시계열분석특수연구(001) 과제#1

1. Exercise on VAR

Choose one of papers in the following that we briefly reviewed in class. Do similar work as in the empirical part of the paper for your own data and a variant of the model: Get empirical results and discuss.

Blanchard and Quah (1989, AER)

- **What is Long-run Restriction?**

Blanchard and Quah (1989) considered the accumulated effects of shocks to the system. In terms of the structural impulse responses in $Y_t = \Theta_0 u_t + \Theta_1 u_{t-1} + \cdots$, they focussed on the total impact matrix

$$\sum_{l=0}^{\infty} \Theta_l = (I_k - A_1 - \cdots - A_p)^{-1} B \equiv A(1)^{-1} B \equiv C$$

where matrices A and B are such that $Y_t = A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + B\epsilon_t$. They identified the structural shocks by placing zero restrictions on this matrix C. In other words, they assumed that some shocks do not have any total long-run effects. In particular, they considered a bivariate system consisting of output growth $dgdp_t$ and an unemployment rate $unrate_t$. They assumed that the structural innovations represent supply and demand shocks. Moreover, they assumed that the demand shocks have only transitory effects on $dgdp_t$ and that the accumulated long-run effect of such shocks on $dgdp_t$ is zero. Placing the supply shocks first and the demand shocks last in the vectors of structural innovations $u_t = (u_t^s, u_t^d)'$, the (1,2)-th element of C, $c_{12} = 0$. Since our case is bivariate VAR case, this zero restriction is enough for identification and in particular it makes the matrix C to be lower triangular. Using the relation $C = (I_k - A_1 - \cdots - A_p)^{-1} B$ and noting that

$$CC' = (I_k - A_1 - \cdots - A_p)^{-1} \Sigma_\epsilon (I_k - A_1 - \cdots - A_p)^{-1}$$

the matrix B can be estimated by premultiplying a Choleski decomposition of the matrix

$$P = chol[(I_k - \hat{A}_1 - \cdots - \hat{A}_p)^{-1} \hat{\Sigma}_\epsilon (I_k - \hat{A}_1 - \cdots - \hat{A}_p)^{-1}]$$

by $(I_k - \hat{A}_1 - \cdots - \hat{A}_p)$:

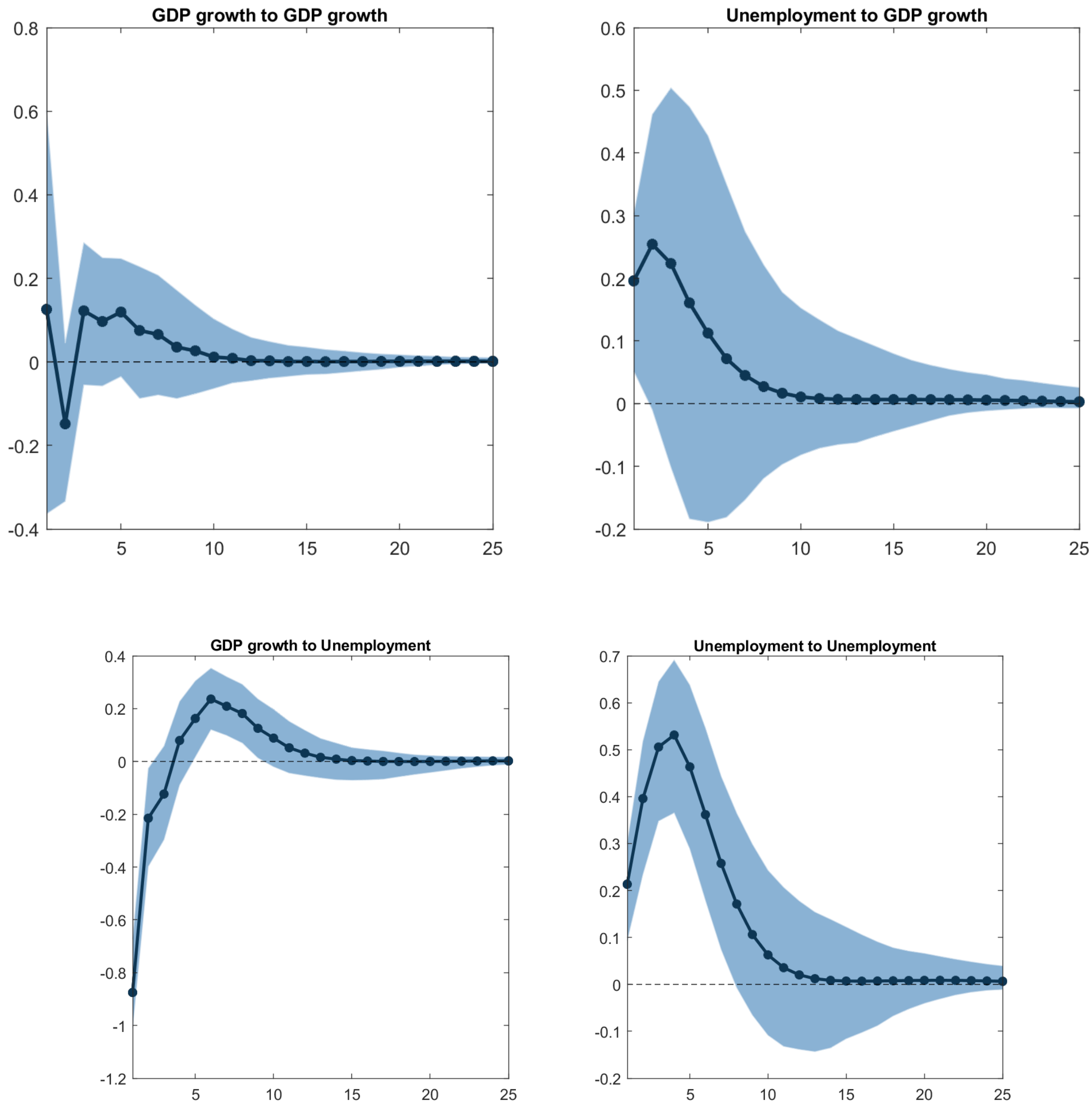
$$\hat{B} = (I_k - \hat{A}_1 - \cdots - \hat{A}_p) P$$

- **Model Specification**

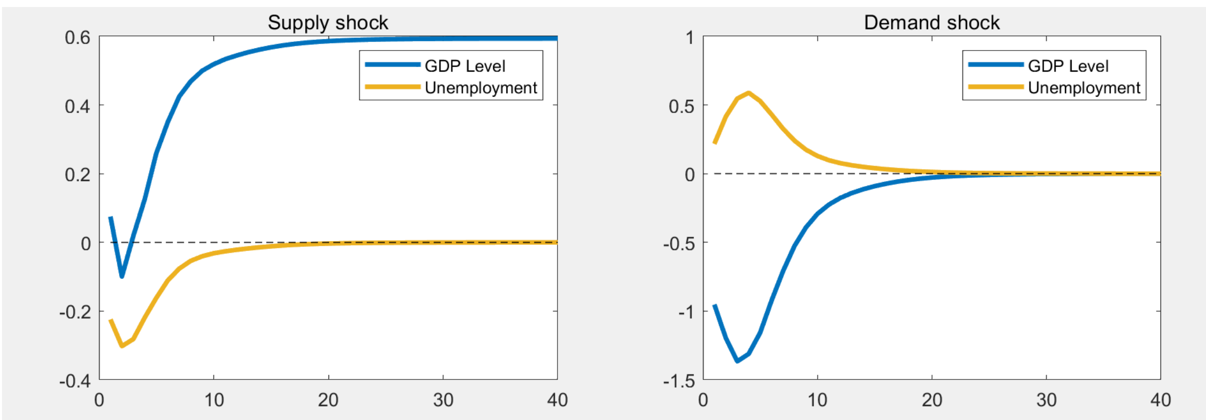
	Blanchard and Quah	This Report	
Model	VAR(8)	VAR(4)	
Data	1948q2~1987q4	FRED-QD 1959~2017 quarterly data	
Variables	GNP	GDPC1	Real Gross Domestic Product, 3 Decimal (Billions of Chained 2012 Dollars)
	Unemployment	UNRATE	Civilian Unemployment Rate (Percent)

- **With the original 1948q2~1987q4 data we have the folloing figures:**

1. Reduced form IRFs. The CI is computed with 1000 rep bootstrap



2. Structural IRF with the Zero Long-run Restriction: “demand shock does not affect output in the long run”.



We should be careful while interpreting the graphs. The Supply shock is a procyclical shock while Demand shock is acyclical.

By the construction the Demand shock does not affect GDP level in the long run: the IRF of GDP to the Demand shock(the 2nd panel blue line) converges to 0.

The overall reaction direction of GDP and unemployment are in the opposite signs.

- **The analysis results are similar even in recent series(1959 1q~2017 4q), however we have somewhat different results in details.**

1. Lag Selection with AIC

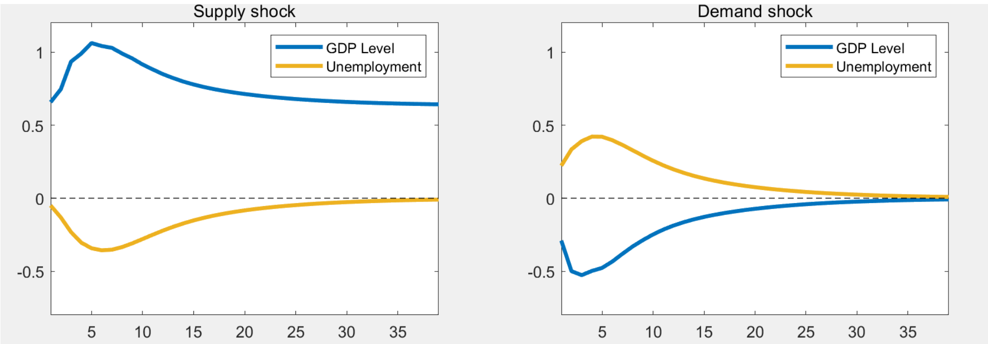
   a. There is no dramatic change in AIC after L(1)→L(2). As it is done in B. Hansen (2022), we choose VAR(4) model.

lag	1	2	3	**4**
AIC	1715.8	1639.2	1624.3	**1613.1**
lag	5	6	7	8
AIC	1603.1	1599.2	1582.1	1574.5
lag	9	10	11	12
AIC	1561.2	1551.7	1551.8	1541.6

   b. VAR coefficints and Identification Matrices

A1			P(choleski)		
	0.0702	-0.8501		0.6334	0
	-0.0957	1.3805		-4.9155	5.4186
A2			B		
	0.2228	1.5161		0.6574	-0.2907
	-0.1053	-0.5479		-0.0500	0.2221
A3			A(1)		
	0.0045	-0.4332		0.6215	-0.0537
	-0.0323	0.1423		0.2391	0.0410
A4					
	0.0809	-0.1791			
	-0.0058	-0.0159			

2. The Structural IRFs



First, we notice that the supply shock always has positive impact on GDP level. It's different from the original result which had a negative sign at horizon 2. Second, the long run effect of supply shock on gdp level(1st panel blue line) is about 0.6 in both data periods. Finally, the demand shock had more impact on gdp (temporary) growth in the past.

To summarize, it seems that in the past, the demand shock was more impactful, while the supply shock was relatively mean, compared to recent days. This may prove that demand shocks like WWII were prevalent in earlier dates, meanwhile, Supply shocks like the introduction of PCs and Internet Service were important elements of the economy.

Next, we present results of bootstrap regression. The 95%-bootstrap Confidence Bands for each shock-variable pair are also included. The bold red line is the median impulse response line of the bootstrap estimates.

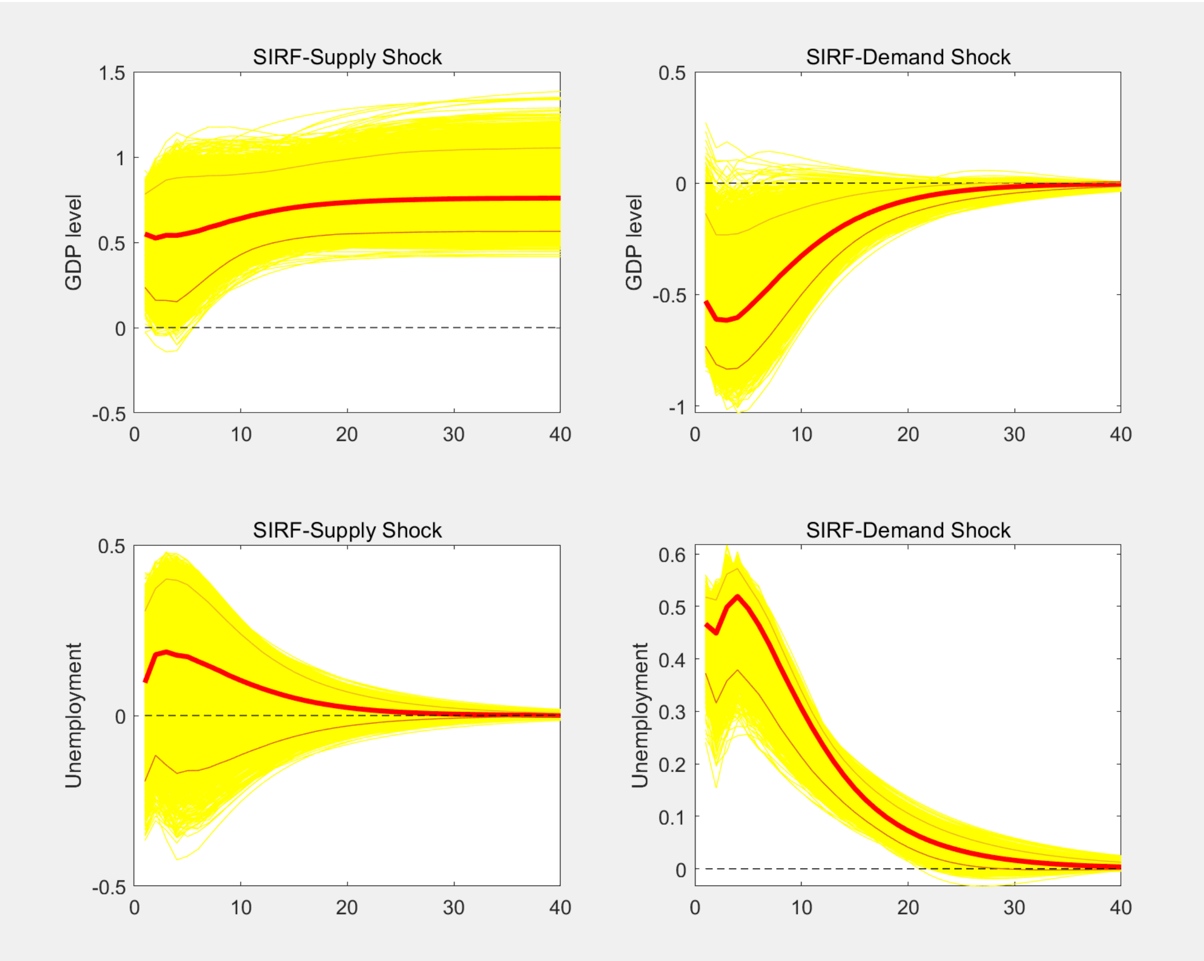
We used the Fixed Design Resgression Bootstrap. The Algorithm is as follows:

1. Set $(Y_t, X_t, \hat{u}_t)$, where $X_t = (Y'_{t-1}, \ldots, Y'_{t-p})'$. Fix the sample values of X_t

2. Simulate u_t^* by iid draw with replacement from the empirical distribution$(\hat{u}_1' - \bar{\hat{u}}', \ldots, \hat{u}_n' - \bar{\hat{u}}')'$.
3. Bootstrap sample is constructed by $Y_t^* = X_t'\hat{\beta} + e_t^*$.
4. Estimate $\hat{\alpha}$'s and $I\hat{R}F$'s for each bootstrap iteration.
5. After the iteration is over, we set 95% bootstrap confidence bands for each IRFs: [0.025 and 0.975 quantiles] for each horizon.

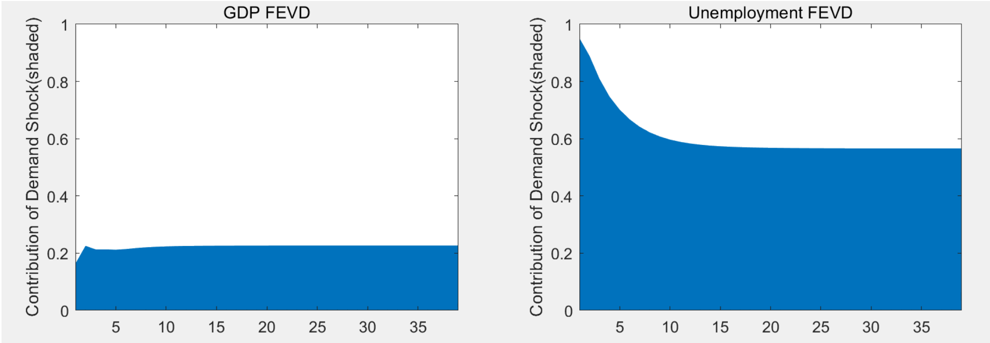
Note that the confidence bands for Unemployment IRFs to Supply shocks(2nd panel of 1st column) are not precise as well as the sign is the opposite to our total sample estimates(In general we expect negative signs in unemployment because if the economy expands, it's more natural that the unemployment rate ascends). Besides, they mostly do not reject H0(zero impact).

We recognize the drawbacks of Fixed Design Regression Bootstrap: this work consistently only when the true error is homoskedastic. Also this method impose too strong condition: conditional independence between X_t and the error. There are other bootstrap methods to compute Confidence Bands. For example the block bootstrap and percentile-t bootstrap CIs may improve finite sample performance and may be more robust to heteroskedasticity, model misspecification, etc.. However we prolong other methods because of the due date.



3. The Structural FEVDs

Next, we present Forecast Error Variance Decomposition of each variable. The shaded area is the proportion of demand shock in the total accumulated variance. The variance of GDP mainly(about 80%) consists of supply shock at (almost) every horizon. The unemployment fluctuation is due to demand shock in the short term. However it converges to about 60% in the long run.



2. Exercise on Time-Varying Volatility

2.1 Apply EGARCH to estimate leverage effects and time-varying volatility in daily data of KOSPI and the won-dollar exchange rate.

Data Source

Variable Name	Source	URL	Time period	Frequency
KOSPI	KRX 정보데이터시템	(http://data.krx.co.kr/contents/MDC/MDI/mdiLoader/index.cmd?menuId=MDC0201010101)	09.04.13-24.04.12	Dayly Data
Exchange Rate	Woori Bank	https://spib.wooribank.com/pib/Dream?withyou=CMCOM0185	03.09.24-24.04.12	Dayly Data
CPI	KOSIS		65.01-24.03	Monthly Data

2.1.1 KOSPI

GARCH model

We implement bottom-up model selection procedure: starting from the simplest model we add further lag terms and check the significance of the estimated coefficient. And stop at the latest model if H0 is not rejected.

The following table is the summary of this procedure. The 2nd column is p-values of the baseline model. The 3rd and 4th columns are p-values of the significance test for the additional terms: GARCH(1,2) and GARCH(2,1). Since they are 0.852 and 0.734, we do not reject H0, and thus stop at GARCH(1,1) model.

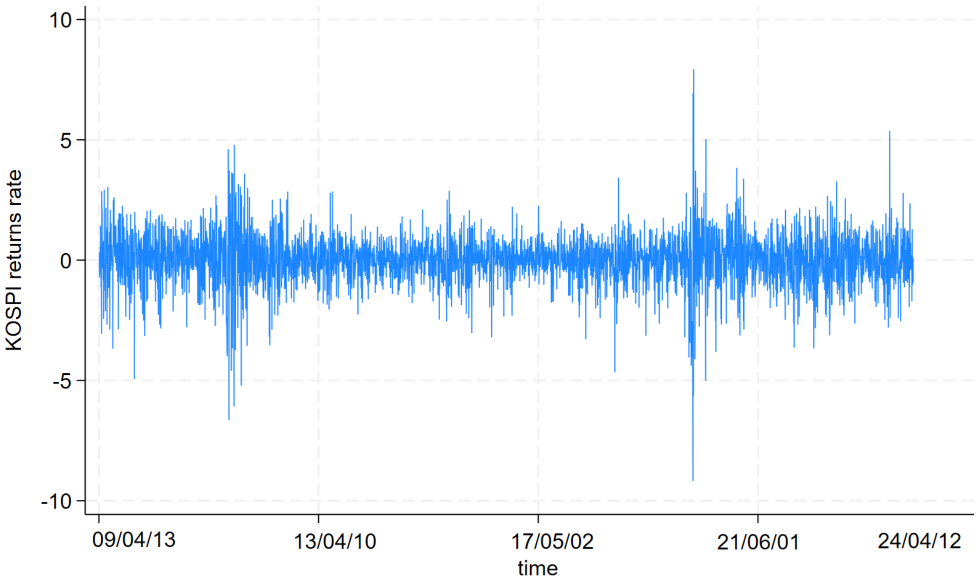
p-value of the last lag	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)
ARCH term	0.000	0.000	**0.734**
GARCH term	0.000	**0.852**	0.000

EGARCH model

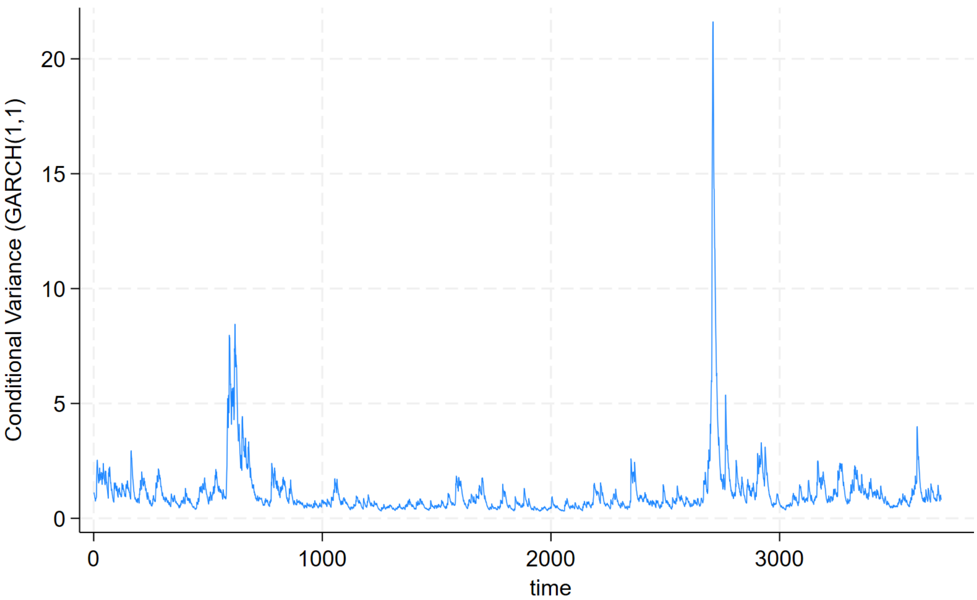
Exponential GARCH model captures asymmetric impacts of positive and negative shocks(news) to conditional variance. By the same model selection algorithm we choose EGARCH(1,1) model.

p-value of the last lag	EGARCH(1,1)	EGARCH(2,1)	EGARCH(1,2)
ARCH term	0.000	0.000	0.000
ARCH(absolute)	0.000	**0.682**	0.000
GARCH term	0.000	0.000	**0.072**

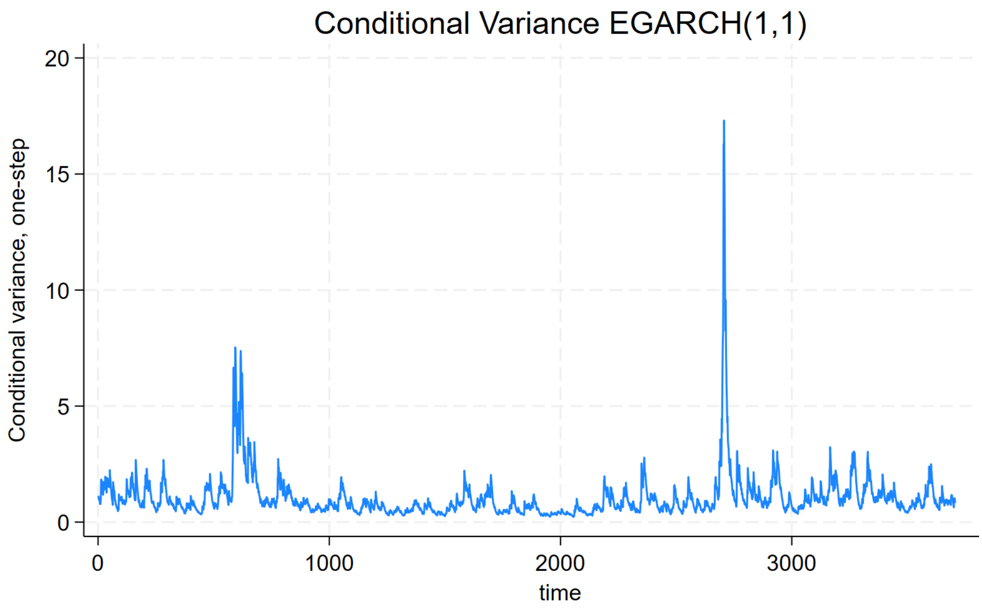
The following fugures are KOSPI returns rate and the estimated conditional variances of KOSPI retursn rate, repectively.



10
5
0
-5
-10
KOSPI returns rate
09/04/13
13/04/10
17/05/02
21/06/01
24/04/12
time



20
15
10
5
0
Conditional Variance (GARCH(1,1))
0
1000
2000
3000
time



The spikes are in 2011 and 2020. These periods are European Sovereign Debt Crisis and COVID-19 Pandemic, respectively.

We choose EGARCH(1,1). The following table is the results this model.

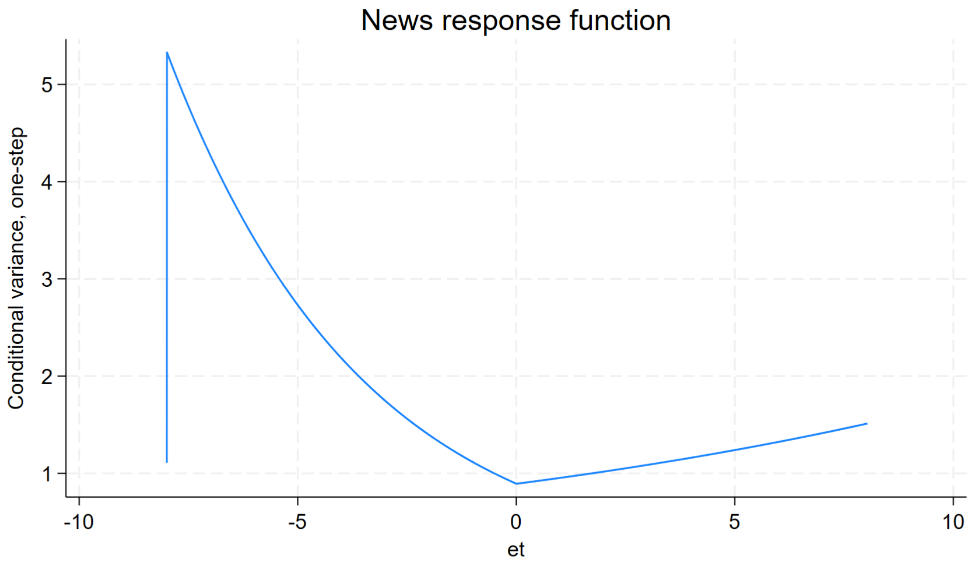
```
Sample: 2 thru 3707                               Number of obs     =       3706
                                                  Wald chi2(.)      =          .
Log likelihood = -4965.965                        Prob > chi2       =          .

------------------------------------------------------------------------------
             |                 OPG
           R | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
R            |
       _cons |   -.002827   .0140239    -0.20   0.840    -.0303133    .0246592
-------------+----------------------------------------------------------------
ARCH         |
       earch |
         L1. |   -.078906   .0067532   -11.68   0.000     -.092142   -.0656699
             |
     earch_a |
         L1. |   .1445585   .0105427    13.71   0.000     .1238952    .1652217
             |
      egarch |
         L1. |   .9729293   .0031314   310.70   0.000     .9667919    .9790667
             |
       _cons |   .0018303   .0018063     1.01   0.311      -.00171    .0053707
------------------------------------------------------------------------------
```

The estimated Variance equation is

$$\ln(\sigma_t^2) = 0.002 + -0.079 z_{t-1} + 0.145(|z_{t-1}| - \sqrt{2/\pi}) + 0.973\ln(\sigma_{t-1}^2)$$

where $z_t = \epsilon_t/\sigma_t$, which is distributed as $N(0,1)$. This indicates that unexpected decrease in KOSPI index(or individual stock prices) are more destabilizing than increases in KOSPI indices. We plot the news response function as below:



This curve shows the resulting conditional variance as a function of unanticipated news, in the form of innovations, that is, the conditional variance σ_t^2 as a function of ϵ_t. Thus we must evaluate σ_t^2 for various values of ϵ_t — say, −8 to 8 — and then graph the result.

Further we estimate GARCH-in-mean model to check whether there is or not negative relation between returns rate and its volatility.

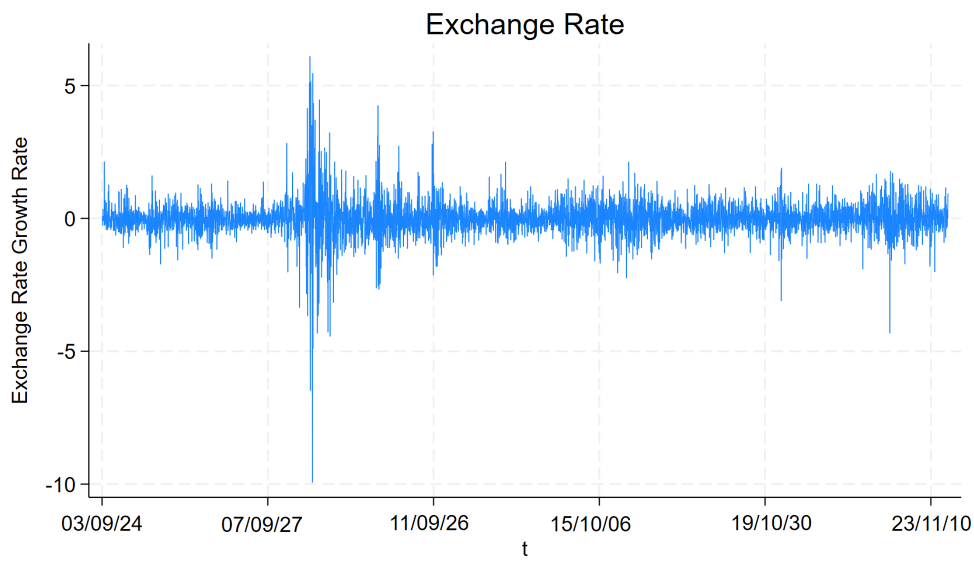
> The leverage effect refers to the generally negative correlation between an asset return and its changes of volatility. A natural estimate consists in using the empirical correlation between the daily returns and the changes of daily volatility estimated from high frequency data. (Ait-Sahalia, Y., Fan, J., & Li, Y. (2013). The leverage effect puzzle: Disentangling sources of bias at high frequency.
> *Journal of financial economics*, *109*(1), 224-249.)

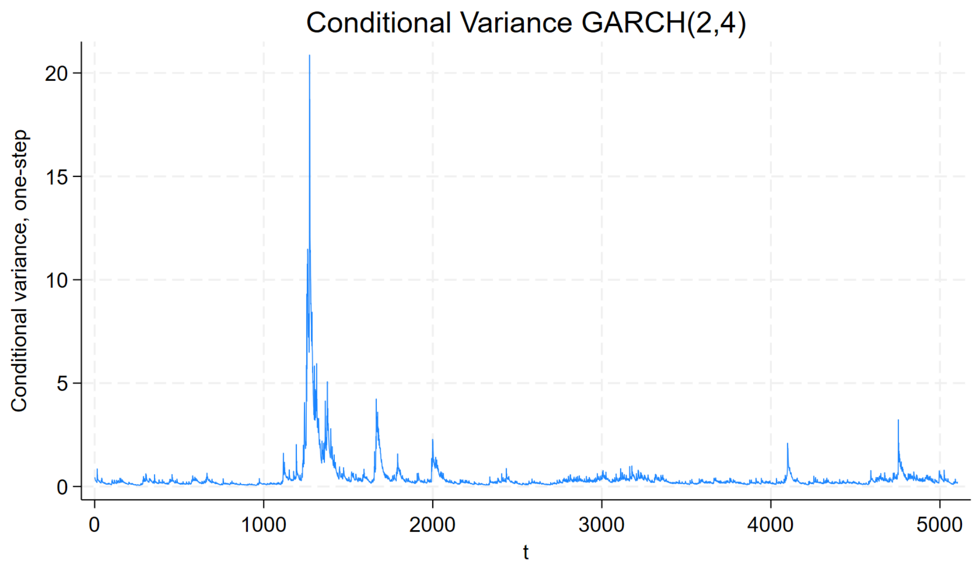
The following table is the summary of mean equation term. We can find that there is evidence of positive correlation between returns rate and conditional variance in KOSPI. The results seem to be contradicting each other. However, notice that the estimated concepts differ subtly.

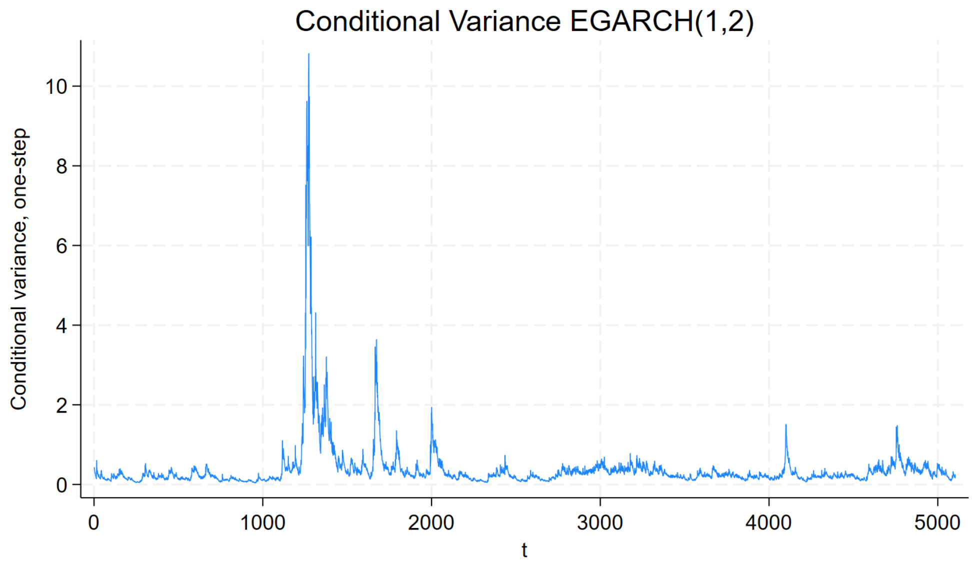
```
Sample: 2 thru 3707                               Number of obs     =      3706
                                                  Wald chi2(1)      =      4.54
Log likelihood = -4963.996                        Prob > chi2       =    0.0332
------------------------------------------------------------------------------
             |                 OPG
           R | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ARCHM        |
      sigma2 |   .0577099   .0270964     2.13   0.033     .0046019    .1108179
------------------------------------------------------------------------------
```

2.1.2 Won/Dollar Exchange Rate

In this subsection we implement the same analysis as above, but with the Won/Dollar Exchange rate. The following figures are Exchange Rate Growth Rate, Conditional Variance estimated with GARCH and EGARCH models respectively. The data is daily time series from September 24th, 2003 to April 12th, 2024. There is a big spike in conditional variance was in 2008. This is related with the Global Financial Crisis.


Exchange Rate
Exchange Rate Growth Rate
5
0
-5
-10
03/09/24
07/09/27
11/09/26
15/10/06
19/10/30
23/11/10
t


Conditional Variance GARCH(2,4)
Conditional variance, one-step
20
15
10
5
0
0
1000
2000
3000
4000
5000
t


Conditional Variance EGARCH(1,2)
Conditional variance, one-step
10
8
6
4
2
0
0
1000
2000
3000
4000
5000
t

p-value of the last lag	GARCH(1,1)	GARCH(1,1)-AR(1)	GARCH(2,1)	GARCH(3,1)
AR term	0.000	**0.960**	0.000	0.000
ARCH term	0.000	0.000	0.000	**0.073**
GARCH term	0.000	0.000	0.000	0.000
p-value of the last lag	GARCH(2,2)	GARCH(2,3)	**GARCH(2,4)**	GARCH(2,5)
AR term	0.000	0.000	0.000	0.000
ARCH term	0.000	0.000	0.000	**0.142**
GARCH term	0.000	0.005	0.016	0.000

p-value of the last lag	EGARCH(1,1)	EGARCH(1,1)-AR(1)	EGARCH(2,1)	**EGARCH(1,2)**
AR term		**0.970**		
ARCH term	0.000	0.000	**0.386**	0.000
ARCH(absolute)	0.000	0.000	0.000	0.000
GARCH term	0.000	0.000	0.000	0.000
p-value of the last lag	EGARCH(1,3)			
ARCH term	0.000			
ARCH(absolute)	0.000			
GARCH term	**0.940**			

We do not utter the same procedure. As a result we choose GARCH(2,4) and EGARCH(1,2) models. The following summary table is the results of EGARCH(1,2) model

```
Sample: 2 thru 5104                              Number of obs     =       5103
                                                 Wald chi2(.)      =          .
Log likelihood = -3777.573                       Prob > chi2       =          .

------------------------------------------------------------------------------
             |                 OPG
         d_e | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
d_e          |
       _cons |  -.0011609   .0061744    -0.19   0.851    -.0132624    .0109406
-------------+----------------------------------------------------------------
ARCH         |
       earch |
         L1. |   .0426248   .0073892     5.77   0.000     .0281422    .0571074
             |
     earch_a |
         L1. |    .231217   .0130198    17.76   0.000     .2056986    .2567354
             |
      egarch |
         L1. |   .5040886   .0748607     6.73   0.000     .3573643    .6508129
         L2. |   .4765245   .0744175     6.40   0.000      .330669      .62238
             |
       _cons |  -.0165214     .00333    -4.96   0.000     -.023048   -.0099948
------------------------------------------------------------------------------
```

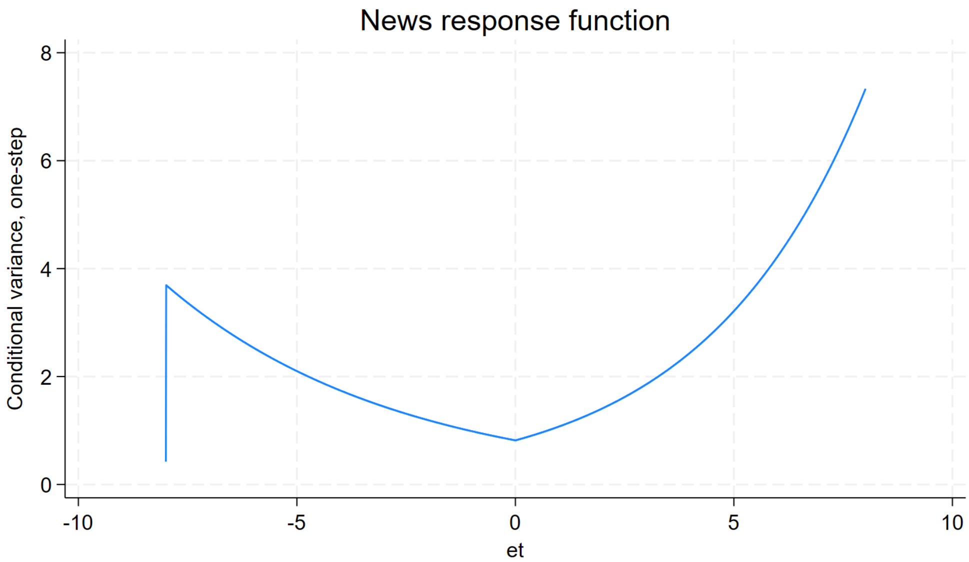
We present a graph with y-axis: predicted variances and x-axis: news, that is a news response function for our estimated EGARCH(1,1) model. We can see the asymmetry in the response depending on the sign of news.

Our result for the variance is

$$\ln(\sigma_t^2) = -0.017 + 0.043 z_{t-1} + 0.231(|z_{t-1}| - \sqrt{2/\pi}) + 0.504 \ln(\sigma_{t-1}^2) + 0.477 \ln(\sigma_{t-2}^2)$$

where $z_t = \epsilon_t/\sigma_t$, which is distributed as $N(0,1)$.

The positive L(1) earch coefficient implies that positive innovations (unanticipated increases in exchange rate) are more destabilizing than negative innovations. The effect appears not very strong (0.043) and is smaller than the symmetric effect (0.231). We can still graphically illustrate that the positive leverage is greater if we plot what is often referred to as the news-response or news-impact function.



Now we check the “leverage effect” of Exchange rate by implementing GARCH-in-mean estimation.

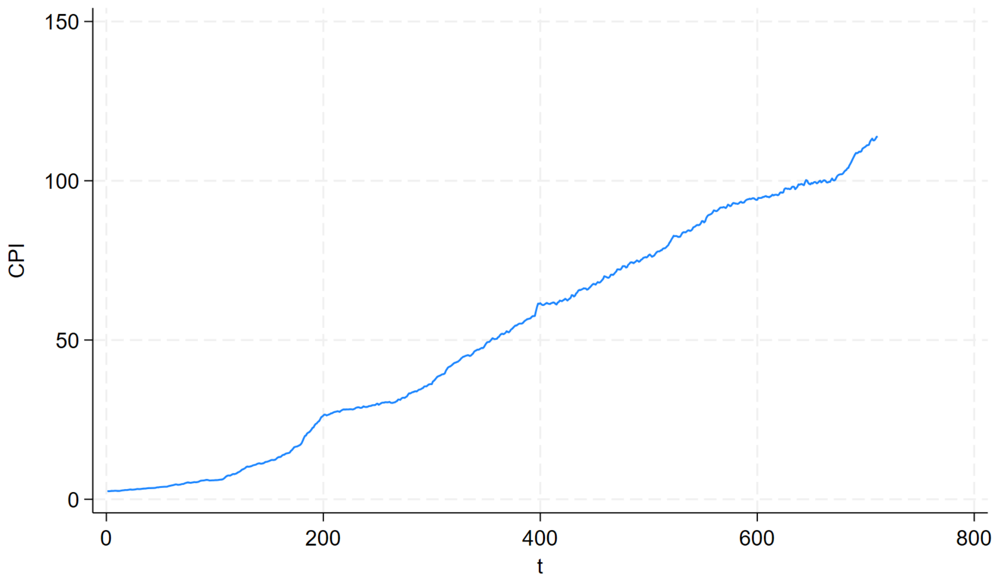
```
Sample: 2 thru 5104                              Number of obs     =       5103
                                                 Wald chi2(1)      =       1.22
Log likelihood = -3782.259                       Prob > chi2       =     0.2698

------------------------------------------------------------------------------
             |                 OPG
         d_e | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ARCHM        |
      sigma2 |  -.0284877   .0258154    -1.10   0.270     -.079085    .0221095
------------------------------------------------------------------------------
```

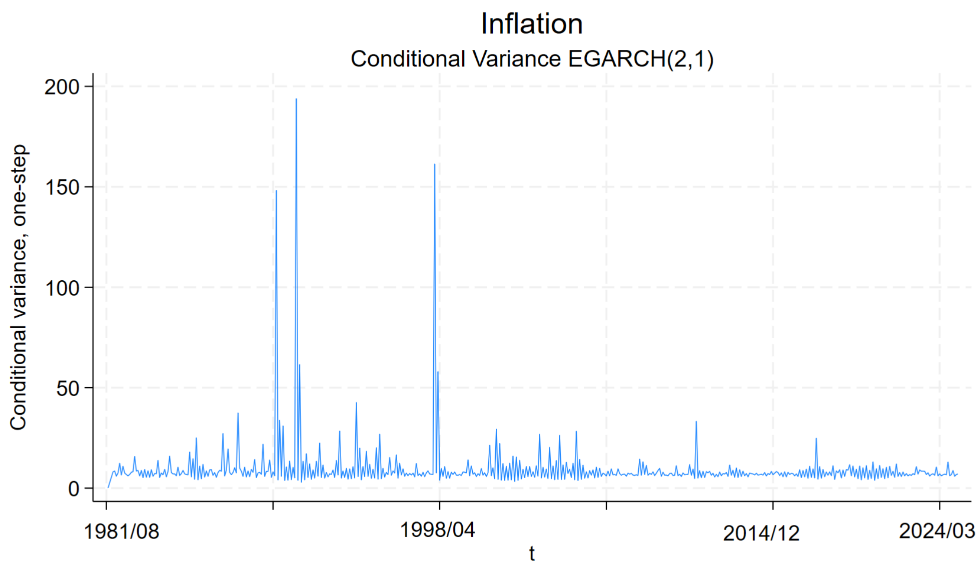
The p-value for arch in mean term is 0.270, thus we do not reject H0:sigma2=0. That is there is not enough evidence of “leverage effect” of Exchange rate.

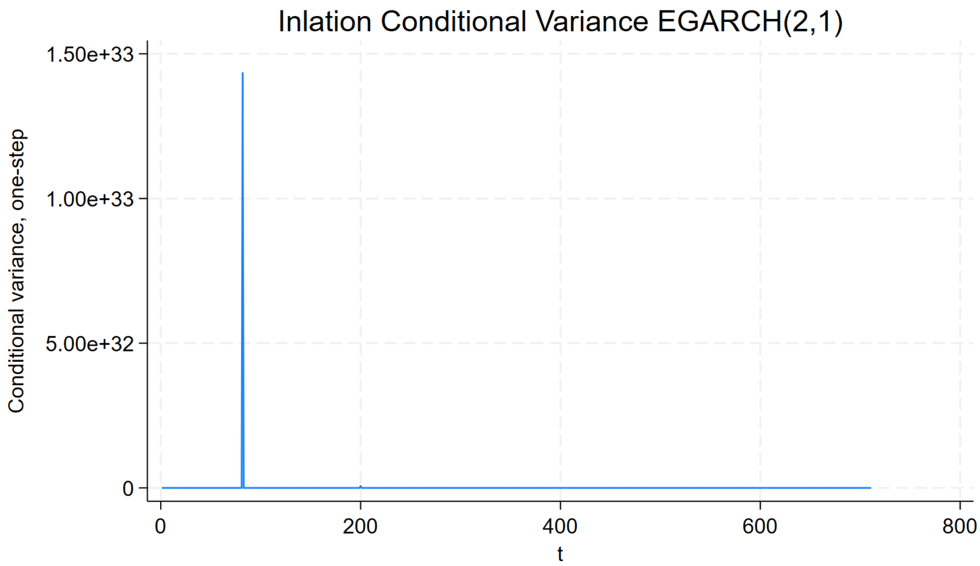
2.2 Get your own data. Apply one of ARCH/GARCH variants to your data to detect and estimate TV-volatility, etc.

We want to check a kind of ‘leverage’ effect in inflation rate in Korea. In this case the inflation might have fluctuated (in response to unexpected inflation shock) due to the distrust of private sectors in the Bank of Korea's inflation targeting system(just one possible channel). We use CPI data from KOSIS from 1965 to 2024. The index is reported in a monthly basis. The 1st figure shows the overall dynamics of CPI. We can find rapid rise in 1980s and in 2023~2024. They are the Oil price shock and the recovering from COVID-19 pandemic period, respectively.



The second plot shows the Inflation rate during the same time period. It is obvious that the movements of inflation before 1980 and after are different. The fluctuations are much less in recent dates. So we plot the 2nd figure that starts from 1981. This figure shows the estimated conditional variance of inflation. We can see here that the volatility has been small for decades. This could advocate the success of BOK to stabilize inflation(inflation targeting). The last figure is the full sample EGARCH(1,1) estimates.





p-value of the last lag	EGARCH(1,1)	**EGARCH(2,1)**	EGARCH(3,1)	EGARCH(2,2)	EGARCH(2,2)-AR(1)
AR term				do not converge	0.000
ARCH term	0.000	0.000	0.107		0.390
ARCH(absolute)	0.000	0.000	0.157		0.523
GARCH term	0.000	0.000	0.000		0.000

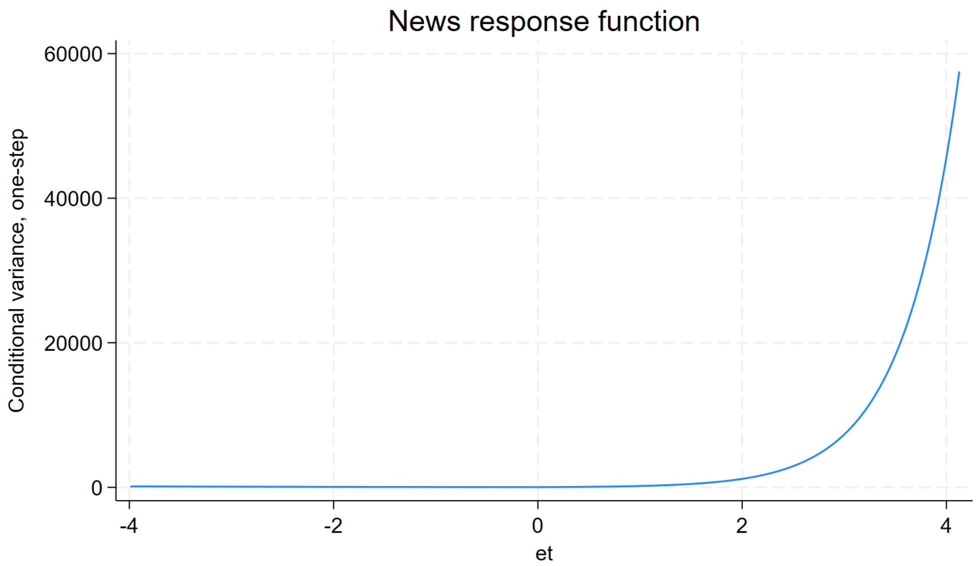
We want to check if there is asymmetry. The following table summarizes the estimation.

```
Sample: 2 thru 711                               Number of obs     =        710
                                                 Wald chi2(.)      =          .
Log likelihood = -1777.421                       Prob > chi2       =          .

------------------------------------------------------------------------------
             |                 OPG
   inflation | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
inflation    |
       _cons |   1.645774   .0996399    16.52   0.000     1.450484    1.841065
-------------+----------------------------------------------------------------
ARCH         |
       earch |
         L1. |   .3816445   .0531121     7.19   0.000     .2775466    .4857423
         L2. |   .3488513   .0542349     6.43   0.000     .2425528    .4551498
             |
     earch_a |
         L1. |   .6665659   .0763349     8.73   0.000     .5169522    .8161797
         L2. |    .433354   .0834537     5.19   0.000     .2697877    .5969203
             |
      egarch |
         L1. |  -.8866396   .0134586   -65.88   0.000     -.913018   -.8602613
             |
       _cons |   4.315675    .103518    41.69   0.000     4.112783    4.518567
------------------------------------------------------------------------------
```

$$\ln(\sigma_t^2) = 4.316 + 0.382 z_{t-1} + 0.349 z_{t-2} + 0.667(|z_{t-1}| - \sqrt{2/\pi}) + 0.433(|z_{t-2}| - \sqrt{2/\pi}) - 0.887 \ln(\sigma_{t-1}^2)$$

Comparing 0.382, 0.349 of earch_a term and 0.667, 0.433 of earch term, it is not easy to say whether the positive news “dominates” the negative news or not. Still we can see that there is obvious asymmetry in the response function. The fluctuation in the prices are more vulnerable to unexpected inflation rather then disinflation.



Next, we present GARCH-in-mean results and check if the mean equation term is significant.

The sign is positive as our claim, and it is significant.

```
Sample: 2 thru 711                                Number of obs     =        710
                                                  Wald chi2(1)      =      62.70
Log likelihood = -1633.242                        Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
   inflation | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ARCHM        |
      sigma2 |   .1743413   .0220175     7.92   0.000     .1311878    .2174947
------------------------------------------------------------------------------
```

Matlab Code

```
clear
close all
clc

addpath(genpath('C:\Users\eunkyu\git\VAR-Toolbox'))
%% Parmeter 정하기
numlag = 4;
vecdim = 2;
h = 39; % horizon
with_constant = 1; % 1이면 상수항 추가 0이면 상수항 없음
with_trend = 1; % 1이면 타임트렌드 추가 0이면 없음


%% Data 불러오기
```

```
load FRED_QD_hw1_1.mat %1959~2017까지의 분기별 데이터임.
gdp = FREDQD.gdpc1;
gdplag = lagmatrix(gdp,1);
dgdp = 100*(gdp-gdplag)./gdp;
% lgdp = log(gdp);
% dgdp = 100*lgdp-lagmatrix(lgdp,1);
unrate = FREDQD.unrate;

Y = [dgdp,unrate];
Y = Y(2:end,:);  %변화율로 변환하면서 초기값이 빠짐. 따라서 2부터 시작

T = size(Y,1);

%% SVAR estimation with Long-run Restriction,
if (with_trend ==0 && with_constant == 1)

    X = ones(T,numlag*vecdim+1);
    for p=1:numlag
        X(:,2*p:2*p+1)=lagmatrix(Y,p);
    end
    Y = Y(p+1:end,:); %lag term을 만들면서 p+1개의 관측치가 날아감
    X = X(p+1:end,:);

    numobs = size(Y,1); %관측치를 새로 정의해줌

    Ahat = (X'*X)\X'*Y; % 우리가 원하는 행렬의 전치행렬로 결과가 나옴

    resid = Y-X*Ahat;
    Sigma = resid'*resid/numobs;

    constant = Ahat(1,:)';
    Ahats = zeros(vecdim,vecdim,numlag);
    for p=1:numlag
        Ahats(:,:,p) = Ahat(2*p:2*p+1,:)'; %각 lag polynomial을 재배열해주기 위해 전치함
    end
    A1 = eye(vecdim);
    for p=1:numlag
        A1 = A1-Ahats(:,:,p);
    end

    P = chol(A1\Sigma/A1')'; %하삼각행렬로 만들어주기 위해 전치함
    B = A1*P;


    %%%%%% VAR(p) process를 VMA(inf) process로 표현하는 작업(Wold Decomposition)

    A = zeros(vecdim*numlag);
    for p=1:numlag
        A(1:2,2*p-1:2*p) = Ahats(:,:,p);
    end
    for p=1:numlag-1
        A(2*p+1:2*p+2,2*p-1:2*p)=eye(vecdim);
    end
```

```
    %%% IRF to demand shock
    d = [0;1];
    dIRF = zeros(vecdim,h+1);
    IR = zeros(vecdim*numlag,1);
    IR(1:vecdim,1) = B*d;
    for i=1:h+1
        dIRF(:,i) = IR(1:vecdim,1);
        IR = A*IR;
    end

    d_irf_gdp = dIRF(1,:);
    d_irf_unrate = dIRF(2,:);

    %%% IRF to supply shock
    s = [1;0];
    sIRF = zeros(vecdim,h+1);

    IR = zeros(vecdim*numlag,1);
    IR(1:vecdim,1) = B*s;
    for i=1:h+1
        sIRF(:,i) = IR(1:vecdim,1);
        IR = A*IR;
    end

    s_irf_gdp = sIRF(1,:);
    s_irf_unrate = sIRF(2,:);

end


if (with_trend ==0 && with_constant == 0)
    T = size(Y,1);
    X = zeros(T,numlag*vecdim);

    for p=1:numlag
    X(:,2*p-1:2*p)=lagmatrix(Y,p);
    end
    Y = Y(p+1:end,:); %lag term을 만들면서 p+1개의 관측치가 날아감
    X = X(p+1:end,:);
    numobs = size(Y,1); %관측치를 새로 정의해줌

    Ahat = (X'*X)\X'*Y; % 우리가 원하는 행렬의 전치행렬로 결과가 나옴

    resid = Y-X*Ahat;
    Sigma = resid'*resid/numobs;

    Ahats = zeros(vecdim,vecdim,numlag);
    for p=1:numlag
        Ahats(:,:,p) = Ahat(2*p-1:2*p,:)'; %각 lag polynomial을 재배열해주기 위해 전치함
    end

    A1 = eye(vecdim);
    for p=1:numlag
        A1 = A1-Ahats(:,:,p);
```

```
    end

    P = chol(A1\Sigma/A1')'; %하삼각행렬로 만들어주기 위해 전치함
    B = A1*P;

    %%%%%%

    A = zeros(vecdim*numlag);
    for p=1:numlag
        A(1:2,2*p-1:2*p) = Ahats(:,:,p);
    end
    for p=1:numlag-1
        A(2*p+1:2*p+2,2*p-1:2*p)=eye(vecdim);
    end

    %%% IRF to demand shock
    d = [0;1];
    dIRF = zeros(vecdim,h+1);
    IR = zeros(vecdim*numlag,1);
    IR(1:vecdim,1) = B*d;
    for i=1:h+1
        dIRF(:,i) = IR(1:vecdim,1);
        IR = A*IR;
    end

    d_irf_gdp = dIRF(1,:);
    d_irf_unrate = dIRF(2,:);

    %%% IRF to supply shock
    s = [1;0];
    sIRF = zeros(vecdim,h+1);

    IR = zeros(vecdim*numlag,1);
    IR(1:vecdim,1) = B*s;
    for i=1:h+1
        sIRF(:,i) = IR(1:vecdim,1);
        IR = A*IR;
    end

    s_irf_gdp = sIRF(1,:);
    s_irf_unrate = sIRF(2,:);

end



if with_trend ==1
    time = 1:T;
    X = ones(T,numlag*vecdim+2);
    X(:,2) = time';
    for p=1:numlag
        X(:,2*p+1:2*p+2)=lagmatrix(Y,p);
    end
    Y = Y(p+1:end,:); %lag term을 만들면서 p+1개의 관측치가 날아감
    X = X(p+1:end,:);
```

```
numobs = size(Y,1); %관측치를 새로 정의해줌

Ahat = (X'*X)\X'*Y; % 우리가 원하는 행렬의 전치행렬로 결과가 나옴

resid = Y-X*Ahat;
Sigma = resid'*resid/numobs;

constant = Ahat(1,:)';
trend = Ahat(2,:)';
Ahats = zeros(vecdim,vecdim,numlag);
for p=1:numlag
    Ahats(:,:,p) = Ahat(2*p+1:2*p+2,:)'; %각 lag polynomial을 재배열해주기 위해 전치함
end
A1 = eye(vecdim);
for p=1:numlag
    A1 = A1-Ahats(:,:,p);
end

P = chol(A1\Sigma/A1')'; %하삼각행렬로 만들어주기 위해 전치함
B = A1*P;

%%%%%%

A = zeros(vecdim*numlag);
for p=1:numlag
    A(1:2,2*p-1:2*p) = Ahats(:,:,p);
end
for p=1:numlag-1
    A(2*p+1:2*p+2,2*p-1:2*p)=eye(vecdim);
end

%%% IRF to demand shock
d = [0;1];
dIRF = zeros(vecdim,h+1);
IR = zeros(vecdim*numlag,1);
IR(1:vecdim,1) = B*d;
for i=1:h+1
    dIRF(:,i) = IR(1:vecdim,1);
    IR = A*IR;
end


d_irf_gdp = dIRF(1,:);
d_irf_unrate = dIRF(2,:);

%%% IRF to supply shock
s = [1;0];
sIRF = zeros(vecdim,h+1);

IR = zeros(vecdim*numlag,1);
IR(1:vecdim,1) = B*s;
for i=1:h+1
    sIRF(:,i) = IR(1:vecdim,1);
    IR = A*IR;
```

```
    end

    s_irf_gdp = sIRF(1,:);
    s_irf_unrate = sIRF(2,:);


end


%%
%%% IRF Graph %%%
figure(1)

subplot(1,2,1)
plot(cumsum(s_irf_gdp,2),'LineWidth',2.5,'Color',cmap(1))
xlim ([1,h])
ylim ([-0.8,1.2])
hold on
plot(s_irf_unrate,'LineWidth',2.5,'Color',cmap(2))
hold on
plot(zeros(h,1),'--k')
legend({'GDP Level';'Unemployment'})
title('Supply shock')
subplot(1,2,2)
plot(cumsum(d_irf_gdp,2),'LineWidth',2.5,'Color',cmap(1))
hold on
plot(d_irf_unrate,'LineWidth',2.5,'Color',cmap(2))
hold on
plot(zeros(h,1),'--k')
title('Demand shock')
legend({'GDP Level';'Unemployment'})
xlim ([1,h])
ylim ([-0.8,1.2])


% %%
% %%% Forecast Error Variance Decomposition %%%
%
% A = zeros(vecdim*numlag);
% for p=1:numlag
%     A(1:2,2*p-1:2*p) = Ahats(:,:,p);
% end
%
% fev_k = zeros(h+1,vecdim,vecdim); % horizon(i), shock(j), variable(k)
% fev_total = zeros(h+1,1,vecdim);
%
% Phi = eye(vecdim*numlag);
% for i=1:h+1
%     for j = 1:vecdim
%         for k = 1:vecdim
%             fev_k(i,j,k) = (Phi(k,1:vecdim)*B(:,j))^2;
%             fev_total(i,1,k) = fev_total(i,1,k)+fev_k(i,j,k);
%         end
%     end
%     Phi=A*Phi;
```

```
% end
%
% fevd = zeros(h+1,vecdim,vecdim);
% for j = 1:vecdim
%     for k = 1:vecdim
%         fevd(:,j,k) = cumsum(fev_k(:,j,k),1)./cumsum(fev_total(:,1,k),1);
%     end
% end
%
% figure(2)
% FigSize(26,8)
% subplot(1,2,1)
% AreaPlot(fevd(:,2,1))
% ylabel 'Contribution of Demand Shock(shaded)'
% title 'GDP FEVD'
% hold on
% subplot(1,2,2)
% AreaPlot(fevd(:,2,2))
% ylabel 'Contribution of Demand Shock(shaded)'
% title 'Unemployment FEVD'

%%
%%% Forecast Error Variance Decomposition
%%% TA session 그대로 따라하기

d_fev_gdp = d_irf_gdp.^2;
d_fev_gdp = cumsum(d_fev_gdp,2);
s_fev_gdp = s_irf_gdp.^2;
s_fev_gdp = cumsum(s_fev_gdp,2);

d_fevd_gdp = d_fev_gdp./(d_fev_gdp+s_fev_gdp);

d_fev_unrate = d_irf_unrate.^2;
d_fev_unrate = cumsum(d_fev_unrate,2);
s_fev_unrate = s_irf_unrate.^2;
s_fev_unrate = cumsum(s_fev_unrate,2);

d_fevd_unrate = d_fev_unrate./(d_fev_unrate+s_fev_unrate);

figure(2)

subplot(1,2,1)
AreaPlot(d_fevd_gdp)
ylabel 'Contribution of Demand Shock(shaded)'
title 'GDP FEVD'
ylim ([0,1])
xlim ([1,h])
hold on
subplot(1,2,2)
AreaPlot(d_fevd_unrate)
ylabel 'Contribution of Demand Shock(shaded)'
title 'Unemployment FEVD'
ylim ([0,1])
xlim ([1,h])
```

```
%%
%%% Bootstrap CIs
%%% Recursive Bootstrap

rep=10000;

if with_trend==1

    boot_save_Ahat = zeros(vecdim*numlag+2,vecdim,rep);
    boot_save_dIRF = zeros(vecdim,h+1,rep);
    boot_save_sIRF = zeros(vecdim,h+1,rep);

    for r=1:rep
        bootsample=randi(numobs,numobs,1);
        resid_centered = resid-mean(resid,1);
        Ub = resid_centered(bootsample,:);
        Yb=X*Ahat+Ub;
        Xb=X;
        for p=1:numlag
            Xb(:,2*p+1:2*p+2)=lagmatrix(Yb,p);
        end
        Xb(:,1) = ones(numobs,1);
        Xb(:,2) = (1:1:numobs)';
        for p=1:numlag
            Xb(1:(2*p-1)*p,2*p+1:2*p+2)=X(1:(2*p-1)*p,2*p+1:2*p+2);
        end

        Ahat_b = (Xb'*Xb)\Xb'*Yb;
        boot_save_Ahat(:,:,r)=Ahat_b;

        resid_b = Yb-Xb*Ahat_b;
        Sigma_b = resid_b'*resid_b/numobs;

        constant_b = Ahat_b(1,:)';
        trend_b = Ahat_b(2,:)';
        Ahats_b = zeros(vecdim,vecdim,numlag);
        for p=1:numlag
            Ahats_b(:,:,p) = Ahat_b(2*p+1:2*p+2,:)'; %각 lag polynomial을 재배열해주기 위해
        end
        A1_b = eye(vecdim);
        for p=1:numlag
            A1_b = A1_b-Ahats_b(:,:,p);
        end

        P_b = chol(A1_b\Sigma_b/A1_b')'; %하삼각행렬로 만들어주기 위해 전치함
        B_b = A1_b*P_b;

        %%%%%%

        A_b = zeros(vecdim*numlag);
        for p=1:numlag
            A_b(1:2,2*p-1:2*p) = Ahats_b(:,:,p);
        end
        for p=1:numlag-1
```

```
            A_b(2*p+1:2*p+2,2*p-1:2*p)=eye(vecdim);
        end

        %%% IRF to demand shock
        d = [0;1];
        dIRF_b = zeros(vecdim,h+1);
        IR = zeros(vecdim*numlag,1);
        IR(1:vecdim,1) = B_b*d;
        for i=1:h+1
            dIRF_b(:,i) = IR(1:vecdim,1);
            IR = A_b*IR;
        end
        boot_save_dIRF(:,:,r)=dIRF_b;

        d_irf_gdp_b = dIRF_b(1,:);
        d_irf_unrate_b = dIRF_b(2,:);

        %%% IRF to supply shock
        s = [1;0];
        sIRF_b = zeros(vecdim,h+1);

        IR = zeros(vecdim*numlag,1);
        IR(1:vecdim,1) = B_b*s;
        for i=1:h+1
            sIRF_b(:,i) = IR(1:vecdim,1);
            IR = A_b*IR;
        end
        boot_save_sIRF(:,:,r)=sIRF_b;

        s_irf_gdp_b = sIRF_b(1,:);
        s_irf_unrate_b = sIRF_b(2,:);

    end

    up95_sIRF = zeros(vecdim,h+1);
    lo95_sIRF = zeros(vecdim,h+1);
    up95_dIRF = zeros(vecdim,h+1);
    lo95_dIRF = zeros(vecdim,h+1);

    boot_save_sIRF_cumsum = cumsum(boot_save_sIRF,2);
    boot_save_dIRF_cumsum = cumsum(boot_save_dIRF,2);
    for i=1:h+1
        up95_sIRF(2,i)=quantile(boot_save_sIRF(2,i,:),0.975,3);
        lo95_sIRF(2,i)=quantile(boot_save_sIRF(2,i,:),0.025,3);
        up95_dIRF(2,i)=quantile(boot_save_dIRF(2,i,:),0.975,3);
        lo95_dIRF(2,i)=quantile(boot_save_dIRF(2,i,:),0.025,3);

        up95_sIRF(1,i)=quantile(boot_save_sIRF_cumsum(1,i,:),0.975,3);
        lo95_sIRF(1,i)=quantile(boot_save_sIRF_cumsum(1,i,:),0.025,3);
        up95_dIRF(1,i)=quantile(boot_save_dIRF_cumsum(1,i,:),0.975,3);
        lo95_dIRF(1,i)=quantile(boot_save_dIRF_cumsum(1,i,:),0.025,3);
    end

    figure(3)
    % title 'Bootstrap Confidence Bands'
```

```
subplot(2,2,4)

plot(lo95_dIRF(2,:))
hold on
plot(up95_dIRF(2,:))
hold on
plot(d_irf_unrate,'LineWidth',2.5,'Color','yellow')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'Unemployment'
title 'SIRF-Demand Shock'

subplot(2,2,2)

plot(lo95_dIRF(1,:))
hold on
plot(up95_dIRF(1,:))
hold on
plot(cumsum(d_irf_gdp,2),'LineWidth',2.5,'Color','yellow')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'GDP level'
title 'SIRF-Demand Shock'

subplot(2,2,3)

plot(lo95_sIRF(2,:))
hold on
plot(up95_sIRF(2,:))
hold on
plot(s_irf_unrate,'LineWidth',2.5,'Color','yellow')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'Unemployment'
title 'SIRF-Supply Shock'

subplot(2,2,1)

plot(lo95_sIRF(1,:))
hold on
plot(up95_sIRF(1,:))
hold on
plot(cumsum(s_irf_gdp,2),'LineWidth',2.5,'Color','yellow')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'GDP level'
title 'SIRF-Supply Shock'


figure(4)

subplot(2,2,1)

plot(lo95_sIRF(1,:))
hold on
```

```
plot(up95_sIRF(1,:))
for r=1:rep
    plot(cumsum(boot_save_sIRF(1,:,r),2),"yellow")
    hold on
end
plot(median(cumsum(boot_save_sIRF(1,:,:),2),3),'LineWidth',2.5,'Color','r')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'GDP level'
title 'SIRF-Supply Shock'

subplot(2,2,3)

plot(lo95_sIRF(2,:))
hold on
plot(up95_sIRF(2,:))
for r=1:rep
    plot(boot_save_sIRF(2,:,r),"yellow")
    hold on
end
plot(median(boot_save_sIRF(2,:,:),3),'LineWidth',2.5,'Color','r')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'Unemployment'
title 'SIRF-Supply Shock'

subplot(2,2,2)

plot(lo95_dIRF(1,:))
hold on
plot(up95_dIRF(1,:))
for r=1:rep
    plot(cumsum(boot_save_dIRF(1,:,r),2),"yellow")
    hold on
end
plot(median(cumsum(boot_save_dIRF(1,:,:),2),3),'LineWidth',2.5,'Color','r')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'GDP level'
title 'SIRF-Demand Shock'

subplot(2,2,4)

plot(lo95_dIRF(2,:))
hold on
plot(up95_dIRF(2,:))
for r=1:rep
    plot(boot_save_dIRF(2,:,r),"yellow")
    hold on
end
plot(median(boot_save_dIRF(2,:,:),3),'LineWidth',2.5,'Color','r')
hold on
plot(zeros(h+1,1),'--k')
ylabel 'Unemployment'
title 'SIRF-Demand Shock'
```

```
    subplot(2,2,2)

    plot(lo95_dIRF(1,:))
    hold on
    plot(up95_dIRF(1,:))

    subplot(2,2,4)

    plot(lo95_dIRF(2,:))
    hold on
    plot(up95_dIRF(2,:))

    subplot(2,2,3)

    plot(lo95_sIRF(2,:))
    hold on
    plot(up95_sIRF(2,:))

    subplot(2,2,1)

    plot(lo95_sIRF(1,:))
    hold on
    plot(up95_sIRF(1,:))


end


rmpath(genpath('C:\Users\eunkyu\git\VAR-Toolbox'))

% recursive bootstrap하다가 포기함
% rep = 1000;
% init = [dgdp(2:numlag+1,1),unrate(2:numlag+1,1)];
% init_cond = zeros(1,vecdim*numlag);
% for p=1:numlag
%     init_cond(:,2*p-1:2*p)=init(p,:);
% end
% Ahat_boot = zeros(2+vecdim*numlag,vecdim,rep);
%
%
% if with_trend ==1
%     init_cond = [1,0,init_cond];
%     for r=1:rep
%         bootsample = randi(numobs,numobs,1);
%         error_boot = resid(bootsample,:);
%         Yb = zeros(numobs,vecdim);
%         Yb(1,:) = init_cond*Ahat;
%         for i=2:numobs
%             Yb(i,:)=
%         end
%     end
% end
```

Stata Code

```
***** KOSPI *****
clear
drop _all

import excel "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\kospi.xlsx", sheet("Sheet1"
rename 종가 kospi
gen time = _n
tsset time
gen R = 100*(kospi-L.kospi)/kospi

twoway (tsline R, lwidth(vthin)), ytitle("KOSPI returns rate") tlabel()
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\KOSPI수익률.gph",replac

** model selection procedure **
arch R, arch(1) garch(1)
arch R, arch(1) garch(1/2)
arch R, arch(1/2) garch(1)

arch R, arch(1) garch(1)
est store GARCH_result
predict var_garch, variance

twoway (tsline var_garch, lwidth(vthin)), ytitle("Conditional Variance (GARCH(1,1)")
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\KOSPI조건부분산garch.gp


** model selection procedure **
arch R, earch(1) egarch(1)
arch R, earch(1/2) egarch(1)
arch R, earch(1) egarch(1/2)

arch R, earch(1) egarch(1)
est store EGARCH_result
predict var_egarch, variance
twoway (tsline var_egarch) , title("Conditional Variance EGARCH(1,1)")
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\KOSPI조건부분산egarch.g

** News Responce Function **
generate et = (_n-1850)/1850*8
predict sigma2, variance at(et 1)
line sigma2 et , m(i) c(l) title(News response function)

** GARCH-in-mean
arch R, earch(1) egarch(1) archm

*************************
***** Exchange Rate *****
*************************

clear
drop _all

import excel "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\ExchangeRate.xlsx", sheet("
```

```
gen t = _n
tsset t
gen d_e = 100*(erate-L.erate)/erate

twoway (tsline d_e, lwidth(vthin)), title("Exchange Rate")
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\ExchangeRate.gph" , r

** model selection procedure **
arch d_e, arch(1) garch(1)
arch d_e, arch(1) garch(1) ar(1)
arch d_e, arch(1/2) garch(1)
arch d_e, arch(1/3) garch(1)
arch d_e, arch(1/2) garch(1/2)
arch d_e, arch(1/2) garch(1/3)
arch d_e, arch(1/2) garch(1/4)
arch d_e, arch(1/2) garch(1/5)

arch d_e, arch(1/2) garch(1/4)
est store GARCH_result
predict var_garch, variance


twoway (tsline var_garch , lwidth(vthin)), title("Conditional Variance GARCH(2,4)")
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\ExchangeRate_garch.gp

** model selection procedure **
arch d_e, earch(1) egarch(1)
arch d_e, earch(1) egarch(1) ar(1)
arch d_e, earch(1/2) egarch(1)
arch d_e, earch(1) egarch(1/2)
arch d_e, earch(1) egarch(1/3)

arch d_e, earch(1) egarch(1/2)
est store EGARCH_result
predict var_egarch, variance
twoway (tsline var_egarch,lwidth(vthin)) , title("Conditional Variance EGARCH(1,2)")
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\ExchangeRate_egarch.g

** News Responce Function **
generate et = (_n-2550)/2550*8
predict sigma2, variance at(et 1)
line sigma2 et , m(i) c(l) title(News response function)

** GARCH-in-mean
arch d_e, earch(1) egarch(1) archm
predict var_egarchm, variance
twoway (tsline var_egarchm)



*******************************
***** Inflation ***************
*******************************

clear
```

```
drop _all

import excel "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\cpi.xlsx", sheet("데이터") fi

gen t = _n
tsset t

twoway(tsline CPI)

gen inflation=400*(CPI-L.CPI)/CPI

twoway(tsline inflation)
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\CPI.gph" , replace

/*
** model selection procedure **
arch inflation, arch(1) garch(1)
arch inflation, arch(1/2) garch(1)
arch inflation, arch(1/3) garch(1)
arch inflation, arch(1/2) garch(1/2)
arch inflation, arch(1/2) garch(1)
arch inflation, arch(1/2) garch(1) ar(1)
arch inflation, arch(1/2) garch(1) ar(1/2)
arch inflation, arch(1/2) garch(1) ar(1/3)
*/

/*
gen d = t<201
arch inflation, arch(1/2) garch(1) ar(1/2)
arch inflation, arch(1/2) garch(1) ar(1/2) archm
arch inflation d, arch(1/2) garch(1) ar(1/2)
drop var_garch
predict var_garch,variance
twoway (tsline var_garch) , tlabel(none)
graph save "Graph" "C:\Users\eunkyu\Dropbox\4-1\시계열분석특수연구\hw1\CPI_garch.gph" , repla
*/

** model selection procedure **
arch inflation, earch(1) egarch(1)
arch inflation, earch(1/2) egarch(1)
arch inflation, earch(1/3) egarch(1)
arch inflation, earch(1/2) egarch(1/2)
arch inflation, earch(1/2) egarch(1) ar(1)

arch inflation, earch(1/2) egarch(1)
predict var_egarch,variance
twoway (tsline var_egarch), title("Inlation Conditional Variance EGARCH(2,1)")
twoway (tsline var_egarch if t>200, lwidth(vthin)), title("Inflation") subtitle("Conditio

** News Responce Function **
generate et = (_n-350)/350*4
predict sigma2, variance at(et 1)
line sigma2 et , m(i) c(l) title(News response function)

** GARCH-in-mean
```

```
arch inflation, arch(1/2) garch(1) archm

predict sigmag,variance
twoway (tsline sigmag)

/*
* 1981.09부터분석함
drop if t < 201
arch inflation, earch(1) egarch(1) ar(1/2)
predict sigmaeg,variance
twoway (tsline sigmaeg)
*/
```