Compute the distance between

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{y} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

using

$$a. < \boldsymbol{x}, \boldsymbol{y} >:= \boldsymbol{x}^T \boldsymbol{y}$$

$$[1, 2, 3] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

assuming the canonical basis, then

$$\boldsymbol{x} = 1\boldsymbol{e_1} + 2\boldsymbol{e_2} + 3\boldsymbol{e_3} = \Sigma_i \lambda_i \boldsymbol{e_i}$$

$$\boldsymbol{y} = -1\boldsymbol{e_1} - 2\boldsymbol{e_2} + 0\boldsymbol{e_3} = \Sigma_j \varphi_j \boldsymbol{e_j}$$

$$\Sigma_i \Sigma_j \lambda_j < \boldsymbol{e_i}, \boldsymbol{e_j} > \varphi_j$$

$$\boldsymbol{x}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{y} = [1 \;\; 2 \;\; 3] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -1 - 2 = -3$$

using

$$b. \boldsymbol{x}^T \boldsymbol{A} \boldsymbol{y}, \boldsymbol{A} := \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$[1, 2, 3] \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = [4 \;\; 4 \;\; 4] \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = -4 - 4 = -8$$