A local and global approach to the minimum vertex cover optimisation problem

Overview

- Minimum Vertex Cover is a NP-hard optimisation problem
- Two local search algorithms and one global stochastic algorithm were implemented in polynomial time and provide a naive solution
- All testing conducted with stage four compile optimisations
- Promising results for the global approach should further research be invested

Local Approach 1

Next-Best Greedy

- Ranks each node based of its degree
- Chooses next node to add to the cover based off highest rank
- No node will be chosen twice in the same cover
- Hash-map randomness ensures a new variation of 'next-best' every trial

| Graph | Target | Average CPU Time (seconds) | Best Cover | Average Cover |
|----------------|--------|----------------------------------|---------------|------------------|
| brock800_1.clq | 777 | 0.0669 | 784 | 786.31 |
| brock800_2.clq | 776 | 0.06638 | 784 | 786.16 |
| brock800_3.clq | 775 | 0.06655 | 785 | 786.3 |
| brock800_4.clq | 774 | 0.06662 | 783 | 786.1 |
| C2000.9.clq | 1922 | 0.16573 | 1943 | 1948.85 |
| C4000.5.clq | 3982 | 5.22589 | 3986 | 3988.15 |
| MANN_a45.clq | 691 | 0.00216 | 700 | 702.93 |
| p_hat1500.clq | 1488 | 0.70672 | 1491 | 1493.4 |

Local Approach 2

Randomised Local Search

- Chooses next node based off a normally distributed random choice
- Each node has the same chance of being picked
- No node will be chosen twice in the same cover
- Identical local minima compared to next-best greedy

| Graph | Target | Average CPU Time (seconds) | Best Cover | Average Cover |
|----------------|--------|----------------------------------|---------------|------------------|
| brock800_1.clq | 777 | 0.0669 | 784 | 786.31 |
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Global Approach

Simulated Annealing

- Outperforms both local search algorithms
- Cooled at a rate of 0.8 for temperature = 100,000
- Iteratively improves its solution at every temperature
- Probability of choosing a worse solution decreases as temperature lowers
- Algorithm approaches better minima as it cools

| Graph | Target | Average CPU Time (seconds) | Best Cover | Average Cover |
|----------------|--------|----------------------------------|---------------|------------------|
| brock800_1.clq | 777 | 2.14652 | 782 | 784.04 |
| brock800_2.clq | 776 | 2.13537 | 782 | 784.15 |
| brock800_3.clq | 775 | 2.16887 | 781 | 784 |
| brock800_4.clq | 774 | 2.14215 | 783 | 784.14 |
| C2000.9.clq | 1922 | 8.97447 | 1941 | 1944.01 |
| C4000.5.clq | 3982 | 240.951 | 3985 | 3986.59 |
| MANN_a45.clq | 691 | 0.186 | 703 | 704.44 |
| p_hat1500.clq | 1488 | 37.5072 | 1490 | 1491.6 |

Conclusions

- Local Search algorithms should be avoided for the anti-greedy Brock collection of graphs
- If close enough is good enough an extension to Simulated Annealing should be chosen
- If benchmark results are required further research is required into different types of algorithms