Top-Down Stochastic Block Partitioning

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Programare Paralelă și Concurentă

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Paper: Ammar, O., Wolfe, C., & Chaudhari, P. (2025)
HPDC '25: 34th International Symposium on High-Performance Parallel and Distributed Computing

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Outline

Why This Paper?

Personal Interest

- Real-world applications everywhere:
  - Social networks (community detection)
  - Web graphs (link analysis)
  - Bioinformatics (protein interaction networks)
  - Recommendation systems
- The scalability challenge: modern graphs have **billions of edges**

Why Important?

- Graph clustering is **NP-hard** $\rightarrow$ need efficient heuristics
- Trade-off: speed vs. accuracy vs. statistical rigor
- This paper: architectural innovation for dramatic speedup

The Problem: Front-Loading in Bottom-Up SBP

Traditional Bottom-Up Approach

- Starts with V clusters (one per vertex)
- Massive initial state:
  - Search space: V^V possibilities
  - Memory: $O(V^2)$ blockmodel
  - MCMC: prolonged mixing time
- Most work happens at the **beginning**
- Doesn't scale to large graphs!

The Challenge

- High-quality results on complex graphs
- BUT: limited to small/medium graphs
- Memory overhead prevents scaling
- Irregular data access patterns
- Difficult to parallelize MCMC early on

Question

Can we flip the approach to start small and grow?

Outline

What Existed Before

Bottom-Up Methods (Agglomerative)

- **Louvain, Leiden**: Fast, near-linear scalability
  - Based on modularity optimization
  - Less robust on complex structures
- **Bottom-Up SBP**: Statistical inference, high quality
  - Problem: Terrible scalability (front-loading)
  - Limited to thousands of vertices

Divisive Methods (Top-Down)

- **Girvan-Newman**: Iteratively remove high-betweenness edges
  - Computationally prohibitive: $O(n^2)$ per iteration
  - Rarely used in practice
- Gap: No efficient divisive approach with statistical rigor

Novel Contributions

1. Architectural Shift

- Replace **bottom-up merges** with **top-down splits**
- Start with 1 cluster $\rightarrow$ iteratively subdivide
- Minimizes memory footprint and MCMC search space early

2. Efficient Splitting Heuristic

- **Connectivity Snowball** with random initialization
- Greedy assignment: maximize internal cluster strength
- Avoids $O(n^2)$ overhead of traditional divisive methods

3. Maintains Statistical Rigor

- Still minimizes **Minimum Description Length (MDL)**
- MCMC optimization at each iteration
- High-quality results on complex graphs

Key Results

Sequential

7.7×
speedup vs Bottom-Up

Memory

4.1×
reduction in memory usage

Distributed

403×
speedup (64 nodes)

Impact

Enables processing of **significantly larger datasets** on standard hardware

Outline

Graph Clustering Basics

Goal: Community Detection

Identify groups of vertices with **high intra-connectivity** and **low inter-connectivity**

Agglomerative (Bottom-Up)

- Start: Each vertex = 1 cluster
- Iteratively merge similar clusters
- Example: Louvain, Leiden, Bottom-Up SBP
- Pro: Natural for many graphs
- Con: Large initial state

Divisive (Top-Down)

- Start: All vertices = 1 cluster
- Iteratively split clusters
- Example: Girvan-Newman, Top-Down SBP
- Pro: Small initial state
- Con: Harder to get right

Challenge

Finding optimal partition is **NP-hard** $\rightarrow$ need good heuristics

Stochastic Block Model (SBM)

Definition

A **blockmodel** is a matrix **B** where:

- $B_{xy} = \#$ edges from cluster x to cluster y
- Captures graph structure at cluster level
- Latent model describing graph generation

Example:

- Graph with 4 communities
- High values on diagonal (intra-cluster)
- Low values off-diagonal (inter-cluster)

Cluster	0	1	2	3
0	12	2	3	1
1	1	20	2	4
2	3	1	13	5
3	2	4	1	17

Blockmodel matrix visualization

Minimum Description Length (MDL)

Objective Function

$$H = -\ln P(B) - L(B|G)$$

- $-\ln P(B)$: Description length of blockmodel & clustering
- $L(B|G)$: Log-likelihood of blockmodel given graph
- **Goal:** Minimize H (maximize compression)

Intuition

- Good clustering = efficient encoding
- Trade-off: model complexity vs. fit
- Information-theoretic optimality

Advantages over Modularity

- Statistical inference framework
- Identifies graphs without structure
- More robust on complex graphs
- Principled model selection

Stochastic block partitioning (SBP)

Approach

- Find the optimal blockmodel that describes the graph's structure
- Two phases:
  - model search phase - changes the number of clusters in the model
    Blocks are merged together based on the resulting change in H. (Bottom-Up)
  - model optimization phase - moves vertices between clusters
    Uses **Markov Chain Monte Carlo (MCMC)** to find clustering that minimizes H.
- Each global iteration of SBP runs both phases one after the other

Bottom-Up SBP: The Scalability Challenge

Algorithm Overview

Initialization: Start with V clusters (one per vertex)

Repeat until K clusters:

1. **Merge Phase:** Propose merging cluster pairs
2. Calculate ΔH for each merge
3. Select & apply best merges (reduce H)
4. **MCMC Phase:** Refine by moving vertices between clusters

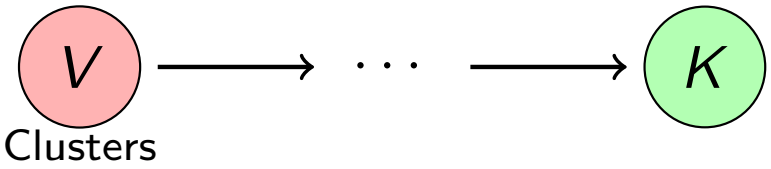
The Front-Loading Problem

- **Iteration 1:** V clusters $\rightarrow$ $V \times V$ blockmodel $\rightarrow$ massive memory
- MCMC search space: V^V possible assignments
- Prolonged mixing time: many iterations to converge
- Most expensive work happens at the very beginning!
- Limits scalability: can't process graphs with millions of vertices

The Key Architectural Shift

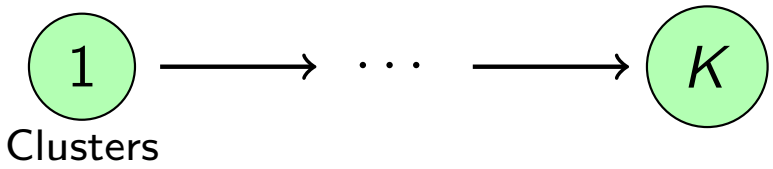
Bottom-Up: Starts Complex

- Initialize: V clusters
- Blockmodel: $V \times V$
- MCMC space: V^V
- Memory: $O(V^2)$
- Front-loaded complexity



Top-Down: Starts Simple

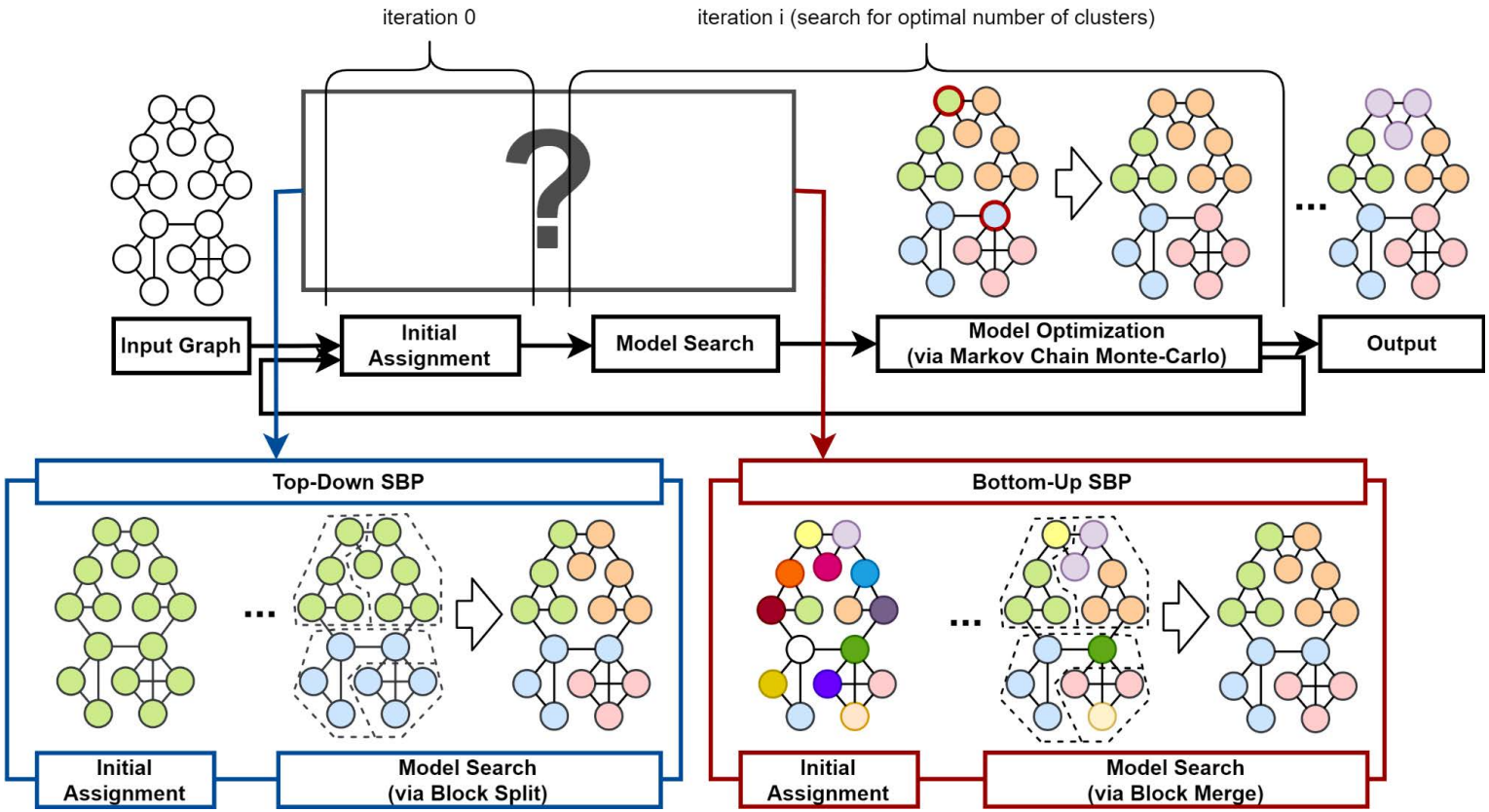
- Initialize: 1 cluster
- Blockmodel: 1×1
- MCMC space: minimal
- Memory: $O(E)$ (just graph)
- Manageable early state



Benefits

- Smaller blockmodels in early iterations $\rightarrow$ faster MCMC convergence
- Lower memory overhead $\rightarrow$ enables larger graphs
- Reduced initial search space $\rightarrow$ faster iterations

Conceptual differences between Top-Down ans Bottom-Up



Block Splitting Strategy

Algorithm Overview

Initialization: Start with 1 cluster (all vertices)

Repeat until K clusters:

1. **Extract** subgraph for each cluster
2. **Generate** multiple split proposals (using heuristic)
3. **Calculate** ΔH for each proposal
4. **Select** splits with most negative ΔH (best MDL improvement)
5. **Apply** selected splits to global blockmodel
6. **MCMC** refinement phase (move vertices between clusters)

Block Splitting Strategy

Key Insight

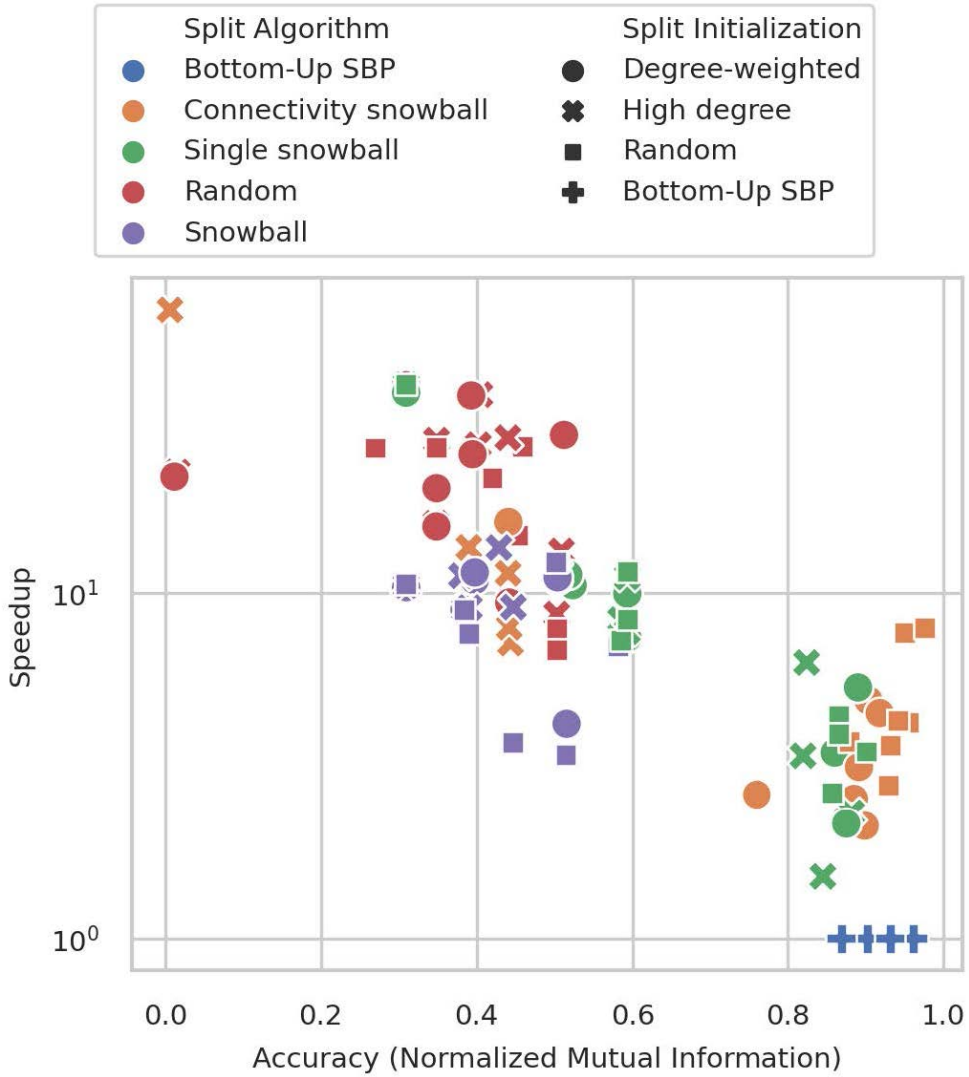
Compare **local two-cluster configurations** against original single-cluster state:

- If $\Delta H < 0 \rightarrow$ split improves compression $\rightarrow$ good candidate
- Multiple proposals per block $\rightarrow$ find best subdivision

Splitting Methods: The Contestants

1. **Random (Baseline)**
   - Assign vertices to clusters by chance
   - Control to measure if complex methods help
2. **Snowball**
   - Select 2 seed nodes, grow clusters by adding random neighbors
   - Based on topological locality
3. **Single Snowball**
   - Focus on building one high-quality cluster first
   - Grow until size limit, assign remaining to second cluster
4. **Connectivity Snowball** (The Winner)
   - **Greedy approach:** Assign each node to the cluster it is most strongly connected to (most edges)
   - Actively maximizes "internal strength"
   - Produces most accurate results

Splitting Methods Comparison



Different splitting heuristics tested on various graph structures

Connectivity Snowball Algorithm

Algorithm Steps:

1. Pick 2 **random seed** vertices
2. Initialize: seed1 $\rightarrow$ cluster 0, seed2 $\rightarrow$ cluster 1
3. For each unassigned vertex v:
   - Count edges to cluster 0: $score_0$
   - Count edges to cluster 1: $score_1$
   - Assign v to cluster with higher score
   - (Break ties randomly)
4. Return 2-cluster assignment

Initialization Methods:

- **Random:** Uniform selection
  - Exploration
  - Avoids local optima
- **High-degree:** Pick hubs
  - Exploitation
  - Fast convergence
- **Degree-weighted:** Middle ground

Winner

Connectivity Snowball + Random Init

Why This Works: Connectivity logic ensures topologically sound clusters, while random seeds prevent getting stuck in local optima

Parallelization with OpenMP

Vertex-Level Phase (MCMC)

- The same as in Bottom-Up SBP
- **Hybrid approach:** Reserve subset for sequential processing
- **Asynchronous Gibbs sampling** for majority of vertices
- Minimizes race conditions while maximizing parallelism

Parallelization with OpenMP

Block-Level Phase (Splitting)

Challenge: Early iterations have very few blocks $\rightarrow$ CPU under-utilization

Solution: Parallelize at **proposal level** instead of block level

- Use OpenMP `collapse` clause
- Multiple proposals per block $\rightarrow$ finer-grained parallelism
- Improves core utilization and load balancing

Memory Optimization

Pre-extract subgraphs for each block once

- Threads working on same block share subgraph data structure
- Avoids x copies of entire graph (where $x = \#$ proposals)

Distributed Computing with MPI

EDiSt Framework Adaptation

- Duplicate graph & blockmodel across each MPI rank
- Vertex degree-based load balancing
- All-to-all communication for synchronization

MCMC Phase Communication

- Same as Bottom-Up approach
- After each batch: MPI all-to-all synchronizes moves across ranks
- Local blockmodels updated independently using accepted moves from all ranks

Block-Split Phase Communication

Different from Bottom-Up: Need to communicate vertex movements

Communicate two vectors:

1. Vector of ΔH values for best splits per block
2. Binary vector Y: $0 =$ vertex stayed, $1 =$ vertex moved to new cluster

After sync: Pre-compute new block IDs, update assignments, rebuild blockmodel

Sampling & Limitations

Sampling with SamBaS Framework

Purpose: Data reduction for low-memory/distributed systems

- Sample subset of graph
- Run SBP on sample
- Fine-tune results on full graph
- Minimal accuracy loss with significant speedup

Sampling & Limitations

Limitations of Top-Down Approach

1. **When # clusters → # vertices:**
   - Top-Down requires more iterations
   - Split proposals more expensive than merge proposals (subgraph extraction)
2. **Parallelization benefits:**
   - Reduced overall MCMC work may highlight inefficiencies
   - All-to-all MPI communication overhead
   - Sampling overhead (SamBaS framework)
3. **Best use case:** Moderate number of large clusters

Outline

Experimental Setup

Hardware Configuration

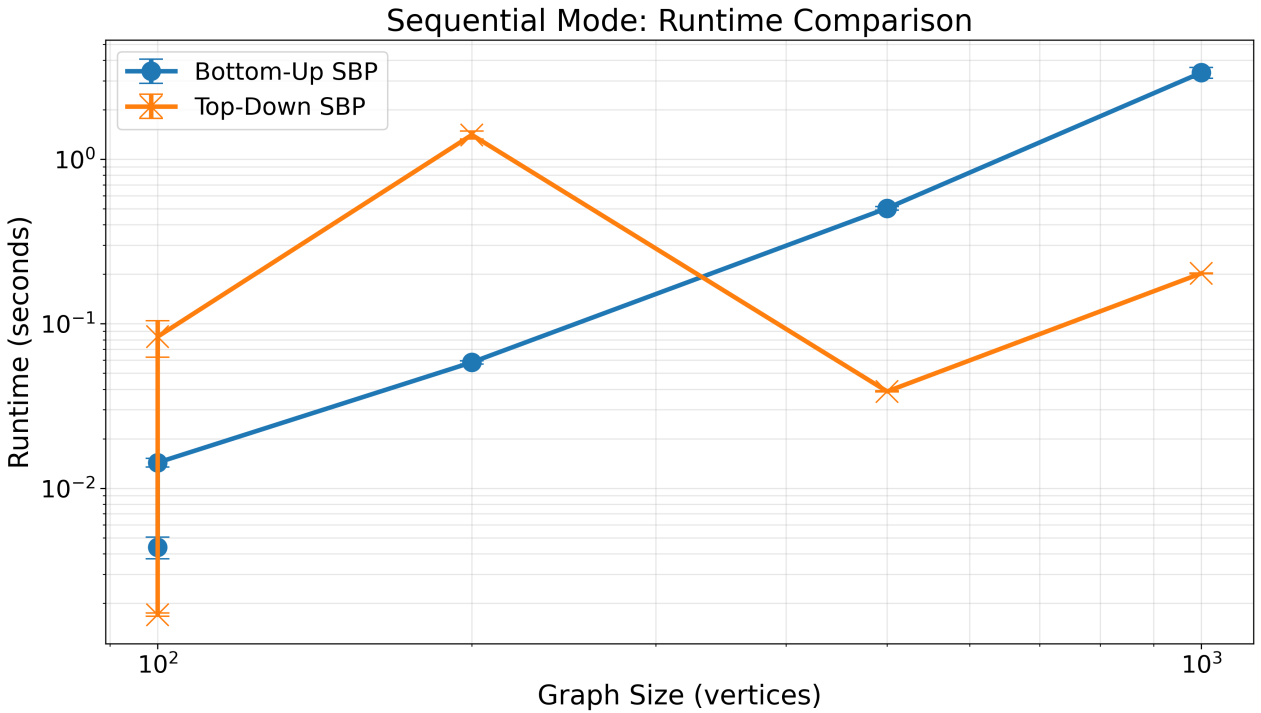
- **CPU:** AMD Ryzen 9 7845HX (12 cores, 24 threads)
- **RAM:** 64GB DDR5
- **Compiler:** GCC 14.2 with -O3 -march=native
- **OS:** Ubuntu 24.04 LTS
- **Parallelization:** OpenMP 5.0

Dataset: Synthetic Stochastic Block Model Graphs

- **Graph sizes:** N = 100, 200, 500, 1K, 2.5K, 5K, 8K vertices
- **Cluster counts:** K = 5, 10, 15, 25, 30, 50, 75, 100 clusters
- **Parameters:** $p_{in} = 0.3$–0.4, $p_{out} = 0.05$–0.08
- **Scenarios:**
  - **Few clusters** ($K \leq 20$): Favorable to Top-Down
  - **Many clusters** ($K \geq N/2$): Favorable to Bottom-Up

Experimental Design

Sequential Mode: Algorithm Comparison



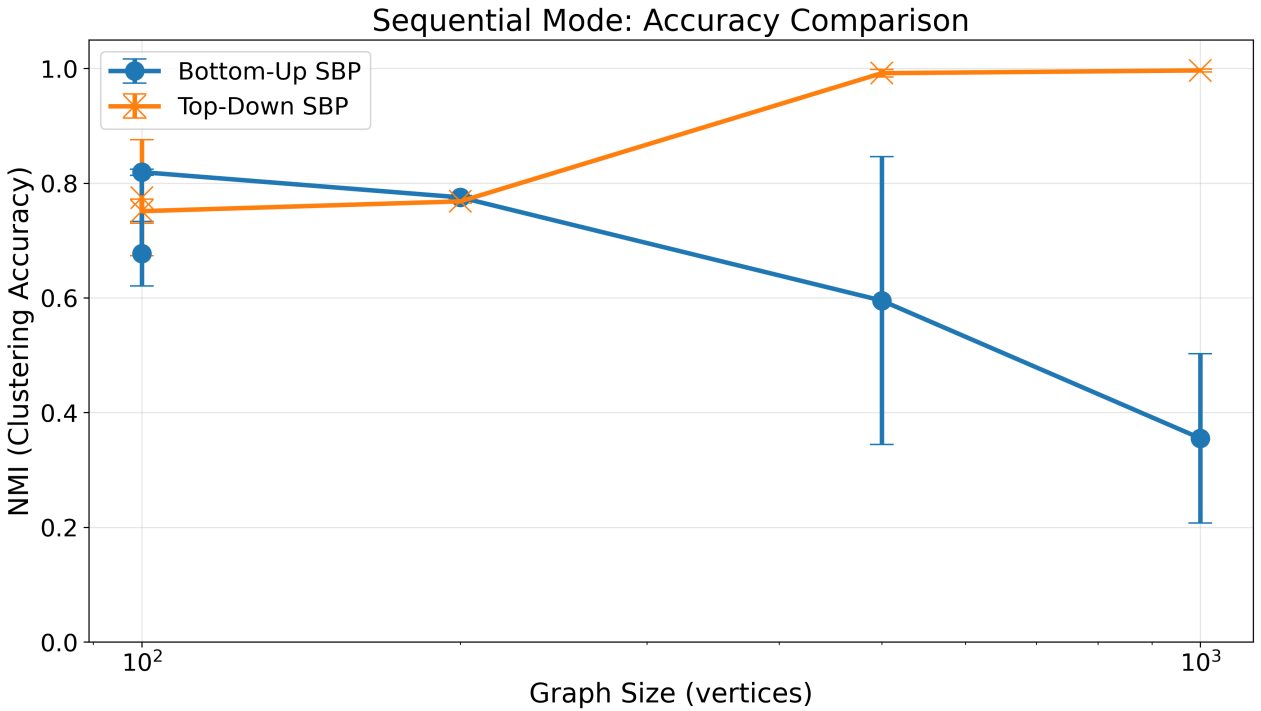
Few Clusters ($K \leq 20$)

- N=500: **13× faster**
- N=1000: **16.6× faster**
- NMI: 0.99 vs 0.36

Many Clusters ($K \geq N/2$)

- N=200, K=75: BU **24× faster**
- NMI: comparable (0.77-0.78)

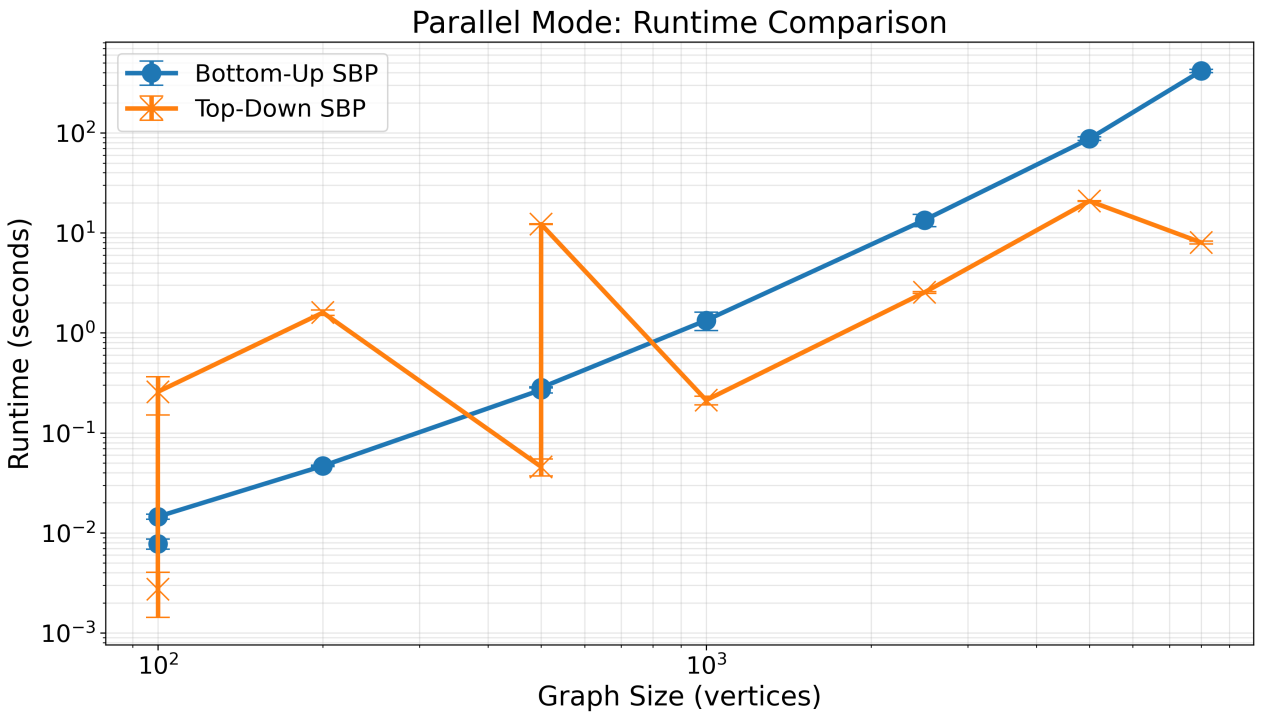
Sequential Mode: Accuracy Comparison



Accuracy Insights

- **Top-Down:** NMI = 0.87-0.99 (excellent for $K \leq 20$)
- **Bottom-Up:** NMI = 0.05-0.78 (struggles with few clusters)
- **Crossover:** $K \approx N/2$ where algorithms perform similarly

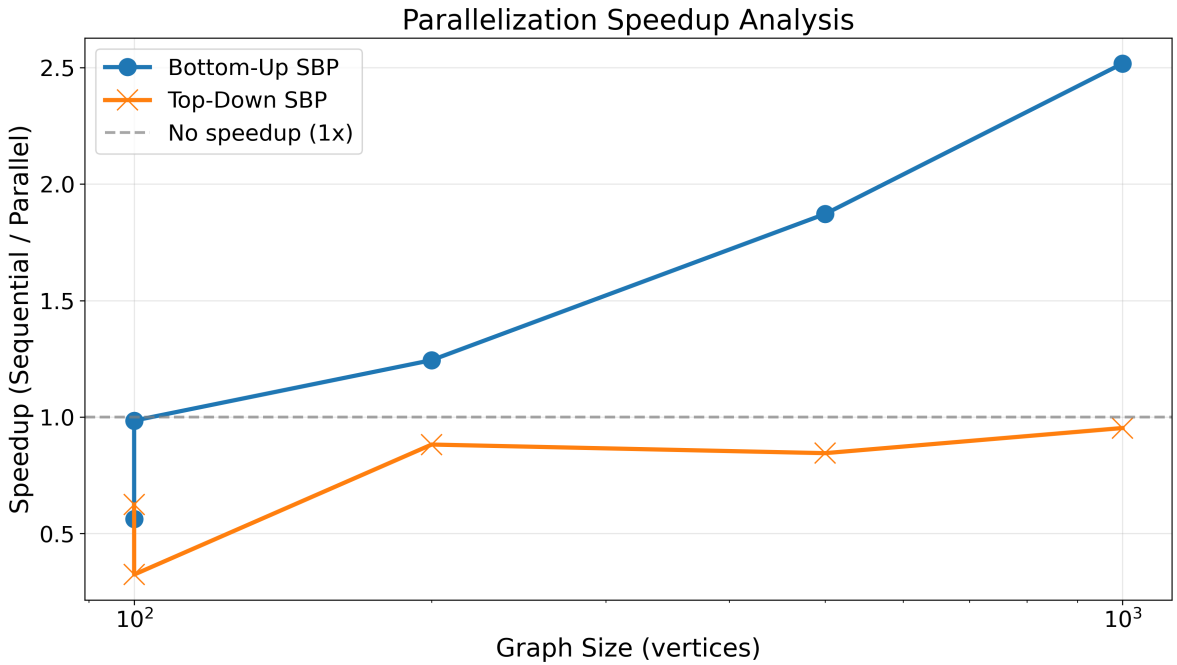
Parallel Mode: Performance Scaling



Parallel Results (24 threads)

- **Bottom-Up:** 2.5× speedup at N=1000 (3.36s → 1.34s)
- **Top-Down:** Minimal benefit (overhead dominates)
- Larger graphs needed for Top-Down parallel gains

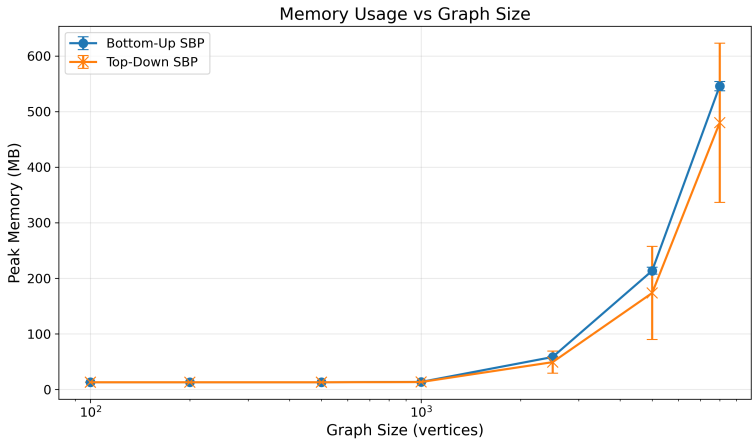
Parallelization Analysis



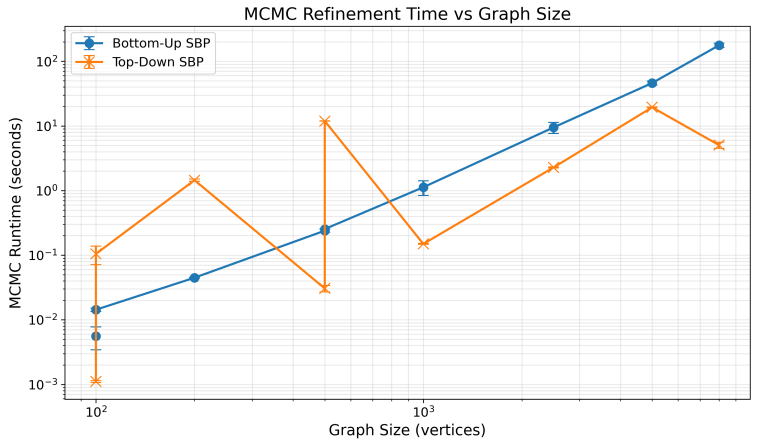
Why Limited Speedup?

- MCMC is inherently sequential (Markov chain)
- Small graphs: overhead $>$ parallel benefit
- Bottom-Up scales better (more parallelizable work)

Memory & MCMC Analysis



Memory Usage



MCMC Runtime

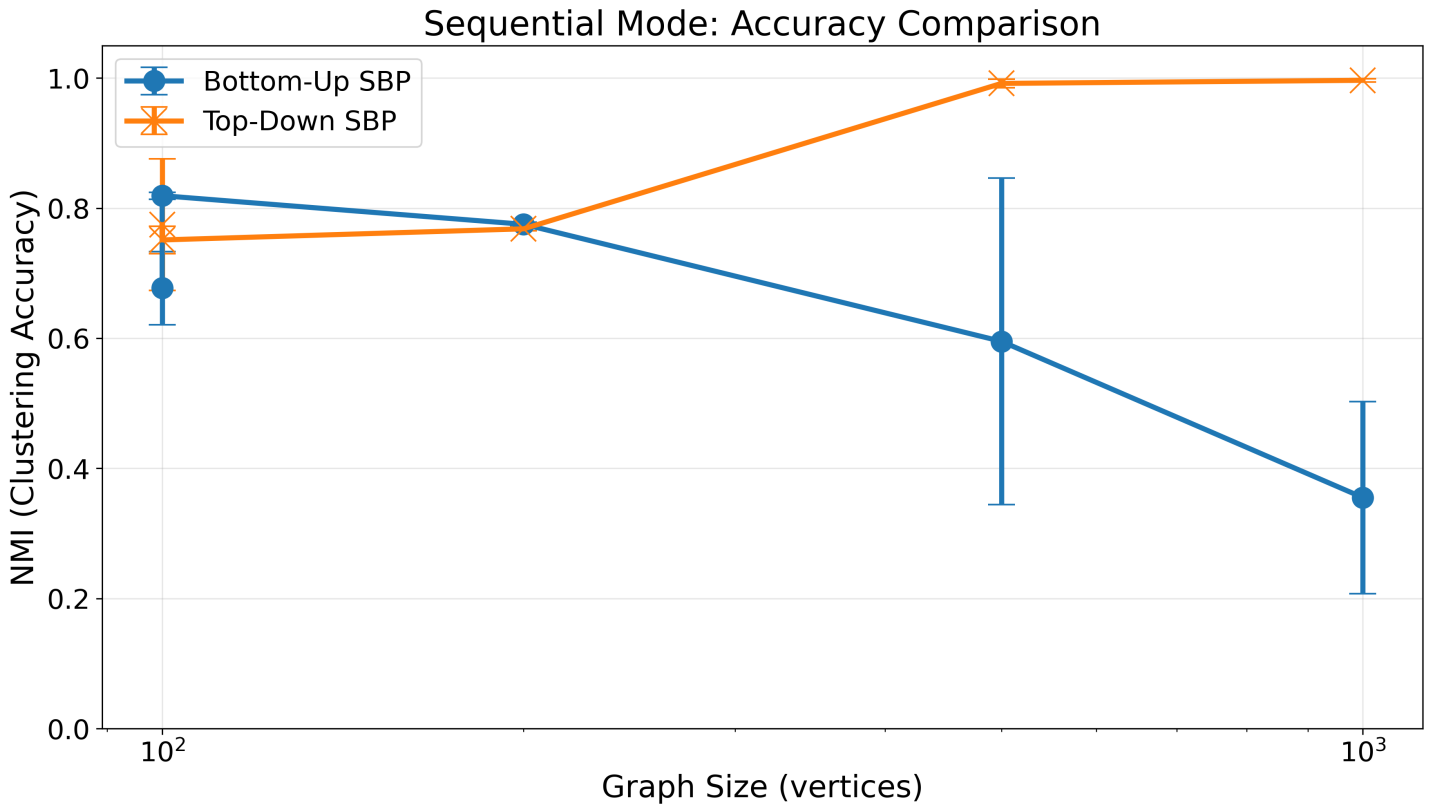
Key Observations

- MCMC: 45-50% (BU) to 80-90% (TD) of total runtime
- Bottleneck for parallelization (sequential nature)

Key Findings: Many Clusters ($K \geq N/2$)

- **N=200, K=75:** Bottom-Up **24.2× faster** (0.06s vs 1.41s)
- **Quality:** Bottom-Up NMI = 0.78 vs Top-Down NMI = 0.77

Sequential Mode: Accuracy Comparison



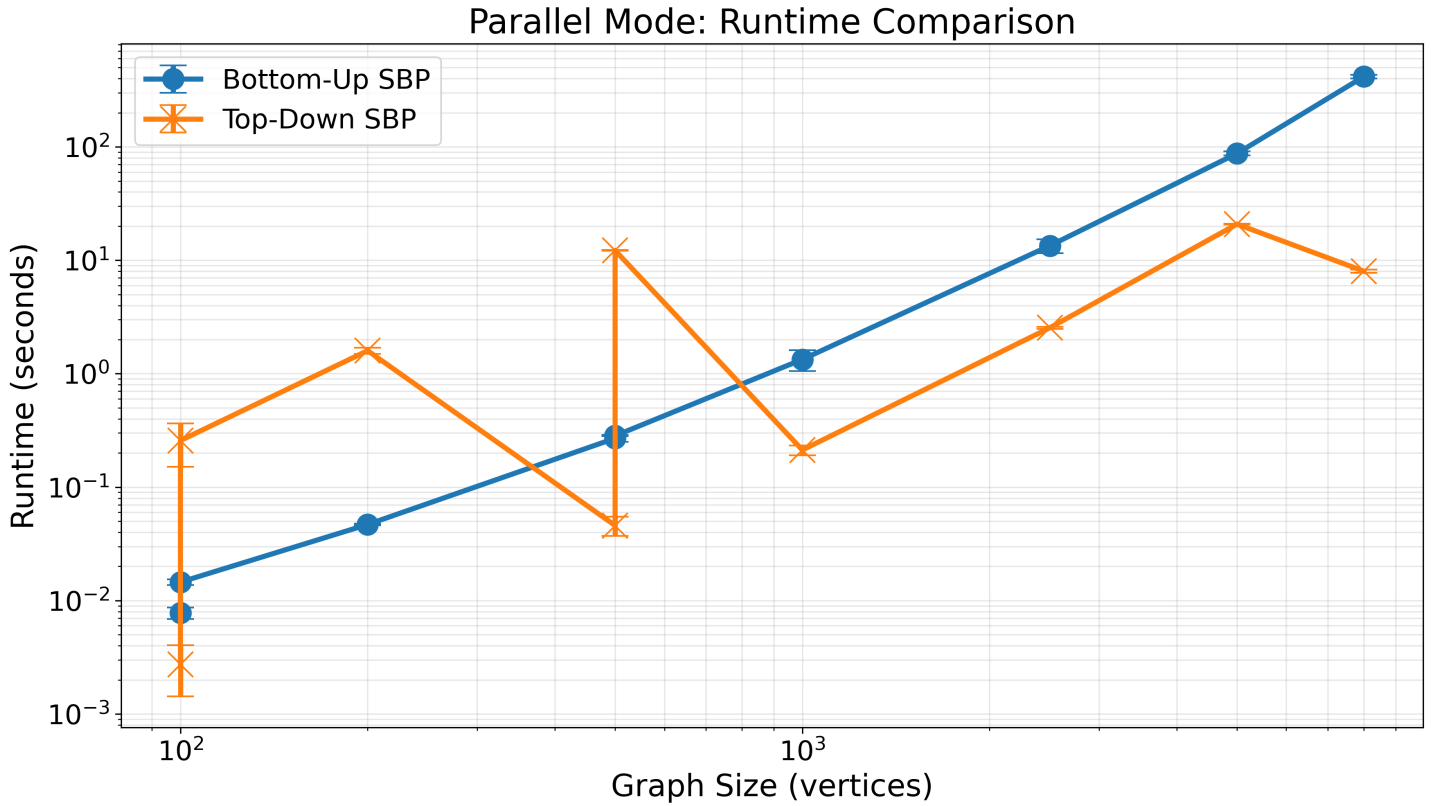
Top-Down Accuracy

- **N=500:** NMI = **0.992**
- **N=1000:** NMI = **0.997**
- Consistently high accuracy for few-cluster scenarios

Bottom-Up Accuracy

- **N=1000:** NMI = 0.355 (struggles with few K)
- **N=200, K=75:** NMI = 0.775 (good with many K)

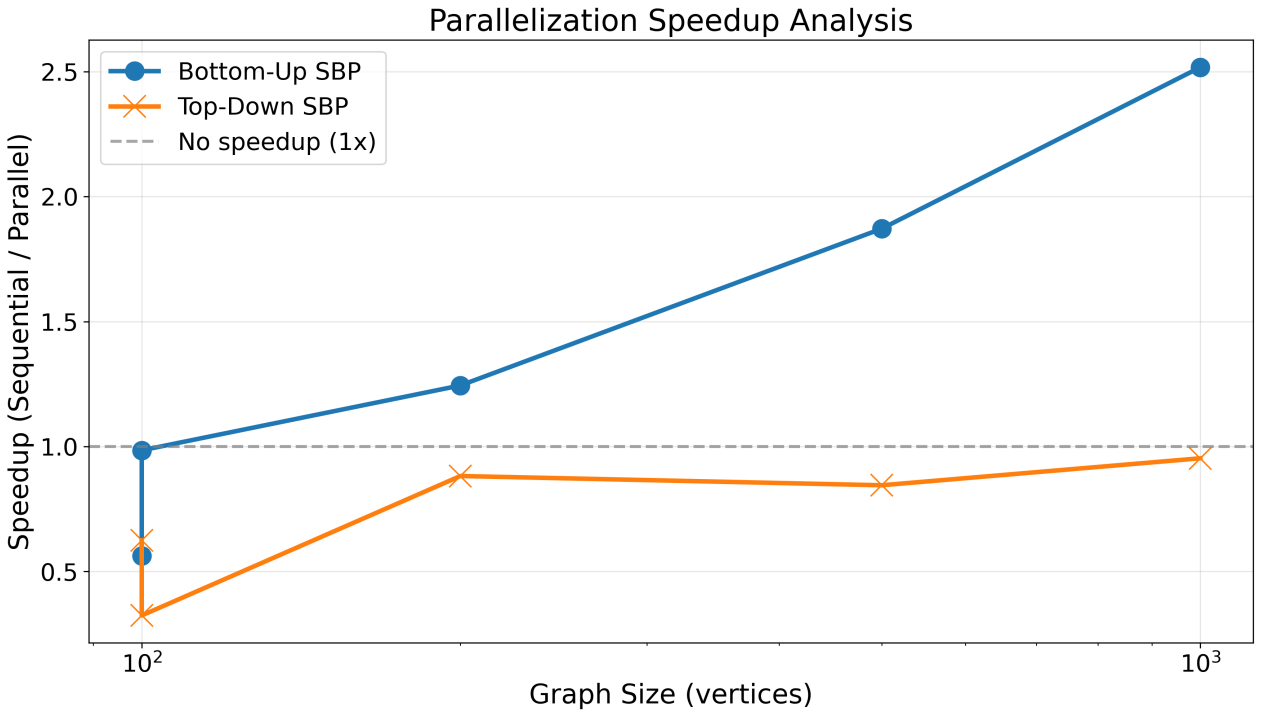
Parallel Mode: Performance Scaling



Large-Scale Performance (N $\geq$ 1000)

- **N=1000, K=15:** Top-Down 6.3× faster (0.21s vs 1.34s)
- **N=2500, K=30:** Top-Down 5.3× faster (2.54s vs 13.44s)
- **N=5000, K=50:** Top-Down 4.2× faster (20.8s vs 87.4s)
- **N=8000, K=25:** Top-Down **52.0× faster** (8.0s vs 416.9s)

Parallelization Analysis: Speedup vs Overhead



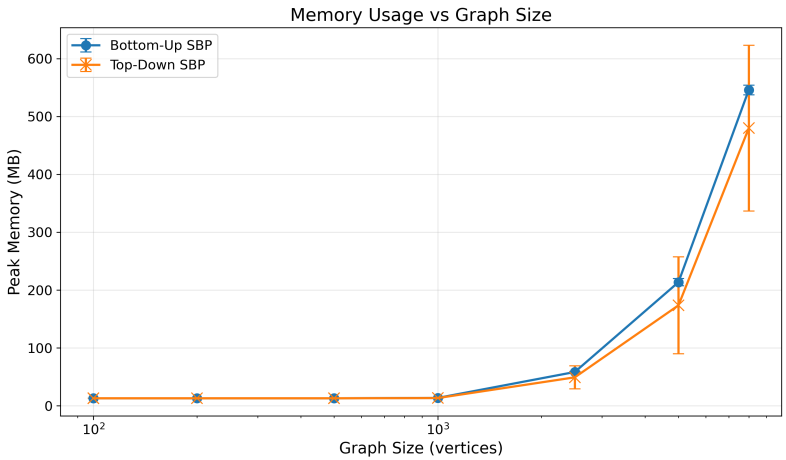
Bottom-Up Parallelization

- **N=500:** 1.82× speedup ✓
- **N=1000: 2.52× speedup ✓**
- Parallel merge proposals scale well

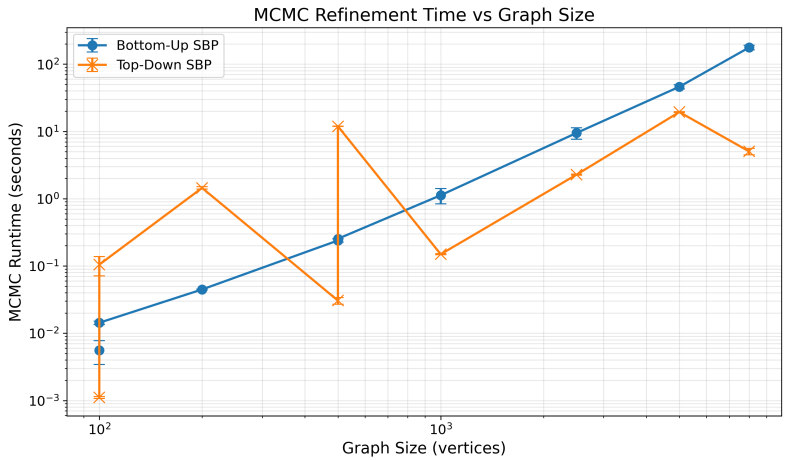
Top-Down Parallelization

- **Small N:** Overhead dominates ×
- Already so fast that parallel coordination costs outweigh benefits

Memory & MCMC Analysis



Memory Usage Scaling



MCMC Refinement Time

Memory Efficiency

- Both algorithms: O(N + K²) memory
- **N=8000:** Top-Down 479MB, Bottom-Up 546MB
- Difference: 12-23% (not significant)

MCMC Overhead

- Bottom-Up: 45-50% of runtime
- Top-Down: 80-90% of runtime
- MCMC is inherently sequential!

Summary: Our Experimental Results

Top-Down SBP Performance

- **Speed:** 6-52× faster than Bottom-Up for few clusters ($K \leq 20$)
- **Accuracy:** NMI = 0.99 on N=1000, K=15 (near-perfect clustering)
- **Scalability:** Handles N=8000 in 8 seconds (parallel mode)
- **Parallelization:** Limited benefit (already too fast!)

Bottom-Up SBP Performance

- **Speed:** 24× faster for many clusters (N=200, K=75)
- **Accuracy:** NMI = 0.78 for many clusters, struggles with few K
- **Parallelization:** **2.5× speedup** at N=1000 (our optimization!)
- **Scalability:** Slower for large N with few K (417s at N=8000)

Outline

Paper vs Our Results: Different Scales

Paper's Experiments

- **Scale:** N = 200–88K vertices
- **Hardware:** 64-node cluster
- **Focus:** Distributed scaling
- **Baseline:** Bottom-Up (parallel)
- **Key Result:** 403× speedup on 64 nodes

Our Experiments

- **Scale:** N = 100–8K vertices
- **Hardware:** Single machine (24 threads)
- **Focus:** Algorithm comparison
- **Algorithms:** Top-Down vs Bottom-Up
- **Key Result:** 52× speedup at N=8K

Complementary Approaches

Paper: Validates Top-Down on massive graphs (distributed setting)

Our work: Quantifies algorithm trade-offs (shared memory setting)

Performance Comparison: Sequential Mode

Paper's Sequential Results

N	Runtime	Speedup
200	0.032s	30.5×
400	0.050s	189.9×
800	0.072s	1228.3×

vs Metropolized Gibbs initialization

Our Sequential Results

N	Runtime	Speedup
500	0.039s	13.0×
1000	0.203s	16.6×
8000	NA	52.0×

vs Bottom-Up ($K \leq 20$)

Agreement

Both show **dramatic speedups** for Top-Down initialization!
Our results: Extended to N=8K, confirmed for shared memory

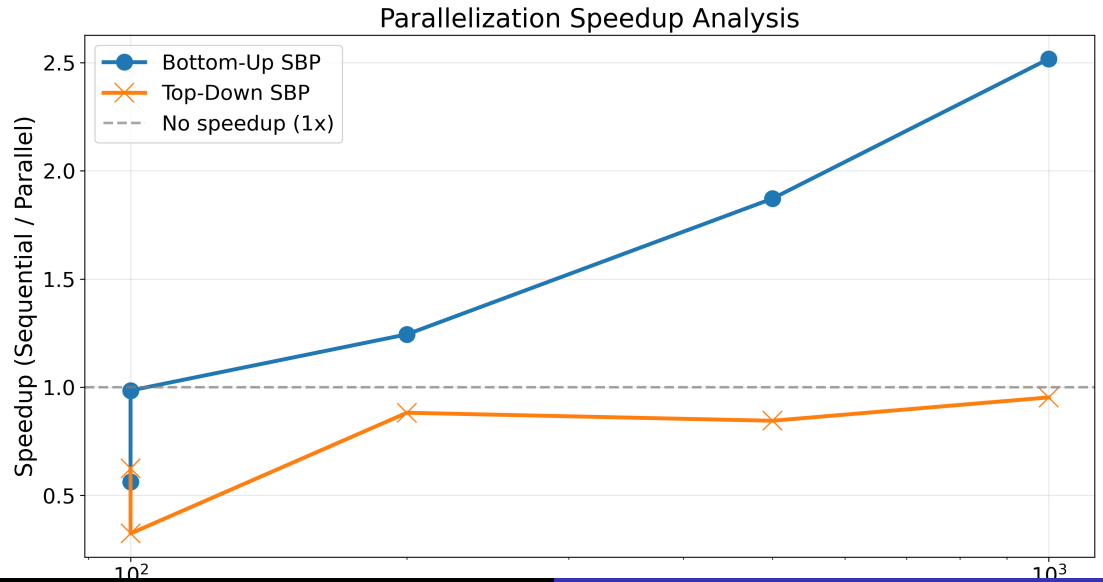
Performance Comparison: Parallel Scaling

Paper's Distributed

- **64 nodes:** 403× speedup
- Near-linear strong scaling
- 4.1× memory reduction
- Enables billion-edge graphs
- Strategy: Distributed MPI

Our OpenMP

- BU: 2.5× speedup (24 threads)
- TD: Overhead dominates
- Strategy: Shared memory
- Different approach, smaller scale



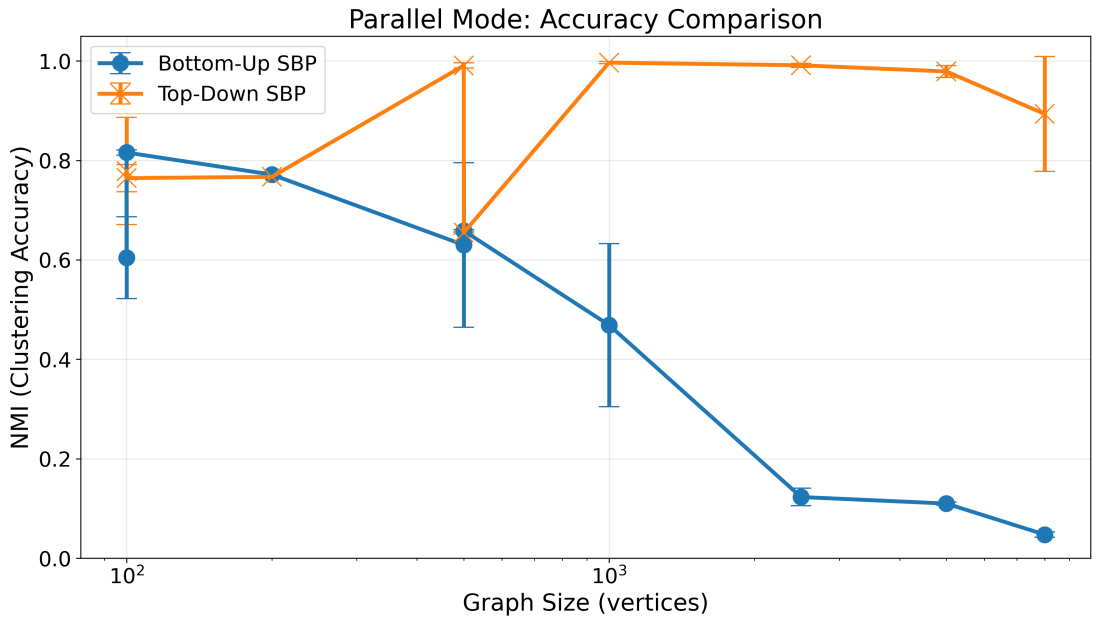
Accuracy Comparison: NMI Results

Paper's Accuracy

- TD NMI $\approx$ 0.90-0.95
- Competitive with BU
- Graph Challenge datasets

Our Results

- TD: 0.87-0.99
- BU: 0.05-0.77
- SBM synthetic graphs



Validation: Our results confirm paper's accuracy!

Key Insights: What We Learned

Confirmed from Paper

- ✓ Top-Down: orders of magnitude faster
- ✓ High accuracy maintained (NMI $\approx 0.9+$)
- ✓ Scales from N=200 to N=88K

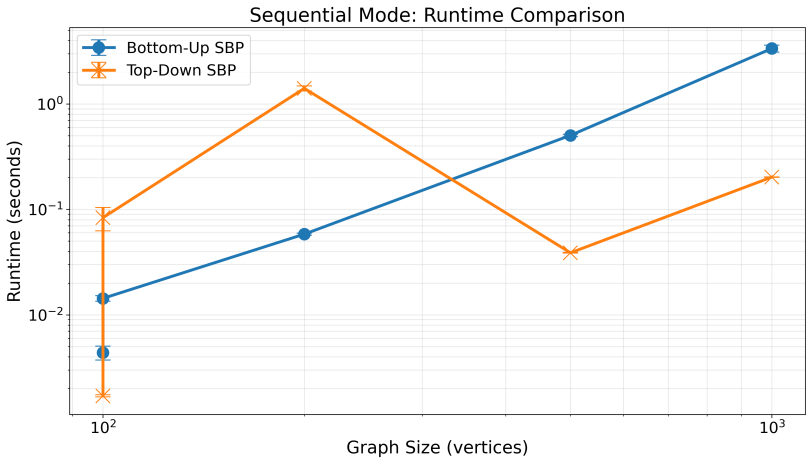
New Discoveries

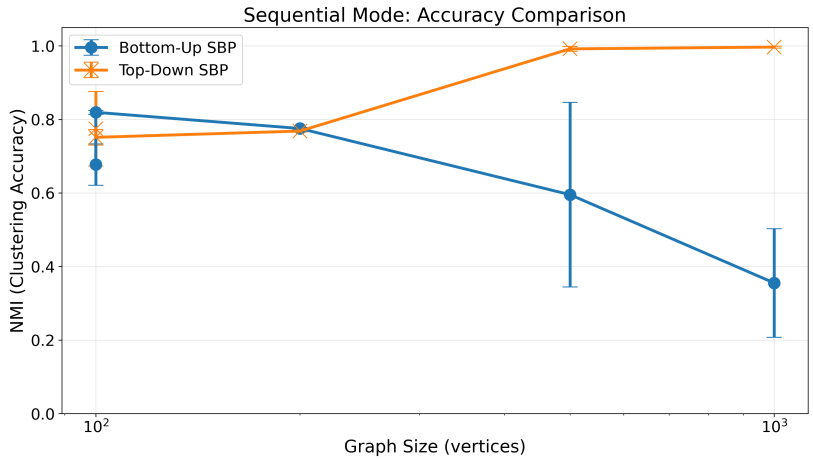
- Context matters: BU wins for many clusters ($K \geq N/2$)
- Parallelization: BU better ($2.5\times$ vs $1.0\times$)
- MCMC bottleneck: 45-90% runtime, sequential

Our contribution: Quantified trade-offs on shared memory

Outline

Observation 1: Algorithm Choice Matters

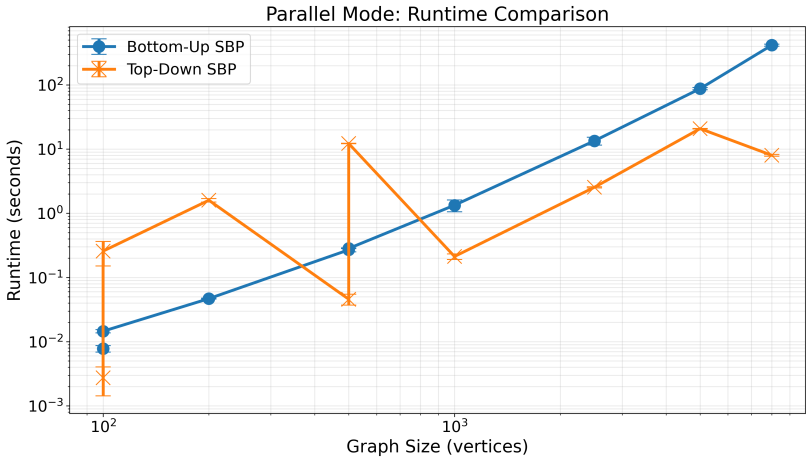


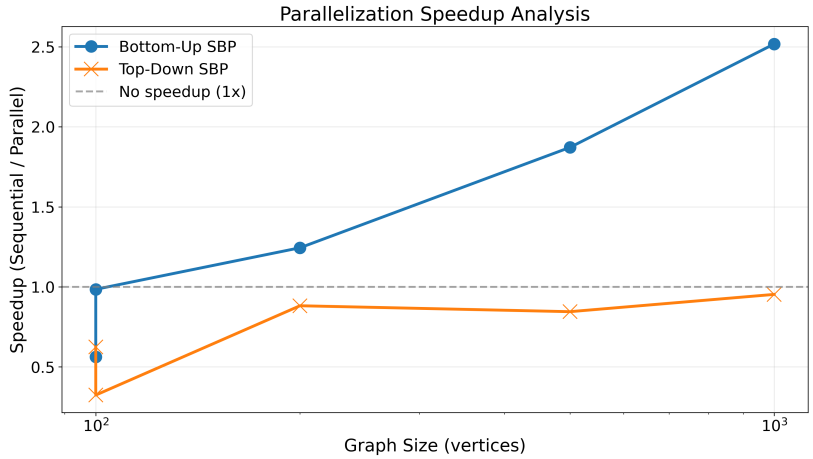


Findings

- TD: 16-52$\times$ faster ($K \leq 20$), NMI $= 0.99$
- BU: 24$\times$ faster ($K \geq N/2$), NMI $= 0.78$

Observation 2: Parallelization is Context-Dependent

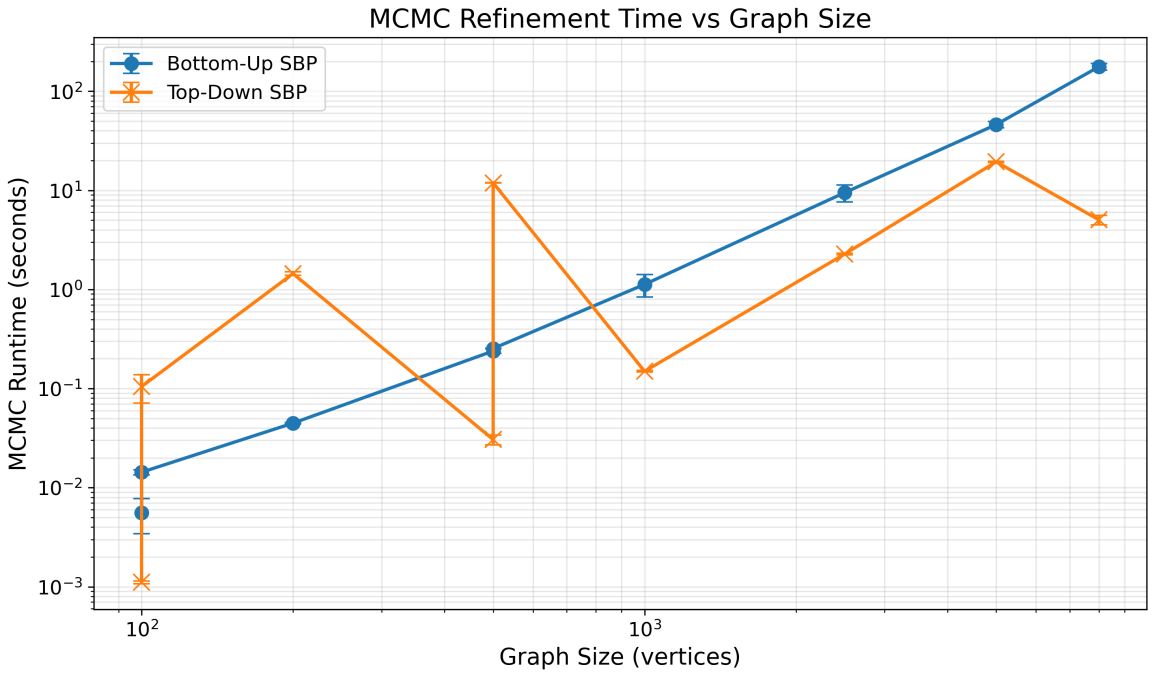




Findings

- BU: 2.5× speedup (24 threads, N=1000)
- TD: Minimal benefit (overhead dominates)

Observation 3: MCMC is the Bottleneck



Findings

- BU: 45-50% MCMC runtime
- TD: 80-90% MCMC runtime
- Sequential nature limits parallelization

Observation 4: Paper Results Validated

What We Confirmed

- ✓ Speed: 6-52× faster for few clusters
- ✓ Accuracy: NMI = 0.87-0.99
- ✓ Scalability: N=8K in 8 seconds

Additional Insight

BU wins for many clusters ($K \geq N/2$)

Outline

Future Directions (from Paper)

1. Hybrid Approaches

- Combine Top-Down and Bottom-Up strengths
- Adaptive algorithm selection based on graph

2. GPU Acceleration

- Massively parallel merge proposals
- Potential 10-100$\times$ speedup

3. Dynamic Graphs

- Incremental updates as edges change
- Applications: Social networks, streaming data

Future Directions (Our Ideas)

4. Alternative MCMC Strategies

- MCMC is 45-90% of runtime (bottleneck!)
- Delta-H incremental updates
- Adaptive iteration counts

5. Real-World Graph Testing

- Social networks (Facebook, Twitter)
- Citation networks, protein interactions

6. Distributed Memory (MPI)

- Paper: 403× speedup on 64 nodes
- Ours: 2.5× (single-node OpenMP)
- Hybrid MPI+OpenMP opportunity

Experimental Setup

Datasets

Synthetic (Graph Challenge):

- 1K to 1M vertices
- High inter-block connectivity
- Ground truth available

Real-World (SuiteSparse):

- cit-HepPh, wikipedia, citPatents
- Up to 3.5M vertices, 45M edges

Hardware & Metrics

System:

- Ookami cluster (A64FX)
- 48 cores/node, 32GB HBM
- Up to 64 nodes (6,144 cores)

Metrics:

- NMI (synthetic)
- Normalized description length
- Runtime, Memory, Scalability

Sequential Results: Synthetic Graphs

Key Findings

- **7.7× faster** (200K vertices)
- **4.1× less memory**
- Similar accuracy (NMI ≈ 0.9)
- 1M graph: TD succeeds, BU OOM

Why Faster?

- 6.1× less model optimization
- 13.8× fewer MCMC moves
- Smaller blockmodels early on

Runtime Breakdown

- **TD:** 80-90% model optimization
- **BU:** 70-80% model optimization
- Model search: higher % in TD

Convergence

- Similar iterations to converge
- TD starts better ($H_{norm} = 1.0$)
- BU starts worse than random

Parallel Results: 48 Cores

Performance

- **4.9× faster** than parallel BU
- Processes larger graphs
- Similar accuracy maintained

Challenges

- Poor strong scaling (both)
- <50% efficiency at 48 cores
- TD: load imbalance early
- BU: better core utilization

Bottlenecks

- Sequential blockmodel updates
- Processor group layout
- Asynchronous Gibbs overhead

Notable Result

1M graph (TD, 48 cores):

- 20 min runtime
- 1.67× slower than BU baseline
- But uses 170× fewer cores!

Distributed Results: Up to 64 Nodes

4-Node Results

- **6.9× faster** than distributed BU
- <2× speedup over 1-node
- <50% distributed efficiency

Communication

TD: More data, less frequent

- Block splits: once/iteration
- 208 AllReduce calls (200K graph)

BU: Less data, more frequent

- Model opt: many/iteration
- 3,184 AllGather calls

64-Node Scaling

- Single-digit efficiency
- Load imbalance dominates
- **403× speedup** (eu-2005)
- **321× speedup** (citPatents)

Key Insight

Most time-consuming collective ≠ most data:

- BU: AllGather 90% time, 0.02% data
- TD: AllReduce 79% time, 0.07% data

⇒ Load balance matters most

Real-World Graphs: Key Results

Performance

- **13.2× faster** than BU (max)
- wiki-topcats: TD succeeds, BU OOM
- Trends match synthetic graphs

Weak Scaling (Poor)

16 nodes efficiency:

- BU: 1.3% (no sampling)
- TD: 3.7% (no sampling)
- Superlinear $O(E \log^2 E)$
- MPI overhead compounds

Sampling Results

- 50% sample size
- **4× speedup** TD vs BU
- Lower than 6.9× without sampling
- TD: higher overhead (finetuning)

Cluster Count

- TD: tends to overestimate
- BU: tends to underestimate
- Similar H_{norm} and NMI
- Both valid solutions

Summary

Paper Contributions

- **Top-Down SBP:** Novel divisive approach to graph clustering
- **Connectivity Snowball:** Efficient splitting heuristic
- **Performance:** 7.7× speedup, 4.1× memory reduction, 403× distributed
- **Impact:** Enables statistical inference on large-scale graphs

Our Experimental Findings

- **Validated paper claims:** Top-Down achieves 6-52× speedup vs Bottom-Up
- **Algorithm trade-offs:** Top-Down for speed, Bottom-Up for many clusters
- **Parallelization insights:** Bottom-Up scales better (2.5× speedup)
- **Bottleneck identified:** MCMC takes 45-90% of runtime

References & Questions

Paper

Ammar, O., Wolfe, C., & Chaudhari, P. (2025). *Top-Down Stochastic Block Partitioning.* HPDC '25.

`https://dl.acm.org/doi/pdf/10.1145/3731545.3731589`

Our Implementation

Top-Down + Bottom-Up with OpenMP parallelization
140 experiments, 9 graph configurations
Complete analysis with plots

Questions?

Thank you for your attention!