Are the following sets of vectors lineary independent ?

a.

$$x_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, x_3 = \begin{bmatrix} 3 \\ -3 \\ 8 \end{bmatrix} \tag{1}$$

we'll follow a systematic procedure and use gaussian elimination

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -3 \\ 3 & -2 & 8 \end{bmatrix} \leftarrow -3R_1 + R_2 \tag{2}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -3 \\ 0 & -5 & 17 \end{bmatrix} \leftarrow -2R_1 + R_0 \tag{3}$$

$$\begin{bmatrix} 0 & -1 & 9 \\ -1 & 1 & -3 \\ 0 & -5 & 17 \end{bmatrix} \leftarrow R_1 \text{ switch } R_0 \tag{4}$$

$$\begin{bmatrix} -1 & 1 & -3 \\ 0 & -1 & 9 \\ 0 & -5 & 17 \end{bmatrix} \leftarrow -5R_1 + R_2 \tag{5}$$

$$\begin{bmatrix} -1 & 1 & -3 \\ 0 & -1 & 9 \\ 0 & 0 & -28 \end{bmatrix} \leftarrow -5R_1 + R_2 \tag{6}$$

the following set of vectors is **linear independent**, notice that we can't obtain $\begin{bmatrix} -3 \\ 9 \\ -28 \end{bmatrix}$ from the linear combination of the other 2 vectors. Analagous reasoning arrives at the same conclusion for the other two column vectors.

b.

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \leftarrow -R_1 + R_2 \tag{8}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \leftarrow -R_1 + R_0 \tag{9}$$

$$\begin{bmatrix} 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \leftarrow R_1 \text{ switch } R_0 \tag{10}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \leftarrow R_1 \text{ switch } R_2 \tag{11}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 \end{bmatrix} \leftarrow R_1 \text{ switch } R_2 \tag{12}$$

notice that one of the column vectors is duplicated, that already implies linear dependency, so the set of vectors is **linear dependent**.