Deriving the reflection coefficient for a (classical) single tone in a cavity experiment

We start from the general quantum coupled equation of motion for the cavity ladder operator $\hat{a}$, as well as an ansatz for the motion of the oscillator position, $x(t)$, which is taken to oscillate at an angular frequency Ω_m, and an amplitude x_0.

$$\dot{\hat{a}} = -\kappa/2\hat{a} + -i(\Delta + G \cdot x)\hat{a} + \sqrt{\kappa_{ext}}\hat{a}_{in}e^{-i\omega_L t} \tag{1}$$

$$x(t) = x_0 \cdot cos(\Omega_m t) \tag{2}$$

The meanings are the same as in [1]. We take a classical form of (1) by taking the expectation value $\hat{a} \to \langle\hat{a}\rangle = \alpha$, and a similar for for $\hat{a}_{in}$. We further take a perturbative approach to the solution, that is assume that we can divide $\alpha(t)$ in two parts: the unperturbed solution $\alpha_0(t)$, as well as the small correction to this solution from the optomechanical interaction, $\alpha_1(t)$. We also assume that $x \approx \mathsf{O}(\alpha_1)$. It is trivial to see that the steady-state unperturbed solution is:

$$\alpha_0(t) = \frac{\sqrt{\kappa_{ext}}}{i\Delta + \kappa/2}\alpha_{in}e^{i\omega_L t}. \tag{3}$$

The perturbed equation can be rearranged to have the form (remembering that $2\cos(A) = e^{iA} + e^{-iA}$:

$$\alpha_1(t) = \frac{1}{i\Delta + \kappa/2} \cdot \frac{Gx_0}{2}\left(e^{i\Omega_m t} + e^{-i\Omega_m t}\right) \tag{4}$$

[1] Kippenberg *et al.* Rev. Mod. Phys. (2014)