Let $F = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 0\}$ and $G = \{(a - b, a + b, a - 3b) \mid a, b \in \mathbb{R}\}$

a . Show that F and G are subspaces of $\mathbb{R}^3$

Again to show a structure is a subspace we need to show the following

- show that is an abelian group
- show associativity of scalar operations for multipliation
- show distributivity of scalar operations for addition
- show distributivity of vector sum operation
- show identity scalar exists in the set

Let's start by showing F and G are groups

- show that an identity element exist for addition
- show that an inverse element exist for addition
- show that associativity is a valid property
- show that any operation between two elements is contained in $\mathbb{R}^3$

identity element

$$x + \mathrm{id} = x \tag{1}$$

- proof for F

$$(x, y, z) + (a, b, c) = (x, y, z) \tag{2}$$

$$(x + a, y + b, z + c) = (x, y, z) \tag{3}$$

thus it must be the case

$$(0, 0, 0) \tag{4}$$

and indeed $0 + 0 - 0 = 0$

- proof for G

$$(a - b, a + b, a - 3b) + (x, y, z) = (a - b, a + b, a - 3b) \tag{5}$$

thus $(0, 0, 0)$ is the identity element and indeed, $0 \in \mathbb{R}$

inverse element

- proof for F

$$x + x^{-1} = 0 \tag{6}$$

$$(x, y, z) + (a, b, c) = (0, 0, 0) \tag{7}$$