



MARMARA UNIVERSITY
FACULTY OF ENGINEERING



Stress Analysis of Fiber Reinforced Polymer Composite Plates Containing Stress Concentrations

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GRADUATION PROJECT REPORT

Department of Mechanical Engineering

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MARMARA UNIVERSITY
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**Stress Analysis of Fiber Reinforced Polymer Composite Plates
Containing Stress Concentrations**

by

Mustafa Kara & Şamil ÇARHACIOĞLU

June 2021; Istanbul

**SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN
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ABSTRACT

Stress Analysis of Fiber Reinforced Polymer Composite Plates Containing Stress Concentrations

Mustafa KARA & Şamil ÇARHACIOĞLU

Today, advanced composite materials are frequently preferred in the aviation industry because of their high strength properties compared to their weight. Geometric discontinuities such as holes in these structures cause stress concentrations. Predicting these stress concentrations in structures is critical because the initiation of failure occurs at or near these points.

The aim of this study is to examine the stress concentrations in multi-directional fiber composite materials which contain elliptical hole and to present a program with a user interface to determine the stress concentration factor in multi-directional laminated composite plates containing elliptical holes, based on the analytical solutions in the literature, and present it to TAI's use.

Within the scope of this study, the analytical, numerical, and experimental studies to determine the stress concentration factor in the literature were examined, and a program with a user interface was created by using a consistent analytical solution. The results were compared with the numerical and experimental results in the literature, and it was seen that they overlapped by over 95%.

Keywords — *Multidirectional composite laminates; Stress concentration factors, Fiber polymer composites*

ÖZET

Delikli Elyaf Takviyeli Kompozit Levhalarda Gerilme Yığılması

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Günümüzde ileri kompozit malzemeler ağırlıklarına oranla sağladığı yüksek mukavemet özelliklerinden dolayı havacılık sektöründe sıkça tercih edilmektedir. Bu yapılarda bulunan delik gibi geometrik süreksizlikler gerilme yığılmalarına sebep olmaktadır. Yapılardaki gerilme yığılmalarının belirlenmesi kritik öneme sahiptir çünkü malzemenin hasar başlangıcı bu noktalarda meydana gelmektedir.

Bu çalışmanın amacı çok yönlü lamine edilmiş elyaf takviyeli delikli kompozit malzemelerde oluşan gerilme yığılmalarını incelemek ve literatürdeki analitik çözümleri esas alarak eliptik şekilde deliğe sahip çok yönlü lamine edilmiş katmanlı yapılarda gerilme yığılma faktörünü belirlemeye yönelik kullanıcı ara yüzüne sahip bir program oluşturarak TUSAŞ' ın kullanımına sunmaktır.

Bu çalışma kapsamında literatürde bulunan gerilme yığılma faktörünü belirlemeye yönelik yapılan analitik, numerik ve deneysel çalışmalar incelenmiş olup, tutarlı bir analitik çözüm kullanılarak, kullanıcı ara yüzüne sahip program oluşturulmuştur. Çalışmada elde edilen sonuçlar literatürdeki numerik ve deneysel sonuçlarla kıyaslanmış olup %95 üzerinde örtüşükleri görülmüştür.

Anahtar kelimeler — Çok eksenli kompozit levhalar, Gerilme yığılma katsayıları, Elyaf takviyeli polimer kompozitler

SYMBOLS

| | |
|---------------|--|
| ρ | : Density (kg/m ³) |
| S | : Tensile strength (GPa) |
| W | : Width of the Plate (mm) |
| d | : Diameter of the hole (mm) |
| θ | : Fiber orientation of unidirectional lamina(degree) |
| l | : Length of the plate (mm) |
| L | : 1 st principal axis (Longitudinal Axis) |
| T | : 2 nd principal axis (Transverse Axis) |
| T' | : 3 rd principal axis (Transverse Axis) |
| σ_i | : Stress in i direction(Pa) |
| ϵ_i | : Extensional strain in i direction |
| G_{ij} | : Shear modulus on ij plane (GPa) |
| τ_{ij} | : Shear stress on ij plane (Pa) |
| γ_{ij} | : Engineering shear strain on ij plane |
| ν_{ij} | : Poisson's ratio on ij plane |
| m_x, m_y | : Cross-coefficients |
| Q_{ij} | : Elements of reduced stiffness matrix |
| S_{ij} | : Elements of compliance matrix |
| K_T^∞ | : Stress Concentration Factor for infinite plate |
| K_T | : Stress Concentration Factor for finite-width plate |
| λ | : Aspect ratio of opening (b/a) |
| E_x | : Tensile modulus of composite plate in x-direction referred to global coordinate |
| E_y | : Tensile modulus of composite plate in y-direction referred to global coordinate. |
| G_{xy} | : Shear modulus of composite plate in xy plane referred to global coordinate. |
| E_1 | : Shear modulus of composite plate in 1 plane referred to local coordinate. |
| E_2 | : Shear modulus of composite plate in 2 plane referred to local coordinate. |
| G_{12} | : Shear modulus of composite plate in 12 plane referred to local coordinate. |

ABBREVIATIONS

UD : Unidirectional

SCF : Stress Concentration Factor

FEA : Finite Element Analysis

PSC : Point Stress Criteria

EPSC : Extended Point Stress Criteria

FWC : Finite Width Correction

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1. INTRODUCTION

Advanced fiber-reinforced composite materials have two major advantages over many others: better strength and stiffness properties when compared with others on a unit weight basis. For instance, owing to composite materials same strength and stiffness as high-strength steel can be achieved in 70% lighter material than high-strength steel. In addition to strength and stiffness properties thanks to the nature of fiber-reinforced composite materials, they can be tailored to meet the design requirements of strength, stiffness, and other parameters all in various directions (controlled anisotropy). These advantages lead to new aircraft and spacecraft designs utilizing composites (Jones, 1999) .

Applications of polymer matrix composites range from tennis racquets to space shuttle. The use of composites has been conservative due to safety reasons in commercial airlines, so they are mainly used for secondary structures in airplanes some examples are rudders and elevators made of graphite/epoxy for the Boeing 767 and landing gear doors made of Kevlar–graphite/epoxy. There are also examples of using composites in primary structures such as the tail fin of Airbus A310-300. The tail consists of graphite/epoxy and an aramid honeycomb. It provided not only 300 kg weight reduction of a tail but also reduced the number of parts from 2000 to 100 Today (Kaw, 2005).

Composites structures are used not only in aerospace industries but also in many other markets and their usage is increasing every year. Table 1.1 shows the market share of composites since 1990 (Kaw, 2005).

Table 4.1 U.S. Composites Shipment in 106 lb., Including Reinforced Thermoset and Thermoplastic Resin Composites, Reinforcements, and Fillers (Kaw, 2005).

| Markets | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|-------------------------------|-------------|-------------|-------------|-------------|---------------|---------------|
| Aircraft/aerospace/military | 39 | 38.7 | 32.3 | 25.4 | 24.2 | 24 |
| Appliance/business equipment | 153 | 135.2 | 143.2 | 147.5 | 160.7 | 166.5 |
| Construction | 468 | 420 | 483 | 530 | 596.9 | 626.9 |
| Consumer products | 165 | 148.7 | 162.2 | 165.7 | 174.8 | 183.6 |
| Corrosion-resistant equipment | 350 | 355 | 332.3 | 352 | 376.3 | 394.6 |
| Electrical/electronic | 241 | 231.1 | 260 | 274.9 | 299.3 | 315.1 |
| Marine | 375 | 275 | 304.4 | 319.3 | 363.5 | 375.1 |
| Transportation | 705 | 682.2 | 750 | 822.1 | 945.6 | 984 |
| Other | 79 | 73.8 | 83.4 | 89.3 | 101.8 | 106 |
| TOTAL | 2575 | 2360 | 2551 | 2726 | 3043.1 | 3176.4 |

Stress concentration is an extremely important issue in both homogeneous isotropic and heterogeneous composite materials because the initiation failure occurs at or near the maximum stress concentration. Cutouts (or openings), discontinuous geometries, joints, mismatch of elastic properties between two adjacent components and voids and damage due to fabrication are reasons for stress concentrations in structures (Tan, 1994).

Although some of the issues related to stress concentration of laminated composites are discussed in the literature much work remains to be done to be able to predict failure under a broad range of general stress state (Tan, 1994).

1.1 Objective of the Thesis

Within the scope of this graduation thesis, it is aimed to investigate the stress concentration effect of an elliptical (circle is also a special version of ellipse) parametrically. With this study, a consistent analysis method will be presented to calculate the maximum stress concentration factor for fiber-reinforced composite plates containing elliptical openings, exposed to tensile loads. A parametric study will be done between plate sizes, fiber direction and hole geometries. The parametric study will be done by changing the fiber direction in composite material. At the end of the study, an application software allowing the user to get the analytical solution of maximum stress concentration factor is created. As a result of this study, more precise calculations can be made by giving lower safety factors while considering the stress concentration in the holes on the plate.

1.2 Outline of the Thesis

The thesis consists of six parts. In the first part, the purpose and outline of the study are given in addition to the introductory information. In the second some general information about composites and literature survey related to the purpose of this study is provided. In the third part theoretical background to get mechanical properties of laminate composites and analytical solutions for stress concentration around hole subjects are presented. In the fourth part, the created app is introduced and numerical and experimental comparisons of the solution is given. In the fifth part the evaluation of the current work from MUDEK perspective has been considered. Finally, discussion and conclusion subjects were given in the in the last part.

2. GENERAL INFORMATION & LITERATURE SURVEY

2.1 Structure of composite material

The composite word in the composite material term signifies those different materials are combined in a macroscopic scale to form a useful material. The composites generally have two main constituents that are fiber and matrix. These two materials are combined in a way to produce the new material with the desired properties.

The strength to density (S/ρ) and stiffness to density (E/ρ) ratios are commonly used as indicators of the effectiveness of a fiber, especially in weight-sensitive applications such as aircraft and space vehicles (Daniel & Ishai, 2006).

The properties of high tensile strength and low cost of the glass fibers (E-Glass, S-Glass) make them commonly used in low to medium performance composites. However, because of their relatively low stiffness, low fatigue endurance and rapid property degradation with exposure to severe hydrothermal conditions, they are limited in high performance composite applications. With a range of stiffness and strengths depending on the manufacturing process carbon fibers are the most widely used for advanced composites and come in many forms (Daniel & Ishai, 2006).

2.1.2 Matrix Materials

Naturally, fibers do not mean anything if they are not bonded. The primary task of matrix materials is binding the constituents of composites and hold them together. The loads are carried mainly by fibers. The matrices act as an insulator against the effects of fibers against each other and the environment against fibers. Even though they have low mechanical strength properties when compared to those of fibers, some special properties of composites are affected by matrix, such as transverse modulus and strength, shear modulus and strength, thermal coefficient of expansion, and properties in compression. Generally, matrices in composites limit the composite's service temperature because of their relatively low melting temperatures with respect to fibers (Bhagwan D., Lawrence J., & K., 2006).

A typical organic epoxy material such as Narcmo 2387 has a density of 11.9 kN/m^3 , compressive strength of .158 GPa, compressive modulus of 3.86 GPa, a tensile strength of .029 GPa and tensile modulus of 3.38 GPa (Jones, 1999).

2.2 Classification of Composite Materials

As illustrated in Figure 2.2 composite materials can be classified into three categories according to type, geometry, and orientation of reinforcement phase.

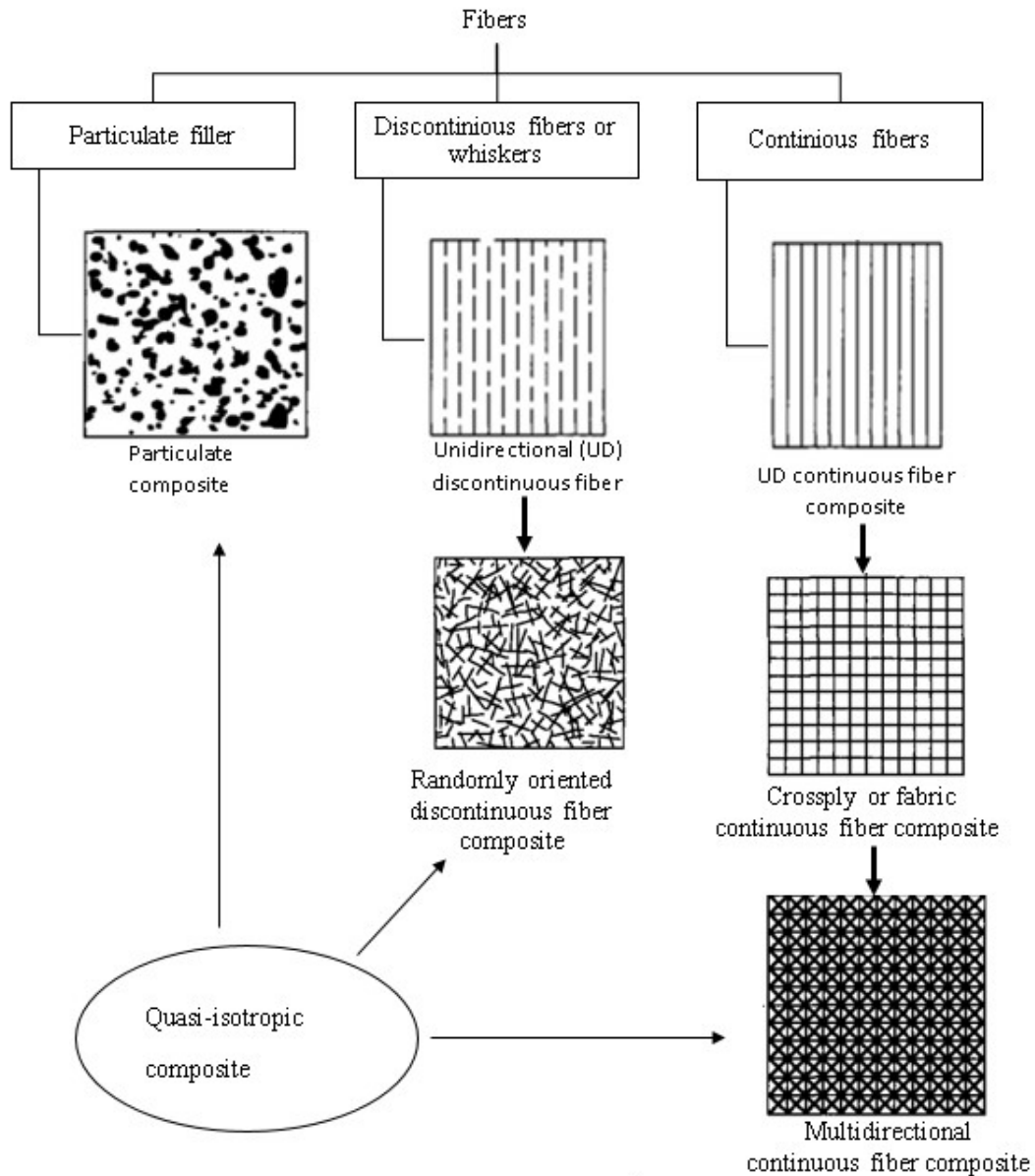


Figure 2.2 Classification of Composite Material Systems (Daniel & Ishai, 2006).

Because of the usual randomness of particles of various sizes in particulate composites, discontinuous fibers in randomly oriented fiber composites, these composites can be regarded as quasi-homogeneous or quasi-isotropic composites. As well as the randomness of them some multidirectional continuous fiber composites with special layup configuration show quasi-isotropic behaviors (Daniel & Ishai, 2006).

Different types of composites appear in several applications. For example, while Some examples of the application of discontinuous fiber composites in literature are disk brakes, gas turbine exhaust flaps and node cones (slideshare, 2007) it is seen that the particulate composites are mostly used in the construction sector and protective organic coating materials (Majer & Náhlík, 2012). When it comes to Continuous-fiber composites, their long continuous fibers

make them suitable for most engineering applications since they are the most efficient ones from the point of stiffness and strength (Daniel & Ishai, 2006).

It is also seen that fiber-reinforced composites can be classified according to the matrix type used, such as polymer-, metal-, ceramic-, and carbon-matrix composites. Although matrix type affects the transverse modulus and strength, shear modulus and strength, thermal coefficient of expansion, and properties in compression as stated in 2.1.2 it seems that the classification of fiber reinforced composites according to matrix materials mainly differentiate the allowable service temperature at which the composite material will be used. While ceramic matrix composites are suitable for very high-temperature applications, polymer matrix composites are best suited for relatively low-temperature applications w.r.t. others. Softening or melting temperature of the metal matrix limits the service temperature of metal matrix composites to a moderate level in regard to others.

2.3 Stress Concentration on Composite materials

To be able to assemble different parts together or for other reasons the composite plates must be drilled in most cases. The hole created imposes stress concentrations on the composite plate and it needs to be investigated. Therefore, the study of the investigation of stress concentration factor on composite plates having different holes was examined by many researchers. The study of examining stress concentration factor unidirectional or multidirectional composite laminates having stress concentration is investigated analytically, numerically, or experimentally by many researchers.

One of the most important studies was held by Lekhnitsskii (Lekhnitsskii, 1968) , he has found the stress concentration factor (SCF) for the infinite plates having elliptical or circular holes analytically using the complex variable method.

Tan (Tan, 1994) found the finite width solutions of the plates having stress concentration by imposing correction factors to ones found by Lekhnitski, and he supported his studies by comparing his approach with published experimental and numerical studies and he improved his theory for different elliptical shapes.

Baek et al (Baek, Kim, Rhee, & Rowland, 2000) presented on the effects of varying in a power series representation of the stress function. They have used photoelastic data by hybridizing it with complex variables and mapping techniques to calculate tangential stress on the boundary of the cutout.

Xiwu et al. (Xiwu, Liangxin, & Xuqi, 1995) obtained an analytical solution concerned with stress concentration around an elliptical hole in a finite composite laminated plate with the help of Fabor series expansion and the least-squares boundary collocation technique, by using complex potential method. They showed that the relative size of the laminated plate (W/d) has a strong effect on stress concentration (The smaller relative size the larger the stress concentration). The approach that as if the finite plate is infinite is reasonable when $W/d \geq 10$. They also showed that while the increase of numbers of ± 45 lamina is beneficial to decrease of stress concentration, the larger eccentricity of hole cause more serious stress concentration. According to their solution even though the stress concentration becomes more serious with the increase of ellipticity, there is a hole of specific ellipticity which causes the smallest stress concentration and in general, it is not a circular hole.

Dirikolu and Aktaş (Dirikolu & Aktaş, 2000) presented a comparative study regarding the determination of stress intensity factors. Based on their experimental data they obtained, different analytical approaches and finite element (FE) results obtained from a finite element fracture analysis tool called FRANC were compared. They showed that the average Stress criterion and FE analysis predict decreasing SIF values as the radius of the hole increase. From their results, it is seen that for infinite plates containing respective central circular holes whose diameters are 4-6 and 8 mm, SCF was around 4, 5 for specified laminate they examined.

Durelli et al. (Durelli, Parks, & Feng, 1966) presented a photoelastic distribution of stresses around a centrally located elliptical hole in a plate of finite width subjected to uniform axial loading in their study. they performed 16 different experiments to investigate SCF. They showed that As the width of the testing specimen becomes narrower, the SCF increases over the whole range of λ (Aspect ratio of the opening) .They compared Curves of SCF versus λ they obtained with H. Neuber's approximate theoretical solution and get an 20% of deviation.

Ueng (Ueng, 1976) determined the stress concentration factors around a circular hole in a composite four-layer (-45/45/45/-45) graphite epoxy laminate. He determined the factors experimentally by means of electrical resistance strain gages and analytically by using hybrid finite element analysis. He showed that SCF of composite plate loaded tension near the hole at $\theta = \pm 90$ was around 5.8 which is considerably higher than the classical factor 3 for an infinite, isotropic plate.

Abu-Farsakh et al. (Abu-Farsakh, Almasri*, & Qa'dan, 2015) have investigated the effect of d/W and L/W ratios (L is plate length) on stresses induced in a plate which is subjected to in-plane axial tensile loading. They studied linear and nonlinear material effects numerically and proposed a new method to incorporate the nonlinearity model into the ANSYS computer program. They have found that, as the plate aspect ratio (L/W) and the hole size ratio (d/b) increase, the influence of nonlinear material behavior becomes more pronounced and has a significant effect on increasing the SCFs.

Özaslan (Özaslan, 2019) has investigated SCFs on multidirectional laminates containing single and double circular holes by using FEA. He also asserted an Extended Point Stress Criteria (EPSC) for prediction of the strength of the perforated laminates for the cases that Point Stress Criteria (PSC) is not applicable which is normally used for the prediction of the strength of laminates containing single holes by modifying PSC. Özaslan showed that when W/d is greater than 6, laminates behave as if they are infinite in terms of SCFs.

Karakılçık et al. (Karakılçık, Özbay, & Adin, 2016) have investigated the effect of fiber reinforcement angle and hole diameter to SCF by using finite element analysis. They obtained the results that as the fiber reinforcement angle increases the stress concentration factor at the vicinity of the hole decreases. They also showed the hole diameter effect to stress concentration, and it is seen that their overall results were quite similar to those obtained by Tan (Tan, 1994).

3. THEORETICAL BACKGROUND

In this section, the theoretical background to be able to get the elastic mechanical properties of multidirectional laminates which consist of unidirectional plies oriented in different directions will be presented. It is seen that micromechanical analysis of laminate composite creates more precise results when compared to micromechanics so, micromechanical analysis of laminate composites will be used in this project. According to micromechanical analysis mechanical the properties of plies used in laminate should be known. See the levels of observation and types of analysis for composite materials in Figure 3.1 (Daniel & Ishai, 2006).

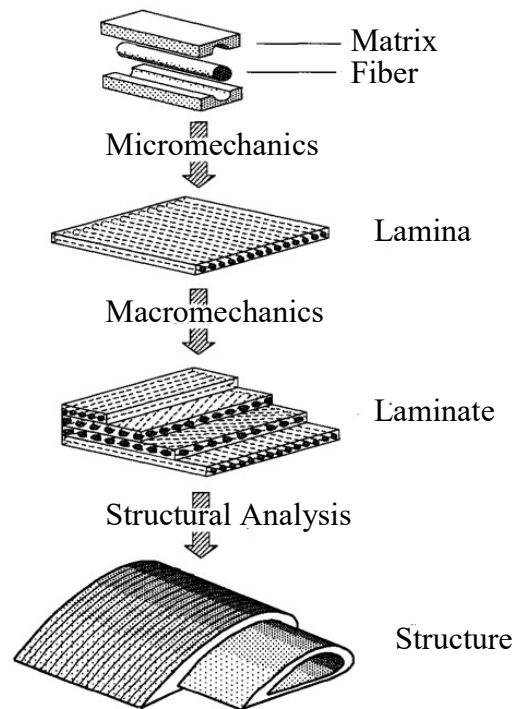


Figure 3.1 Levels of observation and types of analysis for composite materials (Daniel & Ishai, 2006)

3.1 Analysis of an Orthotropic Lamina

3.1.1 Stress-Strain Relations and Engineering Constants

Before going through the mechanics of UD lamina is the principal material axis that comes to the picture in the mechanics of UD lamina. There basically 3 principal material axis in UD laminas or laminates these are L(longitudinal) or 1 axis which represents the fiber direction, T(Transverse) or 2 axis which represent the direction perpendicular to L axis and thickness direction and T' or 3 axis which represents the axis perpendicular to the others (Figure 3.2).

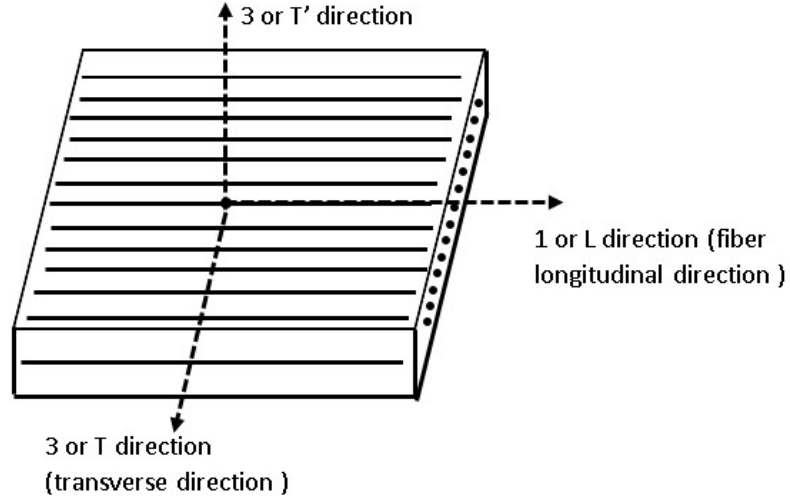


Figure 3.2 Principal axis of UD composite

Due to structure UD composites, they show orthotropic (specially orthotropic) behavior when loaded in principal directions L, T and T') which means if we applied extensional stress in one of the principal directions it would only generate extensional strains likewise if we applied shear stress in a specific plane it would only generate shear strain in the material. Thus, strains generated under these circumstances are found via formulas that are also valid in isotropic materials. In Figures 3.3 and 3.4 UD composite plate loaded in tension in one of its principal axis and UD composite plate loaded in shear in principal material direction are shown respectively.

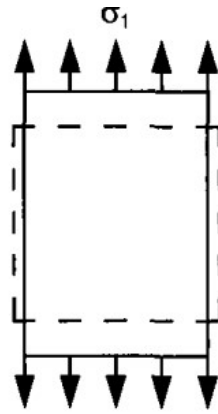


Figure 3.3 UD composite plate loaded in tension in principal material direction (dotted lines shows the deformed shape)

Thus, for the uniaxially loaded UD composite;

$$\varepsilon_1 = \frac{\sigma_1}{E_1} \quad (3.1)$$

$$\varepsilon_2 = -\frac{\nu_{12}\sigma_1}{E_1} \quad (3.2)$$

$$\gamma_{12} = 0 \quad (3.3)$$

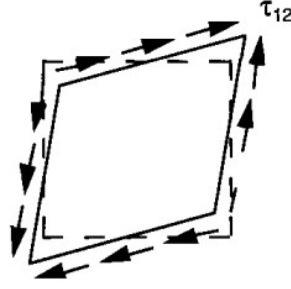


Figure 3.4 UD composite plate loaded in shear in principal material direction (dotted lines shows the deformed shape)

Likewise, strains generated for the orthotropic plate loaded in shear can be given as ;

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} \quad (3.4)$$

$$\varepsilon_1 = \varepsilon_2 = 0 \quad (3.5)$$

By using the superposition rule, the equations for the case of $\sigma_L \neq 0$, $\sigma_T \neq 0$ and $\tau_{LT} \neq 0$ are given in the matrix form as follows.

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} * \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} \quad (3.6)$$

When the reference axes (axis of loading) are different from the lamina axes of symmetry, the lamina is called a generally orthotropic lamina (Figure 3.5). In this circumstance similar to the anisotropic material, extensional forces would generate not only extensional strain but also shear strain and it is also the same for shear force. Since the extensional and shear strain are coupled, in addition to the usual engineering constants associated with the x and y axes (E_x , E_y , G_{xy} , ν_{xy} and ν_{yx}), cross-coefficients m_x and m_y are also required to relate stresses and strains.

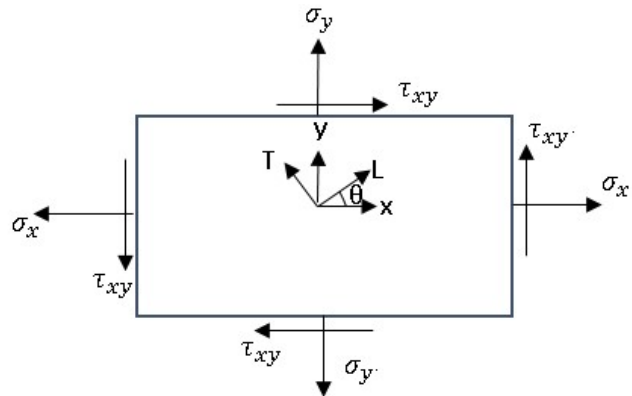


Figure 3.5 Generally orthotropic lamina with applied stresses

In view of the assumptions of linearly elastic material, by applying a similar procedure and applying superposition as in Eqn 3.6 the equations for the case of $\sigma_x \neq 0$, $\sigma_y \neq 0$ and $\tau_{xy} \neq 0$ for generally orthotropic lamina, are given in the matrix form as follows (Bhagwan D., Lawrence J., & K., 2006);

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & -\frac{m_x}{E_L} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{m_y}{E_L} \\ -\frac{m_x}{E_L} & -\frac{m_y}{E_L} & \frac{1}{G_{xy}} \end{bmatrix} * \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (3.7)$$

By using the stress transformation equations normal and shearing stress along are obtained as:

$$\begin{aligned} \sigma_L &= \sigma_x \cos^2 \vartheta \\ \sigma_T &= \sigma_x \sin^2 \vartheta \\ \tau_{LT} &= -\sigma_x \sin(\theta) \cos(\theta) \end{aligned} \quad (3.8)$$

By plugging Eqn. 3.8 into the Eqn. 3.6, the strains in the longitudinal and transverse directions are

$$\begin{aligned} \varepsilon_x &= \sigma_x \left(\frac{\cos^2 \theta}{E_L} - \nu_{TL} \frac{\sin^2 \theta}{E_T} \right) \\ \varepsilon_T &= \sigma_x \left(\frac{\sin^2 \theta}{E_T} - \nu_{LT} \frac{\cos^2 \theta}{E_L} \right) \\ \gamma_{LT} &= -\frac{\sigma_x \sin \theta \cos \theta}{G_{LT}} \end{aligned} \quad (3.9)$$

By using the inverse of the strain-transformation law, the strains in the x and y directions can be written in the expanded form as

$$\begin{aligned} \varepsilon_x &= \varepsilon_L \cos^2 \theta + \varepsilon_T \sin^2 \theta - \gamma_{LT} \sin \theta \cos \theta \\ \varepsilon_y &= \varepsilon_L \sin^2 \theta + \varepsilon_T \cos^2 \theta + \gamma_{LT} \sin \theta \cos \theta \\ \gamma_{xy} &= 2(\varepsilon_L - \varepsilon_T) \sin \theta \cos \theta + \gamma_{LT} (\cos^2 \theta - \sin^2 \theta) \end{aligned} \quad (3.10)$$

By solving the Eqns 3.8-10 together elastic properties along x and y axis is obtained as follows:

$$\begin{aligned} \frac{1}{E_x} &= \frac{\cos^4 \theta}{E_L} + \frac{\sin^4 \theta}{E_T} + \frac{1}{4} \left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L} \right) \sin^2 2\theta \\ \frac{1}{E_y} &= \frac{\sin^4 \theta}{E_L} + \frac{\cos^4 \theta}{E_T} + \frac{1}{4} \left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_L} \right) \sin^2 2\theta \\ \frac{1}{G_{xy}} &= \frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \left(\frac{1}{E_L} + \frac{2\nu_{LT}}{E_L} + \frac{1}{E_T} - \frac{1}{G_{LT}} \right) \cos 2\theta \end{aligned} \quad (3.11)$$

Finally, by solving Eqns 3.7, 3.8 and 3.10 together cross-coefficients m_x (that relates the shearing strain to the normal stress σ_x) and m_y (that relates the shearing strain to normal stress σ_y) are defined as

$$m_x = \sin 2\theta \left[v_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}} - \cos^2 \theta (1 + 2v_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}}) \right] \quad (3.12)$$

$$m_y = \sin 2\theta \left[v_{LT} + \frac{E_L}{E_T} - \frac{E_L}{2G_{LT}} - \sin^2 \theta (1 + 2v_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}}) \right] \quad (3.13)$$

To better evaluate the variation of the elastic constants, they can be plotted as functions of orientation (θ) for specific materials. As an example, variations in E_x , G_{xy} , v_{xy} ($v_{xy} = -\frac{\epsilon_y}{\epsilon_x}$), and m_x , with change in θ are shown in Figure 3.6 for glass-epoxy lamina (Bhagwan D., Lawrence J., & K., 2006).

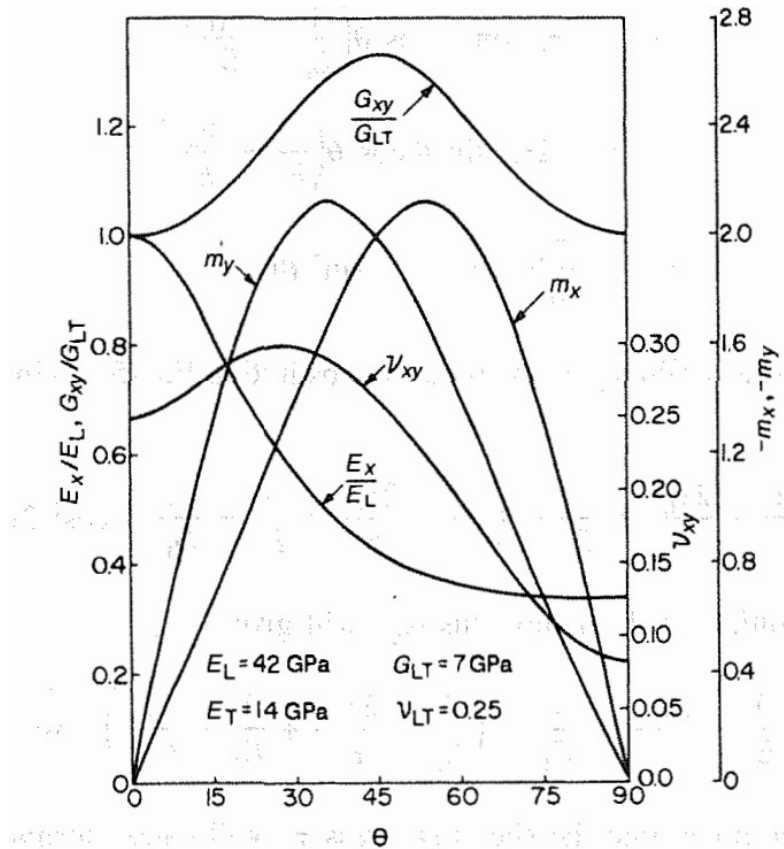


Figure 3.6 Elastic constants of a glass-epoxy lamina: variation with fiber orientation

3.1.2 Hooke's Law and Stiffness and Compliance Matrices

The generalized Hooke's law relating stresses to strains can be written in contracted notation (see Appendix 1) as

$$\sigma_i = C_{ij}\varepsilon_j \quad i, j = 1, \dots, 6 \quad (3.14)$$

where σ_i are the stress components in x, y, and z (or the 1, 2, and 3) coordinates in Figure 3.7, likewise ε_j is the strain components in the same coordinate system.

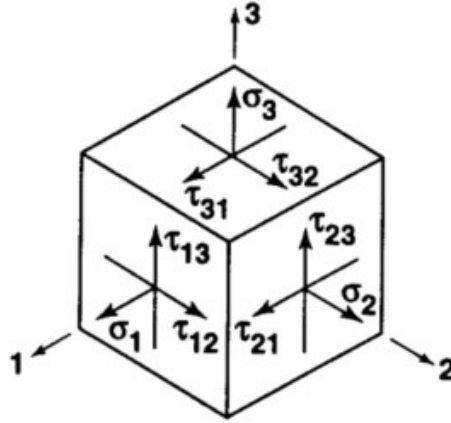


Figure 3.7 Stresses on an element

In Equation 3.14, C represent the stiffness matrix of the material and it has 36 constants. However, less than 36 of the constants is independent for elastic materials when important characteristics of the strain is considered. Namely, when an incremental work is thought on an elastic body since the order of differentiation of W is immaterial, so

$$C_{ij} = C_{ji} \quad (3.15)$$

Thus, the stiffness matrix is symmetric, so only 21 of the constants independent. So, the generalized expression (characterizing materials without isotropy) within the framework of elasticity can be shown as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} \quad (3.16)$$

It has been shown that in especially orthotropic materials (orthotropic material loaded in its material axis) there is no interaction between normal stresses and shearing stresses or in other words they have 2 orthogonal planes of material property. The coefficients that couple normal stresses and shearing strains; and the coefficients that couple shearing stresses and normal

strains should be zero. Thus, the number of coefficients reduces to 9 and the Hook's law for specially orthotropic material can be given as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} \quad (3.17)$$

For the case $\sigma_{33} = \tau_{13} = \tau_{23} = 0$ for specially orthotropic material the equation becomes

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (3.18)$$

Where Q_{ij} are the so called reduced stiffness for plane stress and by comparing Equations 3.17 and 3.18 they are calculated as

$$Q_{ij} = C_{ij} - \frac{C_{i3}C_{j3}}{C_{33}} \quad (2.19)$$

The expression can be also expressed in the form of

$$[\sigma]_{12} = [Q]\{\varepsilon\}_{12} \quad (3.20)$$

Where indices (12) designate the plane stress applied.

In the expression 3.20, dependent variable can $\{\varepsilon\}_{12}$ can be left alone. So,

$$\begin{aligned} [S][\sigma]_{12} &= \{\varepsilon\}_{12} \\ [Q]^{-1} &= [S] \end{aligned} \quad (3.21)$$

In the inverse Equation 3.21, Q matrix is called as compliance matrix and it is designated as $[S]$. So the equation is

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (3.22)$$

The relations 3.18 and 3.22 above can be expressed in terms of engineering constants by noting that.

$$S_{11} = \frac{1}{E_L}$$

$$\begin{aligned}
S_{22} &= \frac{1}{E_T} \\
S_{12} &= -\frac{\nu_{LT}}{E_L} = -\frac{\nu_{TL}}{E_T} \\
S_{66} &= \frac{1}{G_{LT}}
\end{aligned} \tag{3.23}$$

and;

$$\begin{aligned}
Q_{11} &= \frac{E_L}{1 - \nu_{LT}\nu_{TL}} \\
Q_{22} &= \frac{E_T}{1 - \nu_{LT}\nu_{TL}} \\
Q_{12} &= -\frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}} = -\frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}} \\
S_{66} &= G_{LT}
\end{aligned} \tag{3.24}$$

3.1.3 Transformation of [Q] and [S] Matrices

If L, T directions are not aligned to the load direction which is generally the case since a composite material structure is constructed by stacking several unidirectional laminae in a specified sequence of orientation. Stresses and strains can be transformed into another set of axis. The stress transformation equation can be written as (Bhagwan D., Lawrence J., & K., 2006).

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = [T_1] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{3.25}$$

Where the stress-transformation matrix $[T_1]$ is

$$[T_1] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{3.26}$$

The strain transformation equations can be written as

$$\begin{Bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{Bmatrix} = [T_2] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \tag{3.27}$$

Where the stress-transformation matrix $[T_2]$ is;

$$[T_2] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{3.28}$$

To obtain transformed stiffness matrix, we first multiply both sides of Equation 3.25 by inverse of $[T_1]$ so we get

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T_1]^{-1} \begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} \quad (3.29)$$

Substitution of Equation 3.18 into 3.29 gives

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T_1]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} \quad (3.30)$$

Now, the substitution of Equation 3.27 to 3.30 gives

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T_1]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T_2] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \quad (3.31)$$

Thus, equation 3.31 gives stress-strain relation for an orthotropic lamina for arbitrary axes. For the purpose of uniformity $[\bar{Q}]$ is defined that relates stresses to engineering strains. So, Equation 3.31 becomes;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \quad (3.32)$$

By comparing the Equations 3.31 and 3.32 elements of $[\bar{Q}]$ matrix are given as

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta \\ \bar{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta \\ \bar{Q}_{12} &= (Q_{11}+Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta) \\ \bar{Q}_{66} &= (Q_{11}+Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta) \\ \bar{Q}_{16} &= (Q_{11}-Q_{12} - 2Q_{66})\cos^3\theta\sin\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta \end{aligned} \quad (3.33)$$

Similarly, inverse stress-strain relations referring to xy axis can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (3.33)$$

In a similar manner followed in Equation 3.33 to get elements of $[\bar{Q}]$ matrix in terms of elements of $[Q]$ matrix, elements of $[\bar{S}]$ matrix in terms of the elements of the compliance matrix is obtained as follows:

$$\begin{aligned} \bar{S}_{11} &= S_{11}\cos^4\theta + S_{22}\sin^4\theta + 2(S_{12} + 2S_{66})\sin^2\theta\cos^2\theta \\ \bar{S}_{22} &= S_{11}\sin^4\theta + S_{22}\cos^4\theta + 2(S_{12} + 2S_{66})\sin^2\theta\cos^2\theta \\ \bar{S}_{12} &= (S_{11}+S_{22} - 4S_{66})\sin^2\theta\cos^2\theta + S_{12}(\cos^4\theta + \sin^4\theta) \end{aligned} \quad (3.33)$$

$$\begin{aligned}\bar{S}_{66} &= (S_{11} + S_{22} - 2S_{12} - 2S_{66})\sin^2\theta\cos^2\theta + S_{66}(\sin^4\theta + \cos^4\theta) \\ \bar{S}_{16} &= (S_{11} - S_{12} - 2S_{66})\cos^3\theta\sin\theta - (S_{22} - S_{12} - 2S_{66})\cos\theta\sin^3\theta \\ \bar{S}_{26} &= (S_{11} - S_{12} - 2S_{66})\cos\theta\sin^3\theta - (S_{22} - S_{12} - 2S_{66})\cos^3\theta\sin\theta\end{aligned}$$

3.2 Analysis of Laminated Composites

The most important advantage of fibrous composites is that their anisotropy or properties can be controlled by altering the material and manufacturing variables. While the longitudinal properties of unidirectional composites are controlled by fiber properties, transverse properties of unidirectional composites matrix dominated. The transverse properties of unidirectional composites are found to be unsatisfactory in most engineering applications. Thus, laminates made up of several laminae with different fiber orientations are widely used rather than unidirectional composites. In this part, laminates analysis will be presented within the framework of the following assumptions and restrictions.

1. Each lamina of the laminate is orthotropic
2. The laminate is thin and the laminate and its layers (except for their edges) are in a state of plane stress ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$)
3. All displacements are small compared with the thickness of the laminate
4. Displacements are continuous throughout the laminate
5. In-plane displacements vary linearly through the thickness
6. Lines normal to the middle surface remain straight and normal to that surface after deformation. ($\gamma_{xz} = \gamma_{yz} = 0$)
7. Strain-displacement and stress-strain relations are linear.
8. Normal distances from the middle surface remain constant ($\varepsilon_z = 0$)

3.2.1 Laminate Strains

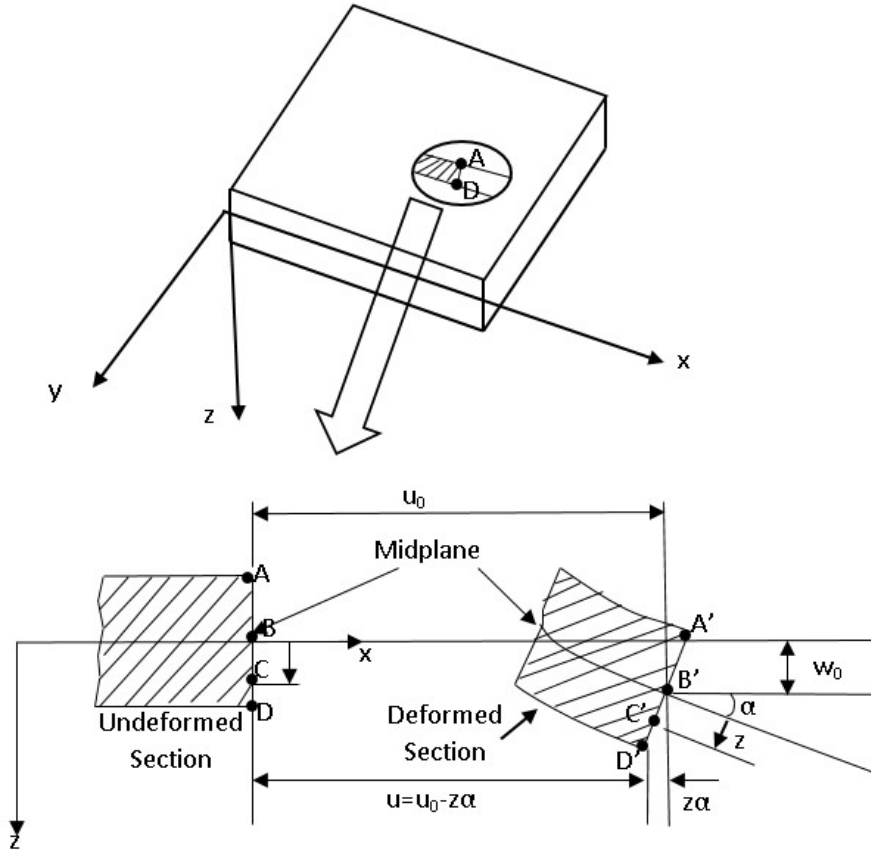


Figure 3.8 Deformation of a line element during bending of the laminate in the xz plane.

Considering the deformation in Figure 3.8 and also further assuming that the point B at the geometric midplane undergoes displacements u_0 , v_0 , and w_0 along x , y , and z directions, respectively, the preceding strain displacement relation can be written in terms of the midplane strains and the plate curvatures as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \quad (3.34)$$

Where the midplane strains are

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma u_0}{\sigma_x} \\ \frac{\sigma v_0}{\sigma_y} \\ \frac{\sigma u_0}{\sigma_y} + \frac{\sigma v_0}{\sigma_x} \end{Bmatrix} \quad (3.35)$$

And the plate curvatures are

$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma^2 w_0}{\sigma x^2} \\ \frac{\sigma^2 w_0}{\sigma y^2} \\ 2 \frac{\sigma^2 w_0}{\sigma x \sigma y} \end{Bmatrix} \quad (3.36)$$

3.2.2 Variation of Stresses in a Laminate

Strains at any point in a laminate can be calculated via Equation 3.36 and they vary linearly across the thickness because of the assumption that the adjacent laminae do not slip over each other. Stresses at any point in the k_{th} layer can be obtained by substituting Equation 3.34 in the stress-strain relation [Equation 3.32] for the lamina as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}_k \quad (3.37)$$

Since the stiffness matrix $[\bar{Q}]$ is a constant for each lamina in the Equation 3.37, it gives linear stress variation across lamina thickness. However, in a laminate, laminae usually have different elastic properties because of different fiber orientations. So, stress variation across the total laminate thickness will be composed of several linear segments. So the variation of stress and strain in a hypothetical three-ply laminate can be drawn as follows (Figure 3.9).

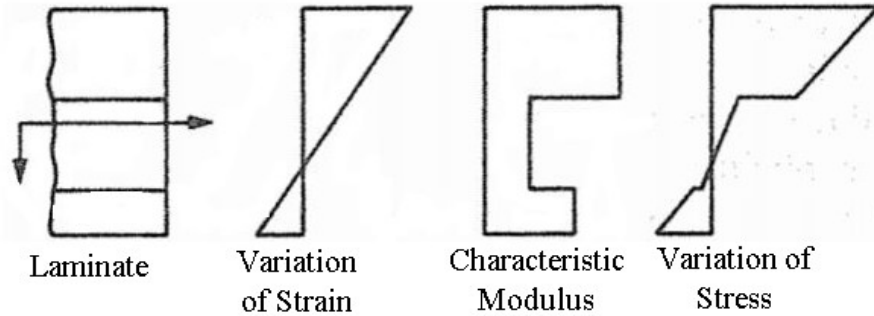


Figure 3.9 Variations of strain and stress

3.2.3 Resultant Force and Moments & ABD matrix

Resultant forces and moments (Appendix 2) acting on a laminate are calculated by corresponding stress through laminate thickness h (Figure 3.10) and corresponding stress times the moment arm with respect to the midplane, respectively.

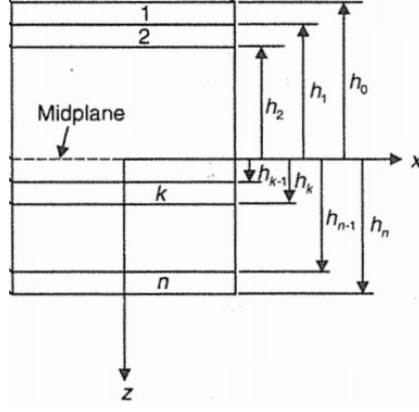


Figure 3.10 Description of a multilayered laminate geometry

. Those expressions can be written in terms of midplane strains and plate curvatures from Equation 3.37 and thus the resultant forces and moments are expressed as: (Bhagwan D., Lawrence J., & K., 2006).

$$\begin{aligned} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \end{aligned} \quad (3.38)$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k - h_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3) \end{aligned} \quad (3.39)$$

Combining both Equations in 3.38 total plate constitutive equation can be written as follows:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k^0 \end{Bmatrix} \quad (3.40)$$

The matrices A, B, and D are called extensional stiffness matrix, coupling stiffness matrix, and bending stiffness matrix respectively. So, these matrices ([A], [B], [D]) connect force and moment resultants to midplane strains and curvatures in a laminate. The presence of B matrix in the plate constitutive equation (Eq.3.40) means that normal and shear forces acting at the midplane of the plate cause not only the in-plane deformations, leading to the midplane strains, but also twisting and bending (produce plate curvatures) and vice versa. Couplings discussed usually are undesirable because of the fact that they may induce plate unwanted curvature and

assembly stresses. Some special laminate constructions reduce means critical elements of the stiffness matrix to zero [B] matrix is zero for symmetric laminates which means bending and extension is no more coupled: or angle ply laminates makes A_{16} and A_{26} zero that means no coupling exists between extensional force and shearing force like (orthotropic material). For further information for construction properties of laminate, the reader is advised to look at reference books given at the end of the report.

3.2.4 Laminate Engineering Properties

3.2.4.1 Symmetric Balanced Laminates

In-plane engineering properties of a symmetrically balanced laminate can be obtained formulas: (Daniel & Ishai, 2006).

$$\begin{aligned}\bar{E}_x &= \frac{1}{h} \left[A_{11} - \frac{A_{12}^2}{A_{22}} \right] \\ \bar{E}_y &= \frac{1}{h} \left[A_{22} - \frac{A_{12}^2}{A_{11}} \right] \\ \bar{\nu}_{xy} &= \frac{A_{12}}{A_{22}} \\ \bar{\nu}_{yx} &= \frac{A_{12}}{A_{11}} \\ \bar{G}_{xy} &= \frac{A_{66}}{h}\end{aligned}\tag{3.41}$$

Where h is the laminate thickness \bar{E}_x , \bar{E}_y , $\bar{\nu}_{xy}$, $\bar{\nu}_{yx}$, and \bar{G}_{xy} are effective laminate properties.

3.2.4.2 General Laminates

For general laminates, the elements of extensional laminate compliance matrix is used which is

$$[a] = [A^{-1}]\tag{4}$$

So,

$$\begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1}\tag{3.42}$$

and following relations are used for deriving in plane engineering properties of general laminates:

$$\begin{aligned}\bar{E}_x &= \frac{1}{ha_{11}} \\ \bar{E}_y &= \frac{1}{ha_{22}} \\ \bar{\nu}_{xy} &= -\frac{a_{12}}{a_{11}}\end{aligned}\tag{3.43}$$

$$\bar{v}_{yx} = -\frac{a_{12}}{a_{22}}$$

$$\bar{G}_{xy} = \frac{1}{ha_{66}}$$

Where h is the laminate thickness \bar{E}_x , \bar{E}_y , $\bar{\nu}_{xy}$, $\bar{\nu}_{yx}$, and \bar{G}_{xy} are effective laminate properties.

3.3. SCF for Composite Plate

After obtaining global laminate properties and ABD matrices, an analytical solution for calculating SCF for a composite plate containing elliptical hole loaded in tension is calculated by analytical solutions available in the literature. The ratio of width(a) to length(b) of the elliptical hole (Figure 3.11) affect SCF of the plate significantly.

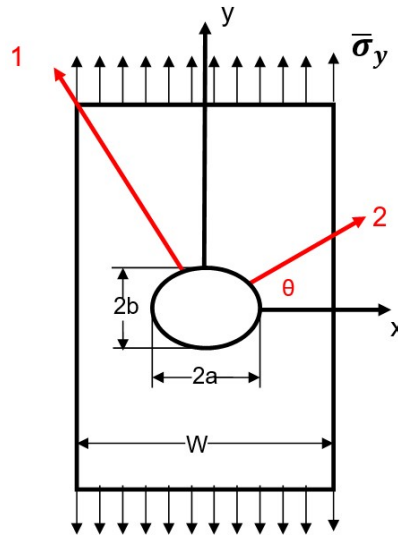


Figure 3.11 Finite-width plate containing a central elliptical hole

When $a/b \geq 4$ following finite-width correction formula is used (Tan, 1994)

$$\begin{aligned} \frac{K_T^\infty}{K_T} = & \frac{\lambda^2}{(1-\lambda)^2} + \frac{(1-2\lambda)}{(1-\lambda)^2} \sqrt{1 + (\lambda^2 - 1)(2a/W)^2} - \frac{\lambda^2}{(1-\lambda)} \frac{(2a/W)^2}{\sqrt{1 + (\lambda^2 - 1)(2a/W)^2}} \\ & + \frac{\lambda^7}{2} \left(\frac{2a}{W}\right)^6 \left(K_T^\infty - 1 - \frac{2}{\lambda}\right) \left\{ \left[1 + (\lambda^2 - 1) \left(\frac{2a}{W}\right)^2 \right]^{-5/2} \right. \\ & \left. - \left(\frac{2a}{W}\right)^2 \left[1 + (\lambda^2 - 1) \left(\frac{2a}{W}\right)^2 \right]^{-7/2} \right\} \end{aligned} \quad (3.44)$$

where $\lambda=b/a$. For the special case $\lambda=1$ (circular hole), L'Hospital rule is employed twice to formulas and following equation is obtained by substituting $\lambda=1$. (Tan, 1994)

$$\frac{K_T^\infty}{K_T} = \frac{\left[2 - \left(\frac{2a}{W}\right)^2 - \left(\frac{2a}{W}\right)^4 \right]}{2} + \frac{\left(\frac{2a}{W}\right)^6 (K_T^\infty - 3) \left[1 - \left(\frac{2a}{W}\right)^2 \right]}{2} \quad (3.45)$$

where K_T^∞ SCF for an infinite plate having an elliptical hole and ;

These solutions do not show good accuracy for predicting SCF for the cases that $a/b < 4$. Tan improved Lekhnitskii' s solution by multiplying $2a/W$ in the Lekhnitskii' s solution by a magnification factor(M) for the cases $a/b < 4$. So, the following relations were developed. (Tan, 1994)

$$\begin{aligned} \frac{K_T^\infty}{K_T} = 1 - \frac{2a}{W}M + Re \left\{ \frac{1}{\mu_1 - \mu_2} \left[\frac{-\mu_2}{1 + i\mu_1\lambda} \left(\frac{2a}{W}M - 1 + i\mu_1\lambda \left(\frac{2a}{W}M \right) \right. \right. \right. \\ \left. \left. + \sqrt{1 - (1 + \mu_1^2\lambda^2) \left(\frac{2a}{W}M \right)^2} \right) + \frac{\mu_1}{1 + i\mu_2\lambda} \right. \\ \left. \left. * \left(\frac{2a}{W}M - 1 + i\mu_2\lambda \left(\frac{2a}{W}M \right) + \sqrt{1 - (1 + \mu_2^2\lambda^2) \left(\frac{2a}{W}M \right)^2} \right) \right] \right\} \end{aligned} \quad (3.46)$$

where,

$$M^2 = \frac{\sqrt{1 - 8 \left[\frac{3 \left(1 - \frac{2a}{W} \right)}{2 + \left(1 - \frac{2a}{W} \right)^3} - 1 \right]} - 1}{2 \left(\frac{2a}{W} \right)^2} \quad (3.47)$$

Here μ_1 and μ_2 are described as the root of the following formula whose imaginary part is positive (Tan, 1994).

$$a_{22}\mu^4 - 2a_{26}\mu^3 + (2a_{11} + a_{66})\mu^2 - 2a_{16}\mu + a_{11} = 0 \quad (3.47)$$

Where a_{ij} , $i,j=1,2,6$ are elements of the laminate compliance matrix obtain in Equation 3.42.

When, a similar procedure which is used to obtain Equation 3.45 is applied to Equation 3.46, the following finite width correction factor equation which is used to obtain determine SCF for the plate having an elliptical hole ($a/b < 4$) is obtained. (Tan, 1994)

$$\frac{K_T^\infty}{K_T} = \frac{3(1 - \frac{2a}{W})}{2 + \left(1 - \frac{2a}{W} \right)^3} + \frac{1}{2} \left(\frac{2a}{W}M \right)^6 (K_T^\infty - 3) \left[1 - \left(\frac{2a}{W}M \right)^2 \right] \quad (3.48)$$

To obtain infinite plate SCF following equation is used (Tan, 1994):

$$K_T^\infty = 1 + \frac{1}{\lambda} \sqrt{\frac{2}{A_{66}} \left[\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}} \right]} \quad (3.49)$$

Where A_{ij} $i,j=1,2,6$ is elements of the A matrix.

By multiplying finite width correction factors found in Equations 3.46 and 3.48 by the result of Equation 3.49 SCF for plate containing an elliptical hole is obtained. (Tan, 1994).

4. RESULTS

4.1 Experimental Comparison

Using the experimental data presented in (Durelli, Parks, & Feng, 1966), the prediction of the maximum tangential stress concentration of a finite-width isotropic plate containing an elliptical opening under uniaxial loading is examined in Figure 4.1.

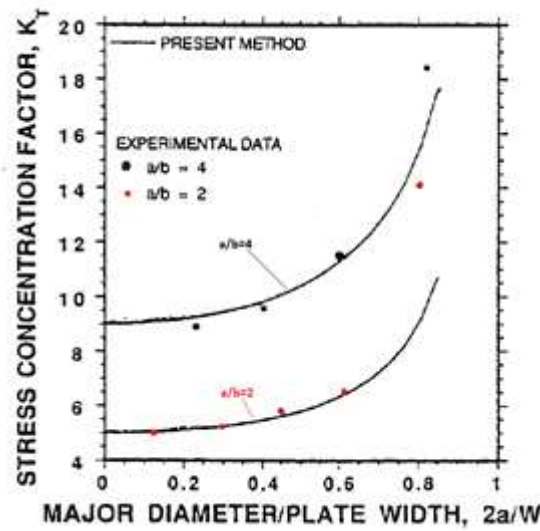


Figure 4.1 The maximum tangential stress concentration of an isotropic finite-width plate containing an elliptical opening (Tan, 1994).

The experimental comparison shows that the present basic approach, Equation (3.44) is highly accurate for $a/b > 1$ with $2a/W < 0.6$ and improved theory, Equation (3.45), has excellent agreement with the data for $a/b < 4$.

4.2 Finite Element Comparison

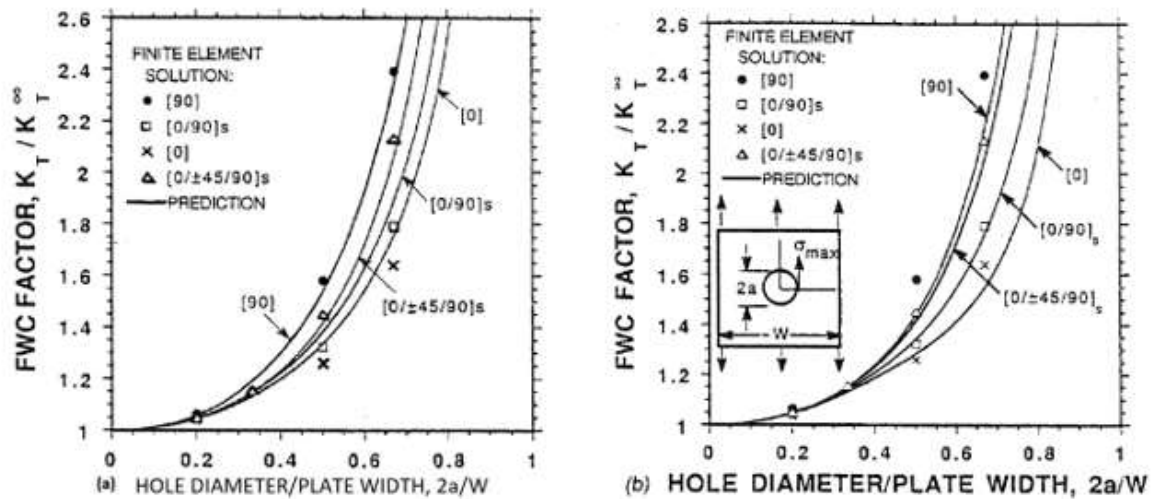


Figure 4.2 Comparison of FWC factors of maximum tangential stress of orthotropic laminate with a circular hole using the improved solution: (a) Equation 3.46; (b) Equation (3.48) and the finite element solution (Tan, 1994).

Table 4.1 Elastic Properties of Typical Graphite/Epoxy Laminae (Tan, 1994)

| Parameter | Example 1 | Example 2 (T300/5208) |
|--------------------------------------|-------------|--------------------------|
| Longitudinal Young's modulus, E_L | 146.900 GPa | 147.548 GPa |
| Transverse Young's modulus, E_T | 10.890 GPa | 11.032 GPa |
| In-Plane shear modulus, G_{LT} | 6.412 GPa | 5.309 GPa |
| In-plane Poisson's ratio, ν_{LT} | 0.38 | 0.29 |

The prediction for the FWC factors of orthotropic laminates containing a circular hole are compared to the finite-element solution [4] in Figure 4.2. The material properties, Example 1, are listed at Table 4.1. It is shown that both improved anisotropic FWC factor, Equation (3.46), and the improved orthotropic solution, Equation (3.48), agree highly accurately with the finite-element solution. Even when $2a/W$ ratio is over 90%, the predictions are still in excellent agreement with the finite element solution. (Table 4.2)

Table 4.2 Comparison of the Improved Theory and the Finite Element Solution [for Anisotropic Laminates] (Example 1 in Table 1) with a Circular Hole and $2a/W=0.91$. (Tan, 1994).

| Laminate Layup | [0/±45/90] _s | [0] | [90] | [0/90] _s |
|---------------------|-------------------------|--------|--------|---------------------|
| FWC, 4.24 | 7.4101 | 3.9393 | 8.5554 | 5.0844 |
| FWC, 4.21 | 7.4101 | 4.6263 | 8.8426 | 5.5033 |
| FWC, Finite Element | 7.5556 | 4.5084 | 8.9068 | 5.4393 |

The two types of solution are used to calculate the stress concentration factor for the composite plate. The finite plate has a stress concentration formulation for two methods. The basic solution has good accuracy for $a/b \geq 4$, while the improved theory has excellent accuracy for the domain $a/b < 4$. For the $a/b \geq 5$, the influences of anisotropy upon the FWC factor vanish. The program is prepared with these conditions in mind. According to user input, the program detects the aspect ratio and width-hole diameter ratio and selects the proper formulation for that user input hole configuration.

The comparison of Özaslan's thesis which is prepared for the examination of the stress concentration factor for a plate with a circular hole is given at Table 4.3.

Table 4.3 Comparison of Results with Özaslan and program (Özaslan, 2019).

| W/D | Finite Element Analysis SCF | Program Outputs SCF | Differences % |
|-----|-----------------------------|---------------------|---------------|
| 12 | 3.03 | 3.02 | 0.33% |
| 6 | 3.11 | 3.09 | 0.65% |
| 3.6 | 3.32 | 3.29 | 0.91% |
| 3 | 3.48 | 3.44 | 1.16% |

The following formula is used to calculate the difference between program outputs and Finite Element analysis results in (Özaslan, 2019).

$$Fark(\%) = \frac{|SCF_{analiz} - SCF_{program}|}{SCF_{program}} * 100 \quad (4.1)$$

The all-program outputs and numerical results were compared. As a result of the comparisons, the consistency of the program outputs was verified. The present methods that we used have excellent accuracy in their ranges. The program can be used to calculate stress concentration factor for composite plate including central hole without not giving so much effort due to finite element analysis.

4.3 Program Demonstration

The program consists of two parts in the user interface. The yellow sides are the input section, and the red sides are the outputs section. First, the user would type the input value such as; laminae properties, hole configuration, lamina thickness, and stacking ratio. Then, the program calculates the stress concentration factor and ABD matrix of the laminate by using required inputs and related formulas. The program flowchart is given at Figure 4.3. Program is a functional and basic interface for users. The green parts in the program interface are the input parts that the user must enter. The red parts are the result parts that the program offers to the user. When the user moves the mouse over the box he wants to enter, he will see a text explaining what value that box is. This case is available in every box in Figure 4.4. The program interface can be seen in Appendix 4.

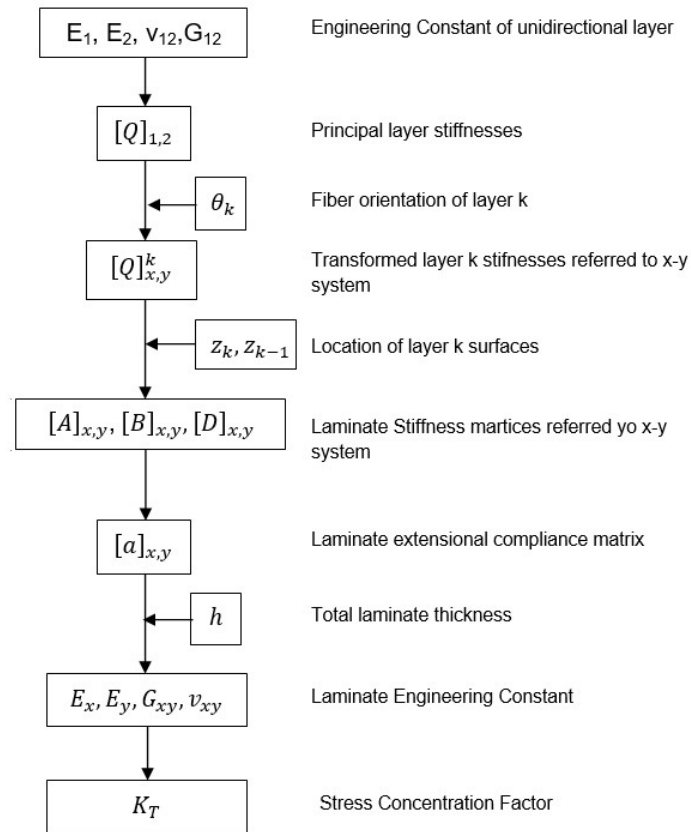


Figure 4.3 Flow Diagram of the program

Lamina Engineering Properties

E₁ GPa

E₂ GPa

G₁₂ GPa

ν₁₂

Thickness mm

Stacking
Top to
bottom

Plate Properties for Stress Concentration

a mm

a/b

W mm

b mm

2a/W

Laminate Properties

E₁ GPa

E₂ GPa

G₁₂ GPa

ν₁₂

Stress Concentration Values

K_{T∞}

FWC Factor

Stress Concentration of Finite-Width Plate

Figure 4.4 Program interface input and output sections

5. EVALUATION OF THE CURRENT WORK FROM MUDEK PERSPECTIVE

5.1. Economic Analysis

In this project, we received a support scholarship within the scope of Tübitak 2209-B. We used our scholarship to upgrade the configurations of the computers we use. Another cost item in our project is that we can get one book for a fee.

5.2. Real Life Conditions

The use of more conservative values than our program outputs will be healthier in terms of the safety of the work to be done. This should be considered because the location of the maximum stress concentration around the hole will always change depending on the configuration.

5.3. Producibility

The program was written at a sufficient level of compatibility within the scope of the graduation thesis. However, by making additions to the program and writing in a different language instead of MATLAB App Designer, the program can be easily used for both commercial and educational purposes. Companies dealing with composites or educational institutions that include composite materials in their curriculum can use the program. The program is very simple to learn and use, thanks to its easy interface. The program can be used online via internet hosting. However, our program is offered as a package program used by users who do not have MATLAB on their computers.

5.4. Constraints

Composite plates in the program are designed according to unidirectional composite plates. The program cannot calculate for particle, woven, or hybrid composite plates. There is no plot showing stress distribution in the program interface. Since there is no study related to failure criteria in the program, the program cannot calculate the strengths of the plates.

6.DISCUSSIONS AND CONCLUSIONS

6.1 Conclusion of the Project

Our project is based on a program that can analytically calculate the stress concentration around the hole in composite plates. Within the scope of the project, analytical studies and formulas in the literature have been investigated. It has been seen that there are different formulas for different hole configurations and these formulas give different accuracy values when compared with numerical and experimental results. It has been optimized according to different aspect ratios for circular and elliptical holes. The results of the formulas used in the program have been compared with the finite element method and the experimental results.

As a result of these comparisons, the findings of this study clearly show that used formulas have accuracy over %95. The elastic engineering properties of the layer can be calculated by entering the stacking sequence and properties of the lamina; the program can give the maximum stress concentration factor.

6.2 Recommendation for Further Works

The program calculates the maximum stress concentration factor in composite plates containing circular or elliptical holes. Our program cannot calculate for elliptical holes in case of bending relative to the principal axis. Calculations suitable for the slanted angle for this ellipse can be integrated into the program. A visual module can be added that shows stress distribution and stress concentration factor for each lamina and overall plate. Calculations can be added for more than one hole and different positions of these holes with respect to each other. Different damage theories can be added to the program, and the strength of the composite plate according to these theories can be determined within the scope of the program.

The program can be taken to a higher level by using different object-oriented programs instead of the MATLAB App designer. Some interface and visual works cannot be done since the MATLAB App designer is a developing program. By rewriting the program in Java, Python, C++, the program can be integrated into ANSYS or CATIA. Thus, the results can be used quickly during the design phase.

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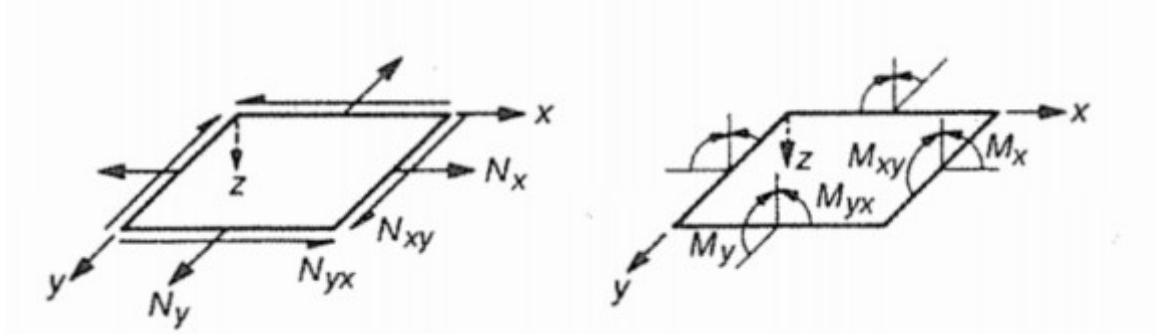
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Appendix 1 Notation of Stresses and Strains

| Stresses | | Strains | |
|---------------------------|---------------------|--|---------------------|
| <i>Tensor Notation</i> | Contracted Notation | <i>Tensor Notation</i> | Contracted Notation |
| $\sigma_{11}(\sigma_1)$ | σ_1 | $\varepsilon_{11}(\varepsilon_1)$ | ε_1 |
| $\sigma_{22}(\sigma_2)$ | σ_2 | $\varepsilon_{22}(\varepsilon_2)$ | ε_2 |
| $\sigma_{33}(\sigma_3)$ | σ_3 | $\varepsilon_{33}(\varepsilon_3)$ | ε_3 |
| $\tau_{23} = \sigma_{32}$ | σ_4 | $\gamma_{23} = 2\varepsilon_{23}$ ¹ | ε_4 |
| $\tau_{31} = \sigma_{31}$ | σ_5 | $\gamma_{31} = 2\varepsilon_{31}$ | ε_5 |
| $\tau_{12} = \sigma_{12}$ | σ_6 | $\gamma_{12} = 2\varepsilon_{12}$ | ε_6 |

¹Note that γ_{ij} represents engineering shear strain whereas $\varepsilon_{ij}(i \neq j)$ represent shear strain

Appendix 2 Sign convention for resultant forces and moments



Appendix 3 Matlab App Designer Codes

```
classdef GraduationApp < matlab.apps.AppBase
```

```
% Properties that correspond to app components
```

```
properties (Access = public)
```

```
    UIFigure                                matlab.ui.Figure
    Image                                    matlab.ui.control.Image
    CalculateButton                          matlab.ui.control.Button
    COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel matlab.ui.control.Label
    ABDMATRIXPanel                          matlab.ui.container.Panel
    Panel_2                                  matlab.ui.container.Panel
    ABD1                                    matlab.ui.control.NumericEditField
    ABD2                                    matlab.ui.control.NumericEditField
    ABD3                                    matlab.ui.control.NumericEditField
    ABD7                                    matlab.ui.control.NumericEditField
    ABD8                                    matlab.ui.control.NumericEditField
    ABD9                                    matlab.ui.control.NumericEditField
    ABD13                                   matlab.ui.control.NumericEditField
    ABD14                                   matlab.ui.control.NumericEditField
    ALabel                                  matlab.ui.control.Label
    ABD15                                   matlab.ui.control.NumericEditField
    GPammLabel                              matlab.ui.control.Label
    Panel_3                                  matlab.ui.container.Panel
    ABD22                                   matlab.ui.control.NumericEditField
    ABD23                                   matlab.ui.control.NumericEditField
    ABD24                                   matlab.ui.control.NumericEditField
    ABD28                                   matlab.ui.control.NumericEditField
    ABD29                                   matlab.ui.control.NumericEditField
    ABD30                                   matlab.ui.control.NumericEditField
    ABD34                                   matlab.ui.control.NumericEditField
    ABD35                                   matlab.ui.control.NumericEditField
    ABD36                                   matlab.ui.control.NumericEditField
    DLabel                                  matlab.ui.control.Label
    NmmLabel                                matlab.ui.control.Label
    Panel_4                                  matlab.ui.container.Panel
    ABD4                                    matlab.ui.control.NumericEditField
    ABD5                                    matlab.ui.control.NumericEditField
    ABD6                                    matlab.ui.control.NumericEditField
    ABD10                                   matlab.ui.control.NumericEditField
    ABD11                                   matlab.ui.control.NumericEditField
    ABD12                                   matlab.ui.control.NumericEditField
    ABD17                                   matlab.ui.control.NumericEditField
    ABD18                                   matlab.ui.control.NumericEditField
    BLabel_3                               matlab.ui.control.Label
    ABD16                                   matlab.ui.control.NumericEditField
    NLabel_2                               matlab.ui.control.Label
    Panel_5                                  matlab.ui.container.Panel
    ABD19                                   matlab.ui.control.NumericEditField
    ABD20                                   matlab.ui.control.NumericEditField
    ABD21                                   matlab.ui.control.NumericEditField
    ABD25                                   matlab.ui.control.NumericEditField
    ABD26                                   matlab.ui.control.NumericEditField
```

| | |
|---|------------------------------------|
| ABD27 | matlab.ui.control.NumericEditField |
| ABD31 | matlab.ui.control.NumericEditField |
| ABD32 | matlab.ui.control.NumericEditField |
| BLabel_2 | matlab.ui.control.Label |
| ABD33 | matlab.ui.control.NumericEditField |
| NLabel | matlab.ui.control.Label |
| LaminaEngineeringPropertiesPanel | matlab.ui.container.Panel |
| Label | matlab.ui.control.Label |
| EL | matlab.ui.control.NumericEditField |
| EEditField_2Label | matlab.ui.control.Label |
| ET | matlab.ui.control.NumericEditField |
| EEditFieldLabel | matlab.ui.control.Label |
| GLT | matlab.ui.control.NumericEditField |
| GEditFieldLabel | matlab.ui.control.Label |
| vLT | matlab.ui.control.NumericEditField |
| vEditField_2Label | matlab.ui.control.Label |
| ThicknessEditField | matlab.ui.control.NumericEditField |
| ThicknessEditFieldLabel | matlab.ui.control.Label |
| StackingToptobottomLabel | matlab.ui.control.Label |
| StackingToptobottomEditField | matlab.ui.control.EditField |
| GPaLabel | matlab.ui.control.Label |
| GPaLabel_2 | matlab.ui.control.Label |
| GPaLabel_3 | matlab.ui.control.Label |
| mmLabel | matlab.ui.control.Label |
| VLTEditFieldLabel_2 | matlab.ui.control.Label |
| VLTEditFieldLabel_3 | matlab.ui.control.Label |
| VLTEditFieldLabel_4 | matlab.ui.control.Label |
| VLTEditFieldLabel_5 | matlab.ui.control.Label |
| LaminatePropertiesPanel | matlab.ui.container.Panel |
| EEditField_3 | matlab.ui.control.NumericEditField |
| EEditField_3Label | matlab.ui.control.Label |
| EEditField_4 | matlab.ui.control.NumericEditField |
| EEditField_4Label | matlab.ui.control.Label |
| GEditField_2 | matlab.ui.control.NumericEditField |
| GEditField_2Label | matlab.ui.control.Label |
| vEditField | matlab.ui.control.NumericEditField |
| vEditFieldLabel | matlab.ui.control.Label |
| VLTEditFieldLabel_6 | matlab.ui.control.Label |
| VLTEditFieldLabel_7 | matlab.ui.control.Label |
| VLTEditFieldLabel_8 | matlab.ui.control.Label |
| VLTEditFieldLabel_9 | matlab.ui.control.Label |
| GPaLabel_4 | matlab.ui.control.Label |
| GPaLabel_5 | matlab.ui.control.Label |
| GPaLabel_6 | matlab.ui.control.Label |
| PlatePropertiesforStressConcetrationPanel | matlab.ui.container.Panel |
| maindia | matlab.ui.control.NumericEditField |
| aEditFieldLabel | matlab.ui.control.Label |
| WEditField | matlab.ui.control.NumericEditField |
| WEditFieldLabel | matlab.ui.control.Label |
| aWEditField | matlab.ui.control.NumericEditField |
| aWEditFieldLabel | matlab.ui.control.Label |
| ab | matlab.ui.control.NumericEditField |
| abEditField_2Label | matlab.ui.control.Label |
| bEditFieldLabel | matlab.ui.control.Label |

```

bEditField matlab.ui.control.NumericEditField
mmLabel_2 matlab.ui.control.Label
mmLabel_3 matlab.ui.control.Label
mmLabel_4 matlab.ui.control.Label
StackingTypesEditFieldLabel matlab.ui.control.Label
StackingTypesEditField matlab.ui.control.EditField
Image2 matlab.ui.control.Image
StressConcentrationValuesPanel matlab.ui.container.Panel
KEditFieldLabel matlab.ui.control.Label
KEditField matlab.ui.control.NumericEditField
FWCFactorEditFieldLabel matlab.ui.control.Label
FWCFactorEditField matlab.ui.control.NumericEditField
StressConcentrationoffFiniteWidthPlateEditFieldLabel matlab.ui.control.Label
StressConcentrationoffFiniteWidthPlateEditField
matlab.ui.control.NumericEditField
Label_3 matlab.ui.control.Label
TLabel matlab.ui.control.Label
TotalThicknessEditField_2Label matlab.ui.control.Label
TotalThicknessEditField matlab.ui.control.NumericEditField
mmLabel_5 matlab.ui.control.Label
Label_2 matlab.ui.control.Label
end

```

```

methods (Access = private)
function QbarM=Qbar(thetad,EL,ET,GLT,VLT)
VTL= VLT*ET/EL;
theta=pi*thetad/180;
C=cos(theta);
S=sin(theta);
Ex=1/((C^4/EL)+(S^4./ET)+0.25*((1/GLT)-(2*VLT/EL))*sin(2*theta)^2);
Ey=1/((S^4/EL)+(C^4/ET)+0.25*((1/GLT)-(2*VLT/EL))*sin(2*theta)^2);
GXY=1/(1/EL+1/ET+(2*VLT/EL)-(1/EL+1/ET+2*VLT/EL-1/GLT)*cos(2*theta)^2);
S11 = 1/EL;
S22 = 1/ET;
S12 = (-VLT/EL);
S66 = 1/GLT;
Q11 = EL/(1-VTL*VLT);
Q22 = ET/(1-VTL*VLT);
Q12 = (VLT*ET)/(1-VLT*VTL);
Q66 = (GLT);
QM=[Q11,Q12,0;Q12,Q22,0;0,0,Q66];
SM=[S11,S12,0;S12,S22,0;0,0,S66];
T1M=[C^2,S^2,2*S*C;S^2,C^2,-2*S*C;-S*C,S*C,C^2-S^2];
T2M=[C^2,S^2,S*C;S^2,C^2,S*C;-2*S*C,2*S*C,C^2-S^2];
Q16=(Q11-Q12-2*Q66)*C^3*S-(Q22-Q12-2*Q66)*C*S^3;
Q26=(Q11-Q12-2*Q66)*C*S^3-(Q22-Q12-2*Q66)*C^3*S;
Qbar66=(Q11+Q22-2*Q12-2*Q66)*S^2*C^2+Q66*(S^4+C^4);
e=inv(T1M)*QM*T2M;
e(3)=Q16;
e(6)=Q26;
e(7)=Q16;
e(8)=Q26;
e(9)=Qbar66;

```

```

QbarM=e;
% SbarM1 =inv(QbarM);
Sbar11=S11*C^4+S22*S^4+2*(2*S12+S66)*S^2*C^2;
Sbar22=S11*S^4+S22*C^4+2*(2*S12+S66)*S^2*C^2;
Sbar12=(S11+S22-S66)*C^2*S^2+S12*(C^4+S^4);
Sbar66=2*(2*S11+2*S22-4*S12-S66)*C^2*S^2+S66*(C^4+S^4);
Sbar16=(2*S11-2*S12-S66)*C^3*S-(2*S22-2*S12-S66)*C*S^3;
Sbar26=(2*S11-2*S12-S66)*S^3*C-(2*S22-2*S12-S66)*S*C^3;
SbarM=[Sbar11,Sbar12,Sbar16;Sbar12,Sbar22,Sbar26;Sbar16,Sbar26,Sbar66];
end
end

```

```

% Callbacks that handle component events
methods (Access = private)

```

```

% Button pushed function: CalculateButton
function CalculateButtonPushed(app, event)

```

```

format short g
% elem1=app.Element1Button.Value;
% elem2=app.Element2Button.Value;
% elem3=app.Element3Button.Value;
% if elem1==1
% EL= 146.9E9;
% ET=10.890E9;
% GLT= 6.412E9;
% VLT= 0.38;
% VTL= VLT*ET/EL;
% app.EL.Value=EL/1E9;
% app.ET.Value=ET/1E9;
% app.GLT.Value=GLT/1E9;
% app.vLT.Value=VLT;
% elseif elem2==1
% EL = 147.548E9;
% ET = 11.032E9;
% GLT= 5.309E9;
% VLT= 0.29;
% VTL= VLT*ET/EL;
% app.EL.Value=EL/1E9;
% app.ET.Value=ET/1E9;
% app.GLT.Value=GLT/1E9;
% app.vLT.Value=VLT;
% elseif elem3==1
EL= app.EL.Value*1E9;
ET= app.ET.Value*1E9;
GLT= app.GLT.Value*1E9;
VLT=app.vLT.Value;
% end

```

```

er=app.StackingToptobottomEditField.Value; %your input is a=[1,2,3,4];
thetad=str2double(strsplit(er','));
% thetad = [0,+45,-45,90,90,-45,+45,0];

```

```

h=app.ThicknessEditField.Value*0.001*ones(1,length(thetad))
z=ones(1,length(h)+1);
for i=2:length(thetad)
z(1)=-sum(h)/2;
z(i)=[-sum(h)/2+(i-1)*h(i)];
z(length(thetad)+1)=sum(h)/2;
end
for i=1:length(thetad)
Q{i}=Qbar(thetad(i),EL,ET,GLT,VLT);
end
for i=1:length(thetad)
A{i}=Q{i}*(z(i+1)-z(i));
end
A=sum(cat(length(thetad),A{:}),length(thetad));
% A=A{1}+A{2}+A{3}
for i=1:length(thetad)
B{i}=0.5*Q{i}*((z(i)^2)-(z(i+1)^2));
end
B=sum(cat(length(thetad),B{:}),length(thetad));
% B=B{1}+B{2}+B{3}
for i=1:length(thetad)
D{i}=(1/3)*Q{i}*((z(i+1)^3)-(z(i)^3));
end
D=sum(cat(length(thetad),D{:}),length(thetad));
% D=D{1}+D{2}+D{3}
ABD=[A,B;B,D];
indiceszero=find(abs(ABD)<10e-4); %Write the zero for specific threshold value
ABD(indiceszero)=0;
ABD;
a=inv(A);
if mod(length(thetad),2)==1;
tt=length(thetad)/2+app.ThicknessEditField.Value*0.5 ;
ttt=tt;
else
tt=length(thetad)/2;
ttt=tt+1;
end
if mod((sum(thetad)),90)==0 & thetad(1:tt)== flip(thetad(ttt:length(thetad)))%
symmetric and balanced check
app.StackingTypesEditField.Value='Your stacking is symmetric and balanced';
E1=(1/sum(h))*(A(1)-((A(2)^2)/A(5)));
E2=(1/sum(h))*(A(5)-((A(2)^2)/A(1)));
G12=A(9)/sum(h);
v12=A(2)/A(1);
elseif thetad(1:tt)== flip(thetad(ttt:length(thetad)))%symetric check
app.StackingTypesEditField.Value='Your stacking is symmetric';
E1=1/(sum(h)*a(1));
E2=1/(sum(h)*a(5));
G12=1/(sum(h)*a(9));
v12=-a(2)/a(1);
else %General Stacking

```



```

app.StackingTypesEditField.Value='Your stacking does not match any special
stacking case';
E1=1/(sum(h)*a(1));
E2=1/(sum(h)*a(5));
G12=1/(sum(h)*a(9));
v12=-a(2)/a(1);
v21=-a(4)/a(5);
end

```

```

app.TotalThicknessEditField.Value= sum(h)*1000; %total thickness of the plate
in mm
%
app.EEditField_3.Value=E1/1E9;
app.EEditField_4.Value=E2/1E9;
app.GEditField_2.Value=G12/1E9;
app.vEditField.Value=v12;
app.ABD1.Value=ABD(1)/1E6;
app.ABD2.Value=ABD(2)/1E6;
app.ABD3.Value=ABD(3)/1E6;
app.ABD4.Value=ABD(4);
app.ABD5.Value=ABD(5);
app.ABD6.Value=ABD(6);
app.ABD7.Value=ABD(7)/1E6;
app.ABD8.Value=ABD(8)/1E6;
app.ABD9.Value=ABD(9)/1E6;
app.ABD10.Value=ABD(10);
app.ABD11.Value=ABD(11);
app.ABD12.Value=ABD(12);
app.ABD13.Value=ABD(13)/1E6;
app.ABD14.Value=ABD(14)/1E6;
app.ABD15.Value=ABD(15)/1E6;
app.ABD16.Value=ABD(16);
app.ABD17.Value=ABD(17);
app.ABD18.Value=ABD(18);
app.ABD19.Value=ABD(19);
app.ABD20.Value=ABD(20);
app.ABD21.Value=ABD(21);
app.ABD22.Value=ABD(22)*1E3;
app.ABD23.Value=ABD(23)*1E3;
app.ABD24.Value=ABD(24)*1E3;
app.ABD25.Value=ABD(25);
app.ABD26.Value=ABD(26);
app.ABD27.Value=ABD(27);
app.ABD28.Value=ABD(28)*1E3;
app.ABD29.Value=ABD(29)*1E3;
app.ABD30.Value=ABD(30)*1E3;
app.ABD31.Value=ABD(31);
app.ABD32.Value=ABD(32);
app.ABD33.Value=ABD(33);
app.ABD34.Value=ABD(34)*1E3;
app.ABD35.Value=ABD(35)*1E3;
app.ABD36.Value=ABD(36)*1E3;
aa=app.maIndia.Value*0.001;

```

```

ab=app.ab.Value;
bb=(1/ab)*aa;
app.bEditField.Value=bb*1000;
W=app.WEditField.Value*0.001;
dW=(2*aa)/W;
app.aWEditField.Value=dW;
M=sqrt((sqrt(1-8*((3*(1-dW))/(2+(1-dW)^3))-1))-1)/(2*(dW)^2);
q=bb/aa;
qq=q^2;
if ab<4 %Improved theory Selection
if aa==bb % Circle Case Stress Concentration 4.24
Ktinf=1+sqrt((2/A(5))*(sqrt(A(1)*A(5))-A(2)+((A(1)*A(5)-(A(2)^2))/(2*A(9))))))
KtinfKt1=((3*(1-dW))/(2+(1-dW)^3))+(0.5*((dW*M)^6)*(Ktinf-3)*(1-(dW*M)^2));
FWC=1/KtinfKt1
Kt=FWC*Ktinf
app.KEditField.Value=Ktinf;
app.StressConcentrationofFiniteWidthPlateEditField.Value=Kt;
app.FWCfactorEditField.Value=FWC;
else %Ellipse Case Stress Concentration 4.21
num= [a(5),(-2*a(6)), (2*a(2)+a(9)),(-2*a(3)),a(1)]
r=roots(num)
rt=find(imag(r)>0);
for ii=1:length(rt)
rp(ii)=r(rt(ii));
end
n1=rp(1)
n2=rp(2)
aos=real(rp(1))
Ktinf=1+(1/q)*sqrt((2/A(5))*(sqrt(A(1)*A(5))-A(2)+((A(1)*A(5)-(A(2)^2))/(2*A(9))))))
i=sqrt(-1);
A1 = sqrt(1-(1+(n1^2*qq))*((dW*M)^2))
A2 = sqrt(1-(1+(n2^2*qq))*((dW*M)^2))
B1 = (dW*M-1+i*n1*q*dW*M)
B2 = (dW*M-1+i*n2*q*dW*M)
C1 = (1/(n1-n2))
C2 = (-n2/(1+i*n1*q))
C3 = (n1/(1+i*n2*q))
%Eq 4.21
ktinfkt=1-(dW*M)+real(C1*(C2*(B1+A1)+C3*(B2+A2)))
FWC = 1/ktinfkt
kt = FWC*Ktinf
app.KEditField.Value=Ktinf;
app.StressConcentrationofFiniteWidthPlateEditField.Value=kt;
app.FWCfactorEditField.Value=FWC;
end
else
if aa==bb
%%Equation 4.17
%%!! If a/b>=4 this equation is more precise
Ktinf1=1+sqrt((2/A(5))*(sqrt(A(1)*A(5))-A(2)+((A(1)*A(5)-(A(2)^2))/(2*A(9))))))
KtinfKt=((2-((dW)^2)-(dW^4))/2)+(dW^6)*((Ktinf-3)*(1-dW^2)/2)
FWC=KtinfKt^(-1)
Kt=FWC*Ktinf1

```

```

app.StressConcentrationofFiniteWidthPlateEditField.Value=Kt;
app.FWCFactorEditField.Value=FWC;
app.KEEditField.Value=Ktinf1;
else
%%Equation 4.16
%%!! If a/b>=4 this equation is more precise
Ktinf1=1+(1/q)*sqrt((2/A(5))*(sqrt(A(1)*A(5))-A(2)+((A(1)*A(5)-
(A(2)^2))/(2*A(9))))))
B12=(1+(qq-1)*(dW)^2);
KtinfKt=(qq/((1-q)^2))+((1-2*q)/((1-q)^2))*sqrt(B12)-((qq/(1-q))*((dW)^2)*...
(B12)^(-0.5))+((q^7)/2)*((dW)^6)*(Ktinf1-1-(2/q))*((B12)^(-5/2))-...
((dW)^2)*(B12^(-7/2)))
FWC=KtinfKt^(-1)
Kt=FWC*Ktinf1
app.StressConcentrationofFiniteWidthPlateEditField.Value=Kt;
app.FWCFactorEditField.Value=FWC;
app.KEEditField.Value=Ktinf1;
end
end
end
end

% Component initialization
methods (Access = private)

% Create UIFigure and components
function createComponents(app)

% Create UIFigure and hide until all components are created
app.UIFigure = uifigure('Visible', 'off');
app.UIFigure.Color = [0.8 0.8 0.8];
app.UIFigure.Colormap = [0.6353 0.0784 0.1843;0.2431 0.1529 0.6745;0.2471 0.1569
0.6863;0.2471 0.1608 0.698;0.251 0.1647 0.7059;0.251 0.1686 0.7176;0.2549 0.1725
0.7294;0.2549 0.1765 0.7412;0.2588 0.1804 0.749;0.2588 0.1843 0.7608;0.2627
0.1882 0.7725;0.2627 0.1922 0.7843;0.2627 0.1961 0.7922;0.2667 0.2 0.8039;0.2667
0.2039 0.8157;0.2706 0.2078 0.8235;0.2706 0.2157 0.8353;0.2706 0.2196
0.8431;0.2745 0.2235 0.851;0.2745 0.2275 0.8627;0.2745 0.2314 0.8706;0.2745
0.2392 0.8784;0.2784 0.2431 0.8824;0.2784 0.2471 0.8902;0.2784 0.2549
0.898;0.2784 0.2588 0.902;0.2784 0.2667 0.9098;0.2784 0.2706 0.9137;0.2784
0.2745 0.9216;0.2824 0.2824 0.9255;0.2824 0.2863 0.9294;0.2824 0.2941
0.9333;0.2824 0.298 0.9412;0.2824 0.3059 0.9451;0.2824 0.3098 0.949;0.2824
0.3137 0.9529;0.2824 0.3216 0.9569;0.2824 0.3255 0.9608;0.2824 0.3294
0.9647;0.2784 0.3373 0.9686;0.2784 0.3412 0.9686;0.2784 0.349 0.9725;0.2784
0.3529 0.9765;0.2784 0.3569 0.9804;0.2784 0.3647 0.9804;0.2745 0.3686
0.9843;0.2745 0.3765 0.9843;0.2745 0.3804 0.9882;0.2706 0.3843 0.9882;0.2706
0.3922 0.9922;0.2667 0.3961 0.9922;0.2627 0.4039 0.9922;0.2627 0.4078
0.9961;0.2588 0.4157 0.9961;0.2549 0.4196 0.9961;0.251 0.4275 0.9961;0.2471
0.4314 1;0.2431 0.4392 1;0.2353 0.4431 1;0.2314 0.451 1;0.2235 0.4549 1;0.2196
0.4627 0.9961;0.2118 0.4667 0.9961;0.2078 0.4745 0.9922;0.2 0.4784 0.9922;0.1961
0.4863 0.9882;0.1922 0.4902 0.9882;0.1882 0.498 0.9843;0.1843 0.502
0.9804;0.1843 0.5098 0.9804;0.1804 0.5137 0.9765;0.1804 0.5176 0.9725;0.1804

```

```

0.5255 0.9725;0.1804 0.5294 0.9686;0.1765 0.5333 0.9647;0.1765 0.5412
0.9608;0.1765 0.5451 0.9569;0.1765 0.549 0.9529;0.1765 0.5569 0.949;0.1725
0.5608 0.9451;0.1725 0.5647 0.9412;0.1686 0.5686 0.9373;0.1647 0.5765
0.9333;0.1608 0.5804 0.9294;0.1569 0.5843 0.9255;0.1529 0.5922 0.9216;0.1529
0.5961 0.9176;0.149 0.6 0.9137;0.149 0.6039 0.9098;0.1451 0.6078 0.9098;0.1451
0.6118 0.9059;0.1412 0.6196 0.902;0.1412 0.6235 0.898;0.1373 0.6275 0.898;0.1373
0.6314 0.8941;0.1333 0.6353 0.8941;0.1294 0.6392 0.8902;0.1255 0.6471
0.8902;0.1216 0.651 0.8863;0.1176 0.6549 0.8824;0.1137 0.6588 0.8824;0.1137
0.6627 0.8784;0.1098 0.6667 0.8745;0.1059 0.6706 0.8706;0.102 0.6745
0.8667;0.098 0.6784 0.8627;0.0902 0.6824 0.8549;0.0863 0.6863 0.851;0.0784
0.6902 0.8471;0.0706 0.6941 0.8392;0.0627 0.698 0.8353;0.0549 0.702
0.8314;0.0431 0.702 0.8235;0.0314 0.7059 0.8196;0.0235 0.7098 0.8118;0.0157
0.7137 0.8078;0.0078 0.7176 0.8;0.0039 0.7176 0.7922;0 0.7216 0.7882;0 0.7255
0.7804;0 0.7294 0.7765;0.0039 0.7294 0.7686;0.0078 0.7333 0.7608;0.0157 0.7333
0.7569;0.0235 0.7373 0.749;0.0353 0.7412 0.7412;0.051 0.7412 0.7373;0.0627
0.7451 0.7294;0.0784 0.7451 0.7216;0.0902 0.749 0.7137;0.102 0.7529
0.7098;0.1137 0.7529 0.702;0.1255 0.7569 0.6941;0.1373 0.7569 0.6863;0.1451
0.7608 0.6824;0.1529 0.7608 0.6745;0.1608 0.7647 0.6667;0.1686 0.7647
0.6588;0.1725 0.7686 0.651;0.1804 0.7686 0.6471;0.1843 0.7725 0.6392;0.1922
0.7725 0.6314;0.1961 0.7765 0.6235;0.2 0.7804 0.6157;0.2078 0.7804 0.6078;0.2118
0.7843 0.6;0.2196 0.7843 0.5882;0.2235 0.7882 0.5804;0.2314 0.7882 0.5725;0.2392
0.7922 0.5647;0.251 0.7922 0.5529;0.2588 0.7922 0.5451;0.2706 0.7961
0.5373;0.2824 0.7961 0.5255;0.2941 0.7961 0.5176;0.3059 0.8 0.5059;0.3176 0.8
0.498;0.3294 0.8 0.4863;0.3412 0.8 0.4784;0.3529 0.8 0.4667;0.3686 0.8039
0.4549;0.3804 0.8039 0.4471;0.3922 0.8039 0.4353;0.4039 0.8039 0.4235;0.4196
0.8039 0.4118;0.4314 0.8039 0.4;0.4471 0.8039 0.3922;0.4627 0.8 0.3804;0.4745
0.8 0.3686;0.4902 0.8 0.3569;0.5059 0.8 0.349;0.5176 0.8 0.3373;0.5333 0.7961
0.3255;0.5451 0.7961 0.3176;0.5608 0.7961 0.3059;0.5765 0.7922 0.2941;0.5882
0.7922 0.2824;0.6039 0.7882 0.2745;0.6157 0.7882 0.2627;0.6314 0.7843
0.251;0.6431 0.7843 0.2431;0.6549 0.7804 0.2314;0.6706 0.7804 0.2235;0.6824
0.7765 0.2157;0.698 0.7765 0.2078;0.7098 0.7725 0.2;0.7216 0.7686 0.1922;0.7333
0.7686 0.1843;0.7451 0.7647 0.1765;0.7608 0.7647 0.1725;0.7725 0.7608
0.1647;0.7843 0.7569 0.1608;0.7961 0.7569 0.1569;0.8078 0.7529 0.1529;0.8157
0.749 0.1529;0.8275 0.749 0.1529;0.8392 0.7451 0.1529;0.851 0.7451 0.1569;0.8588
0.7412 0.1569;0.8706 0.7373 0.1608;0.8824 0.7373 0.1647;0.8902 0.7373
0.1686;0.902 0.7333 0.1765;0.9098 0.7333 0.1804;0.9176 0.7294 0.1882;0.9255
0.7294 0.1961;0.9373 0.7294 0.2078;0.9451 0.7294 0.2157;0.9529 0.7294
0.2235;0.9608 0.7294 0.2314;0.9686 0.7294 0.2392;0.9765 0.7294 0.2431;0.9843
0.7333 0.2431;0.9882 0.7373 0.2431;0.9961 0.7412 0.2392;0.9961 0.7451
0.2353;0.9961 0.7529 0.2314;0.9961 0.7569 0.2275;0.9961 0.7608 0.2235;0.9961
0.7686 0.2196;0.9961 0.7725 0.2157;0.9961 0.7804 0.2078;0.9961 0.7843
0.2039;0.9961 0.7922 0.2;0.9922 0.7961 0.1961;0.9922 0.8039 0.1922;0.9922 0.8078
0.1922;0.9882 0.8157 0.1882;0.9843 0.8235 0.1843;0.9843 0.8275 0.1804;0.9804
0.8353 0.1804;0.9765 0.8392 0.1765;0.9765 0.8471 0.1725;0.9725 0.851
0.1686;0.9686 0.8588 0.1647;0.9686 0.8667 0.1647;0.9647 0.8706 0.1608;0.9647
0.8784 0.1569;0.9608 0.8824 0.1569;0.9608 0.8902 0.1529;0.9608 0.898
0.149;0.9608 0.902 0.149;0.9608 0.9098 0.1451;0.9608 0.9137 0.1412;0.9608 0.9216
0.1373;0.9608 0.9255 0.1333;0.9608 0.9333 0.1294;0.9647 0.9373 0.1255;0.9647
0.9451 0.1216;0.9647 0.949 0.1176;0.9686 0.9569 0.1098;0.9686 0.9608
0.1059;0.9725 0.9686 0.102;0.9725 0.9725 0.0941;0.9765 0.9765 0.0863;0.9765
0.9843 0.0824];
app.UIFigure.Position = [100 100 1287 716];
app.UIFigure.Name = 'MATLAB App';
app.UIFigure.Icon = 'tusas-768x387.png';

```

```

app.UIFigure.Resize = 'off';
app.UIFigure.Scrollable = 'on';

% Create Image
app.Image = uiimage(app.UIFigure);
app.Image.Position = [541 388 422 197];
app.Image.ImageSource = 'tusas-768x387.png';

% Create CalculateButton
app.CalculateButton = uibutton(app.UIFigure, 'push');
app.CalculateButton.ButtonPushedFcn = createCallbackFcn(app,
@CalculateButtonPushed, true);
app.CalculateButton.Icon = 'tusas-768x387.png';
app.CalculateButton.BackgroundColor = [1 0 0];
app.CalculateButton.FontSize = 21;
app.CalculateButton.FontAngle = 'italic';
app.CalculateButton.FontColor = [1 1 1];
app.CalculateButton.Position = [1047 21 199 57];
app.CalculateButton.Text = 'Calculate';

% Create COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel = uilabel(app.UIFigure);
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.BackgroundColor = [0 0 1];
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.HorizontalAlignment =
'center';
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.FontName = 'Times New Roman';
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.FontSize = 20;
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.FontWeight = 'bold';
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.FontAngle = 'italic';
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.FontColor = [1 1 1];
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.Position = [1 682 1287 35];
app.COMPOSITEPLATESTRESSCONCENTRATIONFACTORLabel.Text = 'COMPOSITE PLATE STRESS
CONCENTRATION FACTOR';

% Create ABDMATRIXPanel
app.ABDMATRIXPanel = uipanel(app.UIFigure);
app.ABDMATRIXPanel.Tooltip = {'ABD Matrix of the laminate'};
app.ABDMATRIXPanel.ForegroundColor = [1 0 0];
app.ABDMATRIXPanel.TitlePosition = 'centertop';
app.ABDMATRIXPanel.Title = 'ABD MATRIX';
app.ABDMATRIXPanel.BackgroundColor = [0.9412 0.9412 0.9412];
app.ABDMATRIXPanel.FontAngle = 'italic';
app.ABDMATRIXPanel.FontWeight = 'bold';
app.ABDMATRIXPanel.FontSize = 20;
app.ABDMATRIXPanel.Position = [323 8 672 373];

% Create Panel_2
app.Panel_2 = uipanel(app.ABDMATRIXPanel);

```

```

app.Panel_2.Tooltip = {'Extensional and Shear Stiffness Matrix of the laminate
'};
app.Panel_2.BackgroundColor = [0.6314 0.6235 0.6235];
app.Panel_2.Position = [20 181 309 152];

% Create ABD1
app.ABD1 = uieditfield(app.Panel_2, 'numeric');
app.ABD1.HorizontalAlignment = 'center';
app.ABD1.Position = [32 110 85 34];

% Create ABD2
app.ABD2 = uieditfield(app.Panel_2, 'numeric');
app.ABD2.HorizontalAlignment = 'center';
app.ABD2.Position = [32 70 85 34];

% Create ABD3
app.ABD3 = uieditfield(app.Panel_2, 'numeric');
app.ABD3.HorizontalAlignment = 'center';
app.ABD3.Position = [32 30 85 34];

% Create ABD7
app.ABD7 = uieditfield(app.Panel_2, 'numeric');
app.ABD7.HorizontalAlignment = 'center';
app.ABD7.Position = [122 110 85 34];

% Create ABD8
app.ABD8 = uieditfield(app.Panel_2, 'numeric');
app.ABD8.HorizontalAlignment = 'center';
app.ABD8.Position = [122 70 85 34];

% Create ABD9
app.ABD9 = uieditfield(app.Panel_2, 'numeric');
app.ABD9.HorizontalAlignment = 'center';
app.ABD9.Position = [122 30 85 34];

% Create ABD13
app.ABD13 = uieditfield(app.Panel_2, 'numeric');
app.ABD13.HorizontalAlignment = 'center';
app.ABD13.Position = [214 110 85 34];

% Create ABD14
app.ABD14 = uieditfield(app.Panel_2, 'numeric');
app.ABD14.HorizontalAlignment = 'center';
app.ABD14.Position = [214 70 85 34];

```

```

% Create ALabel
app.ALabel = uilabel(app.Panel_2);
app.ALabel.FontSize = 30;
app.ALabel.FontColor = [1 1 1];
app.ALabel.Position = [8 79 26 36];
app.ALabel.Text = 'A';

% Create ABD15
app.ABD15 = uieditfield(app.Panel_2, 'numeric');
app.ABD15.HorizontalAlignment = 'center';
app.ABD15.Position = [214 30 85 34];

% Create GPammLabel
app.GPammLabel = uilabel(app.Panel_2);
app.GPammLabel.BackgroundColor = [1 1 1];
app.GPammLabel.HorizontalAlignment = 'center';
app.GPammLabel.FontWeight = 'bold';
app.GPammLabel.Position = [134 4 62 22];
app.GPammLabel.Text = '[GPa.mm]';

% Create Panel_3
app.Panel_3 = uipanel(app.ABDMATRIXPanel);
app.Panel_3.Tooltip = {'Bending and Torsional Stiffness Matrix'};
app.Panel_3.BackgroundColor = [1 0 0];
app.Panel_3.Position = [340 12 315 159];

% Create ABD22
app.ABD22 = uieditfield(app.Panel_3, 'numeric');
app.ABD22.HorizontalAlignment = 'center';
app.ABD22.Position = [18 115 85 34];

% Create ABD23
app.ABD23 = uieditfield(app.Panel_3, 'numeric');
app.ABD23.HorizontalAlignment = 'center';
app.ABD23.Position = [18 74 85 34];

% Create ABD24
app.ABD24 = uieditfield(app.Panel_3, 'numeric');
app.ABD24.HorizontalAlignment = 'center';
app.ABD24.Position = [18 33 85 34];

% Create ABD28
app.ABD28 = uieditfield(app.Panel_3, 'numeric');
app.ABD28.HorizontalAlignment = 'center';
app.ABD28.Position = [108 115 85 34];

```

```

% Create ABD29
app.ABD29 = uieditfield(app.Panel_3, 'numeric');
app.ABD29.HorizontalAlignment = 'center';
app.ABD29.Position = [108 74 85 34];

% Create ABD30
app.ABD30 = uieditfield(app.Panel_3, 'numeric');
app.ABD30.HorizontalAlignment = 'center';
app.ABD30.Position = [108 33 85 34];

% Create ABD34
app.ABD34 = uieditfield(app.Panel_3, 'numeric');
app.ABD34.HorizontalAlignment = 'center';
app.ABD34.Position = [199 115 85 34];

% Create ABD35
app.ABD35 = uieditfield(app.Panel_3, 'numeric');
app.ABD35.HorizontalAlignment = 'center';
app.ABD35.Position = [199 74 85 34];

% Create ABD36
app.ABD36 = uieditfield(app.Panel_3, 'numeric');
app.ABD36.HorizontalAlignment = 'center';
app.ABD36.Position = [199 33 85 34];

% Create DLabel
app.DLabel = uilabel(app.Panel_3);
app.DLabel.FontSize = 30;
app.DLabel.FontColor = [1 1 1];
app.DLabel.Position = [288 80 27 36];
app.DLabel.Text = 'D';

% Create NmmLabel
app.NmmLabel = uilabel(app.Panel_3);
app.NmmLabel.BackgroundColor = [1 1 1];
app.NmmLabel.HorizontalAlignment = 'center';
app.NmmLabel.FontWeight = 'bold';
app.NmmLabel.Position = [128 3 47 22];
app.NmmLabel.Text = '[N.mm]';

% Create Panel_4
app.Panel_4 = uipanel(app.ABDMATRIXPanel);
app.Panel_4.Tooltip = {'Extension-Bending Coupling Stiffness Matrix of the Laminates'};
app.Panel_4.BackgroundColor = [0 0 1];
app.Panel_4.Position = [20 12 309 159];

```



```

% Create ABD4
app.ABD4 = uicontrol(app.Panel_4, 'numeric');
app.ABD4.HorizontalAlignment = 'center';
app.ABD4.Position = [35 115 85 34];

% Create ABD5
app.ABD5 = uicontrol(app.Panel_4, 'numeric');
app.ABD5.HorizontalAlignment = 'center';
app.ABD5.Position = [35 74 85 34];

% Create ABD6
app.ABD6 = uicontrol(app.Panel_4, 'numeric');
app.ABD6.HorizontalAlignment = 'center';
app.ABD6.Position = [35 33 85 34];

% Create ABD10
app.ABD10 = uicontrol(app.Panel_4, 'numeric');
app.ABD10.HorizontalAlignment = 'center';
app.ABD10.Position = [125 115 85 34];

% Create ABD11
app.ABD11 = uicontrol(app.Panel_4, 'numeric');
app.ABD11.HorizontalAlignment = 'center';
app.ABD11.Position = [125 74 85 34];

% Create ABD12
app.ABD12 = uicontrol(app.Panel_4, 'numeric');
app.ABD12.HorizontalAlignment = 'center';
app.ABD12.Position = [125 33 85 34];

% Create ABD17
app.ABD17 = uicontrol(app.Panel_4, 'numeric');
app.ABD17.HorizontalAlignment = 'center';
app.ABD17.Position = [217 74 85 34];

% Create ABD18
app.ABD18 = uicontrol(app.Panel_4, 'numeric');
app.ABD18.HorizontalAlignment = 'center';
app.ABD18.Position = [217 33 85 34];

% Create BLabel_3
app.BLabel_3 = uicontrol(app.Panel_4);
app.BLabel_3.FontSize = 30;
app.BLabel_3.FontColor = [1 1 1];
app.BLabel_3.Position = [7 83 26 36];
app.BLabel_3.Text = 'B';

```

```

% Create ABD16
app.ABD16 = uieditfield(app.Panel_4, 'numeric');
app.ABD16.HorizontalAlignment = 'center';
app.ABD16.Position = [217 115 85 34];

% Create NLabel_2
app.NLabel_2 = uilabel(app.Panel_4);
app.NLabel_2.BackgroundColor = [1 1 1];
app.NLabel_2.HorizontalAlignment = 'center';
app.NLabel_2.FontWeight = 'bold';
app.NLabel_2.Position = [152 3 25 22];
app.NLabel_2.Text = '[N]';

% Create Panel_5
app.Panel_5 = uipanel(app.ABDMATRIXPanel);
app.Panel_5.Tooltip = {'Extension-Bending Coupling Stiffness Matrix of the  
Laminate'};
app.Panel_5.BackgroundColor = [0 0 1];
app.Panel_5.Position = [340 181 315 152];

% Create ABD19
app.ABD19 = uieditfield(app.Panel_5, 'numeric');
app.ABD19.HorizontalAlignment = 'center';
app.ABD19.Position = [18 110 85 34];

% Create ABD20
app.ABD20 = uieditfield(app.Panel_5, 'numeric');
app.ABD20.HorizontalAlignment = 'center';
app.ABD20.Position = [18 70 85 34];

% Create ABD21
app.ABD21 = uieditfield(app.Panel_5, 'numeric');
app.ABD21.HorizontalAlignment = 'center';
app.ABD21.Position = [18 30 85 34];

% Create ABD25
app.ABD25 = uieditfield(app.Panel_5, 'numeric');
app.ABD25.HorizontalAlignment = 'center';
app.ABD25.Position = [108 110 85 34];

% Create ABD26
app.ABD26 = uieditfield(app.Panel_5, 'numeric');
app.ABD26.HorizontalAlignment = 'center';
app.ABD26.Position = [108 70 85 34];

```

```

% Create ABD27
app.ABD27 = uieditfield(app.Panel_5, 'numeric');
app.ABD27.HorizontalAlignment = 'center';
app.ABD27.Position = [108 30 85 34];

% Create ABD31
app.ABD31 = uieditfield(app.Panel_5, 'numeric');
app.ABD31.HorizontalAlignment = 'center';
app.ABD31.Position = [199 110 85 34];

% Create ABD32
app.ABD32 = uieditfield(app.Panel_5, 'numeric');
app.ABD32.HorizontalAlignment = 'center';
app.ABD32.Position = [199 70 85 34];

% Create BLabel_2
app.BLabel_2 = uilabel(app.Panel_5);
app.BLabel_2.FontSize = 30;
app.BLabel_2.FontColor = [1 1 1];
app.BLabel_2.Position = [289 71 26 36];
app.BLabel_2.Text = 'B';

% Create ABD33
app.ABD33 = uieditfield(app.Panel_5, 'numeric');
app.ABD33.HorizontalAlignment = 'center';
app.ABD33.Position = [199 30 85 34];

% Create NLabel
app.NLabel = uilabel(app.Panel_5);
app.NLabel.BackgroundColor = [1 1 1];
app.NLabel.HorizontalAlignment = 'center';
app.NLabel.FontWeight = 'bold';
app.NLabel.Position = [139 4 25 22];
app.NLabel.Text = '[N]';

% Create LaminaEngineeringPropertiesPanel
app.LaminaEngineeringPropertiesPanel = uipanel(app.UIFigure);
app.LaminaEngineeringPropertiesPanel.TitlePosition = 'centertop';
app.LaminaEngineeringPropertiesPanel.Title = 'Lamina Engineering Properties';
app.LaminaEngineeringPropertiesPanel.BackgroundColor = [0 1 0];
app.LaminaEngineeringPropertiesPanel.FontAngle = 'italic';
app.LaminaEngineeringPropertiesPanel.FontWeight = 'bold';
app.LaminaEngineeringPropertiesPanel.FontSize = 14;
app.LaminaEngineeringPropertiesPanel.Position = [14 389 300 271];

% Create Label
app.Label = uilabel(app.LaminaEngineeringPropertiesPanel);

```

```

app.Label.Position = [8 218 25 22];
app.Label.Text = '';

% Create EL
app.EL = uieditfield(app.LaminaEngineeringPropertiesPanel, 'numeric');
app.EL.HorizontalAlignment = 'center';
app.EL.Tooltip = {'Young''s modulus of the lamina in the longitudinal direction of fiber'};
app.EL.Position = [108 206 100 22];

% Create EEditField_2Label
app.EEditField_2Label = uilabel(app.LaminaEngineeringPropertiesPanel);
app.EEditField_2Label.HorizontalAlignment = 'center';
app.EEditField_2Label.FontWeight = 'bold';
app.EEditField_2Label.Position = [39 206 25 22];
app.EEditField_2Label.Text = 'E';

% Create ET
app.ET = uieditfield(app.LaminaEngineeringPropertiesPanel, 'numeric');
app.ET.HorizontalAlignment = 'center';
app.ET.Tooltip = {'Young''s modulus of the lamina in the tranverse direction of fiber'};
app.ET.Position = [108 176 100 22];

% Create EEditFieldLabel
app.EEditFieldLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.EEditFieldLabel.HorizontalAlignment = 'center';
app.EEditFieldLabel.FontWeight = 'bold';
app.EEditFieldLabel.Position = [39 176 25 22];
app.EEditFieldLabel.Text = 'E';

% Create GLT
app.GLT = uieditfield(app.LaminaEngineeringPropertiesPanel, 'numeric');
app.GLT.HorizontalAlignment = 'center';
app.GLT.Tooltip = {'In-plane shear modulus of the lamina '};
app.GLT.Position = [108 146 100 22];

% Create GEditFieldLabel
app.GEditFieldLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.GEditFieldLabel.HorizontalAlignment = 'center';
app.GEditFieldLabel.FontWeight = 'bold';
app.GEditFieldLabel.Position = [39 146 25 22];
app.GEditFieldLabel.Text = 'G';

% Create vLT
app.vLT = uieditfield(app.LaminaEngineeringPropertiesPanel, 'numeric');
app.vLT.HorizontalAlignment = 'center';

```

```

app.vLT.Tooltip = {'In-plane Poisson''s ratio'};
app.vLT.Position = [108 116 100 22];

% Create vEditField_2Label
app.vEditField_2Label = uilabel(app.LaminaEngineeringPropertiesPanel);
app.vEditField_2Label.HorizontalAlignment = 'center';
app.vEditField_2Label.FontWeight = 'bold';
app.vEditField_2Label.Position = [39 116 25 22];
app.vEditField_2Label.Text = 'v';

% Create ThicknessEditField
app.ThicknessEditField = uieditfield(app.LaminaEngineeringPropertiesPanel,
'numeric');
app.ThicknessEditField.HorizontalAlignment = 'center';
app.ThicknessEditField.Tooltip = {'Thickness of the lamina'};
app.ThicknessEditField.Position = [108 86 100 22];

% Create ThicknessEditFieldLabel
app.ThicknessEditFieldLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.ThicknessEditFieldLabel.HorizontalAlignment = 'center';
app.ThicknessEditFieldLabel.FontWeight = 'bold';
app.ThicknessEditFieldLabel.Position = [39 86 64 22];
app.ThicknessEditFieldLabel.Text = 'Thickness';

% Create StackingToptobottomLabel
app.StackingToptobottomLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.StackingToptobottomLabel.HorizontalAlignment = 'center';
app.StackingToptobottomLabel.FontWeight = 'bold';
app.StackingToptobottomLabel.Position = [39 35 56 42];
app.StackingToptobottomLabel.Text = {'Stacking'; 'Top to '; 'bottom'};

% Create StackingToptobottomEditField
app.StackingToptobottomEditField =
uieditfield(app.LaminaEngineeringPropertiesPanel, 'text');
app.StackingToptobottomEditField.Tooltip = {'Stacking sequence top to bottom in
degrees'};
app.StackingToptobottomEditField.Position = [108 13 169 64];

% Create GPaLabel
app.GPaLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.GPaLabel.FontWeight = 'bold';
app.GPaLabel.FontAngle = 'italic';
app.GPaLabel.Position = [215 206 30 22];
app.GPaLabel.Text = 'GPa';

% Create GPaLabel_2
app.GPaLabel_2 = uilabel(app.LaminaEngineeringPropertiesPanel);

```

```

app.GPaLabel_2.FontWeight = 'bold';
app.GPaLabel_2.FontAngle = 'italic';
app.GPaLabel_2.Position = [215 176 30 22];
app.GPaLabel_2.Text = 'GPa';

% Create GPaLabel_3
app.GPaLabel_3 = uilabel(app.LaminaEngineeringPropertiesPanel);
app.GPaLabel_3.FontWeight = 'bold';
app.GPaLabel_3.FontAngle = 'italic';
app.GPaLabel_3.Position = [215 146 30 22];
app.GPaLabel_3.Text = 'GPa';

% Create mmLabel
app.mmLabel = uilabel(app.LaminaEngineeringPropertiesPanel);
app.mmLabel.FontWeight = 'bold';
app.mmLabel.FontAngle = 'italic';
app.mmLabel.Position = [214 86 27 22];
app.mmLabel.Text = 'mm';

% Create VLTEditFieldLabel_2
app.VLTEditFieldLabel_2 = uilabel(app.LaminaEngineeringPropertiesPanel);
app.VLTEditFieldLabel_2.VerticalAlignment = 'top';
app.VLTEditFieldLabel_2.FontSize = 9;
app.VLTEditFieldLabel_2.FontWeight = 'bold';
app.VLTEditFieldLabel_2.Position = [56 104 25 22];
app.VLTEditFieldLabel_2.Text = '12';

% Create VLTEditFieldLabel_3
app.VLTEditFieldLabel_3 = uilabel(app.LaminaEngineeringPropertiesPanel);
app.VLTEditFieldLabel_3.VerticalAlignment = 'top';
app.VLTEditFieldLabel_3.FontSize = 9;
app.VLTEditFieldLabel_3.FontWeight = 'bold';
app.VLTEditFieldLabel_3.Position = [58 135 25 22];
app.VLTEditFieldLabel_3.Text = '12';

% Create VLTEditFieldLabel_4
app.VLTEditFieldLabel_4 = uilabel(app.LaminaEngineeringPropertiesPanel);
app.VLTEditFieldLabel_4.VerticalAlignment = 'top';
app.VLTEditFieldLabel_4.FontSize = 9;
app.VLTEditFieldLabel_4.FontWeight = 'bold';
app.VLTEditFieldLabel_4.Position = [58 164 25 22];
app.VLTEditFieldLabel_4.Text = '2';

% Create VLTEditFieldLabel_5
app.VLTEditFieldLabel_5 = uilabel(app.LaminaEngineeringPropertiesPanel);
app.VLTEditFieldLabel_5.VerticalAlignment = 'top';
app.VLTEditFieldLabel_5.FontSize = 9;
app.VLTEditFieldLabel_5.FontWeight = 'bold';

```

```

app.VLTextFieldLabel_5.Position = [58 195 25 22];
app.VLTextFieldLabel_5.Text = '1';

% Create LaminarPropertiesPanel
app.LaminarPropertiesPanel = uipanel(app.UIFigure);
app.LaminarPropertiesPanel.TitlePosition = 'centertop';
app.LaminarPropertiesPanel.Title = 'Laminar Properties ';
app.LaminarPropertiesPanel.BackgroundColor = [1 0 0];
app.LaminarPropertiesPanel.FontAngle = 'italic';
app.LaminarPropertiesPanel.FontWeight = 'bold';
app.LaminarPropertiesPanel.FontSize = 14;
app.LaminarPropertiesPanel.Position = [1012 470 270 180];

% Create EEditField_3
app.EEditField_3 = uieditfield(app.LaminarPropertiesPanel, 'numeric');
app.EEditField_3.HorizontalAlignment = 'center';
app.EEditField_3.Tooltip = {'Young's modulus of the laminate in the x-direction of fiber'};
app.EEditField_3.Position = [75 127 100 22];

% Create EEditField_3Label
app.EEditField_3Label = uilabel(app.LaminarPropertiesPanel);
app.EEditField_3Label.HorizontalAlignment = 'right';
app.EEditField_3Label.FontWeight = 'bold';
app.EEditField_3Label.Position = [35 127 25 22];
app.EEditField_3Label.Text = 'E';

% Create EEditField_4
app.EEditField_4 = uieditfield(app.LaminarPropertiesPanel, 'numeric');
app.EEditField_4.HorizontalAlignment = 'center';
app.EEditField_4.Tooltip = {'Young's modulus of the laminate in the y-direction of fiber'};
app.EEditField_4.Position = [75 90 100 22];

% Create EEditField_4Label
app.EEditField_4Label = uilabel(app.LaminarPropertiesPanel);
app.EEditField_4Label.HorizontalAlignment = 'right';
app.EEditField_4Label.FontWeight = 'bold';
app.EEditField_4Label.Position = [35 90 25 22];
app.EEditField_4Label.Text = 'E';

% Create GEditField_2
app.GEditField_2 = uieditfield(app.LaminarPropertiesPanel, 'numeric');
app.GEditField_2.HorizontalAlignment = 'center';
app.GEditField_2.Tooltip = {'In-plane shear modulus of laminate'};
app.GEditField_2.Position = [75 53 100 22];

```

```

% Create GEditField_2Label
app.GEditField_2Label = uilabel(app.LaminatePropertiesPanel);
app.GEditField_2Label.HorizontalAlignment = 'right';
app.GEditField_2Label.FontWeight = 'bold';
app.GEditField_2Label.Position = [35 53 25 22];
app.GEditField_2Label.Text = 'G';

% Create vEditField
app.vEditField = uieditfield(app.LaminatePropertiesPanel, 'numeric');
app.vEditField.HorizontalAlignment = 'center';
app.vEditField.Tooltip = {'Poisson''s ratio of laminate'};
app.vEditField.Position = [75 17 100 22];

% Create vEditFieldLabel
app.vEditFieldLabel = uilabel(app.LaminatePropertiesPanel);
app.vEditFieldLabel.HorizontalAlignment = 'right';
app.vEditFieldLabel.FontWeight = 'bold';
app.vEditFieldLabel.Position = [35 17 25 22];
app.vEditFieldLabel.Text = 'v';

% Create VLTEditFieldLabel_6
app.VLTEditFieldLabel_6 = uilabel(app.LaminatePropertiesPanel);
app.VLTEditFieldLabel_6.VerticalAlignment = 'top';
app.VLTEditFieldLabel_6.FontSize = 9;
app.VLTEditFieldLabel_6.FontWeight = 'bold';
app.VLTEditFieldLabel_6.Position = [60 115 25 22];
app.VLTEditFieldLabel_6.Text = 'x';

% Create VLTEditFieldLabel_7
app.VLTEditFieldLabel_7 = uilabel(app.LaminatePropertiesPanel);
app.VLTEditFieldLabel_7.VerticalAlignment = 'top';
app.VLTEditFieldLabel_7.FontSize = 9;
app.VLTEditFieldLabel_7.FontWeight = 'bold';
app.VLTEditFieldLabel_7.Position = [60 81 25 22];
app.VLTEditFieldLabel_7.Text = 'y';

% Create VLTEditFieldLabel_8
app.VLTEditFieldLabel_8 = uilabel(app.LaminatePropertiesPanel);
app.VLTEditFieldLabel_8.VerticalAlignment = 'top';
app.VLTEditFieldLabel_8.FontSize = 9;
app.VLTEditFieldLabel_8.FontWeight = 'bold';
app.VLTEditFieldLabel_8.Position = [60 44 25 22];
app.VLTEditFieldLabel_8.Text = 'xy';

% Create VLTEditFieldLabel_9
app.VLTEditFieldLabel_9 = uilabel(app.LaminatePropertiesPanel);
app.VLTEditFieldLabel_9.VerticalAlignment = 'top';
app.VLTEditFieldLabel_9.FontSize = 9;

```



```

app.VLTextFieldLabel_9.FontWeight = 'bold';
app.VLTextFieldLabel_9.Position = [60 5 25 22];
app.VLTextFieldLabel_9.Text = 'xy';

% Create GPaLabel_4
app.GPaLabel_4 = uilabel(app.LaminatePropertiesPanel);
app.GPaLabel_4.FontWeight = 'bold';
app.GPaLabel_4.FontAngle = 'italic';
app.GPaLabel_4.Position = [204 127 30 22];
app.GPaLabel_4.Text = 'GPa';

% Create GPaLabel_5
app.GPaLabel_5 = uilabel(app.LaminatePropertiesPanel);
app.GPaLabel_5.FontWeight = 'bold';
app.GPaLabel_5.FontAngle = 'italic';
app.GPaLabel_5.Position = [204 90 30 22];
app.GPaLabel_5.Text = 'GPa';

% Create GPaLabel_6
app.GPaLabel_6 = uilabel(app.LaminatePropertiesPanel);
app.GPaLabel_6.FontWeight = 'bold';
app.GPaLabel_6.FontAngle = 'italic';
app.GPaLabel_6.Position = [204 55 30 22];
app.GPaLabel_6.Text = 'GPa';

% Create PlatePropertiesforStressConcetrationPanel
app.PlatePropertiesforStressConcetrationPanel = uipanel(app.UIFigure);
app.PlatePropertiesforStressConcetrationPanel.TitlePosition = 'centertop';
app.PlatePropertiesforStressConcetrationPanel.Title = 'Plate Properties for Stress Concetration';
app.PlatePropertiesforStressConcetrationPanel.BackgroundColor = [0 1 0];
app.PlatePropertiesforStressConcetrationPanel.FontAngle = 'italic';
app.PlatePropertiesforStressConcetrationPanel.FontWeight = 'bold';
app.PlatePropertiesforStressConcetrationPanel.FontSize = 14;
app.PlatePropertiesforStressConcetrationPanel.Position = [14 184 300 197];

% Create maindia
app.maindia = uieditfield(app.PlatePropertiesforStressConcetrationPanel, 'numeric');
app.maindia.HorizontalAlignment = 'center';
app.maindia.Tooltip = {'Main radius of the opening in the figure'};
app.maindia.Position = [127 140 100 22];

% Create aEditFieldLabel
app.aEditFieldLabel = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.aEditFieldLabel.HorizontalAlignment = 'center';
app.aEditFieldLabel.FontWeight = 'bold';

```

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app.aEditFieldLabel.Tooltip = {'This value is the main radius of the opening in
the figure at top'};
app.aEditFieldLabel.Position = [59 140 25 22];
app.aEditFieldLabel.Text = 'a';

% Create WEditField
app.WEditField = uieditfield(app.PlatePropertiesforStressConcetrationPanel,
'numeric');
app.WEditField.HorizontalAlignment = 'center';
app.WEditField.Tooltip = {'Width of the plate.'};
app.WEditField.Position = [125 76 100 22];

% Create WEditFieldLabel
app.WEditFieldLabel = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.WEditFieldLabel.HorizontalAlignment = 'center';
app.WEditFieldLabel.FontWeight = 'bold';
app.WEditFieldLabel.Tooltip = {'This is the width of the plate.'};
app.WEditFieldLabel.Position = [57 78 25 22];
app.WEditFieldLabel.Text = 'W';

% Create aWEditField
app.aWEditField = uieditfield(app.PlatePropertiesforStressConcetrationPanel,
'numeric');
app.aWEditField.HorizontalAlignment = 'center';
app.aWEditField.Tooltip = {'Ratio of main diameter and width.'};
app.aWEditField.Position = [125 16 100 22];

% Create aWEditFieldLabel
app.aWEditFieldLabel = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.aWEditFieldLabel.HorizontalAlignment = 'center';
app.aWEditFieldLabel.FontWeight = 'bold';
app.aWEditFieldLabel.Position = [57 16 34 22];
app.aWEditFieldLabel.Text = '2a/W';

% Create ab
app.ab = uieditfield(app.PlatePropertiesforStressConcetrationPanel, 'numeric');
app.ab.HorizontalAlignment = 'center';
app.ab.Tooltip = {'Aspect ratio of main radius and secondary radius'};
app.ab.Position = [126 108 100 22];

% Create abEditField_2Label
app.abEditField_2Label = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.abEditField_2Label.HorizontalAlignment = 'center';
app.abEditField_2Label.FontWeight = 'bold';
app.abEditField_2Label.Tooltip = {'This is the aspect ratio of main radius and
secondary radius'};
app.abEditField_2Label.Position = [59 110 25 22];

```

```

app.abEditField_2Label.Text = 'a/b';

% Create bEditFieldLabel
app.bEditFieldLabel = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.bEditFieldLabel.HorizontalAlignment = 'center';
app.bEditFieldLabel.FontWeight = 'bold';
app.bEditFieldLabel.Position = [58 46 25 22];
app.bEditFieldLabel.Text = 'b';

% Create bEditField
app.bEditField = uieditfield(app.PlatePropertiesforStressConcetrationPanel,
'numeric');
app.bEditField.HorizontalAlignment = 'center';
app.bEditField.Tooltip = {'Secondry radius'};
app.bEditField.Position = [125 45 100 22];

% Create mmLabel_2
app.mmLabel_2 = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.mmLabel_2.FontWeight = 'bold';
app.mmLabel_2.FontAngle = 'italic';
app.mmLabel_2.Position = [242 140 27 22];
app.mmLabel_2.Text = 'mm';

% Create mmLabel_3
app.mmLabel_3 = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.mmLabel_3.FontWeight = 'bold';
app.mmLabel_3.FontAngle = 'italic';
app.mmLabel_3.Position = [242 78 27 22];
app.mmLabel_3.Text = 'mm';

% Create mmLabel_4
app.mmLabel_4 = uilabel(app.PlatePropertiesforStressConcetrationPanel);
app.mmLabel_4.FontWeight = 'bold';
app.mmLabel_4.FontAngle = 'italic';
app.mmLabel_4.Position = [242 47 27 22];
app.mmLabel_4.Text = 'mm';

% Create StackingTypesEditFieldLabel
app.StackingTypesEditFieldLabel = uilabel(app.UIFigure);
app.StackingTypesEditFieldLabel.BackgroundColor = [1 0 0];
app.StackingTypesEditFieldLabel.HorizontalAlignment = 'center';
app.StackingTypesEditFieldLabel.FontWeight = 'bold';
app.StackingTypesEditFieldLabel.FontColor = [1 1 1];
app.StackingTypesEditFieldLabel.Position = [541 627 95 22];
app.StackingTypesEditFieldLabel.Text = 'Stacking Types';

% Create StackingTypesEditField

```

```

app.StackingTypesEditField = ueditfield(app.UIFigure, 'text');
app.StackingTypesEditField.Position = [642 627 337 22];

% Create Image2
app.Image2 = uiimage(app.UIFigure);
app.Image2.Position = [286 389 272 266];
app.Image2.ImageSource = 'xy12.jpg';

% Create StressConcentrationValuesPanel
app.StressConcentrationValuesPanel = uipanel(app.UIFigure);
app.StressConcentrationValuesPanel.TitlePosition = 'centertop';
app.StressConcentrationValuesPanel.Title = 'Stress Concentration Values';
app.StressConcentrationValuesPanel.BackgroundColor = [1 0 0];
app.StressConcentrationValuesPanel.FontAngle = 'italic';
app.StressConcentrationValuesPanel.FontWeight = 'bold';
app.StressConcentrationValuesPanel.FontSize = 14;
app.StressConcentrationValuesPanel.Position = [1012 95 270 372];

% Create KEditFieldLabel
app.KEditFieldLabel = uilabel(app.StressConcentrationValuesPanel);
app.KEditFieldLabel.BackgroundColor = [1 1 1];
app.KEditFieldLabel.HorizontalAlignment = 'center';
app.KEditFieldLabel.FontWeight = 'bold';
app.KEditFieldLabel.FontAngle = 'italic';
app.KEditFieldLabel.Tooltip = {'This is the stress concentration of the infinite plate. It can be used for the calcution of the finite-width plate stress concentrarion factor by multiplying with FWC factor'};
app.KEditFieldLabel.Position = [66 307 118 33];
app.KEditFieldLabel.Text = 'K';

% Create KEditField
app.KEditField = ueditfield(app.StressConcentrationValuesPanel, 'numeric');
app.KEditField.HorizontalAlignment = 'center';
app.KEditField.FontWeight = 'bold';
app.KEditField.FontAngle = 'italic';
app.KEditField.Tooltip = {'Stress concentration of the infinite plate.'};
app.KEditField.Position = [75 278 100 22];

% Create FWCFactorEditFieldLabel
app.FWCFactorEditFieldLabel = uilabel(app.StressConcentrationValuesPanel);
app.FWCFactorEditFieldLabel.BackgroundColor = [1 1 1];
app.FWCFactorEditFieldLabel.HorizontalAlignment = 'center';
app.FWCFactorEditFieldLabel.FontWeight = 'bold';
app.FWCFactorEditFieldLabel.FontAngle = 'italic';
app.FWCFactorEditFieldLabel.Position = [66 223 118 33];
app.FWCFactorEditFieldLabel.Text = 'FWC Factor';

% Create FWCFactorEditField

```

```

app.FWCFactorEditField      =      uieditfield(app.StressConcentrationValuesPanel,
'numeric');
app.FWCFactorEditField.HorizontalAlignment = 'center';
app.FWCFactorEditField.FontWeight = 'bold';
app.FWCFactorEditField.FontAngle = 'italic';
app.FWCFactorEditField.Tooltip = {'Finite-Width Correction Factor'};
app.FWCFactorEditField.Position = [75 194 100 22];

% Create StressConcentrationofFiniteWidthPlateEditFieldLabel
app.StressConcentrationofFiniteWidthPlateEditFieldLabel      =
uicontrol(app.StressConcentrationValuesPanel);
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.BackgroundColor = [1 1
1];
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.HorizontalAlignment =
'center';
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.FontWeight = 'bold';
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.FontAngle = 'italic';
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.Position = [11 119 248
48];
app.StressConcentrationofFiniteWidthPlateEditFieldLabel.Text      =      'Stress
Concentration of Finite-Width Plate';

% Create StressConcentrationofFiniteWidthPlateEditField
app.StressConcentrationofFiniteWidthPlateEditField      =
uicontrol(app.StressConcentrationValuesPanel, 'numeric');
app.StressConcentrationofFiniteWidthPlateEditField.HorizontalAlignment      =
'center';
app.StressConcentrationofFiniteWidthPlateEditField.FontWeight = 'bold';
app.StressConcentrationofFiniteWidthPlateEditField.FontAngle = 'italic';
app.StressConcentrationofFiniteWidthPlateEditField.Tooltip      =      {'Stress
concentration factor of finite plate'};
app.StressConcentrationofFiniteWidthPlateEditField.Position = [85 83 100 22];

% Create Label_3
app.Label_3 = uicontrol(app.StressConcentrationValuesPanel);
app.Label_3.HorizontalAlignment = 'center';
app.Label_3.FontWeight = 'bold';
app.Label_3.FontAngle = 'italic';
app.Label_3.Tooltip = {'This is the stress concentration of the infinite plate.It
can be used for the calcution of the finite-width plate stress concenarion
factor by multiplying with FWC factor'};
app.Label_3.Position = [129 309 20 21];
app.Label_3.Text = '∞ ';

% Create TLabel
app.TLabel = uicontrol(app.StressConcentrationValuesPanel);
app.TLabel.HorizontalAlignment = 'center';
app.TLabel.FontWeight = 'bold';
app.TLabel.FontAngle = 'italic';

```

```

app.TLabel.Tooltip = {'This is the stress concentration of the infinite plate.It
can be used for the calcution of the finite-width plate stress concentrarion
factor by multiplying with FWC factor'};
app.TLabel.Position = [122 318 25 22];
app.TLabel.Text = 'T';

% Create TotalThicknessEditField_2Label
app.TotalThicknessEditField_2Label = uilabel(app.UIFigure);
app.TotalThicknessEditField_2Label.BackgroundColor = [1 0 0];
app.TotalThicknessEditField_2Label.HorizontalAlignment = 'center';
app.TotalThicknessEditField_2Label.FontWeight = 'bold';
app.TotalThicknessEditField_2Label.FontColor = [1 1 1];
app.TotalThicknessEditField_2Label.Position = [543 595 95 22];
app.TotalThicknessEditField_2Label.Text = 'Total Thickness';

% Create TotalThicknessEditField
app.TotalThicknessEditField = uieditfield(app.UIFigure, 'numeric');
app.TotalThicknessEditField.FontWeight = 'bold';
app.TotalThicknessEditField.Position = [644 595 100 22];

% Create mmLabel_5
app.mmLabel_5 = uilabel(app.UIFigure);
app.mmLabel_5.FontWeight = 'bold';
app.mmLabel_5.FontAngle = 'italic';
app.mmLabel_5.Position = [748 592 27 25];
app.mmLabel_5.Text = 'mm';

% Create Label_2
app.Label_2 = uilabel(app.UIFigure);
app.Label_2.HorizontalAlignment = 'center';
app.Label_2.Position = [4 15 328 98];
app.Label_2.Text = {'This program is prepared for TUSAS for the purpose of ';
'prediction of the stress concentration factor of the '; 'composite plate
containing central elliptical hole.'; 'Program's outputs were verified with
avaible test and '; 'numeric analysis results.If any problem occurs,please ';
'contact to mstfkraa@outlook.com or '; 'samil.carhacioglu@gmail.com.'};

% Show the figure after all components are created
app.UIFigure.Visible = 'on';
end
end

% App creation and deletion
methods (Access = public)

% Construct app
function app = GraduationApp

```

```
% Create UIFigure and components
createComponents(app)

% Register the app with App Designer
registerApp(app, app.UIFigure)

if nargin == 0
clear app
end
end

% Code that executes before app deletion
function delete(app)

% Delete UIFigure when app is deleted
delete(app.UIFigure)
end
end
end
```

Appendix 4 Program Interface

COMPOSITE PLATE STRESS CONCENTRATION FACTOR

Lamina Engineering Properties

E_1 GPa

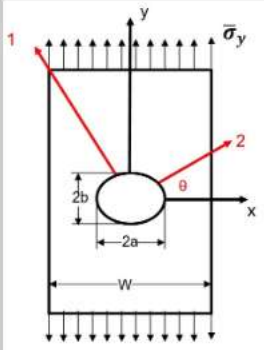
E_2 GPa

G_{12} GPa

ν_{12}

Thickness mm

Stacking Top to bottom



Stacking Types

Total Thickness mm

Laminate Properties

E_x GPa

E_y GPa

G_{xy} GPa

ν_{xy}

Plate Properties for Stress Concentration

a mm

a/b

W mm

b mm

$2a/W$

ABD MATRIX

A

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

[GPa.mm]

B

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

[N]

B

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

[N]

D

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

[N.mm]

Stress Concentration Values

$K_{t_{\infty}}$

FWC Factor

Stress Concentration of Finite-Width Plate

This program is prepared for TUSAS for the purpose of prediction of the stress concentration factor of the composite plate containing central elliptical hole. Program's outputs were verified with available test and numeric analysis results. If any problem occurs, please contact to mstfkraa@outlook.com or samil.carnacioglu@gmail.com.

Calculate

