Problem 6.5

In a Compton scattering experiment, the incident photons have a wavelength of 3×10^{-10} m. Calculate the wavelength of scattered photons if they are viewed at an angle of 60° to the direction of incidence.

[A.U April 2010]

Given data

Wavelength of incident X - rays $\lambda = 3 \times 10^{-10}$ m,

Angle of scattering $\theta = 60°$

$$h = 6.625 \times 10^{-34} \text{ Js}$$

$$m_o = 9.1 \times 10^{-31}$$

$$c = 3 \times 10^{8} \text{ ms}^{-1}$$

Solution:

We know that

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

$$\text{or } \lambda' = \lambda + \frac{h}{m_o c}(1 - \cos\theta)$$

Substituting the given values, we have

$$\lambda' = 3 \times 10^{-10} + \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 60°)$$

$$= 3 \times 10^{-10} + \frac{6.625 \times 10^{-34}}{2.730 \times 10^{-22}}(1 - 0.5)$$

$$= 3 \times 10^{-10} + 2.427 \times 10^{-12} \times 0.5$$

$$= 3 \times 10^{-10} + 1.2132 \times 10^{-12}$$

$$\lambda' = 3.012 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda' = 3.012 \text{ Å}}$$

Problem 6.6

X-rays of 1.0 Å are scattered from a carbon block. Find the wavelength of the scattered beam in a direction making 90° with the incident beam. How much kinetic energy is imparted to the recoiling electron?

[A.U May 2011]

Given data

Wavelength of incident X - rays $\lambda = 1 \text{ Å} = 1 \times 10^{-10}$ m

Angle of scattering $\theta = 90°$

$$h = 6.625 \times 10^{-34} \text{ Js.}$$

$$c = 3.0 \times 10^{8} \text{ ms}^{-1}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ joule.}$$

Solution

The change in wavelength is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

$$\Delta\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 90°)$$

$$\Delta\lambda = 0.242\,(1 - 0) \times 10^{-11}$$

$\Delta\lambda = 0.0242 \times 10^{-10} = 0.0242$ Å

Now $\lambda' = \lambda + \Delta\lambda = 1.0 + 0.0242 = 1.0242$ Å

$= 1.0242 \times 10^{-10}$ m

Energy of incident X-ray photon $= \dfrac{hc}{\lambda}$

Energy of scattered X-ray photon $= \dfrac{hc}{\lambda'}$

$\therefore$ Energy imparted to the recoiling electron

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$= \frac{hc\,(\lambda' - \lambda)}{\lambda\lambda'} = \frac{hc\Delta\lambda}{\lambda\lambda'}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8} \times 0.0242 \times 10^{-10}}{1.0 \times 10^{-10} \times 1.024 \times 10^{-10}}$$

$$= 4.66 \times 10^{-17} \text{ joule}$$

$$= \frac{4.66 \times 10^{-17}}{1.6 \times 10^{-19}} = 291 \text{ eV}$$

Problem 6.7

A neutron of mass 1.675×10^{-27} kg is moving with a kinetic energy 10 keV. Calculate the De-Broglie wavelength associated with it.

[A.U Jan 2011]

Given data

Mass of the neutron $= 1.675 \times 10^{-27}$ kg

Kinetic energy $= 10\text{ keV} = 10 \times 10^{3}$ eV

$$= 10 \times 10^{3} \times 1.6 \times 10^{-19}\text{ J}$$

Planck's constant $h = 6.625 \times 10^{-34}$ Js

Solution:

We know that $\lambda = \dfrac{h}{\sqrt{2mE}}$

Substituting the given values, we have

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 10 \times 10^{3} \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{5.36 \times 10^{-42}}}$$

$$\boxed{\lambda = 2.862 \times 10^{-13}\text{ m}}$$

Problem 6.8

An electron at rest is accelerated through a potential of 5000 V. Calculate de - Broglie wavelength of matter wave associated with it. *[A.U. Jan 2012]*

Given data

Accelerating potential (V) = 5000 V

Solution

We know that $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$\lambda = \frac{12.26}{\sqrt{V}} \times 10^{-10}\text{ m}$$

Substituting the given values, we have

$$\lambda = \frac{12.26 \times 10^{-10}}{\sqrt{5000}}$$

$$\lambda = \frac{12.26 \times 10^{-10}}{70.71}$$

$$\lambda = 0.173 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 0.173 \text{ Å}}$$

Problem 6.9

Calculate de - Broglie wavelength associated with a proton moving with a velocity equal to one-thirtieth of the velocity of light.

(A.U. Dec. 2012)

Given data

Velocity of the proton $v = \frac{1}{30} \times$ velocity of light

$$= \frac{1}{30} \times 3 \times 10^{8} \text{ ms}^{-1}$$

$$= 1 \times 10^{7} \text{ ms}^{-1}$$

Mass of the proton $m = 1.67 \times 10^{-27}$ kg

Planck's constant $h = 6.625 \times 10^{-34}$ J s

Solution

We know that de - Broglie wavelength

$$\lambda = \frac{h}{mv}$$

Substituting the given values, we have

$$\lambda = \frac{6.625 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{7}}$$

$$\boxed{\lambda = 3.97 \times 10^{-14} \text{ m}}$$

Problem 6.10

If the momentum of two particles are in the ratio 1 : 0.25, compare their de - Broglie wave lengths.

(A.U. Jan 2011)

de - Broglie wavelengths associated with two particles of momentum in the ratio 1 : 0.25 are λ_1 and λ_2

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\lambda_1 = \frac{h}{p_1}, \qquad \lambda_2 = \frac{h}{p_2}$$

$$\lambda_1 : \lambda_2$$

$$\frac{h}{p_1} : \frac{h}{p_2}$$

$$\frac{1}{1} : \frac{1}{0.25}$$

$$1 : 4$$

de - Broglie wavelengths are in the ratio

$$\boxed{1 : 4}$$

Problem 6.11

Calculate the de - Broglie's wave length of an electron having a velocity of 10^6 m/sec. (A.U. Dec. 2012)

Given data

Velocity of the electron $v = 10^6 \text{ ms}^{-1}$

Mass of the electron $m = 9.1 \times 10^{-31}$ kg

Planck's constant $h = 6.625 \times 10^{-34}$ Js

Solution

We know that de - Broglie's wavelength $\lambda = \frac{h}{mv}$

Substituting the given values, we have

$$\lambda = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{6}}$$

$$\lambda = 7.28 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 7.28 \text{ Å}}$$

Problem 6.12

Calculate the de - Broglie's wavelength associated with an electron which travels with a velocity 500 km s^{-1}. (A.U. Jan. 2003)

Given data

Velocity of the electron

$$v = 500 \text{ km / sec} = 500 \times 10^{3} \text{ m s}^{-1}$$

Planck's constant $h = 6.625 \times 10^{-34}$ Js

Mass of the electron $m = 9.1 \times 10^{-31}$ kg

Solution

We know that de-Broglie's wavelength associated with electrons

$$\lambda = \frac{h}{mv}$$

Substituting the given values, we have

$$\lambda = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 500 \times 10^{3}}$$

$$\lambda = 0.00145 \times 10^{-6}$$

$$\lambda = 14.5 \times 10^{-10} \text{ m}$$

$$\lambda = 14.5 \text{ Å}$$

Problem 6.13

Calculate the minimum energy an electron can possess in an infinitely deep potential well of width 4nm.

[A.U. Jan 2013]

Given data

Width of potential well $a = 4 \text{ nm} = 4 \times 10^{-9}$ m

For minimum energy, $n = 1$

Mass of the electron $m = 9.1 \times 10^{-31}$ kg

Planck's constant $h = 6.625 \times 10^{-34}$ Js

Solution:

We know that $E_n = \frac{n^2 h^2}{8ma^2}$

Substituting the given values, we have

$$E_1 = \frac{1^2 \times (6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (4 \times 10^{-9})^2}$$

$$E_1 = 3.764 \times 10^{-21} \text{ J}$$

$$E_1 = \frac{3.764 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} \qquad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$\boxed{E_1 = 0.024 \text{ eV}}$$

Problem 6.14

An electron is trapped in a one-dimensional box of length 0.1 nm. Calculate the energy required to excite the electron from its ground state to the fifth excited state. **[A.U. April 2013]**

Given data

Length of the one dimensional box

$$a = 0.1 \text{ nm} = 0.1 \times 10^{-9} \text{ m}$$

For ground state $n = 1$

For 5^{th} excited state, $n = 6$

Solution

We know that $E_n = \frac{n^2 h^2}{8ma^2}$

$$E_1 = \frac{1^2 \times (6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$E_1 = 6.022 \times 10^{-18} \text{ J}$$

For 5^{th} excited state, $n = 6$

$$E_6 = \frac{6^2 \times (6.625 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$E_6 = 2.168 \times 10^{-16} \text{ J}$$

The energy required to excite the electron from its ground state to the fifth excited state is $\Delta E = E_6 - E_1$

$$\Delta E = 2.168 \times 10^{-16} - 6.022 \times 10^{-18}$$

$$= 2.168 \times 10^{-16} - 0.06022 \times 10^{-16} = 2.108 \times 10^{-16} \text{ J}$$

$$= \frac{2.108 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV} \qquad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$\boxed{\Delta E = 1317 \text{ eV}}$$

1. **State Planck's hypothesis.** *(A.U. Dec 2008)*

The atomic oscillators can absorb or emit energy in multiples of a small unit called **quantum**. The quantum of radiation is called **photon**. The energy of the photon (ε) is proportional to the frequency of radiation (ν)

$$\varepsilon \propto \nu$$

i.e., $$\boxed{\varepsilon = h\nu}$$

where h is a constant known as Planck's constant.

2. **State Plank's law of radiation.** *(A.U. Jan 2009)*

The energy density of heat radiation emitted from a black body at temperature T in the wavelength range λ and $\lambda + d\lambda$ is given by

$$E_\lambda = \frac{8\pi hc}{\lambda^5 (e^{h\nu/kT} - 1)}$$

h – Planck's constant

c – Speed of light

ν – Frequency of radiation

k – Boltzmann's constant.

T – Temperature of the black body

3. State compton effect. *(A.U. Jan 2011)*

When a beam of X - rays is scattered by a substance of low atomic number, the scattered radiation consists of two components. One has the same wavelength λ as the incident ray and the other has a slightly longer wavelength λ'. This phenomenon of change in wavelength of scattered X - rays is known as **compton effect**.

4. What is Compton wavelength? *(A.U. Jan 2008)*

The change in wavelength corresponding to scattering angle of 90° obtained in Compton effect is called Compton wavelength.

Mathematically, $\Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta)$

m_o - rest mass of electron $= 9.11 \times 10^{-31}$ kg

When $\theta = 90°$, $\Delta\lambda = \frac{h}{m_o c}(1 - \cos 90°)$

$$= \frac{h}{m_o c}(1 - 0)$$

$$\frac{h}{m_o c} = 0.0243 \text{ Å}$$

This is known as Compton wavelength of electron.

5. What are matter waves? *(A.U. Dec. 2012)*

The waves associated with moving particles of matter (e.g., electrons, photons, etc) are known as matter waves or de-Broglie waves.

6. How De-Broglie justified his concept? *(A.U. May 2012)*

- Our universe is fully composed of light and matter.
- Nature loves symmetry. If radiation like light can act like wave and particle, then material particles (e.g., electron, neutron etc.) should also act as particle and wave.
- Every moving particle has always associated with a wave.

7. Write an expression for the wavelength of matter waves? (or) What is de - Broglie's wave equation? *(A.U. Jan 2010)*

Wavelength for matter waves is

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where $h \rightarrow$ planck's constant

$m \rightarrow$ mass of the particle

$v \rightarrow$ velocity of the particle with which the wave is associated.

$p \rightarrow$ momentum of the particle.

8. Write an expression for the de - Broglie wavelength associated with electrons. *(A.U. Dec. 2011)*

De-Broglie wave length associated with electrons accelerated by the potential V.

$$\lambda = \frac{h}{\sqrt{2 m_0 e V}}$$

where $h \rightarrow$ planck's constant

$e \rightarrow$ charge of the electron

$m \rightarrow$ mass of the electron

$V \rightarrow$ accelerating voltage

9. State the properties of the matter waves. (A.U. Jan 2012)

(i) Lighter is the particle, greater is the wavelength associated with it.

(ii) Smaller is the velocity of the particle, greater is the wavelength associated with it.

(iii) These waves are not electromagnetic waves.

(iv) The velocity of deBroglie wave is equal to the velocity of the material particle.

10. Write down Schroedinger time independent and dependent wave equations. (A.U. Jan 2011)

Schroedinger time independent wave equation is

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Schroedinger time dependent wave equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is Laplacian operator.

ψ – Wave function

m – Mass of the particle.

E – Total energy of the particle.

V – Potential energy.

and $\hbar = \frac{h}{2\pi}$

11. Mention some of the physical significances of the wave function. (A.U. Jan. 2010, May 2011, Jan 2013)

(i) The wave function (ψ) relates the particle and wave nature of matter statistically.

(ii) It is a complex quantity and hence we cannot measure it.

(iii) If the particle is certainly to be found somewhere in a space of dimensions dx, dy, dz, then the probability value is equal to one.

$$\text{i.e., } P = \iiint_V |\psi|^2 \, dx\, dy\, dz = 1$$

12. What are eigen values and eigen function?

(A.U. Jan. 2013)

Energy of a particle moving in one dimensional box of width a is given by

$$\boxed{E_n = \frac{n^2 h^2}{8ma^2}}$$

For each value of n, there is an energy level. Each value of E_n is called an eigen value.

For every quantum state (i.e., for different 'n' values), there is a corresponding wave function ψ_n. This corresponding wave function is called eigen function.

Eigen function associated with an electron in a one dimensional box is given by

$$\boxed{\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right).}$$

13. What is an electron microscope? *(A.U. Dec. 2012)*

- It is a microscope in which the object is illuminated by highly accelerated fast-moving electron beam.
- It has very high magnification of about 100,000 X and very high resolving power.
- It works on the principle of electron diffration.

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ADDITIONAL PART B
'16' MARKS' QUESTIONS

1. With the concepts of quantum theory of black body radiation derive an expression for energy distribution and use it to prove Wien's law and Rayleigh - Jeans law.
2. Obtain the eigen valves and eigen functions for an electron enclosed in a one dimensional potential box.

ASSIGNMENT PROBLEMS

1. In Compton scattering, the incident photons have a wavelength 0.5 nm. Calculate the wavelength of scattered radiation if they are viewed at angle of 45° to the direction of incidence.

   [*Ans:* $\lambda' = 0.5007$ nm]

2. X-rays of 1.0 Å are scattered from a carbon block. Find the wavelength of the scattered beam in a direction making 60° with the incident beam. How much kinetic energy is imparted to the recoiling electron?

   [*Ans:* $\lambda' = 1.0121$ Å K.E. = 149 eV]

3. Find the change in the wavelength of an X-ray photon when it is scattered through an angle of 180°. ***[Ans: 0.0484 Å]***

4. Monochromatic X-rays of wavelength 0.7078 Å are scattered by carbon at an angle of 90° with the direction of incident beam. What is the wavelength of scattered X-rays?

   [Ans: 0.7320 Å]

Estimate the potential difference through which a proton is needed to be accelerated so that its de Broglie wavelength becomes equal to 1 Å.

(Given mass of proton $= 1.673 \times 10^{-27}$ kg.) *[V = 0.082 V]*

6. Calculate the de Broglie wavelength associated with an electron carrying an energy 2000 eV. [$\lambda = 2.74 \times 10^{-11}$ m]

7. Prove that the de Broglie wavelength of an electron accelerated through a potential difference of V volts is

$$\sqrt{\frac{150}{V}}\ \text{Å}.$$

8. Calculate the minimum energy an electron can possess in an infinitely deep potential well of width 4 nm.

[Ans: $E = 0.0236$ eV]

9. Calculate the zero point energy for an electron in a one dimensional box of width 10 Å. *[Ans: 0.376 eV]*