Write

$$\boldsymbol{y} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} \tag{1}$$

as linear combination of

$$\boldsymbol{x_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{x_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{x_3} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \tag{2}$$

$$\Sigma_i^3 \lambda_i \boldsymbol{x_i} = \boldsymbol{y} \tag{3}$$

again we adopt the usual gaussian elimination method so we "simplify" the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & -1 \\ 1 & 3 & 1 \end{bmatrix} \leftarrow -R_1 + R_2 \tag{4}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix} \leftarrow -R_1 + R_0 \tag{5}$$

$$\begin{bmatrix} 0 & -1 & 3 \\ 1 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix} \leftarrow -R_1 \text{ switch } R_0 \tag{6}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \leftarrow -R_1 + R_2 \tag{7}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 4 \\ 0 & 0 & 5 \end{bmatrix} \leftarrow R_1 + R_2 \tag{8}$$

the coefficients are

$$\begin{aligned} \lambda_1 &= -10 \\ \lambda_2 &= 6 \\ \lambda_3 &= 1 \end{aligned} \tag{9}$$