3801ICT Numerical Algorithms – Assignment Part 2

Note:

a) This assignment must be done individually.
b) The programming language to be used is C++ but you may use Python to generate graphs for your reports.
c) For each question requiring a C++ program you must document the algorithm and show any test cases you used. Only submit a single Word document containing the documentation for all questions.
d) The submission time and date are as specified in the Course Profile and the submission method will be communicated during semester.

1. (**10 Marks**) By hand, use Gauss Elimination, LU Decomposition and the Gauss-Jordan method to solve the following system of linear equations to six significant figures. Show each step of your calculations.

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$
$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

2. (**10 Marks**) The trajectory of a thrown ball is defined by the (x, y) coordinates that can be modelled as:

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2 cos^2\theta_0}x^2 + y_0$$

Programmatically using C++ find the appropriate initial angle θ_0, if the initial velocity v_0 = 20 m/s and the distance to the catcher x is 35 m. Note that the ball leaves the thrower's hand at an elevation of y_0 = 2 m and the catcher receives it at an elevation of 1 m. Express the final result in degrees. Use a value of 9.81 m/s^2 for g.

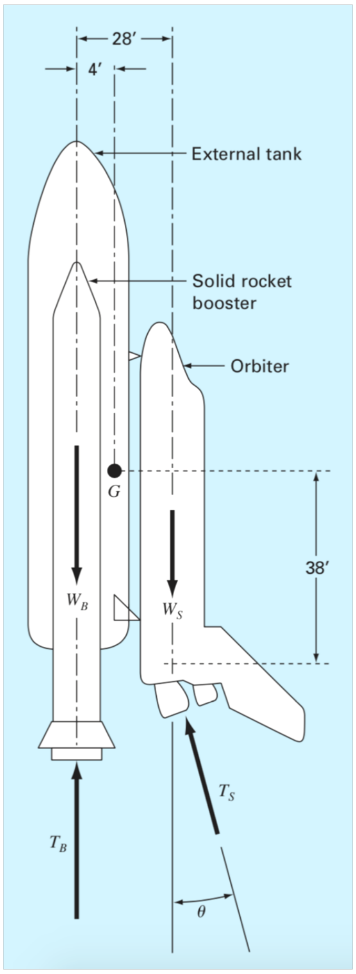
3. (**20 Marks**) The space shuttle, at lift-off from the launch pad, has four forces acting on it, which are shown on the following diagram. The combined weight of the two solid rocket boosters and external fuel tank is W_B = 1.663 × 10^6 lb of which1.300 × 10^6 lb is fuel which burns at a constant rate for 127 seconds. The weight of the orbiter with a full payload is W_S = 0.23 × 10^6 lb. The combined thrust of the two solid rocket boosters is T_B = 5.30 × 10^6 lb. The combined thrust of the three liquid fuel orbiter engines is T_S =1.125×10^6 lb.

At lift-off, the orbiter engine thrust is directed at angle θ to make the resultant moment acting on the entire craft assembly (external tank, solid rocket boosters, and orbiter) equal to zero. With the resultant moment equal to zero, the craft will not rotate about its mass centre *G* at lift-off. With these forces, the craft will have a resultant force with components in both the vertical and horizontal direction. The vertical resultant force component is what allows the craft to lift off from the launch pad and fly vertically.

The horizontal resultant force component causes the craft to fly horizontally. The resultant moment acting on the craft will be zero when θ is adjusted to the proper value. If this angle is not adjusted properly, and there is some resultant moment acting on the craft, the craft will tend to rotate about it's mass centre.

1. Derive the moment equation about point *G*, the centre of mass.
2. Write a C++ program to solve for the angle θ using the Secant method to find the root of the moment equation. Terminate your iterations when the value of θ has better than five significant figures.

3. T_B is constant over the 127 seconds so the fuel is burned at a constant rate which means the moment about point G resulting from T_B increases over time. This requires continual adjustment to θ so there is no rotation. Determine the appropriate values for θ at suitable intervals and interpolate these values using Lagrange Interpolating Polynomials to obtain an equation giving how θ should be changed during the 127 seconds so that the moment equation is always zero.



4. (**10 Marks**) Flow through a tube is related to the diameter and the angle to the horizontal by the following equation:

$$F = aD^bS^c$$

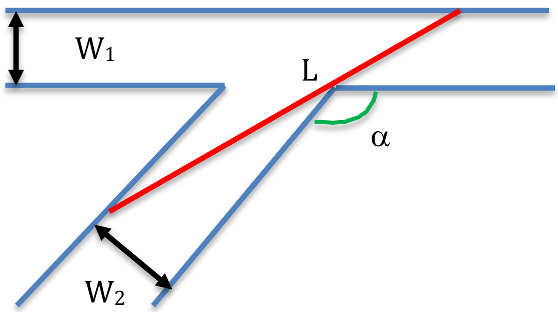
The following table shows results of nine experiments:

Experiment	D	S	F
1	1	0.001	1.4
2	2	0.001	8.3
3	3	0.001	24.2
4	1	0.01	4.7
5	2	0.01	28.9
6	3	0.01	84.0
7	1	0.05	11.1

8	2	0.05	69.0
9	3	0.05	200.0

Using multiple linear regression find F when D = 2.5 and S = 0.025

5. (**10 Marks**) The maximum length L of a plank that can negotiate the corner shown in the sketch below is a function of w_1, w_2 and α. Write a C++ program to develop a plot of L for a range of α values assuming w_1 and w_2 have fixed values.



6. (**40 Marks**) The energy associated with two particles is given by:

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$$V_\rho(r) = \left(e^{\rho(1-r)} - 1\right)^2 - 1$$

where r is the distance between them and ρ is a constant.

a) Plot this energy as a function of r in the range 0.6 to 2.0 with ρ having values 3, 6, 10 and 14.
b) Use a gradient based optimiser to find the minimum of this function. You can obtain the C++ code for the optimiser from the internet.
c) For systems of n = 2 .. 10 particles:

    a. Estimate how many local minima exist as a function of ρ and n.
    b. Identify the global minima configuration of particles for each value of n and ρ = 6.

Present your results in the form of a scientific paper.