APPENDIX PRELIMINARIES

C-0. APPENDIX PRELIMINARIES

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{S^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right)$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}\left(\beta e^{-\beta t}-\alpha e^{-\alpha t}\right)$ $(a \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}\left(1-e^{-\alpha t}\right)$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left(1-e^{-\alpha t}-\alpha t e^{-\alpha t}\right)$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}\left(\alpha t-1+e^{-\alpha t}\right)$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{2}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$
$\frac{s}{(s+\alpha)^2}$	$(1-\alpha t)\,e^{-\alpha t}$

Laplace Transform $F(s)$	Time Function $f(t)$
$\frac{\omega_n}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\frac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\frac{\omega_n^2}{s\left(s^2+\omega_n^2\right)}$	$1-\cos \omega_n t$
$\frac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n \sqrt{\alpha^2+\omega_n^2} \sin\left(\omega_n t+\theta\right)$ where $\theta=\tan^{-1}\left(\omega_n / \alpha\right)$
$\frac{\omega_n}{(s+\alpha)\left(s^2+\omega_n^2\right)}$	$\frac{\omega_n}{\alpha^2+\omega_n^2} e^{-\alpha t}+\frac{1}{\sqrt{\alpha^2+\omega_n^2}} \sin\left(\omega_n t-\theta\right)$ where $\theta=\tan^{-1}\left(\omega_n / \alpha\right)$
$\frac{\omega_n^2}{s^2+2 \zeta \omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t \quad(\zeta<1)$
$\frac{\omega_n^2}{s\left(s^2+2 \zeta \omega_n s+\omega_n^2\right)}$	$1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2} t+\theta\right)$ where $\theta=\cos^{-1} \zeta \quad(\zeta<1)$
$\frac{s \omega_n^2}{s^2+2 \zeta \omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2} t-\theta\right)$ where $\theta=\cos^{-1} \zeta \quad(\zeta<1)$
$\frac{\omega_n^2(s+\alpha)}{s^2+2 \zeta \omega_n s+\omega_n^2}$	$\omega_n \sqrt{\frac{\alpha^2-2 \alpha \zeta \omega_n+\omega_n^2}{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2} t+\theta\right)$ where $\theta=\tan^{-1} \frac{\omega_n \sqrt{1-\zeta^2}}{\alpha-\zeta \omega_n} \quad(\zeta<1)$
$\frac{\omega_n^2}{s^2\left(s^2+2 \zeta \omega_n s+\omega_n^2\right)}$	$t-\frac{2 \zeta}{\omega_n}+\frac{1}{\omega_n \sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2} t+\theta\right)$ where $\theta=\cos^{-1}\left(2 \zeta^2-1\right) \quad(\zeta<1)$