DID & Synthetic Control

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- AERG -

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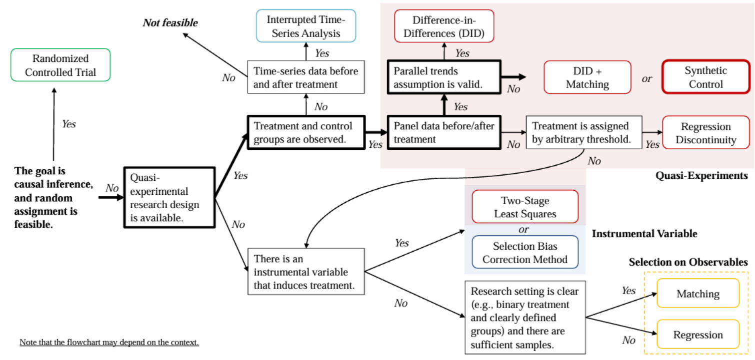
Contents

Intro

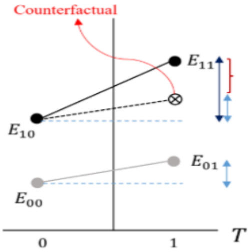


Not feasible
Interrupted Time-Series Analysis
Difference-in-Differences (DID)
Randomized Controlled Trial
Yes
No
Time-series data before and after treatment
Yes
Parallel trends assumption is valid.
No
DID + Matching
or
Synthetic Control
Yes
No
Yes
Treatment and control groups are observed.
Yes
Panel data before/after treatment
No
Treatment is assigned by arbitrary threshold.
Yes
Regression Discontinuity
No
The goal is causal inference, and random assignment is feasible.
No
Quasi-experimental research design is available.
Yes
Quasi-Experiments
Two-Stage Least Squares
or
Selection Bias Correction Method
Instrumental Variable
No
Yes
There is an instrumental variable that induces treatment.
Selection on Observables
No
Research setting is clear (e.g., binary treatment and clearly defined groups) and there are sufficient samples.
Yes
Matching
No
Regression
Note that the flowchart may depend on the context.

Setting & Idea

- DID : Difference between the change in outcome before and after a treatment in a treatment vs control group
- Consider a canonical 2x2 setting:
  - Y_{it} : outcome value of i in t
  - i: 0 if control, 1 if treatment, t: 0 if pretreated, 1 if post-treated

	Treatment group	Control group	Differences
Pre-treated	Y_10	Y_00	Y_10 - Y_00
Post-treated	Y_11	Y_01	Y_11 - Y_01
Differences	Y_11 - Y_10	Y_01 - Y_00	(Y_11 - Y_10) - (Y_01 - Y_00)
Treatment e + Time e		Time e	Treatment e



Potential Outcome Framework(2x2)

- **Causal Effect:**

$$\delta = Y(1)_{it} - Y(0)_{it} \quad \text{(Homogeneous Treatment Effect)}$$

- **Observed Outcomes:**

$$Y_{it} = Y(0)_{it} + [Y(1)_{it} - Y(0)_{it}] \cdot D_{it}$$

- **Untreated Potential Outcome:**

$$Y(0)_{it} = \beta_0 + \beta_1 T_i + \beta_2 P_t + \epsilon_{it}$$

  - T_i: Group Dummy (0 if $i = 0$, 1 if $i = 1$)
  - P_t: Time Dummy (0 if $t = 0$, 1 if $t = 1$)
- **Regression Form:**

$$Y_{it} = \beta_0 + \beta_1 T_i + \beta_2 P_t + \delta D_{it} + \epsilon_{it}$$

- **Interaction Term:**

$$D_{it} = T_i \times P_t$$

- The 2x2 DID design is intuitive, but it does not accommodate the complexity in applications (e.g., treatment for multiple groups, multiple time periods).

Potential Outcome Framework(NXT)

- **Like the 2x2 setting:**

$$\delta = Y(1)_{it} - Y(0)_{it}$$

- **Recall:**

$$Y_{it} = \beta_0 + \beta_1 T_i + \beta_2 P_t + \delta D_{it} + \epsilon_{it} \quad \text{(2x2)}$$

- **Regression Form:**

$$Y_{it} = u_i + v_t + \delta D_{it} + \epsilon_{it} \quad \text{(Two-way Fixed Effects)}$$

- Regression DID is an easy method for estimation and standard error calculation.
- Using FWFE regression models, we may consistently estimate δ.
- **Note:** There is no variation in the timing of treatment implementation.
  - See more on Andrew Goodman-Bacon (2021).

Potential Outcome Framework(NXT)

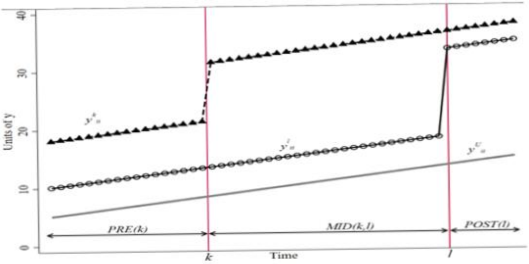


Figure: Timing Heterogeneity

Key Assumption: CTA for DID Identification

- CTA (Common Trend Assumption) is a key assumption for DID identification.
- Roughly speaking, a graph of the time series should look like a set of parallel lines.
- Formally, for $t > 1$:

$$E[Y_{it}(0) - Y_{i(t-1)}(0)] \text{ does not vary across } i.$$

- In the panel data framework:

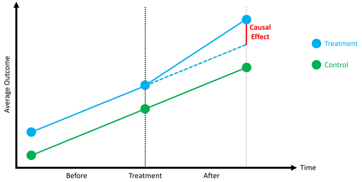
$$Y_{it} = u_i + v_t + \delta D_{it} + \epsilon_{it}$$

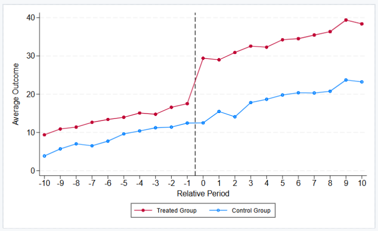
- Exogeneity assumption:

$$E[D_{it}\epsilon_{is}] = 0 \text{ for all } t, s.$$

- This assumption is not fully testable, but it is partially testable.

Key Assumption: CTA for DID Identification





Other assumptions for validation

- **No Confounding Factors:**
  - No other policies were changed concurrently, except for the treatment.
- **No Compositional Differences:**
  - Repeated cross-section data, regional specific data.
  - This is not an issue in the panel data framework.

Replication: Police Forces After a Terrorist Attack (DiTella, 2004)

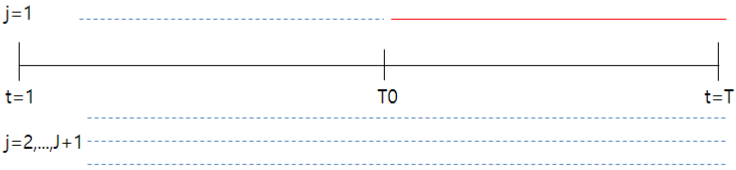
- **DiTella and Schargrodsky (2004)** investigated whether police presence reduces car thefts.
- **Exogenous Event**
  - In 1994, after a terrorist attack in Buenos Aires, the government increased police presence.
  - Using the exogenous event (the terrorist attack) to identify causal effects.
- **Methodology:** DID approach
  - Comparing car theft rates before and after the terrorist attack, and between areas with and without police protection.
- **Heterogeneous Treatment Effect**
  - The presence of police would reduce car thefts, with the effect being stronger closer to the deployed police.
- **Results:** Car thefts decreased in areas with police protection.

Replication: Police Forces After a Terrorist Attack (DiTella, 2004)

- **Timeline of Events**
- **Descriptive Statistics**
- **Estimation Equations**
    - Simple Model
    - Hetero Model
- **Test of Hypothesis**

Synthetic Control

- Aim to estimate the effects of **aggregate interventions.**
- **Aggregate level treatment** affecting a small number of large units:
  - Cities, regions, or countries.
- Useful when the **common trend assumption** is not satisfied.
- When the units of observation are a small number of aggregate entities:
  - A combination of controlled units is often a better choice than any single controlled unit.
- Blue dotted line: untreated, Red solid line: treated.



DID vs Synthetic Control

- In approximating the counterfactual for treated units,
- **Type of counterfactual estimated:**
  - **DID:** 'Counterfactual trend' (Common trend assumption is needed).
  - **Synthetic Control:** 'Counterfactual outcome' weighted combination of untreated units.
- **How units are utilized:**
  - **DID:** Uses the entire control units with equal weight.
  - **Synthetic Control:** Uses a non-uniform weighted average of control units.

DID vs Synthetic Control

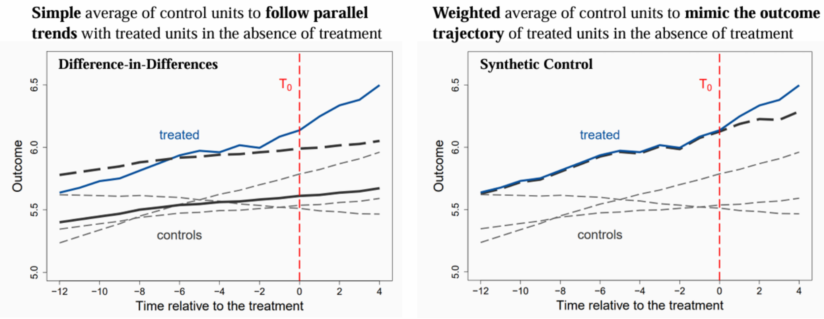


Figure: DID vs Synthetic Control

Synthetic Control: Setting & Idea

- $J+1$ units of data, $j=1$ for treated unit, $j=2,...,J+1$ for untreated ("donor pool").
- T periods of time, with the first T_0 periods before the intervention.
- Y_{jt}: outcome of interest.
- $k \times 1$ vectors $\mathbf{X}_1,...,\mathbf{X}_{J+1}$ contain the values of predictors.
  - $\mathbf{X}_j = (\mathbf{X}_{1j},...,X_{kj})'$
- $k \times J$ matrix $\mathbf{X}_0 = [\mathbf{X}_2,...,\mathbf{X}_{J+1}]$ **consists of row-stacked predictors of untreated units.**

Example: Abadie et al. (2015)

- **Economic Cost of Germany Reunification**
- **Time Horizon:** 1960-2003, where 1990 is the intervention T_0.
- **Donor Pool:** 16 OECD member countries.
- **Predictors (X):**
  - Per capita GDP.
  - Inflation rate.
  - Industry share of value added.
  - Investment rate.
  - Schooling.
  - Measure of trade openness.

Estimation

- Y_{jt}^N: Potential response without intervention.
- Y_{jt}^I: Potential response with intervention.
- **Interest of estimation:**
  - Effect of intervention for the treated unit in period t ($t > T_0$).
- The treatment effect for the treated unit at time t.

$$\tau_{1t} = Y_{1t}^I - Y_{1t}^N$$

- **What we know:**
  - $Y_{1t} = Y_{1t}^I$ for $t > T_0$ (observed outcome after intervention).
- **Need to estimate:**
  - Y_{1t}^N: The counterfactual outcome (i.e., the outcome without the intervention).

Estimation

- Synthetic control can be represented by a $J \times 1$ vector of weights:

$$W = (w_2, \ldots, w_{J+1})'$$

- Synthetic control estimator:

$$\hat{Y}_{1t}^N = \sum_{j=2}^{J+1} w_j Y_{jt}$$

- The treatment effect for the treated unit at time t:

$$\hat{\tau}_{1t} = Y_{1t} - \hat{Y}_{1t}^N$$

- Restrictions for weights: Non-negative, sum to one.
  - Synthetic controls are weighted averages of the units in the donor pool.

Choice of W

- Choose the synthetic control, $\mathbf{W}^* = (W_2^*, \ldots, W_{J+1}^*)'$ under the constraints that minimizes

$$\|\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}\| = \left(\sum_{h=1}^{k} v_h \left(X_{h1} - w_2 X_{h2} - \cdots - w_{J+1} X_{hJ+1}\right)^2\right)^{1/2}$$

  for some v_h's.
- Positive constants $w_2, \ldots, w_{J+1}$ reflect the **relative importance of each of the** j **units** for constructing the synthetic control.
- Positive constants $v_1, \ldots, v_k$ reflect the relative importance of **each of the** k **predictors** for constructing the synthetic control.
- The synthetic control $\mathbf{W}(\mathbf{V}) = (w_2(\mathbf{V}), \ldots, w_{J+1}(\mathbf{V}))$ is a function of $\mathbf{V}$, where $\mathbf{V} = (v_1, \ldots, v_k)$.

Choice of V

- Divide the pre-intervention periods into an initial training period and a subsequent validation period.
- Assume that T_0 is even, and the training and validation periods span:

$$t = 1, \ldots, t_0 \quad \text{(Training period)}$$

and

$$t = t_0 + 1, \ldots, T_0 \quad \text{(Validation period)}$$

where $t_0 = \frac{T_0}{2}$.

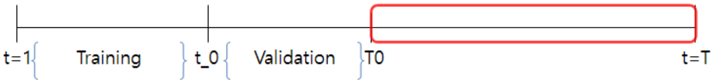


Figure: Training & Validation periods

Choice of V

- For every value of V, let $\tilde{w}_2(\mathbf{V}), \ldots, \tilde{w}_{J+1}(\mathbf{V})$ be the synthetic control weights computed with the **Training Period**, which minimizes:

$$\|\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}\| = \left(\sum_{h=1}^{k} v_h \left(X_{h1} - w_2 X_{h2} - \cdots - w_{J+1} X_{h(J+1)}\right)^2\right)^{1/2}$$

  This can be conducted by quadratic optimization.

- Select a value $\mathbf{V}^*$ such that the Mean Squared Prediction Error (MSPE) for the outcome prediction in the validation set is minimized:

$$\sum_{t=t_0+1}^{T_0} \left(Y_{1t} - \tilde{w}_2(\mathbf{V}) Y_{2t} - \cdots - \tilde{w}_{J+1}(\mathbf{V}) Y_{J+1t}\right)^2$$

- Use the resulting $\mathbf{V}^*$ and data on the predictors for $t = t_0 + 1, \ldots, T_0$ to calculate the final synthetic control weights:

$$\mathbf{W}^* = \mathbf{W}(\mathbf{V}^*)$$

Abadie, Alberto. (2021)

- For the training period 1971-1980, choose $\tilde{\mathbf{W}}$ that minimizes:

$$\|\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}\| = \left(\sum_{h=1}^{k} v_h \left(X_{h1} - w_2 X_{h2} - \cdots - w_{J+1} X_{h(J+1)}\right)^2\right)^{1/2}$$

- For the validation period 1981-1990, minimize the following MSPE:

$$\sum_{t=t_0+1}^{T_0} \left(Y_{1t} - \tilde{w}_2(\mathbf{V}) Y_{2t} - \cdots - \tilde{w}_{J+1}(\mathbf{V}) Y_{J+1t}\right)^2$$

- Use the resulting $\mathbf{V}^*$ and predictors in 1981-1990 (validation period) to choose the final synthetic control weights $\mathbf{W}^*$ that minimize:

$$\|\mathbf{X}_1 - \mathbf{X}_0\mathbf{W}\| = \left(\sum_{h=1}^{k} v_h \left(X_{h1} - w_2 X_{h2} - \cdots - w_{J+1} X_{h(J+1)}\right)^2\right)^{1/2}$$

- Then, the treatment effect is:

$$\hat{\tau}_{1t} = Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt}, \quad \text{where} \quad \mathbf{W}^* = (w_2^*, \ldots, w_{J+1}^*)'$$

Inference: Placebo Test

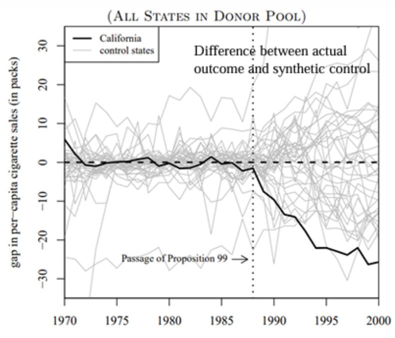
- Tricky: It is not possible to rely on asymptotic theory.
- **Placebo tests:** Apply synthetic control to other units.
- Synthetic control may not fit the trajectory of the outcome for the units in the donor pool.
- **RMSPE** of the synthetic control estimator for unit j and time $t_1, \ldots, t_2$:

$$R_j(t_1, t_2) = \left(\frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} (Y_{jt} - \hat{Y}_{jt}^N)^2 \right)^{1/2}$$

- **Ratio** between the **post-intervention RMSPE** and **pre-intervention RMSPE** for unit j:

$$r_j := \frac{R_j(T_0 + 1, T)}{R_j(1, T_0)}$$

Inference: Placebo Test



Root Mean Squared Prediction Error (RMSPE)

$$R_j(t_1, t_2) = \left(\frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} \left(Y_{jt} - \hat{Y}_{jt}^{N} \right)^2 \right)^{1/2}$$

Figure: Permutation test

Inference: Placebo Test

- Use the permutation distribution of r_j for inference.
- Synthetic control may not fit the trajectory of the outcome for the units in the donor pool.
- Let $H_0 : Y_{jt}^I = Y_{jt}^N$ for each $j = 1, \ldots, J+1$ and $t = 1, \ldots, T$.
- Compute p-value:

$$p = \frac{1}{J+1} \sum_{j=1}^{J+1} I_+(r_j - r_1)$$

- Indicator function:

$$I_+(r_j - r_1) = \begin{cases} 1, & \text{if } r_j \geq r_1 \\ 0, & \text{otherwise} \end{cases}$$

- Reject the null hypothesis if p is less than some pre-specified significance level.

Contextual Requirements

- **Convex Hull Condition:**
  - A convex combination of donor pool measures should approximate the pre-intervention characteristics of the treated unit.
  - (Similar to Parallel pre-trend in DID)
- **No Anticipation:**
  - No forward-looking economic agents react in advance of the intervention.
- **No Confounding Events:**
  - No concurrent confounding shocks.
- **No Interference:**
  - No spillover effects of the intervention of interest.
- **No Structural Breaks:**
  - SCM is predicated on structural stability.

Contextual Requirements

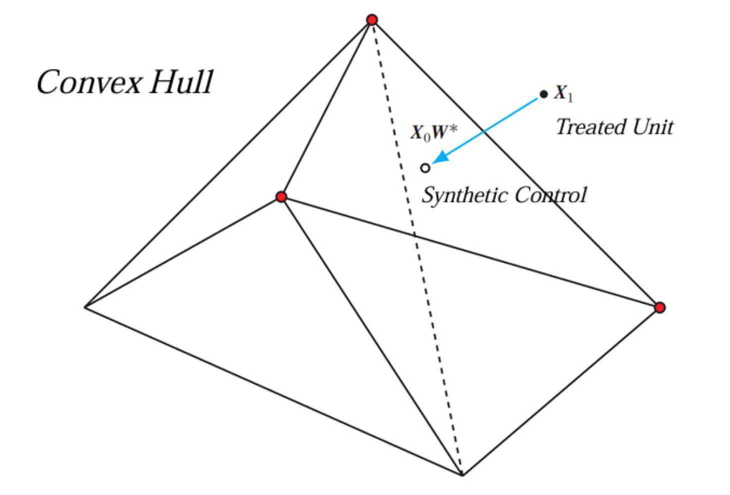
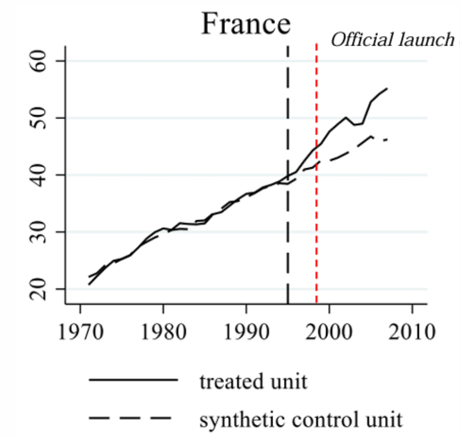


Figure: Convex Hull

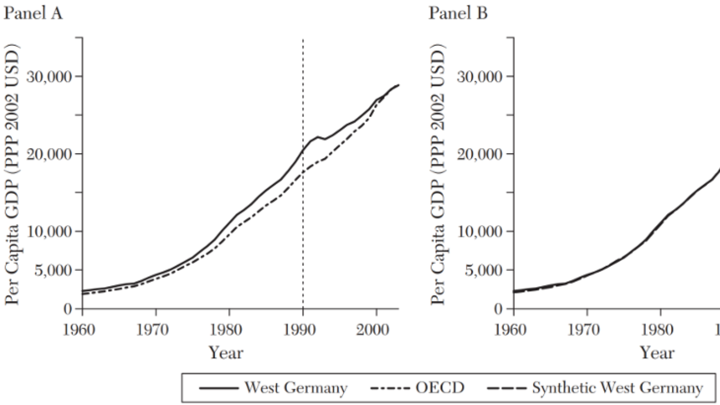
Contextual Requirements



As in the event-study DID model, the pre-treatment gap may be indicative of anticipation effects.

Figure: Anticipation

Application: Impact of Reunification on West Germany (Abadie et al, 2015)



Application: Impact of Reunification on West Germany (Abadie et al, 2015)

TABLE 2
SYNTHETIC CONTROL WEIGHTS FOR WEST GERMANY

Australia	—
Austria	0.42
Belgium	—
Denmark	—
France	—
Greece	—
Italy	—
Japan	0.16
Netherlands	0.09
New Zealand	—
Norway	—
Portugal	—
Spain	—
Switzerland	0.11
United Kingdom	—
United States	0.22

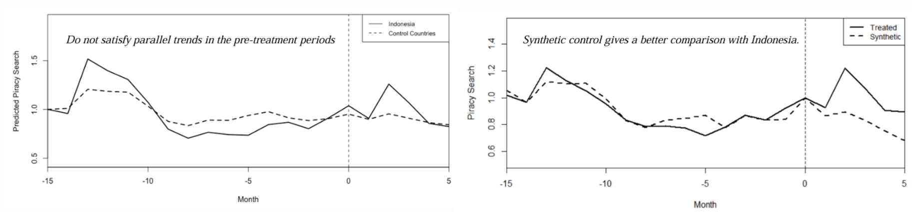
TABLE 3
REGRESSION WEIGHTS FOR WEST GERMANY

Australia	0.12
Austria	0.26
Belgium	0.00
Denmark	0.08
France	0.04
Greece	−0.09
Italy	−0.05
Japan	0.19
Netherlands	0.14
New Zealand	0.12
Norway	0.04
Portugal	−0.08
Spain	−0.01
Switzerland	0.05
United Kingdom	0.06
United States	0.13

Replication: Impact of OTT Services on Piracy Search

- In 2016, Netflix made a major market expansion of its services into 130 countries.
- However, it failed to enter Indonesia due to conflicts with a state-owned telecom company.
- **Synthetic Indonesia:**
  - Constructed using a combination of 40 Asian countries where Netflix was introduced in January 2016 and remained available.

Replication: Impact of OTT Services on Piracy Search



Lu, S., Rajavi, K. and Dinner, I., 2021. The effect of over-the-top media services on piracy search: Evidence from a natural experiment. *Marketing Science*, *40*(3), pp.548-568.

Figure: DID vs Synthetic Control

Replication: Impact of OTT Services on Piracy Search(WP)

1. DID Analysis (Table 1)
2. Checking Parallel Trend Assumption (Fig 2)
3. Weights of Synthetic Indonesia (Table 2)
4. Mean of Pretreatment Characteristics (Table 3)
5. Trends of Piracy Search Volume: Indonesia vs Synthetic Control Country (Figure 3)
6. Gaps in Piracy Search Volume Between Indonesia and Synthetic Control Country (Figure 4)
7. Ratio of Posttreatment MSPE to Pretreatment MSPE Across the Countries (Figure 5)
8. Placebo Test (Figure 6)
9. Others: Treatment Effect, Robustness Check

DID Analysis (Table 1)

- Summarizes the results of the DID analysis for Netflix's market expansion.
- Compares changes in piracy search between treated and control countries.

Table1 DID result.

	(1)
	Piracy Search
Event	-0.075***
	(0.017)
Event x Treated	0.177**
	(0.064)
N	820
R^2	0.382

Standard errors in parentheses * $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

Table: DID Results: Impact of Netflix Expansion on Piracy Search

Checking Parallel Trend Assumption (Fig 2)

- Validates the parallel trend assumption critical for DID analysis.
- Visualizes pre-intervention trends for treated and control groups.

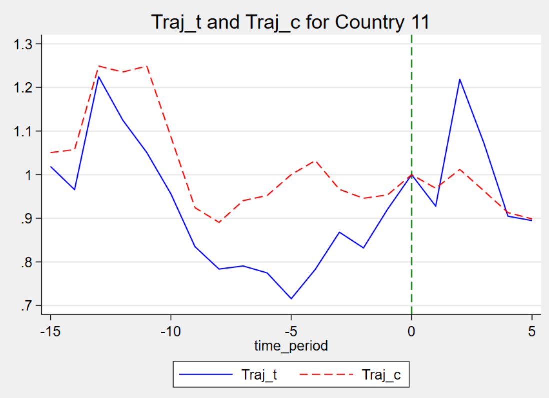


Figure: Parallel Trend Test: Piracy Search Trends Before Netflix Expansion

Weights of Synthetic Indonesia (Table 2)

- Displays weights assigned to countries in constructing Synthetic Indonesia.
- Countries with higher weights resemble Indonesia in pre-intervention characteristics.

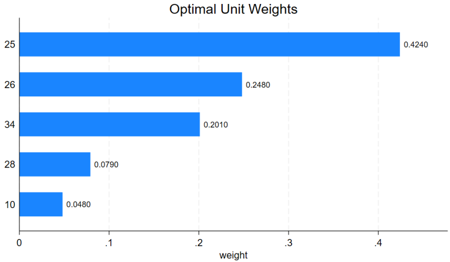


Table: Synthetic Control Weights for Indonesia

Mean of Pretreatment Characteristics (Table 3)

- Compares pre-intervention characteristics of Indonesia and Synthetic Indonesia.
- Evaluates the similarity of treated and synthetic control units.

	Treated	Synthetic
sv_piracy_normalized	.9097321	.9685764
sv_title	468524	694113
sv_netflix	89066.67	109953.4
sv_generalpiracy	92636.67	472578
sv_comp	37726	62753.36
internetuser_mean_2014	4.36e+07	2.65e+07
internetuser_mean_2015	5.01e+07	9169259

Table: Pretreatment Characteristics: Indonesia vs Synthetic Indonesia

Trends of Piracy Search Volume (Figure 3)

- Plots piracy search trends for Indonesia and Synthetic Indonesia.
- Deviations in post-intervention implies the treatment effects.

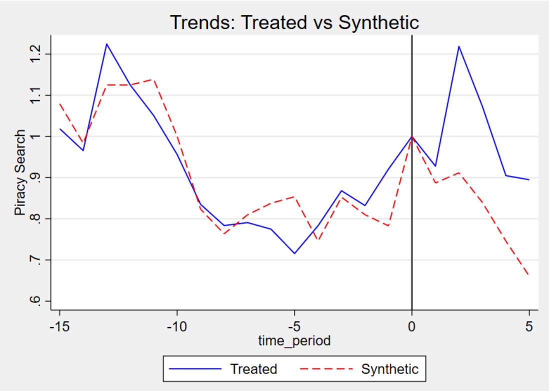


Figure: Piracy Search Trends: Indonesia vs Synthetic Control

Gaps in Piracy Search Volume (Figure 4)

- Visualizes differences in piracy search volume between Indonesia and its synthetic counterpart.

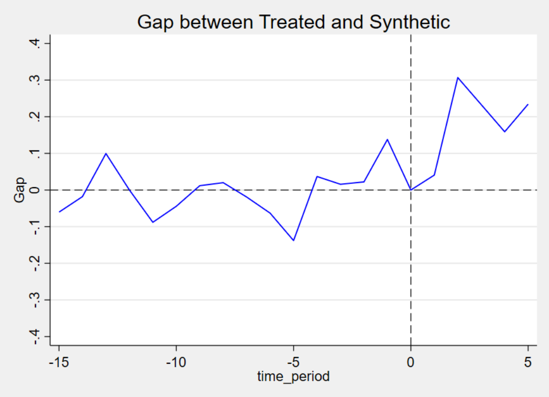


Figure: Gaps in Piracy Search Volume: Indonesia vs Synthetic Control

Ratio of MSPE: Posttreatment to Pretreatment (Figure 5)

- Compares the fit of synthetic control pre- and post-intervention across countries.

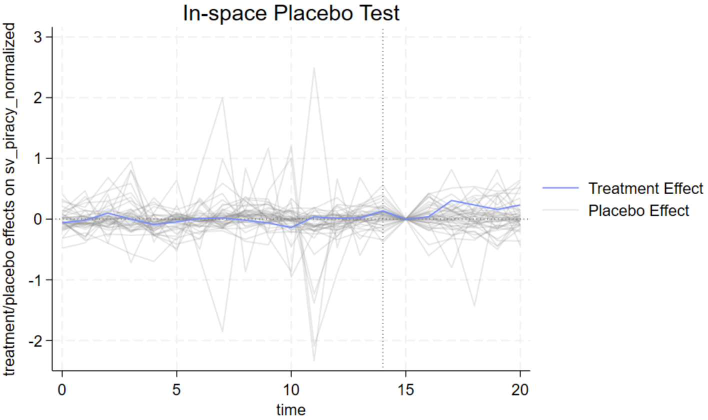


Figure: MSPE Ratio: Post- vs Pre-treatment Across Countries

Placebo Test (Figure 6)

- Tests the robustness of the results by applying the intervention to other countries.
- Helps assess the likelihood of observing the treatment effect by chance.

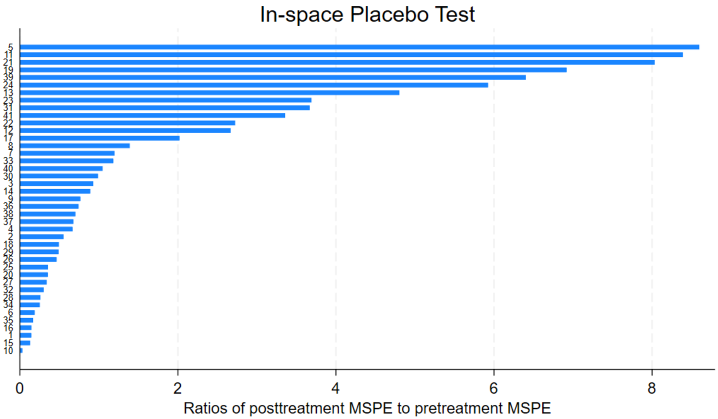


Figure: Placebo Test: Piracy Search Trends Across Countries

Treatment Effect and Robustness Check

- **Treatment Effect:**
  - Estimated impact of Netflix's entry on piracy search in Indonesia.
  - Results highlight the causal effect and policy implications.
- **Robustness Check:**
  - Validate assumptions such as no interference, convex hull, and structural stability.
  - Examine sensitivity to alternative specifications and donor pools.

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