Reduce the following quadratic forms into canonical form by an orthogonal transformation.

1. $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$. *[Ans. $y_1^2 + 4y_2^2 + 4y_3^2$]*

2. $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$. *[Ans. $3y_2^2 + 14y_3^2$]*

3. $6x^2 + 3y^2 + 3z^2 - 4yz + 4zx - 2xy$ *[Ans. $4x^2 + y^2 + z^2$]*

Discuss the nature of the following quadratic forms.

(a) $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_2x_1 + 2x_1x_3$ *[Ans : Indefinite]*

(b) $6x_1^2 + 17x_2^2 + 3x_3^2 - 20x_1x_2 - 14x_2x_3 + 8x_1x_3$

[Ans : Positive semi-definite]

(c) $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$

[Ans : Positive definite]

(d) $x_1^2 + 4x_2^2 + 9x_3^2 - 12x_2x_3 + 6x_1x_3 - 4x_1x_2$

[Ans : Positive semi-definite]

(e) $2x_1x_2 + 2x_2x_3 + 2x_3x_1$ *[Ans : Indefinite]*

❑ NATURE OF QUADRATIC FORM

Let $X'AX$ be the given real quadratic form, where 'A' is the matrix of the quadratic form.

Let the eigenvalues of A be $\lambda_1, \lambda_2, \lambda_3$. Now the quadratic form $X'AX$ is said to be

(a) ***Positive definite*** if all the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ are positive.

(b) ***Negative definite*** if all the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ are negative.

(c) ***Positive semidefinite*** if atleast one eigenvalue is zero and the remaining are positive.

(d) ***Negative semidefinite*** if atleast one eigenvalue is zero and the remaining are negative.

(e) ***Indefinite*** if some eigenvalues are positive and some eigenvalues are negative.