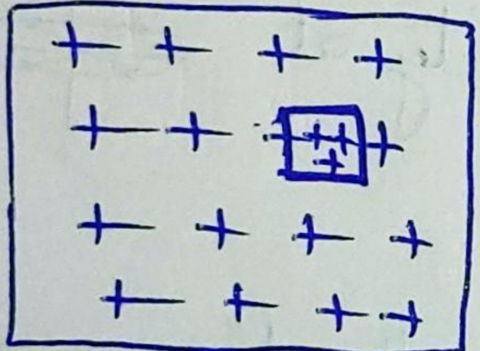
E - Field due to Infinite sheet :-



Infinitely large sheet.

Areal charge density, "σ".

charge per unit area is "σ".

small area $\Rightarrow dA$.

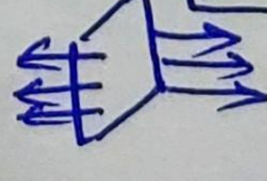
$$\boxed{\sigma = \frac{Q}{A}}$$

one area charge is σ.

For a small dA area, the charge is $dA.\sigma$.

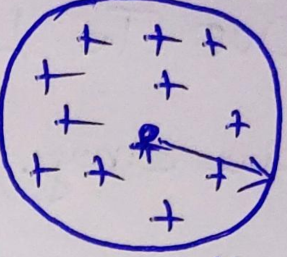
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\sigma}{r^2}\,dA$$

$$\boxed{\therefore E = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma}{r^2}.dA}$$



E- Field in a Volume :-

Volumetric charge density ρ.



$\rho = \frac{Q}{V}$ — $Q \rightarrow$ Total charge. $V \rightarrow$ Total volume.

The charges will not come to the surface, because it is a Non-Conductor.

charge per unit Volume $\Rightarrow \rho$.

$\Rightarrow$ 1 volume have charge $\rightarrow \rho$.

$\therefore$ Small Volume have charge, $dv \Rightarrow \rho . dv$.

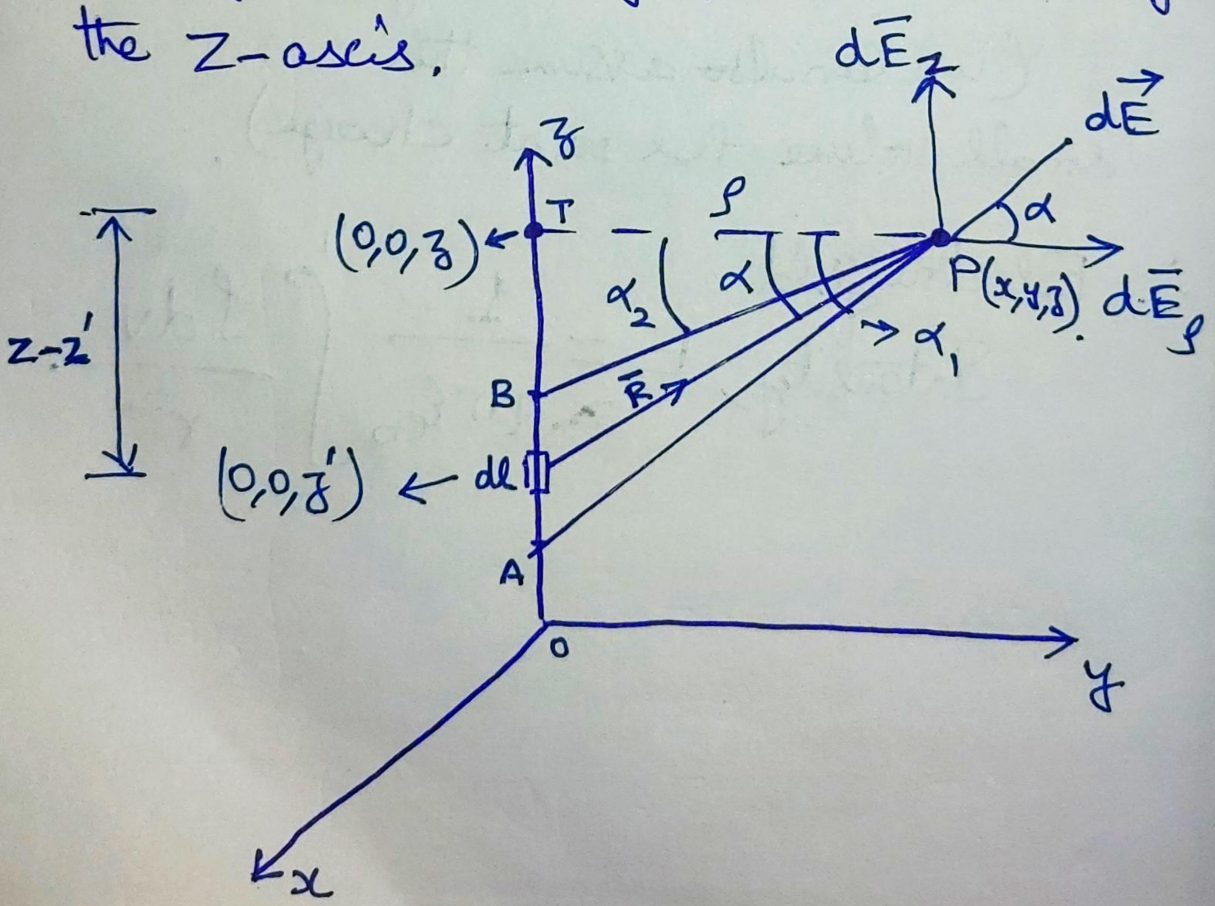
(we can also assume this small volume has point charge).

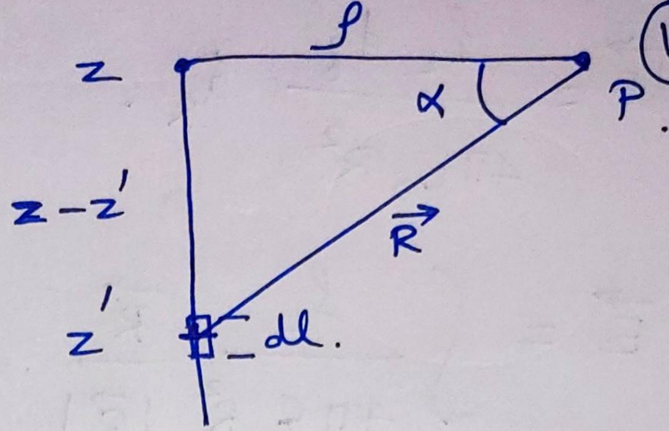
$\therefore$ Electric field Intensity, $E = \frac{1}{4\pi \varepsilon_0} \int_0^V \frac{\rho \, dv}{r^2}$.

1) ELECTRIC FIELD INTENSITY (E) DUE TO LINE CHARGE

1. Line charge, $Q = \int_L \rho_L \, dl$.
2. Surface charge, $Q = \int_S \rho_s \, ds$.
3. Volume charge, $Q = \int_V \rho_v \, dv$.

Consider a line charge, with uniform charge density ρ_L extending from A to B along the Z-axis.



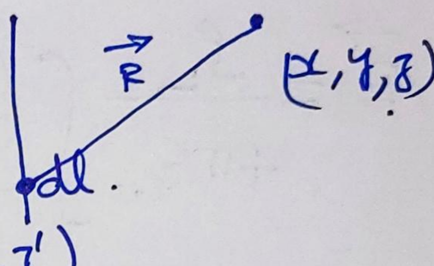


1) $\sin\alpha = \frac{\text{oppo}}{\text{hyp}} = \frac{z-z'}{R}$

$z - z' = R\sin\alpha$.

2) $\cos\alpha = \frac{\text{Adj}}{\text{Hyp}} = \frac{\rho}{R}$

$\rho = R.\cos\alpha$.



3) $\sec\alpha = \frac{R}{\rho}$

4) $\tan\alpha = \frac{OT - z'}{\rho}$

$\therefore z' = OT - \rho\tan\alpha$.

$dz' = -\rho.\sec^2\alpha.d\alpha$.

$\bar{R} = (x-0)\bar{a}_x + (y-0)\bar{a}_y + (z-z')\bar{a}_z$.

$|\bar{R}| = \sqrt{x^2+y^2+(z-z')^2}$

W.K.T. Line charge, $Q = \int_L \rho_L \, dl$.

$R = \sqrt{\rho^2 + (z-z')^2}$

E-Field at $z' \Rightarrow Q = \int_L \rho_L \, dz'$.

$\boxed{\therefore \bar{R} = \rho\bar{a}_\rho + (z-z')\bar{a}_z}$

$\bar{R} = \rho\bar{a}_\rho + (z-z')\bar{a}_z$.

w.k.t. $\vec{E} = \frac{Q}{4\pi\epsilon_0|R|^2}\hat{a}_R$ .N/C (or) V/m

$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\hat{a}_R \Rightarrow \frac{\rho_L dz'}{4\pi\epsilon_0 R^2}\hat{a}_R$

(14)

$$\vec{dE} = \frac{\rho_L \, dz'}{4\pi\epsilon_0 R^2} \hat{a}_R . \qquad \hat{a}_R = \frac{\vec{R}}{|\vec{R}|} .$$

$$\vec{E} = \int \frac{\rho_L}{4\pi\epsilon_0 R^2} \cdot \frac{\vec{R}}{|\vec{R}|} \cdot dz' .$$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \int \frac{(\rho \bar{a}_\rho + (z - \bar{z}) \bar{a}_z)}{R^2 . |R|} dz' .$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int \frac{R . \cos\alpha . \bar{a}_\rho + R \sin\alpha \, \bar{a}_z}{R^3} dz'$$

$\rho = R\cos\alpha .$

$z - z' = R \sin\alpha .$

$\cancel{R = \rho \sec\alpha} .$

$R = \rho \sec\alpha$

$dz' = -\rho \sec^2\alpha \, d\alpha .$

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0} \int \frac{R\cos\alpha \, \bar{a}_\rho + R \sin\alpha \, \bar{a}_z}{R^3} (-\rho \sec^2\alpha) \, d\alpha .$$

$R^3 = \rho^3 \sec^3\alpha .$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int \frac{-\rho \sec^2\alpha \, R\cos\alpha \, \bar{a}_\rho - \rho \sec^2\alpha \, R \sin\alpha \, \bar{a}_z}{R^3} . d\alpha .$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int \frac{(-\rho \sec^2\alpha)(\cos\alpha \, \bar{a}_\rho + \sin\alpha \, \bar{a}_z)}{\rho^2 \sec^2\alpha} d\alpha$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{-(\cos\alpha . \bar{a}_\rho + \sin\alpha . \bar{a}_z)}{\rho} \, d\alpha .$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\cos\alpha . \bar{a}_\rho + \sin\alpha . \bar{a}_z}{\rho} \, d\alpha .$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0 . \rho} \int_{\alpha_1}^{\alpha_2} \left(\cos\alpha . \bar{a}_\rho + \sin\alpha . \bar{a}_z\right) d\alpha .$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0 \rho} \left\{ \left[\sin\alpha\right]_{\alpha_1}^{\alpha_2} \bar{a}_\rho + \left[-\cos\alpha\right]_{\alpha_1}^{\alpha_2} \bar{a}_z \right\} .$$

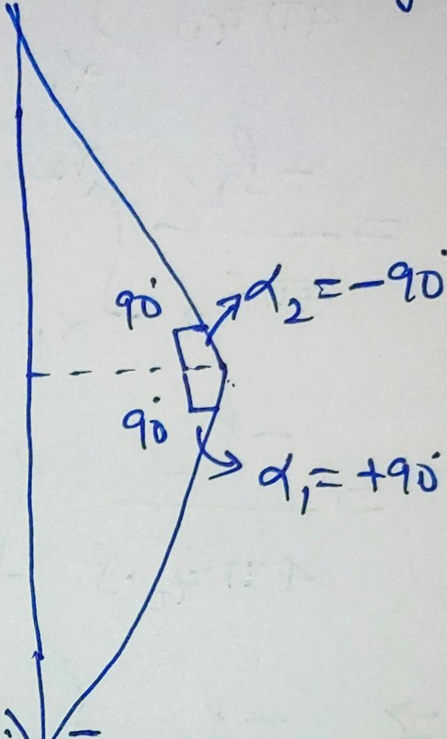
$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0 \rho} \left[(\sin\alpha_2 - \sin\alpha_1)\bar{a}_\rho - (\cos\alpha_2 - \cos\alpha_1)\bar{a}_z \right]$$

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 \rho} \left[(\sin\alpha_1 - \sin\alpha_2)\bar{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\bar{a}_z \right]}$$

$\downarrow$.

$\vec{E}$ Field Intensity for Finite Line charge.

Special Case :- For Infinite Line charge.



w.k.t.

$$E = \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[(\sin\alpha_1 - \sin\alpha_2)\bar{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\bar{a}_z\right].$$

$$\Rightarrow E = \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[(\sin(90°) - \sin(-90°))\bar{a}_\rho + (\cos(-90°) - \cos(90°))\bar{a}_z\right]$$

$$E = \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[[1-(-1)]\bar{a}_\rho + [0-0]\bar{a}_z\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \rho}\left[2\bar{a}_\rho\right]$$

$$\boxed{\therefore \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\cdot\hat{a}_\rho}$$