① A rectangular waveguide with dimensions $a = 2.5\,cm$, $b = 1\,cm$ is to operate below $15.1\,GHz$. How many TE & TM modes can the waveguide transmit if the guide is filled with a medium characterized by $\sigma = 0$, $\varepsilon = 4.\varepsilon_0$; $\mu_r = 1$? Calculate the cut off frequencies of the modes.

Solution:-

w.k.t. Cut off frequency, $f_c = \dfrac{u'}{2}\sqrt{\dfrac{m^2}{a^2} + \dfrac{n^2}{b^2}}$

w.k.t. $u' = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{\sqrt{\mu_r\varepsilon_r}} = \dfrac{c}{\sqrt{(1)(4)}} = \dfrac{c}{2}.$

$$f_c = \dfrac{c}{4}\sqrt{\dfrac{1}{a^2}\left(m^2 + \dfrac{a^2}{b^2}n^2\right)} \Rightarrow \dfrac{c}{4a}\sqrt{m^2 + \dfrac{a^2}{b^2}.n^2}.$$

$$= \dfrac{3\times10^8}{4(2.5\times10^{-2})}\sqrt{m^2 + \dfrac{(2.5\times10^{-2})^2}{(1\times10^{-2})^2}.n^2}$$

$$f_c = 3\sqrt{m^2 + 6.25\,n^2}\ GHz.$$

For different values of "m" & "n" find f_c value below 15.1 GHz.
If "m" is fixed & increasing "n" will quickly reach $f_c > 15.19\,\text{GHz}$

TE_{01} ⟹ $3\sqrt{m^2 + 6.25n^2} = 3\sqrt{0 + 6.25}$

$\{m=0, n=1\}$

$\qquad = 3(2.5) = 7.5\,GHz$.

TE_{02} ⟹ $3\sqrt{0 + (6.25)(4)} = 15\,GHz$.

TE_{03} ⟹ $f_c = 22.5\,GHz$ ⟹ $f_c > 15.1\,GHz$.

Therefore, the maximum value of $\underline{\underline{n=2}}$.

For TE_{10}^{mode} ⟹ $f_c = 3\,GHz$.

$\qquad TE_{20}^{mode}$ ⟹ $f_c = 6\,GHz$.

$\qquad TE_{30}^{mode}$ ⟹ $f_c = 9\,GHz$.

$\qquad TE_{40}^{mode}$ ⟹ $f_c = 12\,GHz$.

$\qquad TE_{50}^{mode}$ ⟹ $f_c = 15\,GHz$ ⟹ Same as for TE_{02} mode.

Therefore, the maximum value of $m = 5$.

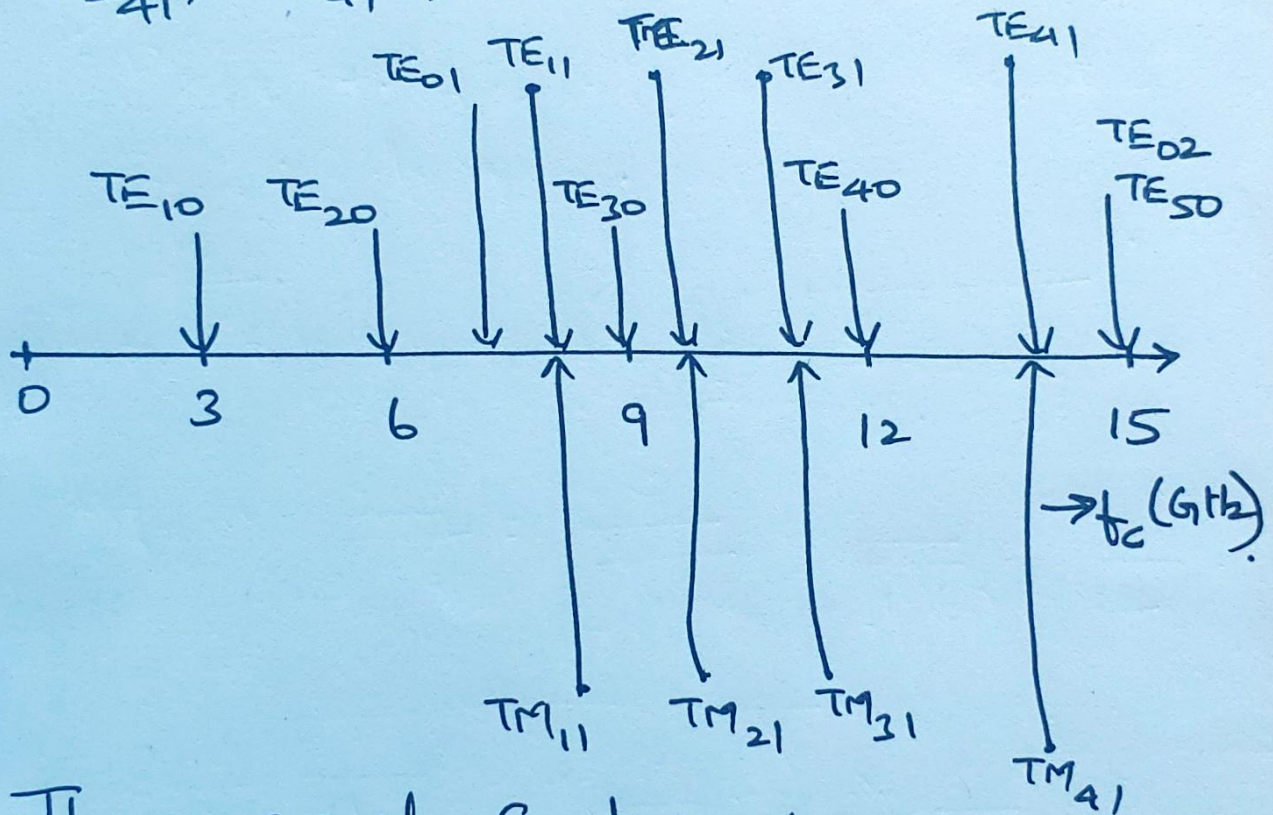$\qquad$ (because for TE_{60} mode, $f_c = 18\,GHz$ ⟹ $f_c > 15.1\,GHz$)

For TE_{11}, TM_{11} (degenerate modes),

$\qquad f_c = 3\sqrt{7.25} = 8.018\,GHz$.

$TE_{21}, TM_{21} \Rightarrow f_c = 3\sqrt{10.25} = 9.6 \, GHz.$

$TE_{31}, TM_{31} \Rightarrow f_c = 11.72 \, GHz.$

$TE_{41}, TM_{41} \Rightarrow f_c = 14.14 \, GHz.$

The waveguide Can transmit the 15 modes with the Cut off frequency of $f_c = 15 \, GHz$ and is illustrated in the above figure.