Coen/Elen 921c
Homework 1 Solution

1.4 Convert each of the following decimal numbers to binary, octal, and hexadecimal numbers.

(a) 27

$$\begin{array}{r|l l l}
2 & 2\ 7 & 1 & \uparrow \text{LSD}\\
2 & 1\ 3 & 1 & \\
2 & 6 & 0 & \\
2 & 3 & 1 & \\
2 & 1 & 1 & \text{MSD}\\
 & 0 & &
\end{array}
\qquad
\begin{array}{r|l l l}
8 & 2\ 7 & 3 & \uparrow \text{LSD}\\
8 & 3 & 3 & \text{MSD}\\
 & 0 & &
\end{array}
\qquad
\begin{array}{r|l l l}
16 & 2\ 7 & B & \uparrow \text{LSD}\\
16 & 1 & 1 & \text{MSD}\\
 & 0 & &
\end{array}$$

$(27)_{10} = (33)_8$ $\qquad$ $(27)_{10} = (1B)_{16}$

$(27)_{10} = (11011)_2$

(b) 915

$$\begin{array}{r|l l l}
8 & 9\ 1\ 5 & 3 & \uparrow \text{LSD}\\
8 & 1\ 1\ 4 & 2 & \\
8 & 1\ 4 & 6 & \\
8 & 1 & 1 & \text{MSD}\\
 & 0 & &
\end{array}
\qquad
\begin{array}{r|l l l}
16 & 9\ 1\ 5 & 3 & \uparrow \text{LSD}\\
16 & 5\ 7 & 9 & \\
16 & 3 & 3 & \text{MSD}\\
 & 0 & &
\end{array}$$

$(393)_{16} = (\underbrace{0011}_{3}\underbrace{1001}_{9}\underbrace{0011}_{3})_2$

Therefore:

$(915)_{10} = (1110010011)_2$

$(915)_{10} = (1623)_8$ $\qquad$ $(915)_{10} = (393)_{16}$

(c) 0.375

$.375 \times 2 \quad = \underline{0}.750$

$.750 \times 2 \quad = \underline{1}.500$ $\qquad$ $.375 \times 8 \quad = \underline{3}.000$ $\qquad$ $.375 \times 16 \quad = \underline{6}.000$

$.500 \times 2 \quad = \underline{1}.000$

$(0.375)_{10} = (0.3)_8$ $\qquad$ $0.375_{10} = (0.6)_{16}$

$(0.375)_{10} = (0.011)_2$

(d) 0.65

$$\begin{aligned} .65 \times 2 &= \underline{1}.3 \\ .3 \times 2 &= \underline{0}.6 \\ .6 \times 2 &= \underline{1}.2 \\ .2 \times 2 &= \underline{0}.4 \\ .4 \times 2 &= \underline{0}.8 \\ .8 \times 2 &= \underline{1}.6 \end{aligned}$$

repeats

$(0.65)_{10} = (0.10\overline{1001})_2$

$$\begin{aligned} .65 \times 8 &= \underline{5}.2 \\ .2 \times 8 &= \underline{1}.6 \\ .6 \times 8 &= \underline{4}.8 \\ .8 \times 8 &= \underline{6}.4 \\ .4 \times 8 &= \underline{3}.2 \end{aligned}$$

repeats

$(0.65)_{10} = (0.5\overline{1463})_8$

$$\begin{aligned} .65 \times 16 &= \underline{10}.4 \\ .4 \times 16 &= \underline{6}.4 \end{aligned}$$

repeats

$(0.65)_{10} = (0.A\overline{4})$

Error. Should be 0.A6

(e) 174.25

Integer part:

$$\begin{array}{rrl|l} 2 & \underline{|\,1\;7\;4} & 0 & \uparrow \text{LSD} \\ 2 & \underline{|\,8\;7} & 1 & \\ 2 & \underline{|\,4\;3} & 1 & \\ 2 & \underline{|\,2\;1} & 1 & \\ 2 & \underline{|\,1\;0} & 0 & \\ 2 & \underline{|\,5} & 1 & \\ 2 & \underline{|\,2} & 0 & \\ 2 & \underline{|\,1} & 1 & \text{MSD} \\ & 0 & & \end{array}$$

$(174)_{10} = (10101110)_2$

$$\begin{array}{rrl|l} 8 & \underline{|\,1\;7\;4} & 6 & \uparrow \text{LSD} \\ 8 & \underline{|\,2\;1} & 5 & \\ 8 & \underline{|\,2} & 2 & \text{MSD} \\ & 0 & & \end{array}$$

$(174)_{10} = (256)_8$

$$\begin{array}{rrl|l} 16 & \underline{|\,1\;7\;4} & 14 & \uparrow \text{LSD} \\ 16 & \underline{|\,1\;0} & 10 & \text{MSD} \\ & 0 & & \end{array}$$

$(174)_{10} = (AE)_{16}$

Fractional part:

$$\begin{aligned} .25 \times 2 &= \underline{0}.5 \\ .5 \times 2 &= \underline{1}.0 \end{aligned}$$

$(0.25)_{10} = (0.01)_2$

$.25 \times 8 = \underline{2}.000$

$(0.25)_{10} = (0.2)_8$

$.25 \times 16 = \underline{4}.000$

$(0.25)_{10} = (0.4)_{16}$

Therefore, combining integer and fractional parts:

$(174)_{10} = (10101110.01)_2$ $\quad (174)_{10} = (256.2)_8$ $\quad (174)_{10} = (AE.4)_{16}$

(f) 250.8

Integer part:

$$\begin{array}{r|l l l}
2 & 250 & 0 & \uparrow \text{LSD} \\
2 & 125 & 1 & \\
2 & 62 & 0 & \\
2 & 31 & 1 & \\
2 & 15 & 1 & \\
2 & 7 & 1 & \\
2 & 3 & 1 & \\
2 & 1 & 1 & \text{MSD} \\
 & 0 & &
\end{array}
\qquad
\begin{array}{r|l l l}
8 & 250 & 2 & \uparrow \text{LSD} \\
8 & 31 & 7 & \\
8 & 3 & 3 & \text{MSD} \\
 & 0 & &
\end{array}
\qquad
\begin{array}{r|l l l}
16 & 250 & 10 & \uparrow \text{LSD} \\
16 & 15 & 15 & \text{MSD} \\
 & 0 & &
\end{array}$$

$(250)_{10} = (372)_8$ $\qquad$ $(250)_{10} = (FA)_{16}$

$(250)_{10} = (11111010)_2$

Fractional part:

$.8 \times 2 = \underline{1}.6$
$.6 \times 2 = \underline{1}.2$
$.2 \times 2 = \underline{0}.4$
$.4 \times 2 = \underline{0}.8$
repeats

$.8 \times 8 = \underline{6}.4$
$.4 \times 8 = \underline{3}.2$
$.2 \times 8 = \underline{1}.6$
$.6 \times 8 = \underline{4}.8$
repeats

$.8 \times 16 = \underline{12}.8$
repeats

$(0.8)_{10} = (0.\overline{C})_{16}$

$(0.8)_{10} = (0.\overline{1100})_2$ $\qquad$ $(0.8)_{10} = (0.\overline{6314})_8$

Therefore, combining integer and fractional parts:

$(250)_{10} = (11111010.\overline{1100})_2$ $\quad$ $(174)_{10} = (372.\overline{6314})_8$ $\quad$ $(174)_{10} = (FA.\overline{C})_{16}$

1.5 Convert each of the following binary numbers to octal, hexadecimal, and decimal numbers using the most appropriate conversion method.

(a) 1101

$\underbrace{001}_{1}\underbrace{101}_{5} = (15)_8$

$\underbrace{1101}_{D} = (D)_{16}$

$2^3 + 2^2 + 2^0 = 8 + 4 + 1 = (13)_{10}$

(b) 101110

$\underbrace{101}_{5}\underbrace{110}_{6} = (56)_8$

$\underbrace{0010}_{D}\underbrace{1110}_{E} = (2E)_{16}$

$2^5 + 2^3 + 2^2 + 2^1 = 32 + 8 + 4 + 2 = (46)_{10}$

(c) 0.101

$.\underbrace{101}_{5} = (.5)_8$

$.\underbrace{1010}_{A} = (.A)_{16}$

$2^{-1} + 2^{-3} = .5 + .125 = (.625)_{10}$

(d) 0.01101

$.\underbrace{011}_{3}\underbrace{010}_{2} = (32)_8$

$.\underbrace{0110}_{6}\underbrace{1000}_{8} = (.68)_{16}$

$2^{-2} + 2^{-3} + 2^{-5} = .25 + .125 + .03125 = (.40625)_{10}$

(e) 10101.11

$\underbrace{010}_{2}\underbrace{101}_{5}.\underbrace{110}_{6} = (25.6)_8$

$\underbrace{0001}_{1}\underbrace{0101}_{5}.\underbrace{1100}_{C} = (15.C)_{16}$

$2^4 + 2^2 + 2^0 + 2^{-1} + 2^{-2} = 16 + 4 + 1 + .5 + .25 = (21.75)_{10}$

(f) 10110110.001

$\underbrace{010}_{2}\underbrace{110}_{6}\underbrace{110}_{6}.\underbrace{001}_{1} = (266.1)_8$

$\underbrace{01011}_{B}\underbrace{0110}_{6}.\underbrace{0010}_{2} = (B6.2)_{16}$

$2^7 + 2^5 + 2^4 + 2^2 + 2^1 + 2^{-3} = 128 + 32 + 16 + 4 + 1 + .125 = (181.125)_{10}$

1.7 Convert each of the following hexadecimal numbers to binary, octal, and decimal using the most appropriate conversion method.

(a) $(4F)_{16}$

$(4F)_{16} = (\underbrace{0100}_{4}\underbrace{1111}_{F})_2$

$\underbrace{001}_{1}\underbrace{001}_{1}\underbrace{111}_{7} = (117)_8$

$4 \times 16 + 15 = 64 + 15 = (79)_{10}$

(b) $(ABC)_{16}$

$(ABC)_{16} = (\underbrace{1010}_{A}\underbrace{1011}_{B}\underbrace{1100}_{C})_2$

$\underbrace{101}_{5}\underbrace{010}_{2}\underbrace{111}_{7}\underbrace{100}_{4} = (5274)_8$

$10 \times 16^2 + 11 \times 16 + 12 = 2,560 + 176 + 12 = (2,748)_{10}$

(c) $(F8.A7)_{16}$

$(F8.A7)_{16} = (\underbrace{1111}_{F}\underbrace{1000}_{8}.\underbrace{1010}_{A}\underbrace{0111}_{7})_2$

$\underbrace{011}_{3}\underbrace{111}_{7}\underbrace{000}_{0}.\underbrace{101}_{5}\underbrace{001}_{1}\underbrace{110}_{6} = (370.516)_8$

$15 \times 16 + 8 + 10 \times 16^{-1} + 7 \times 16^{-2} = 240 + 8 + .625 + .027343750 = (248.6523438)_{10}$

(d) $(2000)_{16}$

$$(2000)_{16} = (\underbrace{0010}_{2}\underbrace{0000}_{0}\underbrace{0000}_{0}\underbrace{0000}_{0})_2$$

$$\underbrace{010}_{2}\underbrace{000}_{0}\underbrace{000}_{0}\underbrace{000}_{0}\underbrace{000}_{0} = (20000)_8$$

$$2 \times 16^3 = (8,192)_{10}$$

(e) $(201.4)_{16}$

$$(201.4)_{16} = (\underbrace{0010}_{2}\underbrace{0000}_{0}\underbrace{0001}_{1}.\underbrace{0100}_{4})_2$$

$$\underbrace{001}_{1}\underbrace{000}_{0}\underbrace{000}_{0}\underbrace{001}_{1}.\underbrace{010}_{2} = (1001.2)_8$$

$$2 \times 16^2 + 1 + 4 \times 16^{-1} = 512 + 1 + .25 = (513.25)_{10}$$

(f) $(3D65E)_{16}$

$$(3D65E)_{16} = (\underbrace{0011}_{3}\underbrace{1101}_{D}\underbrace{0110}_{6}\underbrace{0101}_{5}\underbrace{1110}_{E})_2$$

$$\underbrace{111}_{7}\underbrace{101}_{5}\underbrace{011}_{3}\underbrace{001}_{1}\underbrace{011}_{3}\underbrace{110}_{6} = (753,136)_8$$

$$3 \times 16^4 + 13 \times 16^3 + 6 \times 16^2 + 5 \times 16 + 14 = 196,608 + 2,808 + 1,536 + 80 + 14 = (251,486)_{10}$$

1.8 Find the two's complement of each of the following binary numbers assuming $n = 8$.

(a) 101010

$$\begin{array}{rcrl} N & = & 00101010 & \\ & & 11010101 & \text{- complement} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (11010110)_2 & \end{array}$$

(b) 1101011

$$\begin{array}{rcrl} N & = & 01101011 & \\ & & 10010100 & \text{- complement} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (10010101)_2 & \end{array}$$

(c) 0

$$\begin{array}{rcrl} N & = & 00000000 & \\ & & 11111111 & \text{- complement} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (00000000)_2 & \end{array}$$

(d) 11111111

$$\begin{array}{rcrl} N & = & 11111111 & \\ & & 00000000 & \text{- complement} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (00000001)_2 & \end{array}$$

(e) 10000000

$$\begin{array}{rcrl} N & = & 10000000 & \\ & & 01111111 & \text{- complement} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (10000000)_2 & \end{array}$$

(f) 11000

$$\begin{array}{rcrl} N & = & 00011000 & \\ & & 11100111 & \text{- complement the bits} \\ & & +1 & \text{- add 1} \\ [N]_2 & = & (11101000)_2 & \end{array}$$

1.10 Calculate $A+B$, $A-B$, $-A+B$, and $-A-B$ for each pair of numbers below assuming a two's complement number system and $n = 8$. Check your results by decimal arithmetic. Explain any unusual results.

(a) 1010101, 1010

$$\begin{array}{rlclcrlcl} A = & (0,1010101)_{2cns} & = & +(85)_{10} & , & B = & (0,0001010)_{2cns} & = & +(10)_{10} \\ -A = & (1,0101011)_{2cns} & = & -(85)_{10} & , & -B = & (1,1110110)_{2cns} & = & -(10)_{10} \end{array}$$

$A+B$

$$\begin{array}{rcccccccc} & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ + & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Result: $(0,1011111)_{2cns} = +(95)_{10}$

$A-B$

$$\begin{array}{rcccccccc} & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ + & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline (1) & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}$$

(Discard carry)

Result: $(0,1001011)_{2cns} = +(75)_{10}$

$-A+B$

$$\begin{array}{rcccccccc} & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ + & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Result: $(1,0110101)_{2cns} = -(75)_{10}$

$-A-B$

$$\begin{array}{rcccccccc} & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ + & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline (1) & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

(Discard carry)

Result: $(1,0100001)_{2cns} = -(95)_{10}$

(b) 1101011, 0101010

$A = (0,1101011)_{2cns} = +(107)_{10}$, $B = (0,0101010)_{2cns} = +(42)_{10}$
$-A = (1,0010101)_{2cns} = -(107)_{10}$, $-B = (1,1010110)_{2cns} = -(42)_{10}$

$A+B$

$$\begin{array}{rcccccccc} & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ + & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Overflow condition! Invalid result

$A-B$

$$\begin{array}{rcccccccc} & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ + & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ \hline (1) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

(Discard carry)

Result: $(0,1000001)_{2cns} = +(65)_{10}$

$-A+B$

$$\begin{array}{rcccccccc} & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ + & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ \hline & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Result: $(1,0111111)_{2cns} = -(65)_{10}$

$-A-B$

$$\begin{array}{rcccccccc} & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ + & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ \hline (1) & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}$$

Overflow condition! Invalid result

(c) 11101010, 101111

$A = (1,1101010)_{2cns} = -(22)_{10}$, $B = (0,0101111)_{2cns} = +(47)_{10}$
$-A = (0,0010110)_{2cns} = +(22)_{10}$, $-B = (1,1010001)_{2cns} = -(47)_{10}$

$A+B$

$$\begin{array}{rcccccccc} & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline (1) & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}$$

(Discard carry)

Result: $(0,0011001)_{2cns} = +(25)_{10}$

$A-B$

$$\begin{array}{rcccccccc} & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ + & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline (1) & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \end{array}$$

(Discard carry)

Result: $(1,0111011)_{2cns} = -(69)_{10}$

$-A+B$

$$\begin{array}{rcccccccc} & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ + & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}$$

Result: $(0,1000101)_{2cns} = +(69)_{10}$

$-A-B$

$$\begin{array}{rcccccccc} & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ + & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{array}$$

Result: $(1,1100111)_{2cns} = -(25)_{10}$

(d) 10000000, 01111111

$A = (1,0000000)_{2cns} = -(128)_{10}$, $B = (0,1111111)_{2cns} = +(127)_{10}$
$-A =$ Can't do $+(128)_{10}$ with 8 bits , $-B = (1,0000001)_{2cns} = -(127)_{10}$

$$A + B \qquad \begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ +\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \end{array}$$

Result: $(1,1111111)_{2cns} = -(1)_{10}$

$$A - B \qquad \begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ +\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1 \\ \hline (1)\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1 \end{array}$$

Overflow condition! Invalid result

Cannot do $(-A+B)$ or $(-A-B)$ since $-A$ cannot be represented!

1.14 Encode each of the following character strings in ASCII code. Represent the encoded strings by hexadecimal numbers.

(a) 1980

	1	9	8	0
ASCII =	31	39	38	30

(b) A = b + C

	A		=		b		+		C
ASCII =	41	20	3D	20	62	20	2B	20	43

(c) COMPUTER ENGINEERING

C	O	M	P	U	T	E	R		E	N	G	I	N	E	E	R	I	N	G
43	4F	4D	50	55	54	45	52	20	45	4E	47	49	4E	45	45	52	49	4E	47

(d) The End.

	T	h	e		E	n	d	.
ASCII =	54	68	65	20	45	6E	64	2E

1.18 For the nine-track magnetic tape of Fig 1.7, the following 8-bit messages are to be recorded. Determine the parity bit to establish odd parity for each message.

(a) P10111010 5 1-bits ⇒ P=0
(b) P00111000 3 1-bits ⇒ P=0
(c) P10011001 4 1-bits ⇒ P=1
(d) P01011010 4 1-bits ⇒ P=1