Find all solutions in $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ of the equation system $\boldsymbol{A}x = 12x$, where

$$\boldsymbol{A} = \begin{bmatrix} 6 & 4 & 3 \\ 6 & 0 & 9 \\ 0 & 8 & 0 \end{bmatrix} \tag{1}$$

and $\Sigma_{i=1}^{3} x_i = 1$

considering the augmented matrix

$$\left[\begin{array}{ccc|c} 6 & 4 & 3 & 12x_1 \\ 6 & 0 & 9 & 12x_2 \\ 0 & 8 & 0 & 12x_3 \end{array}\right] \leftarrow -R_0 + R_1 \tag{2}$$

$$\left[\begin{array}{ccc|c} 6 & 4 & 3 & 12x_1 \\ 0 & -4 & 6 & 12(x_2 - x_1) \\ 0 & 8 & 0 & 12x_3 \end{array}\right] \leftarrow -2R_1 + R_2 \tag{3}$$

$$\left[\begin{array}{ccc|c} 6 & 4 & 3 & 12x_1 \\ 0 & -4 & 6 & 12(x_2 - x_1) \\ 0 & 0 & 12 & 12(x_3 + 2x_2 - 2x_1) \end{array}\right] \leftarrow 2R_1 + R_2 \tag{4}$$

note if we try to solve this system of equations we arrive at the following constraints

$$x_1 = x_2, x_3 = \frac{2x_2}{3} \tag{5}$$

$$\left\{ \boldsymbol{x} \in \mathbb{R}^3, \lambda_1 \in R \;\middle|\; \boldsymbol{x} := \lambda_1 \begin{bmatrix} 1 \\ 1 \\ \frac{2}{3} \end{bmatrix} \right\} \tag{6}$$