Consider two subspaces of $\mathbb{R}^4$:

$$U_1 = \text{span}\left(\begin{bmatrix}1\\1\\-3\\1\end{bmatrix}, \begin{bmatrix}2\\-1\\0\\-1\end{bmatrix}, \begin{bmatrix}-1\\1\\-1\\1\end{bmatrix}\right), U_2 = \text{span}\left(\begin{bmatrix}-1\\-2\\2\\1\end{bmatrix}, \begin{bmatrix}2\\-2\\0\\0\end{bmatrix}, \begin{bmatrix}-3\\6\\-2\\-1\end{bmatrix}\right) \tag{1}$$

Determine a basis of $U_1 \cap U_2$

we can use gaussian elimination and see which of these column vectors, from both subspaces, can be used as the basis

note that both aforementioned basis must be present in their intersection, however a subset of them they might be a linear combination

$$\begin{bmatrix}1 & 2 & -1 & -1 & 2 & -3\\ 1 & -1 & 1 & -2 & -2 & 6\\ -3 & 0 & -1 & 2 & 0 & -2\\ 1 & -1 & 1 & 1 & 0 & -1\end{bmatrix} \leftarrow -R_1 + R_3 \tag{2}$$

$$\begin{bmatrix}1 & 2 & -1 & -1 & 2 & -3\\ 1 & -1 & 1 & -2 & -2 & 6\\ -3 & 0 & -1 & 2 & 0 & -2\\ 0 & 0 & 0 & 3 & 2 & -7\end{bmatrix} \leftarrow 3R_1 + R_2 \tag{3}$$

$$\begin{bmatrix}1 & 2 & -1 & -1 & 2 & -3\\ 1 & -1 & 1 & -2 & -2 & 6\\ 0 & -3 & 2 & -4 & -6 & 16\\ 0 & 0 & 0 & 3 & 2 & -7\end{bmatrix} \leftarrow -R_1 + R_0 \tag{4}$$

$$\begin{bmatrix}0 & 3 & -2 & 1 & 4 & -9\\ 1 & -1 & 1 & -2 & -2 & 6\\ 0 & -3 & 2 & -4 & -6 & 16\\ 0 & 0 & 0 & 3 & 2 & -7\end{bmatrix} \leftarrow R_1 \text{ switch } R_0 \tag{5}$$

$$\begin{bmatrix}1 & -1 & 1 & -2 & -2 & 6\\ 0 & 3 & -2 & 1 & 4 & -9\\ 0 & -3 & 2 & -4 & -6 & 16\\ 0 & 0 & 0 & 3 & 2 & -7\end{bmatrix} \leftarrow R_1 + R_2 \tag{6}$$

$$\begin{bmatrix}1 & -1 & 1 & -2 & -2 & 6\\ 0 & 3 & -2 & 1 & 4 & -9\\ 0 & 0 & 0 & -3 & -2 & 7\\ 0 & 0 & 0 & 3 & 2 & -7\end{bmatrix} \leftarrow R_2 + R_3 \tag{7}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 & -2 & 6 \\ 0 & 3 & -2 & 1 & 4 & -9 \\ 0 & 0 & 0 & -3 & -2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow R_2 + R_3 \tag{8}$$

note that if we consider

$$\begin{aligned} \lambda_4 &= -1 \\ \lambda_5 &= -2 \end{aligned} \tag{9}$$

we obtain the last column vector, so a linear dependency is present, we can just remove two of these column vectors and then arrive at a generating set

same can be applied to the second column vector, is a linear combination of third and first column vector.

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$\boldsymbol{x_k} = \{\boldsymbol{x_1}, \boldsymbol{x_3}, \boldsymbol{x_4}\} \tag{11}$$

that's our basis