MARMARA UNIVERSITY

FACULTY OF ENGINEERING

DESIGN OF AN AUTOMOBILE SUSPENSION SYSTEM BASED ON THE 7-DOF FULL CAR MODEL

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GRADUATION PROJECT REPORT

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ISTANBUL,2020

MARMARA UNIVERSITY

FACULTY OF ENGINEERING

Design of an Automobile Suspension System Based on the 7-DOF Full Car Model

by

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June 14,2020,Istanbul

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF

BACHELOR OF SCIENCE

AT

MARMARA UNIVERSITY

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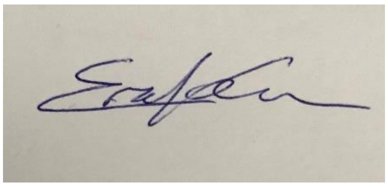

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ACKNOWLEDGEMENT

To start with, I would like to thank my supervisor Assoc. Prof. Dr. Mustafa Özdemir for the advices that he gave me while I am working on this project. His guidance and help was precious and he gave some much ideas while I make progress in this thesis.

June,2020 Eralp Can

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ABSTRACT

Design of an Automobile Suspension System Based on the 7-DOF Full Car Model

Automobile suspension systems developing as the automotive technologies develops rapidly in recent years. Main objective of the suspension systems in the car is reduce road excitations while driving and prevent this excitation to transmit the vehicle body and make a comfortable ride to the driver and passengers in the car. In this thesis it is presented a modelling and simulating of a passive suspension quarter car model and full car models.All the analyzes are made in the Matlab/Simulink to see the design of a 7 degree of freedom full car model which is based on quarter car model with 2 degree of freedom.

SYMBOLS

M_s: Mass of the vehicle body.

M_{ufr}: Front right unsprung mass.

M_{ufl}: Front left unsprung mass.

M_{urr}: Rear right unsprung mass.

M_{url}: Rear left unsprung mass.

$\emptyset$: Roll angle at the center of gravity.

θ: Pitch angle at the center of gravity.

a,b: distance from the center of gravity to front and rear axle respectively.

c,d: distance from the center of gravity to left and right wheels respectively.

I_{xx}: Moment of inertia about X-X axis.

I_{yy}: Moment of inertia about Y-Y axis.

K_{sfr}: Spring stiffness of the front right suspension.

K_{sfl}: Spring stiffness of the front left suspension.

K_{srr}: Spring stiffness of the rear right suspension.

K_{srl}: Spring stiffness of the rear left suspension.

C_{sfr}: Damping coefficient of front right tyre.

C_{sfl}: Damping coefficient of front left tyre.

C_{srr}: Damping coefficient of rear right tyre.

C_{srl}: Damping coefficient of rear left tyre.

K_{tfr}: Tyre stiffness of the front right tyre.

K_{tfl}: Tyre stiffness of the front left tyre.

K_{trr}: Tyre stiffness of the rear right tyre.

K_{trl}: Tyre stiffness of the rear left tyre.

X_{cg}: Displacement of center of gravity of the vehicle body.

X_{frd}: Displacement of the front right tyre.

X_{fld}:Displacement of the front left tyre.

X_{rrd}: Displacement of the rear right tyre.

X_{rld}: Displacement of the rear left tyre.

X_{rfr}: Road input through the front right tyre.

X_{rfl}: Road input through the front left tyre.

X_{rrr}: Road input through the rear right tyre.

X_{rrl}: Road input through the rear left tyre.

Cs : Viscous damping coefficient.

Ks : Spring Stiffness.

Kt : Tyre stiffness.

Xs : Displacement of the sprung mass.

Xu : Displacement of the unsprung mass.

Xr : Road excitations.

ABBREVIATIONS

DOF **:**Degree of Freedom

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1.INTRODUCTION

Suspension systems are links between the wheels and vehicle body and they allow relative motion [1]. Suspension systems have an important role between the road and vehicles. There are some reasons that why we need a suspension in our vehicles. First of all, one of the main purposes of the suspension is absorb the excitements from the road purely and hold the car in the road easily. Secondly and another important role of suspension system is make a comfortable ride for the drive and the passengers in the vehicle. There are already lots of work and research done in this section in the automotive industry. In the designing of suspension systems there are a few models that used for designing. Some of them are; the quarter car 2-DOF (Degree of Freedom) model, half car model and 7-DOF full car model. These common models are used for their purpose of research. In this thesis I am going to design a passive suspension system for the 7-DOF full car model based on 2-DOF quarter car model. In this research I will assume a car in a road for some distance and it passes a bump in a significant height first the two front wheels and then the rare wheels and observe the vertical distance changes in suspension both for quarter car model and full car model. This comparison will help to decide whether if we could design a 7-DOF full car model based on 2-DOF quarter car model or not.

1.1.Components of Suspension Systems

A suspension system consist of two main part. They are the springs and dampers. Springs are categorized in four types; which are leaf springs, coil springs, pneumatic springs and torsion bars.

1.2. Passive, Semi-Active and Active Suspension Systems

There are three way of make a suspension systems. Which are passive, semi-active and active suspension systems. Passive suspension are the most traditional one and the active suspensions system are controlled with electric controller and more expensive than the passive suspension systems.

1.2.1. Passive Suspension Systems

Passive suspension systems used wide area in the world. It control the dynamics of the vehicle's in vertical motion. This motions can be indicated by spinning and tilting. In the other words we can name pitch and roll. Passive word means that components of the suspension system cannot provide any energy to the suspension system. Working principle of this system is that as the system limits the motion of body and wheel, by limiting their relative velocities to a value that gives the required ride comfort. Damping elements placed between the body and wheels of the vehicle. It aims to reduce the vertical body displacement and acceleration and produce effective tire and road contact [2]. Passive suspension systems are the systems that will use in this thesis.

1.2.2. Semi-Active Suspension Systems

This system apply a variable damper or a dissipation component in the suspension system. Some examples for using variable dissipator are twin tube viscous damper and magneto rheological damper. Semi-Active suspension system consumes less power to compared with active suspension systems [3].

1.2.3. Active Suspension Systems

Active suspension systems are the types of suspension systems which are includes electronically controlled actuators in the suspensions [3]. They provide good performance in the suspension and improve the suspension performance.

1.2.3.4 Working Schedule

- Learning of Matlab/Simulink programs (3 weeks).
- Modelling of the quarter car model (2 weeks).
- Applying equations in Simulink and simulate it (1 week).
- Checking from the other researches for the consistency If obtained results are proper, putting it in the report (1 week).
- 04.04.2020-12.04.2020 Midterm week.
- Modelling of the full car model (2 weeks).
- Applying equations in Simulink and simulate it (1 week).
- Checking from the other researches for consistency If obtained results are proper, putting it in the report (1 week).
- Prepration of the report (3 weeks).

2. MODELLING OF VEHICLE SUSPENSION

There are some models use in designing vehicle suspension systems. They are quarter car model, half car model, and full car model. In this thesis I am working on quarter car model and full car model. Because of that these to has given more detailed.

2.1. Quarter Car Model 2-DOF

Quarter car models are the most useful and the basic one for the designing vehicle suspension system.

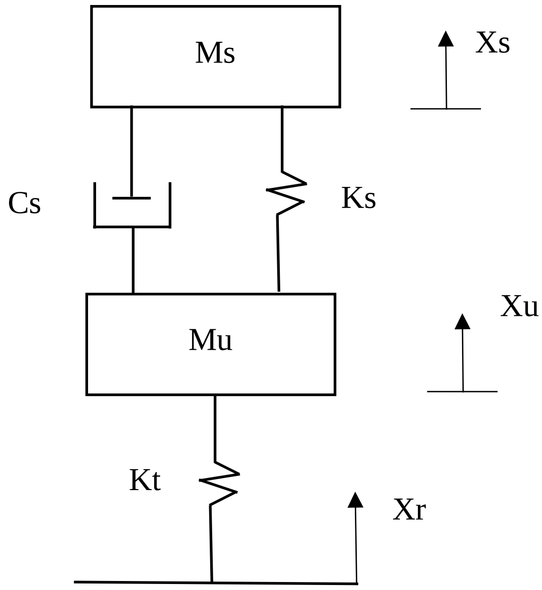

Figure 1: Quarter Car Model

It occurs from two solid masses which is Ms and Mu denoted as sprung and unsprung masses, respectively. The sprung mass Ms in this equation represents ¼ of the body of the vehicle. There are a spring stiffness Ks and a shock absorber with viscous damping coefficient Cs, support the sprung mass and they are called the main suspension. Kt representing here the tire stiffness. Xs, Xu and Xr corresponds to displacement of the sprung mass, displacement of the unsprung mass and road excitations respectrively. The unsprung mass Mu is in direct contact with ground with this spring.[1]

2.1.1.Mathematical Model of Quarter Car 2-DOF

From the newton second law of the motion free body diagram of the model is:

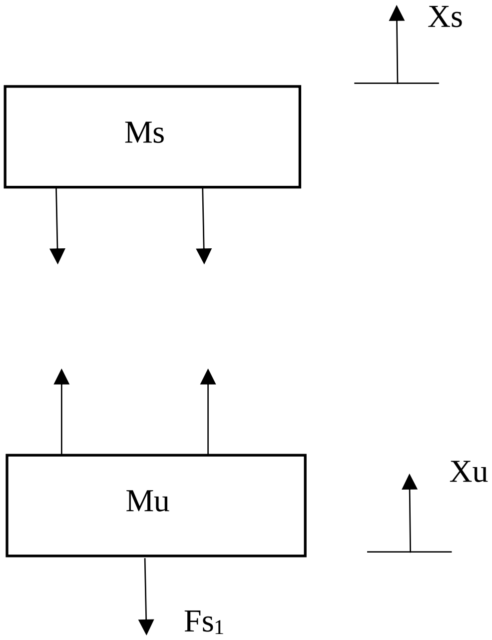

Figure 2: Free body diagram of the quarter car model

$F_d = C_s[\dot{X}_s - \dot{X}_u]$

$F_{s2} = K_s[X_s - X_u]$

$F_{s1} = K_t[X_u - X_r]$

For the sprung mass F=ma;

$F = -F_d - F_{s2} = M_s\ddot{X}_s$

$$M_s\ddot{X}_s + C_s[\dot{X}_s - \dot{X}_u] + K_s[X_s - X_u] = 0 \quad (2.1)$$

$$\ddot{X}_s = \frac{1}{M_s}[C_s[\dot{X}_u - \dot{X}_s] + K_s[X_u - X_s]]$$

For the unsprung mass F=ma;

$F = F_d - F_{s1} = M_u\ddot{X}_u$

$$M_u\ddot{X}_u - K_t[X_r - X_u] - K_s[X_s - X_u] - C_s[\dot{X}_s - \dot{X}_u] = 0 \quad (2.2)$$

$$\ddot{X}_u = \frac{1}{M_u}[K_t[X_r - X_u] + K_s[X_s - X_u] + C_s[\dot{X}_s - \dot{X}_u]]$$

First method that used to solve the quarter car model is state space model.

General form of the spate space model is

$\dot{X} = Ax + Bu$

$\dot{Y} = Cx + Du$

As we take x_1 = Xs, x_2 = Xu, x_3 = $\dot{X}_s$, x_4 =$\dot{X}_u$;

The state space model of the quarter car is:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -Ks/Ms & Ks/Ms & -Cs/Ms & Cs/Ms \\ -Ks/Ms & -(Ks+Kt)/Mu & Cs/Mu & -Cs/Mu \end{bmatrix} \quad [6] \qquad (2.3)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Kt/Ms \end{bmatrix} \qquad (2.4)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \qquad (2.5)$$

$$D = [0] \qquad (2.6)$$

Parameters that used for the quarter car model is given in Table 1.

Parameters	Values	Unit
Sprung mass $\boldsymbol{m_s}$	3000	[kg]
Unsprung mass $\boldsymbol{m_u}$	60	[kg]
Suspension stiffness $\boldsymbol{k_s}$	25000	[N/m]
Tire stiffness $\boldsymbol{k_t}$	$20 \cdot 10^4$	[N/m]
Suspension damping $\boldsymbol{c_s}$	1000	[N/ms]

Table 1. Parameters for Quarter Car Model [5],[7]

Quarter car model is solved analytically by matlab. The excitation from the road selected as step type with the amplitude of 0.2.

The other way that use is MATLAB Simulink blocks. The diagram created with respect to equations (2.1) and (2.2). By doing this way we can validate our values.

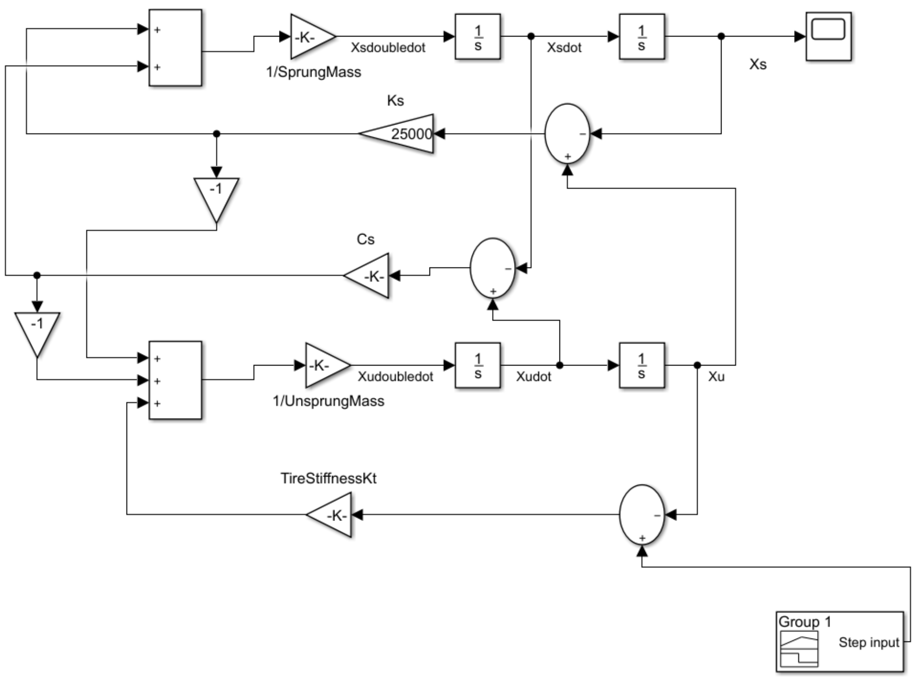

Figure 3: Simulink block diagram of the quarter car model

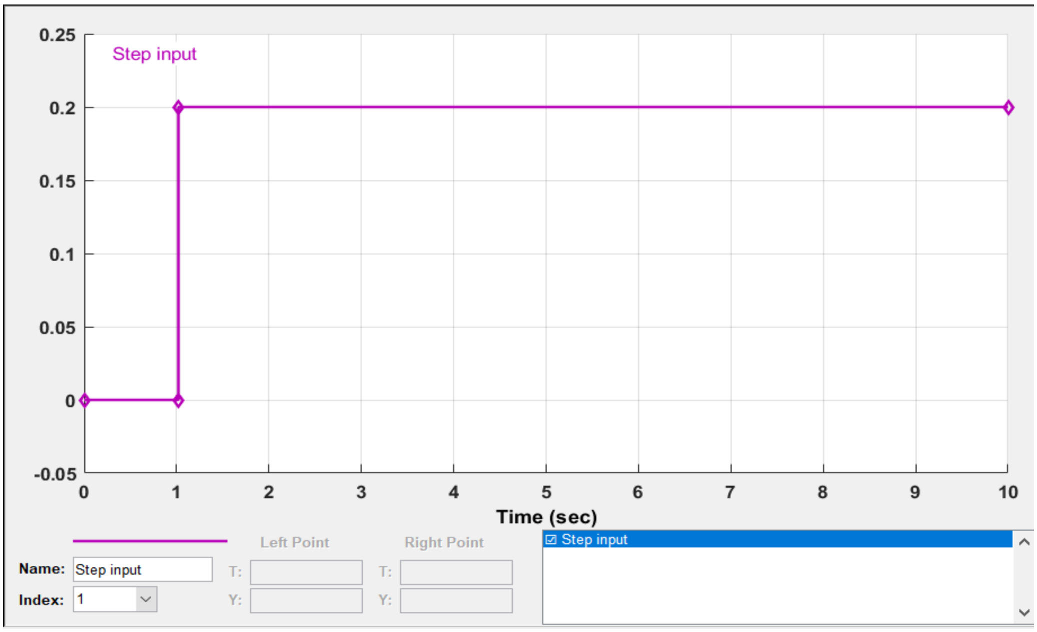

Figure 4: Step input that used with 0.2 amplitude in the Signal Builder

2.2.Full Car Model 7-DOF

This model is a general vibrating model of a vehicle [1].It is occurred from adding link between the sprung mass to four unsprung masses.They are front right, front left, rear right and rear left. In this model from the unsprung masses there are 4 DOF and additional rolling, pitching and bouncing moves are occur different that quarter car model.Therefore in total the model has a 7 DOF.

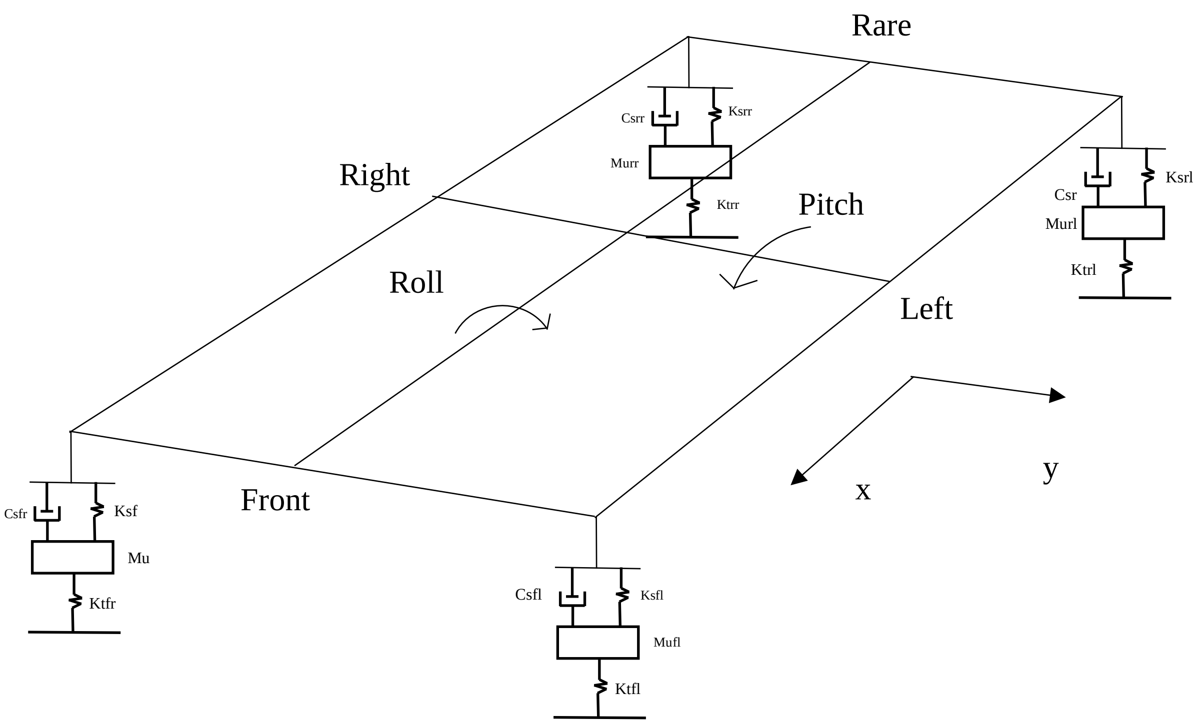

Figure 5: Full car model 7-DOF

From the the figure:

M_s: Mass of the vehicle body.

M_{ufr}: Front right unsprung mass.

M_{ufl}: Front left unsprung mass.

M_{urr}: Rear right unsprung mass.

M_{url}: Rear left unsprung mass.

$\emptyset$: Roll angle at the center of gravity.

θ: Pitch angle at the center of gravity.

a,b: distance from the center of gravity to front and rear axle respectively.

c,d: distance from the center of gravity to left and right wheels respectively.

I_{xx}: Moment of inertia about X-X axis.

I_{yy}: Moment of inertia about Y-Y axis.

K_{sfr}: Spring stiffness of the front right suspension.

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K_{srl}: Spring stiffness of the rear left suspension.

C_{sfr}: Damping coefficient of front right tyre.

C_{sfl}: Damping coefficient of front left tyre.

C_{srr}: Damping coefficient of rear right tyre.

C_{srl}: Damping coefficient of rear left tyre.

K_{tfr}: Tyre stiffness of the front right tyre.

K_{tfl}: Tyre stiffness of the front left tyre.

K_{trr}: Tyre stiffness of the rear right tyre.

K_{trl}: Tyre stiffness of the rear left tyre.

X_{cg}: Displacement of center of gravity of the vehicle body.

X_{frd}: Displacement of the front right tyre.

X_{fld}:Displacement of the front left tyre.

X_{rrd}: Displacement of the rear right tyre.

X_{rld}: Displacement of the rear left tyre.

X_{rfr}: Road input through the front right tyre.

X_{rfl}: Road input through the front left tyre.

X_{rrr}: Road input through the rear right tyre.

X_{rrl}: Road input through the rear left tyre.

In this thesis, because of that we cannot see the pitch and roll motions in the quarter car model. We only focus on bounce effect in the car body. However there will be given all the equations of motions. We will see the bounce effect in the full car model and then we could compare with the quarter car model.

2.2.1.Mathematical Modelling of the Full Car Model

We are going to make 2 modelling just like the quarter car model to see the results. That way it is better for validation.First method that used for modelling is the state space equations and the second one is MATLAB Simulink blocks.

Sprung mass of vehicle body bounce is

$$\begin{aligned}M_s\ddot{X}_{cg} = {} & (Ksfr - Ksfl - Ksrr - Ksrl)X_{cg} + (-Csfr - Csfl - Csrr - Csrl)\dot{X}_{cg} \\ & + \left(K_{sfr}a + K_{sfl}a - K_{srr}b - K_{srl}b\right)\theta \\ & + \left(C_{sfr}a + C_{sfl}a - C_{srr}b - C_{srl}b\right)\dot{\theta} \\ & + \left(K_{sfr}c - K_{sfl}d - K_{srr}c + K_{srl}d\right)\emptyset \\ & + \left(C_{sfr}c - C_{sfl}d + C_{srr}c - C_{srl}d\right)\dot{\emptyset} + K_{sfr}X_{frd} + K_{sfl}X_{fld} + K_{srr}X_{rrd} \\ & + K_{srl}X_{rld} + C_{sfr}\dot{X}_{frd} + C_{sfl}\dot{X}_{fld} + C_{srr}\dot{X}_{rrd} + C_{srl}\dot{X}_{rld} \qquad (2.7\ )\end{aligned}$$

Body pitching motions is

$$\begin{aligned}I_{yy}\ddot{\theta} = {} & \left(K_{sfr}a + K_{sfl}a - K_{srr}b - K_{srl}b\right)X_{cg} + \left(C_{sfr}a + C_{sfl}a - C_{srr}b - C_{srl}b\right)\dot{X}_{cg} \\ & + \left(-K_{sfr}a^2 - K_{sfl}a^2 - K_{srr}b^2 - K_{srl}b^2\right)\theta \\ & + \left(-C_{sfr}a^2 - C_{sfl}a^2 - C_{srr}b^2 - C_{srl}b^2\right)\dot{\theta} \\ & + \left(-K_{sfr}ac + K_{sfl}ad + K_{srr}bc - K_{srl}bd\right)\emptyset \\ & + \left(-C_{sfr}ac + C_{sfl}ad + C_{srr}bc - C_{srl}bd\right)\dot{\emptyset} + \left(-K_{sfr}a\right)X_{frd} \\ & + \left(-K_{sfl}a\right)X_{fld} + (K_{srr}b)X_{rrd} + (K_{srl}b)X_{rld} + \left(-C_{sfr}a\right)\dot{X}_{frd} \\ & + \left(-C_{sfl}a\right)\dot{X}_{fld} + (C_{srr}b)\dot{X}_{rrd} + (C_{srl}b)\dot{X}_{rld} \qquad (2.8)\end{aligned}$$

Body rolling motions is

$$\begin{aligned}I_{xx}\ddot{\emptyset} = {} & (K_{sfr}c - K_{sfl}d + K_{srr}c - K_{srl}d)X_{cg} + (C_{sfr}c - C_{sfl}d + C_{srr}c - C_{srl}d)\,\dot{X}_{cg} \\ & + (-K_{sfr}ac + K_{sfl}ad + K_{srr}bc - K_{srl}bd)\theta + (-C_{sfr}ac + C_{sfl}ad \\ & + C_{srr}bc - C_{srl}bd)\dot{\theta} + (-K_{sfr}c^2 - K_{sfl}d^2 - K_{srr}c^2 - K_{srl}d^2)\emptyset \\ & + (-C_{sfr}c^2 - C_{sfl}d^2 - C_{srr}c^2 - C_{srl}d^2)\dot{\emptyset} + \left(-K_{sfr}c\right)X_{frd} \\ & + \left(K_{sfl}d\right)X_{fld} + (-K_{srr}c)X_{rrd} + (K_{srl}d)X_{rld} + \left(-C_{sfr}c\right)\dot{X}_{frd} \\ & + \left(C_{sfl}d\right)\dot{X}_{fld} + (-C_{srr}c)\dot{X}_{rrd} + (C_{srl}d)\dot{X}_{rld} \qquad (2.9)\end{aligned}$$

Unsprung mass front right is

$$M_{ufr}\ddot{X}_{frd} = K_{sfr}(X_{cg} - \theta a - \emptyset c - X_{frd}) - K_{tfr}(X_{frd} - X_{rfr}) + C_{sfr}(\dot{X}_{cg} - \dot{\theta} a - \dot{\emptyset} c - \dot{X}_{frd}) \quad (2.10)$$

Unsprung mass front left is

$$M_{ufl}\ddot{X}_{fld} = K_{sfl}(X_{cg} - \theta a - \emptyset d - X_{fld}) - K_{tfl}(X_{fld} - X_{rfl}) + C_{sfl}(\dot{X}_{cg} - \dot{\theta} a - \dot{\emptyset} d - \dot{X}_{fld}) \quad (2.11)$$

Unsprung mass rear right is

$$M_{urr}\ddot{X}_{rrd} = K_{srr}(X_{cg} - \theta b - \emptyset c - X_{rrd}) - K_{trr}(X_{rrd} - X_{rrr}) + C_{srr}(\dot{X}_{cg} - \dot{\theta} b - \dot{\emptyset} c - \dot{X}_{rrd}) \quad (2.12)$$

Unsprung mass rear left is

$$M_{url}\ddot{X}_{rld} = K_{srl}(X_{cg} + \theta b + \emptyset d - X_{rld}) - K_{trl}(X_{rld} - X_{rrl}) + C_{srl}(\dot{X}_{cg} + \dot{\theta} b + \dot{\emptyset} c - \dot{X}_{rld}) \quad (2.13)$$

To solve this equations it should be put in a state space form. Therefore general form of the state space equations is

$$\dot{X} = Ax + Bu$$

$$Y = Cx + Du$$

As we take $X_{cg}=X_1$, $\dot{X}_{cg} = X_2$, $\theta = X_3$, $\dot{\theta}=X_4$, $\emptyset=X_5$, $\dot{\emptyset}=X_6$, $X_{frd}=X_7$, $\dot{X}_{frd}=X_8$, $X_{fld}=X_9$, $\dot{X}_{fld}=X_{10}$, $X_{rrd}=X_{11}$, $\dot{X}_{rrd}=X_{12}$, $X_{rld}=X_{13}$, $\dot{X}_{rld}=X_{14}$

Matrix forms are

$$[A] = [A1 \quad A2 \quad A3 \quad A4 \quad A5 \quad A6 \quad A7 \quad A8 \quad A9 \quad A10 \quad A11 \quad A12 \quad A13 \quad A14] \quad (2.14)$$

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{Ktfr}{Mufr} & 0 & \frac{Ktfl}{Mufl} & 0 & \frac{Ktrr}{Murr} & 0 & \frac{Ktrl}{Murl} \end{bmatrix}^T \quad (2.15)$$

$$[U] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad Xrfr \quad 0 \quad Xrfl \quad 0 \quad Xrrr \quad 0 \quad Xrrl]^T \quad (2.16)$$

Output Matix [C];

For X_{cg}:

$$[C] = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (2.17)$$

For θ:

$$[C] = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

For $\emptyset$:

$$[C] = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

In this thesis we analyze the bounce effect there for in the calculations Xcg [C] matrix is taken.

$$[D] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.18}$$

$$[A1] = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.19}$$

$$[A3] = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.20}$$

$$[A5] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.21}$$

$$[A7] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.22}$$

$$[A9] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{2.23}$$

$$[A11] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \tag{2.24}$$

$$[A13] = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] \tag{2.25}$$

$$
[A2] = \begin{bmatrix}
\dfrac{(-Ksfr - Ksfl - Ksrr - Ksrl}{Ms} \\
\dfrac{(-Csfr - Csfl - Csrr - Csrl)}{Ms} \\
\dfrac{\left(Ksfr(a) + Ksfl(a) - Ksrr(b) - Ksrl(b)\right)}{Ms} \\
\dfrac{\left(Csfr(a) + Csfl(a) - Csrr(b) - Csrl(b)\right)}{Ms} \\
\dfrac{\left(Ksfr(c) - Ksfl(d) + Ksrr(c) - Ksrl(d)\right)}{Ms} \\
\dfrac{Csfr(c) - Csfl(d) + Csrr(c) - Csrl(d)}{Ms} \\
\dfrac{Ksfr}{Ms} \\
\dfrac{Csfr}{Ms} \\
\dfrac{Ksfl}{Ms} \\
\dfrac{Csfl}{Ms} \\
\dfrac{Ksrr}{Ms} \\
\dfrac{Csrr}{Ms} \\
\dfrac{Ksrl}{Ms} \\
\dfrac{Csrl}{Ms}
\end{bmatrix}^T \tag{2.26}
$$

$$[A4] = \begin{bmatrix} \dfrac{Ksfr(a) + Ksfl(a) - Ksrr(b) - Ksrl(b)}{Iyy} \\ \dfrac{Csfr(a) + Csfl(a) - Csrr(b) - Csrl(b)}{Iyy} \\ \dfrac{(-Ksfr(a^2)) - Ksfl(a^2) - Ksrr(b^2) - Ksrl(b^2)}{Iyy} \\ \dfrac{(-Csfr(a^2)) - Csfl(a^2) - Csrr(b^2) - Csrl(b^2)}{Iyy} \\ \dfrac{(-Ksfr(ac)) + Ksfl(ad) + Ksrr(bc) - Ksrl(bd)}{Iyy} \\ \dfrac{(-Csfr(ac)) - Csfl(ad) - Csrr(bc) - Csrl(bd)}{Iyy} \\ \dfrac{-Ksfr(a)}{Iyy} \\ \dfrac{-Csfr(a)}{Iyy} \\ \dfrac{-Ksfl(a)}{Iyy} \\ \dfrac{-Csfl(a)}{Iyy} \\ \dfrac{Ksrr(b)}{Iyy} \\ \dfrac{Csrr(b)}{Iyy} \\ \dfrac{Ksrl(b)}{Iyy} \\ \dfrac{Csrl(b)}{Iyy} \end{bmatrix}^T \tag{2.27}$$

$$[A6] = \begin{bmatrix} \dfrac{Ksfr(c) - Ksfl(d) + Ksrr(c) - Ksrl(d)}{Ixx} \\ \dfrac{Csfr(c) - Csfl(d) + Csrr(c) - Csrl(d)}{Ixx} \\ \dfrac{(-Ksfr(ac)) + Ksfl(ad) + Ksrr(bc) - Ksrl(bd)}{Ixx} \\ \dfrac{(-Csfr(ac)) - Csfl(ad) - Csrr(bc) - Csrl(bd)}{Ixx} \\ \dfrac{(-Ksfr(c^2)) - Ksfl(d^2) - Ksrr(c^2) - Ksrl(d^2)}{Ixx} \\ \dfrac{(-Csfr(c^2)) - Csfl(d^2) - Csrr(c^2) - Csrl(d^2)}{Ixx} \\ \dfrac{Ksfr(c)}{Ixx} \\ \dfrac{Csfr(c)}{Ixx} \\ \dfrac{Ksfl(d)}{Ixx} \\ \dfrac{Csfl(d)}{Ixx} \\ \dfrac{-Ksrr(c)}{Ixx} \\ \dfrac{-Csrr(c)}{Ixx} \\ \dfrac{Ksrl(d)}{Ixx} \\ \dfrac{Csrl(d)}{Ixx} \end{bmatrix}^T \tag{2.28}$$

$$[A8] = \begin{bmatrix} \frac{Ksfr}{Mufr} \\ \frac{Csfr}{Mufr} \\ \frac{-Ksfr(a)}{Mufr} \\ \frac{-Csfr(a)}{Mufr} \\ \frac{-Ksfr(c)}{Mufr} \\ \frac{-Csfr(c)}{Mufr} \\ \frac{-Ksfr-Ktfr}{Mufr} \\ \frac{-Csfr}{Mufr} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad [A12] = \begin{bmatrix} \frac{Ksrr}{Murr} \\ \frac{Csrr}{Murr} \\ \frac{Ksrr(b)}{Murr} \\ \frac{Csrr(b)}{Murr} \\ \frac{-Ksrr(c)}{Murr} \\ \frac{-Csrr(c)}{Murr} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-Ksrr-Ktrr}{Murr} \\ \frac{-Csrr}{Murr} \\ 0 \\ 0 \end{bmatrix}^T \qquad (2.29),(2.31)$$

$$[A10] = \begin{bmatrix} \frac{Ksfl}{Mufl} \\ \frac{Csfl}{Mufl} \\ \frac{-Ksfl(a)}{Mulr} \\ \frac{-Csfl(a)}{Mufl} \\ \frac{Ksfl(d)}{Mufl} \\ \frac{Csfl(d)}{Mufl} \\ 0 \\ 0 \\ \frac{-Ksfl-Ktfl}{Mufl} \\ \frac{-Csfl}{Mufl} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad [A14] = \begin{bmatrix} \frac{Ksrl}{Murl} \\ \frac{Csrl}{Murl} \\ \frac{Ksrl(b)}{Murl} \\ \frac{Csrl(b)}{Murl} \\ \frac{Ksrl(d)}{Murl} \\ \frac{Csrl(d)}{Murl} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{-Ksrl-Ktrl}{Murl} \\ \frac{-Csrl}{Murl} \end{bmatrix}^T \ [7] \qquad (2.30),(2,32)$$

As a road excitation it is given 0.2 step input same as quarter car model and therefore this state space model is solved analytically by matlab.

Parameters for the full car model.

Parameters	Values	Unit
Mass of the chassis	$M_s = 1200$	[kg]
Mass of the front tire	$M_{uf} = 60$	[kg]
Mass of the rear tire	$M_{ur} = 60$	[kg]
Roll moment of inertia	$I_{xx} = 4000$	[kg/m^2]
Pitch moment of inertia	$I_{yy} = 950$	[kg/m^2]
Distance of gravity centre from front axle	$a = 1.5$	[m]
Distance of gravity centre from rear axle	$b = 1.5$	[m]
Distance of gravity centre from right axle	$d = 1$	[m]
Distance of gravity centre from left axle	$c = 1$	[m]
Stiffness of the front suspension	$k_f = 25000$	[N/m]
Stiffness of the rear suspension	$k_r = 25000$	[N/m]
Stiffness of the tire	$k_{tf} = k_{tr} = 20 \cdot 10^4$	[N/m]
Damping coefficient	$c_f = c_r = 1000$	[N/ms]

Table 2: Parameters for full car model [5],[7]

```
%Matlab code of full car state space model
Ms=1200;
Mufr=60;
Mufl=60;
Murr=60;
Murl=60;
Ixx=4000;
Iyy=950;
a=1.5;
b=1.5;
c=1;
d=1;
Ktfr=200000;
Ktfl=200000;
Ktrr=200000;
Ktrl=200000;
Csfr=1000;
Csfl=1000;
Csrr=1000;
Csrl=1000;
Ksfr=25000;
Ksfl=25000;
Ksrr=25000;
Ksrl=25000;
B1 = [0 0 0 0 0 0 0 Ktfr/Mufr 0 Ktfl/Mufl 0 Ktrr/Murr 0 Ktrl/Murl];
B = transpose(B1);
C = [1 0 0 0 0 0 0 0 0 0 0 0 0 0];
A1= [0 1 0 0 0 0 0 0 0 0 0 0 0 0];
D = 0;
A3= [0 0 0 1 0 0 0 0 0 0 0 0 0 0];
A5= [0 0 0 0 0 1 0 0 0 0 0 0 0 0];
A7= [0 0 0 0 0 0 0 1 0 0 0 0 0 0];
A9= [0 0 0 0 0 0 0 0 0 1 0 0 0 0];
A11= [0 0 0 0 0 0 0 0 0 0 0 1 0 0];
A13= [0 1 0 0 0 0 0 0 0 0 0 0 0 1];
A2a=[(-Ksfr-Ksfl-Ksrr-Ksrl)/Ms;
   (-Csfr-Csfl-Csrr-Csrl)/Ms;
   (Ksfr*a+Ksfl*a-Ksrr*b-Ksrl*b)/Ms;
   (Csfr*a+Csfl*a-Csrr*b-Csrl*b)/Ms;
   (Ksfr*c-Ksfl*d+Ksrr*c-Ksrl*d)/Ms;
   (Csfr*c-Csfl*d+Csrr*c-Csrl*d)/Ms;
   Ksfr/Ms;Csfr/Ms;Ksfl/Ms;Csfl/Ms;
   Ksrr/Ms;Csrr/Ms;Ksrl/Ms;Csrl/Ms;];
A2=transpose(A2a);
```

```
A4a=[(Ksfr*a+Ksfl*a-Ksrr*b-Ksrl*b)/Iyy;
   (Csfr*a+Csfl*a-Csrr*b-Csrl*b)/Iyy;
   (-Ksfr*a^2-Ksfl*a^2-Ksrr*b^2-Ksrl*b^2)/Iyy;
   (-Csfr*a^2-Csfl*a^2-Csrr*b^2-Csrl*b^2)/Iyy;
   (-Ksfr*a*c+Ksfl*a*d+Ksrr*b*c-Ksrl*b*d)/Iyy;
   (-Csfr*a*c-Csfl*a*d-Csrr*b*c-Csrl*b*d)/Iyy;
   (-Ksfr*a)/Iyy;(-Csfr*a)/Iyy;(-Ksfl*a)/Iyy;
   (-Csfl*a)/Iyy;(Ksrr*b)/Iyy;(Csrr*b)/Iyy;
   (Ksrl*b)/Iyy;(Csrl*b)/Iyy];
A4=transpose(A4a);
A6a=[(Ksfr*c-Ksfl*d+Ksrr*c-Ksrl*d)/Ixx;
   (Csfr*c-Csfl*d+Csrr*c-Csrl*d)/Ixx;
   (-Ksfr*a*c+Ksfl*a*d+Ksrr*b*c-Ksrl*b*d)/Ixx;
   (-Csfr*a*c-Csfl*a*d-Csrr*b*c-Csrl*b*d)/Ixx;
   (-Ksfr*c^2-Ksfl*d^2-Ksrr*c^2-Ksrl*d^2)/Ixx;
   (-Csfr*c^2-Csfl*d^2-Csrr*c^2-Csrl*d^2)/Ixx;
   (Ksfr*c)/Ixx;(Csfr*c)/Ixx;(Ksfl*d)/Ixx;
   (Csfl*d)/Ixx;(-Ksrr*c)/Ixx;(-Csrr*c)/Ixx;
   (Ksrl*d)/Ixx;(Csrl*d)/Ixx];
A6=transpose(A6a);
A8a=[Ksfr/Mufr;Csfr/Mufr;(-Ksfr*a)/Mufr;
   (-Csfr*a)/Mufr;(-Ksfr*c)/Mufr;(-Csfr*c)/Mufr;
   (-Ksfr-Ktfr)/Mufr;(-Csfr)/Mufr;0;0;0;0;0;0];
A8=transpose(A8a);
A10a=[Ksfl/Mufl;Csfl/Mufl;(-Ksfl*a)/Mufl;
   (-Csfl*a)/Mufl;(Ksfl*d)/Mufl;(Csfl*d)/Mufl;
   0;0;(-Ksfl-Ktfl)/Mufl;(-Csfl)/Mufl;0;0;0;0];
A10=transpose(A10a);
A12a=[Ksrr/Murr;Csrr/Murr;(Ksrr*b)/Murr;(Csrr*b)/Murr;
   (-Ksrr*c)/Murr;(-Csrr*c)/Murr;0;0;0;0;(-Ksrr-Ktrr)/Murr;
   (-Csrr)/Murr;0;0];
A12=transpose(A12a);
A14a=[Ksrl/Murl;Csrl/Murl;(Ksrl*b)/Murl;
   (Csrl*b)/Murl;(Ksrl*d)/Murl;(Csrl*d)/Murl;
   0;0;0;0;0;0;(-Ksrl-Ktrl)/Murl;(-Csrl)/Murl];
A14=transpose(A14a);
A = [A1; A2; A3; A4; A5; A6; A7; A8; A9; A10; A11; A12; A13; A14];

sys=ss(A,B,C,D)
opt = stepDataOptions('StepAmplitude',0.2);
step(sys,opt,10)
```

The other method used is MATLAB simulink block for the given equation.The Full car model that is obtained is based on the quartercar model. It occurs from 4 quarter car models linked each other as a single model.

Simulink diagram of the full car model is

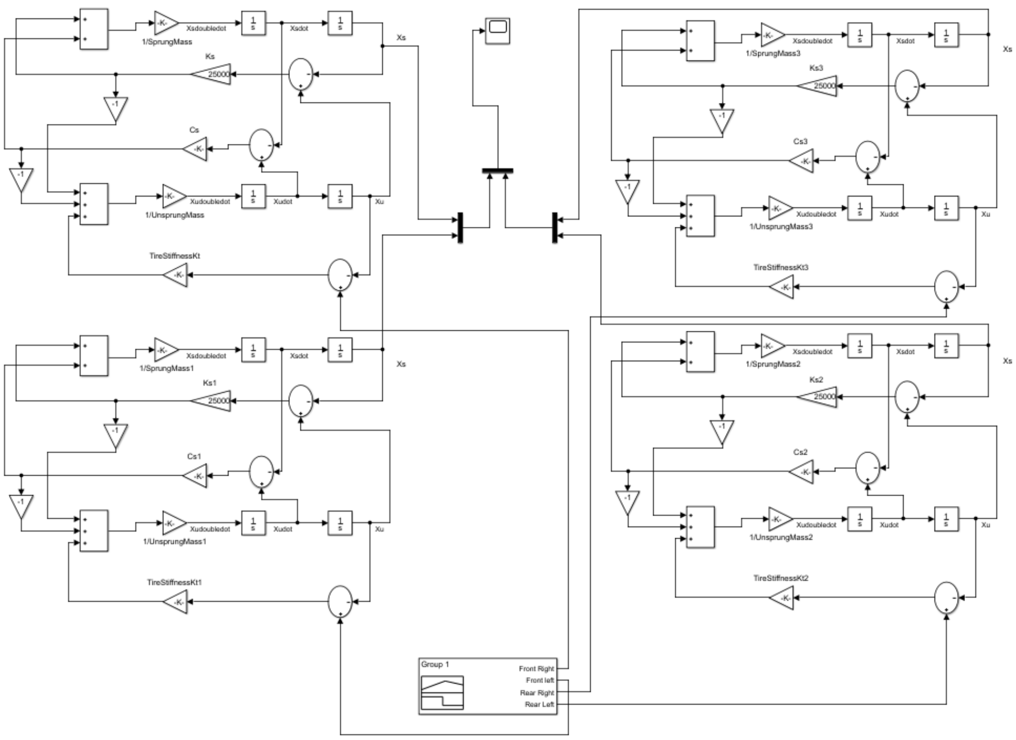

Figure 6: Simulink diagram of the full car model

Input for the full car model is in the figure 8. Step input hits the rear tyres after 1 seconds.

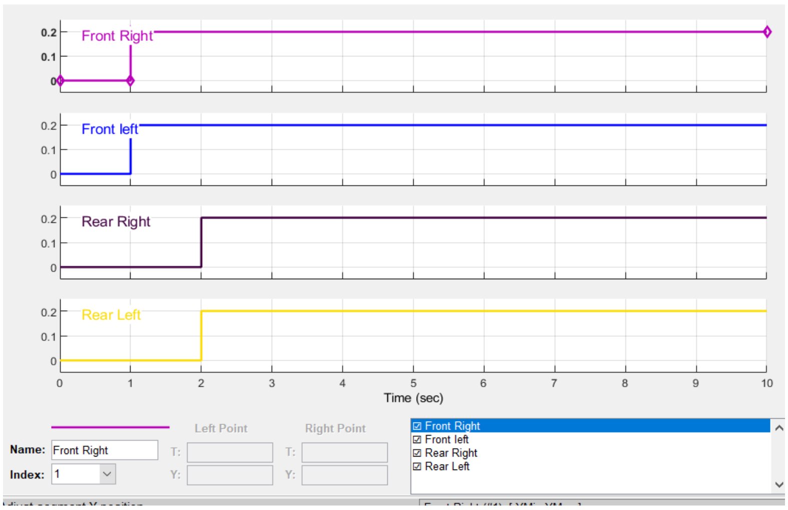

Figure 7:Input for the full car model in signal builder block.

3.RESULTS OF THE FULL CAR AND QUARTER CAR MODELS

3.1.Result of the Quarter Car model

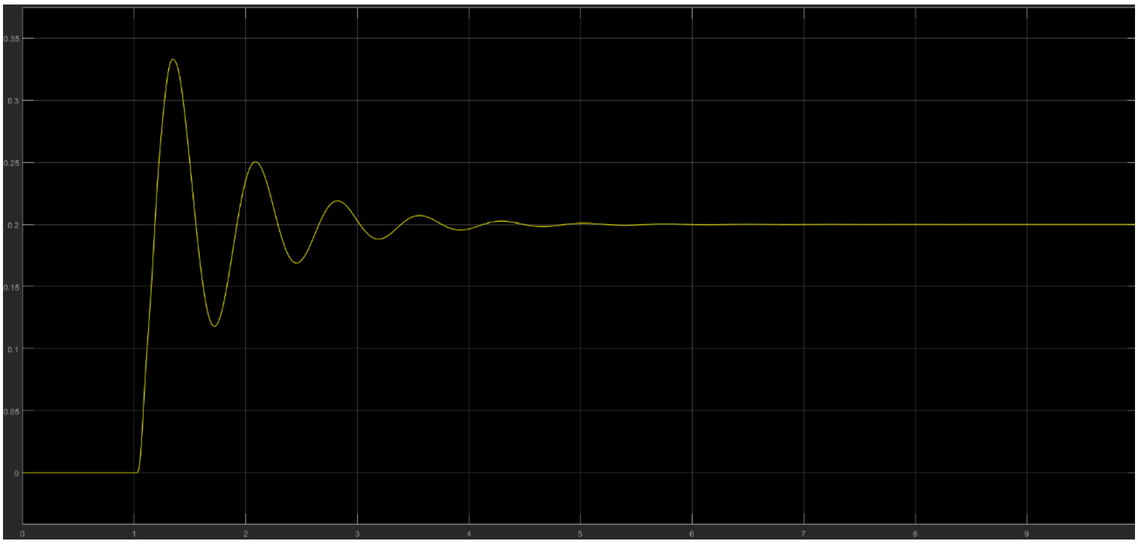

Figure 8: Plot of quarter car model in Simulink model

The figure 8 shows that sprung mass displacement with respect to time. The input excitation was given 0.2 amplitude of step input. It reaches the peak point which is 0.33m in 0.3 seconds and Settling occurs at nearly 3 seconds

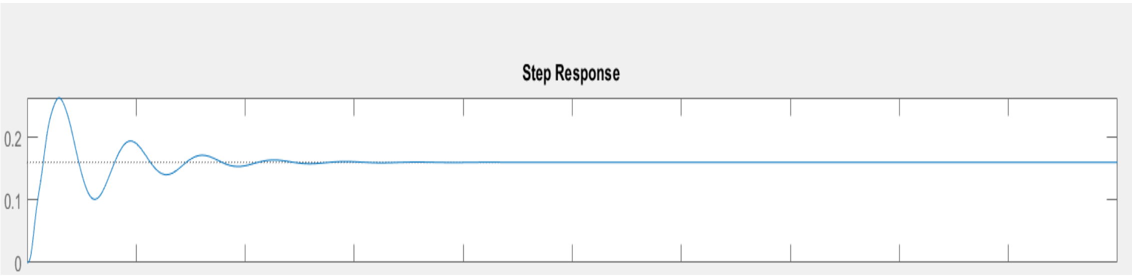

Figure 9: Plot of the quarter car model in Matlab state space matrix model

The figure 9 shows the sprung mass displacement with respect to time and as we can see it is similar. The peak point is 0.26m and it reaches in 0.3 seconds and settling occurs at 2.46 seconds.

As we can see from the both graphs both of the displacements are settled after some amount of time. It supports the theories.

3.2.Results of the Full Car Model

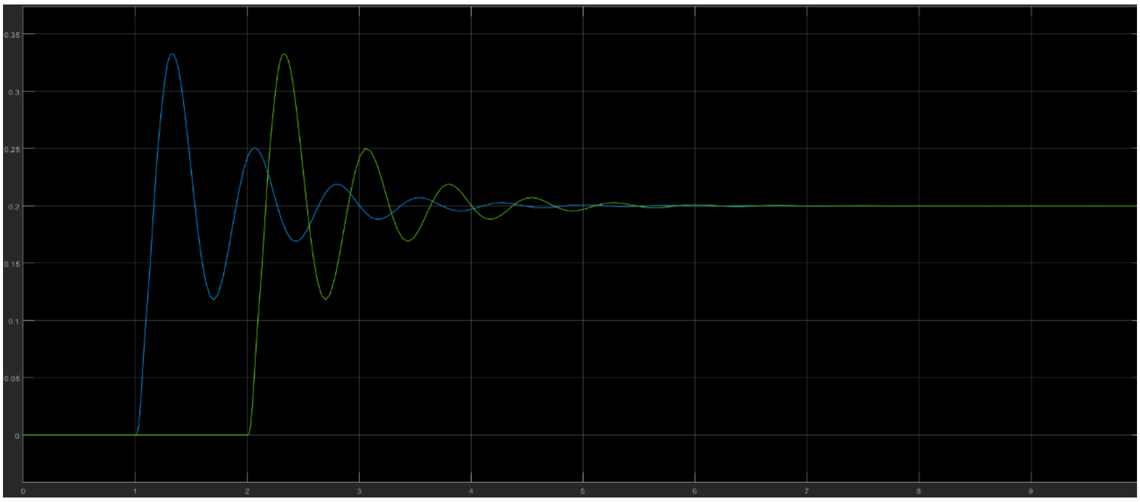

Figure 10: Plot of full car model in Simulink model

Figure 10 shows the front tyres(blue) and rear tyres(green) took a step input with the amplitude of 0.2.

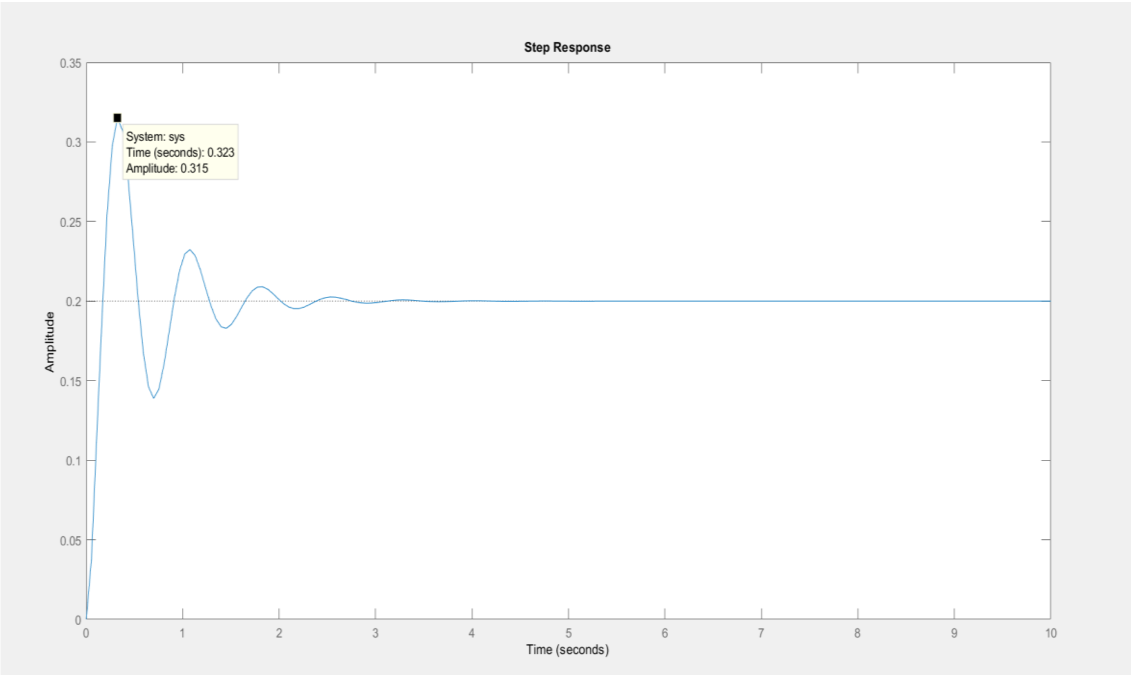

Figure 11: Plot of the quarter car model in Matlab state space matrix model

Figure 11 shows that sprung mass displacement of the car body(bounce motion).The design of the full car model based on the quarter car model. Therefore all(front right-left and rear right-left) the damping coefficients, tyre stiffnesses, spring coefficients and unsprung masses are identical and they used in the quarter car model before.

As we see from the figure 11 peak value is therefore displacement of the sprung mass is equal to 0.315m and the it is reached that value in 0.323 seconds. When we consider the state space models and Simulink models of the quarter car and full car models we can say that there is a similarity between them. That is because this full car model parameters are same in the quarter car parameters and in road excitation as a step input with 0.2 amplitude was same in both.

4.CONCLUSION

In this thesis modelling and the simulating of the passive suspension system in a quarter car model and full car model is presented. Scope of the thesis was to design a 7-DOF full car model with based on quarter car model with 2-DOF.As the result of the modelling and simulating parts, there is a similarity in the peak value, settling times between the two model. Moreover in this thesis we only analyzed bounce effect of the full car model, that is because we cannot see the pitching and rolling motions in the quarter car model. We can say that it is possible to design a full car model which is based on quarter car model. However, to get better results the pitching motion and the rolling motion of the car body should be analyzed while designing a full car model of passive suspension system.

5.REFERENCES

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