Determine the inverse of the following matrices if possible:

a.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \tag{1}$$

considering that the inverse is defined as $\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{I}$ thus consider the following augmented matrix

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 4 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \tag{2}$$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 4 & 5 & 6 & 0 & 0 & 1 \end{array}\right] \leftarrow -R_1 + R_2 \tag{3}$$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & -1 & 1 \end{array}\right] \leftarrow -R_1 + R_2 \tag{4}$$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 & -1 & 1 \end{array}\right] \leftarrow -R_0 + R_1 \tag{5}$$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \end{array}\right] \leftarrow -R_1 + R_2 \tag{6}$$

note that we just proved this matrix is not invertible, last row gives no information or another way of saying it is we have 3 variables, but only two equations, a case of linear dependency so is a singular matrix meaning is non invertible.

b.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \tag{7}$$

repeating the augmented matrix

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{8}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow -R_1 + R_3 \tag{9}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow -R_0 + R_3 \tag{10}$$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow -R_0 - R_1 + R_3 \tag{11}$$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & -1 \end{array}\right] \leftarrow -R_0 - R_1 + R_2 \tag{12}$$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & -1 \end{array}\right] \leftarrow R_0 \text{ switch } R_1 \tag{13}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -2 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & -1 \end{array}\right] \leftarrow R_2 \text{ switch } R_3 \tag{14}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -2 \end{array}\right] \leftarrow R_3 \text{ switch } R_4 \tag{15}$$

and the matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & -1 \\ -1 & -1 & 1 & -2 \end{bmatrix}$ is the inverse of $\boldsymbol{A}$