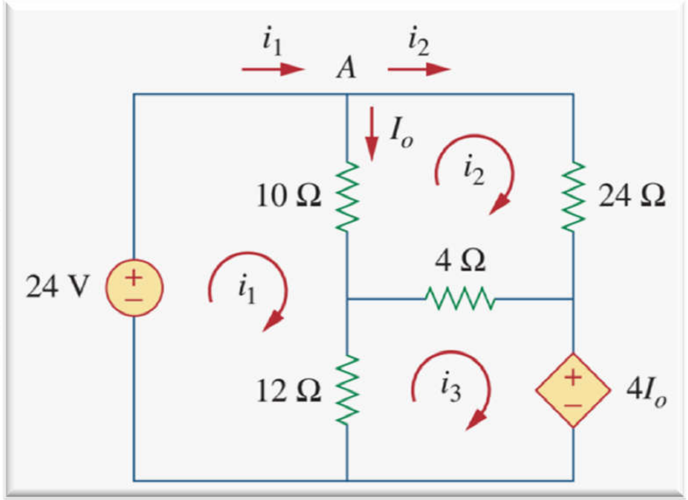
Substitute i_2 and i_3 in eq.1 to get i_1

$$3i_1 - 2 - 2 \times 3 = 1$$

$$\boxed{i_1 = \frac{2+6+1}{3} = 3A}$$

Mesh Analysis

Example 3: Use mesh analysis to find the current in the circuit of the shown figure.



We first obtain the mesh currents using KVL. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

For mesh 2

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$$

For mesh 3

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

but $\quad I_o = (i_1 - i_2)$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

Solve equations 1, 2 and 3 to get i_1 , i_2 and i_2.

Multiply eq. 2 by 3 and then subtract the resulting eq. from eq.1.

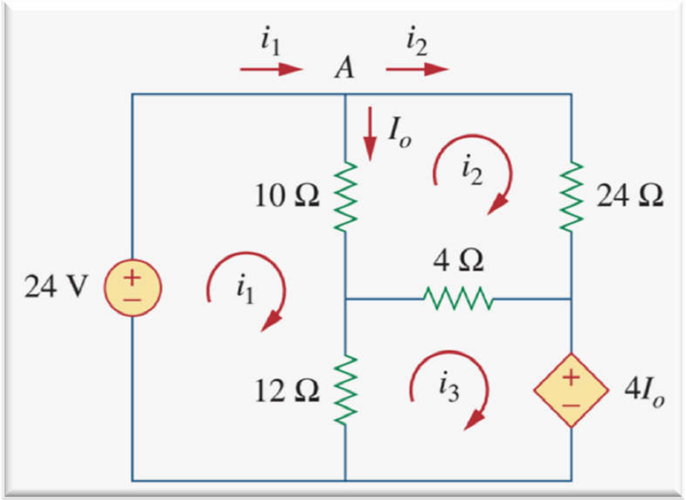
$$11i_1 - 5i_2 - 6i_3 = 12$$

$$-15i_1 + 57i_2 - 6i_3 = 0$$

$$26i_1 - 62i_2 = 12 \qquad (4)$$

Mesh Analysis

Example 3: Use mesh analysis to find the current in the circuit of the shown figure.



Multiply eq. 3 by 3 and then add the resulting eq. from eq.1.

$$11i_1 - 5i_2 - 6i_3 = 12$$

$$-3i_1 - 3i_2 + 6i_3 = 0$$

$$8i_1 - 8i_2 = 12 \qquad (5)$$

Multiply eq. 5 by 62/8 and then subtract the resulting eq. from eq.4.

$$26i_1 - 62i_2 = 12 \qquad (4)$$

$$62i_1 - 62i_2 = 93$$

$$-36i_1 = -81$$

$$\boxed{i_1 = 2.25\text{A}}$$

Substitute i_1 in eq.4 or eq.5 to get i_2

$$8\times2.25 - 8i_2 = 12$$

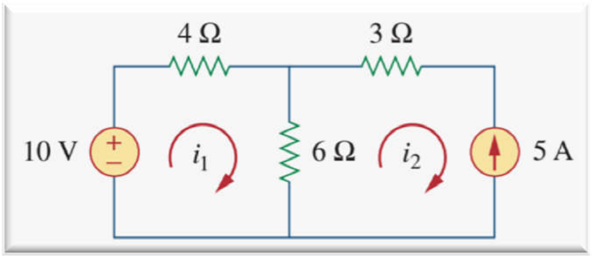
$$\boxed{i_2 = \frac{8\times2.25-12}{8} = 0.75\text{A}}$$

$$\boxed{I_o = i_1 - i_2 = 2.25 - 0.75 = 1.5A}$$

Mesh Analysis with Current Sources

CASE 1: When a current source exists only in one mesh.

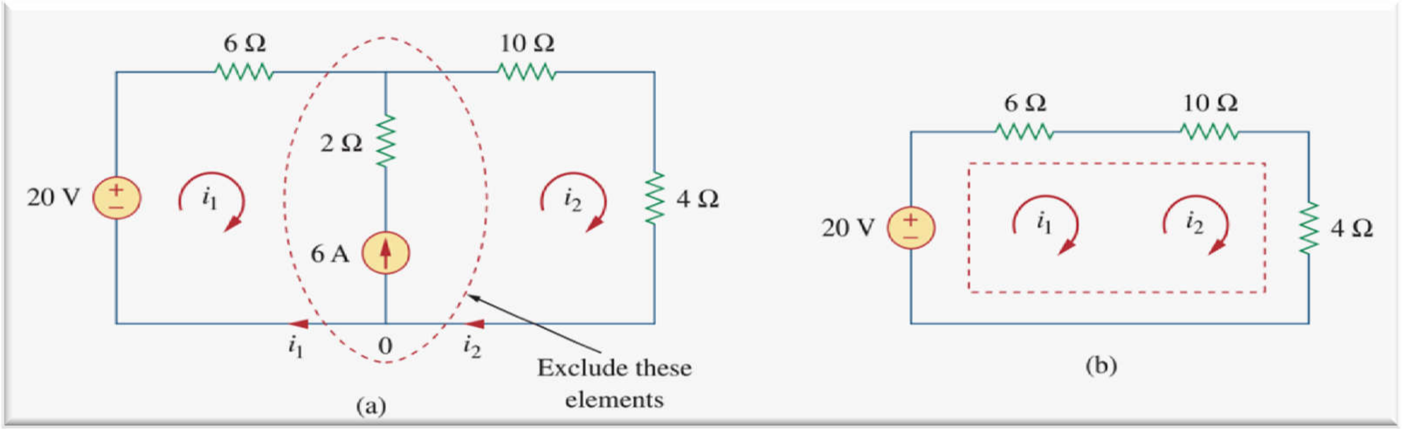
for example. i_2=-5A and write a mesh equation for the other mesh in the usual way; that is,



$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2\ \text{A}$$

CASE 2: When a current source exists between two meshes.

We create a *supermesh* by excluding the current source and any elements connected in series with it.



(a)

(b)

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$i_2 = i_1 + 6$$

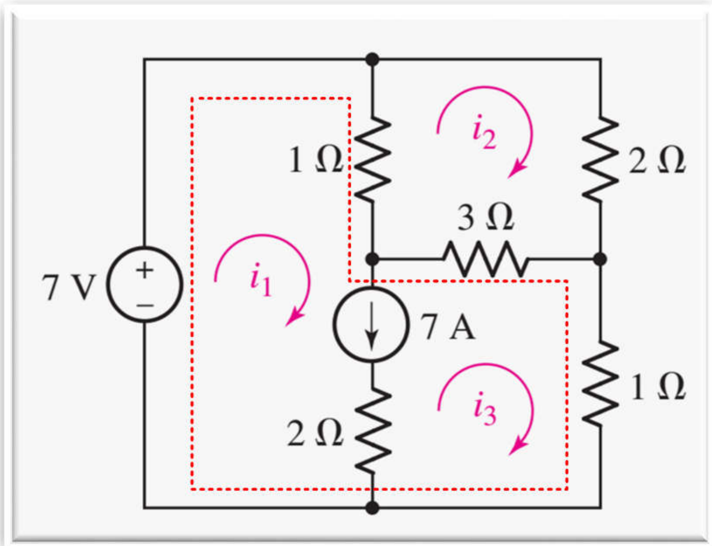
A *supermesh* results when two meshes have a (dependent or independent) current source in common.

Mesh Analysis with Current Sources

Note the following properties of a supermesh:

1. The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

Example: Determine the three mesh currents in the shown figure.



7A independent current source is in the common boundary of two meshes, which leads us to create a supermesh whose interior is that of meshes 1 and 3. Applying KVL about this loop.

$$-7+1(i_1-i_2)+3(i_3-i_2)+1i_3=0$$

$$i_1-4i_2+4i_3=7 \qquad (1)$$

Mesh Analysis with Current Sources

Example 4: Determine the three mesh currents in the shown figure.

For mesh 2

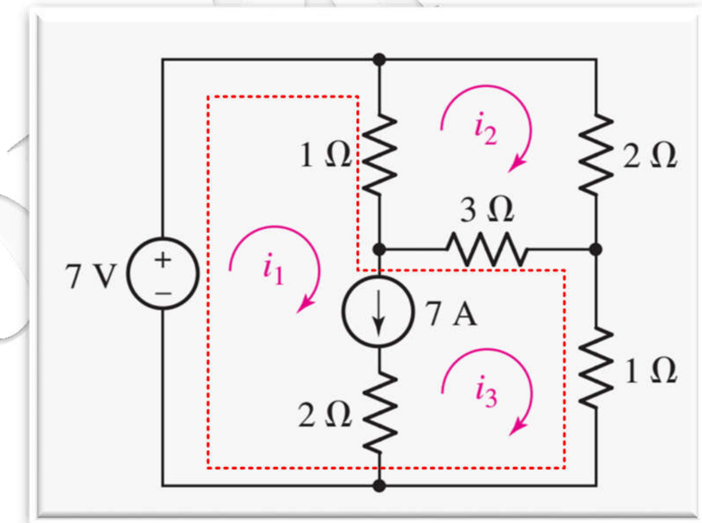
$1(i_2 - i_1)+2i_2+3(i_2 - i_3)=0$

$-i_1 + 6i_2 - 3i_3=0$ **(2)**

the independent source current is related to the mesh currents

$i_1 - i_3=7$ **(3)**

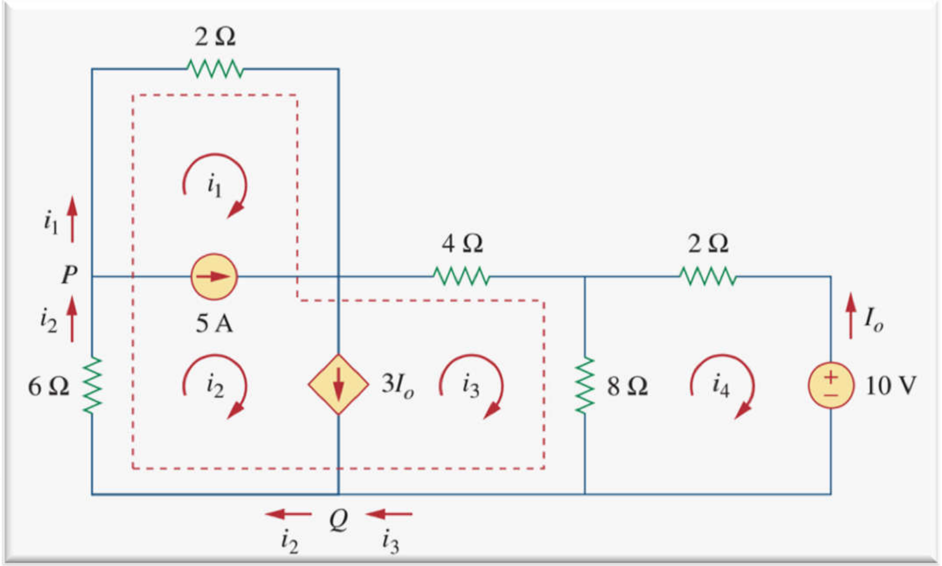
Solve these three equation to get mesh currents



$\boxed{i_1=9A}$ $\boxed{i_2=2.5A}$ $\boxed{i_3=2A}$

Mesh Analysis with Current Sources

Example 4: For the shown circuit, find i_1 to i_4 using mesh analysis.



Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh

$$2i_1+4i_3+8(i_3-i_4)+6i_2=0$$

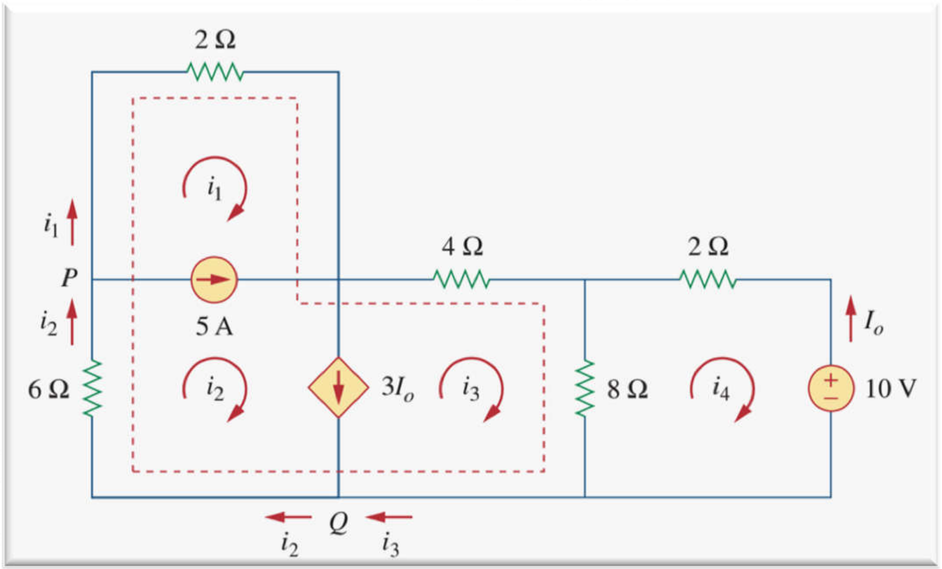
$$2i_1+6i_2+12i_3-8i_4=0 \qquad (1)$$

For the independent current source, we apply KCL to node P:

$$i_2=i_1+5 \qquad (2)$$

Mesh Analysis with Current Sources

Example 4: For the shown circuit, find i_1 to i_4 using mesh analysis.



For the dependent current source, we apply KCL to node Q:

$\boldsymbol{i_2 = i_3 + 3I_o}$

$\boldsymbol{I_o = -\,i_4}$

$\boldsymbol{i_2 = i_3 - 3i_4}$ **(3)**

Applying KVL in mesh 4,

$\boldsymbol{2i_4 + 8(i_4 - i_3) + 10 = 0}$

$\boldsymbol{5i_4 - 4i_3 = -5}$ **(4)**

Solve these four equation to get mesh currents

$\boxed{i_1 = -7.5\text{A}}$ $\boxed{i_2 = -2.5\text{A}}$

$\boxed{i_3 = 3.93\text{A}}$ $\boxed{i_4 = 2.143\text{A}}$

Nodal Versus Mesh Analysis

- Someone may ask: Given a network to be analyzed, how do we know which method is better or more efficient?
- The key is to select the method that results in the smaller number of equations.
- The choice of the better method is dictated by two factors.
  - The first factor is the nature of the particular network. Networks that contain many series-connected elements, voltage sources, or supermeshes are more suitable for mesh analysis, whereas networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis. a circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis.

Nodal Versus Mesh Analysis

- **The second factor is the information required. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.**

- **Since each method has its limitations, only one method may be suitable for a particular problem. For example, mesh analysis is the only method to use in analyzing transistor circuits. But mesh analysis cannot easily be used to solve an op amp circuit, because there is no direct way to obtain the voltage across the op amp itself. For nonplanar networks, nodal analysis is the only option, because mesh analysis only applies to planar networks. Also, nodal analysis is more amenable to solution by computer, as it is easy to program. This allows one to analyze complicated circuits that defy hand calculation.**

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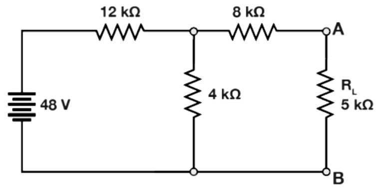
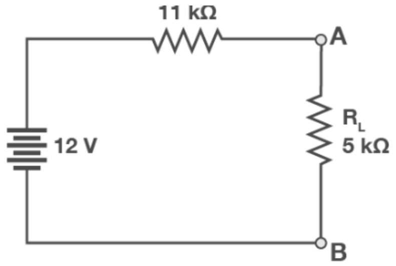
Example :5 Find the value of R_L, using Thevenin's theorem in the circuit Shown.



R_{TH} = 8 kΩ + [(4 kΩ x 12 kΩ) / (4 kΩ + 12 kΩ)]

R_{TH} = 8 kΩ + 3 kΩ

R_{TH} = 11 kΩ



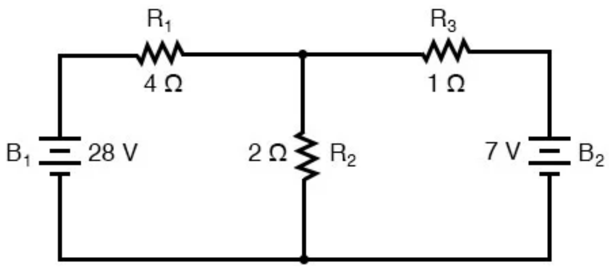
$I_L = V_{TH} / (R_{TH} + R_L)$

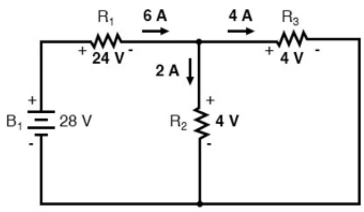
I_L = 12 V / (11 kΩ + 5 kΩ) = 12 V/16 kΩ = 0.75 mA

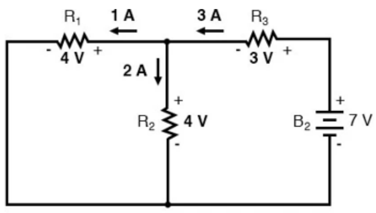
The load voltage is determined as follows:

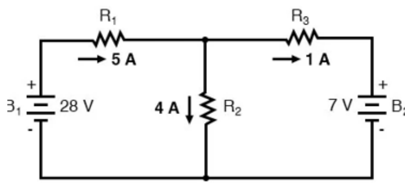
V_L = 0.75 mA x 5 kΩ = 3.75 V

Example 6: For a given circuit, find the current across 2Ω resistor by using super position theorem.

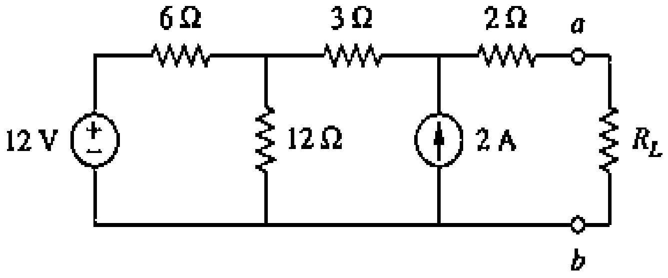








Example 7: Find the value of R_L by the principle of maximum power transfer theorem for the circuit shown.



$$R_T = \left(\frac{6 \times 12}{6+12}\right) + 3$$
$$R_T = 4 + 3 = 7\Omega$$

Apply KCL to the circuit

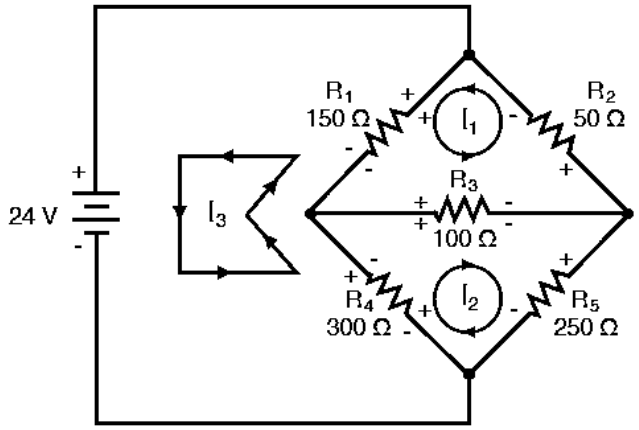
$$\frac{V_1 - 12}{6} + \frac{V_1}{12} = 2$$
$$\Rightarrow 0.166V_1 - 2 + 0.083V_1 = 2$$
$$\Rightarrow 0.2493V_1 = 4$$
$$V_1 = \frac{4}{0.2493} = 16V$$

$$V_2 = V_1 + IR$$
$$V_2 = 16 + (2)3$$
$$V_2 = 22V$$

$$V_{OC} = 22V$$

$$P_{Max} = \frac{\frac{1}{4}(22)^2}{7}$$
$$P_{Max} = 17.28 watt$$

Example:8 For a given circuit, find the current and voltage drop across each resistor.



$$300I_1 + 100I_2 + 150I_3 = 0$$
$$100I_1 + 650I_2 - 300I_3 = 0$$
$$-150I_1 + 300I_2 - 450I_3 = -24$$

Solutions:

$$I_1 = -93.793 \text{ mA}$$
$$I_2 = 77.241 \text{ mA}$$
$$I_3 = 136.092 \text{ mA}$$

$$I_{R1} = I_3 - I_1 = 136.092 \text{ mA} - 93.793 \text{ mA} = 42.299 \text{ mA}$$

$$I_{R2} = I_1 = 93.793 \text{ mA}$$

$$I_{R3} = I_1 - I_2 = 93.793 \text{ mA} - 77.241 \text{ mA} = 16.552 \text{ mA}$$

$$I_{R4} = I_3 - I_2 = 136.092 \text{ mA} - 77.241 \text{ mA} = 58.851 \text{ mA}$$

$$I_{R5} = I_2 = 77.241 \text{ mA}$$

$$E_{R1} = I_{R1}R_1 = (42.299 \text{ mA})(150\Omega) = 6.3448 \text{ V}$$

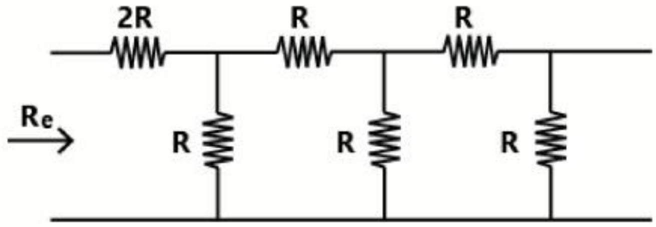
$$E_{R2} = I_{R2}R_2 = (93.793 \text{ mA})(50\Omega) = 4.6897 \text{ V}$$

$$E_{R3} = I_{R3}R_3 = (16.552 \text{ mA})(100\Omega) = 1.6552 \text{ V}$$

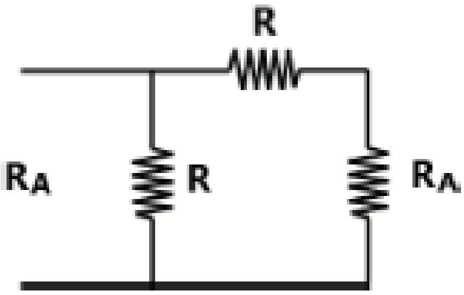
$$E_{R4} = I_{R4}R_4 = (58.851 \text{ mA})(300\Omega) = 17.6552 \text{ V}$$

$$E_{R5} = I_{R5}R_5 = (77.241 \text{ mA})(250\Omega) = 19.3103 \text{ V}$$

Example:8 The ladder network shown in the figure, find the equivalent resistance of R_e.



To find R_e we need to find equivalent resistance of remaining network



$$R_A = \frac{R \times (R + R_A)}{2R + R_A}$$

$$2RR_A + R_A^{\ 2} = R^2 + RR_A$$

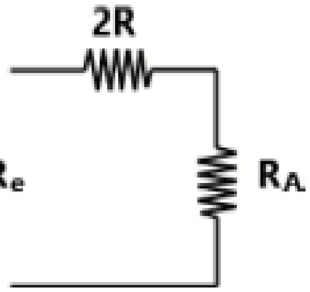
$$R_A^2 + RR_A - R^2 = 0$$

$$R_A = -\frac{-R \pm \sqrt{R^2 + 4R^2}}{2} = \frac{-R + \sqrt{5}R}{2}$$

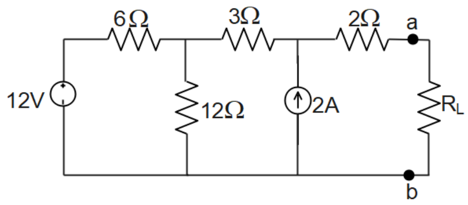
$$R_A = 0.62\ R$$

$$R_e = 2 + 0.62\ R$$

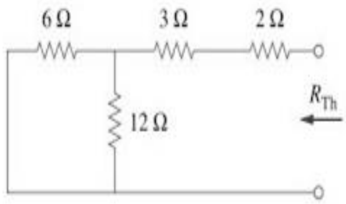
$$\frac{R_e}{R} = 2.62$$

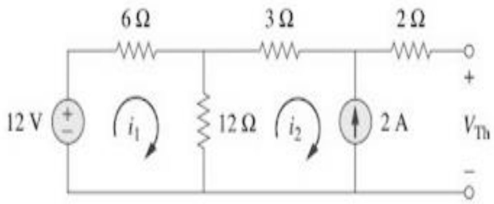


Example:9 Find the value of R_L by the principle of maximum power transfer theorem for the circuit shown.



$$R_{Th} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$





$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get i_1 = -2/3. Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\ \text{V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \text{W}$$

Example 10: For a given circuit, find the current across 20Ω resistor by using super position theorem.