Coen/Elen 21c

Homework 2 Solution

2.6 Simplify the expressions.

$$
\begin{aligned}
\textbf{(a)} \quad f(A,X,Z) &= \underbrace{\bar{X}}\underbrace{(X+Z)}+\bar{A}+AZ && \text{[T5b]}\\
&= \bar{X}Z+\underbrace{\bar{A}}+\underbrace{AZ} && \text{[T5a]}\\
&= \underbrace{\bar{X}Z}+\bar{A}+\underbrace{Z} && \text{[T4a]}\\
&= \bar{A}+Z
\end{aligned}
$$

$$
\begin{aligned}
\textbf{(b)} \quad f(x,y,z) &= (\bar{X}Y+XZ)(X+\bar{Y}) && \text{[P5b]}\\
&= (\bar{X}Y+XZ)X+(\bar{X}Y+XZ)\bar{Y} && \text{[P5b]}\\
&= \underbrace{\bar{X}YX}+XZX+\underbrace{\bar{X}Y\bar{Y}}+XZ\bar{Y} && \text{[P6b]}\\
&= \underbrace{XZX}+XZ\bar{Y} && \text{[T1b]}\\
&= XZ+XZ\bar{Y} && \text{[T4a]}\\
&= XZ
\end{aligned}
$$

$$
\begin{aligned}
\textbf{(c)}\quad f(x,y,z) &= \underbrace{\bar{x}y(z+\bar{y}x)}+\bar{y}z && \text{[P5b]}\\
&= \bar{x}yz+\bar{x}\,\underbrace{y\bar{y}}\,x+\bar{y}z && \text{[P6b]}\\
&= \bar{x}yz+\bar{y}z && \text{[T7a]}\\
&= \bar{x}z+\bar{y}z
\end{aligned}
$$

2.7. Find the simplest switching expression for the functions given below:

$$
\begin{aligned}
\textbf{(a)}\quad f(A,B,C) &= \textstyle\sum m(1,4,5)\\
&= \bar{A}\bar{B}C+A\bar{B}\bar{C}+A\bar{B}C\\
&= \bar{A}\bar{B}C+A\bar{B}\\
&= \bar{B}C+A\bar{B}
\end{aligned}
$$

$$
\begin{aligned}
\textbf{(b)}\quad f(A,B,C,D) &= \textstyle\prod M(0,2,4,5,8,11,15)\\
&= (A+B+C+D)(A+B+\bar{C}+D)(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})\\
&\quad\cdot(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{C}+\bar{D})\\
&= (A+B+D)(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+D)\\
&\quad\cdot(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{C}+\bar{D})\\
&= (A+B+D)(A+\bar{B}+C)(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+\bar{D})\\
&\quad\cdot(\bar{A}+\bar{B}+\bar{C}+\bar{D})\\
&= (A+B+D)(A+\bar{B}+C)(\bar{A}+B+C+D)(\bar{A}+\bar{C}+\bar{D})\\
&= (A+B+D)(A+\bar{B}+C)(B+C+D)(\bar{A}+\bar{C}+\bar{D})
\end{aligned}
$$

2.8. Given the function $f(x,y,z)$ below, write $f(x,y,z)$ as a sum of minterms and as a product of maxterms.

$$
\begin{aligned}
f(x,y,z) &= x\bar{y}+x\bar{z}\\
&= x\bar{y}(\bar{z}+z)+x(\bar{y}+y)\bar{z}\\
&= x\bar{y}\bar{z}+x\bar{y}z+x\bar{y}\bar{z}+xy\bar{z}\\
&= x\bar{y}\bar{z}+x\bar{y}z+xy\bar{z}\\
&= m_4+m_5+m_6\\
&= \textstyle\sum m(4,5,6)\\
&= \textstyle\prod M(0,1,2,3,7)
\end{aligned}
$$

or

$$
\begin{aligned}
f(x,y,z) &= x\bar{y} + x\bar{z} \\
&= x(\bar{y} + \bar{z}) \\
&= (x+\bar{y})(x+y)(\bar{y}+\bar{z}) \\
&= (x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y)(\bar{y}+\bar{z}) \\
&= (x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y+\bar{z})(x+y+z)(\bar{y}+\bar{z}) \\
&= (x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y+\bar{z})(x+y+z)(\bar{x}+\bar{y}+\bar{z})(x+\bar{y}+\bar{z}) \\
&= (x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y+\bar{z})(x+y+z)(\bar{x}+\bar{y}+\bar{z}) \\
&= M_3M_2M_1M_0M_7 \\
&= \prod M(0,1,2,3,7) \\
&= \sum m(4,5,6)
\end{aligned}
$$

2.13. Use Theorem 8 (DeMorgan's) to complement the following expressions:

(a) $X(Y+\bar{Z}(Q+\bar{R}))$

$$
\begin{aligned}
\overline{X(Y+\bar{Z}(Q+\bar{R}))} &= \bar{X} + \overline{(Y+\bar{Z}(Q+\bar{R}))} \\
&= \bar{X} + \bar{Y}(\overline{\bar{Z}(Q+\bar{R})}) \\
&= \bar{X} + \bar{Y}(Z + \overline{(Q+\bar{R})}) \\
&= \bar{X} + \bar{Y}(Z + (\bar{Q}R))
\end{aligned}
$$

(d) $(A+B\bar{C})(\bar{A}+\bar{D}E)$

$$
\begin{aligned}
\overline{(A+B\bar{C})(\bar{A}+\bar{D}E)} &= \overline{(A+B\bar{C})} + \overline{(\bar{A}+\bar{D}E)} \\
&= \bar{A}(\overline{B\bar{C}}) + A(\overline{\bar{D}E}) \\
&= \bar{A}(\bar{B}+C) + A(D+\bar{E})
\end{aligned}
$$

2.16. Find truth tables for the following switching functions:

(a) $f(A,B) = A + \bar{B}$.

$A\ B$	$\bar{B}$	$f(A,B) = A + \bar{B}$
0 0	1	1
0 1	0	0
1 0	1	1
1 1	0	1

(b) $f(A,B,C) = AB + \bar{A}C$

ABC	AB	$\bar{A}C$	$f(A,B,C) = AB + \bar{A}C$
0 0 0	0	0	0
0 0 1	0	1	1
0 1 0	0	0	0
0 1 1	0	1	1
1 0 0	0	0	0
1 0 1	0	0	0
1 1 0	1	0	1
1 1 1	1	0	1

(c) $f(a,b,c) = a\bar{b}c + b\bar{c}$

abc	$a\bar{b}c$	$b\bar{c}$	$f(a,b,c) = a\bar{b}c + b\bar{c}$
0 0 0	0	0	0
0 0 1	0	0	0
0 1 0	0	1	1
0 1 1	0	0	0
1 0 0	0	0	0
1 0 1	1	0	1
1 1 0	0	1	1
1 1 1	0	0	0

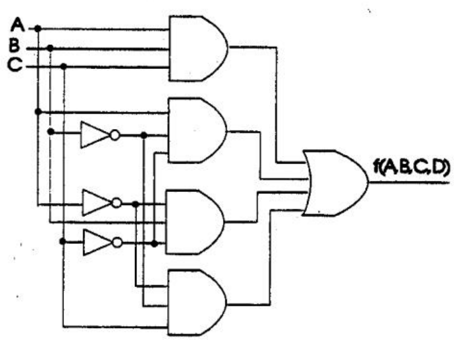
(d) $f(a,b,c) = a(b+\bar{c})(\bar{b}+c)$

abc	$(b+\bar{c})$	$(\bar{b}+c)$	$f(a,b,c) = a(b+\bar{c})(\bar{b}+c)$
0 0 0	1	1	0
0 0 1	0	1	0
0 1 0	1	0	0
0 1 1	1	1	0
1 0 0	1	1	1
1 0 1	0	1	0
1 1 0	1	0	0
1 1 1	1	1	1

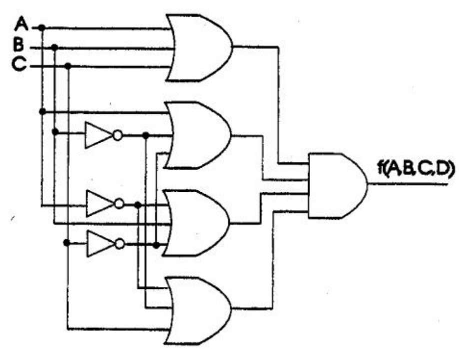
2.23. A long hallway has three doors, one at each end and one in the middle. A switch is located at each door to operate the incandescent lights along the hallway. Label the switches A, B, and C. Design a logic network to control the lights.

ABC	$f(A,B,C)$	
0 0 0	0	- all off initially
0 0 1	1	- 1st switch on
0 1 0	1	- 1st switch on
0 1 1	0	- 2nd switch on
1 0 0	1	- 1st switch on
1 0 1	0	- 2nd switch on
1 1 0	0	- 2nd switch on
1 1 1	1	- 3rd switch on

$$\begin{aligned} f(A,B,C) &= \sum m(1,2,4,7) = \prod M(0,3,5,6) \\ &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\ &= (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C) \end{aligned}$$



AND-OR realization



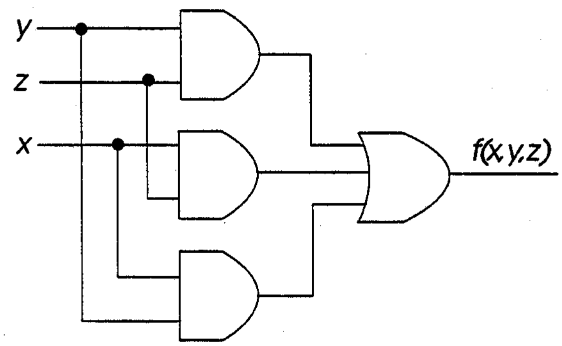
OR-AND realization

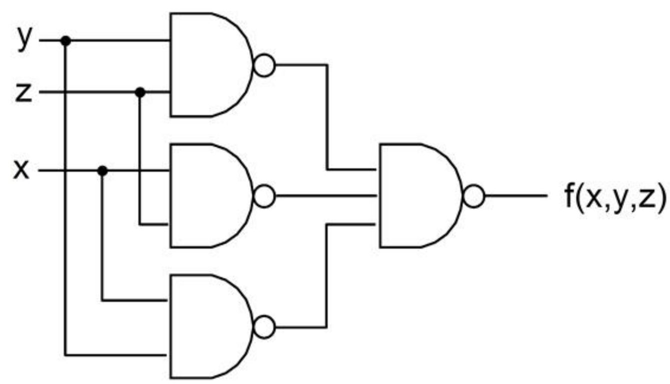
2.31 Joe, Jack, and Jim get together once a week to either go to a movie or go bowling. In order to decide what to do, they vote and a simple majority wins. Assuming a vote for the movie is represented as a 1, design a logic circuit that automatically computes the decision.

Let x = Joe, y = Jack, z = Jim

xyz	$f(x,y,z)$
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

$$\begin{aligned} f(x,y,z) &= \sum m(3,5,6,7) \\ &= \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz \\ &= yz + xz + xy \end{aligned}$$





Problem (non-text)

Given the canonical minterm expression F(A,B,C,D) = ∑m(1,4,6,9,10,13,15)

a) Rewrite F in terms of its variables in the SOP form. (you do not need to try and reduce the equation)

F(A,B,C,D) = A'B'C'D + A'BC'D' + A'BCD' + AB'C'D + AB'CD' + ABC'D + ABCD

b) Write the POS canonical form of F. You only need to show the maxterm form, ie.

F(A,B,C,D) = $\prod$M(A,B,C,D)(0,2,3,5,7,8,11,12,14)

Although not requested in part b, here is the POS form of the equation

F(A,B,C,D) = (A+B+C+D) (A+B+C'+D) (A+B+C'+D') (A+B'+C+D') (A+B'+C'+D') (A'+B+C+D) (A'+B+C'+D') (A'+B'+C+D) (A'+B'+C'+D)