

Compute the angle between

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

using

a. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^T \mathbf{y}$

$$\begin{aligned} \theta &= \arccos\left(\frac{\langle \mathbf{x}, \mathbf{y} \rangle}{\|\mathbf{x}\| \|\mathbf{y}\|}\right) = \arccos\left(-\frac{3}{\sqrt{10}}\right) \\ &= 161.5650 \text{ degree} \end{aligned}$$

b. $\langle \mathbf{x}, \mathbf{y} \rangle := \mathbf{x}^T \mathbf{B} \mathbf{y}, \mathbf{B} := \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$$[1, 2] \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = [3 \ 7] \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -3 - 7 = -10$$

$$[1 \ 2] \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = [3 \ 7] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 3 + 14 = 17$$

$$[-1 \ -1] \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = [-3 \ -4] \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 3 + 4 = 7$$

$$\begin{aligned} \theta &= \arccos\left(-\frac{10}{\sqrt{17 \cdot 7}}\right) \\ &= 156.44 \text{ degree} \end{aligned}$$