# NorMC: a Norm Compliance Temporal Logic Model Checker[⋆]

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**Abstract**

In this paper we describe NorMC, a model checker for Norm Compliance CTL, implemented in the Haskell programming language. NorMC is intended as a tool for students, researchers, and practitioners to learn about and understand normative systems, and as an exploratory tool for researchers in multi-agent systems. The objectives of the paper are twofold. First, to give a system description of NorMC. Second, to argue and demonstrate that the Haskell programming language is a natural and useful alternative for model checking; in particular that the full power of Haskell makes it easy to describe arbitrary state-transition models in a natural way.

*Keywords:* multi-agent systems, normative systems, model checking, haskell

## 1 Introduction

*Normative systems*, or *social laws*, have emerged as a promising and powerful framework for coordinating multi-agent systems [13,14,9,10,1,3,7,2,4]. The starting point is a state-transition model of a multi-agent system, typically a *legacy system*, and the goal is to constrain the behaviour of the agents in the system in

⋆ System description. As the call for papers say that "*We strongly encourage young researchers and students to submit papers, also about experimental and prototypical software tools which are related to modal logics*", we mention that the two first authors are students and that this paper falls into the explicitly mentioned category.

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such a way that the global behaviour of the system exhibits some desirable properties. Such a restriction on agents' behaviour is called a normative system, or a social law. The desirable global properties are typically represented using a (modal) logical formula; the *objective* (typically not satisfied in the initial system).

A key issue in normative systems is the question of *compliance*. Even if a normative system is *effective*, i.e., will ensure that the objective holds, under the assumption that all agents comply with it – how do we know that they will comply? And what happens if they do not? There are several possible reasons for non-compliance [4]: an autonomous and rational agent might choose not to comply because it is not in her best interest; a rational agent might not comply by accident (a failure); an irrational agent might choose not to comply without any particular reason. *Norm compliance* CTL (NCCTL) [4] was developed to reason about normative systems and in particular about (non-)compliance. It can be used, e.g., to model check compliance properties such as "which agents *have to comply* for the objective to hold". NCCTL extends the branching-time temporal logic *Computation-Tree Logic* (CTL) [8] with a unary modality $[P]$, where $P$ is a *coalition predicate*, i.e., a possible property of groups of agents (*coalitions*). The meaning of the expression $\langle P\rangle\phi$ is that if any coalition that satisfies $P$ complies with the normative system, then $\phi$ will hold. Examples of NCCTL expressions include the following, which are evaluated in the context of a model and a normative system:

- $[supseteq(C)]\phi$: if any superset of $C$ comply, $\phi$ will hold ($C$ is *sufficient*)
- $[\neg geq(k)]\neg\phi$: at least $k$ agents have to comply for $\phi$ to hold (the normative system is $k$-*necessary*)
- $[geq(n-k)]\phi \wedge \langle ceq(n-k-1)\rangle\neg\phi$, where $n$ is the total number of agents: $k$ is the largest number of non-compliant agents the normative system can tolerate whilst still being effective for $\phi$ (the *resilience* of the normative system is $k$)

Formally, $[P]$ has a non-standard *update semantics*, which makes it difficult to use standard branching-time model checkers directly to verify normative system properties specified in NCCTL.

In this paper we describe NorMC, a prototype model checker for NCCTL implemented in Haskell [12]. The intended use of NorMC is as a tool to learn about and understand normative systems, and as an exploratory tool for researchers. Even small "toy" examples can be difficult to understand properly without a computational tool, because the number of model modifications (updates) resulting from different groups of agents complying is typically exponential in the size of the model. As a prototype tool intended for academic rather than industrial use, the focus in the implementation of NorMC has not been on efficiency, but rather on clarity, extensibility and ease of use. In particular, standard symbolic model checking optimisation techniques have not been implemented. However, the implementation is still efficient enough for interesting and non-trivial examples.

The objectives of the paper are twofold. First, the paper is a system description of NORMC that demonstrates how it can be used to model check normative systems. Second, we want to argue for and demonstrate the usefulness of Haskell programming language for model checking modal logics, as already suggested by model checkers such as the epistemic logic model checker DEMO [15]. Haskell's native support for discrete structures and lazy evaluation mechanism makes it well suited for programming model checking algorithms. More importantly, that a model checker is implemented in Haskell means that it is possible to have the full power of the Haskell language available for the *user* to *describe models* – in contrast with the restricted model description languages available in popular temporal logic model checkers such as SPIN [11] or NUSMV [5].

The paper is organised as follows. We first review the formal normative systems framework and NCCTL. In Section 3 we describe how to specify a model and execute the model checker via a simple example. In Section 4 we illustrate further with a more complicated example, and show how NORMC was used to find an error in a case study in the literature. The implementation of NORMC is discussed in Section 5, and we conclude in Section 6. NORMC, with source code, can be downloaded from `http://pkazmierczak.github.com/NorMC/`.

## 2 Background

We give an, necessarily terse due to lack of space, overview of the background; see [4] for more details. Assume a set $\Phi$ of propositional variables. As the semantic model we use an *agent-labelled Kripke structure*; a tuple $K = \langle S, s^0, R, A, \alpha, V\rangle$ where $S$ is a finite, non-empty set of *states*; $s^0 \in S$ is the *initial state*; $R \subseteq S \times S$ is a serial binary relation (i.e., $\forall s \exists t\ (s,t) \in R$) on $S$, which we refer to as the *transition relation*; $A$ is a set of *agents*; $\alpha : R \to A$ labels each transition in $R$ with an agent; and $V : S \to \wp(\Phi)$ labels each state with a set of propositional variables.

A *path* $\pi$ over a relation $R$ is an infinite sequence of states $s_0, s_1, s_2, \ldots$ such that $\forall u \in \mathbb{N} : (s_u, s_{u+1}) \in R$. $\pi[0]$ denotes the first element of the sequence, $\pi[1]$ the second, and so on. An $s$-*path* is a path $\pi$ such that $\pi[0] = s$. $\Pi_R(s)$ is the set of $s$-paths over $R$, and we write $\Pi(s)$, if $R$ is clear from the context.

To specify objectives we use CTL. We use an adequate fragment of the language defined by the following grammar:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{E} \bigcirc \varphi \mid \mathsf{E}(\varphi \mathcal{U} \varphi) \mid \mathsf{A}(\varphi \mathcal{U} \varphi)$$

where $p \in \Phi$. The standard derived propositional connectives are used, in addition to standard derived CTL connectives such as $A \bigcirc \phi$ for $\neg E \bigcirc \neg\phi$. Satisfaction is defined as follows (propositional clauses as usual):

$$\begin{aligned}
&K, s \models \mathsf{E} \bigcirc \varphi \text{ iff } \exists \pi \in \Pi(s) : K, \pi[1] \models \varphi;\\
&K, s \models \mathsf{E}(\varphi \mathcal{U} \psi) \text{ iff } \exists \pi \in \Pi(s), \exists u \in \mathbb{N}, \text{s.t. } K, \pi[u] \models \psi\\
&\qquad\qquad\qquad\qquad\qquad \text{and } \forall v, (0 \leq v < u) : K, \pi[v] \models \varphi;\\
&K, s \models \mathsf{A}(\varphi \mathcal{U} \psi) \text{ iff } \forall \pi \in \Pi(s), \exists u \in \mathbb{N}, \text{s.t. } K, \pi[u] \models \psi\\
&\qquad\qquad\qquad\qquad\qquad \text{and } \forall v, (0 \leq v < u) : K, \pi[v] \models \varphi;
\end{aligned}$$

A *normative system* $\eta$ over $K$ is a set of *constraints* on the behaviour of the agents. Formally, $\eta \subseteq R$, such that $R \setminus \eta$ is a total relation, represents the *forbidden* transitions. We say that normative systems are *implemented* on Kripke structures, which means that after the implementation all the transitions that are forbidden according to a normative system are removed from the structure. Formally, if $\eta$ is a normative system over $K$, then $K \dagger \eta$ stands for the Kripke structure obtained from $K$ by removing the transitions forbidden by $\eta$, i.e., if $K = \langle S, s^0, R, A, \alpha, V\rangle$ and $\eta \in N(R)$ (where $N(R)$ is all normative systems over $K$), then $K \dagger \eta = \langle S, s^0, R', A, \alpha', V\rangle$ where $R' = R \setminus \eta$, and $\alpha'$ is the restriction of $\alpha$ to $R'$: if $(s, s') \in R'$ then $\alpha'(s, s') = \alpha(s, s')$. A set $C \subseteq A$ is a *coalition*. Let $\eta$ be a normative system over $K$, then $\eta \upharpoonright C$ is the normative system restricted to the actions of agents in $C$: $\eta \upharpoonright C = \{(s, s') : (s, s') \in \eta \;\&\; \alpha(s, s') \in C\}$.

The language of NCCTL extends the language of CTL with a unary modality $\langle P\rangle$ where $P$ is a *coalition predicate*. The language of coalition predicates is defined by the following grammar:

$$P ::= \mathit{subseteq}(C) \mid \mathit{supseteq}(C) \mid \mathit{geq}(n) \mid \neg P \mid P \vee P$$

where $C \subseteq A$ and $n \in \mathbb{N}$. Satisfaction of a predicate $P$ by a coalition $C_0$, $C_0 \models_{cp} P$, is defined straightforwardly: $C_0 \models_{cp} \mathit{subseteq}(C)$ iff $C_0 \subseteq C$; $C_0 \models_{cp} \mathit{geq}(n)$ iff $|C_0| \geq n$; $C_0 \models_{cp} \mathit{supseteq}(C)$ iff $C_0 \supseteq C$; $C_0 \models_{cp} \neg P$ iff not $C_0 \models_{cp} P$; and $C_0 \models_{cp} P_1 \vee P_2$ iff $C_0 \models_{cp} P_1$ or $C_0 \models_{cp} P_2$.

Formally, the language of the NCCTL is generated as follows:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{E} \bigcirc \varphi \mid \mathsf{E}(\varphi \mathcal{U} \varphi) \mid \mathsf{A}(\varphi \mathcal{U} \varphi) \mid \langle P\rangle\varphi$$

We also write $[P]\varphi$ to denote the dual coalition predicate: $\neg\langle P\rangle\neg\varphi$.

Formulas of NCCTL are interpreted in a triple $(K, \eta, s)$ where $K$ is a Kripke structure, $\eta$ a normative system over $K$ and $s$ is a state of $K$. The clauses for coalition predicates are as follows:

$$K, \eta, s \models \langle P\rangle\varphi \text{ iff } \exists C \subseteq A\ (C \models_{cp} P \text{ and } K \dagger (\eta \upharpoonright C), \eta, s \models \varphi)$$

All other formulas are defined as for CTL, but carrying the normative system in the context: e.g. $K, \eta, s \models \mathsf{E} \bigcirc \varphi$ iff $\exists \pi \in \Pi(s) : K, \eta, \pi[1] \models \varphi$.

## 3 Introducing NORMC by example

In this section we demonstrate how NORMC is used, via a simple example. The core of the model checker is a function called `check`. By importing the NORMC code into a Haskell interpreter, this function can be used as follows:

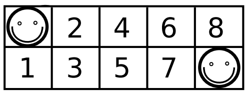
```
*Example> check myModel myNS myFormula
[s1, s3, s7]
```

Here, the arguments `myModel`, `myNS` and `myFormula`, represent the model, the normative system, and the formula respectively, and the function returns the set of (in this case three) states in which the formula holds in the given model and normative system. We now describe the types of these three arguments expected by `check`.

Kripke models are represented by the `Kripke` data structure. To represent sets, such as the set of states in the `Kripke` structure, sorted lists with no duplicate occurrences are used. In NORMC `Kripke` is defined as follows:

```
data (Ord s, Eq p) ⇒ Kripke p s = Kripke {
  agents :: [Int],         states :: [s],
  tr :: FODBR s s,         owner :: (s, s) → Int,
  valuation :: p → [s]  }
```

As an example, we describe a simple model of a pavement, divided into 10 *positions*, with two agents on opposite sides of it. The agents can move (or choose to stand still) alternatingly. The model, in its initial state, can be visualised as follows:



The model is defined by the following declaration.

```
exampleModel :: Kripke Prop State
exampleModel = Kripke [0,1] statespace transition owner val
```

The `transition` relation over the `statespace` describes the possible transitions resulting from agents' actions. In addition to these components we also need to assign the `owner` of each transition step. The proposition symbols and their valuation map complete the definition of the model.

Regarding the type of the propositions, `Prop`, we would like to talk about whether a particular agent is found on the east/west edge of the pavement, or in the north/-south "lane". The type of the states, `State`, must be able to code the current agent and the positions of both the agents. We can represent this by three integers.

```
data Property = N | S | E | W | T deriving Eq

type Prop  = (Property, Int)
type State = (Int, Int, Int)
```

A state is thus a tuple consisting of three integers. The first indicates the position of agent `0`, the second the position of agent 1, and the last indicates which agent's turn it is. The sorted list of possible states of the model can be described by

Haskell's list comprehension syntax succinctly:

```
statespace :: [State]
statespace = [ (p0, p1, i) | p0 ← [0..9], p1 ← [0..9], p0 /= p1, i ← [0,1] ]
```

The valuation function selects the states from `statespace` in which the indicated agent is in one of the appropriate positions.

```
project :: Int → State → Int
project 0 (p0, _, _) = p0
project 1 (_, p1, _) = p1

val :: Prop → [State]
val (N, ag) = filter (even ∘ (project ag)) statespace
val (S, ag) = filter (odd ∘ (project ag)) statespace
val (W, ag) = filter ((`elem` [0,1]) ∘ (project ag)) statespace
val (E, ag) = filter ((`elem` [8,9]) ∘ (project ag)) statespace
val (T, ag) = filter (λ(_,_,i) → i == ag) statespace
```

The `transition` relation indicates which transitions are available to the agents, or simply what 'moves' agents can perform.

```
transition = build [((p0,p1,i), (p0',p1',1-i)) |
                     (p0,p1,i) ← statespace,
                     p0'       ← possibleSteps (i == 0) p0,
                     p1'       ← possibleSteps (i == 1) p1,
                     (p0',p1',1-i) `elem` statespace]
```

`possibleSteps` is a helper function for `transition`, and says that agents can 'stand', 'move' west, east, and so on. If it's not the agent's turn (first argument is `False`) the only possible move is standing.

```
possibleSteps :: Bool → Int → [Int]
possibleSteps False p = [p]
possibleSteps True p = [p-2, p, p+2]++(if even p then [p+1] else [p-1])
```

Normative systems are represented in the same way as the transition relation, through the `FODBR` type. The following relation `illegal` specifies a normative system for the example: any agent must not place himself directly in front of the other agent, and if an agent is able to move in the general direction required for the satisfaction of the goal, he must do so.

```
illegal :: FODBR State State
illegal = build [ ((p0,p1,i),(p0',p1',1-i)) |
                  (p0,p1,i)   ← statespace,
                  (p0',p1',_) ← (find1 (fst transition) (p0,p1,i)),
                  p1' == p0' + 2 || if i == 0 then (p0' < 8 && p0 ≥ p0')
                                              else (p1' > 1 && p1 ≤ p1')]

owner :: (State,State) → Int
owner ((_,_,i),_) = i
```

Finally, formulas are represented by the structures `Formula` and `Coalition`:

```
data (Eq p) => Formula p = Prop p              | Neg  (Formula p)
             | Disj (Formula p) (Formula p) | Conj (Formula p) (Formula p)
             | EX   (Formula p)             | EF   (Formula p)
             | EG   (Formula p)             | EU   (Formula p) (Formula p)
             | CD   Coalition   (Formula p) | CS   Coalition   (Formula p)
               deriving (Show)
```

```
data Coalition = Subseteq [Int]              | Supseteq [Int]
               | Eq [Int]                    | GEQ Int
               | CNeg Coalition              | CDisj Coalition Coalition
                 deriving (Show)

ag,af :: (Eq p) => (Formula p) -> (Formula p)
ag f = Neg (EF (Neg f))
af f = Neg (EG (Neg f))
```

A formula that should be true only in the initial state in the example:

```
initF :: Formula Prop
initF = Conj (Conj (Prop (T, 0)) (Conj (Prop (N, 0)) (Prop (W, 0))))
             (Conj (Prop (S, 1)) (Prop (E, 1)))
```

And now an *objective* formula: agent `0` is east, and `1` is west:

```
goalF :: Formula Prop
goalF = Conj (Prop (E, 0)) (Prop (W, 1))
```

We now have all three arguments for the `check` function:

```
*Ex01> check exampleModel illegal (Conj initF (af goalF))
[]
*Ex01> check exampleModel illegal (CD (GEQ 0) (Conj initF (af goalF)))
[(0,9,0)]
```

We can see that there is no state satisfying both `initF` and `af goalF`, but there exists a coalition which can ensure that we eventually end up satisfying the goal.

## 4 Specifying and model checking a more complex system

A more elaborate example from [4] describes four researches attending a conference who need to share a limited amount of resources. The model in the example has 62500 states and 470596 transitions. We present the essential parts of this example and use it to demonstrate the flexibility in describing more elaborate models in Haskell/NORMC, and that NORMC can model check also larger models. Due to lack of space some details must be left out, but we focus on the essential parts.

The states are tuples $s = \langle O_a, O_b, O_c, O_d, i\rangle$ where for each $i \in \{a, b, c, d\} = A$, $O_i$ represents the resources owned by agent $i$. The resources they need to share are: (i) a printer (the set $R_1$), (ii) two scanners (the set $R_2$), and (iii) three computers (the set $R_3$). The agents do not have the same needs. Agent $a$ needs the printer and a computer, agent $b$ needs a scanner and the printer, agent $c$ needs a scanner and a computer, and agent $d$ only needs the printer. Agents' actions are turn-based. There is no distribution of resources that allows all agents to own the resources they need simultaneously, but there are limitations on the permitted transitions which guarantee that each agent will eventually own all the resources needed.

The first restriction is the implementation of a normative system which sets out some basic rules of behaviour. Since the example is rather elaborate, we have to omit much of the formal definition here and refer to [4] for details. Informal description of the normative system $\eta_0$ [4]:

[...] no agent (i) owns two resources of the same type at the same time, (ii) takes possession of a resource that he does not need, (iii) takes possession of two new resources simultaneously, and (iv) fails to take possession of some useful resource if it is available when it is his turn [. . . ]

We implement an equivalent model where states are represented by seven integers. The first six of these integers represent the owner of, respectively, (1) the printer, (2) scanner$_1$, (3) scanner$_2$, (4) computer$_1$, (5) computer$_2$ and (6) computer$_3$. Each of these can take on a value $0..4$ where 0 represents a *free* resource and a non-zero number indicates the owner of this resource. The seventh integer represent whose turn it is and has a value $1..4$.

```
type State = (Int, Int, Int, Int, Int, Int, Int)
```

The set of states is defined as follows.

```
statespace :: [State]
statespace = sort
            [(p, s1, s2, c1, c2, c3, a) |
                 p  ← [0..4], s1 ← [0..4], s2 ← [0..4],
                 c1 ← [0..4], c2 ← [0..4], c3 ← [0..4],  a  ← [1..4] ]
```

For the transition relation, if the current agent a owns a given resource $r$ (a $== r$), or nobody owns that resource ($r$ ==0), then the current agent can keep (resp. grab) or release (resp. ignore) it, otherwise that resource will not change owner.

```
transition :: FODBR State State
transition = build [((p , s1 , s2 , c1 , c2 , c3 , a),
                     (p′, s1′, s2′, c1′, c2′, c3′, 1 + (a ‘mod‘ 4))) |
                    (p,s1,s2,c1,c2,c3,a) ← statespace,
                    p′  ← if (p  == a || p  == 0) then [0, a] else [p ],
                    s1′ ← if (s1 == a || s1 == 0) then [0, a] else [s1],
                    s2′ ← if (s2 == a || s2 == 0) then [0, a] else [s2],
                    c1′ ← if (c1 == a || c1 == 0) then [0, a] else [c1],
                    c2′ ← if (c2 == a || c2 == 0) then [0, a] else [c2],
                    c3′ ← if (c3 == a || c3 == 0) then [0, a] else [c3] ]
```

We label each transition with its owner. This is a simple projection.

```
owner :: (State, State) → Int
owner ((_,_,_,_,_,_,i), _) = i
```

As in the previous example, we model a proposition symbol by an (agent) index and a keyword. We define the type `Resource` to denote the various resources and give a function which projects from a given state the owner of the given resource.

```
data Resource = Pr | S1 | S2 | C1 | C2 | C3 deriving Eq

project :: Resource → State → Int
project Pr (pr,_,_,_,_,_,_) = pr
project S1 (_,s1,_,_,_,_,_) = s1
-- and so on...
```

The valuation function is now simply defined by removing the states in which given agent does not own the resource in question.

```
type Proposition = (Resource, Int)
```

```
val :: Proposition → [State]
val (res, ag) = filter ((ag == ) ∘ (project res)) statespace
```

The normative system $\eta_0$ is implemented as the union of four separate relations (see the description of $\eta_0$).These are all constructed in a similar way, i.e. restricting the transition relation appropriately, and we show only `component3` which does not allow an agent to grab two resources at the same time.

```
component3 :: FODBR State State
component3 = restrict transition
          (λ(pr,s1,s2,c1,c2,c3,a) (pr', s1', s2', c1', c2', c3', a') →
           (foldr (+) 0 $ zipWith (λx y → if (x /= a) && (y == a) then 1 else 0)
                            [pr , s1 , s2 , c1 , c2 , c3 ]
                            [pr', s1', s2', c1', c2', c3']) > 1)


eta_0 :: FODBR State State
eta_0 = component1 `union` component2 `union` component3 `union` component4
```

Next we define the normative system $\eta_1$: if an agent owns all his useful resources simultaneously (he is 'happy'), he will make them available in the next turn.

```
stHappy :: Int → State → Bool
stHappy 1 (pr, _,  _,  c1, c2, c3, _) = (pr == 1 && (c1 == 1 || c2 == 1 || c3 == 1))
-- and similarly for stHappy 2, 3 and 4, defining the states where agents are happy


ownsSomething :: Int → (State → Bool)
ownsSomething n = (pr, s1, s2, c1, c2, c3, a) → (pr == n || s1 == n || s2 == n ||
                                                c1 == n || c2 == n || c3 == n)

eta_1 :: FODBR State State
eta_1 = restrict transition
        (λs s' → (stHappy (owner (s,s')) s) && (ownsSomething (owner (s,s')) s'))
```

For convenience, we define two example models, one with no system implemented on it ($K_0$), and the second one with $\eta_0$ implemented on it ($K_1 = K_0 \dagger \eta_0$):

```
exampleModel :: Kripke Proposition State
exampleModel = Kripke [1,2,3,4] statespace transition owner val

exampleModel' :: Kripke Proposition State
exampleModel' = ir exampleModel eta_0 [1,2,3,4]
```

Our first objective is that it is always the case that every agent will eventually become happy: $\phi_1 = \mathsf{A}\Box(\bigwedge_{i \in Ag} \mathsf{A}\Diamond happy(i))$, where the proposition $happy(i)$ (code: `fHappy i`) is true iff $i$ is 'happy'.

```
phi_1 :: Formula Proposition
phi_1 =  ag (Conj (af (fHappy 1))(Conj(af (fHappy 2))(Conj(af (fHappy 3))(af (fHappy 4)))))
```

This concludes the implementation of the example model from [4]. We now proceed with model checking. First we check whether `eta_0` is effective.

```
*Ex02> check exampleModel' eta_1 phi_1
[]
```

The model checker outputs an empty set, indicating there are no states satisfying `phi_1` if no agents are required to comply with `eta_1`. In [4] it is claimed that compliance of coalition $\{1, 2, 3\}$ to `eta_1` is sufficient for the objective to hold (assuming that all agents comply with $\eta_0$). Let us check:

```
*Ex02> check exampleModel' eta_1 (CS (Supseteq [1,2,3]) phi_1)
[]
```

Surprisingly, the model checker tells us that the formula is not true in any state. Using the model checker we can produce a trace:

```
[(0,0,0,0,0,0,1),(0,0,0,0,0,1,2),(2,0,0,0,0,1,3),(2,0,0,0,3,1,4),
 (2,0,4,0,3,1,1),(2,0,4,0,3,0,2),(0,2,4,0,3,0,3),(0,2,4,0,0,0,4),
 (0,2,0,0,0,0,1),(0,2,0,0,0,1,2),(2,0,0,0,0,1,3)]
```

In steps 2 and 10 we end up in the same state. We have a loop where all agents comply with both $\eta_0$ and $\eta_1$. Only one agent $(4)$ is ever 'happy' in this loop. So, there is a path $\pi$ in $\Pi(s_0)$ along which it is not the case that every agent will eventually be 'happy'. Thus, there is an error in the example in [4]. This error was not obvious, and this illustrates the benefit of software tools even for "toy" examples.

The problem with the example from [4] is that $\eta_1$ as stated is too weak; as seen above it allows agents to simultaneously grab some resources and release others in an endless loop without ever becoming 'happy'. We now introduce an additional condition that tells the agents to not release any resources they posses until they are 'happy':

```
dont_release :: FODBR State State
dont_release = restrict transition
                (λs@(pr,s1,s2,c1,c2,c3,a) (pr',s1',s2',c1',c2',c3',_) →
                  (not $ stHappy a s) &&
                  (((pr == a) && (pr' /= a)) || ((s1 == a) && (s1' /= a)) ||
                   ((s2 == a) && (s2' /= a)) || ((c1 == a) && (c1' /= a)) ||
                   ((c2 == a) && (c2' /= a)) || ((c3 == a) && (c3' /= a))))
```

Modify the normative system `eta_1` to include the new condition:

```
eta_1' :: FODBR State State
eta_1' = eta_1 `union` dont_release
```

Now $\{1,2,3\}$ is a sufficient[4] coalition (the only minimal sufficient coalition):

```
*Ex02> let is = ((0,0,0,0,0,0,1)::State)
*Ex02> is `elem` check exampleModel' eta_1' (CS (Supseteq [1,2,3]) phi_1)
True
```

## 5 Implementation

We now discuss the design and implementation of NORMC. As mentioned, the function `check` is the core of the model checker. It implements an extension of a standard model checking algorithm for CTL formulas, as described in [6], with additional clauses for coalition predicates. `check` takes a model, normative system and formula as arguments and returns the set of states in which the given formula is satisfied. The function is defined recursively, and the clauses for propositional variables and path-state-quantifiers are defined as in standard CTL algorithms.

[4] $C$ is sufficient for $\varphi$ in the context of $K$ and $\eta$ iff $\forall C' \subseteq A : (C \subseteq C') \Rightarrow [K \dagger (\eta \restriction C') \models \varphi]$. Note that $K \dagger (\eta \restriction C) \models \varphi$ does not in general imply that $K \dagger (\eta \restriction C') \models \varphi$ when $C \subseteq C'$ [4].

For the coalition predicates we need some semantic update functions: `nsimplement` implements a normative system on a Kripke structure, `nsrestrict` implements the $\upharpoonright$ operator, and `ir` combines the two, implementing a given normative system restricted to the given coalition predicate:

```
nsimplement :: (Ord s, Eq p) ⇒ (Kripke p s) → (FODBR s s) → (Kripke p s)
nsimplement mod sys = mod { tr = (tr mod) `minus` sys }

nsrestrict :: (Ord s, Eq p) ⇒ (Kripke p s) → (FODBR s s) → [Int] → (FODBR s s)
nsrestrict mod sys coa = restrict sys (λs s' → ((owner mod (s,s')) `elem` coa))

ir :: (Ord s, Eq p) ⇒ (Kripke p s) → (FODBR s s) → [Int] → (Kripke p s)
ir mod sys coa = nsimplement mod (nsrestrict mod sys coa)
```

We now describe some minor optimisation tricks used to make NORMC usable for non-trivial examples.

Model checking NCCTL involves quantification over coalitions. As discussed, for each subformula $\phi$ the algorithm computes the set of satisfying states (call it $sat(\phi)$). By the semantics of NCCTL, we have that $sat([P]\phi) = \{s \in S : \forall C \subseteq A(C \models_{cp} P \Rightarrow K \dagger (\eta \upharpoonright C), \eta, s \models \phi)$. A naive implementation of this involves testing every coalition against the predicate, once for each state. However, it is easy to see that the quantifier can be moved out (and similarly for $\langle P \rangle \varphi$):

**Lemma 5.1** *$sat([P]\phi) = \bigcup_{C \models_{cp} P}\{s \in S : K \dagger (\eta \upharpoonright C), \eta, s \models \phi)$ and $sat(\langle P \rangle \phi) = \bigcap_{C \models_{cp} P}\{s \in S : K \dagger (\eta \upharpoonright C), \eta, s \models \phi)$.*

Thus we only have to test the predicate once for each coalition, and save a considerable amount of time in practice. This is made use of as follows:

```
check mod sys (CD c f) = foldl' nubunion [] $
                         map (λmod → (check mod sys f)) $
                             map (ir mod sys) $ coasGivenCP mod c

check mod sys (CS c f) = foldl' nubisect (states mod) $
                         map (λmod → (check mod sys f)) $
                             map (ir mod sys) $ coasGivenCP mod c
```

Much of the core functionality of NORMC is found in the handling of the transition relation, and performing a model update operation is performing an appropriate restriction on this relation. We only mention the implementation of relation handling briefly here, due to lack of space and to keep focus on more high-level considerations. Our relation handling implementation, FODBR (Finite Ordered Domain Binary Relation), is an efficient representation of a binary relation by two binary search trees. We require that the domain is finite and consists of elements with an ordering, and we represent the binary relation by two multifunctions: $src : \mathcal{D} \to \wp(\mathcal{D}')$ and $trg : \mathcal{D} \to \wp(\mathcal{D}')$. Both these functions are stored in a binary search tree where each node contains a value of the type $(\mathcal{D}, [\mathcal{D}'])$ ('key' and 'value set', respectively). The functionality provided by FODBR requires that argument lists are sorted and contain no duplicate elements, and guarantees that any returned lists also satisfy these properties. This allows us to implement the usual set theoretic operations of union, intersection and difference efficiently.

## 6 Conclusions

In this paper we have described NorMC, a prototype model checker for Norm Compliance CTL (NCCTL). NCCTL extends CTL with a family of modalities with update semantics. We have also aimed to demonstrate that the Haskell programming language is a natural and useful alternative for model checking; in particular that the full power of Haskell makes it easy to describe arbitrary state-transition models in a natural way. While NorMC has not been optimised for industrial use, we have seen that it is efficient enough to be used on interesting and non-trivial examples. If higher efficiency is needed, there are two natural options. The first is to extend NorMC with standard symbolic model checking techniques such as binary decision diagrams (BDDs) and partial order reduction. The other is to change NorMC into a front-end for an efficient existing CTL model checker. For future work, we are currently implementing a graphical front-end to NorMC.

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## References

[1] Ågotnes, T., W. van der Hoek, C. S. Juan A. Rodriguez-Aguilar and M. Wooldridge, *On the logic of normative systems*, in: *Proceedings of IJCAI 2007*, pp. 1175–1180.

[2] Ågotnes, T., W. van der Hoek, M. Tennenholtz and M. Wooldridge, *Power in normative systems*, in: *Proceedings of AAMAS 2009*, pp. 145–152.

[3] Ågotnes, T., W. van der Hoek and M. Wooldridge, *Normative system games*, in: *Proceedings of AAMAS 2007*, pp. 876–883.

[4] Ågotnes, T., W. van der Hoek and M. Wooldridge, *Robust normative systems and a logic of norm compliance*, Logic Journal of the IGPL **18** (2009), pp. 4–30.

[5] Cimatti, A., E. Clarke, E. Giunchiglia, F. Giunchiglia, M. Pistore, M. Roveri, R. Sebastiani and A. Tacchella, *NuSMV Version 2: An OpenSource Tool for Symbolic Model Checking*, in: *Proc. International Conference on Computer-Aided Verification (CAV 2002)*, LNCS **2404** (2002).

[6] Clark, E. M., E. A. Emerson and A. P. Sistla, *Automatic verification of finite-state concurrent systems using temporal logic specifications*, ACM Trans. on Progr. Lang. and Systems **8** (1986), pp. 244–263.

[7] Dellunde, P., *On the multimodal logic of normative systems*, in: *Proc. of COIN'07* (2008), pp. 261–274.

[8] Emerson, E. A., *Temporal and modal logic*, in: J. van Leeuwen, editor, *Handbook of Theoretical Computer Science Volume B: Formal Models and Semantics*, Elsevier, 1990.

[9] Fitoussi, D. and M. Tennenholtz, *Choosing social laws for multi-agent systems: Minimality and simplicity*, Artificial Intelligence **119** (2000), pp. 61–101.

[10] Hoek, W., M. Roberts and M. Wooldridge, *Social laws in alternating time: Effectiveness, feasibility, and synthesis*, Synthese **156** (2007), pp. 1–19.

[11] Holzmann, G., *The Spin model checker*, IEEE Trans. on Software Eng. **23** (1997), pp. 279–295.

[12] Jones, S. P., “Haskell 98 language and libraries: the Revised Report,” Cambridge University Press, 2003.

[13] Shoham, Y. and M. Tennenholtz, *On the synthesis of useful social laws for artificial agent societies*, in: *Proceedings of AAAI 1992*.

[14] Shoham, Y. and M. Tennenholtz, *On social laws for artificial agent societies: Off-line design*, in: *Computational Theories of Interaction and Agency*, The MIT Press: Cambridge, MA, 1996 pp. 597–618.

[15] van Eijck, J., *Dynamic epistemic modelling*, Technical report, CWI/ILLC, Amsterdam, Uil-OTS, (2005).