Image Denoising: Homework 4

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chapter 9: The global EPLL Algorithm:

Exercice n 9.1:

We have P obeys a GMM model and $\tilde{P}$ is a noisy observation of P with a gaussian isotropic noise of variance σ^2.

Thus,
$$\mathbb{P}(\tilde{P} \mid P) = \frac{1}{(2\pi)^{d/2} \det(\Sigma_k)^{1/2}} e^{-\frac{1}{2}(P-\tilde{P})^T \Sigma_k^{-1} (P-\tilde{P})}$$
$$= \frac{1}{(2\pi)^{d/2} \sigma^d} e^{-1/2\sigma^2 (P-\tilde{P})^T (P-\tilde{P})}$$
$$= \frac{1}{(2\pi)^{d/2} \sigma^d} e^{-\frac{1}{2\sigma^2} \|P-\tilde{P}\|^2}$$

Now, minimizing the energy corresponds to:

$$\min_P E(P \mid \tilde{P}) = \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P))$$
$$\Rightarrow \min_P \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P))$$
$$\Leftrightarrow \max_P \log(\mathbb{P}(P)) - \frac{1}{2\sigma^2}\|P-\tilde{P}\|^2$$
$$\Leftrightarrow \max_P \log\left(\mathbb{P}(P) \times e^{-\frac{1}{2\sigma^2}\|P-\tilde{P}\|^2}\right)$$
$$\Leftrightarrow \max_P \mathbb{P}(P) \times e^{-\frac{1}{2\sigma^2}\|P-\tilde{P}\|^2}$$
$$\Leftrightarrow \max_P \mathbb{P}(P) \times \frac{e^{-\frac{1}{2\sigma^2}\|P-\tilde{P}\|^2}}{(2\pi)^{d/2}\sigma^d}$$
$$\Leftrightarrow \max_P \mathbb{P}(P) \times \mathbb{P}(\tilde{P} \mid P)$$
$$\Leftrightarrow \max_P \cancel{\mathbb{P}(P)} \times \frac{\mathbb{P}(P \mid \tilde{P})\, \mathbb{P}(\tilde{P})}{\cancel{\mathbb{P}(P)}}$$

using Bayes Rule

$$\iff \max_{P} \mathbb{P}(P \mid \check{P}) \cdot \mathbb{P}(\check{P})$$

$$\iff \max_{P} \mathbb{P}(P \mid \check{P})$$

$$\Rightarrow \min_{P} E(P \mid \check{P}) \iff \max_{P} \mathbb{P}(P \mid \check{P})$$

We found that minimizing the above energy amounts to compute a maximum a posteriori estimate of P given $\check{P}$.

Exercice n 9.2 :

→ The likelihood of the image is given by:

$$P(U) = \mathbb{P}\Big(\bigcap_{P \in \mathcal{P}} \{PU\}\Big)$$

→ The log-likelihood of the image is given by:

$$\log(\mathbb{P}(U)) = \log\Big(\mathbb{P}\Big(\bigcap_{P \in \mathcal{P}} \{PU\}\Big)\Big)$$

In the case of independent patches:

$$\begin{aligned}
\log(\mathbb{P}(U)) &= \log\Big(\mathbb{P}\Big(\bigcap_{P \in \mathcal{P}} \{PU\}\Big)\Big) \\
&= \log\Big(\prod_{P \in \mathcal{P}} \mathbb{P}(\{PU\})\Big) \\
&= \sum_{P \in \mathcal{P}} \log(\mathbb{P}(\{PU\})) \\
&= EPLL_{\mathbb{P}}(U)
\end{aligned}$$

Thus, the $EPLL_{\mathbb{P}}(U)$ can be interpreted as the

log-likelihood of the image U only in the case when all the patches in the image are independent which is not always the case.

In the general case, we could not interpret the EPLL as a log-likelihood of the image.