Day 1

I) Implementation of different estimators

In [29]:
```
import numpy as np
import pandas as pd
import yfinance as yf
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings("ignore")
```

Import of SPY data

In [30]:
```
data = yf.download('SPY', start='2000-01-01', end='2014-01-01')
```

```
[*********************100%%**********************]  1 of 1 completed
```

Close to close estimator

In [31]:
```
def close_to_close(data):
    data['Close-to-close'] = np.abs(np.log(data['Close'] / data['Close'].shift(1)))
    Close_to_close=data['Close-to-close']
    return Close_to_close
```

Parkinson estimator

In [32]:
```
def Parkinson(data):
    Parkinson = np.sqrt(1 / (4 * np.log(2)) * np.log(data['High'] / data['Low']) ** 2)
    return Parkinson
```

Garman Klass estimator

In [33]:
```
def garman_klass__estimator(data):
    sigma_GK = 0.5 * (np.log(data['High']/data['Low']))**2 - (2*np.log(2)-1) * (np.log(d
    sigma_GK = np.sqrt(sigma_GK)
    return sigma_GK
```

Roger Satchell estimator

In [34]:
```
def rogers_satchell_volatility(data):
    high = data['High']
    low = data['Low']
    close = data['Close']
    open = data['Open']

    log_hc = np.log(high / close)
    log_ho = np.log(high / open)
    log_lc = np.log(low / close)
    log_lo = np.log(low / open)

    sigma_rs = np.sqrt(log_hc * log_ho - log_lc * log_lo)
    return sigma_rs
```

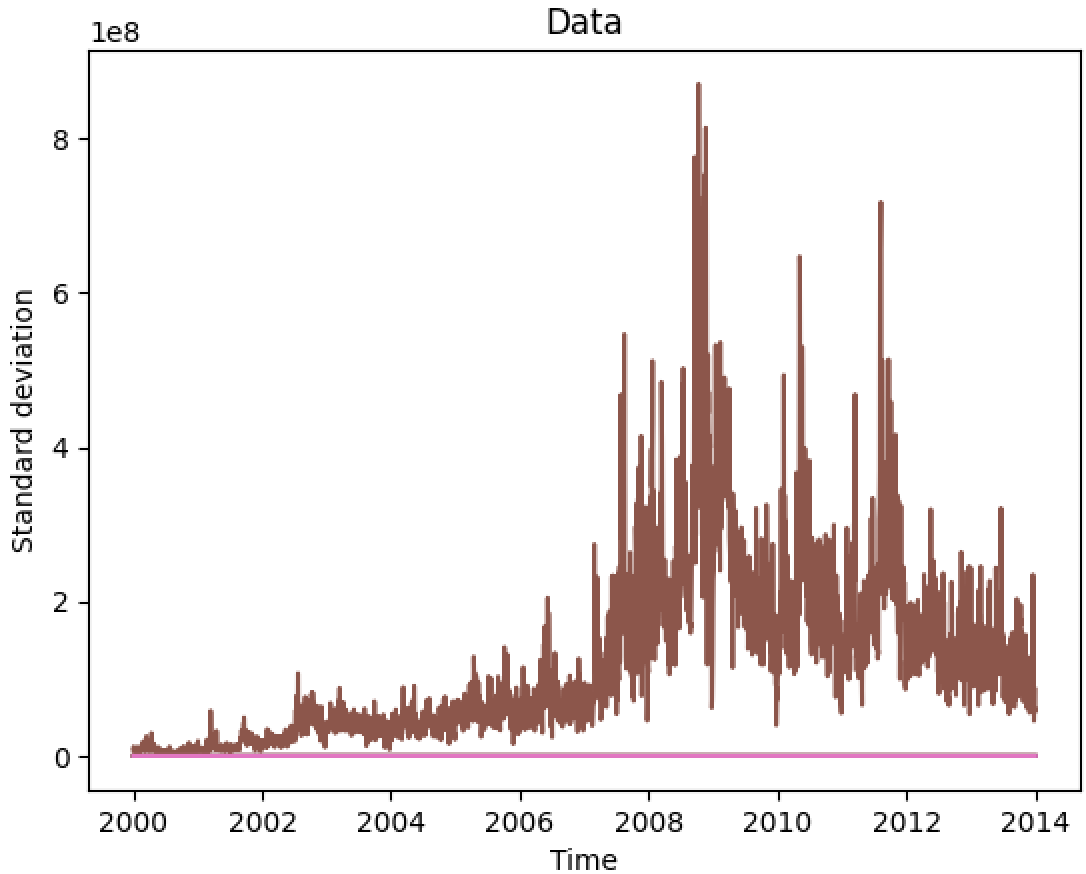
Yang-Zhang estimator

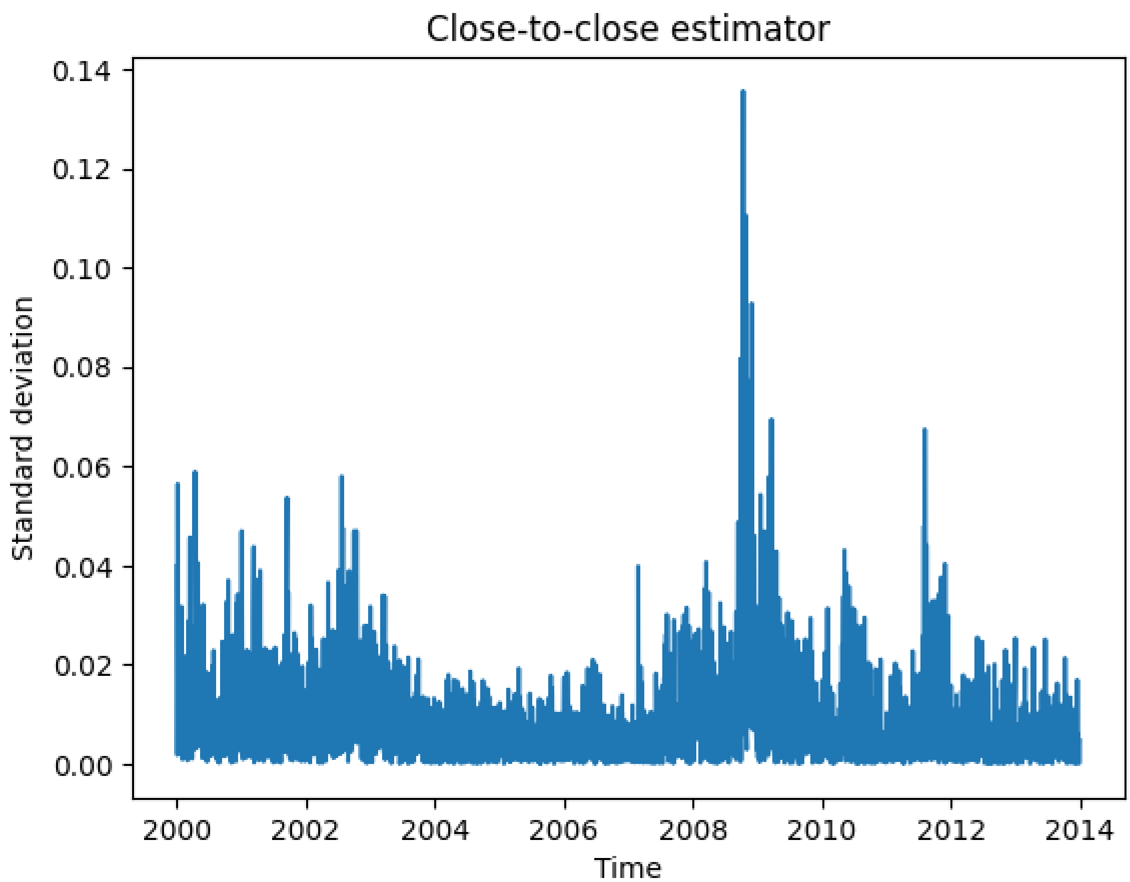
In [35]:
```
def calculate_yang_zhang(data, k):
```

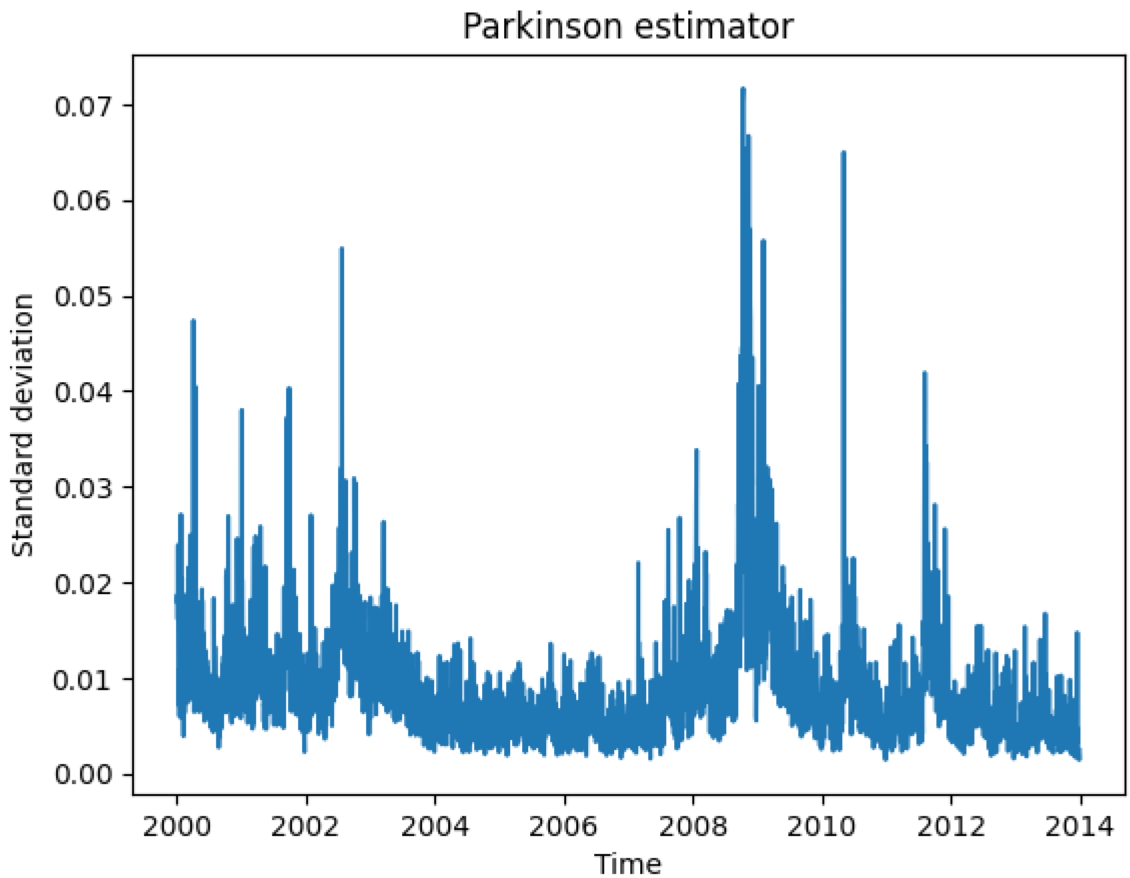
```
    """
    Calcule l'indicateur Yang-Zhang pour une valeur donnée de k.
    """
    # Calculer l'indicateur Yang-Zhang
    yang_zhang = np.sqrt(
        np.log(data['Open'] / data['Close'].shift(1)) ** 2
        + k * np.log(data['Open'] / data['Close']) ** 2
        + (1 - k) * (np.log(data['High'] / data['Close']) * np.log(data['High'] / data['
                    + np.log(data['Low'] / data['Close']) * np.log(data['Low'] / data['Ope

    return yang_zhang
```

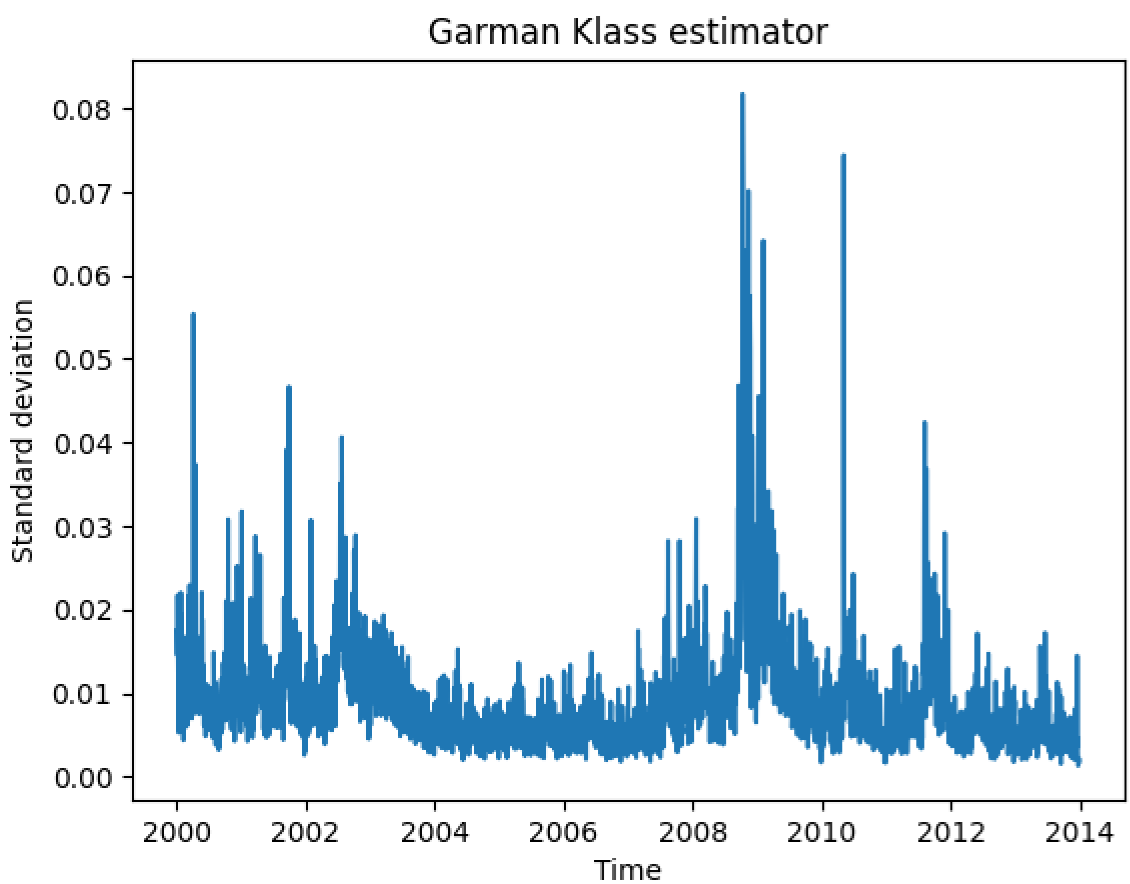
Graphs

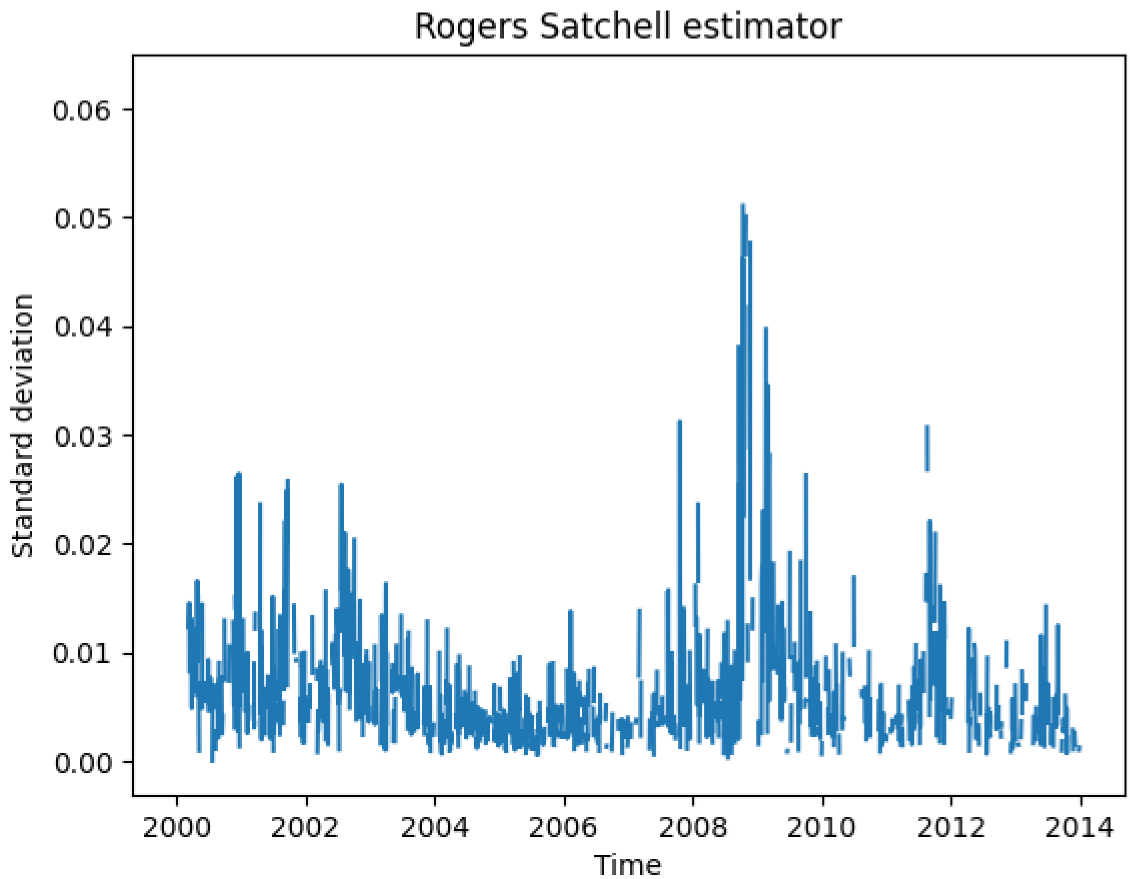
In [36]:
```
parkinson_data = Parkinson(data)
garman_data = garman_klass__estimator(data)
rogers_data = rogers_satchell_volatility(data)
close_data = close_to_close(data)

plt.plot(data)
plt.title('Data')
plt.xlabel('Time')
plt.ylabel('Standard deviation')
plt.show()

plt.plot(close_data)
plt.title('Close-to-close estimator')
plt.ylabel('Standard deviation')
plt.xlabel('Time')
plt.show()


plt.plot(parkinson_data)
plt.title('Parkinson estimator')
plt.ylabel('Standard deviation')
plt.xlabel('Time')
plt.show()


plt.plot(garman_data)
plt.title('Garman Klass estimator')
plt.ylabel('Standard deviation')
plt.xlabel('Time')
plt.show()

plt.plot(rogers_data)
plt.title('Rogers Satchell estimator')
plt.ylabel('Standard deviation')
plt.xlabel('Time')
plt.show()

yangzhang= calculate_yang_zhang(data,k=0.5)

plt.plot(yangzhang)
plt.title('Yang-Zhang estimator')
plt.ylabel('Standard deviation')
plt.xlabel('Time')
plt.show()
```

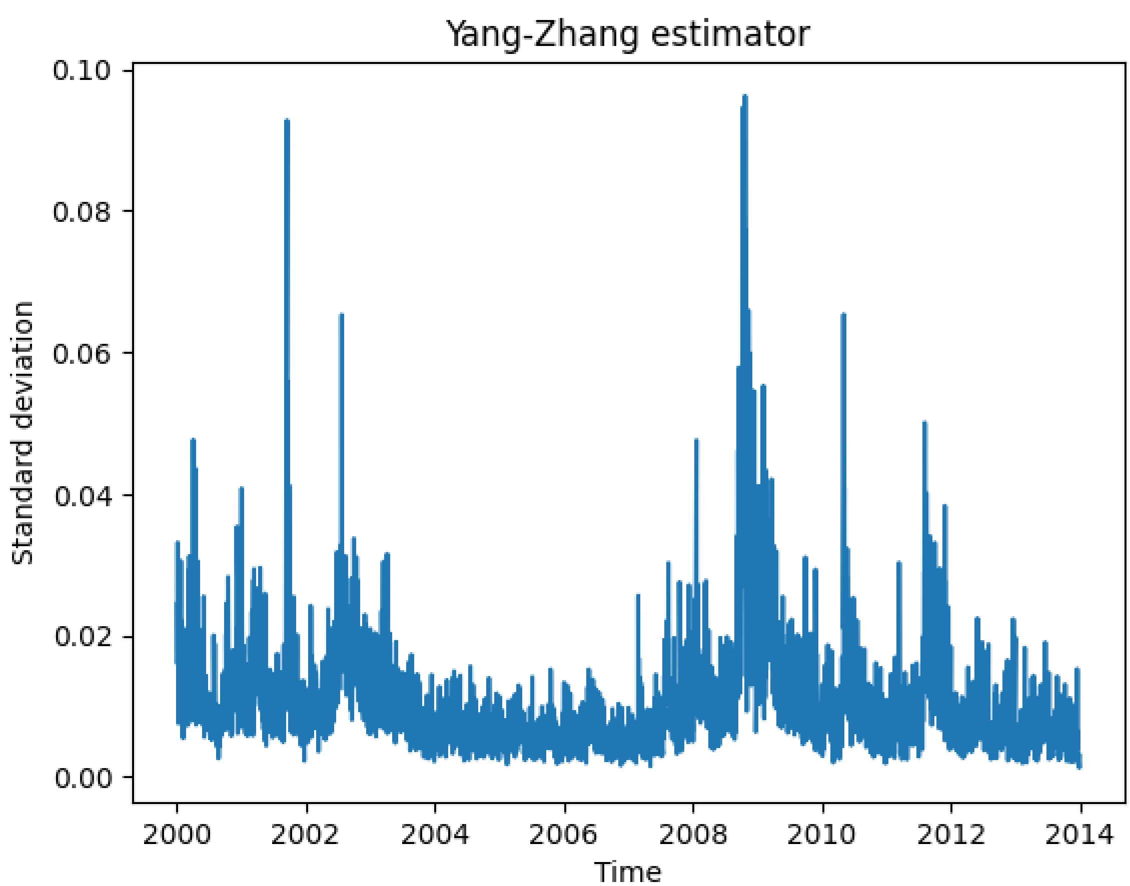

Data
1e8
8
6
4
2
0
Standard deviation
2000
2002
2004
2006
2008
2010
2012
2014
Time


Close-to-close estimator
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Standard deviation
2000
2002
2004
2006
2008
2010
2012
2014
Time


Parkinson estimator
Standard deviation
Time
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
2000
2002
2004
2006
2008
2010
2012
2014


Garman Klass estimator
Standard deviation
Time
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
2000
2002
2004
2006
2008
2010
2012
2014





II) Comparison between estimators

Correlation matrix

In [53]:
```
# Correlation between the five volatility estimators
```

```
k = 1 # Set the value k

data = yf.download('SPY', start='2000-01-01', end='2014-01-01')

A = pd.Series(close_to_close(data))
B = garman_klass__estimator(data)
C = pd.Series(Parkinson(data))
D = rogers_satchell_volatility(data)
E = calculate_yang_zhang(data, k)

# A list of the 5 estimators (as dataframes)
estimators = [A,B,C,D,E]

for i, est in enumerate(estimators):
    if not isinstance(est, pd.Series):
        print(f"Estimator {i} is not a pandas Series.")
    if np.isinf(est).any():
        print(f"Estimator {i} contains infinite values.")

# Computing the correlation matrix
correlation_matrix = np.zeros((5,5))
for i in range(len(estimators)):
    for j in range(len(estimators)):
        correlation_value = estimators[i].corr(estimators[j])
        if not pd.isna(correlation_value):
            correlation_matrix[i, j] = correlation_value



labels = ['Close to close', 'Garman Klass', 'Parkinson', 'Roger Satchell', 'Yang Zhang']

df_correlation = pd.DataFrame(correlation_matrix, index=labels, columns=labels)

print(df_correlation)
```

```
[*********************100%%**********************]  1 of 1 completed
                Close to close  Garman Klass  Parkinson  Roger Satchell  \
Close to close        1.000000      0.621091   0.731183        0.441660
Garman Klass          0.621091      1.000000   0.967018        0.820563
Parkinson             0.731183      0.967018   1.000000        0.728378
Roger Satchell        0.441660      0.820563   0.728378        1.000000
Yang Zhang            0.848369      0.695364   0.828181        0.479573

                Yang Zhang
Close to close    0.848369
Garman Klass      0.695364
Parkinson         0.828181
Roger Satchell    0.479573
Yang Zhang        1.000000
```

Correlation with true volatility

In [38]:
```
# Téléchargement des données pour SPY comme précédemment
data = yf.download('SPY', start='2000-01-01', end='2014-01-01')

# Simulation simple de la vraie volatilité (ici nous utilisons l'écart type des prix de
true_volatility = data['Close'].rolling(window=30).std()

# Assurons que true_volatility est une série complète pour la période (sans NaN au début
true_volatility = true_volatility.fillna(method='bfill')
```

```
[*********************100%%**********************]  1 of 1 completed
```

In [39]:
```
# Initialisation du tableau pour stocker les corrélations
```

```python
correlation_with_true_vol = np.zeros(5)

# Calcul de la corrélation pour chaque estimateur avec la vraie volatilité
for i in range(len(correlation_with_true_vol)):
    correlation_with_true_vol[i] = estimators[i].corr(true_volatility)

# Création du DataFrame pour afficher les résultats
df_corr_true_vol = pd.DataFrame([correlation_with_true_vol], columns=labels)

print("Correlation with True Volatility:\n", df_corr_true_vol)
```

```
Correlation with True Volatility:
    Close to Close  Garman Klass  Parkinson  Roger Satchell  Yang Zhang
0        0.443313      0.622307   0.615934        0.552727    0.504924
```

Estimators efficiency

In [40]:
```python
# Calcul de la variance de la vraie volatilité pour la période considérée
true_variance = true_volatility.var()

# Initialisation du tableau pour stocker les efficiences
efficiency_vector = np.zeros(5)

# Calcul de l'efficacité de chaque estimateur par rapport à la vraie volatilité
for i in range(len(efficiency_vector)):
    efficiency_vector[i] = true_variance / estimators[i].var()

# Création du DataFrame pour afficher les résultats
df_efficiency = pd.DataFrame([efficiency_vector], columns=labels)

print("Efficiency of Estimators:\n", df_efficiency)
```

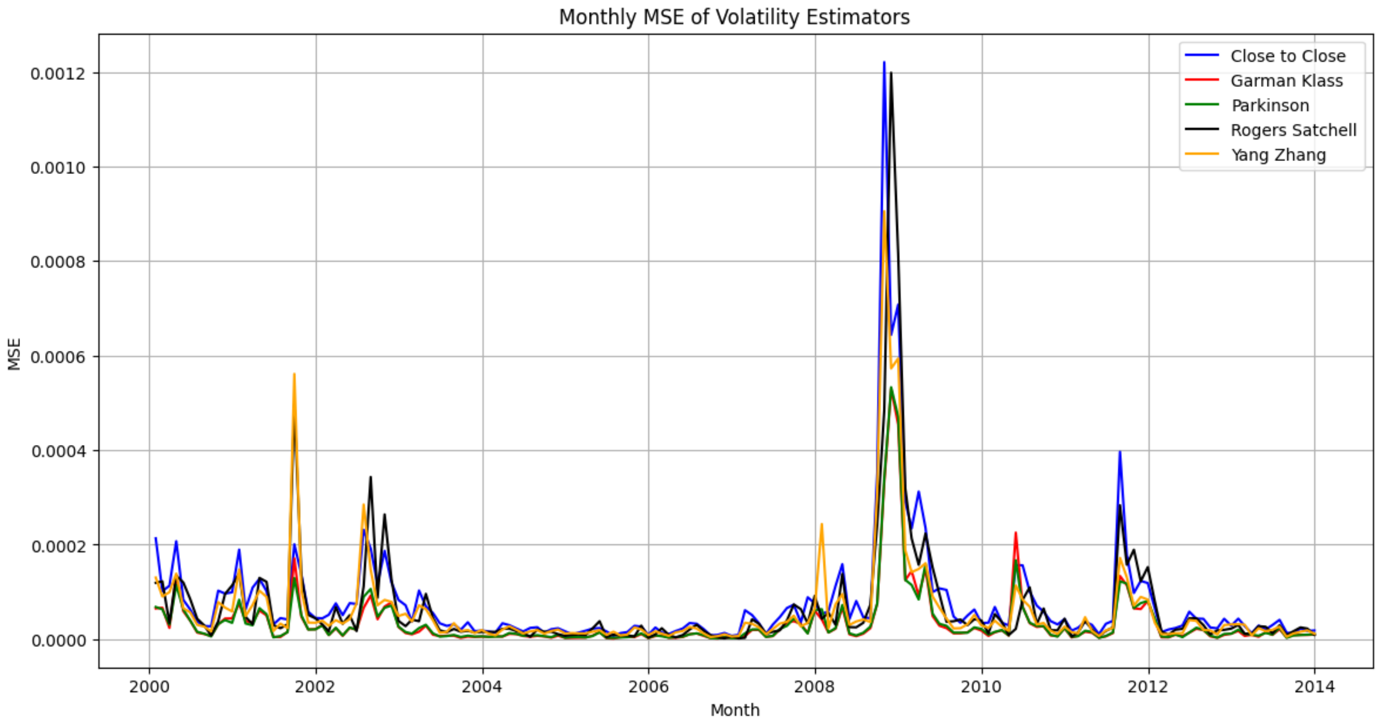
```
Efficiency of Estimators:
    Close to Close  Garman Klass     Parkinson  Roger Satchell    Yang Zhang
0    20253.973378  42809.182226  43364.971547    54721.679971  24670.783472
```

Monthly MSE of Volatility Estimators

In [54]:
```python
data['Date'] = data.index
data['Month'] = data['Date'].dt.to_period('M')
```

In [55]:
```python
# Assurez-vous que l'index est de type DateTimeIndex
data.index = pd.to_datetime(data.index)

# Calculer les estimateurs de volatilité
data['Estimator1'] = pd.Series(close_to_close(data))
data['Estimator2'] = pd.Series(garman_klass__estimator(data))
data['Estimator3'] = pd.Series(Parkinson(data))
data['Estimator4'] = pd.Series(rogers_satchell_volatility(data))
data['Estimator5'] = pd.Series(calculate_yang_zhang(data, k=1))

# Calculer la vraie volatilité mensuelle (par exemple, l'écart-type mensuel des rendemen
data['True_Volatility'] = data['Close'].pct_change().rolling(window=30).std()
data = data.fillna(method='bfill')

# Grouper par mois et calculer la volatilité réelle mensuelle
monthly_true_volatility = data['True_Volatility'].resample('M').mean()

# Calculer le MSE mensuel pour chaque estimateur
def calculate_monthly_mse(estimateur, true_volatility):
    return ((estimateur - true_volatility) ** 2).resample('M').mean()
monthly_mse = pd.DataFrame({
    'Estimator1': calculate_monthly_mse(data['Estimator1'], data['True_Volatility']),
    'Estimator2': calculate_monthly_mse(data['Estimator2'], data['True_Volatility']),
    'Estimator3': calculate_monthly_mse(data['Estimator3'], data['True_Volatility']),
```

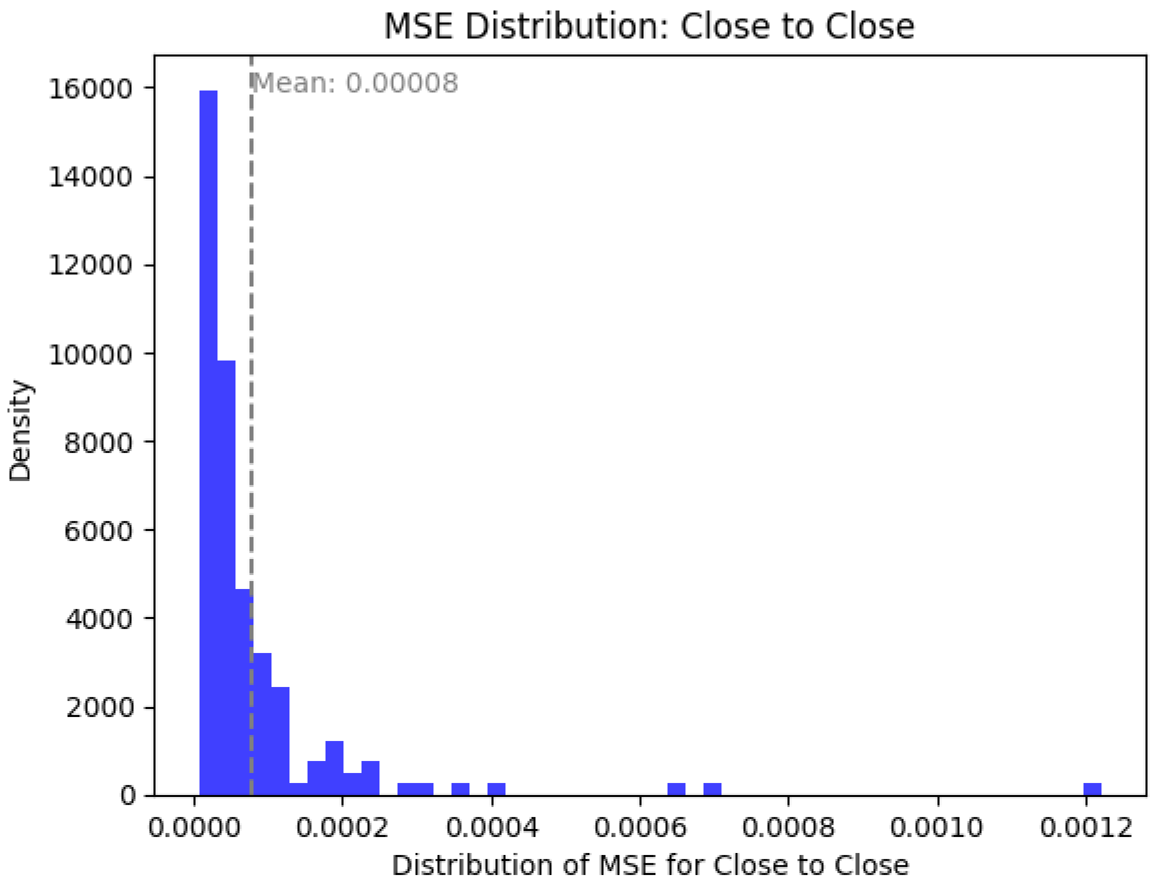
```
    'Estimator4': calculate_monthly_mse(data['Estimator4'], data['True_Volatility']),
    'Estimator5': calculate_monthly_mse(data['Estimator5'], data['True_Volatility'])
})

# Tracer les données
plt.figure(figsize=(14, 7))

# Ajouter une étiquette de légende explicite et définir les couleurs pour chaque estimat
plt.plot(monthly_mse.index.tolist(), monthly_mse['Estimator1'].tolist(), label='Close to
plt.plot(monthly_mse.index.tolist(), monthly_mse['Estimator2'].tolist(), label='Garman K
plt.plot(monthly_mse.index.tolist(), monthly_mse['Estimator3'].tolist(), label='Parkinso
plt.plot(monthly_mse.index.tolist(), monthly_mse['Estimator4'].tolist(), label='Rogers S
plt.plot(monthly_mse.index.tolist(), monthly_mse['Estimator5'].tolist(), label='Yang Zha

plt.title('Monthly MSE of Volatility Estimators')
plt.xlabel('Month')
plt.ylabel('MSE')
plt.legend()
plt.grid(True)
plt.show()
```

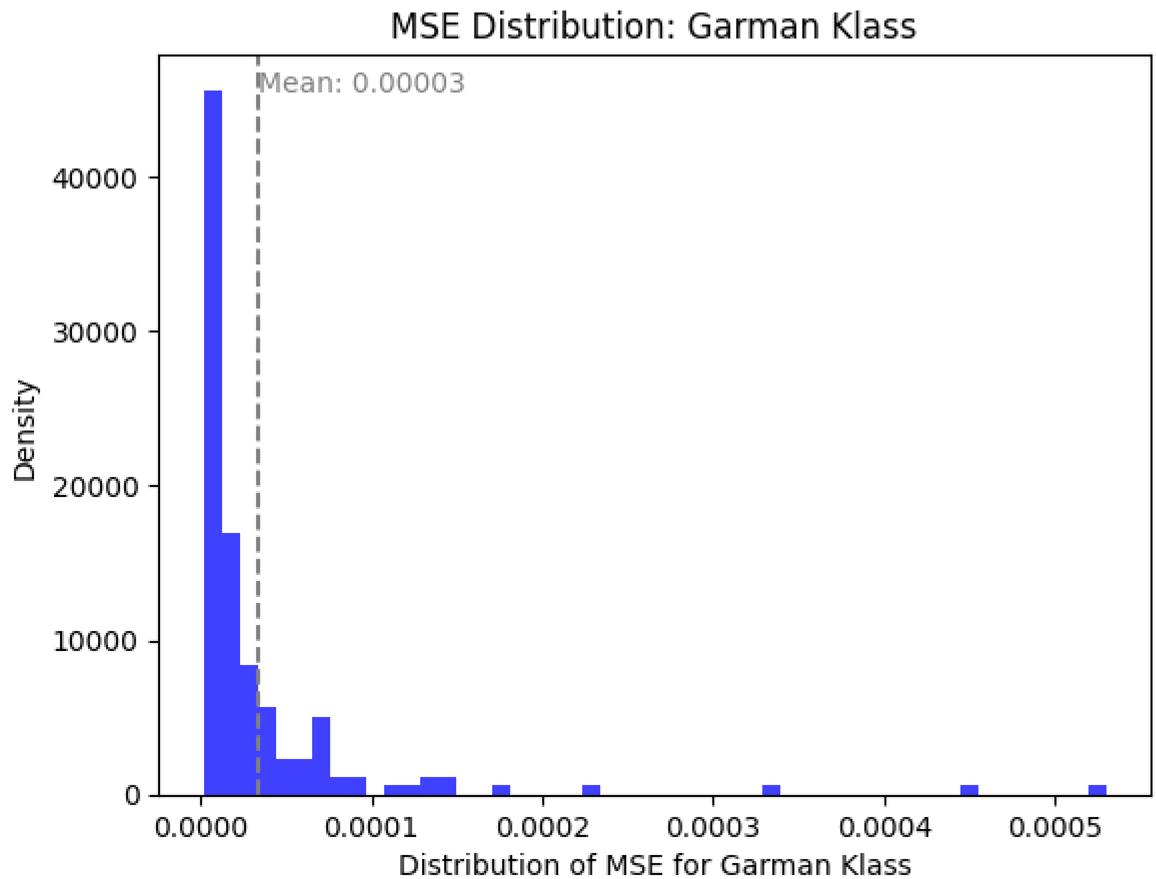


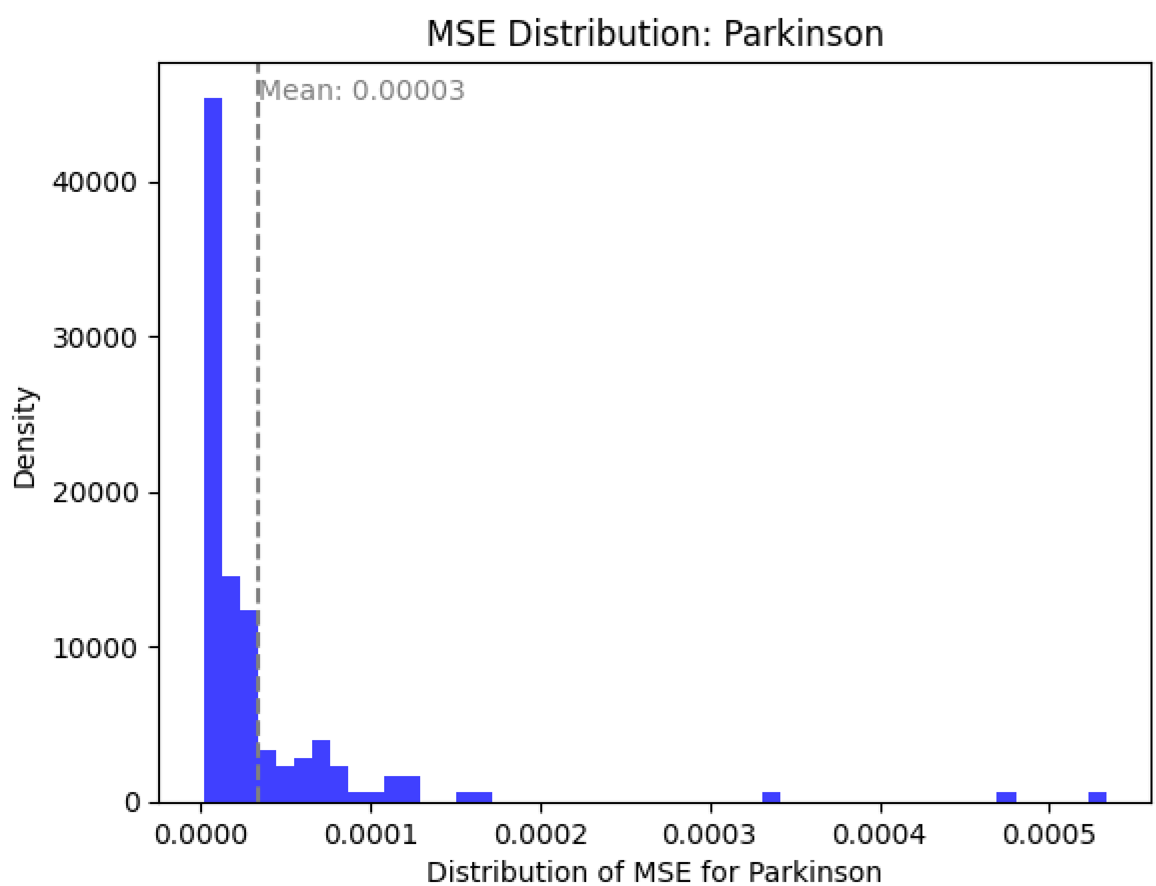
MSE distribution

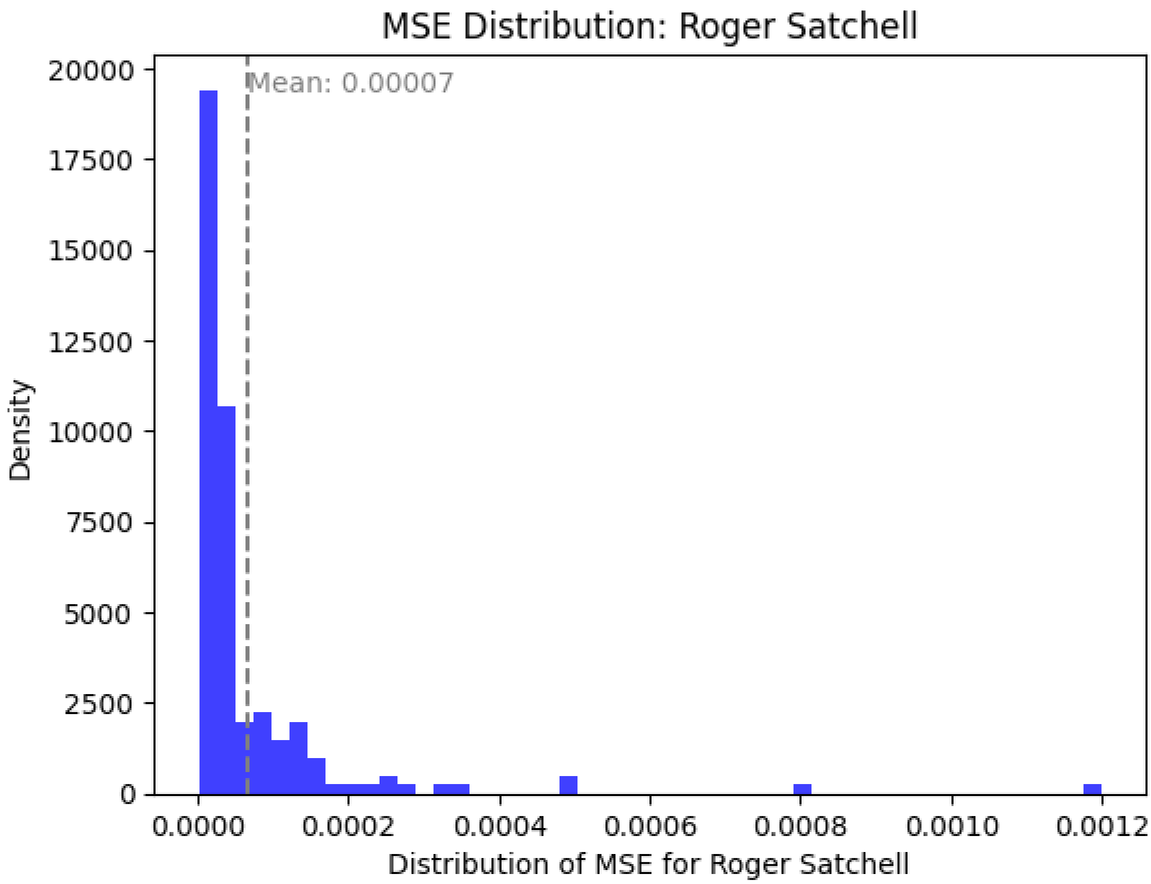
In [56]:
```
import matplotlib.pyplot as plt

legends = ['Close to Close', 'Garman Klass', 'Parkinson', 'Roger Satchell', 'Yang Zhang'


mse_means = {}

for i in range(1, 6):
    x = f'Estimator{i}'
    liste = monthly_mse[x].tolist()

    mean_mse = np.mean(liste)
    mse_means[legends[i-1]] = mean_mse  # Stocker la moyenne dans le dictionnaire avec l

    density, bins, _ = plt.hist(liste, bins=50, alpha=0.75, color='blue', density=True)
    plt.xlabel(f'Distribution of MSE for {legends[i-1]}')
    plt.ylabel('Density')
    plt.axvline(x=mean_mse, color='gray', linestyle='--')
```

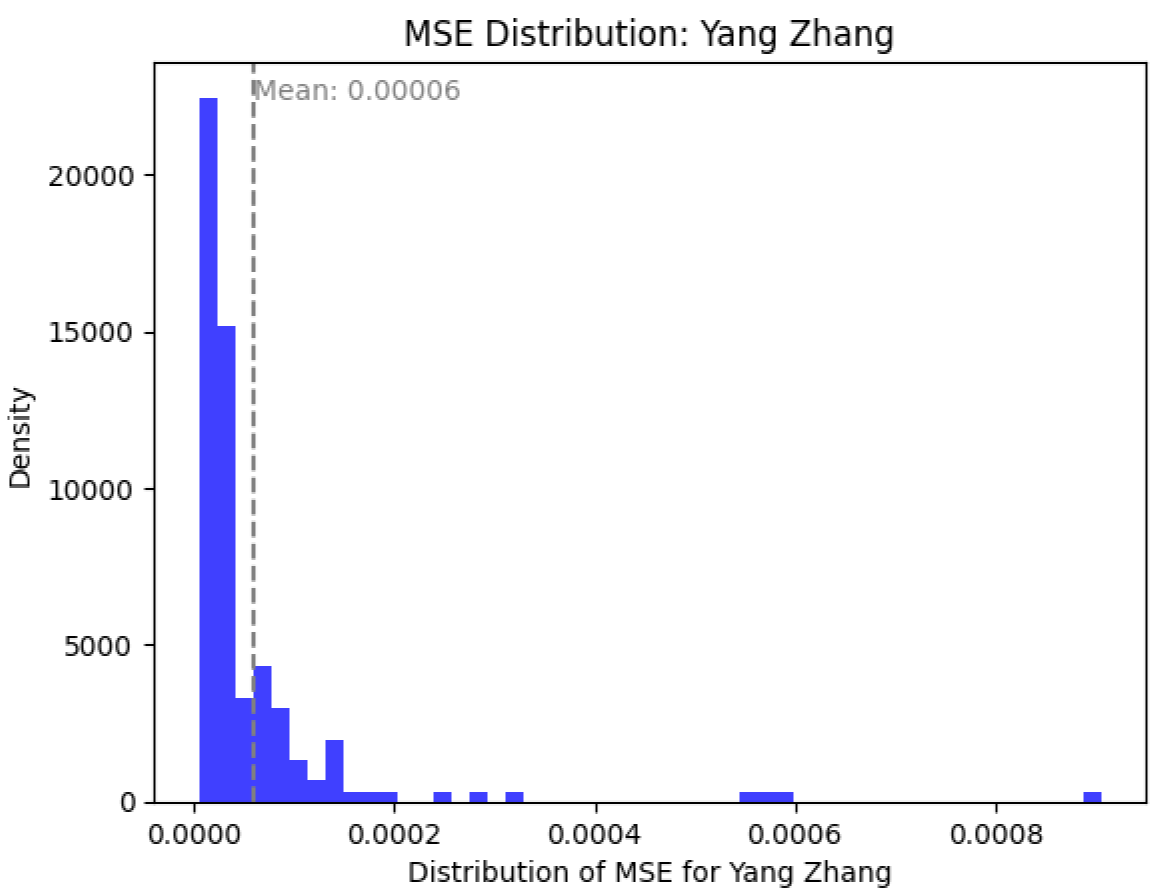
```
    plt.text(mean_mse, plt.ylim()[1] * 0.95, f'Mean: {mean_mse:.5f}', color='gray', ha='
    plt.title(f'MSE Distribution: {legends[i-1]}')
    plt.show()

sorted_means = sorted(mse_means.items(), key=lambda item: item[1])
print("Estimators sorted from least to most biased based on MSE:")
for estimator, mse in sorted_means:
    print(f"{estimator}: {mse:.5f}")
```




MSE Distribution: Garman Klass
Mean: 0.00003
Density
40000
30000
20000
10000
0
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
Distribution of MSE for Garman Klass


MSE Distribution: Parkinson
Mean: 0.00003
Density
40000
30000
20000
10000
0
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
Distribution of MSE for Parkinson






```
Estimators sorted from least to most biased based on MSE:
Garman Klass: 0.00003
Parkinson: 0.00003
Yang Zhang: 0.00006
Roger Satchell: 0.00007
Close to Close: 0.00008
```

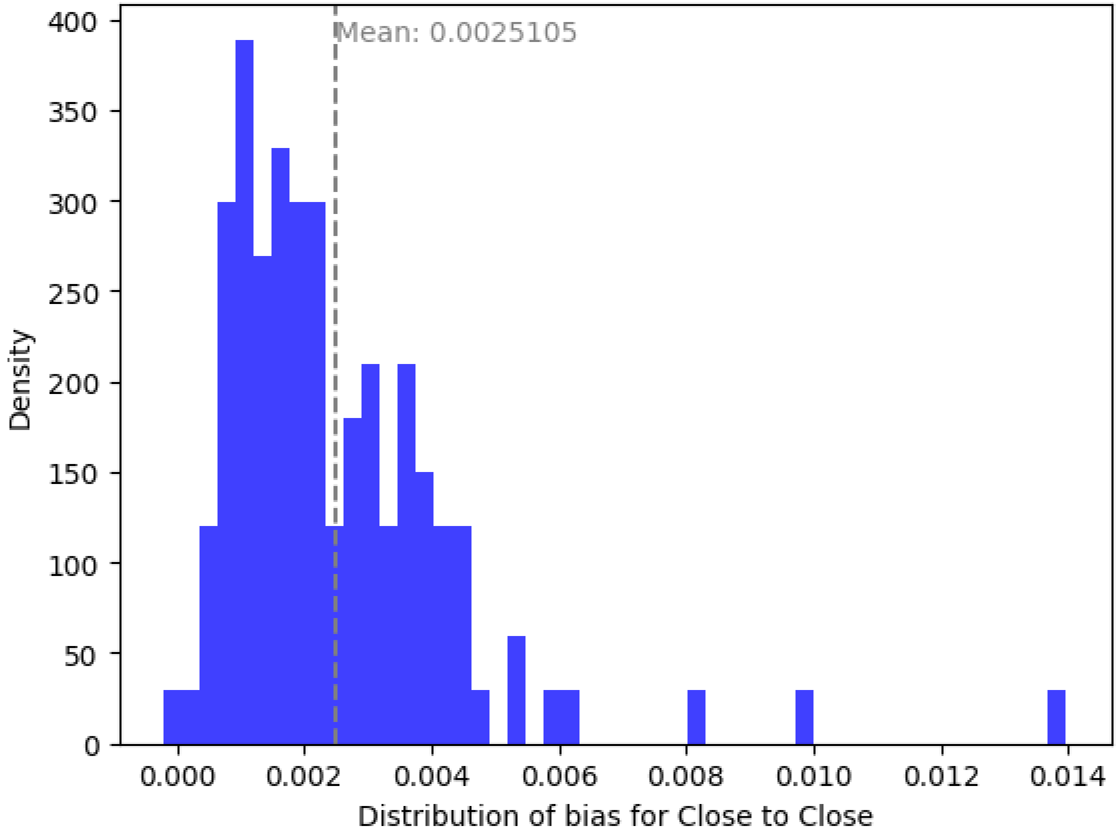
Bias distribution

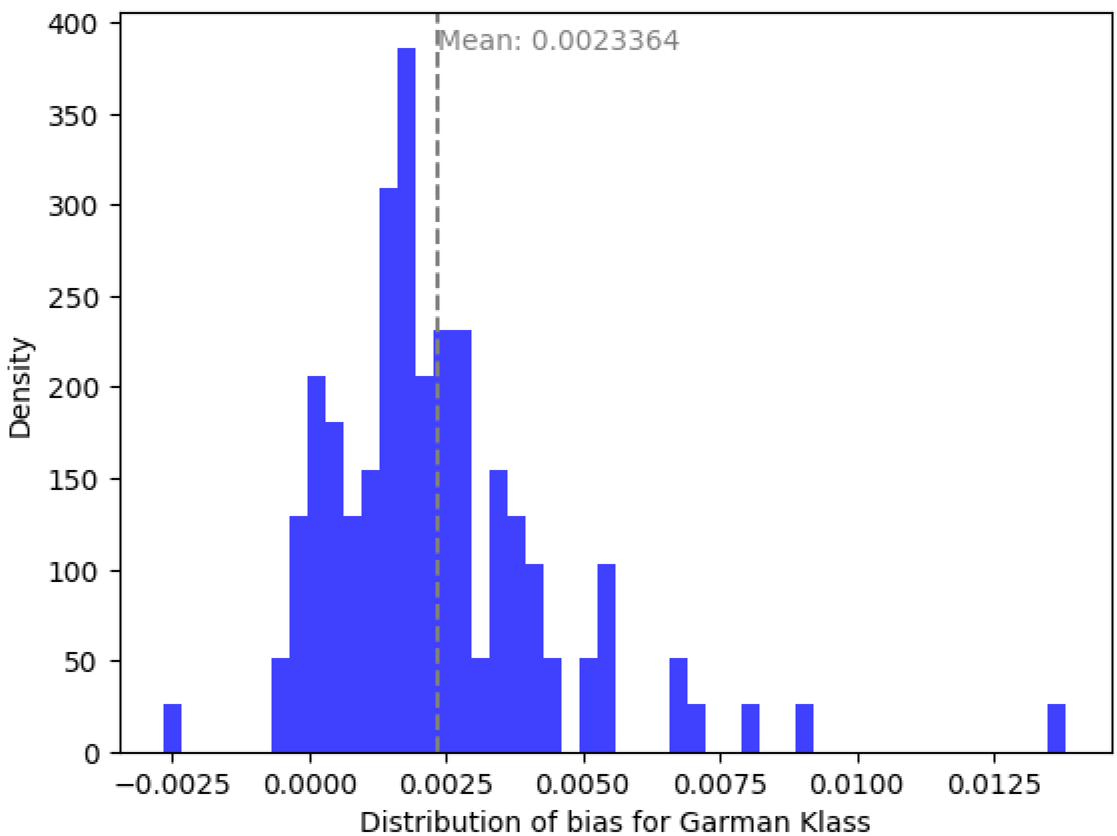
In [57]:

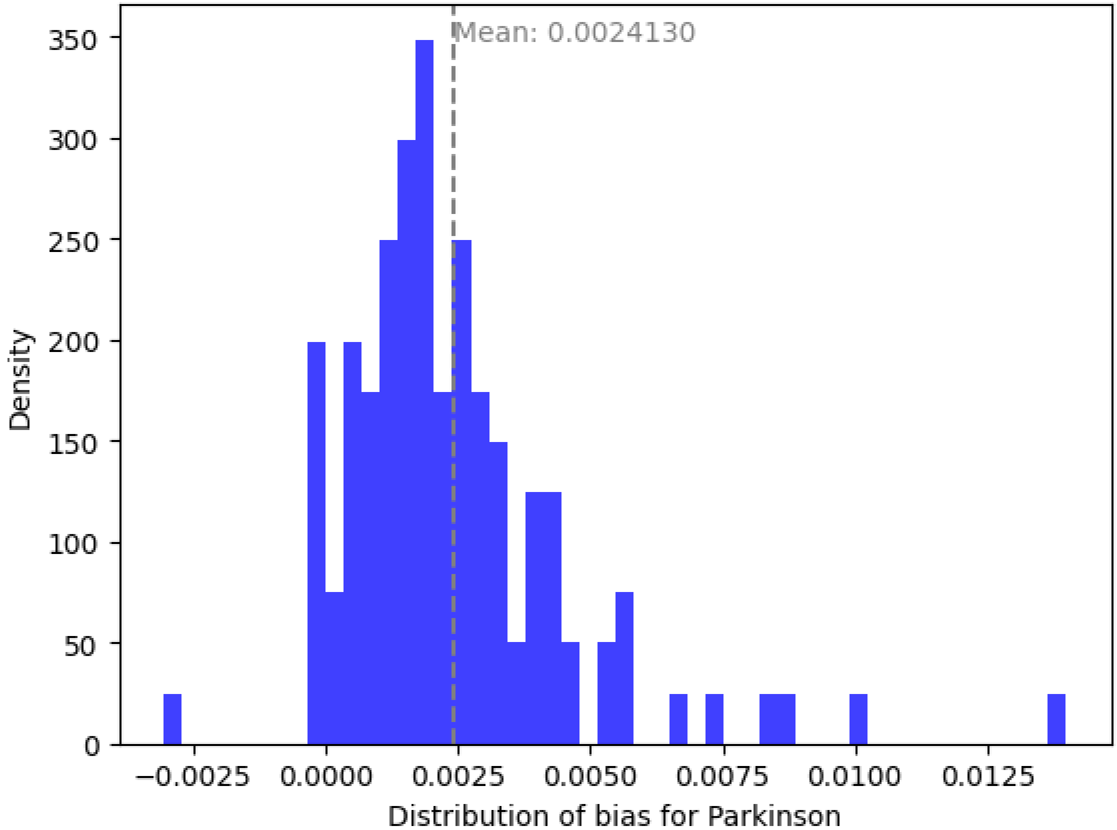
```
monthly_true_volatility = data['True_Volatility'].resample('M').mean()
data['True_Volatility'] = data['Close'].pct_change().rolling(window=30).std()

def calculate_monthly_mean(estimateur):
    return estimateur.resample('M').mean()

def calculate_monthly_bias(estimator, true_volatility):
    return (true_volatility-calculate_monthly_mean(estimator))

monthly_bias = pd.DataFrame({
    'Estimator1': calculate_monthly_bias(data['Estimator1'], data['True_Volatility']),
    'Estimator2': calculate_monthly_bias(data['Estimator2'], data['True_Volatility']),
    'Estimator3': calculate_monthly_bias(data['Estimator3'], data['True_Volatility']),
    'Estimator4': calculate_monthly_bias(data['Estimator4'], data['True_Volatility']),
    'Estimator5': calculate_monthly_bias(data['Estimator5'], data['True_Volatility'])
})
```

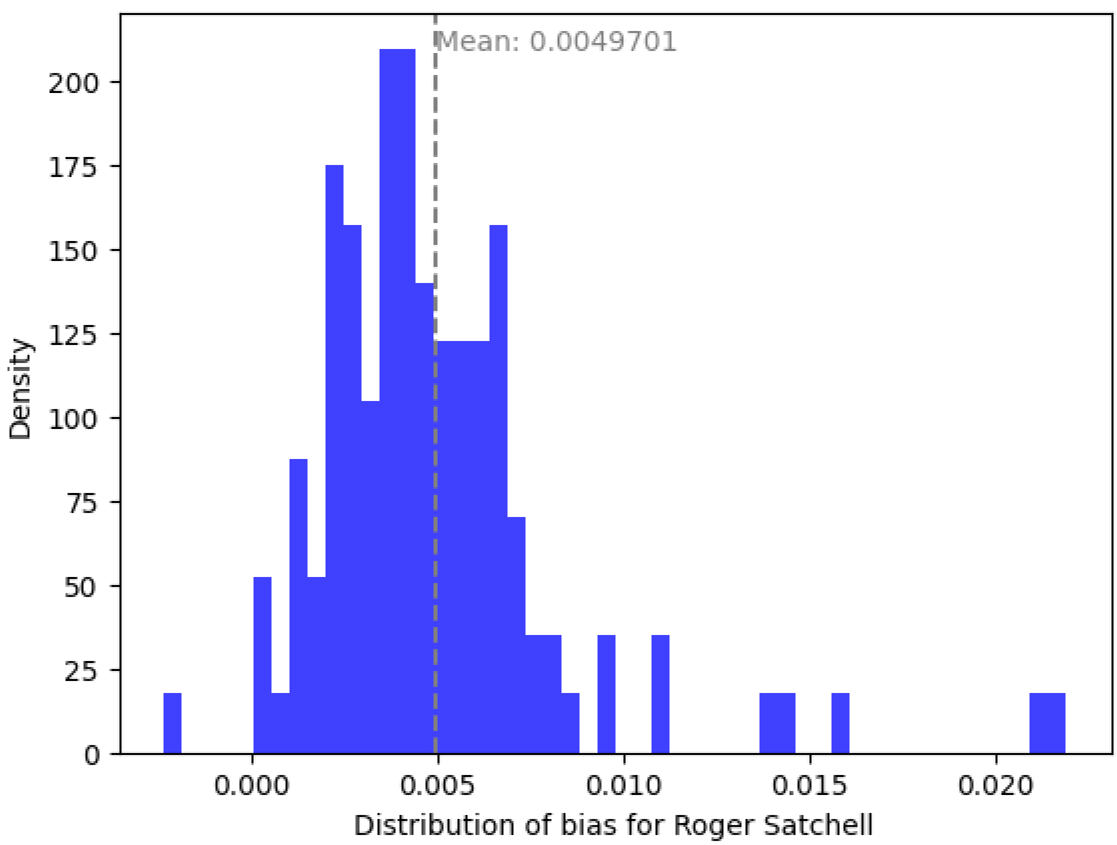
In [58]:

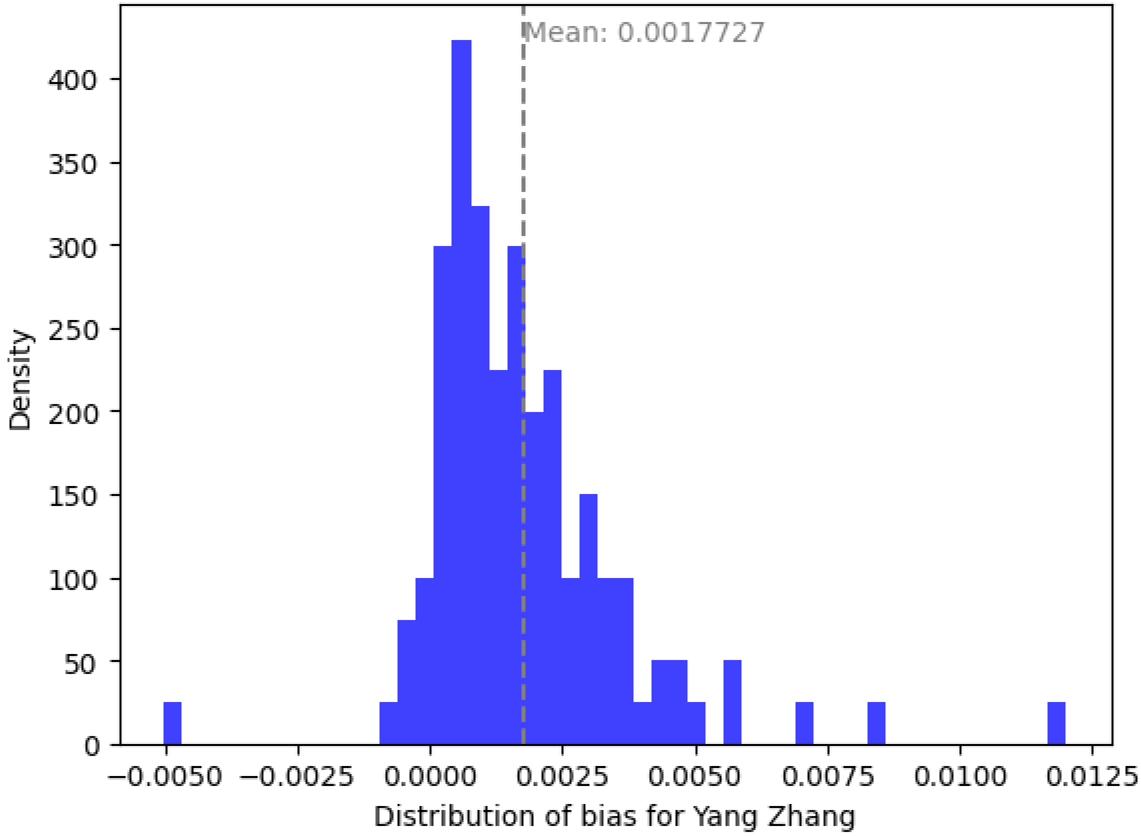
```
import matplotlib.pyplot as plt

bias_means = {}

for i in range (1,6):

    x = f'Estimator{i}'
    liste = monthly_bias[x].tolist()
    bias_means[legends[i-1]] = np.nanmean(liste)

    density, bins, _ = plt.hist(liste, bins=50, alpha=0.75, color='blue', density=True)
    bin_centers = 0.5 * (bins[1:] + bins[:-1])
    plt.xlabel(f'Distribution of bias for {legends[i-1]}')
    plt.ylabel('Density')
    plt.axvline(x=np.nanmean(liste),color='gray',linestyle='--')
    plt.text(np.nanmean(liste), plt.ylim()[1] * 0.95, f'Mean: {np.nanmean(liste):.7f}',
    plt.show()

sorted_estimators = sorted(bias_means.items(), key=lambda item: item[1])

# Affichage des estimateurs triés
print("Estimators sorted from least to most biased based on MSE:")
for estimator, bias in sorted_estimators:
    print(f"{estimator}: {bias:.7f}")
```


Mean: 0.0025105
Density
0
50
100
150
200
250
300
350
400
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014
Distribution of bias for Close to Close


Mean: 0.0023364
Density
0
50
100
150
200
250
300
350
400
−0.0025
0.0000
0.0025
0.0050
0.0075
0.0100
0.0125
Distribution of bias for Garman Klass


Mean: 0.0024130
Density
Distribution of bias for Parkinson


Mean: 0.0049701
Density
Distribution of bias for Roger Satchell



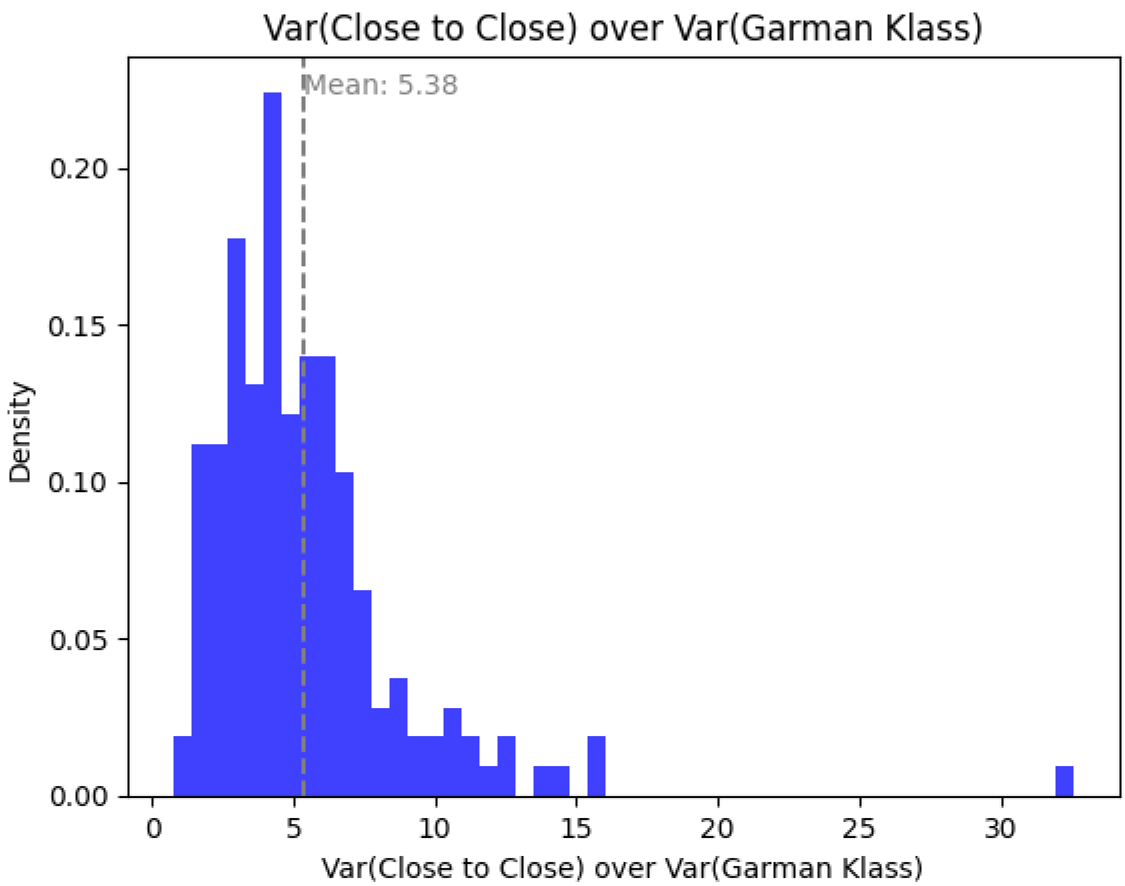

```
Estimators sorted from least to most biased based on MSE:
Yang Zhang: 0.0017727
Garman Klass: 0.0023364
Parkinson: 0.0024130
Close to Close: 0.0025105
Roger Satchell: 0.0049701
```

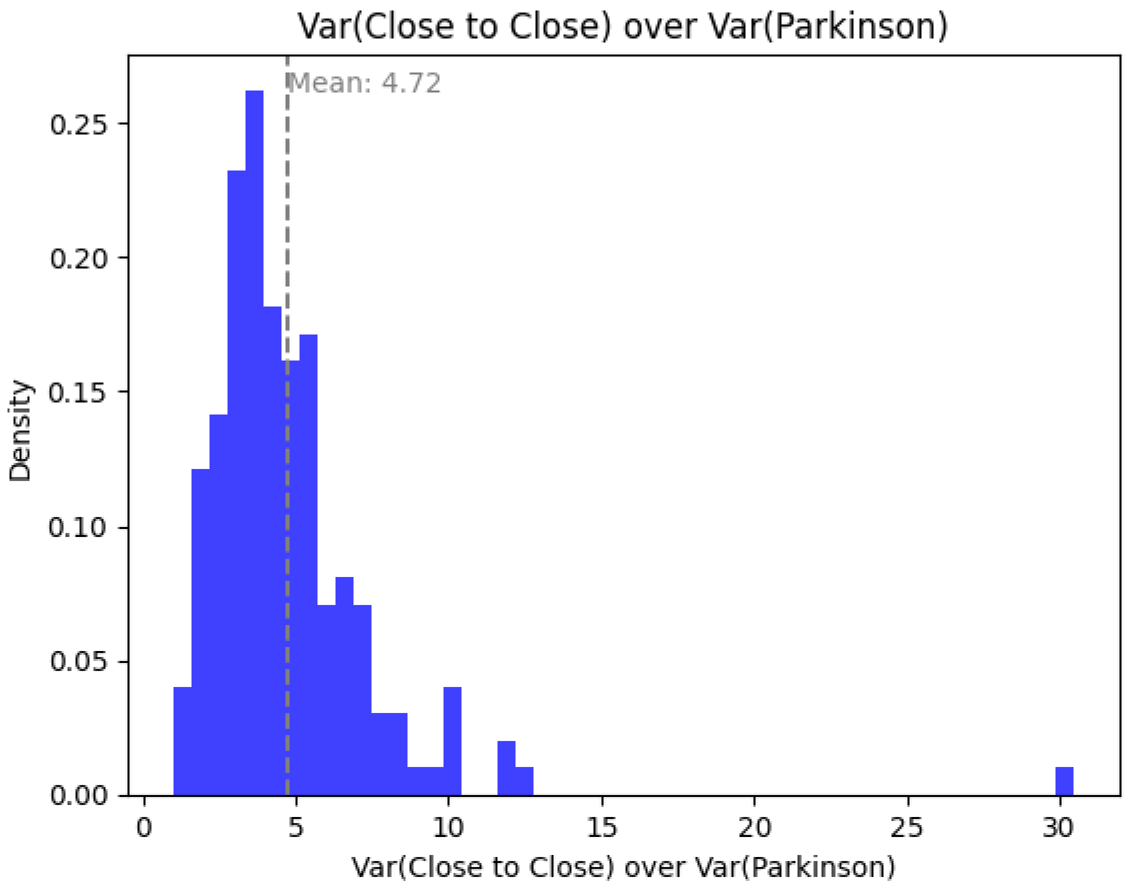
Visualization of Mean Variance Ratios between estimator pairs

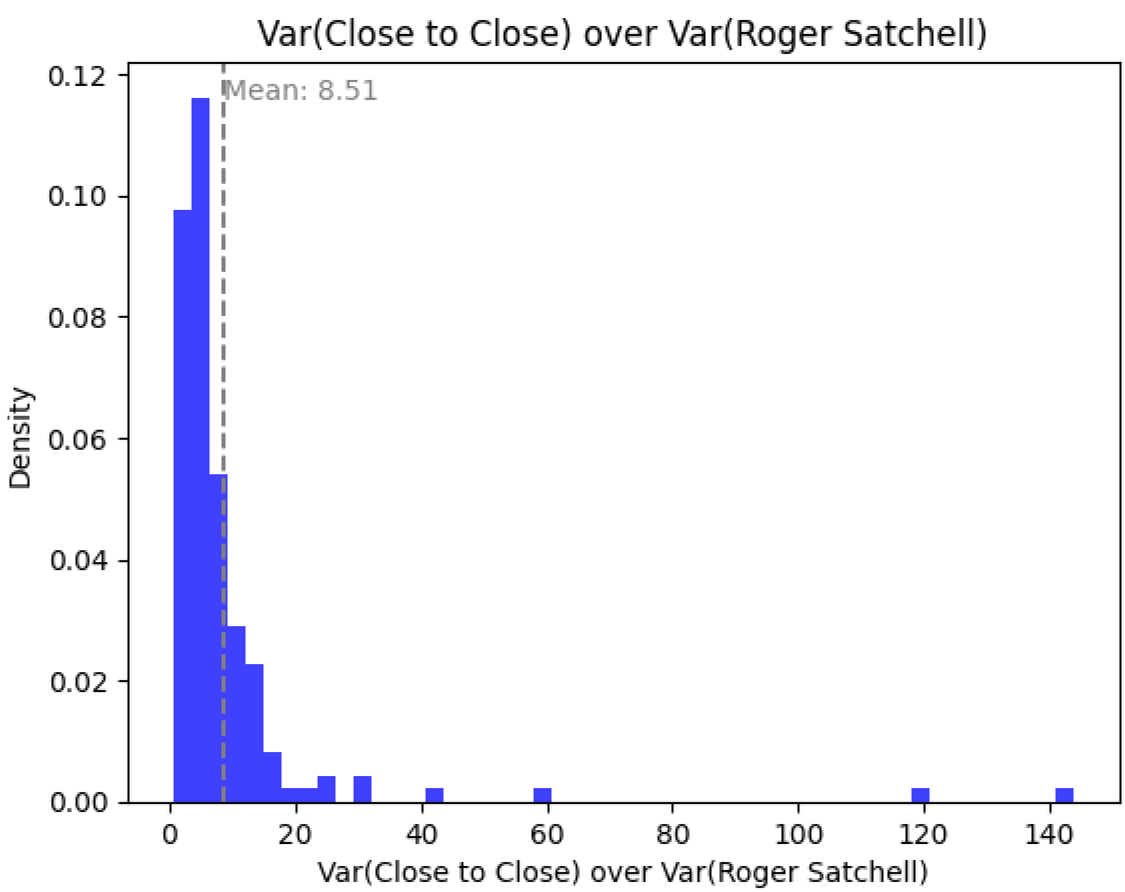
In [59]:

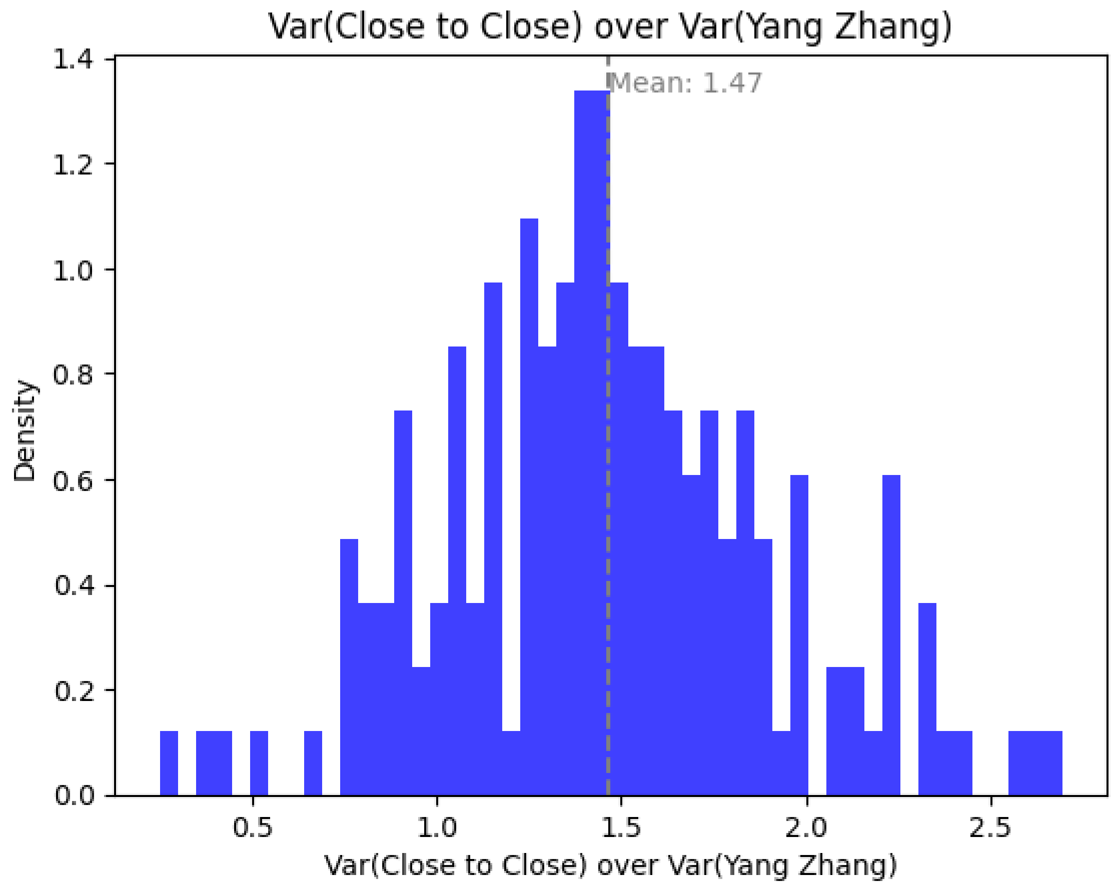
```
def calculate_monthly_var(estimator):
    return estimator.resample('M').var()

# Création d'un DataFrame avec la variance mensuelle pour chaque estimateur
monthly_var = pd.DataFrame({
    'Estimator1': calculate_monthly_var(data['Estimator1']),
    'Estimator2': calculate_monthly_var(data['Estimator2']),
    'Estimator3': calculate_monthly_var(data['Estimator3']),
    'Estimator4': calculate_monthly_var(data['Estimator4']),
    'Estimator5': calculate_monthly_var(data['Estimator5'])
})

# Fonction pour diviser les listes élément par élément
div = lambda l1, l2: [l1[k]/l2[k] for k in range(len(l1))]

# Dictionnaire pour stocker les moyennes des rapports de variance
variance_ratios_means = {}

# Boucle à travers chaque paire d'estimateurs
for i in range(1, 6):
    for j in range(i+1, 6):
        x = f'Estimator{i}'
        y = f'Estimator{j}'
        liste_x = monthly_var[x].tolist()
        liste_y = monthly_var[y].tolist()
        ratio_list = div(liste_x, liste_y)

 # Calcul de la moyenne du ratio et stockage dans le dictionnaire
        mean_ratio = np.nanmean(ratio_list)
```

```
        key_name = f'Var({legends[i-1]}) over Var({legends[j-1]})'
        variance_ratios_means[key_name] = mean_ratio

        # Affichage des histogrammes
        density, bins, _ = plt.hist(ratio_list, bins=50, alpha=0.75, color='blue', densi
        plt.xlabel(key_name)
        plt.ylabel('Density')
        plt.axvline(x=mean_ratio, color='gray', linestyle='--')
        plt.text(mean_ratio, plt.ylim()[1] * 0.95, f'Mean: {mean_ratio:.2f}', color='gra
        plt.title(key_name)
        plt.show()

# Affichage des moyennes triées par valeur
sorted_means = sorted(variance_ratios_means.items(), key=lambda item: item[1])
for name, mean in sorted_means:
    print(f"{name}: {mean:.2f}")
```

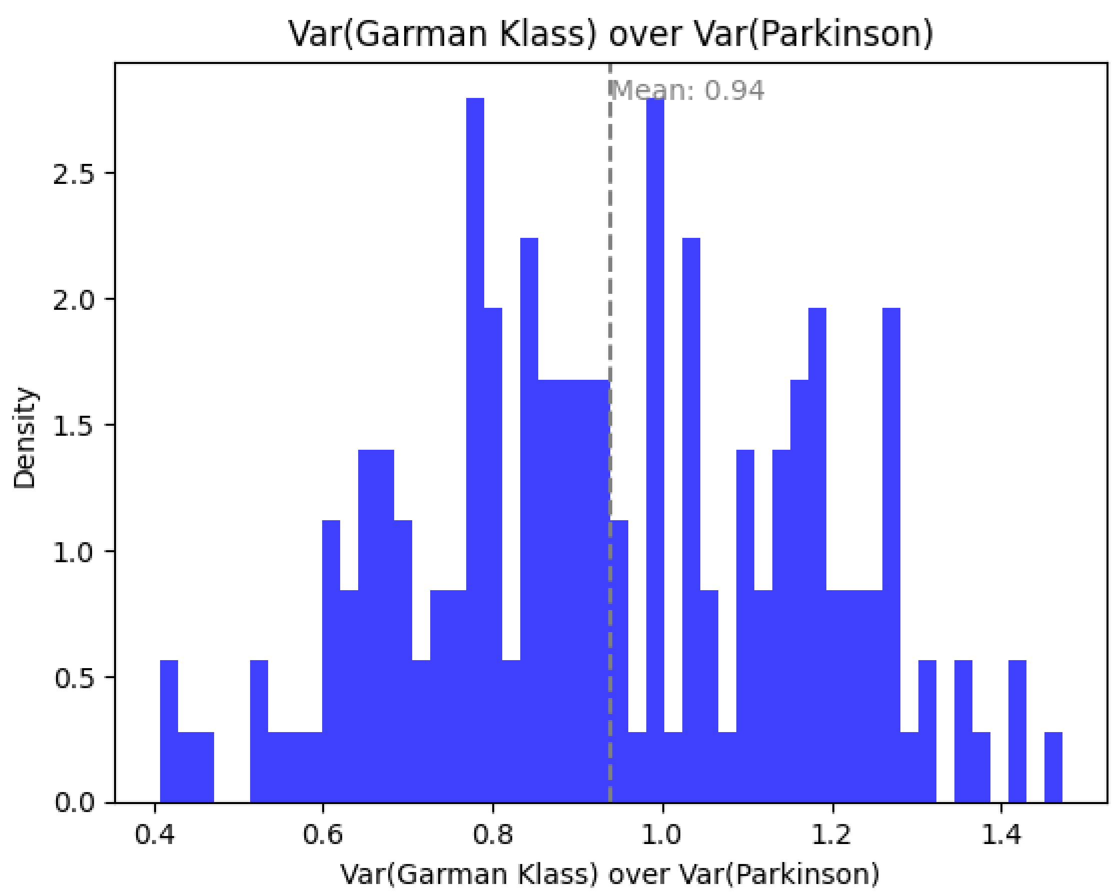



Var(Close to Close) over Var(Parkinson)
Mean: 4.72
Density
0.00
0.05
0.10
0.15
0.20
0.25
0
5
10
15
20
25
30
Var(Close to Close) over Var(Parkinson)

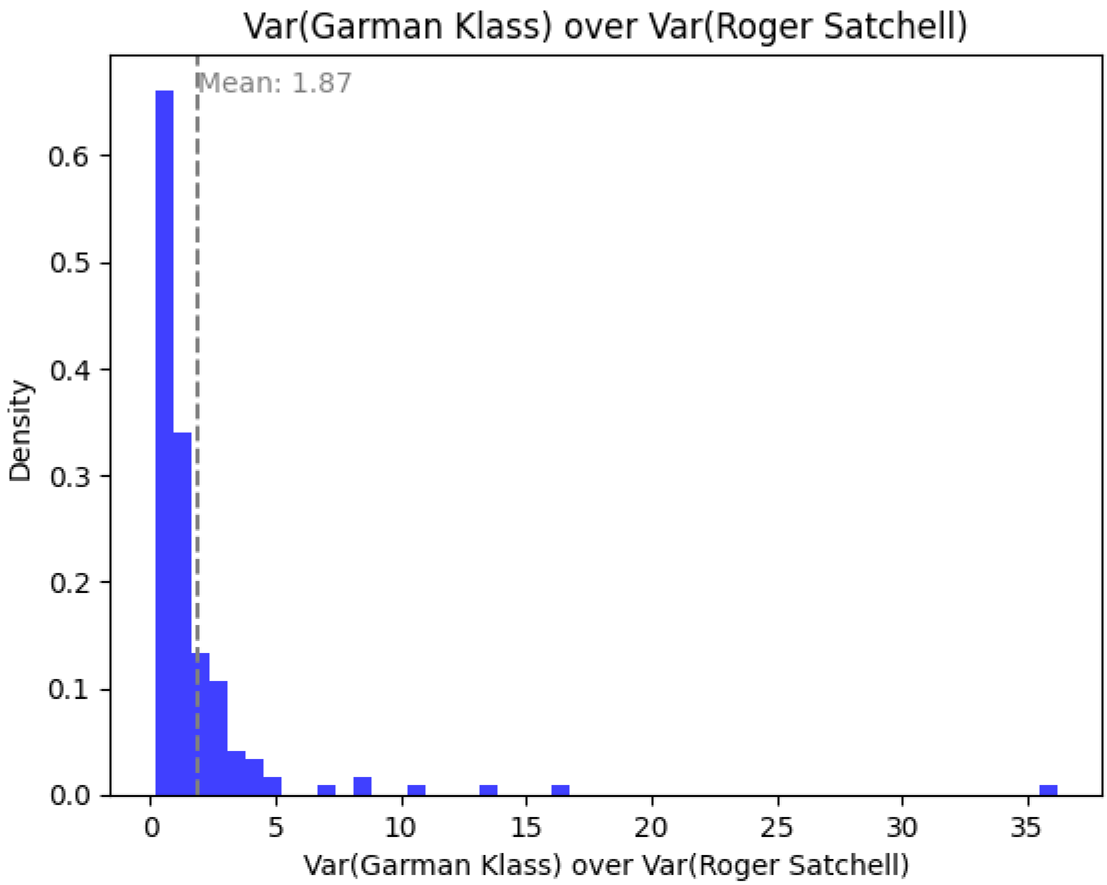

Var(Close to Close) over Var(Roger Satchell)
Mean: 8.51
Density
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
20
40
60
80
100
120
140
Var(Close to Close) over Var(Roger Satchell)



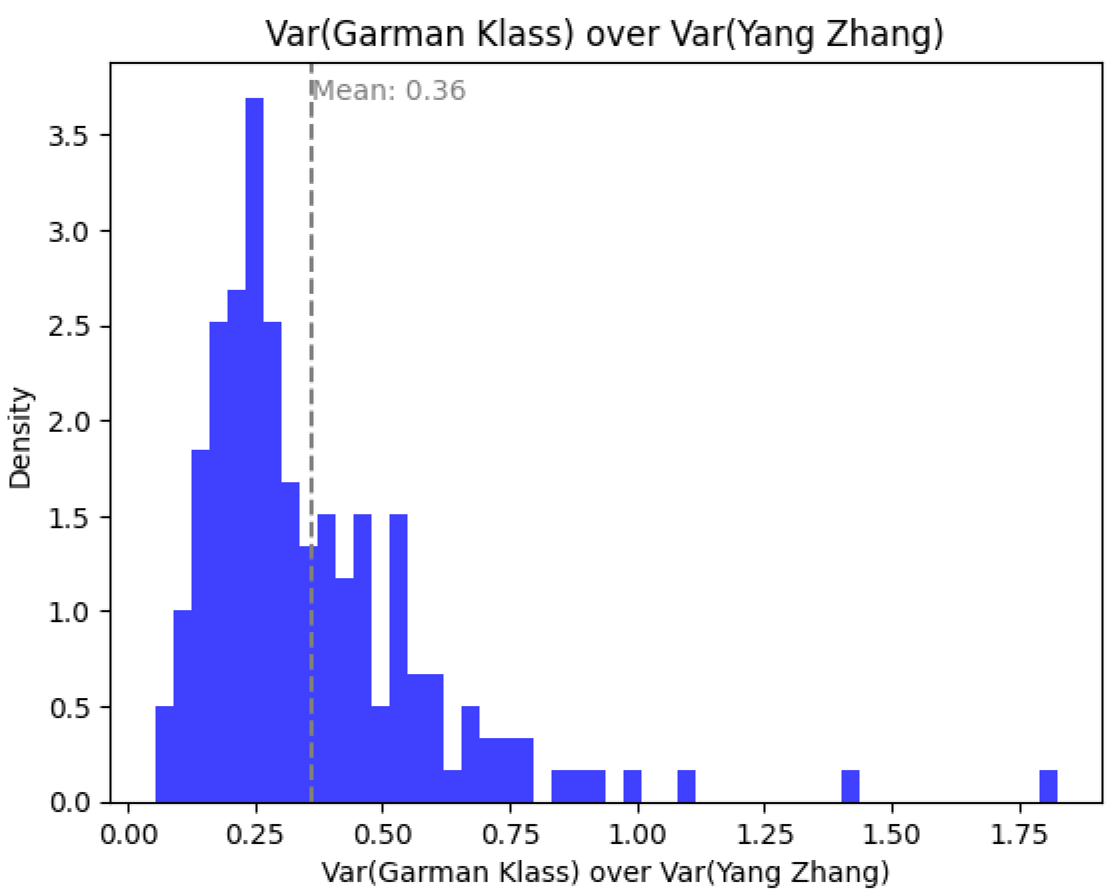
Var(Close to Close) over Var(Yang Zhang)
Mean: 1.47
Density
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.5
1.0
1.5
2.0
2.5
Var(Close to Close) over Var(Yang Zhang)



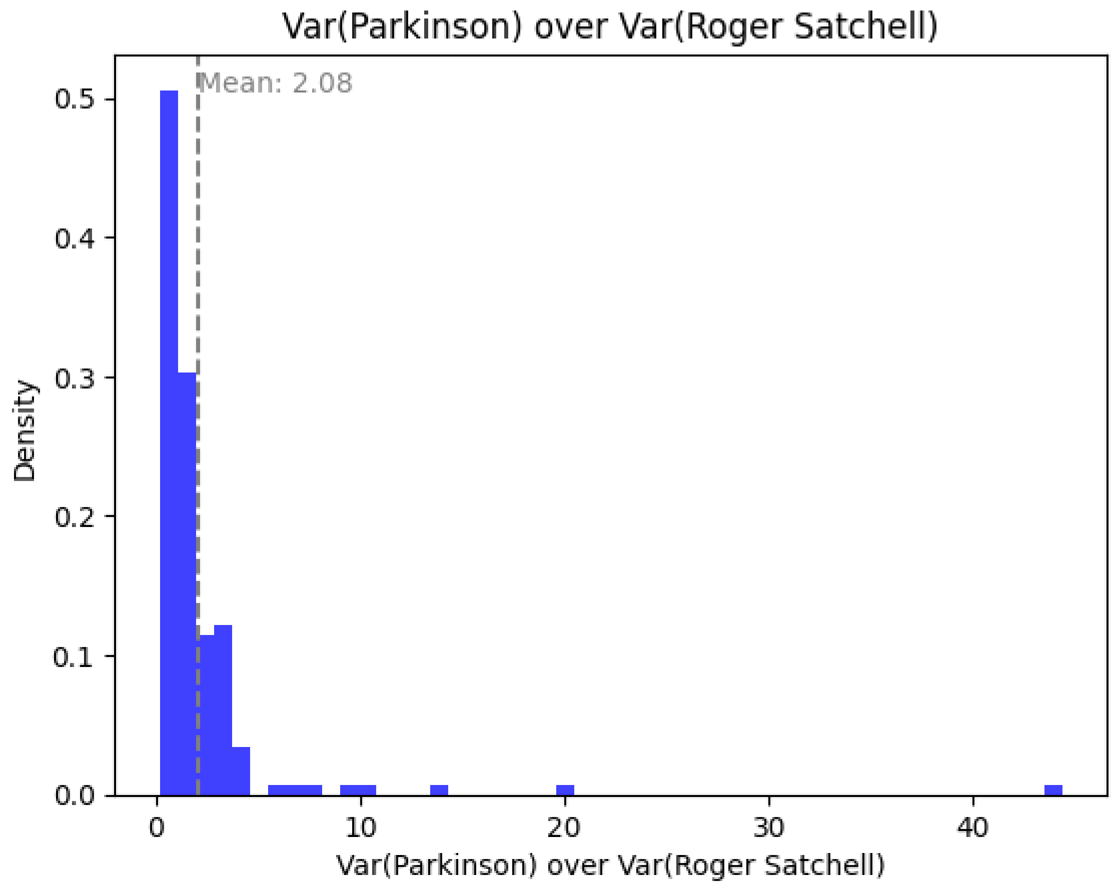
Var(Garman Klass) over Var(Parkinson)
Mean: 0.94
Density
0.0
0.5
1.0
1.5
2.0
2.5
0.4
0.6
0.8
1.0
1.2
1.4
Var(Garman Klass) over Var(Parkinson)

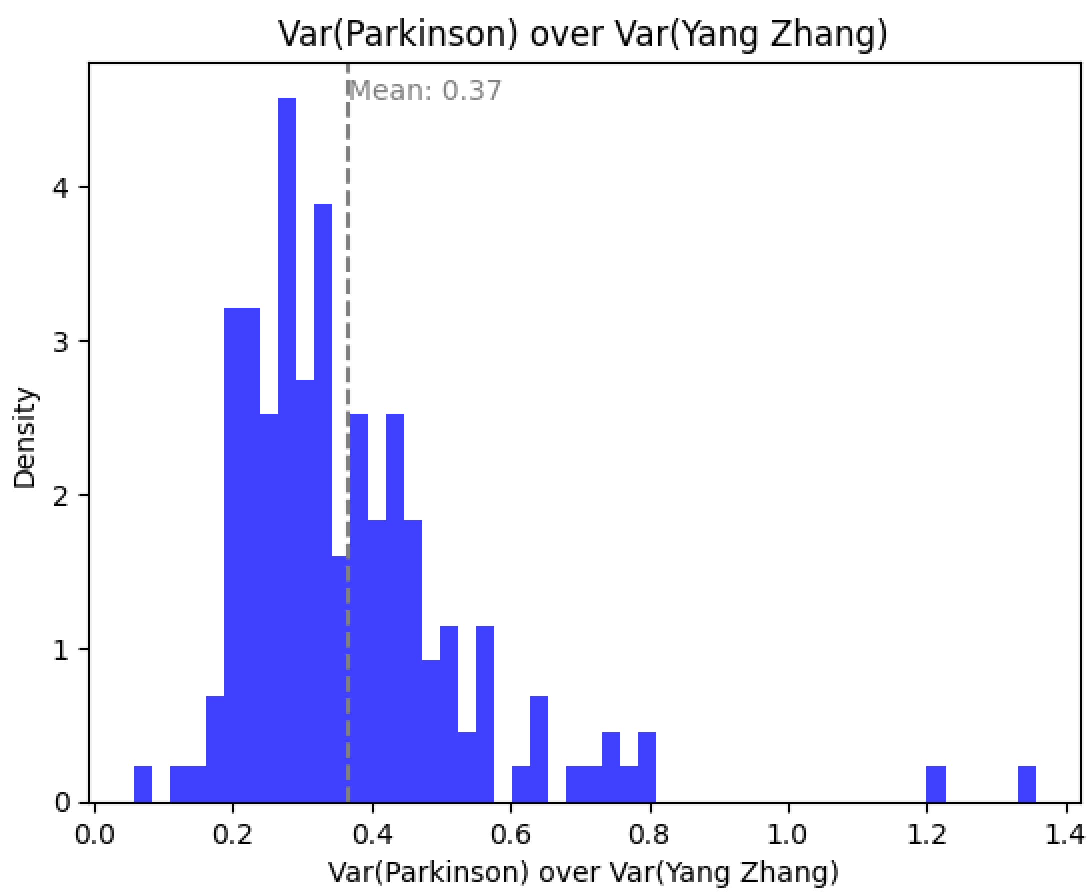


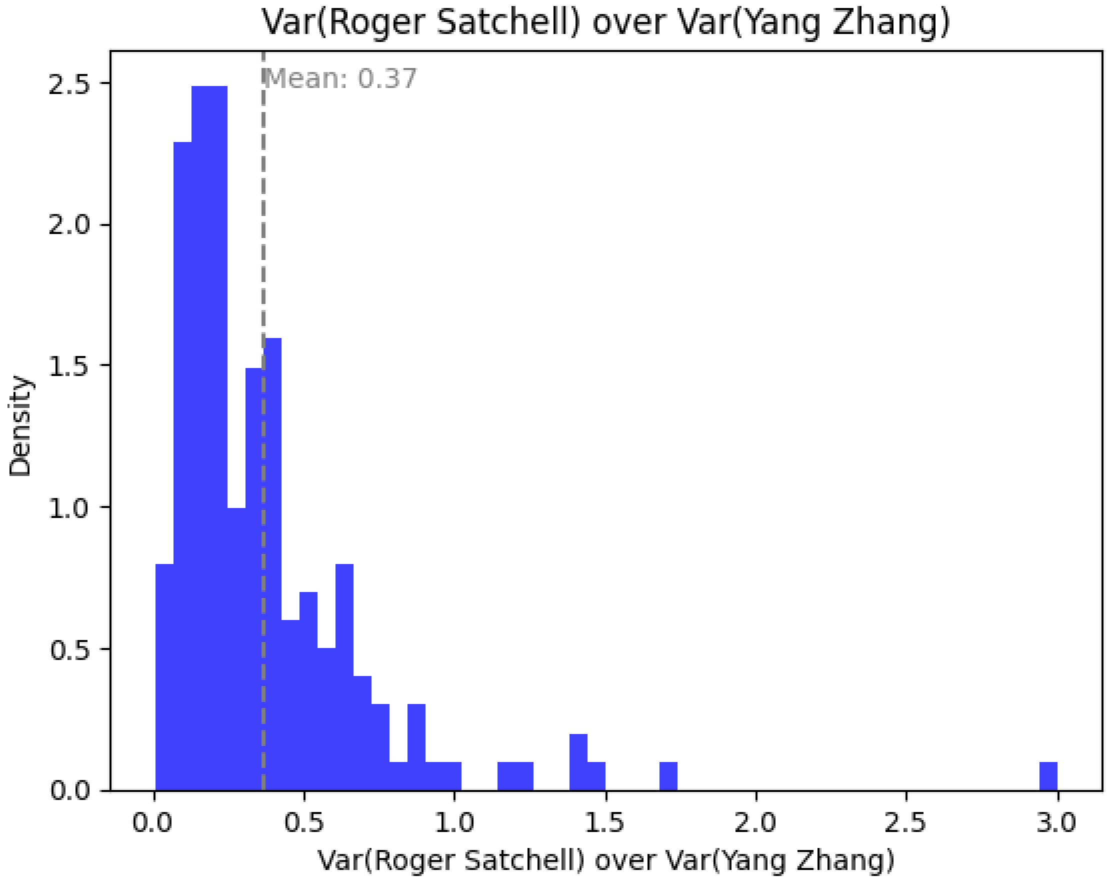
Var(Garman Klass) over Var(Roger Satchell)
Mean: 1.87
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
5
10
15
20
25
30
35
Var(Garman Klass) over Var(Roger Satchell)



Var(Garman Klass) over Var(Yang Zhang)
Mean: 0.36
Density
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
Var(Garman Klass) over Var(Yang Zhang)


Var(Parkinson) over Var(Roger Satchell)
Mean: 2.08
Density
0.0
0.1
0.2
0.3
0.4
0.5
0
10
20
30
40
Var(Parkinson) over Var(Roger Satchell)

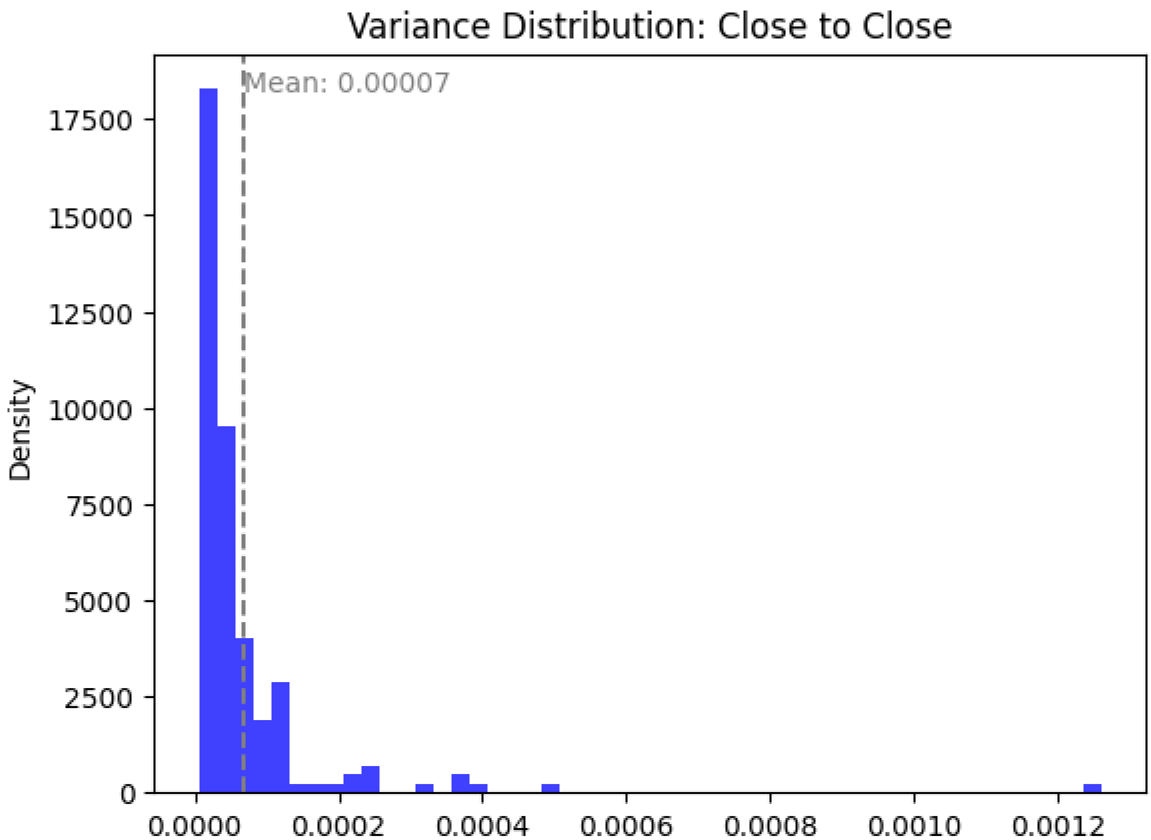

Var(Parkinson) over Var(Yang Zhang)
Mean: 0.37
Density
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Var(Parkinson) over Var(Yang Zhang)

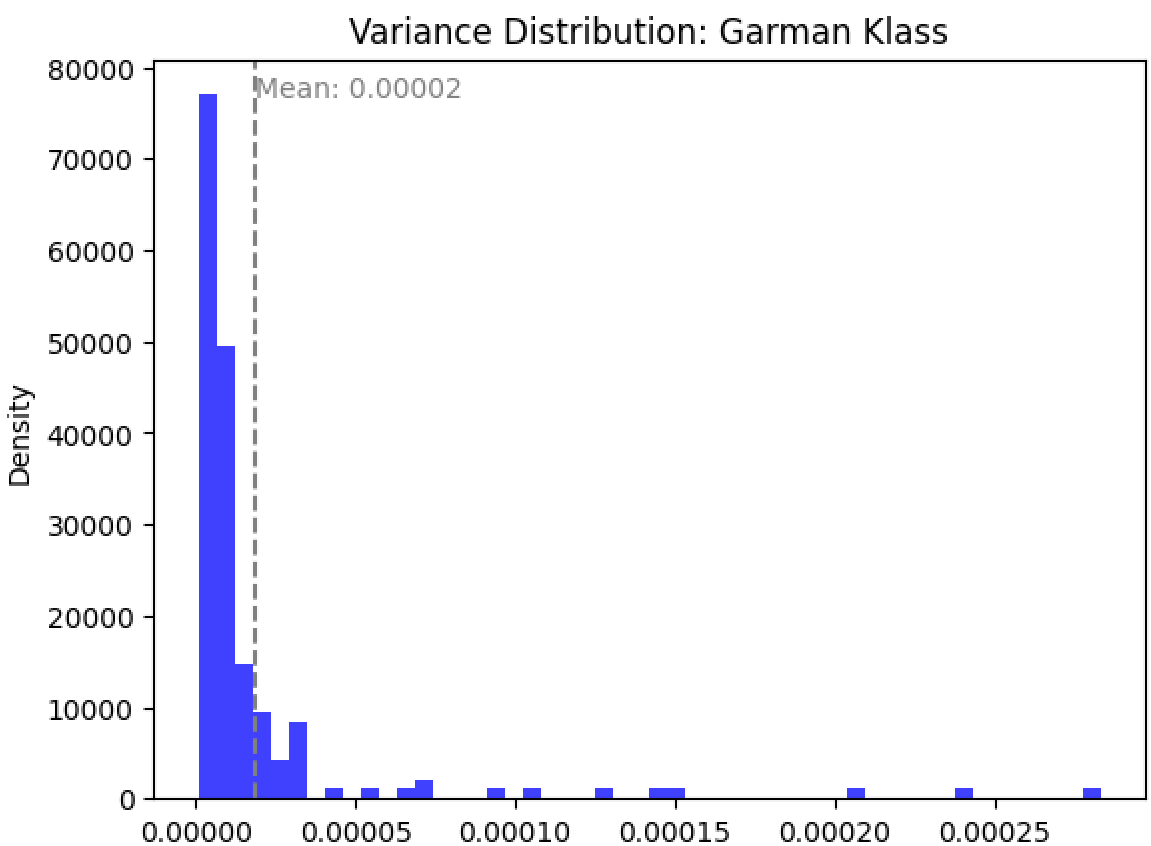


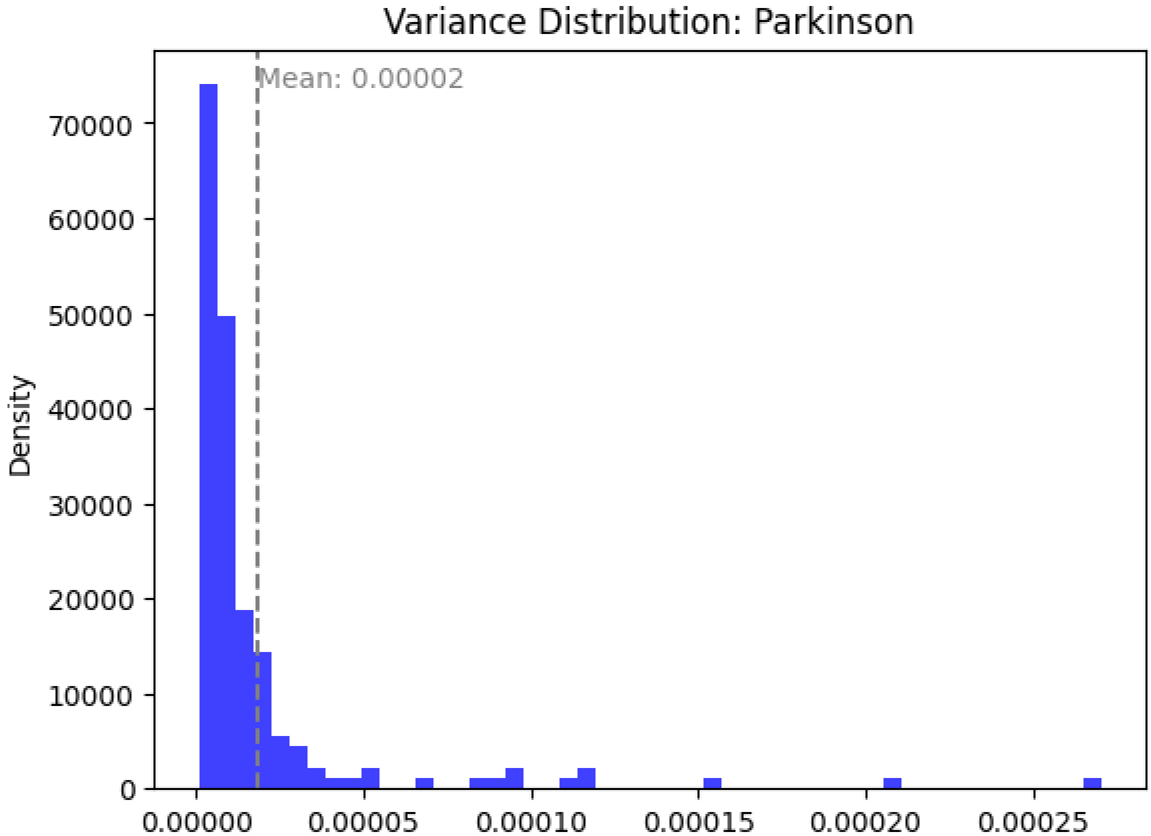

```
Var(Garman Klass) over Var(Yang Zhang): 0.36
Var(Parkinson) over Var(Yang Zhang): 0.37
Var(Roger Satchell) over Var(Yang Zhang): 0.37
Var(Garman Klass) over Var(Parkinson): 0.94
Var(Close to Close) over Var(Yang Zhang): 1.47
Var(Garman Klass) over Var(Roger Satchell): 1.87
Var(Parkinson) over Var(Roger Satchell): 2.08
Var(Close to Close) over Var(Parkinson): 4.72
Var(Close to Close) over Var(Garman Klass): 5.38
Var(Close to Close) over Var(Roger Satchell): 8.51
```

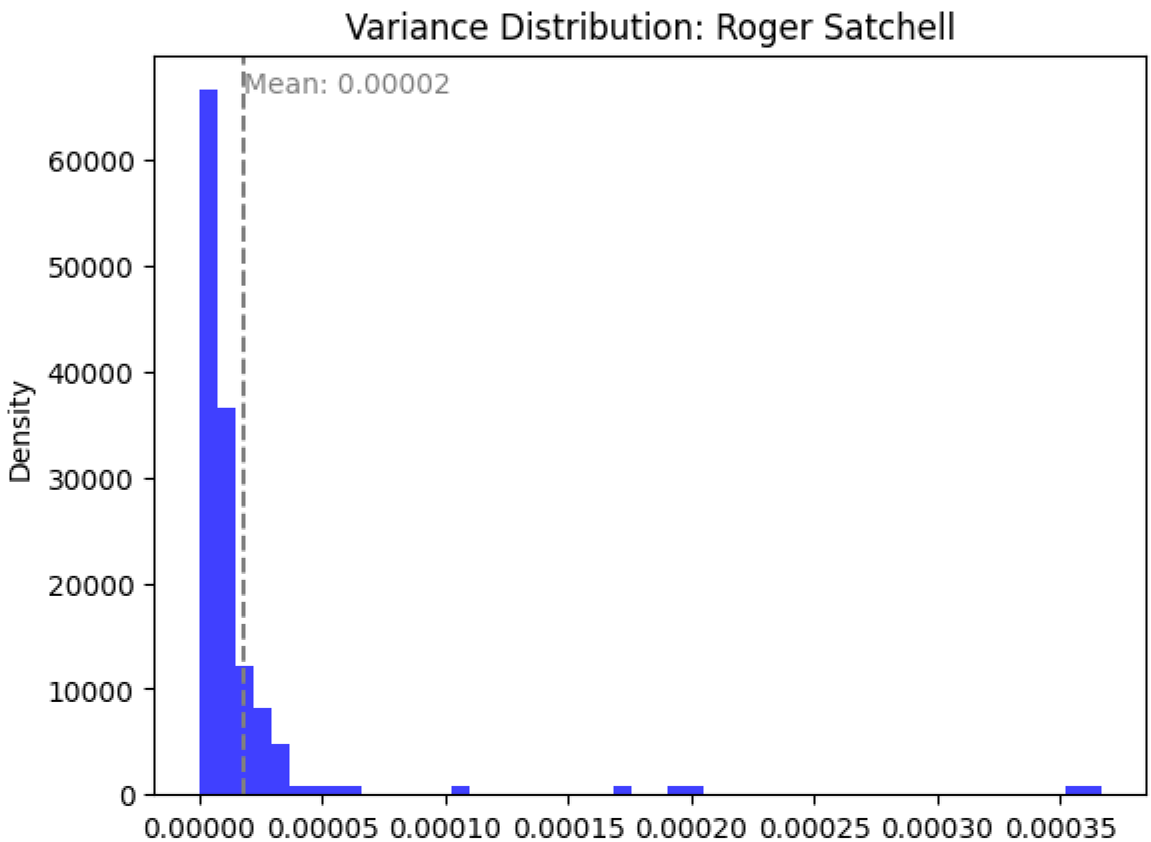
Variance distribution

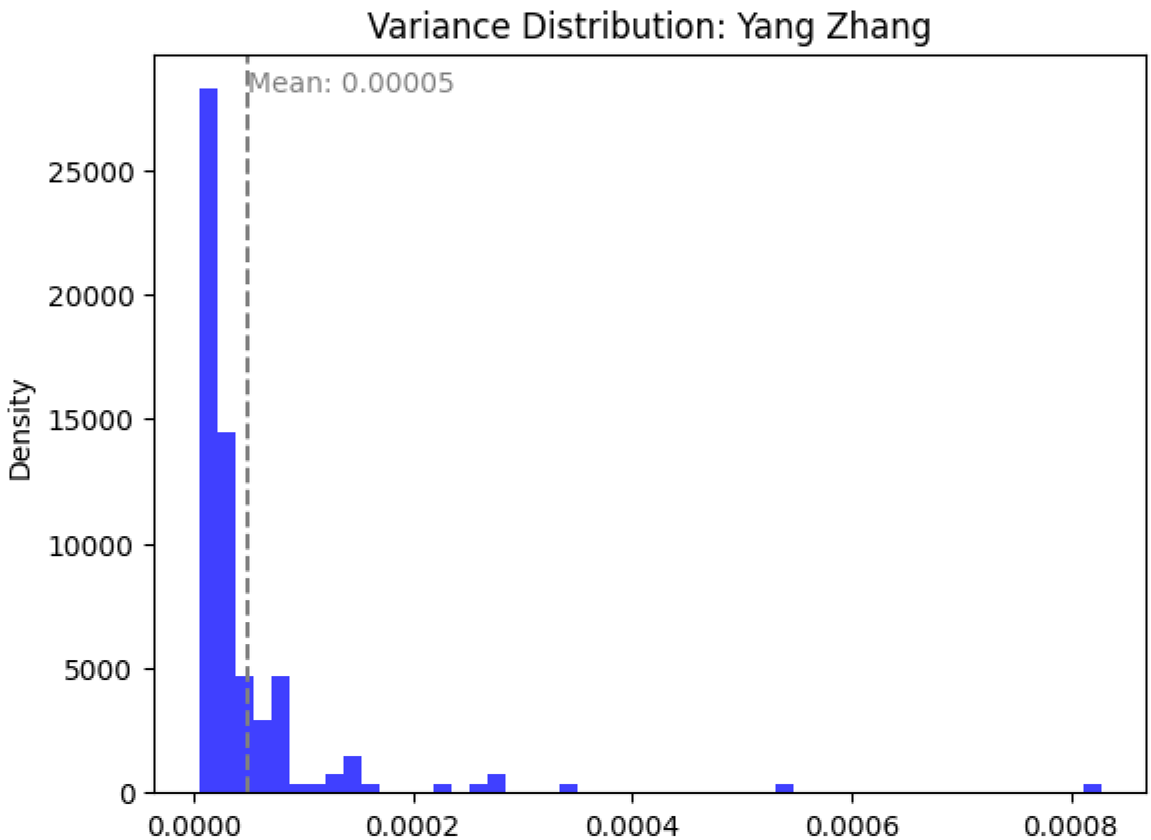
In [60]:
```
var_means = {}

for i in range(1, 6):
    x = f'Estimator{i}'
    liste = monthly_var[x].tolist()

    mean_var = np.mean(liste)
    var_means[legends[i-1]] = mean_var
    density, bins, _ = plt.hist(liste, bins=50, alpha=0.75, color='blue', density=True)
    plt.ylabel('Density')
    plt.axvline(x=mean_var, color='gray', linestyle='--')
    plt.text(mean_var, plt.ylim()[1] * 0.95, f'Mean: {mean_var:.5f}', color='gray', ha='
    plt.title(f'Variance Distribution: {legends[i-1]}')
    plt.show()

sorted_means = sorted(var_means.items(), key=lambda item: item[1])
for estimator, variance in sorted_means:
    print(f"{estimator}: {variance:.5f}")
```


Variance Distribution: Close to Close
Mean: 0.00007
Density
17500
15000
12500
10000
7500
5000
2500
0
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012


Variance Distribution: Garman Klass
Mean: 0.00002
Density
80000
70000
60000
50000
40000
30000
20000
10000
0
0.00000
0.00005
0.00010
0.00015
0.00020
0.00025


Variance Distribution: Parkinson
Mean: 0.00002
Density
70000
60000
50000
40000
30000
20000
10000
0
0.00000
0.00005
0.00010
0.00015
0.00020
0.00025


Variance Distribution: Roger Satchell
Mean: 0.00002
Density
60000
50000
40000
30000
20000
10000
0
0.00000
0.00005
0.00010
0.00015
0.00020
0.00025
0.00030
0.00035



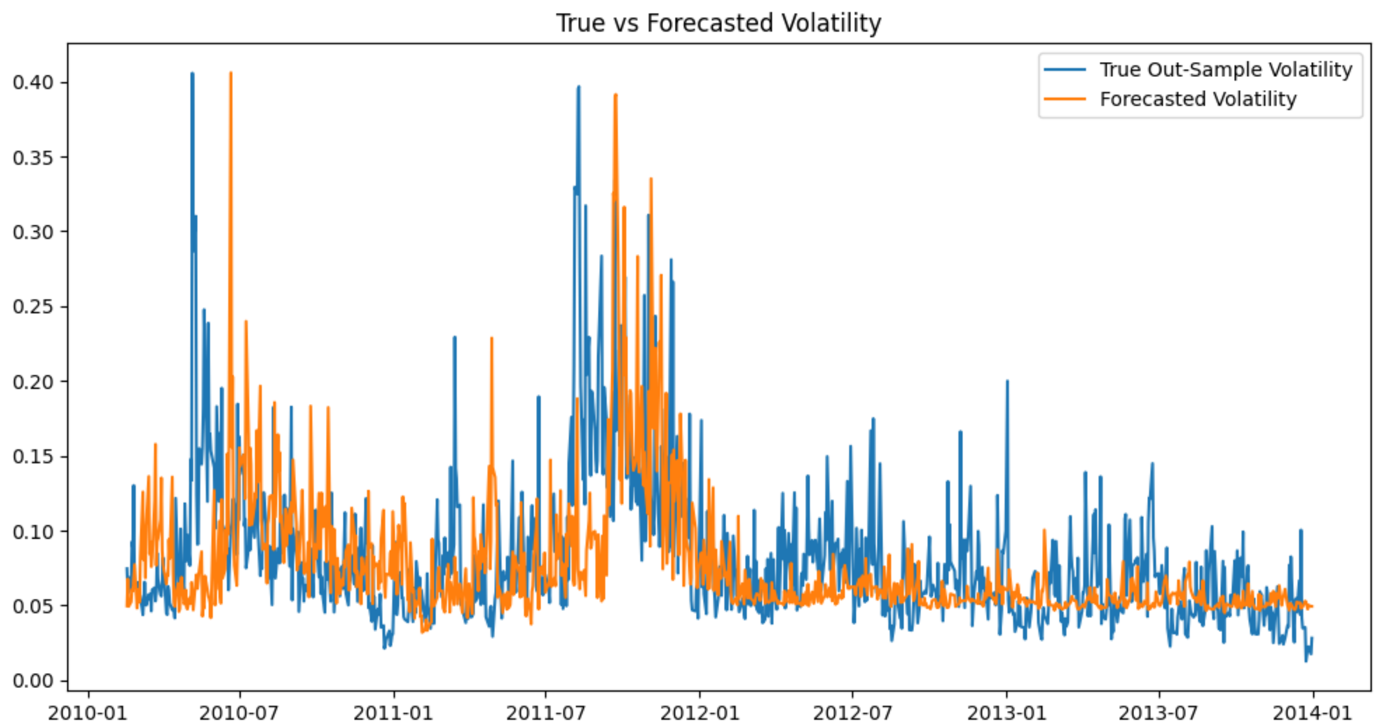

```
Parkinson: 0.00002
Garman Klass: 0.00002
Roger Satchell: 0.00002
Yang Zhang: 0.00005
Close to Close: 0.00007
```

III) Garch model

Garch prediction

In [45]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from arch import arch_model
import warnings
warnings.filterwarnings("ignore")

# Load data
data = pd.read_csv('RealizedVarianceData.csv', low_memory=False)

# Convert dates and handle invalid values
data['Unnamed: 0'] = pd.to_datetime(data['Unnamed: 0'], errors='coerce')
data = data.dropna(subset=['Unnamed: 0', 'Realized Variance (5-minute)'])

# Convert 'Realized Variance (5-minute)' to numeric and handle errors
data['Realized Variance (5-minute)'] = pd.to_numeric(data['Realized Variance (5-minute)'
data = data.dropna(subset=['Realized Variance (5-minute)'])

# Rename the date column
data.rename(columns={'Unnamed: 0': 'Date'}, inplace=True)
data.set_index('Date', inplace=True)

# Use the realized variance directly
data['Realized Volatility (5-minute)'] = np.sqrt(data['Realized Variance (5-minute)']*10

# Split data into in-sample and out-sample sets
split_date = '2010-01-01'
```

```
in_sample = data.loc[:split_date]
out_sample = data.loc[split_date:]

a=1
b=0
# Rolling window forecast
window_size = 30
forecasts = []

for start in range(len(out_sample) - window_size):
    end = start + window_size
    train_data = pd.concat([in_sample['Realized Volatility (5-minute)'], out_sample['Rea

    # Fit the GARCH model
    model = arch_model(train_data, vol='Garch', p=a, q=b)
    model_fit = model.fit(disp='off')
    if b==0:
       model_fit.params['beta[1]']=0
    # Manually forecast the volatility
    last_volatility = train_data.iloc[-1]  # Last observed volatility
    forecast_variance = model_fit.params['omega'] + model_fit.params['alpha[1]'] * (last
    forecast_volatility = np.sqrt(forecast_variance)
    forecasts.append(forecast_volatility)

# Convert forecasts to a Series with correct index
forecasts = pd.Series(forecasts, index=out_sample.index[window_size:])

# Plot the true out-sample data and the forecasted data
plt.figure(figsize=(12, 6))
plt.plot(out_sample.index[window_size:], out_sample['Realized Volatility (5-minute)'][wi
plt.plot(forecasts.index, forecasts, label='Forecasted Volatility')
plt.legend()
plt.title('True vs Forecasted Volatility')
plt.show()
```



Garch study

In [47]:
```
from arch import arch_model
warnings.filterwarnings("ignore")
def download_data(ticker='SPY', start='2000-01-01', end='2013-12-31'):
    data = yf.download(ticker, start=start, end=end)
```

```
    data = data[['Open', 'High', 'Low', 'Close']]
    return data

# Application des modèles de séries temporelles GARCH
def apply_garch_model(log_returns, p, q):
    p, q = int(p), int(q)  # S'assurer que p et q sont des entiers
    model = arch_model(log_returns, vol='Garch', p=p, q=q, rescale=False)
    model_fit = model.fit(disp="off")
    forecasts = model_fit.forecast(horizon=1, start=0)
    predicted_vol = np.sqrt(forecasts.variance.values.flatten())
    return predicted_vol

# Application des modèles de séries temporelles ARCH
def apply_arch_model(log_returns, p):
    p = int(p)  # S'assurer que p est un entier
    model = arch_model(log_returns, vol='ARCH', p=p, rescale=False)
    model_fit = model.fit(disp="off")
    forecasts = model_fit.forecast(horizon=1, start=0)
    predicted_vol = np.sqrt(forecasts.variance.values.flatten())
    return predicted_vol

# Calcul de la MSE hebdomadaire
def calculate_weekly_mse(realized_vol, predicted_vol):
    mse_weekly = ((realized_vol - predicted_vol) ** 2).resample('W').mean()
    return mse_weekly

# Programme principal
if __name__ == "__main__":
    data = download_data()

    # Calcul des rendements logarithmiques
    log_returns = np.log(data['Close'] / data['Close'].shift(1)).dropna()

    # Calcul de la volatilité réalisée comme la valeur absolue des rendements logarithmi
    realized_vol = log_returns.abs()

    # Initialisation des résultats
    results = []

    # Comparaison des modèles GARCH pour p, q dans [1, 2, 3]
    for p in range(1, 4):
        for q in range(1, 4):
            predicted_vol_garch = apply_garch_model(log_returns, p, q)
            predicted_vol_garch = predicted_vol_garch[-len(realized_vol):]
            mse_weekly_garch = calculate_weekly_mse(realized_vol, predicted_vol_garch)
            avg_mse_garch = mse_weekly_garch.mean()
            results.append(('GARCH', p, q, avg_mse_garch))

    # Comparaison des modèles ARCH pour p dans [1, 2, 3]
    for p in range(1, 4):
        predicted_vol_arch = apply_arch_model(log_returns, p)
        predicted_vol_arch = predicted_vol_arch[-len(realized_vol):]
        mse_weekly_arch = calculate_weekly_mse(realized_vol, predicted_vol_arch)
        avg_mse_arch = mse_weekly_arch.mean()
        results.append(('ARCH', p, 0, avg_mse_arch))  # q=0 pour ARCH

    # Conversion des résultats en DataFrame pour affichage
    results_df = pd.DataFrame(results, columns=['Model', 'p', 'q', 'Avg_MSE'])

    # Affichage des résultats
    print(results_df)

    # Calcul du nombre de lignes et de colonnes pour les subplots
    num_models = len(results_df)
    num_cols = 4
    num_rows = (num_models + num_cols - 1) // num_cols  # Calcul du nombre de lignes néc
```

```
    # Préparation des histogrammes pour chaque modèle
    fig, axs = plt.subplots(num_rows, num_cols, figsize=(18, num_rows * 6))
    fig.suptitle('Histogramme de la MSE hebdomadaire pour des modèles GARCH et ARCH')

    for index, (model, p, q, avg_mse) in enumerate(results):
        if model == 'GARCH':
            predicted_vol = apply_garch_model(log_returns, p, q)
        else:
            predicted_vol = apply_arch_model(log_returns, p)

        predicted_vol = predicted_vol[-len(realized_vol):]
        mse_weekly = calculate_weekly_mse(realized_vol, predicted_vol)

        ax = axs[index // num_cols, index % num_cols]
        ax.hist(mse_weekly.dropna(), bins=50, edgecolor='black')
        ax.set_title(f'{model} (p={p}, q={q})')
        ax.set_xlabel('MSE')
        ax.set_ylabel('Fréquence')
        ax.text(0.95, 0.95, f'Avg MSE: {avg_mse:.6f}', horizontalalignment='right', vert

    # Supprimer les axes vides
    for i in range(num_models, num_rows * num_cols):
        fig.delaxes(axs.flatten()[i])

    plt.tight_layout(rect=[0, 0.03, 1, 0.95])
    plt.show()
```

```
[*********************100%%**********************]  1 of 1 completed
C:\Users\pauld\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0
\LocalCache\local-packages\Python311\site-packages\arch\univariate\base.py:766: Converge
nceWarning: The optimizer returned code 4. The message is:
Inequality constraints incompatible
See scipy.optimize.fmin_slsqp for code meaning.

  warnings.warn(
C:\Users\pauld\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0
\LocalCache\local-packages\Python311\site-packages\arch\univariate\base.py:766: Converge
nceWarning: The optimizer returned code 4. The message is:
Inequality constraints incompatible
See scipy.optimize.fmin_slsqp for code meaning.

  warnings.warn(
```

```
    Model  p  q   Avg_MSE
0   GARCH  1  1  0.000066
1   GARCH  1  2  0.000057
2   GARCH  1  3  0.000053
3   GARCH  2  1  0.023285
4   GARCH  2  2  0.000066
5   GARCH  2  3  0.000066
6   GARCH  3  1  0.000076
7   GARCH  3  2  0.000069
8   GARCH  3  3  0.000069
9    ARCH  1  0  0.000051
10   ARCH  2  0  0.000054
11   ARCH  3  0  0.000056
```

```
C:\Users\pauld\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0
\LocalCache\local-packages\Python311\site-packages\arch\univariate\base.py:766: Converge
nceWarning: The optimizer returned code 4. The message is:
Inequality constraints incompatible
See scipy.optimize.fmin_slsqp for code meaning.

  warnings.warn(
C:\Users\pauld\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0
\LocalCache\local-packages\Python311\site-packages\arch\univariate\base.py:766: Converge
```

```
nceWarning: The optimizer returned code 4. The message is:
Inequality constraints incompatible
See scipy.optimize.fmin_slsqp for code meaning.

  warnings.warn(
```

Histogramme de la MSE hebdomadaire pour des modèles GARCH et ARCH