



MARMARA UNIVERSITY  
FACULTY OF ENGINEERING



**VEHICLE SEAT SUSPENSION SYSTEM: MODELING  
and ANALYSIS**

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Fevzi Doğaner

**GRADUATION PROJECT REPORT**

Department of Mechanical Engineering

Supervisor  
Assoc. Prof. Dr. Mehmet Berke GÜR

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ISTANBUL, 2024



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**Vehicle Seat Suspension System: Modeling and Analysis**

by

**Fevzi Doğaner**

**June 2024, Istanbul**

**SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN  
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**June 2024**    Fevzi Doğaner

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## **ABSTRACT**

### **Vehicle Seat Suspension System: Modeling and Analysis**

This thesis focuses on the modeling, analysis and improvement of vehicle seat suspension systems. The objective is to develop a mathematical modeling equation incorporating physical engineering systems and real-world scenarios, and to assess the system response using numerical simulations and analytical methods. And comparison between system responses to different parameters and inputs are included in the thesis. MATLAB/Simulink has been utilized simulating the model.

This thesis is limited by mathematical modeling equations, simulations and certain defined inputs, limiting its scope to the parameters and conditions defined within these simulations.

## **SYMBOLS**

$\Delta$ : Delta

$\Sigma$ : Summation

$\omega$ : Frequency

$\phi$ : Phase Angle

$\zeta$ : Damping ratio

$m_1$ : Mass of seat

$m_2$ : Mass of deriver

$b_1$ : Viscous damping coefficient of suspension damper

$b_2$ : Viscous damping coefficient of seat cushion

$k_1$ : Stiffness of the suspension spring

$k_2$ : Stiffness of the seat cushion

$z_0$ : Vehicle's floor displacement

$z_1$ : Seat displacement

$z_2$ : Driver displacement

## **ABBREVIATION**

FBD: Free Body Diagram

ODE: Ordinary Differential Equations

SSR: State-Space Representation

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## **1. INTRODUCTION**

Passengers and drivers of all vehicles experience vibrations due to road irregularities, which drop off ride comfort and adversely affect the vehicles' road-holding capacity. Therefore, vehicle suspension systems are designed to ensure adequate ride comfort and road-holding capacity (Ufuk Demircioğlu, 2023).

To begin with, we chose an agricultural tractor as the vehicle in order to model and analyze its seat suspension system. The agricultural tractor operators are exposed to unwanted vibrations while the tractor is operating, which disturb (makes them feel uncomfortable) them and negatively affect their health. (Prasad et al., 1995; Park et al., 2013; Cvetanovic and Zlatkovic, 2013; Cutini et al., 2016; Scarlett et al., 2007). Tractors have their own unique geometrical characteristics that differs them any other vehicles or cars and their seats are placed in a high position, that means the operators sitting in a high position. Therefore, even small motions create big motion effects on the operators (Crosby and Allen, 1974). There are so many researchers had works on tractors with the aim of understanding the vibrations of them (Prasad et al., 1995; Ahmed and Goupillon, 1997; Tewari and Prasad, 1999).

There are 3 different suspension systems has been established for tractors and they are as follows: (1) Seat suspension, (2) cab suspension and (3) axle suspension (Kyuhyun Sim, Hwayoung Lee, Ji Won Yoon, Chanho Choi, Sung-Ho Hwang, 2016).

Since they affect critical things, such as driving dynamics, safety and comfort, suspension systems are essential to investigate and develop.

Although seat suspension systems affect the driver positively, it is difficult to ensure comfort for the driver. The performance of the suspension varies according to the weight on the seat suspension, that is, according to the mass of the driver. And this influences the deflection and natural frequency of the seat suspension (Suggs and Huang, 1969).

The main objective of this project is to perform the mathematical modeling of a seat suspension system with the aim of isolating vibration transmitted from the floor of the vehicle to the driver, simulating this model using MATLAB and Simulink, analysis of the system's responses to different inputs and the developing the system based on these analyses. We will also perform a comparison between system's responses with different parameters.

## **2. SUSPENSIONS**

In this part, we will create a section containing general information about suspension and suspension types.

### **2.1. Suspension Systems**

It is clear that road unevenness produces oscillations which will be transmitted to drivers. The role of the suspension system is to reduce as much vibrations and shocks occurring in the operation (Andronic Florin, Manolache-Rusu Ioan-Cozmin, Pătuleanu Liliana). Suspension system supply a tender ride over rough roads while establishing that the wheels remain in contact with the ground and vehicle roll is minimized. The suspension systems are generally contained three main components: a structure maintaining the vehicle's weight, a spring transforming kinematic energy into potential energy or vice versa; and a shock absorber, a mechanical device to dissipate kinetic energy (Goodarzi A, Khajepour A., 2017).

There are three sorts of vehicle suspension reducing or absorbing the vibration namely, active, semi-active, and passive suspension that various specialists have made utilizing kind of methods and algorithms. Contrasted with the semi-active and active suspension, the passive suspension has no vehicle steadiness. The dynamic behavior of passive suspension systems is determined by choosing the suitable spring stiffness (due to the spring type and its characteristic) and shock absorbers characteristic represented by the damping coefficient (Iyasu JIREGNA, Goftila SIRATA, 2020)

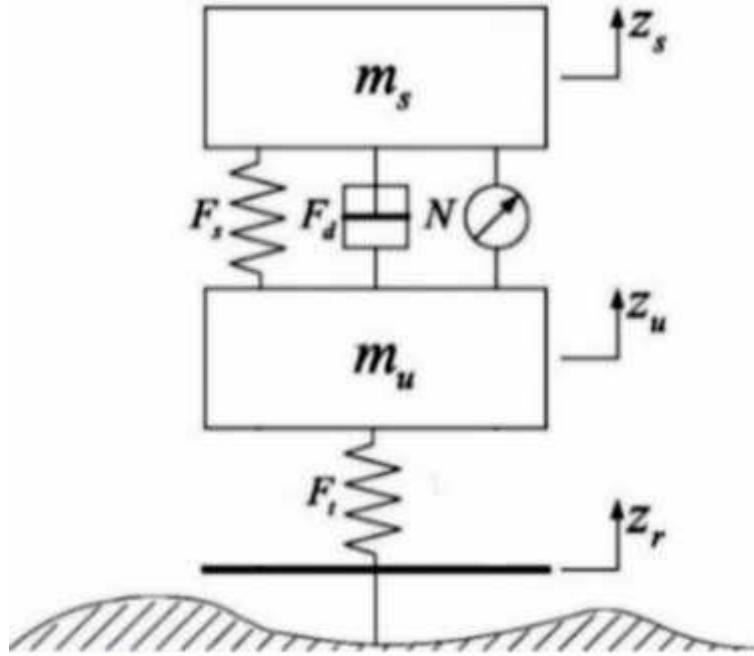
Passive systems are the most common ones, so we have covered a passive-suspension system to work on isolating vibration, which affects driver, a bump caused.

#### **2.1.1. Active-Suspension Systems**

Active suspensions are equipped with electronic control frameworks that control the activity of the suspension components. The active suspension, represented in Figure 2.1, consists of the actuator, the mechanical spring and the shock absorber. They do not have a bounded performance as passive suspensions and create a new progress to remove the issue of the designing a compromise present in passive suspensions.

In active suspension systems, the actuator facilitates the suspension to absorb the wheel's acceleration energy, so minimizing the vehicle body's acceleration. These systems have boosted sensitivity to induced vertical forces arise from unpredictable road distortion, as the actuator force governs both shock absorbers. The actuator functions by either possessing or distributing system power, and its operation can be regulated by various types of

controllers, conditioned on the specific design. Active suspension systems can achieve a sensitive balance between vehicle handling comfort and driving stability, leading to an enhanced suspension design when coupled with appropriate control methods.



*Figure 2.1: Representation of an active-suspension system*

### 2.1.2. Semi-Active Suspension Systems

Semi-active suspension system is almost identical with the passive suspension system. This type of suspension has a spring and governable damper in which the spring element is used to store the energy meanwhile the controllable damper is used to dissipate the energy. Some of the semi-active suspension systems use the passive damper and the controllable spring, the controllable damper usually acts with limited capability to produce a controlled force when dissipating energy.

A semi-active suspension is identical to passive suspension system with only varying damping coefficient and constant spring constant one without active force sources. Hence, the mechanical design of a semi-active suspension is equal to a passive one. Consequently, this gives the possibility of the damper reaction forces. A semi-active suspension can be remotely electrically switched to either soften or stiffen the suspension. Its damping coefficient can be changed continuously or discontinuously (Pramod A. Yadav, Mohsin G. Mulla, Ansar A. Mulla, 2015).

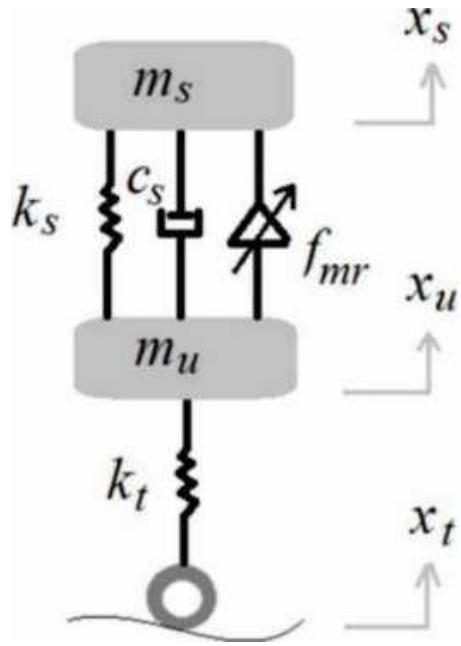


Figure 2.2: Representation of Semi-active suspension system.

### 2.1.3 Passive Suspension Systems

Most seat suspensions employ a passive mechanism to reduce the influence of the vehicle vibration which are created by rough road surface on the driver body. The passive suspension as a conventional suspension system contains a spring with a damping shock absorber to regulate undesired vibrations (Stein G., Ballo I., 1991). The passive suspension system is shown in Figure 2.3 to present the fundamental concepts. A passive control system can absorb structural vibrations and remove energy from the dynamic system without an external energy source. The method is to connect the structure to elements and materials with damping characteristics that overcome vibrations. A passive system is regularly implemented with springs and dampers with time-invariant stiffness and damping coefficients (Nathan, J.F. Tang, T.Q. Ghabra R., 2007).

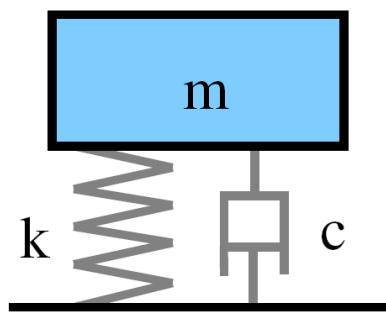


Figure 2.3: Representation of passive suspension system

### 3. DYNAMIC MODEL OF THE PASSIVE SEAT SUSPENSION SYSTEM

This section will include mathematical modeling of the passive seat suspension system, Simulink diagrams and graphs of the system responses.

#### 3.1. Mathematical Modeling Equations

The first step in analysis and design of the dynamics systems is invariably formulating a mathematical modeling equation. Therefore, we will start with developing a mathematical equation of the seat suspension system. (Assumption: Springs and dampers are ideal)

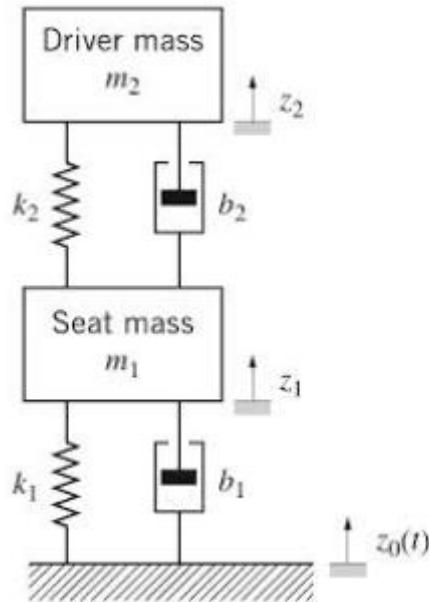


Figure 3.1: Mechanical representation for the system

Here, the seat is connected to vehicle's floor with an ideal spring  $k_1$  and viscous damper  $b_1$ . Mass of the seat and driver are represented with  $m_1$  and  $m_2$  respectively. Spring constant  $k_2$  stands for the stiffness and the viscous friction  $b_2$  stands for damping of the seat cushion. As shown in Figure 3.1,  $z_1$  and  $z_2$  are vertical displacements for seat mass and driver mass in that order and both of them are measured from their static equilibrium and  $z_0(t)$  is vertical displacement of the vehicle's floor.

If we assume the relative displacement,  $z_1 - z_0$  and  $z_1 - z_2$  are positive, then spring  $k_1$  is in tension and spring  $k_2$  is in compression and the both reaction forces according to these displacements are downward for the seat mass  $m_1$ . For the driver mass  $m_2$ , the reaction force compressed spring  $k_2$  caused is upward. To determine the friction forces that viscous

dampers bring; relative velocities are taken into account. Again, if we assume relative velocities  $\dot{z}_1 - \dot{z}_0$  and  $\dot{z}_1 - \dot{z}_2$  are positive, then the reaction forces that viscous damper  $b_1$  and  $b_2$  caused for the seat mass  $m_1$  are downward. For the driver mass  $m_2$ , the reaction force that viscous damper  $b_2$  caused is upward according to the same assumptions. Appropriately to these assumptions and determinations free body diagram (FBD) of the system becomes as shown in Figure 3.2.

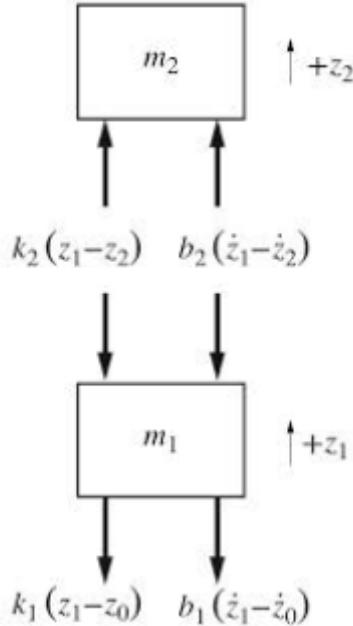


Figure 3.2: FBD of the seat suspension system.

According to all these data, the force equation of the system will be as follows:

For Seat Mass  $m_1$ :

$$+\uparrow \Sigma F: m_1 \ddot{z}_1 = -k_1(z_1 - z_0) - b_1(\dot{z}_1 - \dot{z}_0) - k_2(z_1 - z_2) - b_2(\dot{z}_1 - \dot{z}_2) \quad (1)$$

For Driver Mass  $m_2$ :

$$+\uparrow \Sigma F: m_2 \ddot{z}_2 = k_2(z_1 - z_2) + b_2(\dot{z}_1 - \dot{z}_2) \quad (2)$$

Here,  $z_0$  is input variable, so if we rearrange these equations with the input variables on the right-hand side and the others on the left-hand side, we have

$$m_1 \ddot{z}_1 + b_1 \dot{z}_1 + b_2(\dot{z}_1 - \dot{z}_2) + k_2(z_1 - z_2) + k_1 z_1 = b_1 \dot{z}_0(t) + k_1 z_0(t) \quad (3)$$

$$m_2 \ddot{z}_2 - k_2(z_1 - z_2) - b_2(\dot{z}_1 - \dot{z}_2) = 0 \quad (4)$$

As seen in the equations above, the model consists of two second-order linear Ordinary Differential Equations (ODE). Then, the convenient attitude to system analysis is to utilize state-space representation (SSR). To obtain the SSR, first we

need to define state variables. For an  $n$ th order system, we need  $n$  state variables. The model consists of 2 second order ODEs so the system is 4<sup>th</sup> order. For this reason, we need 4 state variables. As state variables  $x_1, x_2, x_3$  and  $x_4$  are assigned for dynamic variables  $z_1, \dot{z}_1, z_2$  and  $\dot{z}_2$  respectively. Inputs are displacement of the vehicle's floor  $u_1 = z_0(t)$  and velocity of the vehicle's floor  $u_2 = \dot{z}_0(t)$ .

$$\dot{x}_1 = \dot{z}_1 = x_2 \quad (5)$$

$$\dot{x}_2 = \ddot{z}_1 = \frac{-(k_1 + k_2)}{m_1}x_1 + \frac{-(b_1 + b_2)}{m_1}x_2 + \frac{k_2}{m_2}x_3 + \frac{b_2}{m_1}x_4 + \frac{k_1}{m_1}u_1 + \frac{b_1}{m_1}u_2 \quad (6)$$

$$\dot{x}_3 = \dot{z}_2 = x_4 \quad (7)$$

$$\dot{x}_4 = \ddot{z}_2 = \frac{k_2}{m_2}x_1 + \frac{b_2}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b_2}{m_2}x_4 \quad (8)$$

The matrix vector format of the full state equation becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(k_1 + k_2)}{m_1} & \frac{-(b_1 + b_2)}{m_1} & \frac{k_2}{m_1} & \frac{b_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & \frac{-k_2}{m_2} & \frac{-b_2}{m_2} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (9)$$

Desired outputs are driver's displacement  $z_2$  and driver's acceleration  $\ddot{z}_2$ .

So that, for the first output (displacement of the driver)  $y_1 = z_2 = x_3$  and for the second output (acceleration of the driver)  $y_2 = \ddot{z}_2 = \dot{x}_4$ . And output equations matrix becomes

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & -\frac{k_2}{m_2} & -\frac{b_2}{m_2} \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (10)$$

State and output equations can be presented as follows:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (11)$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} \quad (12)$$

Finally, the SSR is obtained as shown above. Now we can simulate the system and observe system response, and we can receive the graph of the outputs.

## 3.2. System Response

System response will be analyzed for 2 types of inputs in this section. One is an pulse input created by a bump which causes sudden displacement of the vehicle's floor the vehicle is assumed to pass over. The other one is sinusoidal inputs created by equally spaced bumps the vehicle moving with a constant speed is assumed to pass over. We will use impulse response and frequency response method to obtain system's response. This part also includes parametric sensitivity analysis.

### 3.2.1. Impulse Response

Free (natural or transient) response of the system depends on the roots of the characteristic equation that is equivalent to eigenvalues of the system's state-matrix A. Hence, we started with calculating the eigenvalues.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k_1 + k_2) & -(b_1 + b_2) & \frac{k_2}{m_1} & \frac{b_2}{m_1} \\ \frac{m_1}{m_1} & \frac{m_1}{m_1} & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b_2}{m_2} & \frac{-k_2}{m_2} & \frac{-b_2}{m_2} \end{bmatrix}$$

Table 3.1: Parameters for the Seat-Suspension System

System Parameters	Values
Seat Mass, $m_1$	15 kg
Driver Mass, $m_2$	85 kg
Suspension Stiffness, $k_1$	8000 N/m
Seat Cushion Stiffness, $k_2$	9000 N/m
Suspension Friction Coefficient, $b_1$	1850 N-s/m
Seat Cushion Friction Coefficient, $b_2$	180 N-s/m

By using numerical values in Table 3.1, state-matrix A becomes as follow:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1133.33 & 135.33 & 600 & 12 \\ 0 & 0 & 0 & 1 \\ 105.88 & 2.118 & -105.88 & -2.118 \end{bmatrix}$$

We computed eigenvalues by using 'eig(A)' command in MATLAB and obtained four eigenvalues (characteristic roots).

```

>> eig(A)

ans =

    -126.44 +      0i
    -5.6945 +      0i
    -2.656 +      8.4483i
    -2.656 -      8.4483i

```

Figure 3.3: Screenshot of eigenvalues taken from MATLAB screen

$$r_1 = -126.44 \quad r_2 = -5.6945 \quad r_3 = -2.656 + j8.4483 \quad r_4 = -2.656 - j8.4483$$

There are 2 real and negative roots and 2 complex roots with negative real parts. Hence, natural (free) response decays to zero when the system reaches its steady-state value. Natural response becomes generalized form as follows:

$$y_H(t) = c_1 e^{-126.44t} + c_2 e^{-5.6945t} + c_3 e^{-2.656t} \cos(8.4483t + \phi) \quad (13)$$

Root  $r_1$  coincides with an exponential function that decays to zero in  $t_{s(1)} = 0.0316s$ .

Root  $r_2$  and  $r_{3,4}$  die out in about  $t_{s(2)} = 0.7024s$  and  $t_{s(3,4)} = 1.51s$  respectively. The natural response decays to zero in  $1.51s$  (slowest root) after the impulse input. Complex roots are dominant ones, so the natural response of the system exhibits underdamped response characteristic.

By multiplying complex conjugate terms, the underdamped part of the characteristic equation is found as follows:

$$r^2 + 5.312r + 78.428 = 0 \quad (14)$$

The undamped natural frequency is  $\omega_n = \sqrt{78.428} = 8.86 \text{ rad/s}$  and the damping ratio is  $\zeta = 5.312/2\omega_n = 0.3$ .

In order to obtain the seat-suspension system's response to inputs, Simulink is used. First, we made some assumptions to prepare Simulink model

#### **Assumptions (for inputs $z_0$ and $\dot{z}_0$ ):**

A spike input that causes sudden displacement of the vehicle's floor:  $z_0(t) = 0.045m$ . This input is created by integrating a sequence of constant-velocity pulses.

Constant vertical bump rate  $\dot{z}_0$ :  $7.5 \text{ m/s (rising)}$  and  $-7.5 \text{ m/s (descending)}$

Initial step function  $\dot{z}_0 = 7.5 \text{ m/s}$  (applied at  $t = 0.5\text{s}$ )

Second step function:  $\dot{z}_0 = -15\text{m/s}$  (applied at  $t = 0.5+\Delta t$ )

Third (last) step function:  $\dot{z}_0 = 7.5 \text{ m/s}$  (applied at  $t = 0.5+2*\Delta t$ )

Input  $\dot{z}_0(t)$  is created by the three step functions and the pulse input  $z_0(t)$  created by integrating the velocity pulses.

Here,  $\Delta t = z_0/\dot{z}_0 = 0.045\text{m}/7.5 \text{ m/s} = 0.006 \text{ s} = 6ms$

Total duration of the pulse is  $2 * \Delta t = 2 * 0.006 \text{ s} = 0.012\text{s} = 12 \text{ ms}$

Initial condition vector is  $x_0 = [0 \ 0 \ 0 \ 0]^T$

Simulink model is shown in Figure 3.4 below.

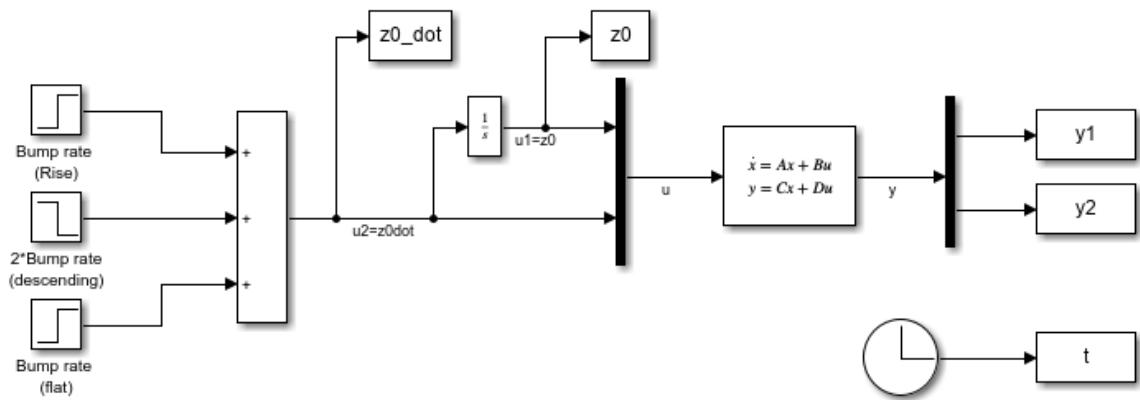


Figure 3.4: Simulink diagram for the seat suspension system (Impulse input)

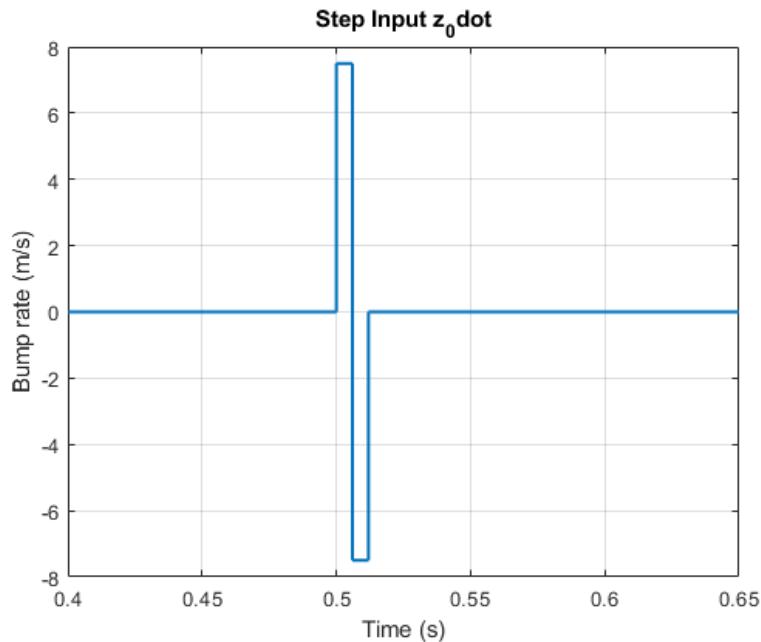


Figure 3.5: Seat suspension system velocity-pulse input  $u_2(t)$

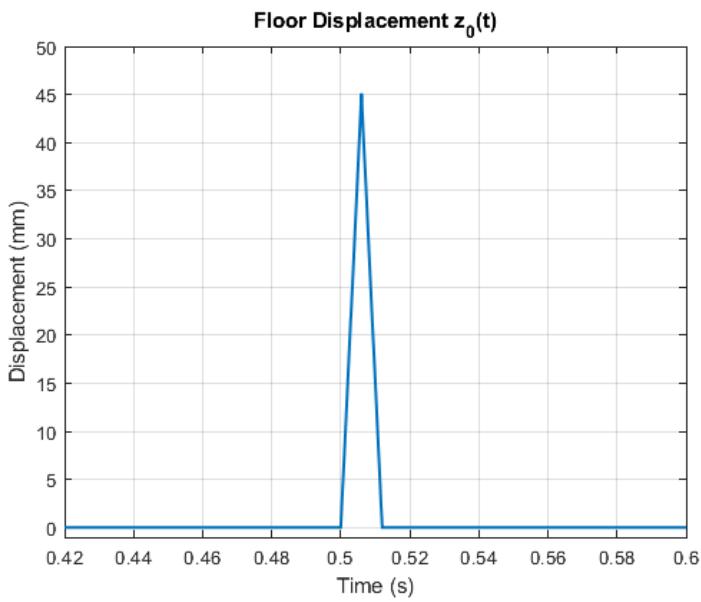


Figure 3.6: Triangular pulse input  $u_1(t)$

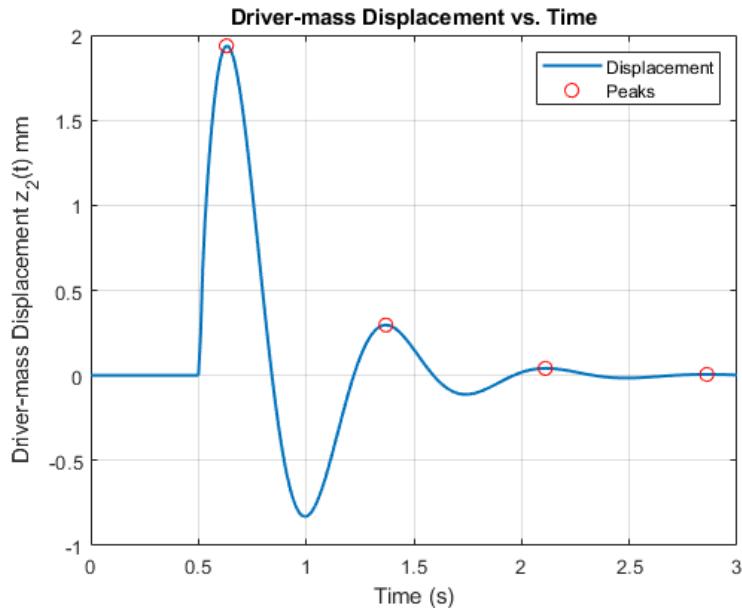


Figure 3.7: Impulse response of the seat-suspension system (Driver displacement  $z_2$ )

Time (s)	PeakValue (mm)
0.63	1.9372
1.37	0.29586
2.11	0.041427
2.86	0.0057511

Table 3.2: Time and peak value table of output  $y_1 = z_0(t)$

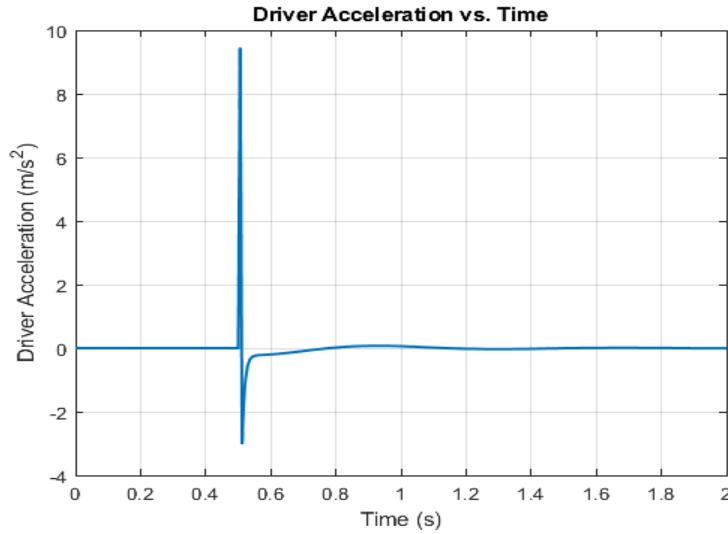


Figure 3.8: Impulse response of the seat-suspension system (Driver acceleration  $\ddot{z}_2$ )

### 3.2.2. Frequency Response

In this part, we will build Simulink diagram to observe the system's frequency response to sinusoidal inputs. In order to obtain and analyze a frequency response we assumed the vehicle traveling over a rough road that resulted in a periodic input. The repeating road vibrations will create sinusoidal input that is modeled as  $z_0(t) = A \sin(\omega t)$  for floor displacement.

Second input  $\dot{z}_0(t)$  (floor vertical velocity) is produced by differentiating the first input  $z_0(t)$ .

A: Amplitude of the vehicle's floor vibrations in (m)

$\omega$ : Input frequency in (rad/s)

We considered that the amplitude A is fixed at 0.03m. Input frequency differs with vehicle's moving speed and spacing between bumps, for that reason we will use different  $\omega$  values to compare and observe the response of the system.

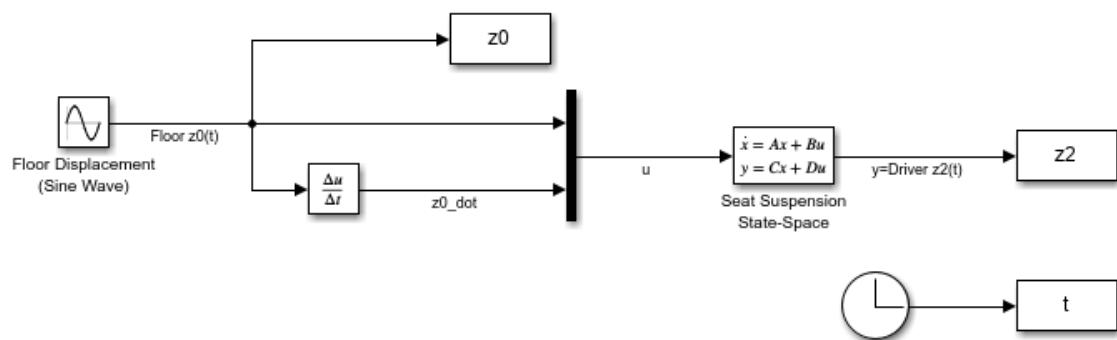
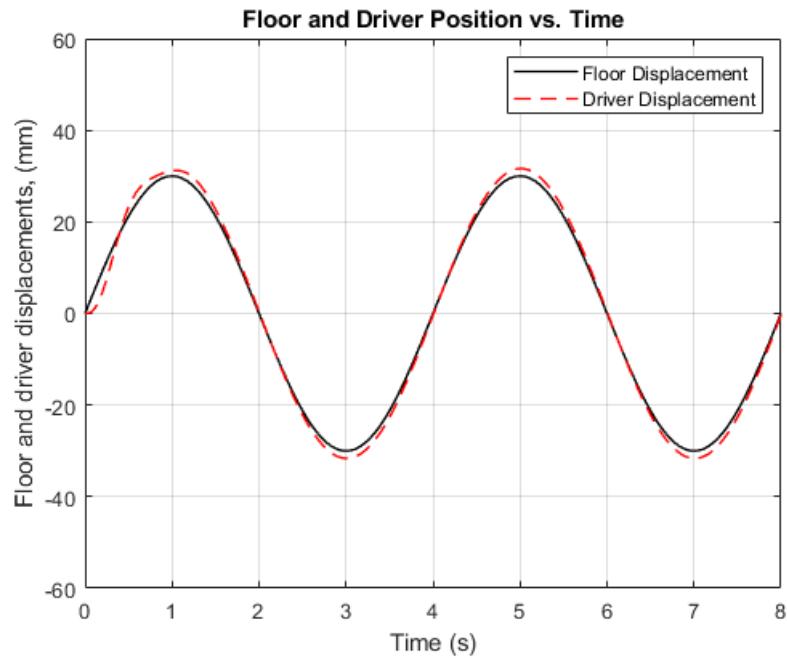
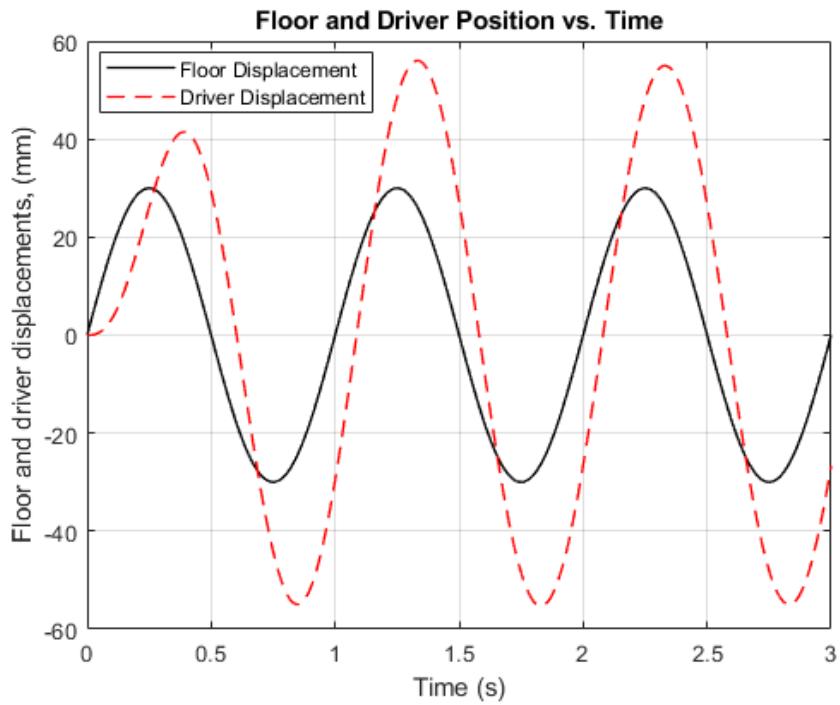


Figure 3.9: Simulink diagram for the system (sinusoidal input)

As we run the Simulink diagram shown in Figure 3.9, we had the system's responses in graphs as shown in figures below.



*Figure 3.10: System's response to 0.25 Hz frequency input*



*Figure 3.11: System's response to 1 Hz frequency input*

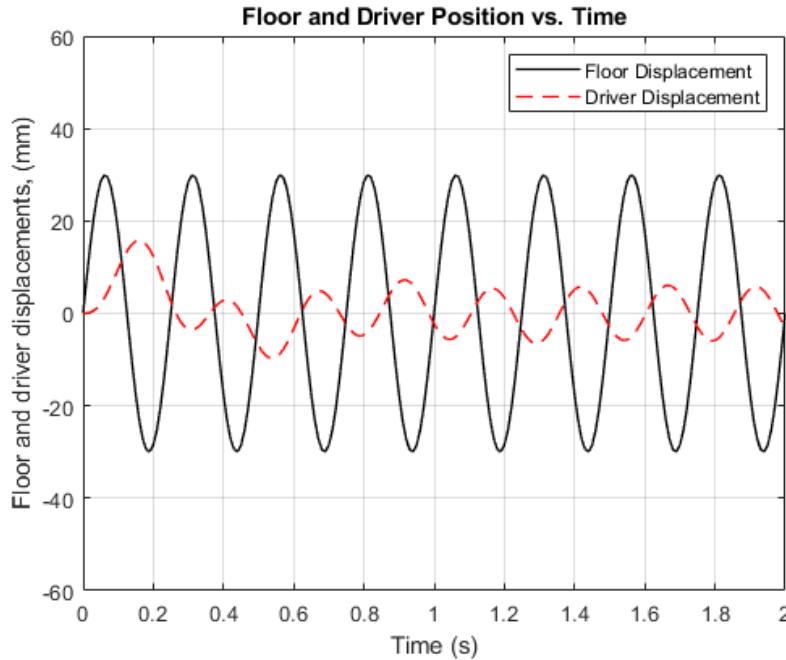


Figure 3.12: System's response to 4 Hz frequency input

### 3.2. Sensitivity Analysis

The aim of the seat-suspension system, as mentioned before, is to prevent the vibration of the vehicle floor which is transmitted to the operator. There are so many parameters that affects the performance of the system. In order to observe the effect each parameter create on the performance of the system; we use transmissibility which means the performance measure of the parametric analysis. Transmissibility is defined by the amplitude ratio of the frequency response output  $z_2(t)$  and sinusoidal input  $z_0(t)$ . Transmissibility is essentially the magnitude plot from the bode diagram for output (Craig A. Kluever, 2015). We will measure transmissibility for input frequencies in a certain range and different stiffness and viscous friction values. We will use MATLAB to obtain the results.

First, input matrix B will be rearranged because bode diagram exhibits data for single input and single output ( $u = z_0(t)$  and  $y = z_2$ )

$$\text{State variable } x_2 \text{ is defined as } x_2 = \dot{z}_1 - \frac{b_1}{m_1} z_0$$

New input matrix B becomes as follows:

$$B = \begin{bmatrix} b_1/m_1 \\ \frac{k_1}{m_1} - \frac{b_1^2}{m_1^2} - \frac{b_1 b_2}{m_1^2} \\ 0 \\ b_1 b_2 / m_1 m_2 \end{bmatrix}$$

We defined the frequency input range from 0.1 Hz to 5 Hz and three different stiffness and viscous damping coefficients to observe the effects each of them has on the system's response.

Stiffness values for  $k_1 = 4000 \text{ N/m}$ ,  $8000 \text{ N/m}$ ,  $12000 \text{ N/m}$

Stiffness values for  $k_2 = 4500 \text{ N/m}$ ,  $9000 \text{ N/m}$ ,  $13500 \text{ N/m}$

Viscous friction values for  $b_1 = 925 \text{ N.s/m}$ ,  $1850 \text{ N.s/m}$ ,  $2775 \text{ N.s/m}$

Utilizing the parameters given above, we received the graphs from MATLAB as shown in figures below.

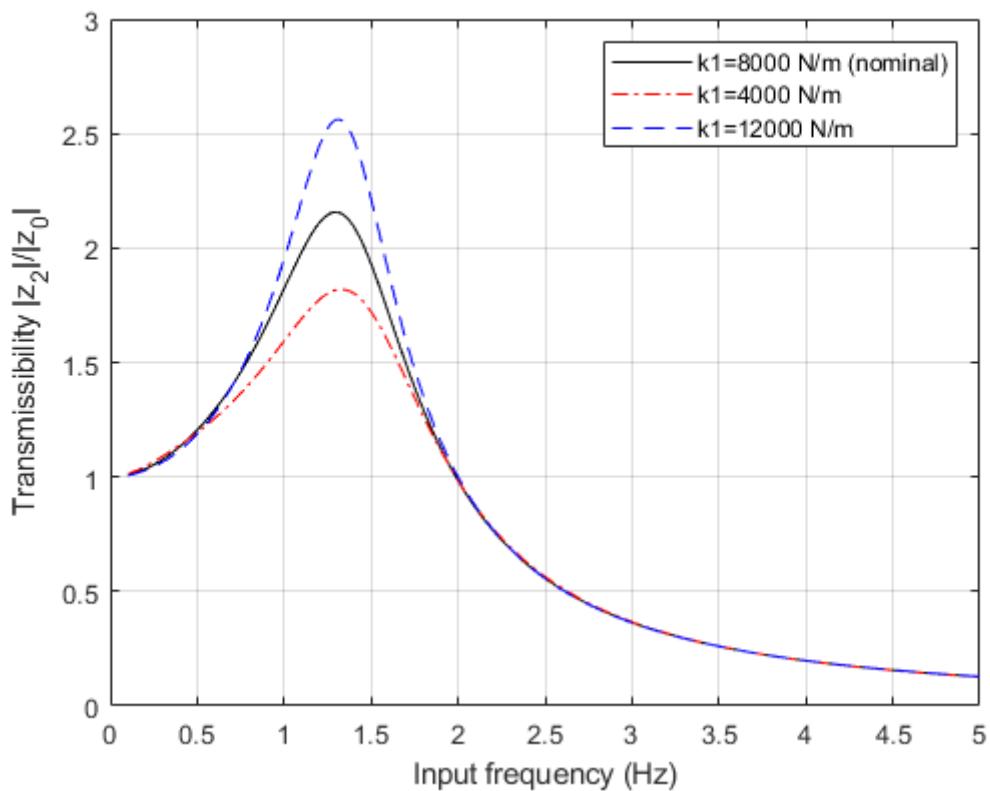


Figure 3.13: Transmissibility  $|z_2|/|z_0|$  for different stiffness values of  $k_1$

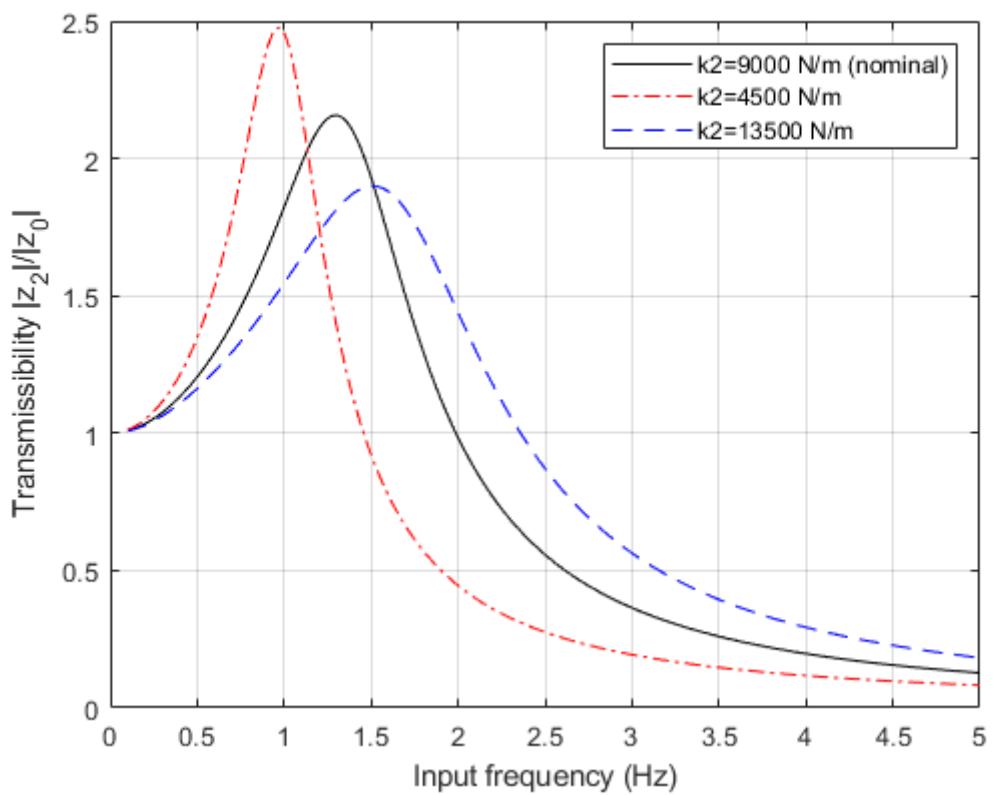


Figure 3.14: Transmissibility  $|z_2|/|z_0|$  for different stiffness values of  $k_2$

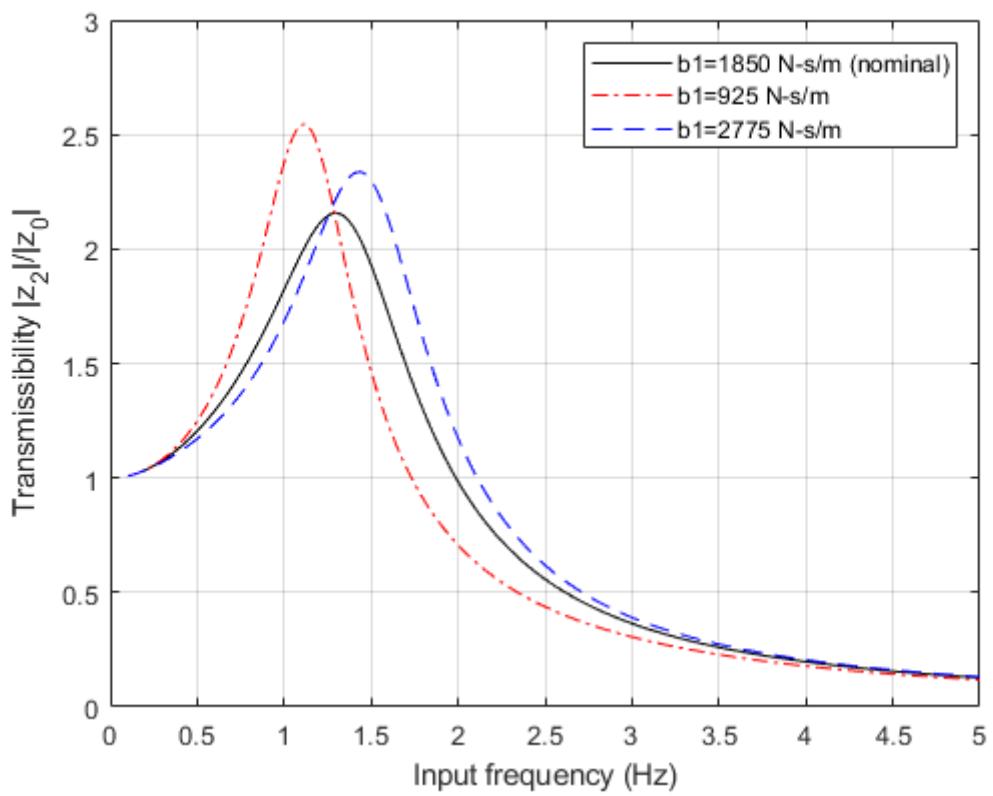


Figure 3.15: Transmissibility  $|z_2|/|z_0|$  for different viscous friction values of  $b_1$

## 4. RESULTS AND DISCUSSION

### Impulse Response

In the studies conducted in the impulse response section, the effect on the driver of a 45-millimeter vertical displacement of the vehicle floor, which occurs in 12 milliseconds, is shown graphically. According to the calculations and the data obtained from Figure 3.7, the vibration caused by this displacement was completely eliminated by the suspension system in approximately 1.51 seconds, which is the settling time  $t_s$ , and the system returned to its initial equilibrium state at  $t = 2$  seconds.

As shown in Figure 3.7, a sudden change in the vertical position of the vehicle floor created an initial peak of 1.94 mm at  $t = 0.63$ s. We can see these values in Table 3.2, obtained from the graph in Figure 3.7. After this peak, the second peak was observed at time  $t = 1.37$ s, 0.3 mm. This large change in the second peak after the first peak shows that the suspension system is performing an effective damping. Further work can be done to reduce the first peak and achieve a smoother damping.

### Frequency Response

From figure 3.10 which has shown in the frequency response section, we can observe that at a low frequency of 0.25 Hz ( $\omega = 1.57$  rad/s), the driver displacement and the vehicle floor displacement are in phase and have the same amplitude and the phase angle is almost zero. The fact that the system has the same amplitude tells us that there is no attenuation and amplification in the system. This shows that the suspension system we designed does not fully work at low frequencies.

From Figure 3.11, we can find that the amplitude ratio is  $56/30=1.87$  and the steady state drive displacement is 87% larger when we set the input frequency  $\omega$  to 1 Hz (6.283 rad/s). We can say that this is because the input frequency approached the resonant frequency of the system. The resonant frequency of the system is

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} = 8.02 \text{ rad/s} = 1.277 \text{ Hz.}$$

The closer we get to this value, the more the system will resonate. Also, the output sinusoid is slightly delayed compared to the input.

When we enter the input frequency  $\omega$  as 4 Hz (25.133 rad/s), we obtain the ratio of output amplitude to input amplitude in steady-state response as  $6.05/30 = 0.2017 = 20.17\%$ . The fact that the output/input amplitude ratio is less than 1 indicates that the suspension system we designed has the capacity to damp the vibrations. The fact that the output peaks almost

match the input valley points also shows that the phase angle is almost 180 degrees ( $\pi$  rad). As a result, our system works efficiently at this frequency input value.

### Sensitivity Analysis

In this section we investigated the effect of variation in suspension stiffness on the sensitivity analysis. Analyzing the graph obtained in Figure 3.13, we see that a stiffer suspension spring increases the peak of the transmissibility. We can observe that varying the spring stiffness has no effect after a certain frequency input. This value corresponds to approximately 12.57 rad/s (2 Hz) in the graph. We can choose lighter suspension springs for a comfortable ride, as it ensures that the peak point is lower than the others.

The seat cushion, which has a higher stiffness than the others, decreased the peak of the transmissibility and increased the resonance frequency, according to the data obtained from the graph shown in figure 3.14. The stiffer spring constant increased the vibrations transmitted at high frequency inputs. Vehicle vibrations are predominantly in the 2.5 Hz (15.71 rad/s) range (Craig A. Kluever, 2015), so if a lower transmittance point is desired, a seat with a spring constant of nominal stiffness at a frequency of 2.5 Hz should be selected. If a lower transmitted vibration is desired, a softer spring constant can be selected.

When we look at the effect of viscous dampers with different damping coefficients on the transmissibility in the graph in Figure 3.15, we observe that the damping with nominal value gives the best performance. Although we increase the damping coefficient, the transmissibility peak increases, indicating that there is an optimum damping coefficient for this system. As can be seen from the graph, this optimum damping coefficient is the nominal value of 1850 N.s/m. The damper with a value of 2775 N.s/m has both increased the transmittance coefficient and increased the resonant frequency. However, at high frequencies, as can be seen from the graph, the variable damping coefficient does not have much effect on the transmissibility.

## 5. CONCLUSION

In this study, the vibration isolation characteristics of the suspension system were extensively investigated. First, the impulse response analysis shows that a vertical displacement of 45 mm at the base of the vehicle is fully damped by the suspension system in about 1.51 seconds and returns to the initial equilibrium state within 2 seconds. This indicates that the dynamic response of the system is at the satisfactory level.

In the frequency response analysis, it is found that at low frequency (0.25 Hz), the driver and vehicle floor displacements are in phase and have the same amplitude, showing that there is no amplification or damping in the system. However, at higher frequencies, when the resonance frequency of the system is approached (1 Hz), it is found that the driver displacement is 87% larger than the input at steady state and the phase angle is lightly lagged relative to the input. At 4 Hz (25.133 rad/s), the damping ability of the system increased and the phase angle was nearly 180 degrees. This shows that our system works effectively at high frequency vibrations.

In the sensitivity study, the effect of the stiffness of the suspension spring on the transmissibility was examined. It is found that the transmissibility peak increases with increasing spring stiffness, but there is no change beyond a certain frequency. The analysis with various damping coefficients of the viscous dampers shows that the nominal damping coefficient gives the best performance and its effect is not significant at high frequencies.

Regarding the results of this work, they present valuable information for the optimized design of the suspension system. It is emphasized that the system parameters should be carefully adjusted, particularly in those applications where it is important to control low- and high-frequency vibrations. Future studies can focus on refining the system performance and investigating the system behavior in different road conditions.

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## APPENDICES

### Matlab Codes

#### **Impulse Response**

```
clear all

m1 = 15; % seat mass, kg
m2 = 85; % driver mass, kg
k1 = 8000; % suspension stiffness, N/m
k2 = 9000; % seat cushion stiffness, N/m
b1 = 1850; % suspension friction, N-s/m
b2 = 180; % seat cushion friction, N-s/m
% SSR matrices
```

```

% States: x = [ z1 z1-dot z2 z2-dot ]'
% x1 = z1 (seat mass position, m)
% x2 = z1dot, m/s
% x3 = z2 (driver mass position, m)
% x4 = z2dot (driver mass velocity, m/s)
% Inputs: u1 = z0
% u2 = z0dot (= u1dot)
A = [0 1 0 0 ; (-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ; 0 0 0 1 ; k2/m2 b2/m2 -
k2/m2 -b2/m2];
B = [0 0 ; k1/m1 b1/m1 ; 0 0 ; 0 0];

% Output: y1 = z2 = driver position, m
% y2 = z2ddot = driver acceleration, m/s^2
C = [0 0 1 0 ; k2/m2 b2/m2 -k2/m2 -b2/m2];
D = zeros(2,2);

% initial state (equilibrium)
x0 = [0 0 0 0]';

% triangular pulse input

z0dot = 7.5; % vertical rate of triangular pulse, m/s
peak_z0 = 0.045; % max height of triangle pulse, m
peak_time = peak_z0/z0dot; % time to reach peak of triangle pulse, sec
% end time
t_end = 3;

sim seat_susp_project

% outputs and plots
figure(1)
plot(t,y1*1e3,'LineWidth',1.5)
grid on
title('Driver-mass Displacement vs. Time')
xlabel('Time (s)')
ylabel('Driver-mass Displacement z_2(t) mm')

figure(2)
plot(t,y2,'LineWidth',1.5)
xlim([0 2])
grid on
title('Driver Acceleration vs. Time')
xlabel('Time (s)')
ylabel('Driver Acceleration (m/s^2)')

figure(3)
plot(t,z0*1e3,'LineWidth',1.5)
grid on
xlim([0.42 0.6])
ylim([-1 max(z0)*1e3+5])
xlabel('Time (s)')
ylabel('Displacement (mm)')
title('Floor Displacement z_0(t)')

figure(4)
plot(t,z0_dot,'LineWidth',1.5)
grid on
xlim([0.4 0.65 ])

```

```

ylim([-8 8])
xlabel('Time (s)')
ylabel('Bump rate (m/s)')
title('Step Input z_0dot')

% To find peak values
peakIndices = [];
for i = 2:length(y1)-1
    if y1(i) > y1(i-1) && y1(i) > y1(i+1)
        peakIndices = [peakIndices i];
    end
end

peakValues = y1(peakIndices)*1e3;

% Marking the peak points on the graph
figure(5)
plot(t, y1*1e3, 'LineWidth', 1.5)
grid on
hold on;
plot(t(peakIndices), peakValues, 'ro', 'MarkerSize', 7);
grid on
hold off;
title('Driver-mass Displacement vs. Time')
xlabel('Time (s)')
ylabel('Driver-mass Displacement z_2(t) mm')
legend('Displacement', 'Peaks');
T = table([t(peakIndices)], [peakValues], 'VariableNames', {'Time (s)', 'PeakValue (mm)'})

```

## Frequency Response

```

% mechanical system parameters
m1 = 15; % seat mass, kg
m2 = 85; % driver mass, kg
k1 = 8000; % suspension stiffness, N/m
k2 = 9000; % seat cushion stiffness, N/m
b1 = 1850; % suspension friction, N-s/m
b2 = 180; % seat cushion friction, N-s/m

A = [ 0 1 0 0 ; (-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ; 0 0 0 1 ; k2/m2 b2/m2 -
k2/m2 -b2/m2 ];
B = [ 0 0 ; k1/m1 b1/m1 ; 0 0 ; 0 0 ];
C = [ 0 0 1 0];
D = [0 0];
% initial state (equilibrium)
x0 = [0 0 0 0]';
% floor displacement amplitude, m
z0_amp = 0.03; % m amplitude A
% frequency
f = input('Enter input frequency in hertz (Hz)');
w_rps = 2*pi*f; % input frequency in rad/s
% end time
t_end = input('Enter simulation time ');
% run Simulink model
sim frequency_response.slx
% plot
figure(1)
plot(t,z0*1e3,'k','LineWidth',1)

```

```

hold on
plot(t,z2*1e3,'--r','LineWidth',1)
title('Floor and Driver Position vs. Time')
hold off
grid
axis([0 t_end -60 60 ])
xlabel('Time (s)')
ylabel('Floor and driver displacements, (mm)')
legend('Floor Displacement','Driver Displacement')

```

## Sensitivity Analysis (Transmissibility)

```

% mechanical system parameters
m1 = 15; % seat mass, kg
m2 = 85; % driver mass, kg
k1 = 8000; % suspension stiffness, N/m
k2 = 9000; % seat cushion stiffness, N/m
b1 = 1850; % suspension friction, N-s/m
b2 = 180; % seat cushion friction, N-s/m
% SSR:
A = [ 0 1 0 0;-(k1+k2)/m1 -(b1+b2)/m1 k2/m1 b2/m1;0 0 0 1; k2/m2 b2/m2 -k2/m2 -
b2/m2 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;
% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 1000;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz
for i=1:Npts
    w_in = w_Hz(i)*2*pi; % input frequency in rad/s
    [mag,phase] = bode(sys,w_in);
    TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency

plot(w_Hz,TR,'k')
grid
xlabel('Input frequency, Hz')
ylabel('Transmissibility')
hold on

m1 = 15; % seat mass, kg
m2 = 85; % driver mass, kg
k1 = 8000; % suspension stiffness, N/m
k2 = 9000; % seat cushion stiffness, N/m
b1 = 925; % suspension friction, N-s/m
b2 = 180; % seat cushion friction, N-s/m
% SSR
A = [ 0 1 0 0;-(k1+k2)/m1 -(b1+b2)/m1 k2/m1 b2/m1;0 0 0 1; k2/m2 b2/m2 -k2/m2 -
b2/m2 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;

```

```

% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 1000;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz
for i=1:Npts
    w_in = w_Hz(i)*2*pi; % input frequency in rad/s
    [mag,phase] = bode(sys,w_in);
    TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency
plot(w_Hz,TR,'-.r')
grid
xlabel('Input frequency, Hz')
ylabel('Transmissibility')
hold on

m1 = 15; % seat mass, kg
m2 = 85; % driver mass, kg
k1 = 8000; % suspension stiffness, N/m
k2 = 9000; % seat cushion stiffness, N/m
b1 = 2775; % suspension friction, N-s/m
b2 = 180; % seat cushion friction, N-s/m
% SSR
A = [ 0 1 0 0; -(k1+k2)/m1 -(b1+b2)/m1 k2/m1 b2/m1; 0 0 0 1; k2/m2 b2/m2 -k2/m2 -
b2/m2 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;
% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 1000;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz

for i=1:Npts
    w_in = w_Hz(i)*2*pi; % input frequency in rad/s
    [mag,phase] = bode(sys,w_in);
    TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency
plot(w_Hz,TR,'--b')
hold off
grid on
xlabel('Input frequency (Hz)')
ylabel('Transmissibility |z_2|/|z_0|')

hleg=legend('b1=1850 N-s/m (nominal)', 'b1=925 N-s/m', 'b1=2775 N-s/m');

```