

ISD 2023 - Week 2 Assignment

There are 10 questions for a total of 20 marks.

1. (2 marks) Consider a rectangular lattice where the atoms are spaced periodically with a lattice constant a . It is convenient to represent this lattice in the reciprocal (Fourier) space. The various segments in a reciprocal lattice are referred to as the Brillouin zones. Which of the following statements is/are true in a reciprocal lattice?

A. the periodicity is $2\pi/a$ in k -space

B. the periodicity in k -space is a

C. the first Brillouin zone is from $-\pi/a$ to $+\pi/a$

D. the periodicity in k -space is $2a$

$$f(E) = \frac{1}{1 + e^{E - E_F / kT}}$$

2. (2 marks) Consider an intrinsic Silicon (bandgap 1.12 eV) at room temperature (300 K). Let the probability that a state located at the bottom of the conduction band is filled be (f_c), and the probability that a state located at the top of the valence band is empty be (f_v). Which of the following is true?

A. $f_c > f_v$ **B. $f_c = f_v$** C. $f_c < f_v$ D. $f_c + f_v = 0.5$

Intrinsic $\rightarrow E_F$ @ center of E_c / E_v .

3. (2 marks) Consider intrinsic silicon at $T=300$ K. What is the Fermi level position relative to the bottom of the conduction band? Assume $N_c = 3 \times 10^{19} \text{ cm}^{-3}$, $n_i = 1 \times 10^{10} \text{ cm}^{-3}$.

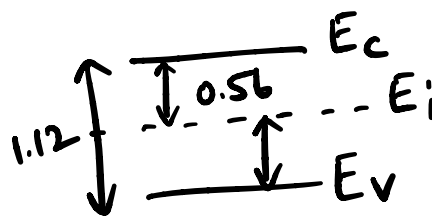
A. 0.56 eV below the conduction band

B. 0.56 eV above the conduction band

C. 0.15 eV above the conduction band

D. 0.15 eV below the conduction band

Intrinsic $\rightarrow E_F$ @ center of Bandgap.



4. (2 marks) Consider n-type silicon at $T=300$ K. What is the Fermi level position relative to the conduction band edge? Assume $N_c = 3 \times 10^{19} \text{ cm}^{-3}$, $n_i = 1 \times 10^{10} \text{ cm}^{-3}$ and $N_D = 1 \times 10^{17} \text{ cm}^{-3}$.

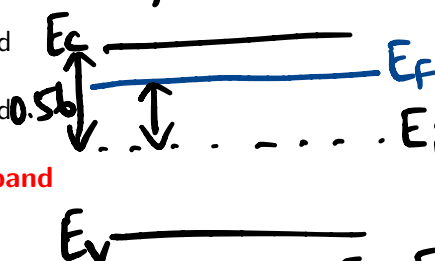
A. 0.56 eV below the conduction band

B. 0.56 eV above the conduction band

C. 0.15 eV above the conduction band

D. 0.15 eV below the conduction band

Doped \rightarrow shifts E_F near to CB edge

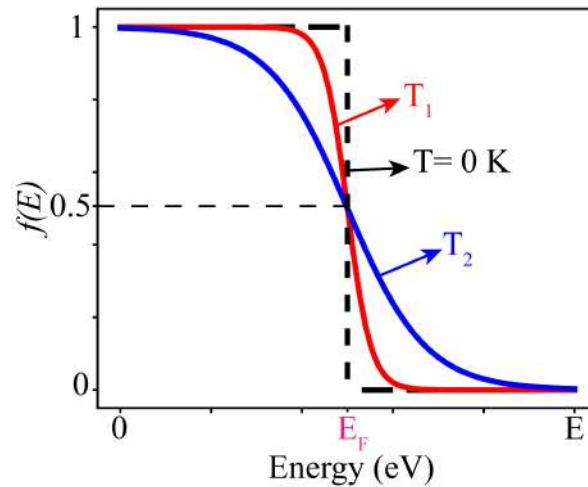


$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$$

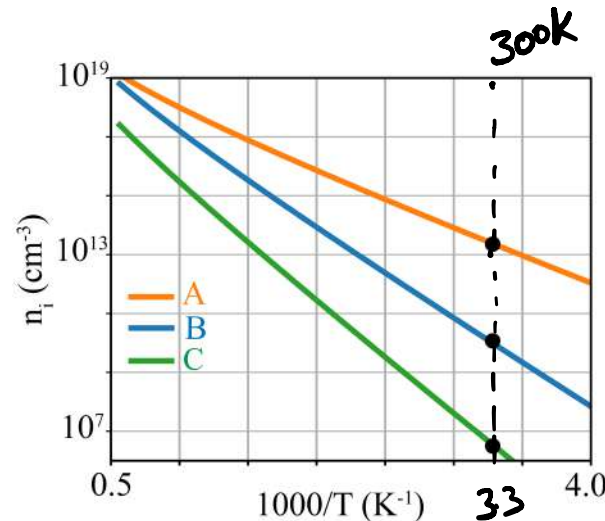
$$\approx 0.41 \text{ eV}$$

$$E_c - E_F = 0.56 - 0.41 \approx 0.15 \text{ eV}$$

5. (2 marks) Consider the Fermi-Dirac distribution function ($f(E)$) at different temperatures as shown in the figure. Which of the following statements is true regarding T_1, T_2 ?



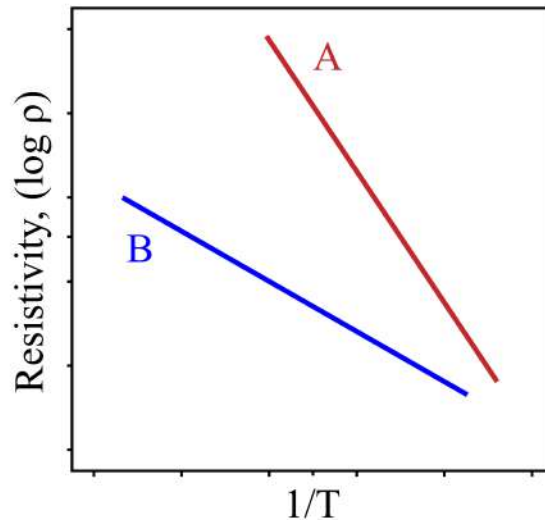
- A. $T_2 = T_1 \neq 0$ **B. $T_2 > T_1$** C. $T_2 < T_1$ D. $T_2 = T_1 = 0$
6. (2 marks) Consider three typical semiconductors A, B and C with bandgap E_{gA} , E_{gB} and E_{gC} respectively. The intrinsic carrier concentration n_i of three materials as a function of temperature $1/T$ is shown in the figure. Which of the following is true regarding their bandgap?



- A. $E_{gA} < E_{gB} < E_{gC}$** B. $E_{gB} < E_{gA} < E_{gC}$ C. $E_{gA} > E_{gB} > E_{gC}$ D. $E_{gB} > E_{gA} > E_{gC}$

Reflect and remember: The intrinsic carrier concentration increased by over 4 orders of magnitude as the temperature increased by 150 K. Why does the intrinsic carrier density increase with increasing temperature? Also, notice that at high temperatures n_i seems to converge to a particular number. What is this limit of n_i at very high temperatures?

7. (2 marks) Consider two non-degenerate semiconductors A and B with bandgap E_{gA} and E_{gB} respectively. The resistivities ρ of these two materials (ρ_A and ρ_B) reduce with the reciprocal of temperature $1/T$ as shown in the figure. Which of the following is true regarding their bandgap?



$$\log n_i \propto -E_g/2kT$$

- A. $E_{gA} = E_{gB} \neq 0$ **B. $E_{gA} > E_{gB}$** C. $E_{gA} < E_{gB}$ D. $E_{gA} = E_{gB} = 0$

8. (2 marks) (GATE-EC2022) In a non-degenerate bulk semiconductor with electron density $n = 10^{16} \text{ cm}^{-3}$, the value of $E_C - E_{Fn} = 200 \text{ meV}$, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume the thermal voltage as 26 mV and the intrinsic carrier concentration is 10^{10} cm^{-3} . For $n = 0.5 \times 10^{16} \text{ cm}^{-3}$, the closest approximation of the value of $(E_C - E_{Fn})$, among the given options is _____ meV.

A. 165

B. 235

C. 218

D. 182

$$\frac{1 \times 10^{16}}{0.5 \times 10^{16}} = \frac{N_c \cdot e^{-(0.2)/kT}}{N_c \cdot e^{-(E_C - E_F)/kT}} = 2$$

$$n = N_c \cdot e^{-(E_C - E_F)/kT}$$

$$\frac{E_C - E_F - 0.2}{kT} = \ln 2$$

$$E_C - E_F = 0.218 \text{ eV}$$

9. (2 marks) (GATE-EC2023) In a semiconductor, the Fermi energy level lies 0.35 eV above the valence band. The effective density of states in the valence band at $T=300 \text{ K}$ is $1 \times 10^{19} \text{ cm}^{-3}$. The thermal equilibrium hole concentration in silicon at 400 K is _____ $\times 10^{13} \text{ cm}^{-3}$. Given $kT = 0.026 \text{ eV}$. (Recall, effective density of states N_v depends on effective mass and temperature)

A. 36 B. 92 **C. 63** D. 25

$$p = N_v \exp(E_v - E_F)/kT$$

$$p|_{400K} = \left(\frac{400}{300}\right)^{3/2} \cdot 1 \times 10^{19} \times \exp\left(\frac{-0.35}{0.026 \times \frac{400}{300}}\right) = 6.2 \times 10^{14} / \text{cm}^3$$

$$E_F = 0.35 \text{ eV} + E_v$$

No

10. (2 marks) Consider a silicon sample doped with 10^{17} cm^{-3} phosphorus atoms. Assume the donor energy level is 45 meV below E_c for phosphorus. We have seen that at $T = 0 \text{ K}$ the dopant atoms are not ionized (i.e., the excess electron is at the dopant site and cannot move around the lattice.). At higher temperatures, some of the dopants are ionized and they contribute to electrons in the conduction band. Estimate the probability of 'finding' an electron at the donor energy level at $T = 300 \text{ K}$ by calculating $E_D - E_F$. Based on this can you infer what fraction of the donors are "not ionized"? You may assume that an electron at E_D implies that the corresponding fraction of dopant atoms are not ionized.

Handwritten calculations and energy level diagram:

Left side calculations:

$$E_F - E_i = kT \ln \frac{10^{17}}{10^{10}} \approx 0.42 \text{ eV}$$

Options:

- A. 100 %
- B. 2.3 %
- C. 65 %
- D. 20 %

Handwritten formula for the probability of finding an electron at the donor level:

$$f(E_D) = \frac{1}{1 + e^{(E_D - E_F)/kT}} = \frac{1}{1 + e^{0.095/0.026}} = 0.023$$

Energy level diagram (right side):

The diagram shows the following energy levels and distances:

- E_c (Conduction band edge)
- E_D (Donor level) is 0.045 eV below E_c .
- E_F (Fermi level) is 0.095 eV below E_c .
- E_i (Intrinsic level) is 0.42 eV below E_F .
- E_v (Valence band edge) is 0.56 eV below E_c .

Reflect and remember: You will notice that as N_d increases, E_F moves toward E_D , and the probability of non-ionization can become quite large. In reality, the impurity level broadens into an impurity band that merges with the conduction band in heavily doped semiconductors (i.e., when donors or acceptors are close to one another). This happens for the same reason energy levels broaden into bands when atoms are brought close to one another to form a crystal. The electrons in the impurity band are also in the conduction band.

Therefore, the assumption of $n = N_d$ (or complete ionization) is reasonable even at very high doping densities. The same holds true in P-type materials.

Refer Donald Neamen, Semiconductor Physics and Devices, 4th Edition, page no. 118-120 for additional details.