Coen/Elen 21c
Homework 3 Solution

2.18. Find the minterm and maxterm list forms for the switching functions of Problem 2.16. Use any method.

(a) $f(A, B) = A + \bar{B}$

$$\begin{aligned}
A + \bar{B} &= A(\bar{B} + B) + (\bar{A} + A)\bar{B} \\
&= A\bar{B} + AB + \bar{A}\bar{B} + A\bar{B} \\
&= A\bar{B} + AB + \bar{A}\bar{B} \\
&= m_2 + m_3 + m_0 \\
&= \sum m(0, 2, 3) \\
&= \prod M(1)
\end{aligned}$$

(b) $f(A, B, C) = AB + \bar{A}C$

$$\begin{aligned}
AB + \bar{A}C &= AB(\bar{C} + C) + \bar{A}(\bar{B} + B)C \\
&= AB\bar{C} + ABC + \bar{A}\bar{B}C + \bar{A}BC \\
&= m_6 + m_7 + m_1 + m_3 \\
&= \sum m(1, 3, 6, 7) \\
&= \prod M(0, 2, 4, 5)
\end{aligned}$$

2.20. Expand the following function into canonical SOP form.

$$\begin{aligned}
f(x_1, x_2, x_3) &= x_1x_3 + x_2x_3 + x_1x_2x_3 \\
&= x_1(\bar{x}_2 + x_2)\bar{x}_3 + (\bar{x}_1 + x_1)x_2\bar{x}_3 + x_1x_2x_3 \\
&= x_1\bar{x}_2\bar{x}_3 + x_1x_2\bar{x}_3 + \bar{x}_1x_2\bar{x}_3 + x_1x_2\bar{x}_3 + x_1x_2x_3 \\
&= x_1\bar{x}_2\bar{x}_3 + x_1x_2\bar{x}_3 + \bar{x}_1x_2\bar{x}_3 + x_1x_2x_3 \\
&= m_4 + m_6 + m_2 + m_7 \\
&= \sum m(2, 4, 6, 7)
\end{aligned}$$

2.32 Derive the logic equations for a circuit that will subtract two 2-bit binary numbers, $(X_1X_0)_2 - (Y_1Y_0)_2$ and produce as an output the resulting number $(D_1D_0)_2$ and borrow condition B_1.

$x_1x_0y_1y_0$	$b_2d_1d_0$
1 1 1 0	0 0 1
1 1 1 1	0 0 0

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0001

2.22. A burglar alarm is designed so that it senses four input signal lines. Line A is from the secret control switch, line B is from a pressure sensor under a steel safe in a locked closet, line C is from a battery powered clock, and line D is connected to a switch on the locked closet door. The following conditions produce a logic 1 voltage on each line:

A: The control switch is closed.

B: The safe is in its normal position in the closet.

C: The clock is between 1000 and 1400 hours.

D: The closet door is closed.

Write the switching expression for the burglar alarm which produces a logic 1 (rings a bell) when the safe is moved and the control switch is closed, or when the closet is opened after banking hours, or when the closet is opened with the control switch open.

$$f(A,B,C,D) = \bar{B}A + D\bar{C} + \bar{D}\bar{A}$$

$$\begin{aligned}
b_2 &= \sum m(1,2,3,6,7,11) \\
&= x_1x_0y_1y_0 + x_1x_0y_1\bar{y}_0 + \bar{x}_1x_0y_1y_0 + \bar{x}_1x_0y_1\bar{y}_0 + \bar{x}_1x_0y_1y_0 + x_1\bar{x}_0y_1y_0 \\
&= x_1x_0y_0 + x_1x_0y_1 + \bar{x}_1x_0y_1 + \bar{x}_0y_1y_0 \\
&= \bar{x}_1\bar{x}_0y_0 + \bar{x}_1y_1 + \bar{x}_0y_1y_0
\end{aligned}$$

$$\begin{aligned}
d_1 &= \sum m(1,2,6,7,8,11,12,13) \\
&= x_1x_0y_1y_0 + \bar{x}_1\bar{x}_0y_1\bar{y}_0 + \bar{x}_1x_0y_1\bar{y}_0 + \bar{x}_1x_0y_1y_0 + x_1x_0\bar{y}_1y_0 + x_1x_0y_1y_0 + x_1x_0\bar{y}_1\bar{y}_0 + x_1x_0\bar{y}_1y_0 \\
&= x_1x_0\bar{y}_1y_0 + \bar{x}_1y_1\bar{y}_0 + \bar{x}_1x_0y_1 + x_1\bar{y}_1y_0 + x_1x_0y_1y_0 + x_1x_0y_1
\end{aligned}$$

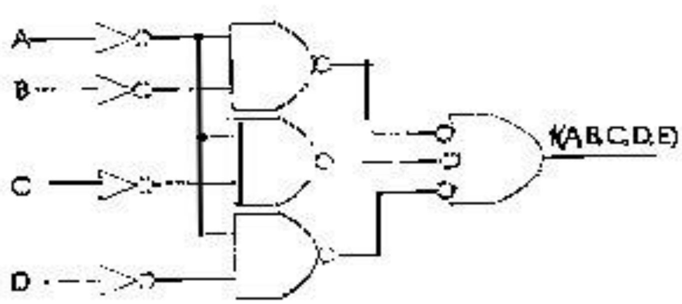
$$\begin{aligned}
d_0 &= \sum m(1,3,4,6,9,11,12,14) \\
&= \bar{x}_1\bar{x}_0\bar{y}_1y_0 + x_1\bar{x}_0y_1y_0 + x_1x_0\bar{y}_1y_0 + x_1x_0y_1\bar{y}_0 + x_1\bar{x}_0\bar{y}_1y_0 + x_1\bar{x}_0y_1y_0 + x_1x_0\bar{y}_1y_0 + x_1x_0y_1\bar{y}_0 \\
&= \bar{x}_1\bar{x}_0y_0 + \bar{x}_1x_0\bar{y}_0 + x_1x_0y_0 + x_1x_0\bar{y}_0 \\
&= \bar{x}_0y_0 + x_0\bar{y}_0
\end{aligned}$$

2.36 The input to a logic circuit consists of four signal lines A, B, C, and D. These lines represent a 4-bit binary number where A represents the most significant bit and D the least significant bit. Design the logic circuit such that the output is high only when the binary input is less than $(0111)_2 = 7_{10}$. Use any desired logic gates.

$ABCD$	$f(A,B,C,D)$
0 0 0 0	1
0 0 0 1	1
0 0 1 0	1
0 0 1 1	1
0 1 0 0	1
0 1 0 1	1
0 1 1 0	1
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
1 1 1 1	0

Producing a minimal SOP expression from minterms:

$$
\begin{aligned}
f(A,B,C,D) &= \sum m(0,1,2,3,4,5,6) \\
&= \bar{A}\bar{B}\bar{C}D + AB\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + ABCD + AB\bar{C}D + \bar{A}BC\bar{D} \\
&= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{D}
\end{aligned}
$$



Or, considering maxterms and generating a minimimal POS expression:

$$f(A,B,C,D) = \prod M(7,8,9,10,11,12,13,14,15)$$

$$
\begin{aligned}
&= (A+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)(\bar{A}+B+C+D)(A+B+C+D) \\
&\quad\cdot(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+C+D)(A+\bar{B}+C+D) \\
&\quad\cdot(A+\bar{B}+\bar{C}+\bar{D}) \\
&= (A+B+C+D)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(A+B+C)(\bar{A}+\bar{B}+\bar{C}) \\
&= (A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B)(A+\bar{B}) \\
&= (A+\bar{B}+\bar{C}+\bar{D})\bar{A} \\
&= (\bar{B}+\bar{C}+\bar{D})\bar{A}
\end{aligned}
$$