Computing Risk Parity Portfolio Weights*

*As a part of Algorithmic Trading Course

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***Abstract*—This project is aimed to compute risk-parity portfolio weights for further analysis. In addition to hierarchical risk parity technique, two methods of Sharpe Ratio calculation, Markowitz's Minimum-Variance, and Inverse-Variance portfolios were computed to make a comparison between traditional and newly proposed approaches. The paper is divided into sections: the Introduction demonstrates the motivation of the research topic choice, the existing problem, proposed solution to this problem, project plan, and project details which consists of tools used for implementations, roadmap, and dataset information; the Related work discusses different methodologies and their characteristics; the Algorithm illustrates all the steps done; the Results show the computed weights of different approaches; and the Conclusion highlights important information, and concludes the paper.**

***Index Terms*—portfolio weights, hierarchical risk parity, Sharpe ratio, stocks clustering**

I. Introduction

A. Motivation

Constructing profitable portfolios is one of the difficult task for Portfolio Managers. A portfolio is an investment which aimed to earn return in some period of time [1]. The key factors of the portfolio are risk tolerance and time horizon [1]. In this project only risk will be analyzed and optimized.

There are a lot of different methods for minimizing risk and increasing return of the portfolio. The most popular are Sharpe Ratio, Risk-Parity, and Markowitz's Efficient Frontier approaches. Additionally, in this project a new technique called Marcos Lopez de Prado approach which is based on [2] was used to make a performance comparison.

B. Problem

Each portfolio investment methodology has challenges as well as advantages. The Markowitz's approach is dependent on quadratic optimization which might lead to highly concentrated results [2]. On the other hand, traditional risk parity ignores covariances which can be useful for optimization.

C. Solution

To address the above mentioned challenges [2] proposes to use covariances instead of forecasted returns as the Markowitz's approach does, and to cluster correlated stocks which provides less weight allocation to similar assets.

Acknowledgements to Professor Selim Temizer

D. Project Plan

1) Related research 1 week
2) Implementation 1 week
3) Report preparation 1 week

E. Project details

1) Tools: Python programming language and Google Colab notebook

2) Roadmap:
- Apply hierarchical clustering
- Build portfolio by picking one asset with equal weights from each cluster
- Compare the portfolio's risk with traditional methodologies for portfolio optimization

3) Dataset: Portfolio consists of 10 assets, including Facebook, Amazon, Apple, Google, Netflix, Tesla, Microsoft, IBM, Oracle and Atlantis Technology Group companies.

4) Trading time period: almost 3 years, starting from 1st of January 2019 and finishing by 23rd of April 2021.

II. Related work

A. Hierarchical Risk Parity

Risk Parity is a portfolio optimization strategy which provides lower risk [4]. Such portfolio achieves higher Sharpe Ratio, and has more immune against market downtrends than traditional portfolio [3]. The aim of risk parity is to earn return at the optimal risk [4].

The above mentioned approach Marcos Lopez de Prado is based on Modern Graph Theory and Machine Learning techniques [2]. Lopez de Prado is one of the top authors who writes about finance and fintech topics [2].

In more details, Marcos Lopez de Prado approach is a Hierarchical Risk Parity (HRP) portfolio optimization which generates diversified portfolio, and uses clustering to show the closest pairs among stocks [5].

The algorithm works in three steps:

1) Tree clustering: grouping similar stocks into clusters based on their correlation matrix. Hierarchical structure provides stability improvements for quadratic optimizer during inversion.

2) Quazi-diagonalization: the covariance matrix is reorganized such that similar assets are placed together. The matrix

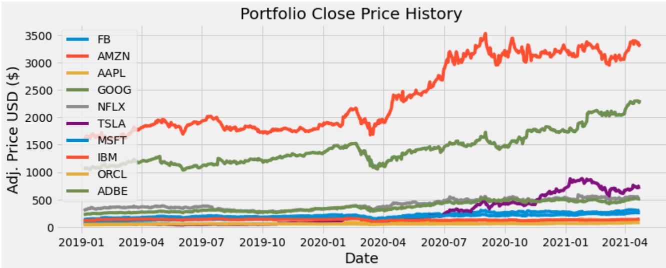

Fig. 1. Portfolio Close Price History

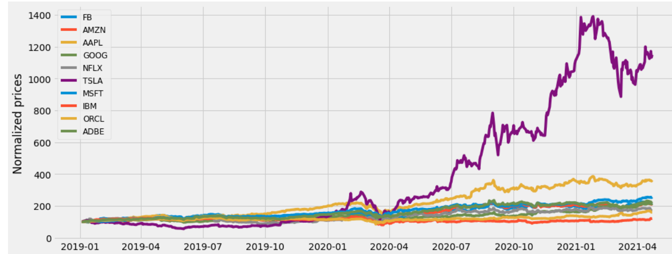

Fig. 2. Normalized prices of 10 assets

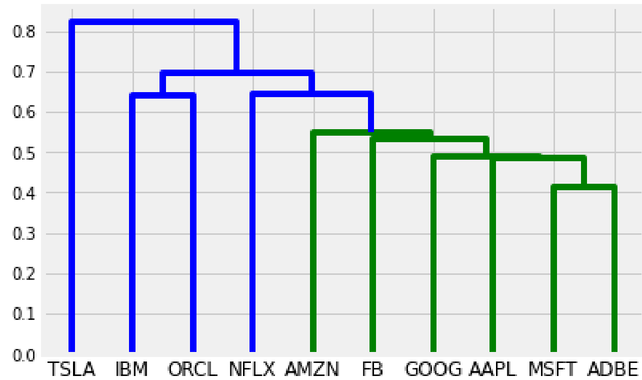

Fig. 3. Cluster Tree of portfolio assets

diagonalization provides optimized weights distribution which is followed by inverse-variance.

3) Recursive bisection: based on the cluster covariance information the allocation is distributed through the recursive bisection.

B. Traditional portfolio management methodologies

- Markowitz's Minimum-Variance Portfolio (MVP) is an approach which focuses on the price volatility minimization [6]. Volatility measures price's movements up or down, and it is often interchangeable with market risk.
- Inverse-Variance Portfolio (IVP) aims to inverse the proportion of each weighted asset with respect to their volatility such that the sum of all weights equals to 1 [7].
- Sharpe Ratio is also used to analyze portfolio returns with respect to its risk [8]. The ratio is calculated average return in risk-free rate per unit of total risk. Volatility here is also a measure of price fluctuations.

III. Algorithm

A. Data Storage

The weight for each assets were divided equally, i.e 10 assets with 0.1 weight. The data were collected from "Yahoo Finance". The "Close prices" from 04 January 2019 to 23 March 2021 were parsed with help of DataReader library. The "Fig.1" shows the portfolio close price history for 10 assets which were chosen, while the "Fig.2" represents the normalized prices to see the variance of prices. The assets are easily changeable.

B. Financial Calculation

Marcos and Sharpe Ration (SR) use return values, in that case DataFrame with return values for 10 assets were created with "pct change()" method. Also, the co-variance matrix is calculated to see the how the assets vary or move together. The diagonal values in the co-variance matrix are variance and others are co-variances. To follow "DRY" (Don't repeat yourself) principle "get cal(new weight to calculate)" method was created with expected portfolio variance and volatility.

$$variance = w^T * \Omega * w \tag{1}$$

$$volatility = \sqrt{^T * \Omega * w} \tag{2}$$

Expected portfolio variance is calculated through equation 1, where Ω is Covariance Matrix, w is vector of weights, w^T is transposed vector of weights. Expected portfolio volatility is square rooted portfolio variance, which is the equation 2.

C. Marcos Lopez de Prado's hierarchical risk parity

Firstly, a hierarchical portfolio is created with a distance matrix based on correlation. This is a proper distance metric $distance = ((1 - correlation)/2)^{0.5}$. Then, the items are correlated by distance. The "Fig.3" shows how assets are pair clustered. Then the method "getRecBipart" allocates bisection based on cluster co-variance.

L.Campos [2] compared well-known Markowitz's Minimum-Variance Portfolio (MVP) and Traditional risk parity's Inverse-Variance Portfolio (IVP) with Marcos López de Prado's hierarchical risk parity (HRP) (Table 1).

TABLE I
Portfolio Weights in Three Methods

	MVP	IVP	HRP
FB	0.001391	0.091199	0.088826
AMZN	0.313225	0.131955	0.110712
AAPL	0.000173	0.094182	0.089643
GOOG	0.110440	0.125388	0.122125
NFLX	0.060202	0.073837	0.061951
TSLA	0.000328	0.024089	0.040477
MSFT	0.000080	0.116318	0.060353
IBM	0.278698	0.123083	0.206822
ORCL	0.235407	0.127069	0.170899
ADBE	0.000056	0.092880	0.048192

The results show that MVP concentrates 4 assets which are AMZN (31%), GOOG (11%), IBM(28%) and ORCL (24%), while HRP concentrates the trade off between risk and return value. On the other hand, IVP evenly spreads weight through all assets, ignoring the correlation structure.

D. Sharpe Ratio

Optimization of the portfolio were done with "pyportfolioopt" library by Robert Ansrew Martin. Then, Sharpe Ratio optimized to maximal level with "EfficientFrontier.max sharpe" method from the library. The result of portfolio weights are AAPL: 0.49066, TSLA: 0.50934, for other 0 weights.

The second method was provided due to that the first method with "pyportfolioopt" library has shown the maximal values which were not included to other assets. The second method is calculated manually, and it stores the Sharpe Ratio with formula $SR = return/volatility$. The risk free rate element is excluded for simplicity. The results for second method are FB: 0.080892, AMZN: 0.159303 , AAPL: 0.235033, GOOG: 0.125475, NFLX: 0.009099, TSLA: 0.242085, MSFT: 0.037267 , IBM: 0.032898, ORCL: 0.064871, ABDE: 0.013077.

IV. Results

The Expected annual return : 47.0%, Annual volatility/standard deviation/risk : 29% and Annual variance : 8.0% were for the equal or initial weights. The maximal optimized Sharpe ratio increased the Expected annual return to 98% and Annual volatility to 48.0%, which has high return value, but risky. The second method for Sharpe ratio showed the increasing for Expected annual return to 66.0% and Annual volatility to 33.0%, which also has high return value, but not risky.

By optimizing with new method by Marcos Lopez de Prado, the portfolio has Expected annual return at 37.0%, Annual volatility at 31.0% and Annual variance : 10.0%. Additionally, IVP and MVP were tested. The results shows Expected annual returns are 39% and 28 %, Annual risks are 27% and 25% respectively.

Conclusion

Overall, building successful portfolio is indeed challenging and risky process. A variety of factors should be observed and analyzed to make high profit with a few loses. Portfolio Managers construct different techniques to make this task come true. In this project most popular and well-analyzed approaches were picked to demonstrate a performance comparison between traditional and newly suggested methodologies. It was discovered that Sharpe Ratio, Inverse-Variance, Minimum-Variance, and Hierarchical Risk-Parity use risk measurement as a direction for portfolio weights optimization. Although, the results shows promising numbers (high expected annual return and low volatility), it is risky to start investing in this portfolio because there always a necessity to make a deeper analysis with other than risk factor as a focus.

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