The Brusselator: A Nonlinear Two-Variable ODE System

The *Brusselator* is a two-dimensional chemical-kinetics model that exhibits both stable equilibria and self-sustained oscillations depending on parameter values. It is defined by

$$\frac{dx}{dt} = A - (B+1)\,x + x^2 y,$$
$$\frac{dy}{dt} = B\,x - x^2 y,$$

with parameters $A > 0$, $B > 0$. For $A = 1$, there is a single equilibrium at $(x^*, y^*) = (1,\, B)$. A qualitative change occurs at $B = 2$: for $B < 2$ the equilibrium is stable; for $B > 2$ it is unstable and a stable limit cycle appears.

Instructions. Use Python and `scipy.integrate.solve_ivp`. Unless noted otherwise, take $A = 1$, integrate on $t \in [0, 50]$, and start from $(x_0, y_0) = (1.2,\, 2.8)$ (feel free to try others).

1. **Numerically integrate the system.**
   - Solve for two parameter choices: $B = 1.5$ (stable fixed point) and $B = 3.0$ (limit cycle).
   - Use the default method `RK45` to start (set tolerances as you see fit).
   - Plot $x(t)$ and $y(t)$ on the same time interval.
2. **Phase portraits.**
   - Plot the trajectory in the (x, y) plane for each B.
   - Mark the equilibrium $(x^*, y^*) = (1,\, B)$ and comment on the qualitative behavior: spiral-in for $B < 2$ vs. closed orbit for $B > 2$.

5. **Parameter sweep in B.**
   - Sweep B over a range, e.g., $1.2 \le B \le 3.5$ (a dozen values is plenty).
   - For each B, run long enough for transients to decay.
   - Decide whether the long-time behavior is equilibrium or sustained oscillation.
   - If oscillatory, estimate the *period* and an *amplitude* measure (e.g., peak-to-peak of $x(t)$). You may use `scipy.signal.find_peaks`.
   - Make a plot of period (and/or amplitude) vs. B to visualize the onset of the limit cycle.
6. **Method comparison and solver behavior.**
   - Re-run key cases with `method='RK45'` and `method='BDF'`.
   - For each run, record: number of internal steps, any solver messages, and qualitative accuracy of the trajectory.
   - Briefly discuss when the implicit method (BDF) is advantageous (e.g., larger B, longer horizons, sharper oscillations).