

# Coen/Elen 21c

## Homework 5 Solution

Problem 1. Text Problem 3.12b (b only)

Min term	List 1			Min-terms	List 2			Min-terms	List 3		
	ABCD	Flags		ABCD	Flags				ABCD	Flags	
2	0010	$\alpha$	✓	2,3	001-	$\alpha$	$PI_5$	4,5,6,7	01	$\beta$	$PI_1$
4	0100	$\beta$	✓	4,5	010-	$\beta$	✓	4,6,12,14	-1-0	$\beta$	$PI_2$
3	0011	$\alpha$	✓	4,6	01-0	$\beta$	✓	5,7,13,15	-1-1	$\alpha$	$PI_3$
5	0101	$\alpha\beta$	✓	4,12	-100	$\beta$	✓	12,13,14,15	11	$\alpha$	$PI_4$
6	0110	$\beta$	✓	3,7	0-11	$\alpha$	$PI_6$				
12	1100	$\alpha\beta$	✓	5,7	01-1	$\alpha\beta$	$PI_7$				
7	0111	$\alpha\beta$	✓	5,13	-101	$\alpha$	✓				
13	1101	$\alpha$	✓	6,7	011-	$\beta$	✓				
14	1110	$\alpha\beta$	✓	6,14	-110	$\beta$	✓				
15	1111	$\alpha$	✓	12,13	110-	$\alpha$	✓				
				12,14	11-0	$\alpha\beta$	$PI_8$				
				7,15	-111	$\alpha$	✓				
				13,15	11-1	$\alpha$	✓				
				14,15	11-1	$\alpha$	✓				

$$\begin{aligned}
 PI_{\alpha\beta} &= \{PI_7 = ABD, PI_8 = AB\bar{D}\} \\
 PI_{\alpha} &= \{PI_3 = BD, PI_4 = AB, PI_5 = \bar{A}\bar{B}C, PI_6 = ACD\} \\
 PI_{\beta} &= \{PI_1 = AB, PI_2 = B\bar{D}\}
 \end{aligned}$$

Problem 2. Text Problem 3.14

3.14 Use the Quine-McCluskey method to minimize the following functions with don't cares:

(a)  $f(A, B, C, D) = \sum m(0, 6, 9, 10, 13) + d(1, 3, 8)$

List 1			List 2			List 3		
Minterm	ABCD		Minterms	ABCD		Minterms	ABCD	
0	0000	✓	0,1	000-	✓	0,1,8,9	-00-	$PI_1$
1	0001	✓	0,8	-000	✓			
8	1000	✓	1,3	00-1	$PI_2$			
3	0011	✓	1,9	-001	✓			
6	0110	$PI_5$	8,9	100-	✓			
9	1001	✓	8,10	10-0	$PI_3$			
10	1010	✓	9,13	1-01	$PI_4$			
13	1101	✓						

	✓ 0	✓ 6	✓ 9	✓ 10	✓ 13
$*PI_1$	⊗		×		
$PI_2$					
$*PI_3$				⊗	
$*PI_4$			×		⊗
$*PI_5$		⊗			

$$\begin{aligned}
 PI_{\alpha\beta} &= PI_1 + PI_3 + PI_4 + PI_5 \\
 &= \bar{B}\bar{C} + A\bar{B}\bar{D} + A\bar{C}D + \bar{A}BC\bar{D}
 \end{aligned}$$

(b)  $f(A, B, C, D) = \sum m(1, 4, 7, 10, 13) + d(5, 14, 15)$

List 1			List 2			List 3		
Minterm	$ABCD$		Minterms	$ABCD$		Minterms	$ABCD$	
1	0001	✓	1,5	0-01	$PI_2$	5,7,13,15	-1-1	$PI_1$
4	0100	✓	4,5	010-	$PI_3$			
5	0101	✓	5,7	01-1	✓			
10	1010	✓	5,13	-101	✓			
7	0111	✓	10,14	1-10	$PI_4$			
13	1101	✓	7,15	-111	✓			
14	1110	✓	13,15	11-1	✓			
15	1111	✓	14,15	111-	$PI_5$			

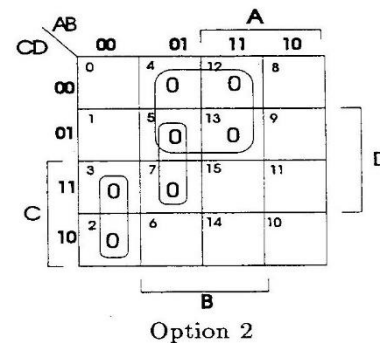
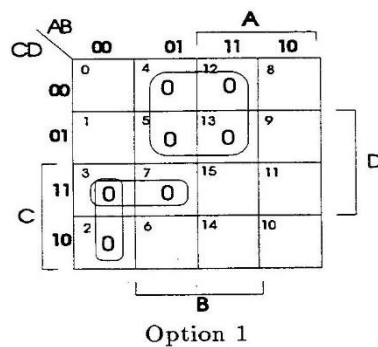
	✓ 1	✓ 4	✓ 7	✓ 10	✓ 13
$*PI_1$			⊗		⊗
$*PI_2$	⊗				
$*PI_3$		⊗			
$*PI_4$				⊗	
$PI_5$					

$$\begin{aligned}
 PI_{\alpha\beta} &= PI_1 + PI_2 + PI_3 + PI_4 \\
 &= BD + \bar{A}\bar{C}D + \bar{A}B\bar{C} + AC\bar{D}
 \end{aligned}$$

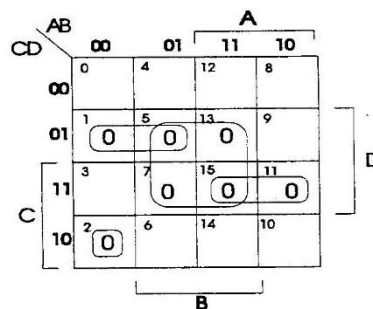
Problem 3. Text Problem 3.45 (a & b)

3.45 Find the minimum POS form for the following functions.

$$\begin{aligned}
 \text{(a)} \quad f(A, B, C, D) &= \prod M(2, 3, 4, 5, 7, 12, 13) \\
 &= (\bar{B} + C)(A + B + \bar{C})(A + \bar{C} + \bar{D}) \quad \text{- Option 1} \\
 &= (\bar{B} + C)(A + B + \bar{C})(A + \bar{B} + \bar{D}) \quad \text{- Option 2}
 \end{aligned}$$



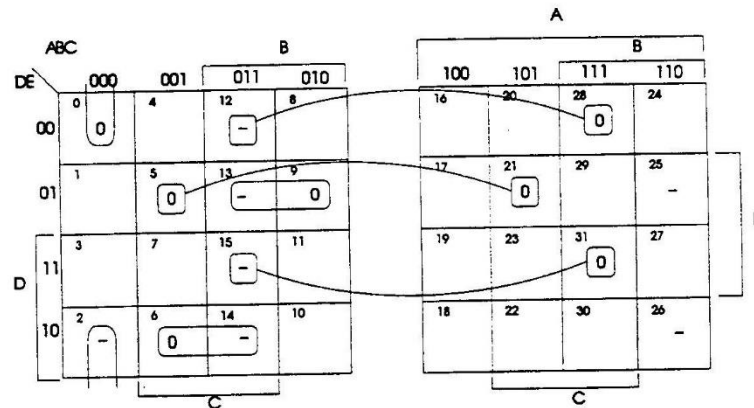
$$\begin{aligned}
 \text{(b)} \quad f(A, B, C, D) &= \prod M(1, 2, 5, 7, 11, 13, 15) \\
 &= (\bar{B} + \bar{D})(A + C + \bar{D})(\bar{A} + \bar{C} + \bar{D})(A + B + \bar{C} + D)
 \end{aligned}$$



Problem 4. Text Problem 3.55

3.55 Find the minimum POS form for the following function.

$$\begin{aligned}
 f(A, B, C, D, E) &= \prod M(0, 5, 6, 9, 21, 28, 31) \cdot D(2, 12, 13, 14, 15, 25, 26) \\
 &= (A + B + C + E)(B + \bar{C} + D + \bar{E})(A + \bar{C} + \bar{D} + E)(\bar{B} + \bar{C} + D + E) \\
 &\quad \cdot (\bar{B} + \bar{C} + \bar{D} + \bar{E})(A + \bar{B} + D + \bar{E}) \quad \text{- Opt 1} \\
 &= (A + B + C + E)(B + \bar{C} + D + \bar{E})(A + \bar{C} + \bar{D} + E)(\bar{B} + \bar{C} + D + E) \\
 &\quad \cdot (\bar{B} + \bar{C} + \bar{D} + \bar{E})(\bar{B} + C + D + \bar{E}) \quad \text{- Opt 2}
 \end{aligned}$$



Option 1

Problem 5

Please indicate which of the following are or are not XOR or XNOR functions.

For those functions that are XOR or XNOR, write their the algebraic expression for that function.

a.  $f(A, B, C) = \sum m(1, 3, 4, 6)$

A \ BC	BC			
	00	01	11	10
0	0	1	1	0
1	1	0	0	1

b.  $f(A,B,C,D) = \sum m(0,2,5,7,8,13,15)$

AB \ CD				
	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	0

**NO**, it is not XOR or XNOR. If minterm m10 were = 1, it would be an **XNOR**  $\therefore (B \wedge D)'$

c.  $f(A,B,C,D) = \sum m(2,3,4,5,10,11,12,13)$

AB \ CD				
	00	01	11	10
00	0	0	1	1
01	1	1	0	0
11	1	1	0	0
10	0	0	1	1

**YES**, it is an XOR function:  $f(A,B,C,D) = B \wedge C$

d.  $f(A,B,C,D) = \sum m(1,3,4,6,8,10,13,15)$

AB \ CD				
	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	0	1	1	0
10	1	0	0	1

Signature checkerboard pattern in groups of two 1's gives an XOR pattern (since 0000 cell = 0)

$F(A,B,C,D) = A \wedge B \wedge D$

e.  $f(A,B,C) = \sum m(0,2,4,6)$

A \ BC				
	00	01	11	10
0	1	0	0	1
1	1	0	0	1

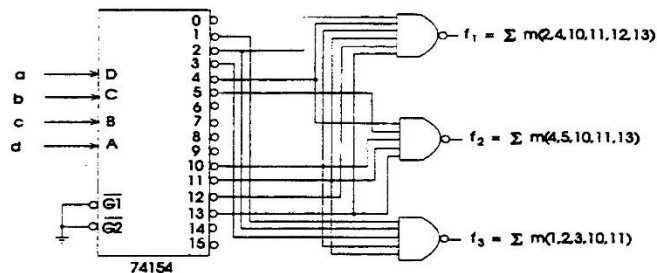
**NO**. This function is not an XOR or XNOR pattern.

$F(A,B,C) = C$

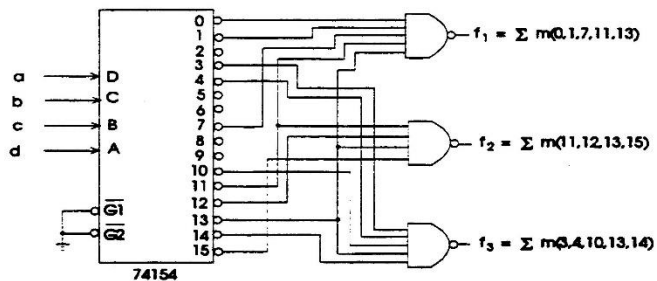
# Problem 6 Text Problem 4.4

4.4 Realize each of the following sets of functions using only a single 74154 decoder module and output logic gates (choose NAND or AND gates to minimize the fan-in of the output gates).

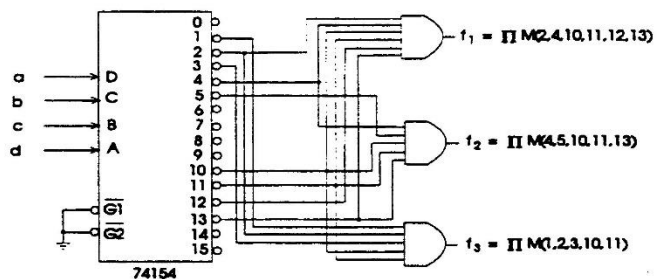
$$\begin{aligned} \text{(a)} \quad f_1(a, b, c, d) &= \sum m(2, 4, 10, 11, 12, 13) \\ f_2(a, b, c, d) &= \prod M(0-3, 6-9, 12, 14, 15) = \sum m(4, 5, 10, 11, 13) \\ f_3(a, b, c, d) &= bc + \bar{a}bd = \sum m(1, 2, 3, 10, 11) \end{aligned}$$



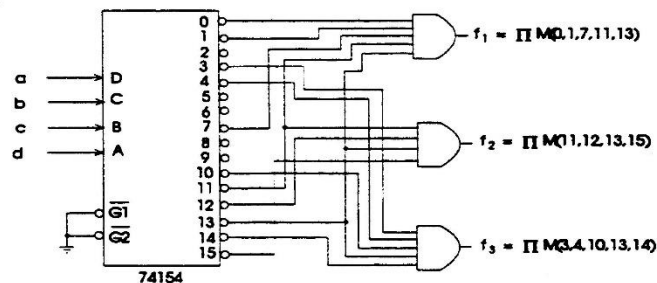
$$\begin{aligned} \text{(b)} \quad f_1(a, b, c, d) &= \sum m(0, 1, 7, 13) \\ f_2(a, b, c, d) &= ab\bar{c} + acd = \sum m(11, 12, 13, 15) \\ f_3(a, b, c, d) &= \prod M(0-2, 5-9, 11, 12, 15) = \sum m(3, 4, 10, 13, 14) \end{aligned}$$



c. Repeat part (a) for the complements of the three functions.



d. Repeat part (b) for the complements of the three functions.



4.15 Design an 8-to-1 multiplexer, using only 4-to-1 multiplexer modules without enable lines.  
(Do not use any additional gates.)

