Linear Regression Eric Johnson

```
library(fpp) library(fpp2) library(corrplot)
fuel summary(fuel)
library(fpp)
## Loading required package: forecast
## Warning: package 'forecast' was built under R version 4.0.5
## Registered S3 method overwritten by 'quantmod':
##
     method
     as.zoo.data.frame zoo
## Loading required package: fma
## Warning: package 'fma' was built under R version 4.0.5
## Loading required package: expsmooth
## Warning: package 'expsmooth' was built under R version 4.0.5
## Loading required package: lmtest
## Loading required package: zoo
## Warning: package 'zoo' was built under R version 4.0.5
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
## Loading required package: tseries
library(fpp2)
```

-- Attaching packages ------ fpp2 2.4 --

```
## v ggplot2 3.3.2
##
##
## Attaching package: 'fpp2'
## The following objects are masked from 'package:fpp':
##
##
       ausair, ausbeer, austa, austourists, debitcards, departures,
       elecequip, euretail, guinearice, oil, sunspotarea, usmelec
library(corrplot)
## corrplot 0.84 loaded
library(data.table)
library(faraway)
##
## Attaching package: 'faraway'
## The following objects are masked from 'package:fma':
##
       airpass, eggs, ozone, wheat
##
library(car)
## Loading required package: carData
## Registered S3 methods overwritten by 'car':
##
     method
                                      from
##
     influence.merMod
                                      lme4
##
     cooks.distance.influence.merMod lme4
##
     dfbeta.influence.merMod
                                      lme4
     dfbetas.influence.merMod
                                      lme4
##
## Attaching package: 'car'
## The following objects are masked from 'package:faraway':
##
##
       logit, vif
library(MASS)
##
## Attaching package: 'MASS'
## The following objects are masked from 'package:fma':
##
       cement, housing, petrol
##
```

Load data and summary of data:

 $\bullet\,$ plan is to predict Carbon based on other variables:

fue:

##		Model	Cylinders	Litres	Barrels	City
##	20	Chevrolet Aveo	4	1.6	12.2	25
##	21	Chevrolet Aveo 5	4	1.6	12.2	25
##	27	Chevrolet Cobalt	4	2.2	12.7	24
##	120	Chevrolet Colorado 2WD	4	2.9	17.1	18
##	127	Chevrolet Colorado 2WD	5	3.7	18.0	17
##	133	Chevrolet Colorado Cab Chassis inc 2WD	5	3.7	20.1	15
##	121	Chevrolet Colorado Crew Cab 2WD	4	2.9	17.1	18
##	128	Chevrolet Colorado Crew Cab 2WD	5	3.7	18.0	17
##	95	Chevrolet HHR FWD	4	2.0	14.9	19
##	96	Chevrolet HHR Panel FWD	4	2.0	14.9	19
##	43	Chevrolet Malibu	4	2.4	13.2	22
##	48	Chevrolet Malibu	4	2.4	13.7	22
##	15	Chevrolet Malibu Hybrid	4	2.4	11.8	26
##	113	Chrysler PT Cruiser	4	2.4	16.3	19
##	122	Chrysler PT Cruiser	4	2.4	16.3	18
##	60	Chrysler Sebring	4	2.4	14.3	21
##	74	Chrysler Sebring Convertible	4	2.4	14.9	20
##	61	Dodge Avenger	4	2.4	14.3	21
##	40	Dodge Caliber	4	2.0	14.3	23
##	71	Dodge Caliber	4	2.4	14.9	21
##	110	Dodge Journey 2WD	4	2.4	16.3	19
##	79	Ford Escape FWD	4	2.5	14.9	20
##	4	Ford Escape Hybrid FWD	4	2.5	10.7	34
##	28	Ford Focus FWD	4	2.0	12.7	24
##	80	Ford Fusion FWD	4	2.3	14.9	20
##	114	Ford Ranger Pickup 2WD	4	2.3	16.3	19
##	123	GMC Canyon 2WD	4	2.9	17.1	18
##	129	GMC Canyon 2WD	5	3.7	18.0	17
##	134	GMC Canyon Cab Chassis Inc 2WD	5	3.7	20.1	15
##	124	GMC Canyon Crew Cab 2WD	4	2.9	17.1	18
##	130	GMC Canyon Crew Cab 2WD	5	3.7	18.0	17
##	62	Honda Accord	4	2.4	14.3	21
##	63	Honda Accord Coupe	4	2.4	14.3	21
##	19	Honda Civic	4	1.8	11.8	25
##	2	Honda Civic Hybrid	4	1.3	8.2	40
##		Honda CR-V 2WD	4	2.4	14.9	20
##		Honda Element 2WD	4	2.4	15.6	20
##		Honda Fit	4	1.5	11.4	27
##		Honda Fit	4	1.5	11.0	28
##		Hyundai Accent	4	1.6	11.8	26
##		Hyundai Elantra	4	2.0	12.2	25
##		Hyundai Elantra Touring	4	2.0	13.2	23
##		Hyundai Sonata	4	2.4	13.7	22
##		Hyundai Tucson 2WD	4	2.0	15.6	20
##		Jeep Compass 2WD	4	2.0	14.3	23
##		Jeep Compass 2WD	4	2.4	14.9	21
##	42	Jeep Patriot 2WD	4	2.0	14.3	23

##	73	Jeep Patriot 2WD	4	2.4	14.9	21
##	47	Kia Optima	4	2.4	13.7	22
##	14	Kia Rio	4	1.6	11.4	26
##	89	Kia Rondo	4	2.4	15.6	20
##	30	Kia Spectra	4	2.0	12.7	24
##	93	Kia Sportage 2WD	4	2.0	15.6	20
##	49	Mazda 3	4	2.0	13.7	22
##	54	Mazda 3	4	2.3	14.3	22
##	68	Mazda 5	4	2.3	14.9	21
##	64	Mazda 6	4	2.5	14.3	21
##	115	Mazda B2300 2WD	4	2.3	16.3	19
##	81	Mazda Tribute FWD	4	2.5	14.9	20
##	5	Mazda Tribute Hybrid 2WD	4	2.5	10.7	34
##	82	Mercury Mariner FWD	4	2.5	14.9	20
##	6	Mercury Mariner Hybrid FWD	4	2.5	10.7	34
##	83	Mercury Milan	4	2.3	14.9	20
	107	Mitsubishi Eclipse	4	2.4	15.6	19
	108	Mitsubishi Eclipse Spyder	4	2.4	15.6	19
	90	Mitsubishi Galant	4	2.4	14.9	20
##		Mitsubishi Lancer	4	2.0	14.3	22
	69	Mitsubishi Lancer	4	2.4	14.9	21
	70	Mitsubishi Lancer Sportback	4	2.4	14.9	21
	94	Mitsubishi Outlander 2WD	4	2.4	15.6	20
##		Nissan Altima	4	2.5	13.2	23
	37	Nissan Altima Coupe	4	2.5	13.2	23
##		Nissan Altima Goupe Nissan Altima Hybrid	4	2.5	10.1	35
	131	Nissan Frontier 2WD	4	2.5	18.0	17
	58	Nissan Rogue FWD	4	2.5	14.3	22
	32	Nissan Kogue rwb Nissan Sentra	4	2.5	13.2	24
	12	Nissan Versa	4	1.8	11.8	27
	31	Nissan Versa	4	1.8	12.7	24
				1.6		25
	22	Pontiac G3 Wave	4	1.6	12.2 12.2	25 25
	23	Pontiac G3 Wave 5	4			
	29	Pontiac G5	4	2.2	12.7	24
##		Pontiac G5 GT	4	2.2	13.2	23
	44	Pontiac G6	4	2.4	13.2	22
	50	Pontiac G6	4	2.4	13.7	22
	102	Pontiac Solstice	4	2.0	16.3	19
	116	Pontiac Solstice	4	2.4	16.3	19
	18	Pontiac Vibe	4	1.8	12.2	26
	65	Pontiac Vibe	4	2.4	14.3	21
	103	Saab 9-3 Convertible	4	2.0	15.6	19
	97	Saab 9-3 Sport Sedan	4	2.0	15.6	19
	104	Saab 9-3 SportCombi	4	2.0	15.6	19
	125	Saab 9-5 Sedan	4	2.3	17.1	17
	126	Saab 9-5 SportCombi	4	2.3	17.1	17
	33	Saturn Astra 2DR Hatchback	4	1.8	12.7	24
	34	Saturn Astra 4DR Hatchback	4	1.8	12.7	24
	45	Saturn Aura	4	2.4	13.2	22
	16	Saturn Aura Hybrid	4	2.4	11.8	26
	105	Saturn SKY	4	2.0	16.3	19
	117	Saturn SKY	4	2.4	16.3	19
	109	Saturn Vue FWD	4	2.4	15.6	19
##	25	Saturn Vue Hybrid	4	2.4	12.2	25

```
## 66
                                        Scion tC
                                                                        14.3
                                                                2.4
                                                                                21
## 56
                                        Scion xB
                                                           4
                                                                2.4
                                                                        14.3
                                                                                22
## 17
                                        Scion xD
                                                                1.8
                                                                        12.2
                                                                                26
## 132
                                                           4
                                                                        18.0
                             Suzuki Equator 2WD
                                                                2.5
                                                                                17
## 111
                            Suzuki Grand Vitara
                                                           4
                                                                2.4
                                                                        16.3
                                                                                19
## 51
                                      Suzuki SX4
                                                           4
                                                                2.0
                                                                        13.7
                                                                                22
## 38
                                Suzuki SX4 Sedan
                                                                2.0
                                                                        13.2
                                                                                23
## 52
                                Suzuki SX4 Sport
                                                           4
                                                                2.0
                                                                        13.7
                                                                                22
## 59
                                    Toyota Camry
                                                           4
                                                                2.4
                                                                        13.7
                                                                                21
## 7
                                                           4
                                                                2.4
                                                                        10.1
                                                                                33
                            Toyota Camry Hybrid
                                  Toyota Corolla
## 10
                                                                1.8
                                                                        11.4
                                                                                27
## 53
                                                                2.4
                                  Toyota Corolla
                                                           4
                                                                        13.7
                                                                                22
## 26
                                                           4
                                                                        12.2
                                   Toyota Matrix
                                                                1.8
                                                                                25
## 67
                                                           4
                                                                2.4
                                                                        14.3
                                   Toyota Matrix
                                                                                21
## 1
                                                           4
                                                                1.5
                                                                         7.4
                                                                                48
                                    Toyota Prius
## 57
                                Toyota RAV4 2WD
                                                           4
                                                                2.5
                                                                        14.3
                                                                                22
## 112
                                                           4
                                                                2.7
                                                                        16.3
                                                                                19
                              Toyota Tacoma 2WD
## 8
                                    Toyota Yaris
                                                           4
                                                                1.5
                                                                        11.0
                                                                                29
## 75
                               Volkswagen Jetta
                                                           5
                                                                2.5
                                                                        14.3
                                                                                20
## 76
                   Volkswagen Jetta SportWagen
                                                           5
                                                                2.5
                                                                        14.3
                                                                                20
## 77
                          Volkswagen New Beetle
                                                           5
                                                                2.5
                                                                        14.9
                                                                                20
## 84
             Volkswagen New Beetle Convertible
                                                           5
                                                                2.5
                                                                        14.9
                                                                                20
## 78
                                                                        14.3
                                                           5
                                                                2.5
                                                                                20
                              Volkswagen Rabbit
## 85
                                   Volvo C30 FWD
                                                           5
                                                                2.4
                                                                        14.9
                                                                                20
## 98
                                                           5
                                                                2.5
                                                                        14.9
                                                                                19
                                   Volvo C30 FWD
## 118
                          Volvo C70 Convertible
                                                           5
                                                                2.5
                                                                        16.3
                                                                                18
## 86
                                   Volvo S40 FWD
                                                           5
                                                                2.4
                                                                        14.9
                                                                                20
## 99
                                   Volvo S40 FWD
                                                           5
                                                                2.5
                                                                        14.9
                                                                                19
                                                           5
                                                                2.4
                                                                        16.3
## 119
                                   Volvo S60 FWD
                                                                                18
## 100
                                                           5
                                   Volvo S60 FWD
                                                                2.5
                                                                        14.9
                                                                                19
## 106
                                   Volvo S60 FWD
                                                           5
                                                                2.5
                                                                        15.6
                                                                                19
## 87
                                   Volvo V50 FWD
                                                           5
                                                                2.4
                                                                        14.9
                                                                                20
## 101
                                   Volvo V50 FWD
                                                           5
                                                                2.5
                                                                        14.9
                                                                                19
##
       Highway Cost Carbon
## 20
             34 1012
                         6.6
             34 1012
## 21
                         6.6
## 27
             33 1049
                         6.8
## 120
             24 1418
                         9.2
## 127
             23 1491
                         9.6
## 133
             20 1667
                        10.8
## 121
             24 1418
                         9.2
## 128
             23 1491
                         9.6
## 95
             29 1233
                         8.0
## 96
             29 1233
                         8.0
## 43
             33 1091
                         7.1
## 48
                         7.3
             30 1134
                978
                         6.3
## 15
             34
## 113
             24 1349
                         8.7
## 122
             24 1349
                         8.7
## 60
                         7.7
             30 1182
## 74
             29 1233
                         8.0
## 61
             30 1182
                         7.7
## 40
             27 1182
                         7.7
## 71
             25 1233
                         8.0
```

##	110	25	1349	8.7
##	79	28	1233	8.0
##	4	31	887	5.7
##	28	33	1049	6.8
##	80	28	1233	8.0
##	114	24	1349	8.7 9.2
## ##	123 129	24 23	1418 1491	9.2
##	134	20	1667	10.8
##	124	24	1418	9.2
##	130	23	1410	9.6
##	62	30	1182	7.7
##	63	30	1182	7.7
##	19	36	978	6.3
##	2	45	675	4.4
##	88	27	1233	8.0
##	91	25	1290	8.3
##	11	33	944	6.1
##	9	35	916	5.9
##	13	35	978	6.3
##	24	33	1012	6.6
##	39	30	1091	7.1
##	46	32	1134	7.3
##	92	25	1290	8.3
##	41	27	1182	7.7
##	72	25	1233	8.0
##	42	27	1182	7.7
##	73	25	1233	8.0
##	47	32	1134	7.3
##	14	35	944	6.1
##	89	27	1290	8.3
##	30	32	1049	6.8
##	93	25	1290	8.3
##	49	30	1134	7.3
##	54	28	1182	7.7
##	68	27	1233	8.0
##	64	30	1182	7.7
##	115	24	1349	8.7
##	81	28	1233	8.0
##	5	31	887	5.7
##	82	28	1233	8.0
##	6	31	887	5.7
##	83	28	1233	8.0
##	107	26	1290	8.3
##	108	26	1290	8.3
##	90	27	1233	8.0
##	55	28	1182	7.7
##	69	27	1233	8.0
##	70	27	1233	8.0
##	94	25	1290	8.3
##	36	31	1091	7.1
##	37	31	1091	7.1
##	3	33	833	5.4
##	131	22	1491	9.6

шш	58	27	1182	7.7
##				
##	32	30	1091	7.1
##	12	33	978	6.3
##	31	32	1049	6.8
##	22	34	1012	6.6
##	23	34	1012	6.6
##	29	33	1049	6.8
##	35	32	1091	7.1
##	44	33	1091	7.1
##	50	30	1134	7.3
		27		
##	102		1349	8.7
##	116	24	1349	8.7
##	18	31	1012	6.6
##	65	29	1182	7.7
##	103	27	1290	8.3
##	97	28	1290	8.3
##	104	27	1290	8.3
##	125	27	1418	9.2
##	126	27	1418	9.2
##	33	30	1049	6.8
##	34	30	1049	6.8
##	45	33	1091	7.1
##	16	34	978	6.3
##	105	27	1349	
				8.7
##	117	24	1349	8.7
##	109	26	1290	8.3
##	25	32	1012	6.6
##	66	29	1182	7.7
##	56	28	1182	7.7
##	17	32	1012	6.6
##	132	22	1491	9.6
##	111	25	1349	8.7
##	51	30	1134	7.3
##	38	31	1091	7.1
##	52	30	1134	7.3
##	59	31	1134	7.3
##	7	34	833	5.4
##	10	35	944	6.1
##	53	30	1134	7.3
##	26	31	1012	6.6
##	67	29	1182	7.7
##	1	45	615	4.0
##	57	28	1182	7.7
##	112	25	1349	8.7
##	8	35	916	5.9
##	75	29	1182	7.7
##	76	29	1182	7.7
##	77	29	1233	8.0
##	84	28	1233	8.0
##	78	29	1182	7.7
##	85	28	1233	8.0
##	98	28		
			1233	8.0
##	118	26	1349	8.7
##	86	28	1233	8.0

```
## 99
            28 1233
                        8.0
## 119
            26 1349
                        8.7
## 100
            28 1233
                        8.0
## 106
            27 1290
                        8.3
## 87
            28 1233
                        8.0
## 101
            28 1233
                        8.0
```

summary(fuel)

```
##
                               Model
                                          Cylinders
                                                            Litres
##
   Volvo S60 FWD
                                  : 3
                                               :4.000
                                                               :1.30
                                        Min.
                                                        Min.
## Chevrolet Colorado 2WD
                                    2
                                        1st Qu.:4.000
                                                        1st Qu.:2.00
## Chevrolet Colorado Crew Cab 2WD:
                                    2
                                        Median :4.000
                                                        Median:2.40
## Chevrolet Malibu
                                  :
                                    2
                                        Mean
                                               :4.157
                                                        Mean
                                                              :2.31
## Chrysler PT Cruiser
                                    2
                                        3rd Qu.:4.000
                                                        3rd Qu.:2.50
## Dodge Caliber
                                  : 2
                                        Max.
                                               :5.000
                                                        Max.
                                                               :3.70
   (Other)
##
                                  :121
##
      Barrels
                                                       Cost
                        City
                                      Highway
## Min.
          : 7.40
                  Min.
                        :15.00
                                  Min.
                                         :20.00
                                                  Min.
                                                         : 615
  1st Qu.:13.20
                   1st Qu.:19.00
                                   1st Qu.:27.00
                                                  1st Qu.:1091
##
## Median :14.30
                   Median :21.00
                                   Median :28.00
                                                  Median:1182
##
  Mean
         :14.31
                   Mean
                         :21.97
                                   Mean
                                         :28.89
                                                  Mean
                                                         :1185
   3rd Qu.:15.60
                   3rd Qu.:23.75
                                   3rd Qu.:31.00
                                                  3rd Qu.:1290
##
          :20.10
                          :48.00
                                         :45.00
  Max.
                   Max.
                                   Max.
                                                  Max.
                                                         :1667
##
##
       Carbon
         : 4.000
  Min.
##
   1st Qu.: 7.100
## Median : 7.700
## Mean
         : 7.671
## 3rd Qu.: 8.300
## Max.
         :10.800
##
```

Checking for correlation and visualizing data

```
library(GGally)
```

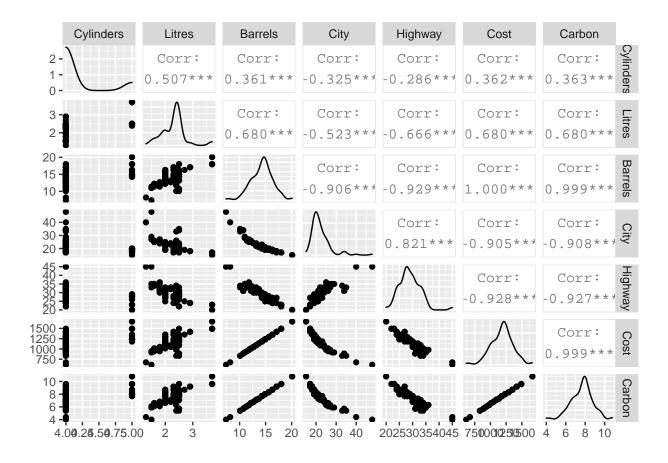
```
## Registered S3 method overwritten by 'GGally':
##
     method from
##
     +.gg
            ggplot2
##
## Attaching package: 'GGally'
## The following object is masked from 'package:faraway':
##
##
       happy
## The following object is masked from 'package:fma':
##
##
       pigs
```

?ggpairs

starting httpd help server ...

done

ggpairs(fuel,c(2:8))



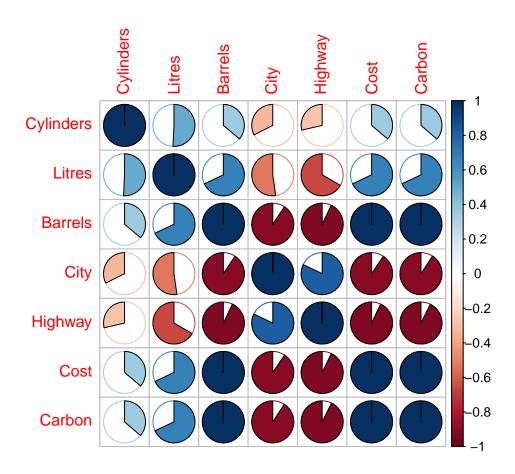
#lower = list(mapping = aes(color = Quarter), continuous = 'smooth', combo = 'facetdensity'))

- above we see there are strong correlations with Carbon and Barrels, City, Highway, and Cost. We see weaker correlations between Carbon and Cylinders and Litres. There is likely colinearity with City and Highway when it comes to determining Carbon; probably only one of those variables is needed to predict Carbon. Further the use of the Cost or Barrels variable would be redundant since clearly as the mpg of the vehicle increases, the cost or barrels would decrease. We will use the City variable to predict Carbon on it's own.
- Further with the relationship of City and Carbon, we can see from the plot the relationship is a slightly curved line indicating an exponential decay. This is non-linear and the data needs transformation to be linear. We will use the natural log of both the independent variable (City) and the reponse variable (Carbon) for constructing our model.

```
f1 <- fuel
fuelmat <- f1[,c(2:8)]
cor(fuelmat)</pre>
```

Correlation matrix and plot:

```
##
              Cylinders
                                                              Highway
                             Litres
                                       Barrels
                                                      City
                                                                             Cost
## Cylinders 1.0000000 0.5072599 0.3612070 -0.3246731 -0.2863089 0.3615217
## Litres 0.5072599 1.0000000 0.6795817 -0.5228023 -0.6663893 0.6796274
## Barrels 0.3612070 0.6795817 1.0000000 -0.9062413 -0.9288892 0.9999567
## City -0.3246731 -0.5228023 -0.9062413 1.0000000 0.8209571 -0.9052234
## Highway -0.2863089 -0.6663893 -0.9288892 0.8209571 1.0000000 -0.9284566
## Cost 0.3615217 0.6796274 0.9999567 -0.9052234 -0.9284566 1.0000000 ## Carbon 0.3633336 0.6798169 0.9994928 -0.9079411 -0.9267689 0.9994937
##
                 Carbon
## Cylinders 0.3633336
## Litres 0.6798169
## Barrels
              0.9994928
## City
             -0.9079411
## Highway -0.9267689
## Cost
              0.9994937
## Carbon
            1.0000000
fuelmat <- cor(fuelmat)</pre>
corrplot(fuelmat, method = 'pie')
```



• The above is a correlation matrix and plot to show the strength of relationships of each variable.

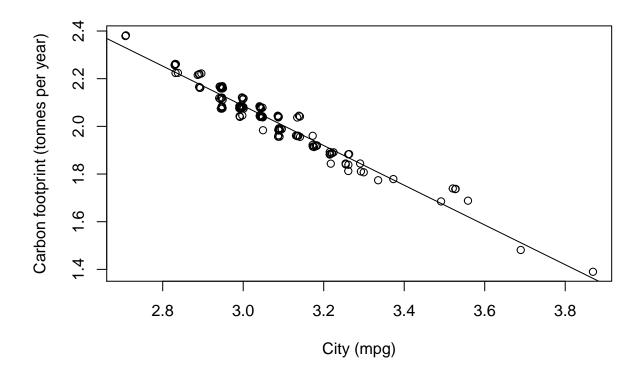
fitting a model to predict Carbon

```
cfit <- lm(log(Carbon) ~ log(City), data = fuel)</pre>
summary(cfit)
##
  lm(formula = log(Carbon) ~ log(City), data = fuel)
##
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
## -0.06330 -0.02093 -0.01033 0.03070 0.09282
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 4.58581
                            0.05043
                                      90.94
                                              <2e-16 ***
                                     -50.84
## log(City)
                            0.01639
## ---
## Signif. codes:
                  0 '*** 0.001 '** 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03408 on 132 degrees of freedom
```

```
## Multiple R-squared: 0.9514, Adjusted R-squared: 0.9511
## F-statistic: 2585 on 1 and 132 DF, p-value: < 2.2e-16</pre>
```

- the above summary of the model shows a high F-statistic with a low p-value indicating the model can be used to predict Carbon.
- below is a plot of Carbon and City after transformation of the data to form a linear relationship along with a superimposed line generated from the fitted data of the model; we can see the model is overall a good fit:

```
plot(jitter(log(Carbon)) ~ jitter(log(City)), xlab="City (mpg)",
ylab="Carbon footprint (tonnes per year)", data=fuel)
abline(cfit)
```

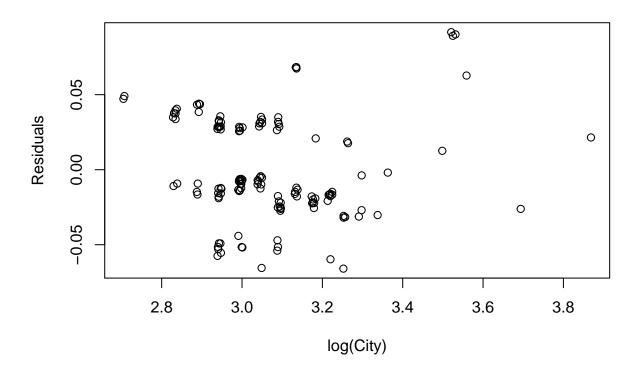


Let's check the residuals:

• the plots below show a fairly good fit of the model and there are no obvious patterns. The QQ-plot reveals a slightly longer right-tail so there is not a normal distribution.

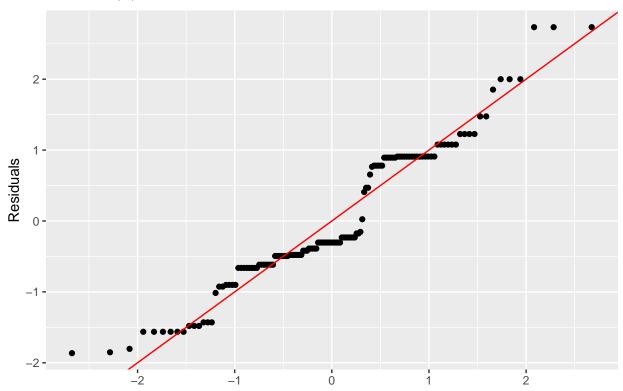
```
res <- residuals(cfit)

plot(jitter(res, amount=.005) ~ jitter(log(City)),
    ylab="Residuals", xlab="log(City)", data=fuel)</pre>
```



```
cfitmod <- fortify(cfit)
qplot(sample = scale(.resid), data = cfitmod) + geom_abline(intercept = 0, slope = 1, color = 'red') +
labs(title = 'Normal QQ Plot', y = 'Residuals')</pre>
```

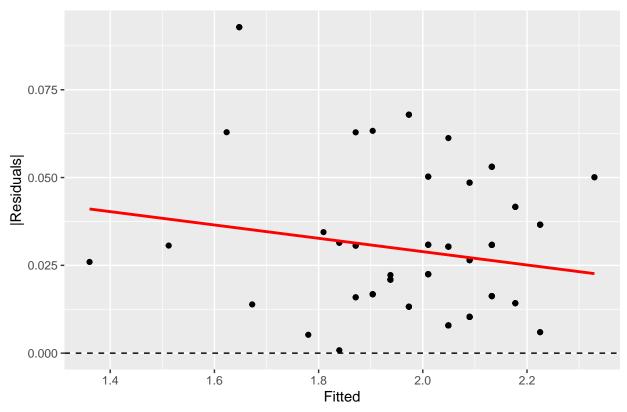
Normal QQ Plot



```
cfitmod <- fortify(cfit)
qplot(.fitted, abs(.resid), data = cfitmod) + geom_hline(yintercept = 0, linetype = 'dashed') +
labs(title = 'Scale Location Plot', x = 'Fitted', y = '|Residuals|' ) +
geom_smooth(method = 'lm', color = 'red', se = F)</pre>
```

'geom_smooth()' using formula 'y ~ x'

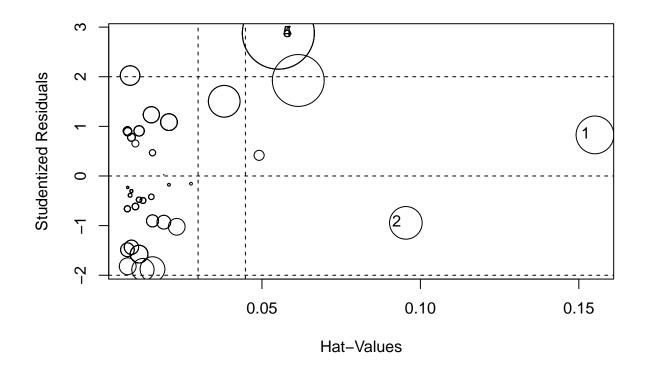
Scale Location Plot



Cook's distance:

- lets look if there are outliers that are influencing the model which we may want to remove:
- we can see what the highest Cook's distance values are with a maximum of .2292:

influencePlot(cfit)



```
## StudRes Hat CookD
## 4 2.8779233 0.05517210 0.22917792
## 2 -0.9446127 0.09541329 0.04709672
## 5 2.8779233 0.05517210 0.22917792
## 1 0.8278611 0.15509206 0.06305240

cook <- cooks.distance(cfit)
summary(cook)</pre>
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 5.850e-06 1.305e-03 2.708e-03 1.187e-02 8.926e-03 2.292e-01
```

• below we can combine the standard residual and cooks distance with the fuel data:

```
r <- stdres(cfit)
fuelcook <- cbind(fuel, cook, r)</pre>
```

• we can determine which rows in the data correspond to any cook's distance greater than 4/n: There are 10 rows that fit that criteria:

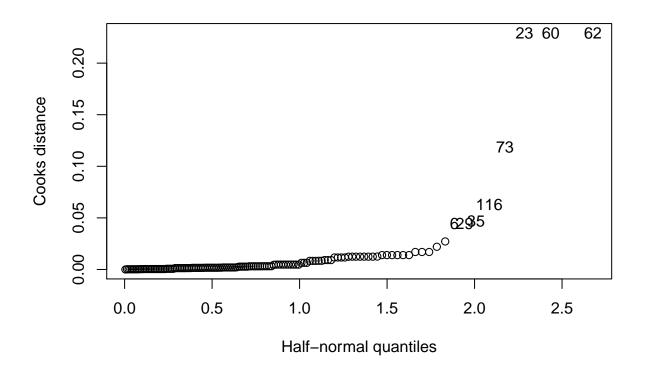
```
fuelcook[cook > 4/134,]
```

Model Cylinders Litres Barrels City

```
## 133 Chevrolet Colorado Cab Chassis inc 2WD
                                                          5
                                                               3.7
                                                                       20.1
                                                                              15
## 4
                                                          4
                        Ford Escape Hybrid FWD
                                                               2.5
                                                                       10.7
                                                                              34
## 134
                                                          5
                GMC Canyon Cab Chassis Inc 2WD
                                                               3.7
                                                                       20.1
                                                                              15
## 2
                             Honda Civic Hybrid
                                                          4
                                                                        8.2
                                                               1.3
                                                                              40
## 5
                      Mazda Tribute Hybrid 2WD
                                                          4
                                                               2.5
                                                                       10.7
                                                                              34
## 6
                    Mercury Mariner Hybrid FWD
                                                          4
                                                               2.5
                                                                       10.7
                                                                              34
## 3
                          Nissan Altima Hybrid
                                                          4
                                                               2.5
                                                                       10.1
                                                                              35
## 1
                                   Toyota Prius
                                                               1.5
                                                                        7.4
                                                                              48
##
       Highway Cost Carbon
                                   cook
## 133
             20 1667
                       10.8 0.04445043
                                         1.4982420
## 4
             31
                 887
                        5.7 0.22917792
                                         2.8016766
             20 1667
## 134
                       10.8 0.04445043
                                         1.4982420
## 2
                 675
             45
                        4.4 0.04709672 -0.9449983
## 5
             31
                 887
                        5.7 0.22917792
                                         2.8016766
## 6
             31
                 887
                        5.7 0.22917792
                                         2.8016766
## 3
             33
                 833
                        5.4 0.11881856
                                          1.9050509
## 1
             45
                 615
                        4.0 0.06305240
                                         0.8288496
```

• the plot below can also show which are the highest influencers on the model.

```
halfnorm(cook, 8, ylab = 'Cooks distance')
```



• further let's sort for Cook's distance from high to low:

```
fuelcook <- cbind(fuel, cook, r)
sortedfuelcook <- fuelcook[order(-cook),]</pre>
```

• A look at the first 8 rows: These are the vehicles with Cook's distance > 4/n

```
sortedfuelcook[1:8,c(1,9)]
```

```
## 4 Ford Escape Hybrid FWD 0.22917792
## 5 Mazda Tribute Hybrid 2WD 0.22917792
## 6 Mercury Mariner Hybrid FWD 0.22917792
## 3 Nissan Altima Hybrid 0.11881856
## 1 Toyota Prius 0.06305240
## 2 Honda Civic Hybrid 0.04709672
## 133 Chevrolet Colorado Cab Chassis inc 2WD 0.04445043
## 134 GMC Canyon Cab Chassis Inc 2WD 0.04445043
```

Fitting a model with outlier's (maximum Cook's distance) removed:

 below is a new model with the max cook's distance datapoints removed and a comparison of summary between the two models:

```
cfit1 <- lm(log(Carbon) ~ log(City), data = fuel, subset = (cook < (4/134)))
summary(cfit)</pre>
```

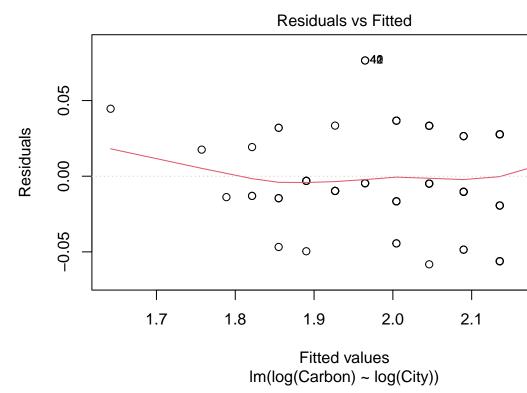
```
##
## Call:
## lm(formula = log(Carbon) ~ log(City), data = fuel)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                           Max
## -0.06330 -0.02093 -0.01033 0.03070 0.09282
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                          0.05043
                                    90.94
## (Intercept) 4.58581
                                            <2e-16 ***
                          0.01639 -50.84
                                            <2e-16 ***
## log(City)
              -0.83320
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.03408 on 132 degrees of freedom
## Multiple R-squared: 0.9514, Adjusted R-squared: 0.9511
## F-statistic: 2585 on 1 and 132 DF, p-value: < 2.2e-16
summary(cfit1)
```

```
##
## Call:
## lm(formula = log(Carbon) ~ log(City), data = fuel, subset = (cook <
## (4/134)))</pre>
```

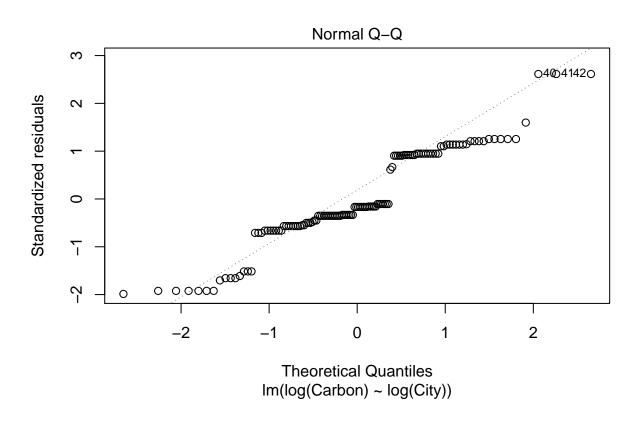
```
##
## Residuals:
##
        Min
                   1Q
                         Median
## -0.058230 -0.016617 -0.004884 0.027430 0.076491
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                                   77.59
## (Intercept) 4.76945
                         0.06147
                                           <2e-16 ***
## log(City) -0.89451
                          0.02012 -44.45
                                           <2e-16 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02942 on 124 degrees of freedom
## Multiple R-squared: 0.941, Adjusted R-squared: 0.9405
## F-statistic: 1976 on 1 and 124 DF, p-value: < 2.2e-16
```

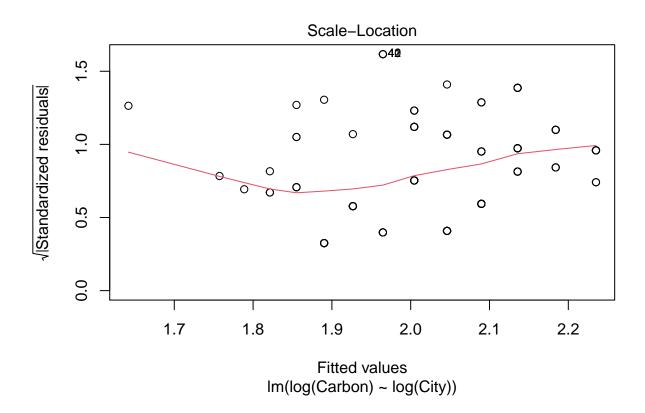
• we can see the second model has a high F-statistic with low p-value and the residual standard error is lower than the first model.

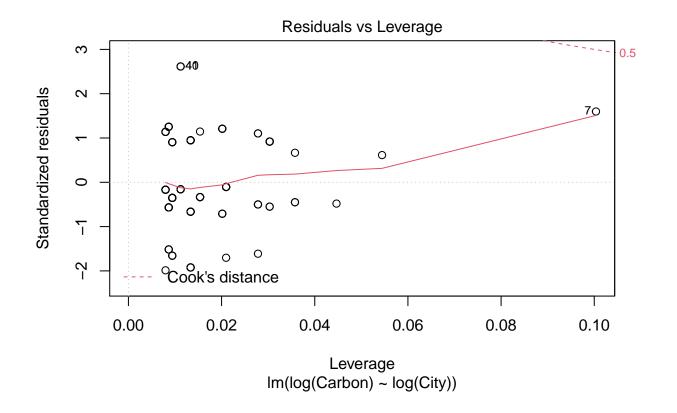
```
plot(cfit1)
```



Let's check the residuals now:







```
Anova(cfit1, cfit)
```

Let's do an Anova test of the 2 models:

```
## Anova Table (Type II tests)
##
## Response: log(Carbon)
## Sum Sq Df F value Pr(>F)
## log(City) 1.71072  1 1472.7 < 2.2e-16 ***
## Residuals 0.15334 132
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1</pre>
```

• it shows the model with the outliers removed is a better fit with a high F-statistic and low p-value.