# Degree distributions and power laws

Matthew J. Salganik

Social Network (Soc 204)
Spring 2017
Princeton University

2/20/17

Logistics:

- questions about homework
- please save frequently and submit when complete

Questions?

Your feedback is important

Your feedback is important

- candy

Your feedback is important

- candy
- slides posted right before class

# Exploring the Role of High School Friends in the Transition to College

Zitsi Mirakhur

300 Wallace Hall

Tuesday, February 21

http://opr.princeton.edu/seminars/Spring/2017

Vote:

1. Watts, Chapter 4, 101-114.
2. Barabasi, A.L. and Bonabeau, E. (2003) Scale-free networks. *Scientific American*, 50-59.
3. Barabasi, A.L. and Albert, R. (1999) The emergence of scaling in random networks. *Science*, 286:509-512.
4. Liljeros, F. et al. (2001). The web of human sexual contacts. *Nature*, 411:907-908 with comment and rejoinder.

Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks

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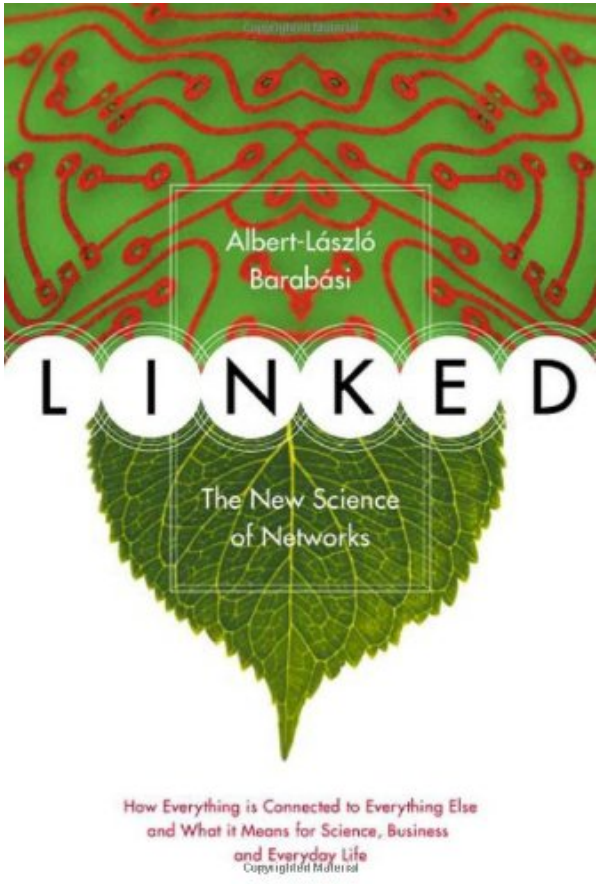
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)


Albert-László
Barabási
LINKED
The New Science
of Networks
How Everything is Connected to Everything Else
and What it Means for Science, Business
and Everyday Life
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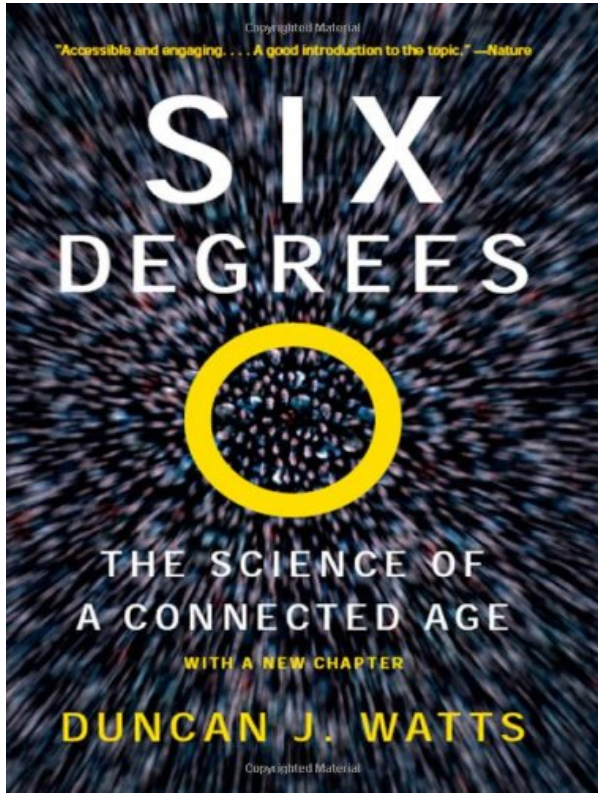
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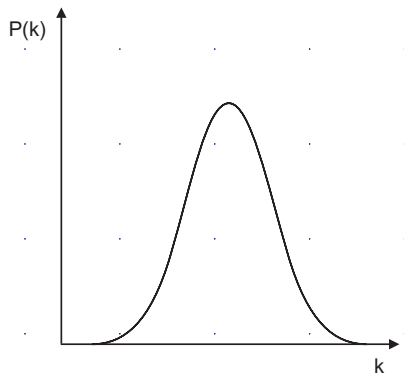
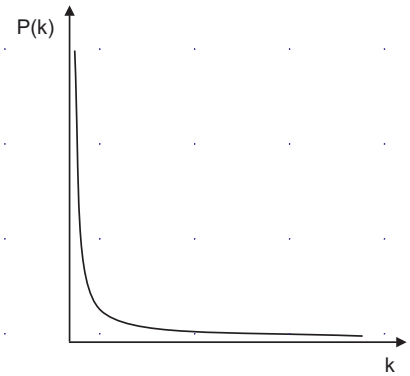
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees

4.1



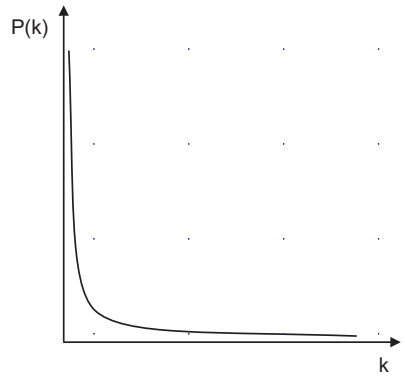

(a) Normal

4.2



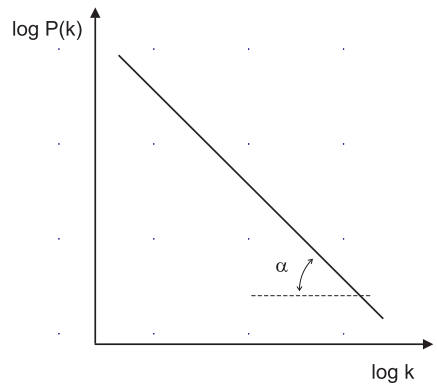

(b) Power law

4.2




(a) Power law

4.3




(b) log-log Power law

It turns out that many degree distributions follow a power law distribution (which Barabasi calls "scale-free")

$p(k) \sim \frac{1}{k^{\gamma}}$

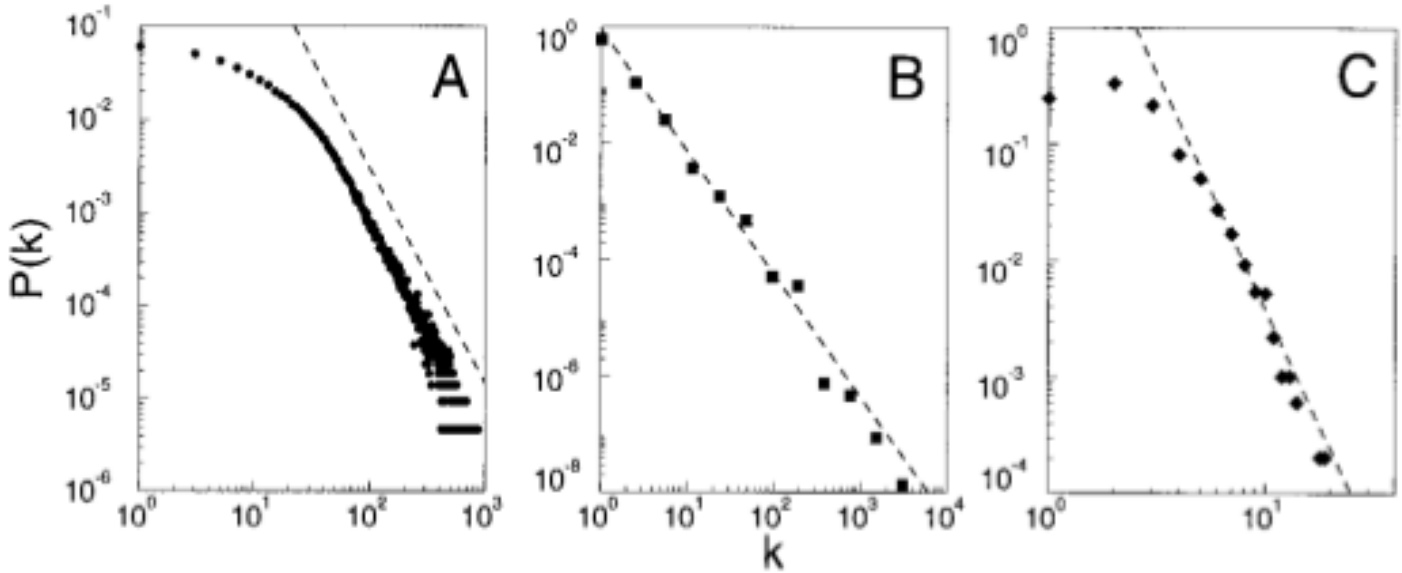
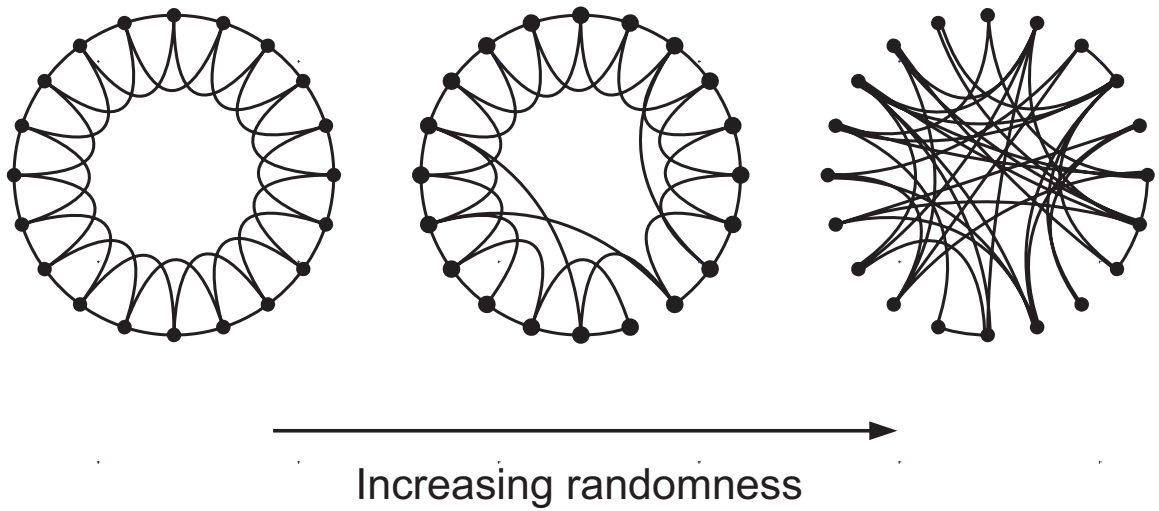



**Fig. 1.** The distribution function of connectivities for various large networks. (**A**) Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. (**B**) WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (*6*). (**C**) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

# Does $\beta$ model produce power law degree distribution?

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

NetLogo Demo

Follow up work:

- implications
- empirical
- modeling

# Implication

**Epidemic Spreading in Scale-Free Networks**

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]

[1]*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*

[2]*The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*

(Received 20 October 2000)

http://dx.doi.org/10.1103/PhysRevLett.86.3200

# Implication

The Internet has a very complex connectivity recently modeled by the class of scale-free networks. This feature, which appears to be very efficient for a communications network, favors at the same time the spreading of computer viruses. We analyze real data from computer virus infections and find the average lifetime and persistence of viral strains on the Internet. We define a dynamical model for the spreading of infections on scale-free networks, finding the absence of an epidemic threshold and its associated critical behavior. This new epidemiological framework rationalizes data of computer viruses and could help in the understanding of other spreading phenomena on communication and social networks.

# Implication

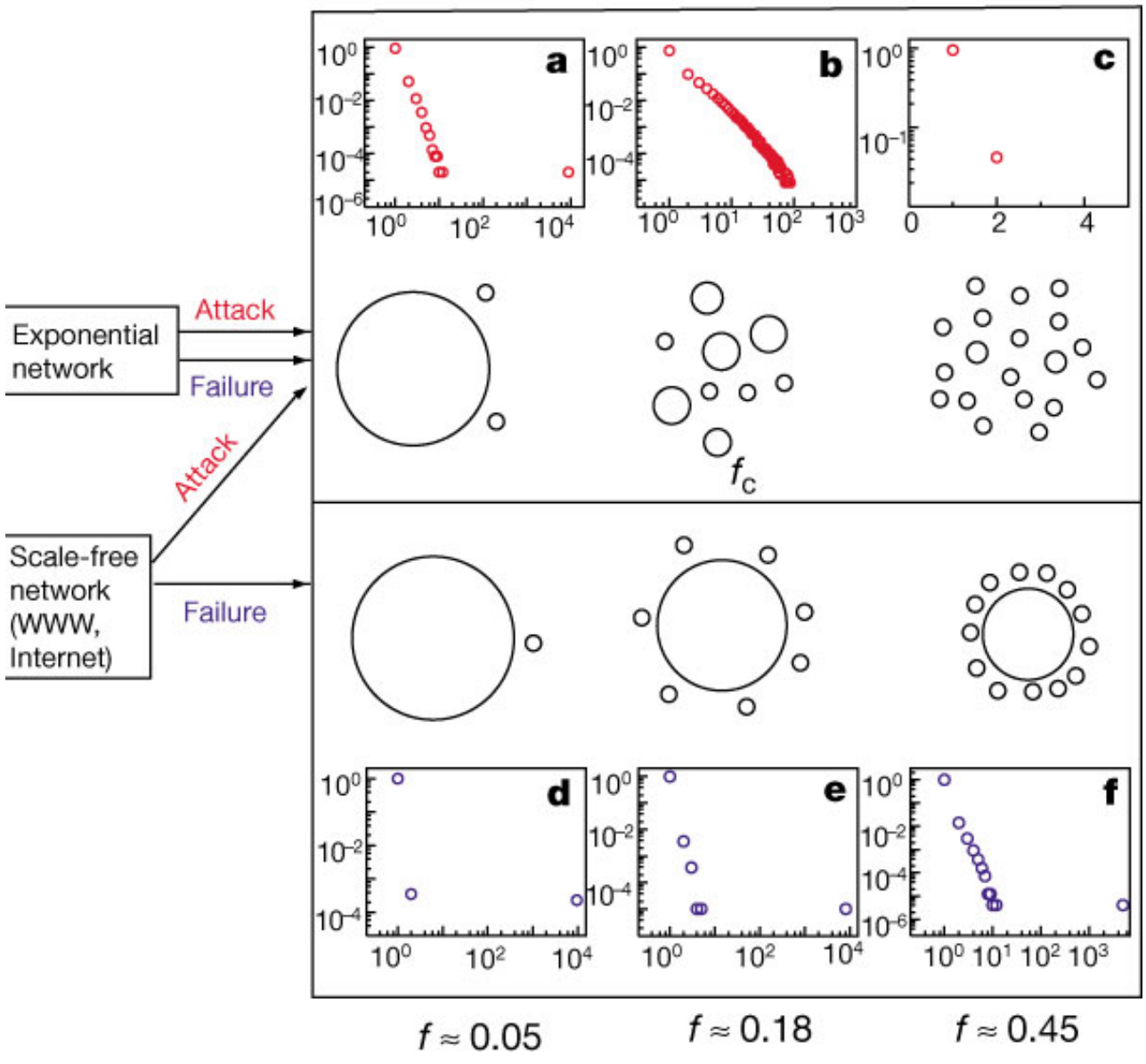
## Error and attack tolerance of complex networks

**Réka Albert, Hawoong Jeong & Albert-László Barabási**

*Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA*

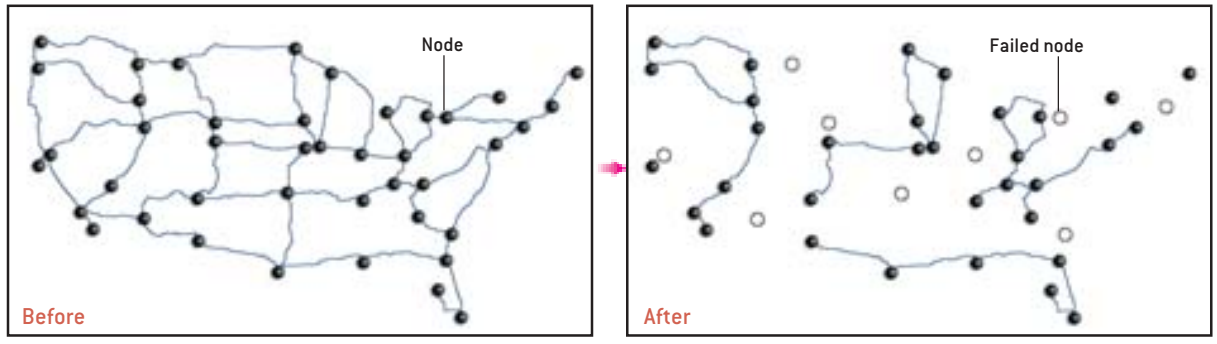
http://dx.doi.org/10.1038/35019019

# Implication

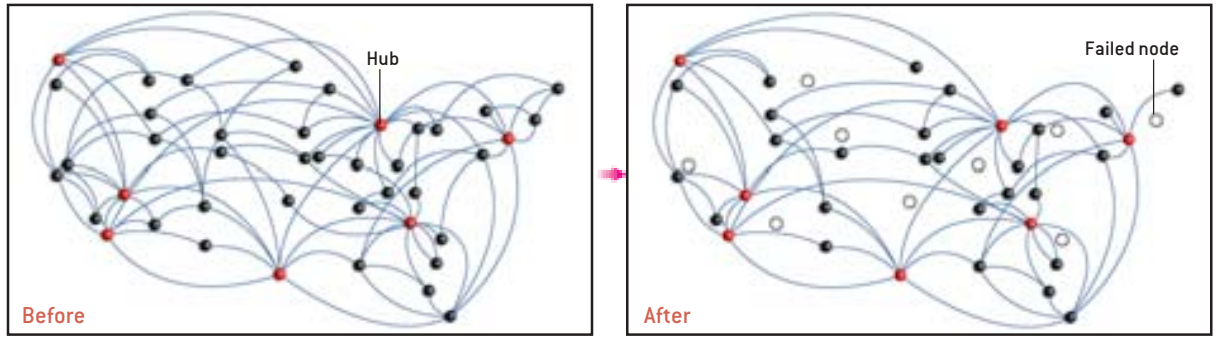


# Implication

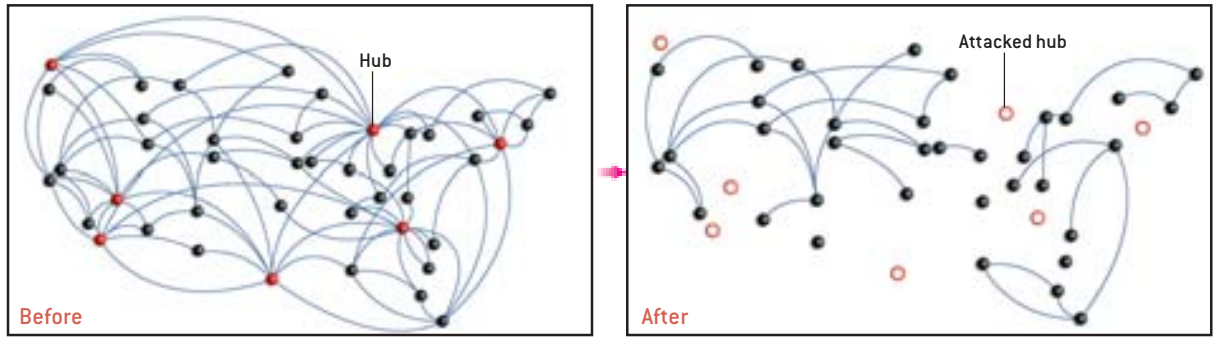
**Random Network, Accidental Node Failure**




**Scale-Free Network, Accidental Node Failure**




**Scale-Free Network, Attack on Hubs**

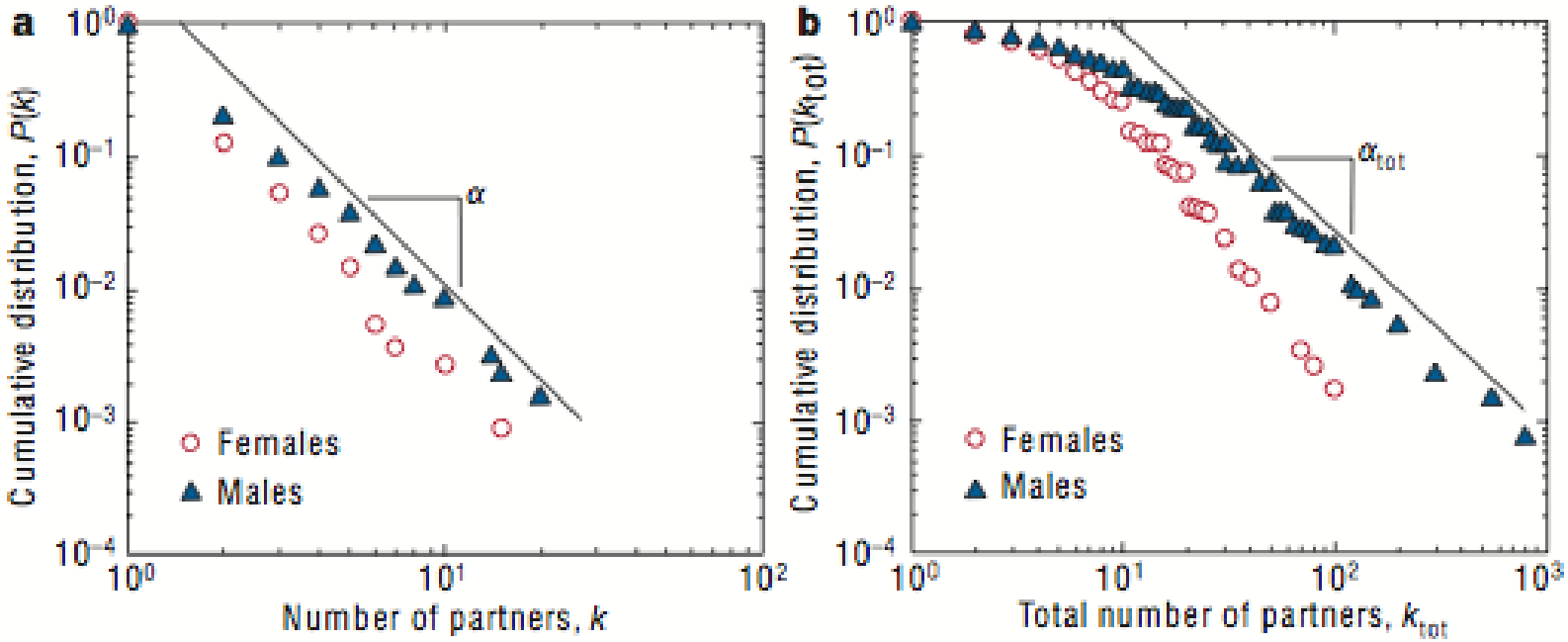


# Emprical

## The web of human sexual contacts

Promiscuous individuals are the vulnerable nodes to target in safe-sex campaigns.

# Emprical


a
b
Cumulative distribution, P(k)
Cumulative distribution, P(k_tot)
Number of partners, k
Total number of partners, k_tot
α
α_tot
Females
Males
Females
Males

# Empirical

**POWER-LAW DISTRIBUTIONS IN EMPIRICAL DATA**

AARON CLAUSET*, COSMA ROHILLA SHALIZI†, AND M. E. J. NEWMAN‡

http://dx.doi.org/10.1137/070710111

# Empirical

## Measuring preferential attachment in evolving networks

**H. Jeong [1,2] - Z. Néda [1[*]] - A. L. Barabási [1]**

[1] Department of Physics, University of Notre Dame - Notre Dame, IN 46616, USA

[2] Department of Physics, Korea Advanced Institute of Science and Technology Taejon, 305-701, Korea

http://iopscience.iop.org/0295-5075/61/4/567

# Modeling

**Organization of growing random networks**

P. L. Krapivsky and S. Redner
*Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, Massachusetts 02215*
(Received 7 November 2000; published 24 May 2001)

Generalizes growth process

https://doi.org/10.1103/PhysRevE.63.066123

# Modeling

**Scale-Free Networks from Varying Vertex Intrinsic Fitness**

G. Caldarelli,[1] A. Capocci,[2] P. De Los Rios,[3,4] and M. A. Muñoz[5]

[1]*INFM UdR ROMA1 Dipartimento Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro 2 00185, Roma, Italy*
[2]*Département de Physique, Université de Fribourg-Pérolles, CH-1700 Fribourg, Switzerland*
[3]*Institut de Physique Théorique, Université de Lausanne, CH-1004 Lausanne, Switzerland*
[4]*INFM UdR Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[5]*Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain*
(Received 15 July 2002; published 3 December 2002)

https://doi.org/10.1103/PhysRevLett.89.258702

# Modeling

A new mechanism leading to scale-free networks is proposed in this Letter. It is shown that, in many cases of interest, the connectivity power-law behavior is neither related to dynamical properties nor to preferential attachment. Assigning a quenched fitness value $x_i$ to every vertex, and drawing links among vertices with a probability depending on the fitnesses of the two involved sites, gives rise to what we call a good-get-richer mechanism, in which sites with larger fitness are more likely to become hubs (i.e., to be highly connected).

Question from Piazza:
"Is it possible for hubs to exist even where a network doesn't follow a power-law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?"

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"Is it possible for hubs to exist even where a network doesn't follow a power-law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?" A note on terminology:

- power law
- scale-free
- hubs

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- many (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks
- networks with power law degree distribution are robust to random failure but fragile to targeted attack

http://bit.ly/socnet204

# http://bit.ly/socnet204

Wednesday:

- Gladwell, M. (1999). Six degrees of Lois Weisberg. The New Yorker.
- Watts, Chapter 4, 114-129.
- Feld, S.L. (1981) The focused organization of social ties. American Journal of Sociology.