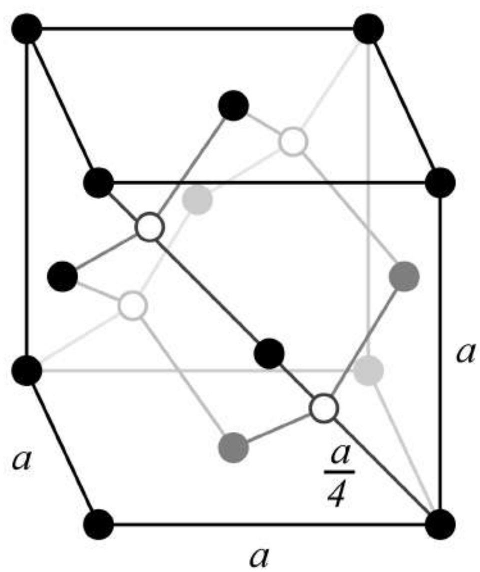
ISD 2023 - Week 1 Assignment

There are 10 questions for a total of 20 marks.

1. (2 marks) Silicon has a diamond-cubic lattice structure with a lattice constant $a = 5.43$ Å as shown in the figure below. The silicon crystal is formed by periodic repetition of the unit cells in the x, y and z directions. In each unit cell, 8 atoms at the corners are shared with 8 adjacent cells. Hence, the corner atoms are considered to contribute one-eighth of the unit cell. The 6 atoms in center of each face are shared by two adjacent unit cells. Hence, they contribute one-half to the unit cell. Lastly, the 4 atoms within the body of the unit cell contribute fully. Hence the effective number of atoms per unit cell is $\frac{1}{8} \times 8 + \frac{1}{2} \times 6 + 4 \times 1 = 8$ atoms. Determine the density of atoms is ______ $\# \, atoms/cm^3$. (Recall: density= # atoms per unit cell/volume of unit cell)

Volume $= a^3$



Density $= \frac{\text{No. of atoms}}{\text{Volume}} = \frac{8}{(5.43 \times 10^{-8})^3} = 4.99 \times 10^{22}$ #

A. 1.6×10^{22}

B. 5×10^{22}

C. 2.5×10^{22}

D. 1×10^{22}

> **Reflect and remember:** The density of atoms in the crystal lattice is important when we discuss doping later in the course. By doping we replace the Si atoms by another element (B, P, As, etc). Also, note the units - we use cm^{-3} to denote density throughout this course. Can you guess the density of atoms for other semiconductors like Ge and GaAs?

2. (2 marks) Given the uncertainty in position of an electron, $\Delta x = 8\ \mathring{A}$. If the nominal value of momentum is $p = 1.2 \times 10^{-23}\ kg - m/s$, determine the corresponding uncertainty in kinetic energy (in eV).
(Recall: The uncertainty in kinetic energy can be found from $E = p^2/2m \Rightarrow \Delta E = dE/dp\, \Delta p = p\Delta p/m$)

$\Delta x \cdot \Delta p \geq \hbar/2$

$\Delta p \geq \hbar/2 \cdot \Delta x$.

A. 0.548 **B. 5.48** C. 0.058 D. 1.25

> **Reflect and remember:** Why are the units of eV used to denote energy? What happens when $\Delta x = 1\mu m$, instead of $8\ \mathring{A}$?

$\Delta E = \frac{p}{m} \cdot \frac{\hbar}{2 \cdot \Delta x} = 5.48\ eV$.

3. (2 marks) An electron and a photon have the same energy. At what value of photon energy (in keV) will the wavelength of the photon be 10 times that of the electron?
(Recall: $E_{photon} = \frac{hc}{\lambda} = \frac{1.24}{\lambda_p[um]}\ eV$ and $E_{electron} = \frac{p^2}{2m}$; $\lambda_e = \frac{h}{p}$)

$\lambda_p = 10\lambda_e$

$\frac{hc}{10\lambda_e\ \cancel{\lambda_p}} = \frac{h^2}{\lambda_e^2 \cdot 2m}$

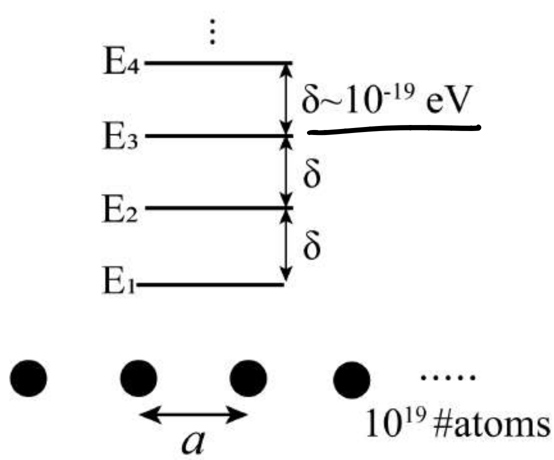
$\lambda_p = 1.2 \times 10^{-10}\ m$

$\lambda_e = \frac{5h}{cm} = 1.2 \times 10^{-11}\ m$

A. 1.25 **B. 10.25** C. 100 D. 50 $\quad hc/\lambda \rightarrow 10.2\ keV$

> **Reflect and remember:** The momentum of the photon is very small compared to the momentum of the electron. This plays a key role in transitions from the valance band to the conduction band.

(For Q4-Q5): Consider a system with 10^{19} one-electron atoms at an equilibrium inter-atomic spacing. Due to interatomic interactions, the energy levels of individual atoms hybridize and form an energy band. If the energy band is $1\ eV$ wide, then spacing between individual levels is $10^{-19}\ eV$ as shown in the figure. Each electron in the system occupies a different energy level.



4. (2 marks) Which of the following statement(s) are true?

   A. The energy band represents the forbidden gap of the energy band diagram

   B. The energy band can be considered to be quasi-continuous due to the small spacing between energy levels

   C. The energy band cannot be considered to be quasi-continuous due to the small spacing between energy levels

   D. The discrete nature of the energy levels in the band is important in studying the bulk electronic properties.

$KE = \frac{1}{2} m v^2$ $= 10^5 (m/s)$

5. (2 marks) Calculate the change in kinetic energy (in eV) of an electron travelling with a velocity $10^7\ cm/s$ and changes by a small amount increases by a value of $100\ cm/s$.

$\Delta KE = m v . \Delta v.$

$= \frac{9.1 \times 10^{-31} \times 10^5 \times 1}{1.6 \times 10^{-19}}$

Very small energy $= 5.6 \times 10^{-7}\ eV.$

   A. $5.6 \times\ 10^{-12}$

   B. $5.6 \times\ 10^{-7}$

   C. $5.6 \times\ 10^{-3}$

   D. $5.6 \times\ 10^{-9}$

> **Reflect and remember**: A small change in velocity results in an extremely small energy change, $\Delta E \approx 10^{-8}\ eV$. This is extremely large compared to the energy difference between the energy levels in the allowed energy band $\Delta \approx 10^{-19}$. Hence, the discrete energies within the allowed band can be treated as a quasi-continuous distribution.

6. (2 marks) Consider an electron initially at rest. If it starts moving with a velocity $10^7\ cm/s$, what is the approximate kinetic energy of such electron (in meV)? Assume rest mass of electron $m_0 = 9.1 \times 10^{-31}\ kg$.

A. 0 **B. 28** C. 100 D. 10

$KE = \frac{1}{2} m v^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times 10^{10} \times \frac{1}{1.6 \times 10^{-19}} = 28.4$ meV.

> **Reflect and remember**: The thermal energy ($k_B T$) of an electron at $T = 300\ K$ is $26\ meV$. The kinetic energy ($KE = \frac{1}{2} m v^2$) of an electron that starts from rest and accelerates to a thermal velocity of $10^7\ cm/s$ is $28\ meV$. This is not surprising since thermal motion involves frequent collisions between electrons which cause them to accelerate and stop randomly. We will use $v_{th} = 10^7\ cm/s$ and $k_B T = 26\ meV$ in this course. Sometimes we will approximate $k_B T \approx 25\ meV$ for ease of calculations.

7. (2 marks) Consider an electron with an energy of $1\ eV$ impinging on a potential barrier of height $V_0 = 5\ eV$ and width $a = 0.5\ nm$. Assume tunnelling probability, in this case, is T_1. Consider the same electron impinging on a potential barrier of height $V_0 = 5\ eV$ and width $a = 2\ nm$. The tunnelling probability, in this case, is T_2. The relation between T_1 and T_2 is ______.

$x_1 = 0.5$
$x_2 = 2$

$$\frac{T_1}{T_2} \simeq e^{-2\gamma(x_1 - x_2)} \simeq e^{2\gamma \Delta x} \gg 1$$

A. $T_2 = T_1$ B. $T_2 >> T_1$ **C.** $T_2 << T_1$ D. $T_2 = T_1 = 1$

> **Reflect and remember**: What is the functional dependence of tunnelling probability on the height and width of the barrier? The tunnelling of electrons across a barrier has a significant impact on the gate leakage current in modern MOSFETs. We will revisit this when we discuss MOSCAPs and MOSFETs.

8. (2 marks) Consider a particle in a 1D potential box of width L. Let E_1 is the energy of the ground state and E_n is the energy of the state with quantum number n. What is the ratio of energy emitted when the particle makes a transition from $n = 3$ to its ground state ($E_{3\rightarrow 1}$) and $n = 5$ to its ground state ($E_{5\rightarrow 1}$).

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \qquad \frac{E_{3\rightarrow 1}}{E_{5\rightarrow 1}} = \frac{3^2 - 1}{5^2 - 1} = 8/24 = 1/3$$

A. 9/25

B. 1/3

C. 8

D. 25

> **Reflect and remember**: Transitions between energy levels define the absorption/emission of photons within a semiconductor.

9. (2 marks) Consider two schematic band structures of Si and GaAs as shown in the figure.

S1: Silicon is an indirect bandgap, and GaAs is a direct bandgap semiconductor.

S2: Silicon is a direct bandgap, and GaAs is an indirect bandgap semiconductor.

S3: In a direct bandgap semiconductor, momentum is conserved by when an electron makes a transition between bands, and in an indirect semiconductor, it is conserved by simultaneous absorption/emission of phonon and photons.

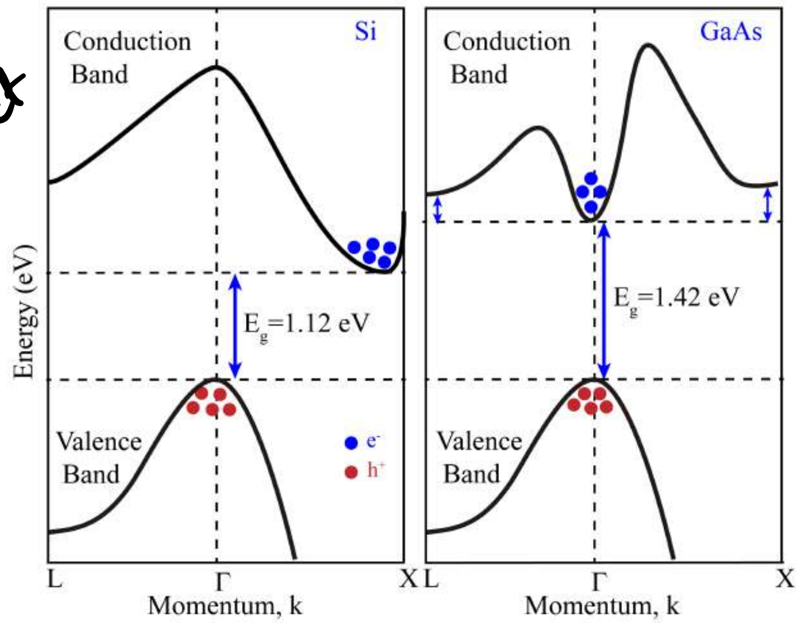
S4: In an indirect semiconductor, momentum is conserved by photon emission and in a direct semiconductor, it is conserved by both phonon and photon emission.

S5: Recombination is much more efficient in GaAs than Si and hence used for optoelectronic devices.

S6: Recombination is less efficient in GaAs than Si and hence is not used for optoelectronic devices.

Which of the following options is true?

Indirect



Direct

A. S1, S3, S5

B. S2, S4, S6

C. S1, S4, S6

D. S2, S4, S5

10. (2 marks) **(EC-GATE 2014)** Cut off wavelength (in μm) of light that can be used for intrinsic excitation of a semiconductor material of band gap 1.1 eV.

$$\lambda_g = \frac{1.24}{E_g(eV)} \cdot \mu m = \frac{1.24}{1.1} \mu m$$

A. 0.85

B. 1.125

C. 1.45

D. 2.25