Let $n \in \mathbb{N}$ and let $x_1, ..., x_n > 0$ be n positive real numbers so that $x_1 + ... + x_n = 1$. Use the Cauchy-Schawrz inequality and show that

a. $\Sigma_i^n x_i^2 \geq \frac{1}{n}$

Remember the inquality states $| < x, y > | \leq \|x\| \ \|y\|$

remember that $\|\boldsymbol{x}\| \ \|\boldsymbol{x}\| > 1$ and $|\Sigma_i^n x_i^2| > 1$

$$|\Sigma_i^n x_i^2| \leq \|\boldsymbol{x}\| \ \|\boldsymbol{x}\|$$

$$|\Sigma_i^n x_i^2| \leq |\Sigma_i^n x_i^2| \ \|\boldsymbol{x}\| \ \|\boldsymbol{x}\|$$

$$\frac{|\Sigma_i^n x_i^2|}{\|\boldsymbol{x}\| \ \|\boldsymbol{x}\|} \leq |\Sigma_i^n x_i^2|$$

$$\frac{1}{\|\boldsymbol{x}\| \ \|\boldsymbol{x}\|} \leq |\Sigma_i^n x_i^2|$$

$$\frac{1}{n} \leq \Sigma_i^n x_i^2$$

and

$$|\Sigma_i^n x_i| \leq \|\boldsymbol{x}\| \Rightarrow n \leq \|\boldsymbol{x}\|^2$$

$$\frac{1}{n} > \|\boldsymbol{x}\|^2$$

$$\frac{1}{n} \leq \frac{1}{\|\boldsymbol{x}\|^2}$$

b. $\Sigma_i^n \frac{1}{x_i} \geq n^2$

note that $1 + ... + \frac{x_j}{x_i} = \frac{1}{x_i}$

and $1 + ... + \frac{x_j}{x_k} = \frac{1}{x_k}$

$$n + \Sigma_i^n x_i \left(\Sigma_{j \neq i}^{n-1} \left(\frac{1}{x_j} \right) \right) = \Sigma_i^n \left(\frac{1}{x_i} \right)$$

$$n + \Sigma_i^n x_i - \frac{1}{x_i} + \left(\frac{1}{x_i} + \left(\Sigma_{j \neq i}^{n-1} \left(\frac{1}{x_j} \right) \right) \right) = \Sigma_i^n \left(\frac{1}{x_i} \right)$$

$$n + \Sigma_i^n x_i - \Sigma_i^n \frac{1}{x_i} + \Sigma_i^n \frac{1}{x_i} = \Sigma_i^n \left(\frac{1}{x_i} \right)$$

$$2n = \Sigma_i^n \left(\frac{1}{x_i} \right)$$

$$n < \Sigma_i^n \left(\frac{1}{x_i} \right)$$

$$\Sigma_i^n x_i^2 n \geq 1$$

$$\Sigma_i^n x_i n \geq n$$

$$\Sigma_i^n 1 \geq n$$

$$\Sigma_i^n 1 n \geq n^2$$

$$\Sigma_i^n \frac{1}{x_i} \geq n^2$$