To establish chaos we will wish to develop a semi-conjugacy from some simpler shift-space. Since we have already established the orbit of $x = 1/2$ divides the interval into a finite number of sub-intervals and that applying the map to these intervals induce a system of F-coverings. [INSERT INFO ABOUT THE COVERING HERE]

We can then construct a string space, using an alphabet where each element in the alphabet is the name of one of the intervals. One can exclude a set of two cylinders by excluding all cylinders of the form $I_j I_k$ where $f(I_j)$ does not cover I_k. This then leads to an adjacency matrix A and a sub-shift of finite type Σ_A. If one could verify that (Σ_A, σ) contains a chaotic invariant set Ω then this suggests we could hopefully construct a semi-conjugacy between these spaces.

Assuming that (Σ_A, σ) is chaotic then we construct a function $\psi : \Sigma_A \to X$. For some $s \in \Sigma_A$, we search for $x_0 \in X$ such that for each $x_n = f^n(x_0) \in \mathcal{O}(x)$, $x_n \in \bar{I_n}$ where I_n is the interval in the n-th place of s. Lemma 20 guarantees that such a point exists and if it possible to show that this is unique then this produces a well-defined function.

This function is obviously a semi-conjugacy if it is well-defined and continuous, so given these assumptions then theorem 17 guarantees that $\psi(\Omega)$ that this is either a chaotic invariant set or a single periodic orbit. If there are distinct periodic sequences that do not belong to the same orbit in Σ_A then these s_0, s_1 must correspond to x_0, x_1 which are not part of the same orbit in X. If this is the case then $\psi(\Omega)$ must not be a single periodic orbit in X and so $\psi(\Omega)$ must be chaotic. Thus we have established our desired result.