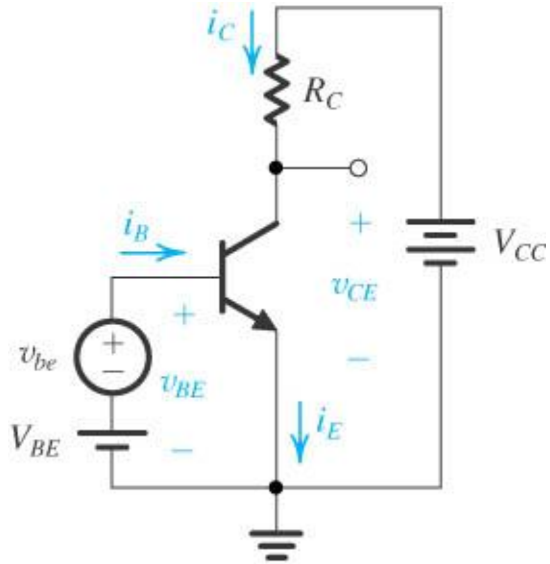
SMALL SIGNAL OPERATION AND MODEL

Chapter 5.6



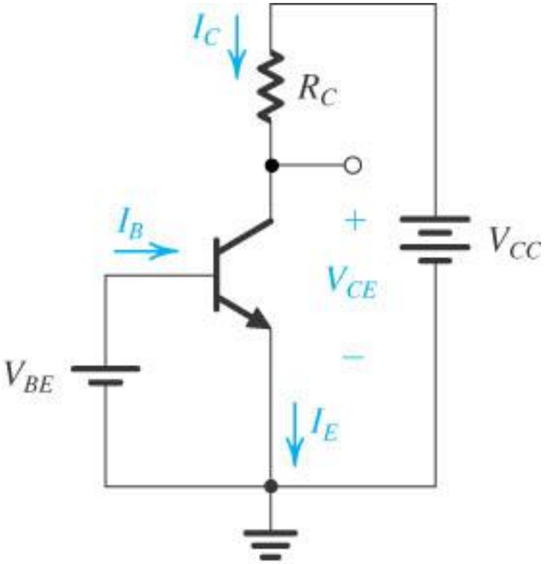
(a)

- Base-Emitter Junction is forward biased using V_{BE} (Battery)

- Collector-Base junction is reverse biased using power supply V_{CC} through R_C

- The input signal is v_{be}

Consider only the dc bias condition by letting v_{be}=0
The voltage and current relations are given by



(b)

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

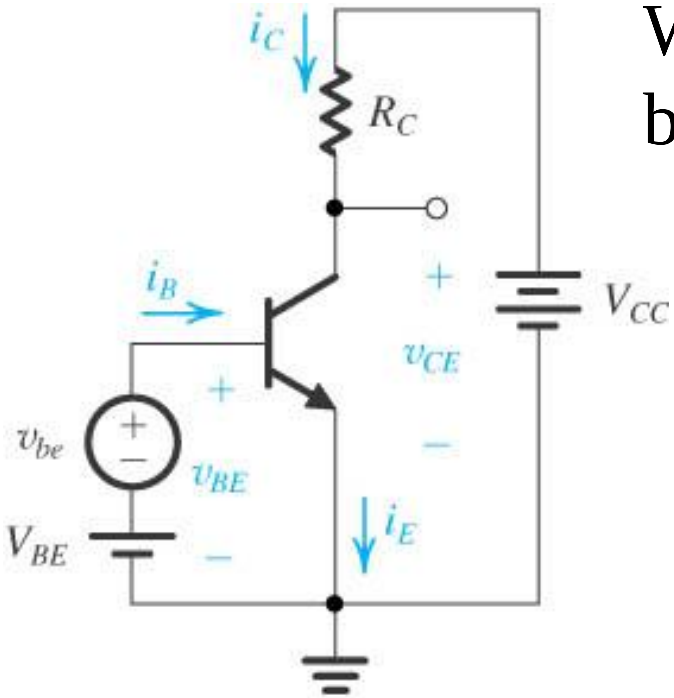
$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

$$V_C = V_{CE} = V_{CC} - I_C R_C$$

For active mode operation V_C should be greater than (V_B-0.4)

COLLECTOR CURRENT AND TRANSCONDUCTANCE



(a)

When v_{be} is applied, total instantaneous base emitter voltage v_{BE} becomes

$$v_{BE} = V_{BE} + v_{be}$$

The collector current becomes

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} = I_S e^{\frac{V_{BE}+v_{be}}{V_T}}$$

$$= I_S e^{\frac{V_{BE}}{V_T}} e^{\frac{v_{be}}{V_T}}$$

Using $I_C = I_S e^{\frac{V_{BE}}{V_T}}$ we get $i_C = I_C e^{\frac{v_{be}}{V_T}}$

If $v_{be} <<<< V_T$, we can approximate the above as

$$i_C = I_C(1+\frac{v_{be}}{V_T})$$

This approximation which is valid only for v_{be} less than 10mV is referred to as **<u>small signal approximation</u>**

Under this approximation , the total collector current is given by

$$i_C = I_C + \frac{I_C v_{be}}{V_T}$$

Signal Component $\boldsymbol{i_c}$

DC BIAS

$$i_c = \frac{I_C}{V_T} v_{be}$$

The above relation relates signal current to the corresponding base emitter signal voltage. This can be written as

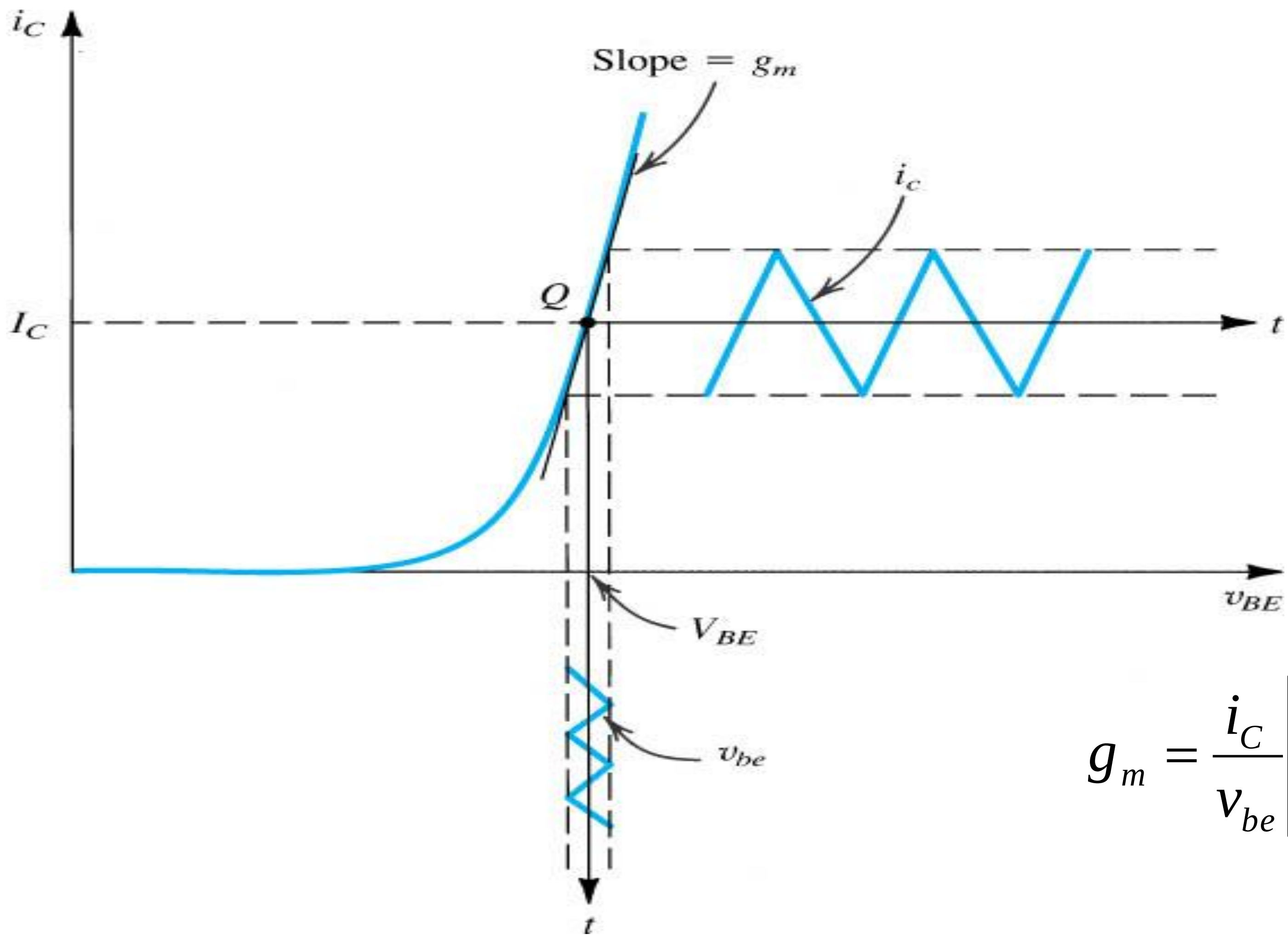
$$i_c = g_m v_{be}$$

Where g_m is called as **transconductance** , $g_m = \frac{I_C}{V_T}$

This is directly proportional to the collector bias current I_C

BJTs have high transconductance. Eg I_C=1mA, V_T=25mV

$g_m \approx 40$mA/V

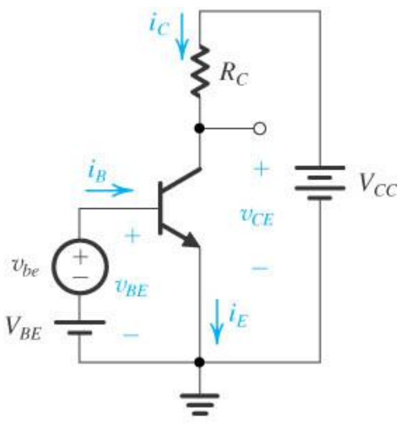


$$g_m = \left.\frac{i_C}{v_{be}}\right|_{i_C = I_C}$$

The Small signal approximation implies keeping the signal amplitude sufficiently small, so that operation is restricted to an almost linear segment of the i_C-v_{BE} exponential curve.

This analysis shows that transistor behaves as a voltage controlled current source.

The BASE CURRENT AND INPUT RESISTANCE AT BASE



(a)

In this circuit , what is the resistance seen by v_{be}

Let is evaluate base current i_B

$$i_B = \frac{i_C}{\beta} \quad and\, we\ know \;\; i_C = I_C + \frac{I_C}{V_T} v_{be}$$

$$i_B = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T} v_{be}$$

$$total\;\; base\; current\;\; i_B = I_B + i_b$$

$$where\;\; I_B = \frac{I_C}{\beta} \;\; and\;\; i_b \;\; is\, signal\;\; component$$

$$i_b = \frac{1}{\beta}\frac{I_C}{V_T} v_{be}$$

$$Substitute \quad \frac{I_C}{V_T} = g_m \quad , we\, get \quad i_b = \frac{g_m}{\beta} v_{be}$$

The small signal input resistance between base and emitter looking into base is defined by

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$$

EMITTER CURRENT AND INPUT RESISTANCE AT EMMITER

$$The\ total\ emitter\ current\ is$$

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

$$total\ emitter\ current \quad i_E = I_E + i_e$$

$$We\ know\ that\ i_c = \frac{I_C}{V_T} v_{be}$$

$$i_E = \frac{I_C}{\alpha} + \frac{1}{\alpha}\frac{I_C}{V_T} v_{be}$$

$$comparing\ the\ equations\ for\ i_E\ ,\ we\ get$$

$$I_E = \frac{I_C}{\alpha} \quad and\ \ Signal\ current\ i_e = \frac{1}{\alpha}\frac{I_C}{V_T} v_{be}$$

$We\ can\ write$

$$i_e = \frac{I_E}{V_T} v_{be} \qquad \because \; I_E = \frac{I_C}{\alpha}$$

If we denote small signal resistance between base and emitter looking onto emitter by r_e

$$r_e = \frac{v_{be}}{i_e} = \frac{V_T}{I_E}$$

We know that $g_m = \frac{I_C}{V_T}$ and $I_C = I_E \alpha$

using above $we\ can\ write$ $r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$

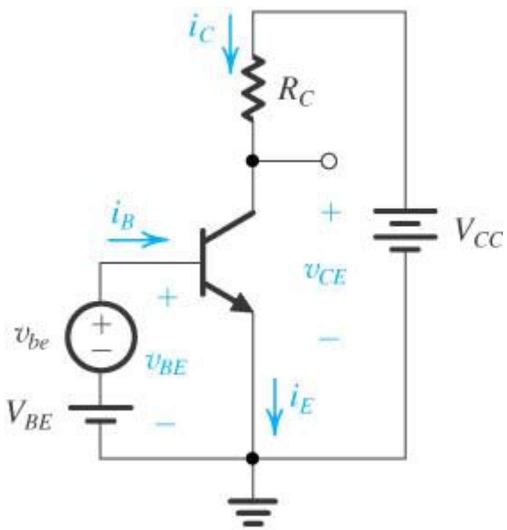
What is the relation between $\boldsymbol{r_\pi}$ and $\boldsymbol{r_e}$

$$v_{be} = i_b r_\pi = i_e r_e$$

$$r_\pi = \left(\frac{i_e}{i_b} \right) r_e \qquad i.e \quad r_\pi = (\beta + 1)\, r_e$$

VOLTAGE GAIN

Transistor is acting as a voltage controlled current source. To obtain output voltage signal , current has to be flown through the resistor. Considering the circuit below



(a)

The total collector voltage v_C

$$v_C = V_{CC} - i_C R_C$$

$$= V_{CC} - (I_C + i_c) R_C$$

$$= V_{CC} - I_C R_C - i_c R_C$$

$$= V_C - i_c R_C$$

We know that total collector current is equal to dc component plus signal component, accordingly we can write $v_C = V_C + v_c$

Comparing above with $v_C = V_C - i_c R_C$

DC BIAS VOLTAGE AT THE COLLECTOR

$v_c = -i_c R_C$ using relation $i_c = g_m v_{be}$ we get

$$v_c = (-g_m R_C) v_{be}$$

Thus voltage gain of the amplifier is $\dfrac{v_c}{v_{be}} = -g_m R_C = A_v$

Substituting for, $g_m = \frac{I_C}{V_T}$ we get $A_v = -\frac{I_C R_C}{V_T}$

E.g.

A BJT having β=100 is biased at a dc collector current of 1mA. Find the value of g_m , r_e and r_π at the bias point

Ans: g_m=40mA/V r_e=25Ω r_π=2.5KΩ

E.g.

In the circuit shown below, V_{BE} is adjusted to yield a dc collector current of 1mA. Let $V_{CC}=15V$, $R_C=10K\Omega$ and $\beta=100$. Find the voltage gain v_c/v_{be}

If $v_{be} = 0.005 \sin wt$ volts find $v_C(t)$ and $i_B(t)$

Ans : Av=-400 V/V

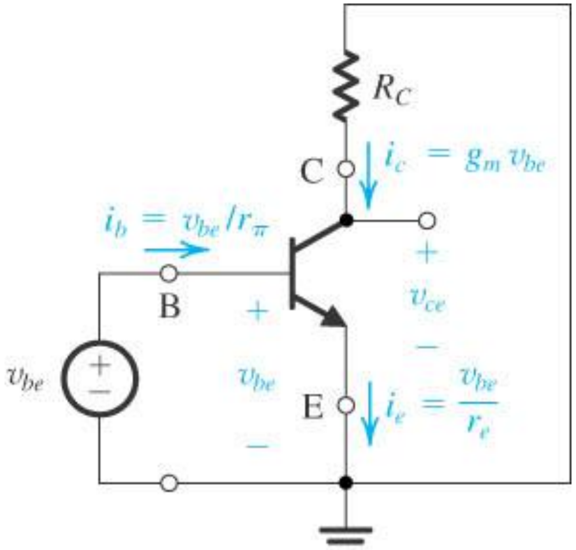
SEPERATING SIGNAL FROM THE DC QUANTITIES

The analysis we have seen consist of two components , i. e dc component and the signal component

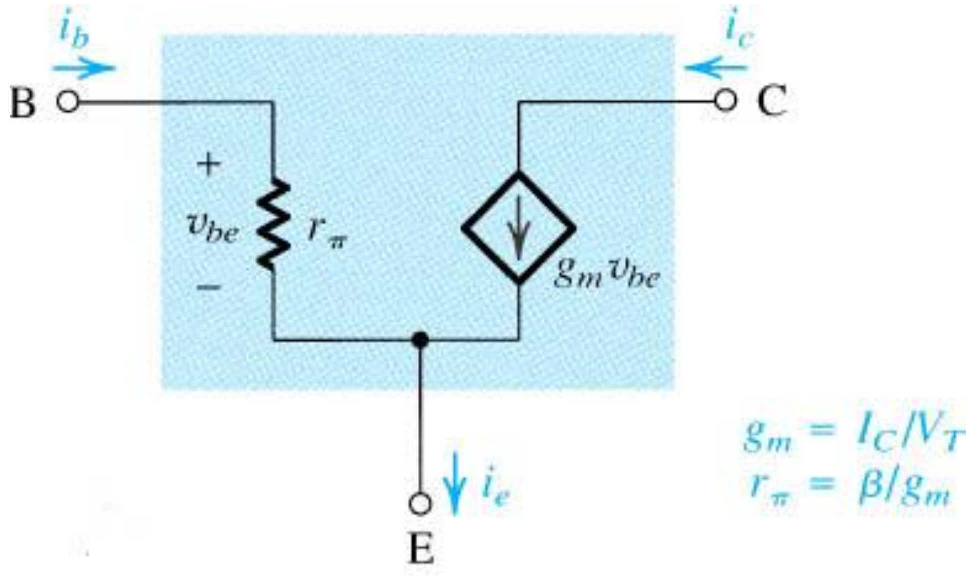
$$i_C = I_C + i_c \quad and \quad v_{BE} = V_{BE} + v_{be}$$

Signal operation can also be represented by eliminating the dc sources as shown

This is only the representation as it is not an amplifier due to no dc bias



THE HYBIRD π MODEL



(a)

This model represents the BJT as a voltage controlled current source and explicitly includes the input resistance looking into the base , r_π

In this model $i_c = g_m v_{be}$

$$i_b = \frac{v_{be}}{r_\pi}$$

The current $i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}(1 + g_m r_\pi)$

But $g_m r_\pi = \beta$, we get $i_e = \frac{v_{be}}{r_\pi}(1+\beta)$

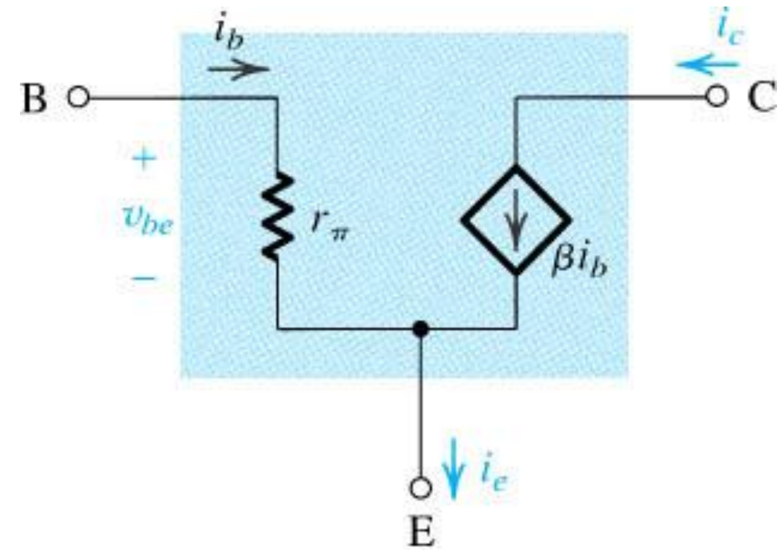
We can rearrange the above equation as

$$i_e = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right) = \frac{v_{be}}{r_e}$$

Slightly different model can be obtained by expressing the current of the controlled source ($g_m v_{be}$) interms of base current i_b

$$g_m v_{be} = g_m(i_b r_\pi) = (g_m r_\pi) i_b = \beta i_b$$

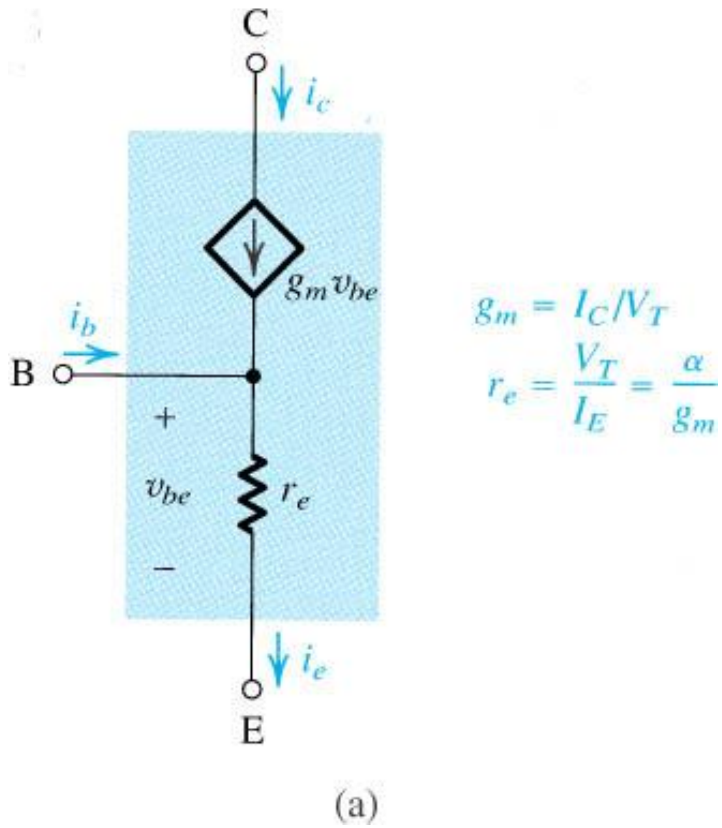
Equivalent circuit model is



(b)

Now the transistor circuit is modeled as current controlled current source.

THE T MODEL



The resistance between base and emitter is shown. This model will yield current expressions for i_c and i_e

We note that

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$$

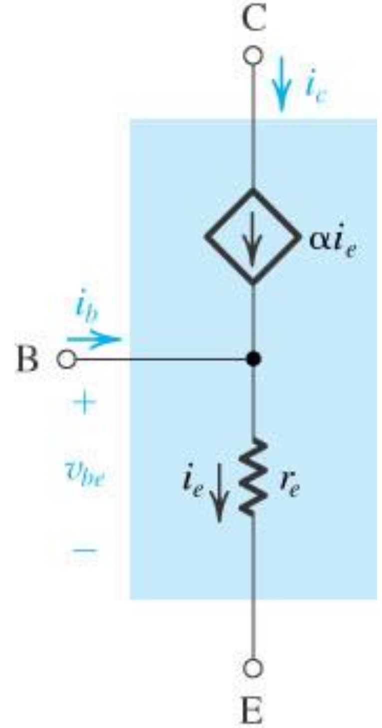
$$= \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{r_e}(1 - \frac{\beta}{\beta+1})$$

$$= \frac{v_{be}}{(\beta+1)r_e} = \frac{v_{be}}{r_\pi}$$

The current of the controlled source can be expressed in terms of the emitter current.

$$g_m v_{be} = g_m (i_e r_e) = (g_m r_e) i_e = \alpha i_e$$

This will yield an alternative T model, represented as a current controlled current source.



(b)

APPLICATION OF THE SMALL SIGNAL EQUIVALENT CIRCUIT

➢**To analyze the transistor amplifier circuits in systematic process**

The Process consist of following steps

1. Determine the DC operating point of the BJT and in particular dc collector current I_C

2. Calculate the values of the small signal model parameters such as

$$g_m = \frac{I_C}{V_T},\ r_\pi = \frac{\beta}{g_m},\ and\ r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

3. Eliminating the dc sources by replacing each dc voltage sources with short circuit and each dc current source with an open circuit.

4. Replace the BJT with one of its small signal equivalent model

5. Analyze the resulting circuit to determine the required quantities. (e.g. voltage gain input resistance)