Reg. No.

SRM Institute of Science & Technology
Faculty of Engineering & Technology
Cycle Test-I

SET-A

Sub. Code: 18MAB101T
Sub. Title: Calculus & Linear Algebra
Date: 08-08-2018

Max. Marks: 25
Duration: 1 Period
Slot: C2

Answer All the Questions

Part-A (3×4 marks=12 marks)

1. Find the Eigen values of the matrix $A=\begin{pmatrix}1 & 2\\5 & 4\end{pmatrix}$ and that of $A-3I$.
2. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$ and find A^4.
3. Determine the nature of the Quadratic form $x_1^2+3x_2^2+6x_3^2+2x_1x_2+2x_2x_3+4x_3x_1$ without reducing it to the canonical form.

Part-B (1×13 marks=13 marks)

4. Reduce the Quadratic form $x^2+y^2+z^2-2xy-2yz-2zx$ to canonical form by orthogonal reduction. Hence find the rank, index and signature of the Quadratic form.

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Part-A   Answers $\lambda^3-3\lambda^2+0\lambda+4=0$

1) Characteristic Eq. $|A-\lambda I|=0$ → (1m)

Eigen values are $\lambda_1=-1,\ \lambda_2=6$ → (2m)

∴ Eigen values of $A-3I$ is

$\lambda_1-k=-4,\ \lambda_2-k=3$ → (1m)

Characteristic Eq. is $\lambda^2-5=0$ → (1m)

We need to S.T. $A^2-5I=0$ → (2m)

$\therefore A^4=\begin{pmatrix}25 & 0\\0 & 25\end{pmatrix}$ → (1m)

Symmetric matrix $A=\begin{pmatrix}1 & 1 & 2\\1 & 3 & 1\\2 & 1 & 6\end{pmatrix}$ → (1m)

$D_1=1,\ D_2=2,\ D_3=3$ → (2m)

Nature of Q.F. is +ve definite → (1m)

Part-B

Symmetric matrix of Given Q.F. is

$A=\begin{pmatrix}1 & -1 & -1\\-1 & 1 & -1\\-1 & -1 & 1\end{pmatrix}$ → (1m)

Eigen values of A are

$\lambda_1=-1,\ \lambda_2=\lambda_3=2$ → (2m)

Eigen vectors of A are

$x_1=\begin{pmatrix}1\\1\\1\end{pmatrix},\ x_2=\begin{pmatrix}0\\1\\-1\end{pmatrix},\ x_3=\begin{pmatrix}-2\\1\\1\end{pmatrix}$ → (3m)

Normalized modal matrix is

$N=\begin{pmatrix}1/\sqrt3 & 0 & -2/\sqrt6\\1/\sqrt3 & 1/\sqrt2 & 1/\sqrt6\\1/\sqrt3 & -1/\sqrt2 & 1/\sqrt6\end{pmatrix}$ → (1m)

$D=N^TAN$

$=\begin{pmatrix}-1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{pmatrix}$ → (2m)

The canonical form is

$Y^TDY=-y_1^2+2y_2^2+2y_3^2$ → (2m)

Rank (A) = 3
Index (s) = 2
Signature = 1
→ (2m)

Nature ~~[illegible]~~

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Reg. No.

SRM Institute of Science & Technology
Faculty of Engineering & Technology
Cycle Test-I

SET-B

Sub. Code: 18MAB101T
Sub. Title: Calculus & Linear Algebra
Date: 08-08-2018

Max. Marks: 25
Duration: 1 Period
Slot: C2

Answer All the Questions

Part-A (3×4 marks=12 marks)

1. Find the sum of the squares of the Eigen values of $A=\begin{pmatrix}1&7&5\\0&2&9\\0&0&5\end{pmatrix}$.

2. Determine the nature of the Quadratic form $6x^2+3y^2+14z^2+4xy+4yz+18zx$ without reducing it to the canonical form.

3. Find the Eigen values and Eigen vectors of the matrix $A=\begin{pmatrix}-2&2\\2&1\end{pmatrix}$.

Part-B (1×13 marks=13 marks)

4. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1&2&3\\2&-1&4\\3&1&-1\end{pmatrix}$ and hence find A^{-1} and A^4.

Part- A

Answers

1) Eigen values of A are

$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5 \rightarrow (2m)$

The sum of the squares of the Eigenvalues $= 1+4+25 = 30 \rightarrow (2m)$

2) Symmetric matrix $A=\begin{pmatrix}6&2&9\\2&3&2\\9&2&14\end{pmatrix} \rightarrow (1m)$

$D_1 = 6, D_2 = 14, D_3 = 1 \rightarrow (2m)$

$\therefore$ Nature of Q.F. is +ve definite $\rightarrow (1m)$

3) Eigen values of A are

$\lambda_1 = -3, \lambda_2 = 2 \rightarrow (2m)$

Eigen vectors of A are

$x_1 = \begin{pmatrix}-2\\1\end{pmatrix}, x_2 = \begin{pmatrix}1\\2\end{pmatrix} \rightarrow (2m)$

Part- B

4) characteristic Eq. of A is

$\lambda^3+\lambda^2-18\lambda-40=0 \rightarrow (2m)$

$\therefore$ we need to S.T.

$A^3+A^2-18A-40I=0 \rightarrow$ (4m)

where $A^2=\begin{pmatrix}14&3&5\\12&9&-2\\2&4&14\end{pmatrix}$, $A^3=$ [illegible] 54 52

$A^{-1} = \frac{1}{40}(A^2+A-18I)$

$= \frac{1}{40}\begin{pmatrix}-3&\cancel{-4}\ 5&11\\14&-10&2\\5&5&-5\end{pmatrix}$

$A^4 = -A^3-18A^2-40A$

$= \begin{pmatrix}248&101&218\\272&109&50\\204&98&204\end{pmatrix}$

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prepared by

[signature] 9/11/18

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Reg. No.

SRM Institute of Science & Technology
Faculty of Engineering & Technology
Cycle Test-I

SET-C

Sub. Code: 18MAB101T
Sub. Title: Calculus & Linear Algebra
Date: 08-08-2018

Max. Marks: 25
Duration: 1 Period
Slot: C2

Answer All the Questions
Part-A (3×4 marks=12 marks)

1. If 3 and 2 are the Eigen values of the matrix $A=\begin{pmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{pmatrix}$, find the Eigen values of A^{-1}, A^3.
2. Using Cayley-Hamilton theorem find the inverse of the matrix $\begin{pmatrix}2 & 1\\ 1 & -5\end{pmatrix}$.
3. Write the Quadratic form as product of matrices $x_1^2-2x_2^2+3x_3^2-4x_1x_2+5x_2x_3+6x_1x_3$.

Part-B (1×13 marks=13 marks)

4. Reduce the matrix $A=\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}$ to a diagonal form using orthogonal transformation.

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Answers

Part – A

1) $\lambda_1+\lambda_2+\lambda_3 = Tr(A) \to (1m)$

$\lambda_3 = 2 \to (1m)$

Eigen values of A^{-1} are $\frac{1}{2}, \frac{1}{2}, \frac{1}{3} \to (1m)$

Eigen values of A^3 are $8, 8, 27 \to (1m)$

2) Characteristic Eq. of A is

$\lambda^2+3\lambda-11=0$
By C-H theorem, $\Big\} \to (2m)$

$A^2+3A-11I=0$

$A^{-1}=\frac{1}{11}(A+3I)=\frac{1}{11}\begin{pmatrix}5 & 1\\ 1 & -2\end{pmatrix} \to (2m)$

3) The symmetric matrix $A=\begin{pmatrix}1 & -2 & 3\\ -2 & -2 & 5/2\\ 3 & 5/2 & 3\end{pmatrix} \to (2m)$

The required Q.F. is

$Q=(x_1\ x_2\ x_3)\begin{pmatrix}1 & -2 & 3\\ -2 & -2 & 5/2\\ 3 & 5/2 & 3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} \to (2m)$

Part – B

4) Characteristic Eq. of A is

$\lambda^3-18\lambda^2+45\lambda=0 \to (2m)$

The Eigen values and vectors are

$\lambda_1=0,\ \lambda_2=3$ and $\lambda_3=15$

$x_1=\begin{pmatrix}1\\2\\2\end{pmatrix},\ x_2=\begin{pmatrix}2\\1\\-2\end{pmatrix},\ x_3=\begin{pmatrix}2\\-2\\1\end{pmatrix} \to (6m)$

Normalized Matrix

$N=\begin{pmatrix}1/3 & 2/3 & 2/3\\ 1/3 & 1/3 & -2/3\\ 1/3 & -2/3 & 1/3\end{pmatrix} \to (2m)$

$D=N^TAN \to (1m)$

$D=\begin{pmatrix}0 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 15\end{pmatrix} \to (2m)$

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Reg.No

B1 - SET A

SRM Institute of Science and Technology
Department of Mathematics
Cycle Test I-Aug 2018
18MAB101T – Calculus and Linear Algebra

Duration : 1Period

Marks: 25

PART A

Answer All Questions (3 * 4 = 12)

1. Find Eigen values of the Matrix $A = \begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

2. Find the sum of the squares of the Eigen values of $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$. Also find Eigen values of A^{-1}.

3. Determine the nature of the Quadratic form $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_3x_1$ without reducing to Canonical form.

PART B

Answer the following Question (1*13=13)

4. Verify Cayley Hamilton theorem and find inverse of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

SET - C

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

18MAB101T - CALCULUS AND LINEAR ALGEBRA

SLOT – C1

DATE : 08 – 08 – 18

MAX. MARKS: 25

DURATION : 1 PERIOD

PART – A (3 * 4 = 12)

1. Find the sum and product of the eigen values of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

2. Using Cayley Hamilton theorem , find the inverse of A = $\begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$.

3. Discuss the nature of the Quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_1x_3 + 2x_2x_1$, without reducing to canonical form.

PART – B (1 * 13 = 13)

4. Reduce the Quadratic form $3x_1^2 - 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_1$ to the Canonical form by an orthogonal transformation and find its rank, index, signature, nature.

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Reg. No.:

SRM Institute of Science and Technology
Department of Mathematics
Cycle Test- 1
18MAB101T-Calculus and Linear Algebra

B

Time: 1 Period (50 Mins.) **Max. Marks: 25**
Combo 2, Batch 1 (C1-Slot)

[Attach the Question Paper with the answer sheet]

Part – A (3× 4=12)
Answer ALL the Questions

1. **Find the eigenvalues and eigenvectors of** $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

2. **Find the eigenvalues of** A^{-1} **and** A^3**, if** $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

3. **Determine the nature of the quadratic form** $6x^2 + 3y^2 + 14z^2 + 4yz + 18xz + 4xy$ **without reducing it to canonical form.**

Part – B (1× 13=13)
Answer any one Question

4. **Verify Cayley-Hamilton theorem for the matrix** $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. **Use it to find** A^{-1} **and** A^4.

*** * * * * * ALL THE BEST * * * * * ***

CT#1-Answer Key for Set-**B**-C1 Slot 18MAB101T-Calculus and Linear Algebra

1. Here $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.
Characteristic equation is $|A - \lambda I| = 0$
$\implies \lambda^2 - 6\lambda + 5 = 0.$ **->(1m)**.
Eigenvalues are 5, 1. **->(1m)**
Corresponding eigenvectors are $X_1 = (1 \quad 1)^T$ and $X_2 = (1 \quad -3)^T$ **->(2m)**

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2. Here $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.
Characteristic equation is $|A - \lambda I| = 0 \implies \lambda^3 - 2\lambda^2 - 4\lambda + 8 = 0.$ **->(1m)**.
Eigenvalues are $-2, 2, 2$. **->(1m)**
Eigenvalues of A^{-1} are $-1/2, 1/2, 1/2$. **->(1m)**
Eigenvalues A^3 are $-8, 8, 8$. **->(1m)**

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3. Here $A = \begin{bmatrix} 6 & 2 & 9 \\ 2 & 3 & 2 \\ 9 & 2 & 14 \end{bmatrix}$.
Now $D_1 = 6 > 0$, $D_2 = \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} = 14 > 0$ and $D_3 = \det(A) = 1$. $\therefore$ The Q.F. is positive definite. **->(4m)**

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4. Here $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ Characteristic equation is $\lambda^3 - 5\lambda^2 + 9\lambda - 1 = 0$, **->(4m)** Cayley-Hamilton theorem verification **->(5m)**
$A^{-1} = A^2 - 5A + 9I = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ **->(2m)**
$A^4 = 5A^3 - 9A^2 + A = \begin{bmatrix} -55 & 104 & 24 \\ -20 & -15 & 32 \\ 32 & -42 & -23 \end{bmatrix}$ **->(2m)**

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B1 - SET C

Reg.No

SRM Institute of Science and Technology
Department of Mathematics
Cycle Test I-Aug 2018
18MAB101T – Caiculus and Linear Algebra

Duration : 1Period Marks: 25

PART A

Answer All Questions (3 * 4 = 12)

1. Find Eigen values and Eigen Vector of the Matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.

2. Two of the Eigen values of $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -6 \\ -1 & -2 & 3 \end{pmatrix}$ are 3 and 6.

   Find the Eigen values of A^2 and A^{-1}.

3. Verify Cayley Hamilton theorem for the Matrix of $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

PART B

Answer the following Question (1*13=13)

4. Reduce the Quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_3x_1$ to a canonical form by orthogonal reduction.

B1 - SET B

Reg.No

SRM Institute of Science and Technology
Department of Mathematics
Cycle Test I-Aug 2018
18MAB101T – Calculus and Linear Algebra

Duration : 1Period Marks: 25

PART A

Answer All Questions (3 * 4 = 12)

1. Find Eigen values of the Matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.

2. Find the sum and product of the Eigen values of $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

3. Verify Cayley Hamilton theorem for the Matrix of $A = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$

PART B

Answer the following Question (1*13=13)

4. Reduce the Quadratic form $7x_1^2 + 10x_2^2 + 7x_3^2 - 4x_1x_2 + 2x_1x_3 - 4x_3x_2$ to a canonical form by orthogonal reduction.