MARMARA UNIVERSITY
FACULTY OF ENGINEERING

MODELING AND STABILIZATION CONTROL OF A BATTLE TANK TURRET SYSTEM

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Abdülsamet KASSAR Sinan ÖZTÜRÜN

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ABSTRACT

MODELING AND STABILIZATION CONTROL OF A BATTLE TANK TURRET SYSTEM

In this study, we aim the gun-barrel target's deviation which is caused by the barrel flexibility in the main battle tank to minimize by improving the stabilization control system. The mathematical model of the battle tank gun system is derived to examine using Simulink®.
Firstly, the elevation dynamics of the gun barrel system, driven by an electric drive consisting of 6 degrees of freedom, is developed. In the same way, the azimuth drive of the gun barrel dynamics, governed by turret rotation, driven by an electric drive consisting of 7 degrees of freedom, is developed.
Then, we design the controllers based on parameters such as oscillation and settling time to construct a reference tracking system. Finally, we create the block diagram of the control system and try to balance it in terms of stabilization.

Key words: Gun-barrel stabilization, dynamic modeling, linear control, elevation and traverse axis gun turret drive modelling

ÖZ

SAVAŞ TANKI KULE SİSTEMİNİN MODELLEME VE STABİLİZASYON KONTROLÜ

Bu çalışmada, ana muharebe tankındaki namlu esnekliğinin neden olduğu silah-namlu hedefinin sapmasını, stabilizasyon kontrol sistemini geliştirerek en aza indirmeyi hedefliyoruz. Muharebe tankı silah sisteminin matematiksel modeli Simulink® kullanılarak incelenmek üzere türetilmiştir.
İlk olarak, 6 serbestlik derecesinden oluşan bir elektrikli tahrik ile sürülen top namlu sisteminin yükseklik dinamikleri geliştirildi. Aynı şekilde, 7 derece serbestlikten oluşan bir elektrikli tahrik tarafından tahrik edilen, kule dönüşü ile yönetilen silah namlusu dinamiklerinin azimut tahriki geliştirilmiştir.
Daha sonra kontrol cihazlarını, bir referans takip sistemi oluşturmak için salınım ve çökelme zamanı gibi parametrelere göre tasarlıyoruz. Son olarak, kontrol sisteminin blok şemasını oluşturuyoruz ve stabilizasyon açısından dengelemeye çalışıyoruz.

Anahtar kelimeler: Silah namlusu stabilizasyonu, dinamik modelleme, lineer kontrol, yükseklik ve travers eksenli top taret sürücü modellemesi

ABBREVIATIONS

FCS: Fire Control System

GCS: Gun Control System

MBT: Main Battle Tank

LOS: Line of Sight

LOF: Line of Fire

GP: Gun part

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1. INTRODUCTION

In today's modern combat technology, main battle tanks are very common thanks to their finishing power and mobility. The widespread use of tanks had led to many development initiatives. One of the most important problems was stabilization. In World War II, the tank must to stop before it could shoot. This made them open target. Improving of stabilization control systems contribute to give big advantage tank for shooting on the move and safe driving.

Main battle tanks require effective weapon control system and gun system in order to achieve the highest hit probability under all battlefield conditions, in the shortest possible reaction time from a stationary or moving tank to a stationary or moving target. Weapon control system is composed of two main parts. These are fire control system (FCS) and gun control system (GCS). Weapon control systems used in main battle tanks (MBTs) stabilise the line of sight (LOS) and line of fire (LOF) in order to increase the firing accuracy while the MBT is on the move. On the other hand, gun control system implements the gun and turret motion by the help of elevation and azimuth (Figure 1.1) drivers and stabilization control systems.

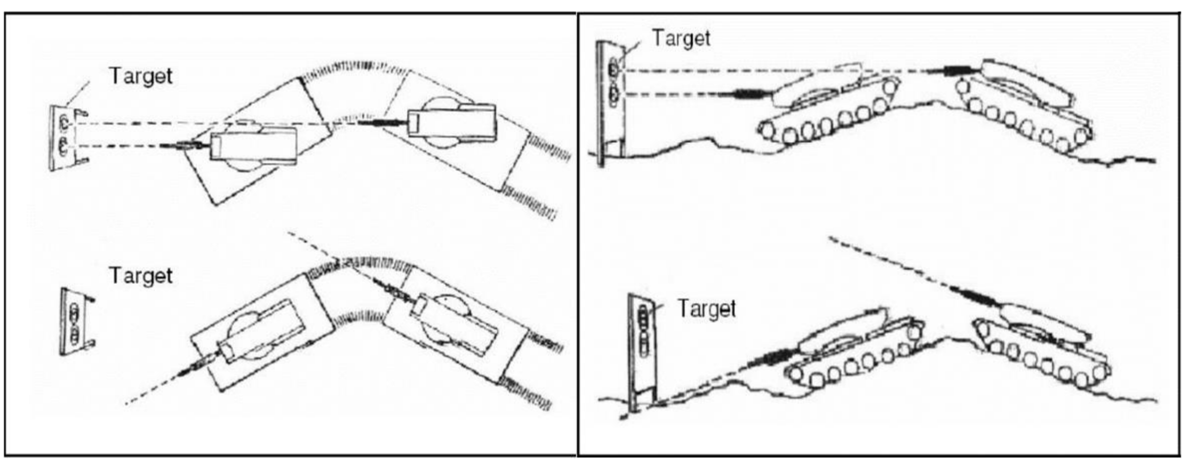

Figure 1.1. Azimuth and Elevation stabilization effect

Systems aimed to do this stabilization basically closed loop servo systems which control the movement of the barrel relative to the earth by using gyroscopes and feedback signals produced by them. Simplest stabilization systems are assigned to cope with the elevation changes and made up of a single closed loop system with a rate gyroscope mounted on the barrel to sense its angular velocity. The “basic systems” involve two closed-loop servo systems, one for the elevation and the other for the azimuth axis of the gun. Each loop contains a gyroscope to sense the velocities in respective axis. Any difference between the sensed and the commanded velocities by the operator, cause to start the servo motors to nullify the error, hence to stabilize the turret. Two axis control systems are proved to be effective, but it was not easy for the operators to correct the stabilization errors and the systems were not rapid enough to reduce the pointing errors to a sufficiently low level while the tank moves on a rough terrain.

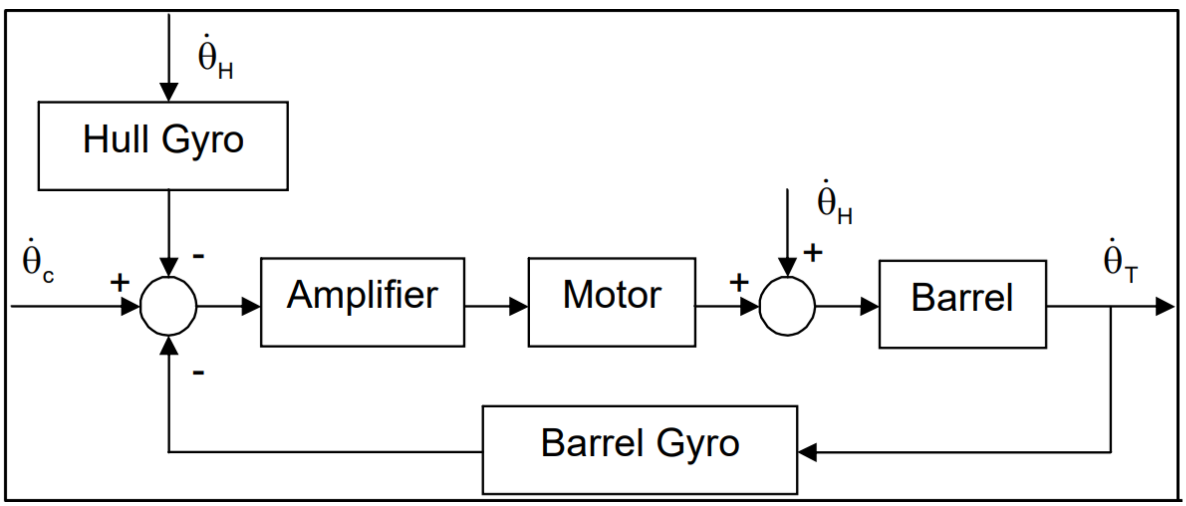

Figure1.2. Second Generation Stabilization System

One of the most important vehicles is gyroscope for our study. All gyros are the basis of a feedback loop that sends a signal to move the gun against the direction that the body of the vehicle is moving. Gyro stabilization system helps to give us information about angular velocity and effect of disturbance which is coming from ground. With gyro we can analyze referance tracking more detailed. After the development of second-generation system which is used 2 gyrospoce to analyze angular veloctiy and controlling of elevation-azimuth systems improve the stabilization and reduce the error of feedforwad open loops.

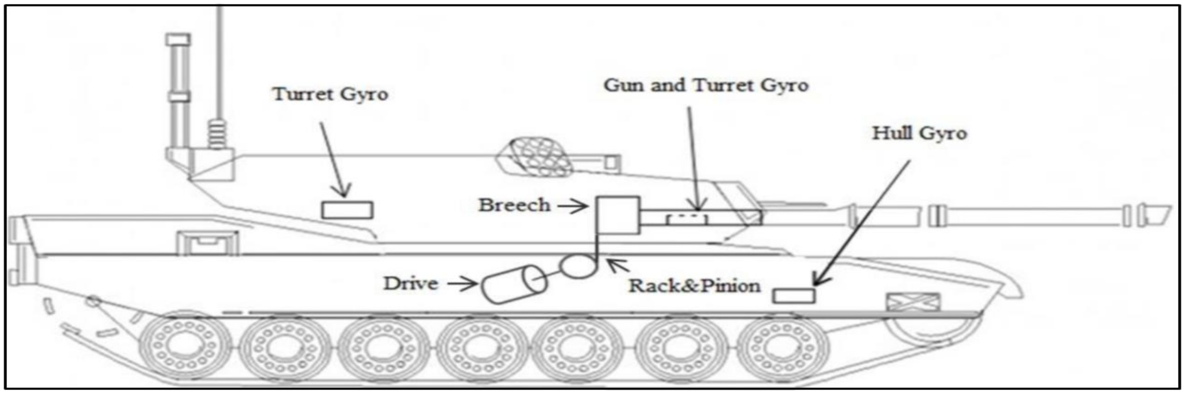

Figure 1.3. MBT and Gyro Locations

1.2. Motivation and Objective

Main purpose of this study is that design the control systems that ensures best shoot and motion stabilization of the tank. We use P, I and D controller and disturbance to design systems. Also, we seperate barrel into 5 degree of freedom for more precise results when we use observer-based method. In aplication of observer based contol, the controller generates estimate of state variable of the system to be controlled, using the measured output and known input of the system. This estimate is generated by state observer the system.

1.3. Organization

In chapter1,Information about stabilization, closed loop servo and gyro described. Researches about development of stabilization main battle tanks are explained and information of about aim of thesis is given.

In chapter 2,Mathematical modelling of azimuth and elevation are derived to design control systems in SIMULINK.

In chapter 3,DC motor are designed to fund for controllers

In chapter 4 Disturbances modelling are created to referance tracking and analyzing muzzle and breech

In chapter 5)Control system created for referance tracking,

In chapter 6) Observer based control method applied and the results were evaluated

In chapter 7) Information was given about the project results and succes.

In chapter 7) Information was given about the project results and succes.

In chapter 8) Steps that can be taken for better results are explained

2. SYSTEM MODEL

The first step in a controller design begins with modeling. A good modeling represents the behavior of the actual system. With this model, tests can be made on the model without any damage risk to the system. This will also give a chance to select the controller parameters for the system.

While modeling, it should not be forgotten that the more the properties introduced to the model the more the problems occur. If all the parameters of the real system were tried to be included in the model, this would probably generate many problems such as the increase in the number of the logical operators, time spent for the evaluation, solver errors caused by the time step and more computer capabilities need etc. All the real world can not be modeled in the same way, so we should make assumptions.

This section is broken down into two subsections, the first subsection examines the 6-DOF elevation axis model of drive line and gun barrel of MBT in Figure 2.1, and other subsection examine the 7-DOF traverse axis model of drive line and gun barrel of MBT in Figure 2.2.

2.1. 6-DOF Elevation System Model

The elevation system consists of two sections is shown in Figure 2.1. The first section includes the driveline of turret elevational motion. The second section includes the gun-barrel of the battle tank. This model is created according to the lumped parameter beam formulation. The gun-barrel is divided into five separate lumped masses. (m1, m2, m3, m4, m5). This helps us to examine the flexibility behavior of the model.

In a simple way, the operating principle of the mechanism is as follows. With the supplied volts, the electric motor activates the drive and pinion to rotate. This leads to elevational movement of the gun-barrel.

This model was created considering the flexibility of the gun-barrel and the axial and rotational movement of the body.

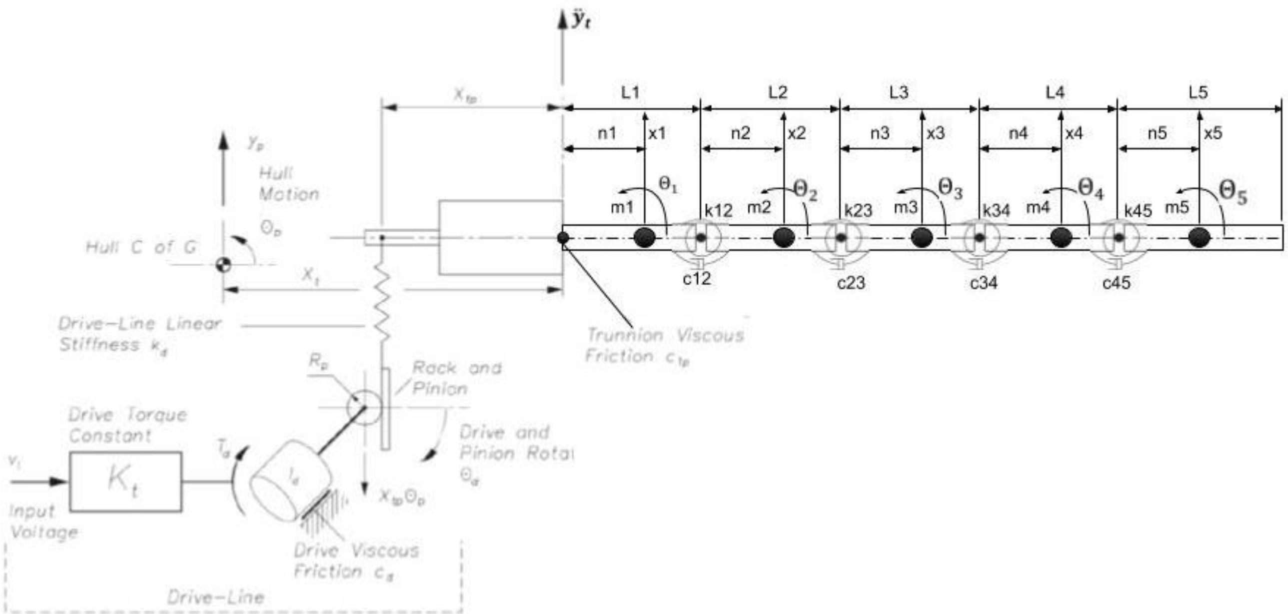

Figure 2.1. 6-DOF Elevation Drive Model

Descriptions of the system parameters for elevation system are as follows;

θ_d: Rotational DoF for Drive (rad)

θ_1: Rotational DoF for Mass 1 (rad)

θ_2: Rotational DoF for Mass 2 (rad)

θ_3: Rotational DoF for Mass 3 (rad)

θ_4: Rotational DoF for Mass 4 (rad)

θ_5: Rotational DoF for Mass 5 (rad)

θ_p: Rotational DoF for Hull (rad)

Note that all rotational DoF (Degree of Freedom) are considered respect to their center of gravity locations and in the direction of counter clock-wise.

I_d: Inertia of drive (kg*m^2)

m_1, I_1: Mass and inertia of gun part 1 and include breech section (kg & kg*m^2)

m_2, I_2: Mass of inertia gun part 2 (kg & kg*m^2)

m_3, I_3: Mass of inertia gun part 3 (kg & kg*m^2)

m_4, I_4: Mass of inertia gun part 4 (kg & kg*m^2)

m_5, I_5: Mass of gun-barrel part 5 (kg & kg*m^2)

Note that m denote the mass and I denote the inertia.

c_d, k_d: Elevation drive viscous friction and stiffness (N*m*s/rad & N*m/rad)

c_{12}, k_{12}: Torsional viscous friction and stiffness between GP (gun-barrel part) 1 and GP 2 (N*m*s/rad & N*m/rad)

c_{23}, k_{23}: Torsional viscous friction and stiffness between GP 1 and GP 2 (N*m*s/rad & N*m/rad)

c_{34}, k_{34}: Torsional viscous friction and stiffness between GP 1 and GP 2 (N*m*s/rad & N*m/rad)

c_{45}, k_{45}: Torsional viscous friction and stiffness between GP 1 and GP 2 (N*m*s/rad & N*m/rad)

c_{1p}: Trunnion viscous friction (N*m*s/rad)

R_p: Pinion's radius (m)

K_t: Drive torque constant

Other parameters denote distance in meter as shown in figure 2.1.

2.1.1 Rotational Mathematical Models for Elevation Dynamics

Applying the Newton's second law, rotational motion of drive system, the breech (m_1) and muzzle (m_2, m_3, m_4, m_5) sections of the gun-barrel are derived mathematical models as below.

Assume that $\theta_d > \theta_1 > \theta_2 > \theta_3 > \theta_4 > \theta_5 > \theta_p$.

1. <u>Drive line:</u>

$$I_d\ddot{\theta}_d + c_d\dot{\theta}_d + k_d\left[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}\right]R_p = K_t v_i$$

2. <u>Mass 1:</u>

$$I_1\ddot{\theta}_1 + c_{1p}\left(\dot{\theta}_1 - \dot{\theta}_p\right) + c_{12}\left(\dot{\theta}_1 - \dot{\theta}_2\right) + k_{12}(\theta_1 - \theta_2) - k_d\left[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}\right]\left(X_{tp} + n_1\right) + F_t n_1 + F_{12}(\,l_1 - n_1) = 0$$

3. Mass 2:

$$I_2\ddot{\theta}_2 - c_{12}(\dot{\theta}_1 - \dot{\theta}_2) - k_{12}(\theta_1 - \theta_2) + c_{23}(\dot{\theta}_2 - \dot{\theta}_3) + k_{23}(\theta_2 - \theta_3) + F_{12}n_2 + F_{23}(l_2 - n_2) = 0$$

4. Mass 3:

$$I_3\ddot{\theta}_3 - c_{23}(\dot{\theta}_2 - \dot{\theta}_3) - k_{23}(\theta_2 - \theta_3) + c_{34}(\dot{\theta}_3 - \dot{\theta}_4) + k_{34}(\theta_3 - \theta_4) + F_{23}n_3 + F_{34}(l_3 - n_3) = 0$$

5. Mass 4:

$$I_4\ddot{\theta}_4 - c_{34}(\dot{\theta}_3 - \dot{\theta}_4) - k_{34}(\theta_3 - \theta_4) + c_{45}(\dot{\theta}_4 - \dot{\theta}_5) + k_{34}(\theta_4 - \theta_5) + F_{34}n_4 + F_{45}(l_4 - n_4) = 0$$

6. Mass 5:

$$I_5\ddot{\theta}_5 - c_{45}(\dot{\theta}_4 - \dot{\theta}_5) - k_{34}(\theta_4 - \theta_5) + F_{45}n_5 = 0$$

2.1.2 Translational Mathematical Models for Elevation Dynamics

Applying Newton's second law, translational motion of the breech and muzzle sections of the gun-barrel are derived mathematical models as below respectively.

$$m_1\ddot{x}_1 - F_t + [(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}] + F_{12} = 0$$

$$m_2\ddot{x}_2 - F_{12} + F_{23} = 0$$

$$m_3\ddot{x}_3 - F_{23} + F_{34} = 0$$

$$m_4\ddot{x}_4 - F_{34} + F_{45} = 0$$

$$m_5\ddot{x}_5 - F_{45} = 0$$

F_t are the reaction force applied by the trunnion to the breech and the other reaction forces are applied the the separation points of the gun barrel.

2.1.3 Geometric Constraints Equations for Elevation Dynamics

Using the trigonometric equations, we derive the elevation dynamics.

$$x_1 = y_t + sin\theta_1 n_1$$

$$x_2 = y_t + sin\theta_1 l_1 + sin\theta_2 n_2$$

$$x_3 = y_t + sin\theta_1 l_1 + sin\theta_2 l_2 + sin\theta_3 n_3$$

$$x_4 = y_t + sin\theta_1 l_1 + sin\theta_2 l_2 + sin\theta_3 l_3 + sin\theta_4 n_4$$

$$x_5 = y_t + sin\theta_1 l_1 + sin\theta_2 l_2 + sin\theta_3 l_3 + sin\theta_4 l_4 + sin\theta_5 n_5$$

Assuming small value for θ_i, $sin\ \theta_i = \theta_i$.

$$x_1 = y_t + \theta_1 n_1$$

$$x_2 = y_t + \theta_1 l_1 + \theta_2 n_2$$

$$x_3 = y_t + \theta_1 l_1 + \theta_2 l_2 + \theta_3 n_3$$

$$x_4 = y_t + \theta_1 l_1 + \theta_2 l_2 + \theta_3 l_3 + \theta_4 n_4$$

$$x_5 = y_t + \theta_1 l_1 + \theta_2 l_2 + \theta_3 l_3 + \theta_4 l_4 + \theta_5 n_5$$

Integrating twice by time.

$$\ddot{x}_1 = \ddot{y}_t + \ddot{\theta}_1 n_1$$

$$\ddot{x}_2 = \ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2$$

$$\ddot{x}_3 = \ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3$$

$$\ddot{x}_4 = \ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4$$

$$\ddot{x}_5 = \ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5$$

Using the geometric constraints in section 2.1.3 for eliminating the translational mathematical model and leaving forces alone,

$$F_{45} = m_5\ddot{x}_5$$

$$\boldsymbol{F_{45} = m_5\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5\right)}$$

$$F_{34} = m_4\ddot{x}_4 + F_{45}$$

$$F_{34} = m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right) + F_{45}$$

$$\boldsymbol{F_{34} = m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right) + m_5\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5\right)}$$

$$F_{23} = m_3\ddot{x}_3 + F_{34}$$

$$F_{23} = m_3\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3\right) + F_{34}$$

$$\boldsymbol{F_{23} = m_3\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3\right) + m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right)}$$
$$\boldsymbol{+ m_5\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5\right)}$$

$$F_{12} = m_2\ddot{x}_2 + F_{23}$$

$$F_{12} = m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2) + F_{23}$$

$$\boldsymbol{F_{12} = m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2) + m_3\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3\right) + m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right)}$$
$$\boldsymbol{+ m_5\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5\right)}$$

$$F_t = m_1\ddot{x}_1 + k_d\left[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}\right] + F_{12}$$

$$F_t = m_1\left(\ddot{y}_t + \ddot{\theta}_1 n_1\right) + F_{12} + k_d\left[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}\right]$$

$$\boldsymbol{F_t = m_1(\ddot{y}_t + \ddot{\theta}_1 n_1) + m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2) + m_3\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3\right) + m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right)}$$
$$\boldsymbol{+ m_5\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5\right) + k_d\left[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}\right]}$$

2.1.4 Multivariable Matrix Form for Elevation Dynamics

Finally, eliminating the forces into the equations of rotational mathematical model in section 2.1.1. Equations are became as following for obtaning the systen dynamics in multivariable matrix form.

1. **<u>For mass 5:</u>**

$$I_5\ddot{\theta}_5 - c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) - k_{34}(\theta_4 - \theta_5) + F_{45}n_5 = 0$$

$$I_5\ddot{\theta}_5 - c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) - k_{34}(\theta_4 - \theta_5) + m_5(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5)n_5 = 0$$

$$\boldsymbol{\ddot{\theta}_5\left(I_5 + m_5 n_5^2\right) + m_5 n_5\left(\ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4\right) - c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) - k_{34}(\theta_4 - \theta_5) = -\ddot{y}_t m_5 n_5}$$

2. **<u>For mass 4:</u>**

$$I_4\ddot{\theta}_4 - c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) - k_{34}(\theta_3 - \theta_4) + c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) + k_{34}(\theta_4 - \theta_5) + F_{34}n_4 + F_{45}(l_4 - n_4) = 0$$

$$I_4\ddot{\theta}_4 - c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) - k_{34}(\theta_3 - \theta_4) + c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) + k_{34}(\theta_4 - \theta_5) + \left(m_4\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4\right) + F_{45}\right)n_4$$
$$+ (F_{45})(l_4 - n_4) = 0$$

$$I_4\ddot{\theta}_4 - c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) - k_{34}(\theta_3 - \theta_4) + c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) + k_{34}(\theta_4 - \theta_5) + m_4(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4)n_4 + m_5(\ddot{y}_t + \ddot{\theta}_1 l_1$$
$$+ \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5)l_4 = 0$$

$$\boldsymbol{\ddot{\theta}_5 m_5 n_5 l_4 + \ddot{\theta}_4\left(I_4 + m_4 n_4^2 + m_5 l_4^2\right) + \ddot{\theta}_3(m_4 l_3 n_4 + m_5 l_3 l_4) + \ddot{\theta}_2(m_4 l_2 n_4 + m_5 l_2 l_4) + \ddot{\theta}_1(m_4 l_1 n_4 + m_5 l_1 l_4)}$$
$$\boldsymbol{- c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) - k_{34}(\theta_3 - \theta_4) + c_{45}\left(\dot{\theta}_4 - \dot{\theta}_5\right) + k_{34}(\theta_4 - \theta_5) = -\ddot{y}_t(m_4 n_4 + m_5 l_4)}$$

3. **For mass 3=**

$$I_3\ddot{\theta}_3 - c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) - k_{23}(\theta_2 - \theta_3) + c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) + k_{34}(\theta_3 - \theta_4) + F_{23}n_3 + F_{34}(l_3 - n_3) = 0$$
$$I_3\ddot{\theta}_3 - c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) - k_{23}(\theta_2 - \theta_3) + c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) + k_{34}(\theta_3 - \theta_4) + \left(m_3\left(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3\right) + F_{34}\right)n_3$$
$$+ F_{34}(l_3 - n_3) = 0$$
$$I_3\ddot{\theta}_3 - c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) - k_{23}(\theta_2 - \theta_3) + c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) + k_{34}(\theta_3 - \theta_4) + m_3(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3)n_3 + (m_4(\ddot{y}_t + \ddot{\theta}_1 l_1$$
$$+ \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4) + m_5(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5))l_3 = 0$$
$$\boldsymbol{\ddot{\theta}_5 m_5 n_5 l_3 + \ddot{\theta}_4(m_4 n_4 l_3 + m_5 l_4 l_3) + \ddot{\theta}_3[I_3 + m_3 n_3^2 + (m_4 + m_5)l_3^2] + \ddot{\theta}_2[m_3 l_2 n_3 + (m_4 + m_5)l_2 l_3] + \ddot{\theta}_1[m_3 l_1 n_3}$$
$$\boldsymbol{+ (m_4 + m_5)l_1 l_3] - c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) - k_{23}(\theta_2 - \theta_3) + c_{34}\left(\dot{\theta}_3 - \dot{\theta}_4\right) + k_{34}(\theta_3 - \theta_4)}$$
$$\boldsymbol{= -\ddot{y}_t[m_3 n_3 + (m_4 + m_5)l_3]}$$

4. **For mass 2=**

$$I_2\ddot{\theta}_2 - c_{12}\left(\dot{\theta}_1 - \dot{\theta}_2\right) - k_{12}(\theta_1 - \theta_2) + c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) + k_{23}(\theta_2 - \theta_3) + F_{12}n_2 + F_{23}(l_2 - n_2) = 0$$
$$I_2\ddot{\theta}_2 - c_{12}\left(\dot{\theta}_1 - \dot{\theta}_2\right) - k_{12}(\theta_1 - \theta_2) + c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) + k_{23}(\theta_2 - \theta_3) + (m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2) + F_{23})n_2 + F_{23}(l_2 - n_2) = 0$$
$$I_2\ddot{\theta}_2 - c_{12}\left(\dot{\theta}_1 - \dot{\theta}_2\right) - k_{12}(\theta_1 - \theta_2) + c_{23}\left(\dot{\theta}_2 - \dot{\theta}_3\right) + k_{23}(\theta_2 - \theta_3) + m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2)n_2$$
$$+ \big(m_3(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3) + m_4(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4) + m_5(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4$$
$$+ \ddot{\theta}_5 n_5)\big)l_2 = 0$$

$$\boldsymbol{\ddot{\theta}_5 m_5 n_5 l_2 + \ddot{\theta}_4(m_4 n_4 l_2 + m_5 l_4 l_2) + \ddot{\theta}_3[m_3 n_3 l_2 + (m_4 + m_5) l_3 l_2] + \ddot{\theta}_2[I_2 + m_2 n_2^2 + (m_3 + m_4 + m_5) l_2^2] + \ddot{\theta}_1[m_2 l_1 n_2 + (m_3 + m_4 + m_5) l_1 l_2] - c_{12}(\dot{\theta}_1 - \dot{\theta}_2) - k_{12}(\theta_1 - \theta_2) + c_{23}(\dot{\theta}_2 - \dot{\theta}_3) + k_{23}(\theta_2 - \theta_3) = -\ddot{y}_t[m_2 n_2 + (m_3 + m_4 + m_5) l_2]}$$

5. **<u>For mass 1=</u>**

$$I_1\ddot{\theta}_1 + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}](X_{tp} + n_1) + F_t n_1 + F_{12}(\,l_1 - n_1) = 0$$

$$I_1\ddot{\theta}_1 + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}](X_{tp} + n_1) + (m_1(\ddot{y}_t + \ddot{\theta}_1 n_1) + F_{12} + k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}])n_1 + (F_{12}\,)(\,l_1 - n_1) = 0$$

$$I_1\ddot{\theta}_1 + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}](X_{tp} + n_1) + (m_1\ddot{x}_1 + k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}])n_1 + (F_{12})l_1 = 0$$

$$I_1\ddot{\theta}_1 + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}]X_{tp} + m_1\ddot{x}_1 n_1 + (F_{12})l_1 = 0$$

$$I_1\ddot{\theta}_1 + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}]X_{tp} + m_1(\ddot{y}_t + \ddot{\theta}_1 n_1)n_1 + [m_2(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 n_2) + m_3(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 n_3) + m_4(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 n_4) + m_5(\ddot{y}_t + \ddot{\theta}_1 l_1 + \ddot{\theta}_2 l_2 + \ddot{\theta}_3 l_3 + \ddot{\theta}_4 l_4 + \ddot{\theta}_5 n_5)]l_1 = 0$$

$$\boldsymbol{\ddot{\theta}_5 m_5 n_5 l_1 + \ddot{\theta}_4(m_4 n_4 l_1 + m_5 l_4 l_1) + \ddot{\theta}_3[m_3 n_3 l_1 + (m_4 + m_5) l_3 l_1] + \ddot{\theta}_2[m_2 n_2 l_1 + (m_3 + m_4 + m_5) l_2 l_1] + \ddot{\theta}_1[I_1 + (m_1 n_1^2 + m_2 + m_3 + m_4 + m_5) l_1^2] + c_{1p}(\dot{\theta}_1 - \dot{\theta}_p) + c_{12}(\dot{\theta}_1 - \dot{\theta}_2) + k_{12}(\theta_1 - \theta_2) - k_d[(\theta_d R_P) - (\theta_1 - \theta_P)X_{tp}]X_{tp} = -\ddot{y}_t[m_1 n_1 + (m_2 + m_3 + m_4 + m_5) l_1]}$$

Multivariable matrix form:

$$M\ddot{\boldsymbol{\theta}} + D\boldsymbol{\theta} + K\boldsymbol{\theta} = F\boldsymbol{u}$$

Where $\boldsymbol{\theta} \coloneqq (\theta_d\theta_1\theta_2\theta_3\theta_4\theta_5)^T$ and $u\!:= \left(v_i\ddot{y}_t\dot{\theta}_P\theta_P\right)^T$. Matrices M, D, K and F are shown in the appendix in matlab code. Finally, elevation system of gun-barrel's dynamics can be represented by the stata-space equations

$$\dot{\xi} = A\xi + Bu \ \ and\ y = C\xi + Du$$

With the states $\xi = (\theta_d\theta_1\theta_2\theta_3\theta_4\theta_5\dot{\theta}_d\dot{\theta}_1\dot{\theta}_2\dot{\theta}_3\dot{\theta}_4\dot{\theta}_5)$, and with the system and input matrices

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ M^{-1}F \end{pmatrix}$$

Matrix C depends on the output(s) of interest or on the measurement(s) available and matrix D is zero matrix.

2.2. 7-DOF Traverse System Model

In traverse system consists of two parts is shown in Figure 2.2.

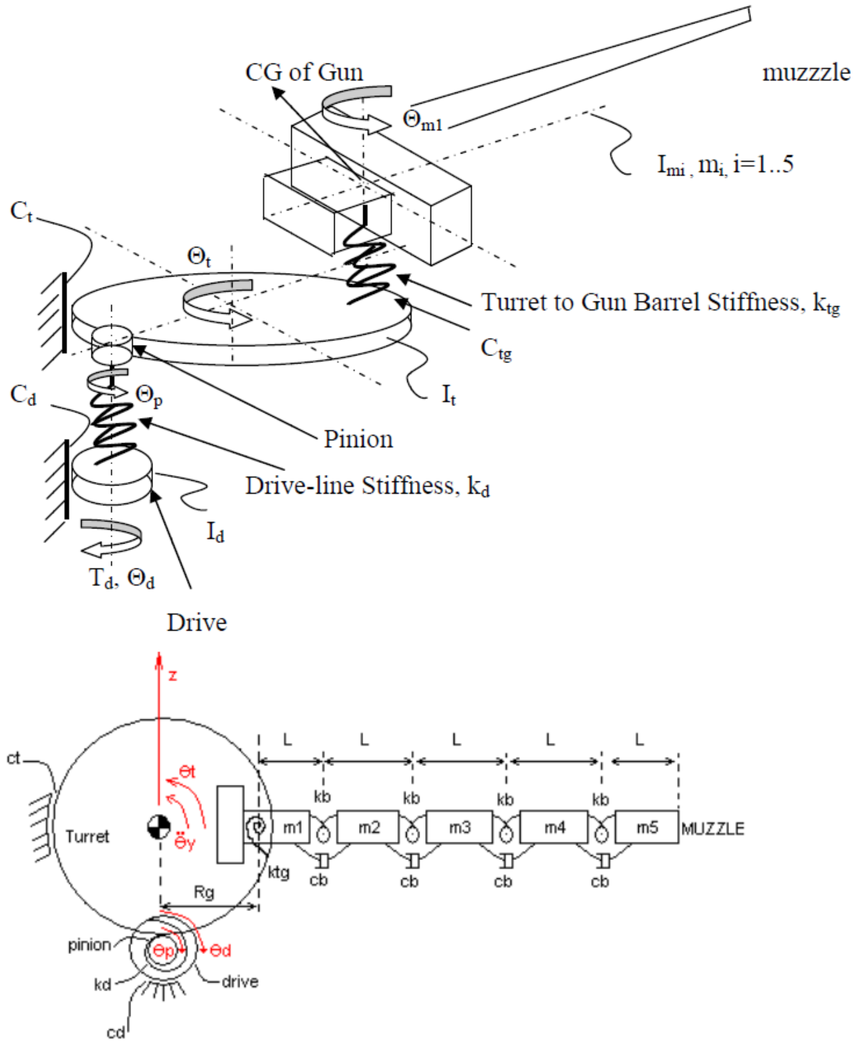

Figure 2.2. 7-DOF Traverse Drive Model

Descriptions of the system parameters for elevation system are as follows;

Rp: Pinion pitch circle radius (m)

Rt: Turret pitch circle radius (m)

Rg: Turret rotation center to gun roatation center (trunnion joint center) (m)

Id: Drive Inertia (kg*m2)

It: Turret inertia (kg*m2)

Ia: Total inertia in azimuth (kg*m2)

cd: Drive viscous friction (N*m*s/rad)

kd: Drive-line stiffness (N*m/rad)

ct: Turret ring gear total viscous friction (N*m*s/rad)

ktg: Turret to gun barrel stiffness

kb: Barrel part structural connection stiffness (N*m/rad)

cb: Barrel part structural connection viscous damping (N*m*s/rad)

m1: Mass of gun part 1 (includes gun breech) (kg)

m2: Mass of gun part 2 (kg)

m3: Mass of gun part 3 (kg)

m4: Mass of gun part 4 (kg)

m5: Mass of gun part 5 (includes any equipment mounted at muzzle) (kg)

I1: Inertia of gun part 1 (kg*m2)

I2: Inertia of gun part 2 (kg*m2)

I3: Inertia of gun part 3 (kg*m2)

I4: Inertia of gun part 4 (kg*m2)

I5: Inertia of gun part 5 (kg*m2)

θ_d: Drive rotation w.r.t ground fixed frame (rad)

θ_p: Pinion rotation w.r.t ground fixed frame (rad)

θ_y: Hull rotation w.r.t ground fixed frame (rad)

θ_t: Turret rotation w.r.t ground fixed frame (rad)

θ_{m1}: m1 rotation w.r.t ground fixed frame (rad)

θ_{m2}: m2 rotation w.r.t ground fixed frame (rad)

θ_{m3}: m3 rotation w.r.t ground fixed frame (rad)

θ_{m4}: m4 rotation w.r.t ground fixed frame (rad)

θ_{m5}: m5 rotation w.r.t ground fixed frame (rad)

Td: Drive actuator torque (N*m)

fd: Pinion to turret ring gear force (N)

z: Linear degree of freedom in sway axis (m)

L: Length of each lumped barrel part (m)

η1: Distance from gun rotation center to m1 center of gravity (m)

2.2.1 Rotational Mathematical Models for Travers Dynamics

Applying the Newton's second law, rotational motion of drive system, turret, the breech (m_1) and muzzle (m_2, m_3, m_4, m_5) sections of the gun-barrel are derived mathematical models as below.

Assume that $\theta_d > \theta_t > \theta_y > \theta_1 > \theta_2 > \theta_3 > \theta_4 > \theta_5 > \theta_p$.

1. Drive system:

$$I_{d_A}\ddot{\theta}_{d_A} + c_{d_A}\dot{\theta}_{d_A} + k_d(\theta_d - \theta_p) = T_d$$

2. Pinion:

$$k_d(\theta_d - \theta_p) = f_d R_p$$

3. Turret:

$$I_t\ddot{\theta}_t + c_t(\dot{\theta}_t - \dot{\theta}_y) + k_{tg}(\theta_t - \theta_{m1}) - f_d\, R_t + f_{tm1}R_g = 0$$

4. Mass 1:

$$I_1\ddot{\theta}_{m1} + c_b(\dot{\theta}_{m1} - \dot{\theta}_{m2}) + k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) - f_{tm1}n_1 + f_{m1m2}(n1 + L) = 0$$

5. Mass 2:

$$I_2\ddot{\theta}_{m2} - c_b(\dot{\theta}_{m1} - \dot{\theta}_{m2}) - k_b(\theta_{m1} - \theta_{m2}) + c_b(\dot{\theta}_{m2} - \dot{\theta}_{m3}) + k_b(\theta_{m2} - \theta_{m3}) + \frac{1}{2}L(f_{m1m2} + f_{m2m3}) = 0$$

6. Mass 3:

$$I_3\ddot{\theta}_{m3} - c_b(\dot{\theta}_{m2} - \dot{\theta}_{m3}) - k_b(\theta_{m2} - \theta_{m3}) + c_b(\dot{\theta}_{m3} - \dot{\theta}_{m4}) + k_b(\theta_{m3} - \theta_{m4}) + \frac{1}{2}L(f_{m2m3} + f_{m3m4}) = 0$$

7. Mass 4:

$$I_4\ddot{\theta}_{m4} - c_b\left(\dot{\theta}_{m3} - \dot{\theta}_{m4}\right) - k_b(\theta_{m3} - \theta_{m4}) + c_b\left(\dot{\theta}_{m4} - \dot{\theta}_{m5}\right) + k_b(\theta_{m4} - \theta_{m5}) + \frac{1}{2}L(f_{m3m4} + f_{m4m5}) = 0$$

8. Mass 5:

$$I_5\ddot{\theta}_{m5} - c_b\left(\dot{\theta}_{m4} - \dot{\theta}_{m5}\right) - k_b(\theta_{m4} - \theta_{m5}) + \frac{1}{2}Lf_{m4m5} = 0$$

2.2.2 Translational Mathematical Models for Travers Dynamics

Applying Newton's second law, translational motion of the breech and muzzle sections of the gun-barrel are derived mathematical models as below respectively.

$$m_1\ddot{z}_1 - f_{tm1} + f_{m1m2} = 0$$

$$m_2\ddot{z}_2 - f_{m1m2} + f_{m2m3} = 0$$

$$m_3\ddot{z}_3 - f_{m2m3} + f_{m3m4} = 0$$

$$m_4\ddot{z}_4 - f_{m3m4} + f_{m4m5} = 0$$

$$m_5\ddot{z}_5 - f_{m4m5} = 0$$

2.2.3 Geometric Constraints Equations for Travers Dynamics

$$z_1 = R_g\left(\theta_t + \theta_y\right) - n_1\sin\left(\theta_{m1}\right)$$

$$z_2 = R_g(\theta_t + \theta_y) + L\left[sin\theta_{m1} + \frac{sin\theta_{m2}}{2}\right]$$

$$z_3 = R_g(\theta_t + \theta_y) + L\left[sin\theta_{m1} + sin\theta_{m2} + \frac{sin\theta_{m3}}{2}\right]$$

$$z_4 = R_g(\theta_t + \theta_y) + L\left[sin\theta_{m1} + sin\theta_{m2} + sin\theta_{m3} + \frac{sin\theta_{m4}}{2}\right]$$

$$z_5 = R_g(\theta_t + \theta_y) + L\left[sin\theta_{m1} + sin\theta_{m2} + sin\theta_{m3} + sin\theta_{m4} + \frac{sin\theta_{m5}}{2}\right]$$

Integrating twice (assuming small θ so $sin\theta = \theta$)

$$\ddot{z}_1 = R_g(\ddot{\theta}_t + \ddot{\theta}_y) - n_1\ddot{\theta}_{m1}$$

$$\ddot{z}_2 = R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]$$

$$\ddot{z}_3 = R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]$$

$$\ddot{z}_4 = R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]$$

$$\ddot{z}_5 = R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]$$

Using the geometric constraints in section 2.2.3 for eliminating the translational mathematical model and leaving forces alone.

$$f_{m4m5} = m_5 \ddot{z}_5$$

$$f_{m4m5} = m_5 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2} \right] \right]$$

$$f_{m3m4} = m_4 \ddot{z}_4 + f_{m4m5}$$

$$f_{m3m4} = m_4 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2} \right] \right] + m_5 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2} \right] \right]$$

$$f_{m2m3} = m_3 \ddot{z}_3 + f_{m3m4}$$

$$f_{m2m3} = m_3 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2} \right] \right] + m_4 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2} \right] \right]$$

$$+ m_5 \left[R_g \left(\ddot{\theta}_t + \ddot{\theta}_y \right) + L \left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2} \right] \right]$$

$$f_{m1m2} = m_2\ddot{z}_2 + f_{m2m3}$$

$$f_{m1m2} = m_2\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]\right] + m_3\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right]$$
$$+ m_4\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right] + m_5\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]$$

$$f_{tm1} = m_1\ddot{z}_1 + f_{m1m2}$$

$$f_{tm1} = m_1\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) - n_1\ddot{\theta}_{m1}\right] + m_2\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]\right] + m_3\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right]$$
$$+ m_4\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right] + m_5\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]$$

$$k_d\left(\theta_d - \theta_p\right) = f_d R_p$$
$$f_d = \frac{k_d\left(\theta_d - \theta_p\right)}{R_p}$$

Note that geometric constraint

$$R_p\theta_p = R_t\theta_t$$

$$\theta_p = \frac{R_t}{R_p}\theta_t$$

Hence

$$f_d = \frac{k_d\left(\theta_d - \frac{R_t}{R_p}\theta_t\right)}{R_p} = \theta_d\left(\frac{k_d}{R_p}\right) - \theta_t\left(\frac{k_d R_t}{R_p^2}\right)$$

2.2.4 Multivariable Matrix Form for Travers Dynamics

Finally, eliminating the forces into the equations of rotational mathematical model in section 2.2.1. Equations are became as following for obtaning the systen dynamics in multivariable matrix form.

1. **For mass 5:**

$$I_5\ddot{\theta}_{m5} - c_b(\dot{\theta}_{m4} - \dot{\theta}_{m5}) - k_b(\theta_{m4} - \theta_{m5}) + \frac{1}{2}Lf_{m4m5} = 0$$

$$I_5\ddot{\theta}_{m5} - c_b(\dot{\theta}_{m4} - \dot{\theta}_{m5}) - k_b(\theta_{m4} - \theta_{m5}) + \frac{1}{2}Lm_5\left[R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right] = 0$$

$$\boldsymbol{\ddot{\theta}_t\left(\frac{1}{2}Lm_5Rg\right)+\ddot{\theta}_{m5}\left(I_5+\frac{1}{4}L^2m_5\right)+\ddot{\theta}_{m4}\left(\frac{1}{2}L^2m_5\right)+\ddot{\theta}_{m3}\left(\frac{1}{2}L^2m_5\right)+\ddot{\theta}_{m2}\left(\frac{1}{2}L^2m_5\right)+\ddot{\theta}_1\left(\frac{1}{2}L^2m_5\right)-c_b(\dot{\theta}_{m4}-\dot{\theta}_{m5}) - k_b(\theta_{m4}-\theta_{m5})=-\ddot{\theta}_y\left(\frac{1}{2}Lm_5Rg\right)}$$

2. **For mass 4:**

$$I_4\ddot{\theta}_{m4}-c_b(\dot{\theta}_{m3}-\dot{\theta}_{m4})-k_b(\theta_{m3}-\theta_{m4})+c_b(\dot{\theta}_{m4}-\dot{\theta}_{m5})+k_b(\theta_{m4}-\theta_{m5})+\frac{1}{2}L(f_{m3m4}+f_{m4m5})=0$$

$$\begin{aligned}&I_4\ddot{\theta}_{m4}-c_b(\dot{\theta}_{m3}-\dot{\theta}_{m4})-k_b(\theta_{m3}-\theta_{m4})+c_b(\dot{\theta}_{m4}-\dot{\theta}_{m5})+k_b(\theta_{m4}-\theta_{m5})\\&+\frac{1}{2}L\left\{m_4\left[R_g(\ddot{\theta}_t+\ddot{\theta}_y)+L\left[\ddot{\theta}_{m1}+\ddot{\theta}_{m2}+\ddot{\theta}_{m3}+\frac{\ddot{\theta}_{m4}}{2}\right]\right]\right.\\&\left.+2m_5\left[R_g(\ddot{\theta}_t+\ddot{\theta}_y)+L\left[\ddot{\theta}_{m1}+\ddot{\theta}_{m2}+\ddot{\theta}_{m3}+\ddot{\theta}_{m4}+\frac{\ddot{\theta}_{m5}}{2}\right]\right]\right\}=0\end{aligned}$$

$$\boldsymbol{\begin{aligned}&\ddot{\theta}_t\left(LR_g(\frac{m_4}{2}+m_5)\right)+\ddot{\theta}_{m5}\left(\frac{L^2m_5}{2}\right)+\ddot{\theta}_{m4}\left(I_4+L^2\left(\frac{m_4}{4}+m_5\right)\right)+\ddot{\theta}_{m3}\left(L^2\left(\frac{m_4}{2}+m_5\right)\right)+\ddot{\theta}_{m2}\left(L^2\left(\frac{m_4}{2}+m_5\right)\right)\\&+\ddot{\theta}_{m1}\left(L^2\left(\frac{m_4}{2}+m_5\right)\right)-c_b(\dot{\theta}_{m3}-\dot{\theta}_{m4})-k_b(\theta_{m3}-\theta_{m4})+c_b(\dot{\theta}_{m4}-\dot{\theta}_{m5})+k_b(\theta_{m4}-\theta_{m5})\\&=-\ddot{\theta}_y\left(LR_g(\frac{m_4}{2}+m_5)\right)\end{aligned}}$$

3. For mass 3:

$$I_3\ddot{\theta}_{m3} - c_b\big(\dot{\theta}_{m2} - \dot{\theta}_{m3}\big) - k_b(\theta_{m2} - \theta_{m3}) + c_b\big(\dot{\theta}_{m3} - \dot{\theta}_{m4}\big) + k_b(\theta_{m3} - \theta_{m4}) + \frac{1}{2}L(f_{m2m3} + f_{m3m4}) = 0$$

$$\begin{aligned} I_3\ddot{\theta}_{m3} &- c_b\big(\dot{\theta}_{m2} - \dot{\theta}_{m3}\big) - k_b(\theta_{m2} - \theta_{m3}) + c_b\big(\dot{\theta}_{m3} - \dot{\theta}_{m4}\big) + k_b(\theta_{m3} - \theta_{m4}) \\ &+ \frac{1}{2}L\left\{m_3\left[R_g\big(\ddot{\theta}_t + \ddot{\theta}_y\big) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right] + 2m_4\left[R_g\big(\ddot{\theta}_t + \ddot{\theta}_y\big) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right]\right. \\ &\left. + 2m_5\left[R_g\big(\ddot{\theta}_t + \ddot{\theta}_y\big) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]\right\} = 0 \end{aligned}$$

$$\begin{aligned} &\boldsymbol{\ddot{\theta}_t\left(LR_g\Big(\frac{m_3}{2} + m_4 + m_5\Big)\right) + \ddot{\theta}_{m5}\left(\frac{L^2 m_5}{2}\right) + \ddot{\theta}_{m4}\left(L^2\Big(\frac{m_4}{2} + m_5\Big)\right) + \ddot{\theta}_{m3}\left(I_3 + L^2\Big(\frac{m_3}{4} + m_4 + m_5\Big)\right)} \\ &\quad \boldsymbol{+ \ddot{\theta}_{m2}\left(L^2\Big(\frac{m_3}{2} + m_4 + m_5\Big)\right) + \ddot{\theta}_{m1}\Big(\frac{m_3}{2} + m_4 + m_5\Big) - c_b\big(\dot{\theta}_{m2} - \dot{\theta}_{m3}\big) - k_b(\theta_{m2} - \theta_{m3})} \\ &\quad \boldsymbol{+ c_b\big(\dot{\theta}_{m3} - \dot{\theta}_{m4}\big) + k_b(\theta_{m3} - \theta_{m4}) = -\ddot{\theta}_y\left(LR_g\Big(\frac{m_3}{2} + m_4 + m_5\Big)\right)} \end{aligned}$$

4. **For mass 2:**

$$I_2\ddot{\theta}_{m2} - c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) - k_b(\theta_{m1} - \theta_{m2}) + c_b\left(\dot{\theta}_{m2} - \dot{\theta}_{m3}\right) + k_b(\theta_{m2} - \theta_{m3}) + \frac{1}{2}L(f_{m1m2} + f_{m2m3}) = 0$$

$$I_2\ddot{\theta}_{m2} - c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) - k_b(\theta_{m1} - \theta_{m2}) + c_b\left(\dot{\theta}_{m2} - \dot{\theta}_{m3}\right) + k_b(\theta_{m2} - \theta_{m3}) + \frac{1}{2}L\left\{m_2\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]\right] + 2m_3\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right] + 2m_4\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right] + 2m_5\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]\right\} = 0$$

$$\boldsymbol{\ddot{\theta}_t\left(LR_g(\frac{m_2}{2} + m_3 + m_4 + m_5)\right) + \ddot{\theta}_{m5}\left(\frac{L^2 m_5}{2}\right) + \ddot{\theta}_{m4}\left(L^2\left(\frac{m_4}{2} + m_5\right)\right) + \ddot{\theta}_{m3}\left(L^2\left(\frac{m_3}{2} + m_4 + m_5\right)\right)}$$
$$\boldsymbol{+ \ddot{\theta}_{m2}\left(I_2 + L^2\left(\frac{m_2}{4} + m_3 + m_4 + m_5\right)\right) + \ddot{\theta}_{m1}L^2\left(\frac{m_2}{2} + m_3 + m_4 + m_5\right) - c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right)}$$
$$\boldsymbol{- k_b(\theta_{m1} - \theta_{m2}) + c_b\left(\dot{\theta}_{m2} - \dot{\theta}_{m3}\right) + k_b(\theta_{m2} - \theta_{m3}) = -\ddot{\theta}_y\left(LR_g(\frac{m_2}{2} + m_3 + m_4 + m_5)\right)}$$

5. **For mass 1:**

$$I_1\ddot{\theta}_{m1} + c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) + k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) - f_{tm1}n_1 + f_{m1m2}(n1 + L) = 0$$

$$I_1\ddot{\theta}_{m1} + c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) + k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) - \left(m_1\left(\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) - n_1\ddot{\theta}_{m1}\right]\right) + f_{m1m2}\right)n_1 + f_{m1m2}(n1 + L) = 0$$

$$I_1\ddot{\theta}_{m1} + c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) + k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) - m_1\left(\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) - n_1\ddot{\theta}_{m1}\right]\right)n_1 + f_{m1m2}L = 0$$

$$\begin{aligned} &I_1\ddot{\theta}_{m1} + c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right) + k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) - m_1\left(\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) - n_1\ddot{\theta}_{m1}\right]\right)n_1 \\ &\quad + L\left\{m_2\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]\right] + m_3\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right]\right. \\ &\quad + m_4\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right] \\ &\quad \left. + m_5\left[R_g\left(\ddot{\theta}_t + \ddot{\theta}_y\right) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]\right\} = 0 \end{aligned}$$

$$\begin{aligned} &\boldsymbol{\ddot{\theta}_t\left[R_g(-m_1n_1 + L(m_2 + m_3 + m_4 + m_5))\right] + \ddot{\theta}_{m5}\left(\frac{L^2m_5}{2}\right) + \ddot{\theta}_{m4}\left(L^2\left(\frac{m_4}{2} + m_5\right)\right) + \ddot{\theta}_{m3}\left(L^2\left(\frac{m_3}{2} + m_4 + m_5\right)\right)} \\ &\quad \boldsymbol{+ \ddot{\theta}_{m2}\left(L^2\left(\frac{m_2}{2} + m_3 + m_4 + m_5\right)\right) + \ddot{\theta}_{m1}\left(I_1 + m_1n_1^2 + L^2(m_2 + m_3 + m_4 + m_5)\right) + c_b\left(\dot{\theta}_{m1} - \dot{\theta}_{m2}\right)} \\ &\quad \boldsymbol{+ k_b(\theta_{m1} - \theta_{m2}) - k_{tg}(\theta_t - \theta_{m1}) = -\ddot{\theta}_y\left[R_g(-m_1n_1 + L(m_2 + m_3 + m_4 + m_5))\right]} \end{aligned}$$

6. For turret:

$$I_t\ddot{\theta}_t - I_a\ddot{\theta}_y + c_t\left(\dot{\theta}_t - \dot{\theta}_y\right) + k_{tg}(\theta_t - \theta_{m1}) - f_d\, R_t + f_{tm1}R_g = 0$$

$$I_t\ddot{\theta}_t - I_a\ddot{\theta}_y + c_t(\dot{\theta}_t - \dot{\theta}_y\ ) + k_{tg}(\theta_t - \theta_{m1}) - \left[\theta_d\left(\frac{k_d}{R_p}\right) - \theta_t\left(\frac{k_d R_t}{R_p^2}\right)\right] R_t$$

$$+ \left[m_1[R_g(\ddot{\theta}_t + \ddot{\theta}_y) - n_1\ddot{\theta}_{m1}] + m_2\left[R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \frac{\ddot{\theta}_{m2}}{2}\right]\right]\right.$$

$$+ m_3\left[R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \frac{\ddot{\theta}_{m3}}{2}\right]\right] + m_4\left[R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \frac{\ddot{\theta}_{m4}}{2}\right]\right]$$

$$\left. + m_5\left[R_g(\ddot{\theta}_t + \ddot{\theta}_y) + L\left[\ddot{\theta}_{m1} + \ddot{\theta}_{m2} + \ddot{\theta}_{m3} + \ddot{\theta}_{m4} + \frac{\ddot{\theta}_{m5}}{2}\right]\right]\right] R_g = 0$$

$$\boldsymbol{\ddot{\theta}_t[I_t + R_g^2(m_1 + m_2 + m_3 + m_4 + m_5)] + \ddot{\theta}_{m5}\left(\frac{LR_g m_5}{2}\right) + \ddot{\theta}_{m4}\left[LR_g\left(\frac{m_4}{2} + m_5\right)\right] + \ddot{\theta}_{m3}\left[LR_g\left(\frac{m_3}{2} + m_4 + m_5\right)\right]}$$

$$\boldsymbol{+ \ddot{\theta}_{m2}\left[LR_g\left(\frac{m_2}{2} + m_3 + m_4 + m_5\right)\right] + \ddot{\theta}_{m1}[R_g(-m_1 n_1 + L(m_2 + m_3 + m_4 + m_5))] + c_t\dot{\theta}_t}$$

$$\boldsymbol{+ k_{tg}(\theta_t - \theta_{m1}) - \left[\theta_d\left(\frac{k_d}{R_p}\right) - \theta_t\left(\frac{k_d R_t}{R_p^2}\right)\right] R_t = -\ddot{\theta}_y[-I_a + Rg^2(m_1 + m_2 + m_3 + m_4 + m_5)] + c_t\dot{\theta}_y}$$

7. **For drive-line**

$$I_{d_A}\ddot{\theta}_{d_A} + c_{d_A}\dot{\theta}_{d_A} + k_d(\theta_d - \theta_p) = T_d$$

$$\boldsymbol{I_{d_A}\ddot{\theta}_{d_A} + c_{d_A}\dot{\theta}_{d_A} + k_d\left(\theta_d - \frac{R_t}{R_p}\theta_t\right) = T_d}$$

Multivariable matrix form:

$$M\ddot{\boldsymbol{\theta}} + D\boldsymbol{\theta} + K\boldsymbol{\theta} = F\boldsymbol{u}$$

Where $\boldsymbol{\theta} \coloneqq \left(\theta_{d_A}\theta_t\theta_{m1}\theta_{m2}\theta_{m3}\theta_{m4}\theta_{m5}\right)^T$ and $u := \left(T_d\dot{\theta}_y\ddot{\theta}_y\right)^T$. Matrices M, D, K and F are shown in the appendix in matlab code. Finally, elevation system of gun-barrel's dynamics can be represented by the stata-space equations

$$\dot{\xi} = A\xi + Bu \;\; and \; y = C\xi + Du$$

With the states $\xi = (\theta_{d_A}\theta_t\theta_{m1}\theta_{m2}\theta_{m3}\theta_{m4}\theta_{m5}\dot{\theta}_{d_A}\dot{\theta}_t\dot{\theta}_{m1}\dot{\theta}_{m2}\dot{\theta}_{m3}\dot{\theta}_{m4}\dot{\theta}_{m5})$, and with the system and input matrices

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ M^{-1}F \end{pmatrix}$$

Matrix C depends on the output(s) of interest or on the measurement(s) available and matrix D is zero matrix.

3. DC SERVOMOTOR MODELLING

In order to obtain a more realistic result in the modeling of the actual system, it should be detailed. In the previous section, azimuth and elevation dynamics have been modeled mathematically to include the driveline and the gun barrel flexibilities. While deriving the equation of motion for the drive line in Part 2.1.1, we obtained the input torque value by multiplying the incoming input voltage and the drive torque constant. The driving torque constant is the simplified model of the dc motor and gearbox. It affects the system as gain. Therefore, DC motor and gearbox need to be modeled in more detail,instead of a drive toruqe constant. In this section, modeling of armature control of dc servomotors and gearbox is examined.

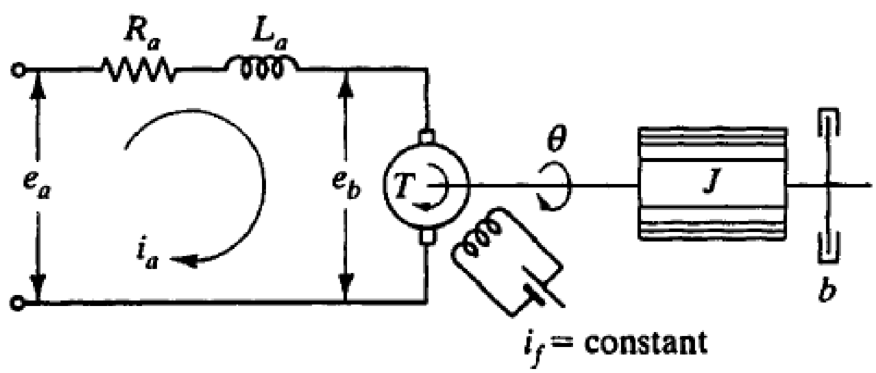

Figure 3.1. Armature-controlled DC Servomotor.

There is electrical and mechanical system in modeling of armature dc servo motor. The electrical system consists of the armature and the field circuit .The speed of DC motor is directly proportional to armature voltage and inversely proportional to flux in field winding. If the field is excited by a constant flux, the desired speed can be obtained by varying the armature voltage which is the input voltage in armature-controlled DC motors. Therefore, the electrical system can be considered only consists the armature circuit.The mechanical system consists of the rotating part of the motor and load connected to the shaft of the motor. The armature-controlled DC motor speed control system is shown in the figure 3.1.

In this figure3.1:

R_a = armature resistance, Ω
L_a = armature inductance, H
i_a = I= armature current, A
i_f = field current,A
e_a = applied armature voltage, V
e_b = back emf, V
θ= angular displacement of the motor shaft, rad
T = torque developed by the motor,N-m
J = moment of inertia of the motor and load referred to the motor shaft, kg-m^2
b = viscous-friction coefficient of the motor and load referred to the motor shaft, N*m/$rads^{-1}$

3.1 Mathematical Models

The armature-controlled DC motor speed control system modeling is shown below.

According to Kirchoff Laws, armeture circuit equation is,

$$-e_{\mathrm{R,a}} - e_{L,a} - e_b + e_a = 0$$

where the voltages for resistor,inductance, back emf and input,Volt. Then, the mathematical model is obtained,

$$L\dot{I} + RI + K_b\dot{\theta} = e_a \qquad Eq. 2.3.1$$

where K_b is the back-emf constant,V*s/rad.

According to Newton Second's Laws,mechanic parts of equation is,

$$\Sigma T = T_m - b\dot{\theta}$$

$$K_m I - b\dot{\theta} = J\ddot{\theta}$$

then, the mathematical model is obtained ,

$$J\ddot{\theta} + b\dot{\theta} = K_m I \qquad Eq. 2.3.2$$

where K_m is the motor torque constant, N*m/A.

Assuming that all initial conditions are zero and the Laplace transforms of mathematical model equations 2.3.1 and 2.3.2 are taken.Then we obtained the transfer function for the DC servomotor considered the $E_a(s)$ as the input and I(s) as the output by eliminated Θ (s).

Finally, the transfer function of DC motor is obtained:

$$G(s) = \frac{\mathrm{I}(s)}{E_a(s)} = \frac{Js + b}{JLs^2 + (RJ + Lb)s + Rb + K_b K_m}$$

The signals generated by the controllers are a function of the voltage. These signals are the input signals of the DC motor. These signals from the controllers must be in a suitable range for the DC motor which is used in the azimuth and elevation systems. A DC motor running with 15 Volts will be enough for both systems.

In summary, we have obtained the transfer function of the DC motor to be used in the Simulink and we have defined the range for input voltage of the motor. In our DC-motor modeling, the output signal is current. However, the input signal of the plant must be torque. Therefore, a gear train (or gearbox) should be used in the continuation of the Dc motor. This gear train wil be provided a torque constant which can be used for finding the input torque. With this constant, DC motor output current is multiplied, and input torque can be obtained.

$$T_{in} = K_g * I$$

where K_g is gear train torque constant,N*m/A.

Gear trains are frequently used in mechanical systems to reduce speed, to magnify torque output of the motor. By increasing the gear ratio, we can achieve the desired torque. But this design is not the scope of this thesis. A certain gear train torque constant will be determined according to the torque value of the plant.

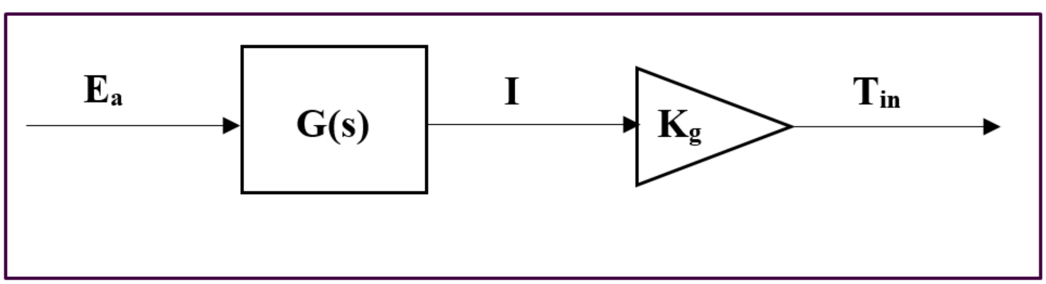

Figure 3.2. Block Diagram of DC Servomotor and Geartrain

4. Disturbance Modelling

We have already mentioned that the gun barrel of MB tanks has a flexible structure. In order to observe the oscillations caused by this flexibility, we made system models of 6 Dof for elevation and 7 Dof for azimuth. Many disturbance factors are observed in the tank, both due to the lands in which these tanks are used and due to the vibrations caused by the unbalance masses. In the design studies of the stabilization contreller, the effect of the disturbances formed on the flexible structure gun-barrel a will be simulated and examined. The disturbances (which are pitch disturbance on elevation axis and yaw disturbance on azimuth axis) used in this study were simulated by looking at the disturbances obtained as a result of experiments on the Leopard A1 tank.

First of all, let's talk briefly about these experiments. Measurements were made with feedforward gyros placed on the hull and turret of the tank and two different experiments were carried out. One of them is Aberdeen Proven Ground (APG) Course in NATO standards and the other one is sinusoide course. In this experiment, we will use the results of azimuth and elevation disturbances from APG course.

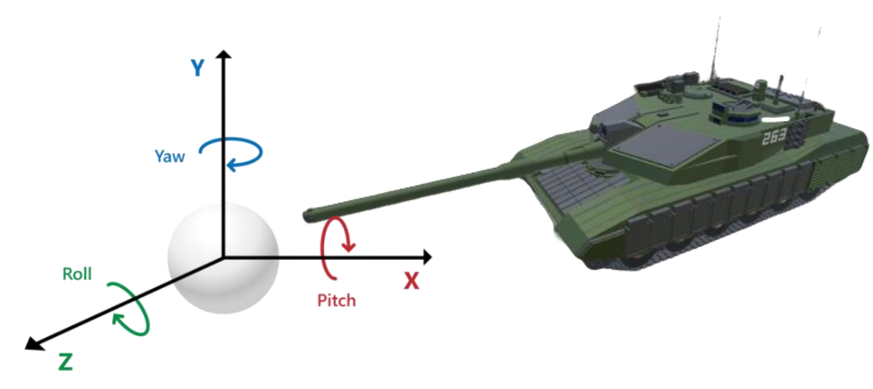

Figure 4.1. Pitch-Roll-Yaw Axes of MBT

4.1. Elevation Disturbance Modelling

There are two different disturbances that affect Elevation dynamics. The first of these is the vertical disturbance of the tank that acts linearly from the ground to the hull. This disturbance directly affects the gun barrel. We simulated (Figure 4.2) these disturbances as acceleration fluctuations in the vertical direction (y axis) that change depending on time.

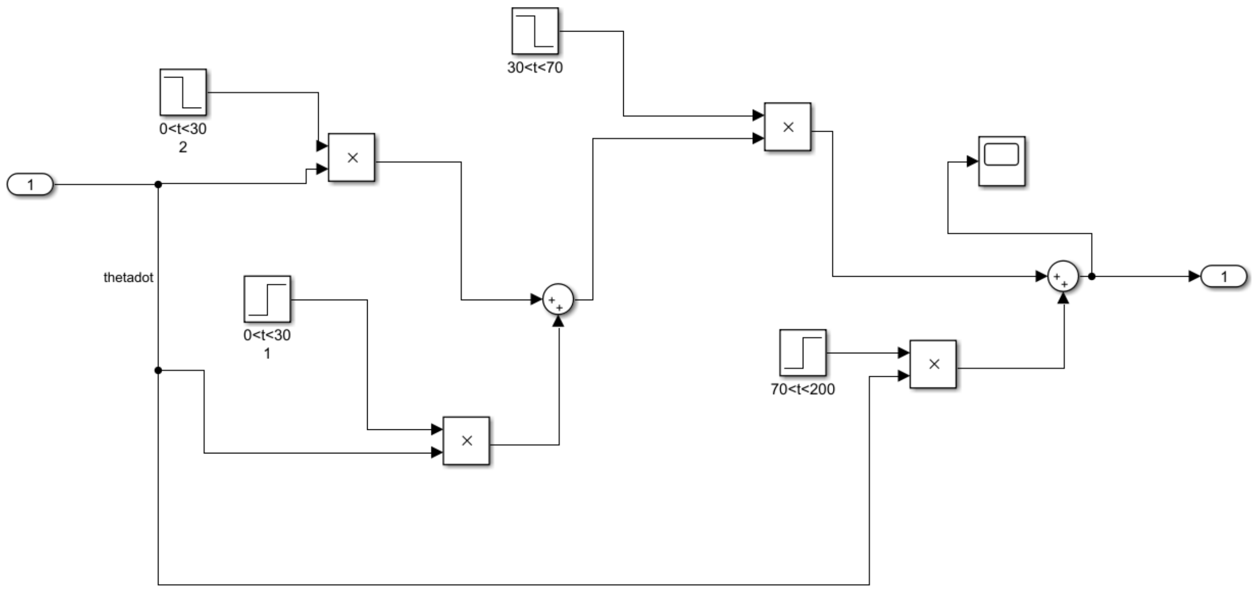

Figure 4.2. Simulink Model for Disturbace of $\ddot{y}_t$

The vertical disturbance is showed in Figure 4.3.($\ddot{y}_t$)

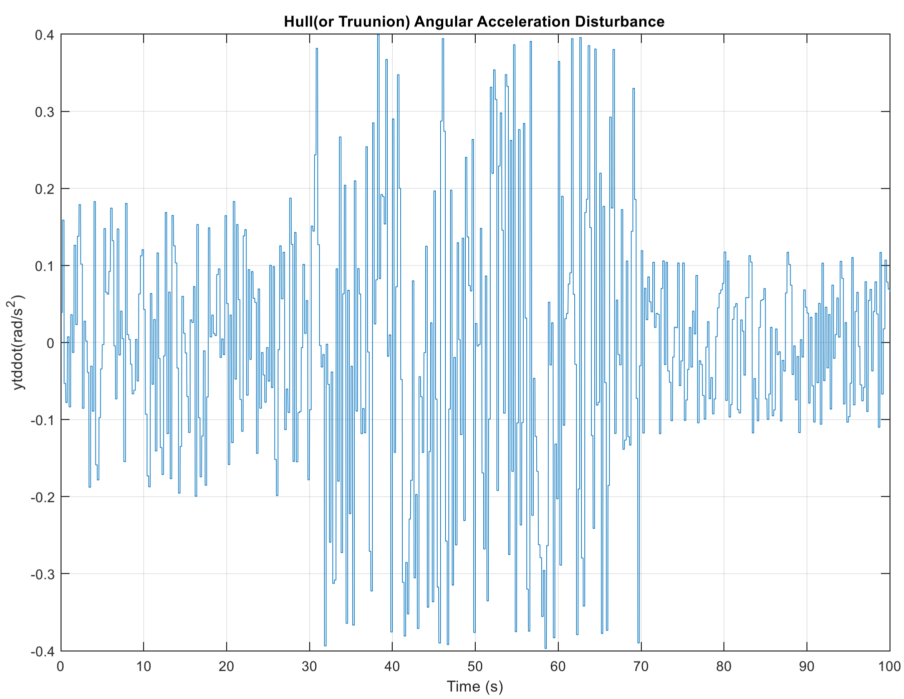

Figure 4.3. Trunnion Angular Acceleration Disturbance, $\ddot{y}_t$

In the APG Course experiment, it was observed that the tank passed over the bump and created disturbance in the pitch rotation axis of the hull. These disturbances were measured with feedforward gyro and it were recorded as angular velocity fluctuations at pitch rotation axis of the hull. We analyzed that disturbance and simulated it similarly vertical disturbance. This disturbance acts in elevation dynamics by changing the angle and angular velocity in the pitch rotation axis of the body. The pitch angular velocity disturbances are showed below in figure 4.3. By integrating the pitch angular velocity, the angle disturbance is obtained.

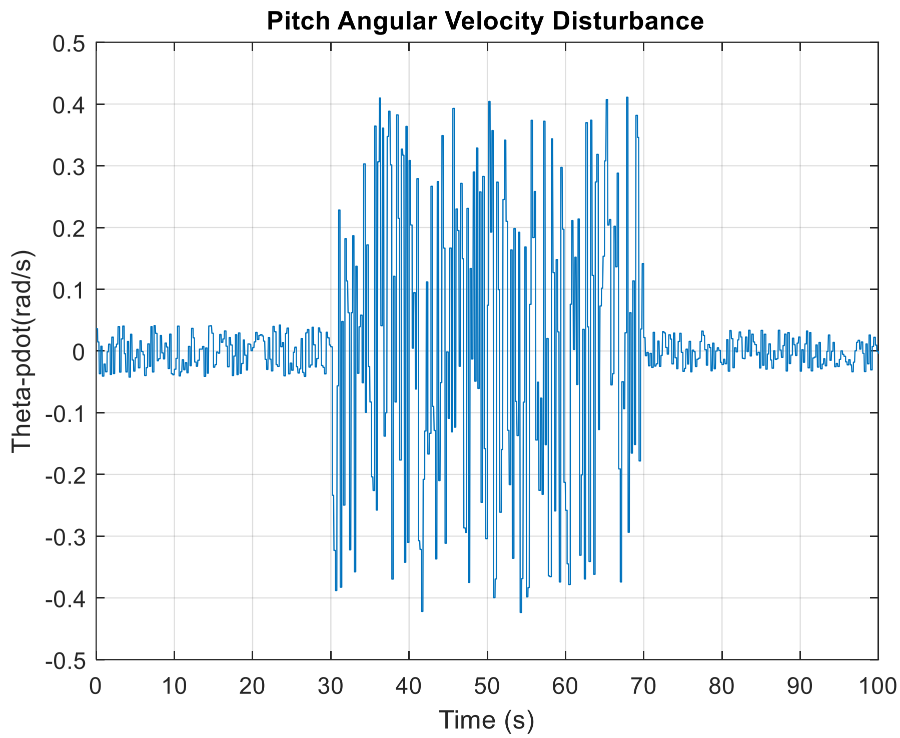

Figure 4.4. Pitch Angular Velocity Disturbance, $\dot{\theta}_{\rho}$

4.2. Azimuth Disturbance Modelling

Again, the APG Course experiment, it was observed that the tank passed over the bump and this time it created disturbance in the *roll* rotation axis of the hull. These disturbances were measured with feedforward gyro and it were recorded as angular velocity fluctuations at *yaw* rotation axis of the hull. We again analyzed that disturbance and simulated it similarly vertical disturbance and pitch. This disturbance acts in azimuth dynamics by changing the angular velocity and angular acceleretion in the yaw rotation axis of the turret. The yaw angular velocity disturbances are showed below in figure 4.5**.** By derivating the yaw angular velocity, the angular acceleretion disturbance is obtained.

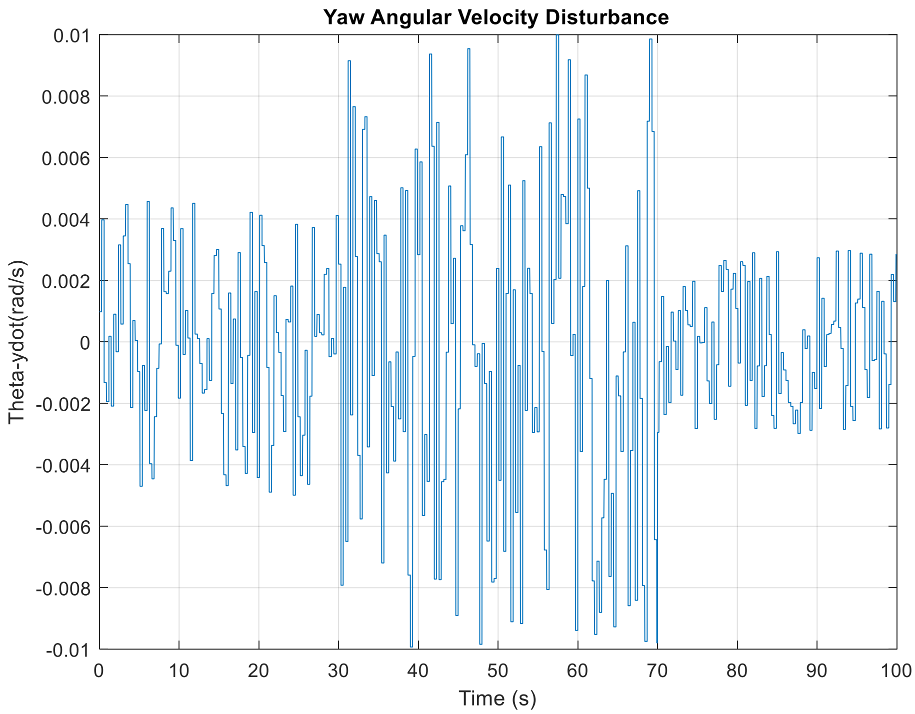

Figure 4.5. Yaw Angular Velocity Disturbance, $\dot{\theta}_y$

4.3. Simulations

In this section, disturbance data will be given to the system model. All controller inputs will be zero. There will be no controller, so the disturbance rejectance without controller will be observed. A Simulink model is prepared for the simulation. Response of the muzzle is plot for both axes.

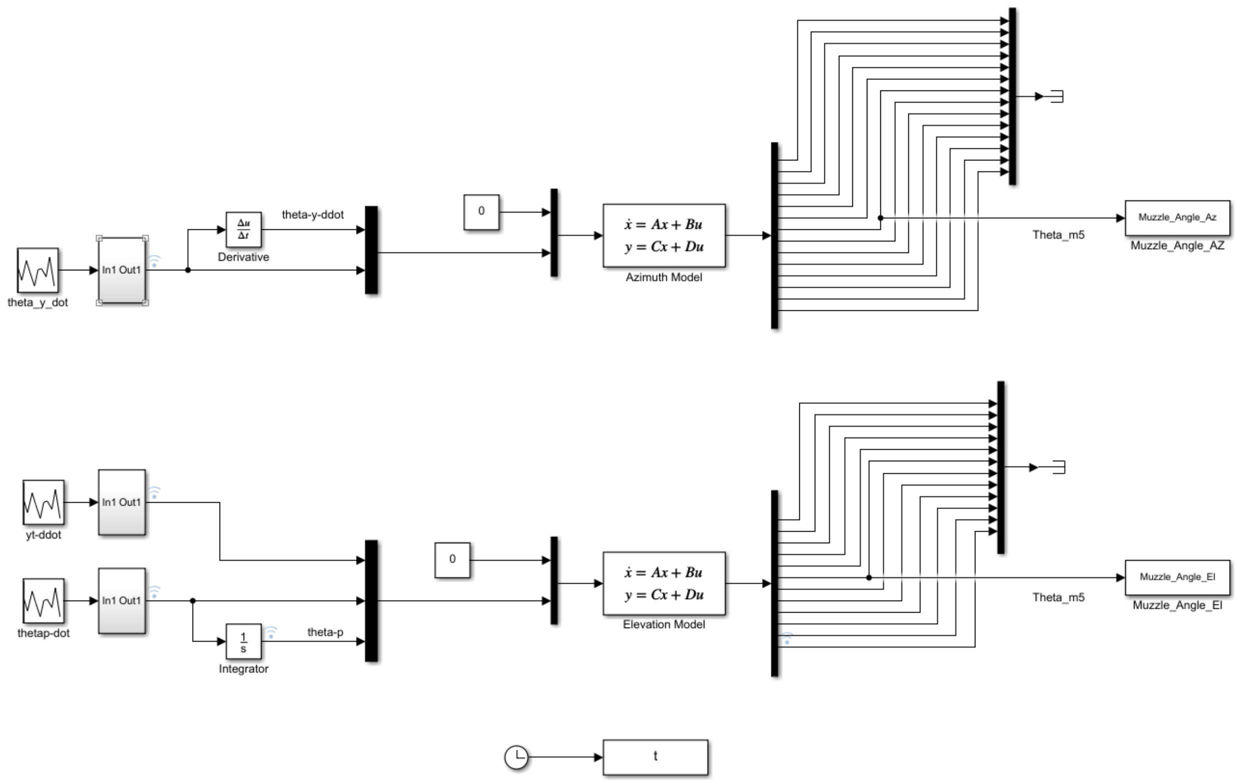

Figure 4.6. Simulink Model for Disturbance Response w/o Controller

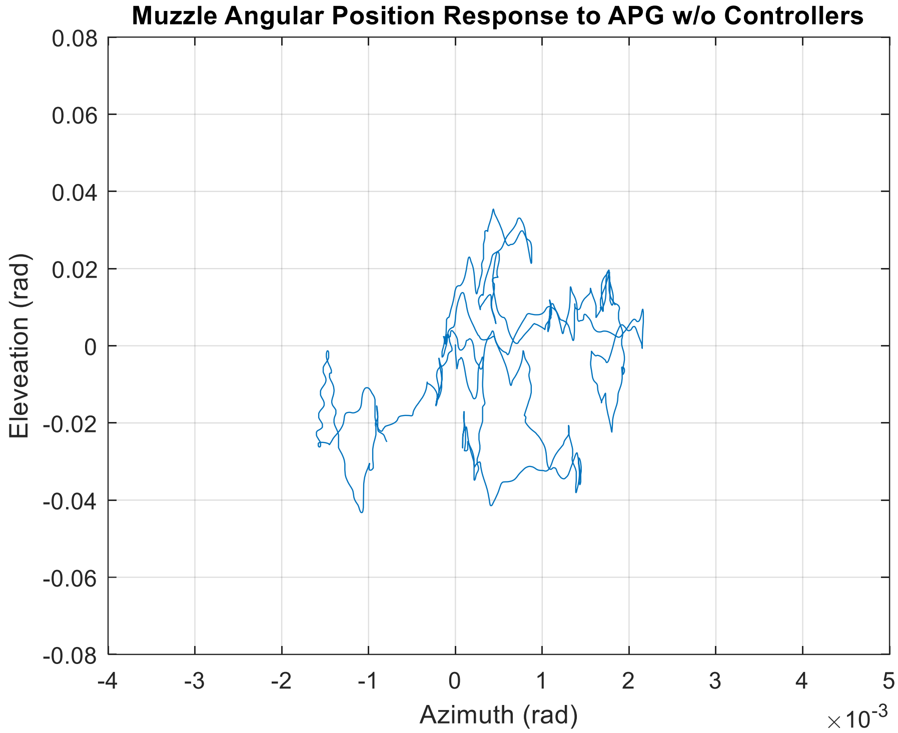

Figure 4.7. APG Angular Position Response of the Muzzle w/o Controller

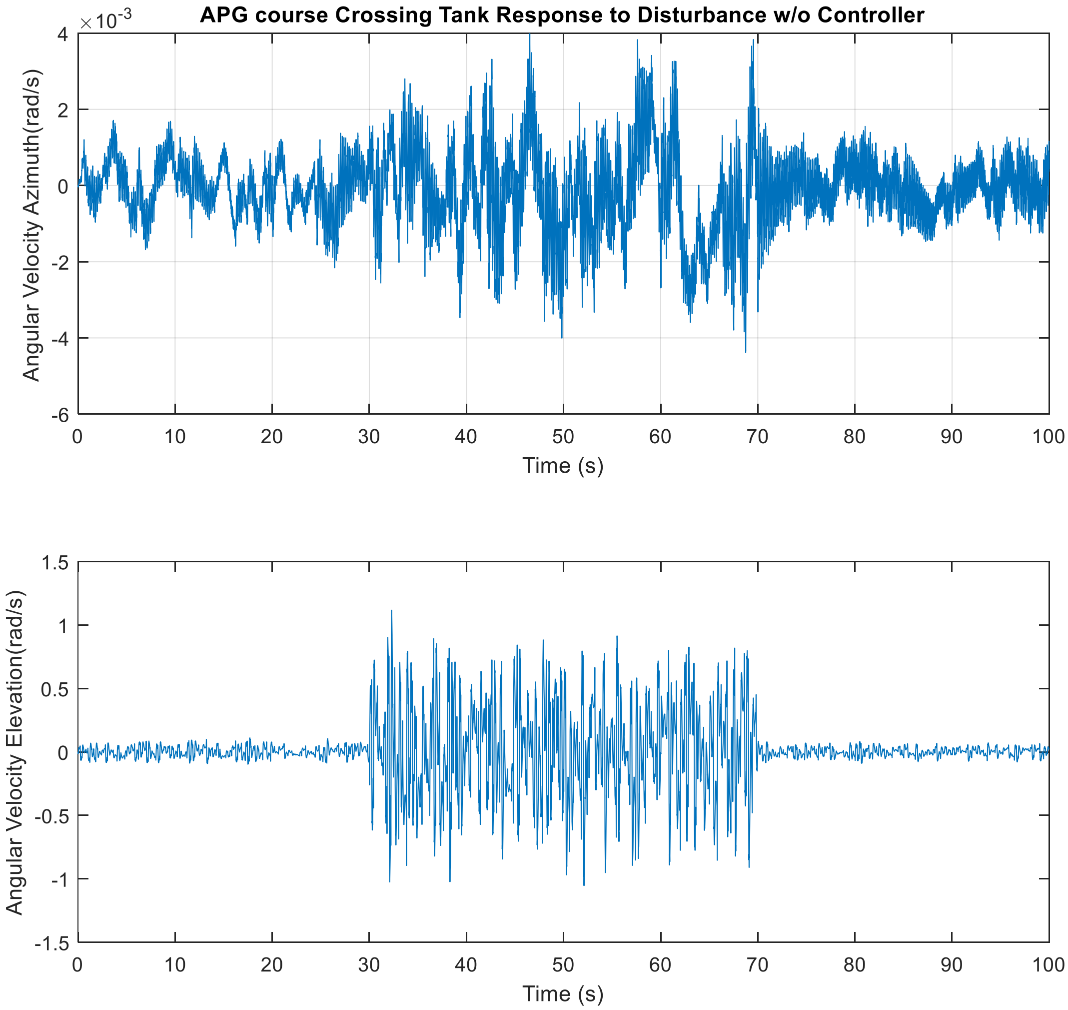

Figure 4.8. APG Response of the Muzzle w/o Controller

5. CONTROLLER DESIGN

In this section, we will design two controllers, one for position and one for speed, for our barrel with 5 degrees of freedom to reach the reference value. The feedback controller will be designed by step response. When designing the controllers, different chracteristic responses such as settling time overshott will be considered as appropriate values, and a steady status will be attempted at a reference input value close to our reference value.
Aim of this thesis is to study mainly the effects of the flexibilities between the muzzle and the trunnion. It should not be forgotten that the angle at the end of the barrel will be smaller than the angle at the beginning. The reason for this is to use one spring per mass. Under these conditions, the flexibity status will be examined and appropriate controllers will be designed.

5.1. Elevation controller design

There are lots of method to design controller.One of them is root locus.But root locus not suitable for the controller design of this system. 5-DOF transfer function elevation model has high order that cause to make analysis difficult.According to this ,controller will be design with MATLAB PID tune. Therefore, system must be converted state space model form.Firstly,disturbances will be zero and 5-DOF state-space elevation model is driven by a step input having 30-degree amplitude for position.Also inpulse input with amplitude 3 degree/s will be used for speed control.

When all the above requirements are met, MATLAB PID tunner give very high controller values which are 15000 for Kp and 180000 for K_I.This is not realistic because the controll effort effect must be observed here. The control effort is the amount of energy or power necessary for the controller to perform its duty.With these controller control effort will reach about 40.000. There is no resource in the system to meet this much value.So that saturation limit with amplitude 15 will be used and DC motor will be preferred to create needed source.

One of the most important thing that,speed and position is not independent from each other.

To keep position desired value which 30-degree, speed must reach 0 after a while.So derivative controller should be used to prevent this problem

Another case is integral windup.Integral windup, also known as integrator windup or reset windup,refers to the situation in a PID feedback controller where a large change in setpoint occurs (say a positive change) and the integral term accumulates a significant error during the rise (windup), thus overshooting and continuing to increase as this accumulated error is unwound (offset by errors in the other direction). The specific problem is the excess overshooting. Integral windup particularly occurs as a limitation of physical systems, compared with ideal systems, due to the ideal output being physically impossible (process saturation: the output of the process being limited at the top or bottom of its scale, making the error constant).

Once all system requirements are met, position and speed controller designed thanks to MATLAB PID tune ;

Position controller

Kp=250
Kd=150

Speed controller
Kp=950
KI=250
Kd=200

With these controller systems simulated without disturbances like below.

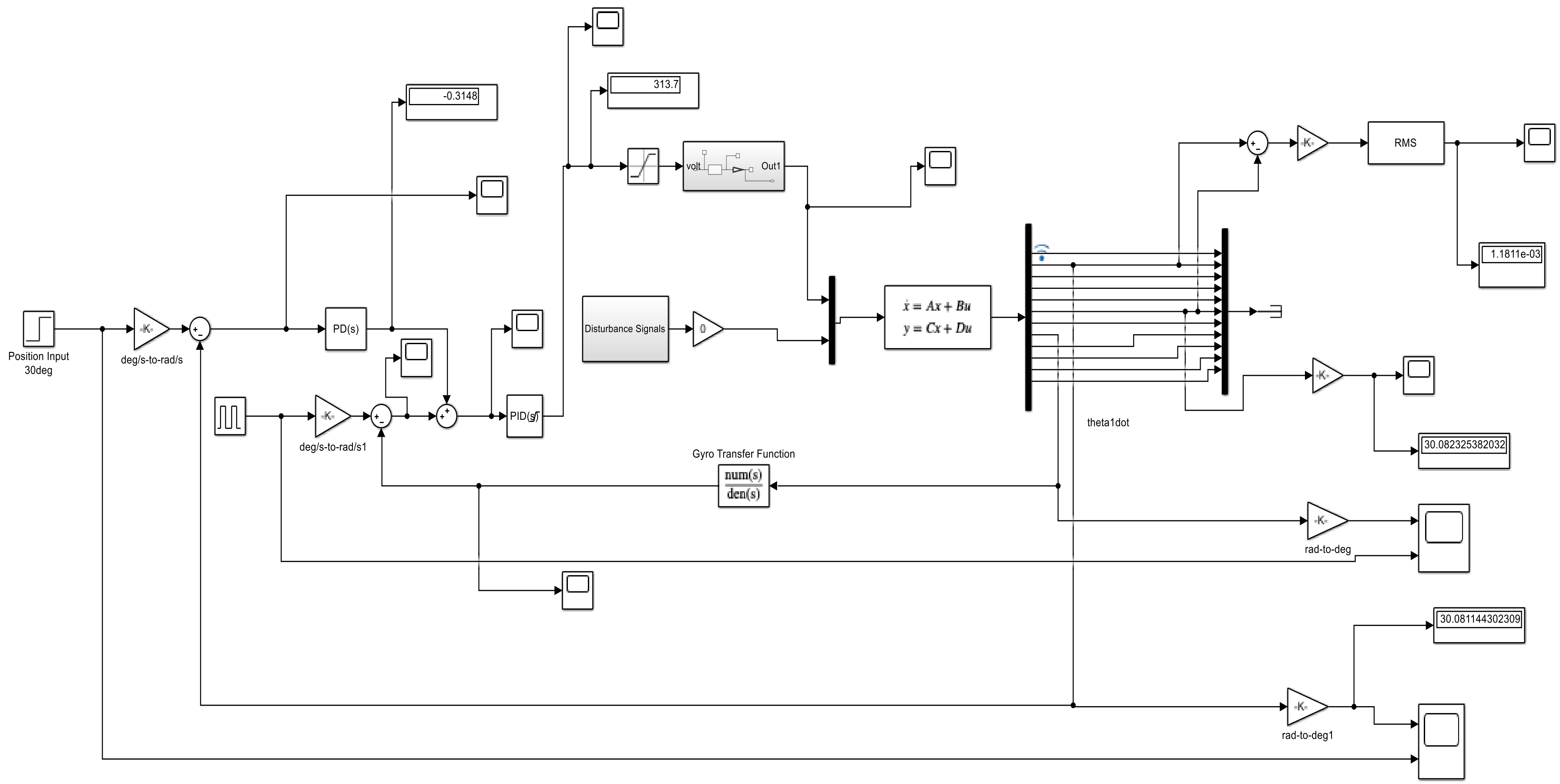

Figure 5.1. Simulink Design with Controller (Disturbances Passive)

As it seems ,Muzlle accuracy reach 0.00118-degree ,Muzzle position is 30.0823 degree and finally breech position is 30.0814 degree.

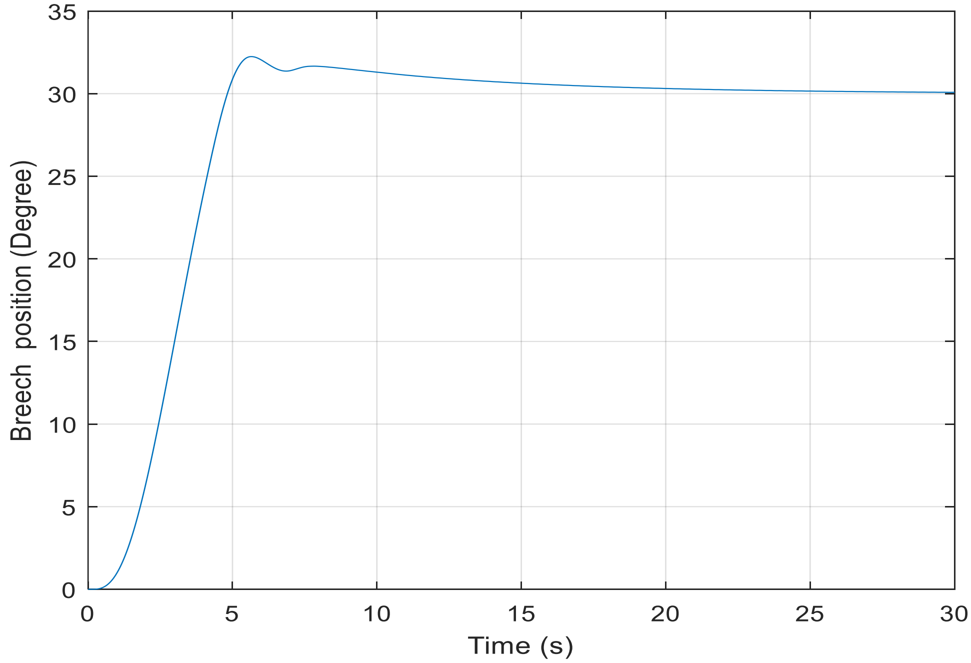

Figure 5.2. Breech Position Without Disturbances

Characteristic Responses
Overshoot=% 0.04
Settling time =16s

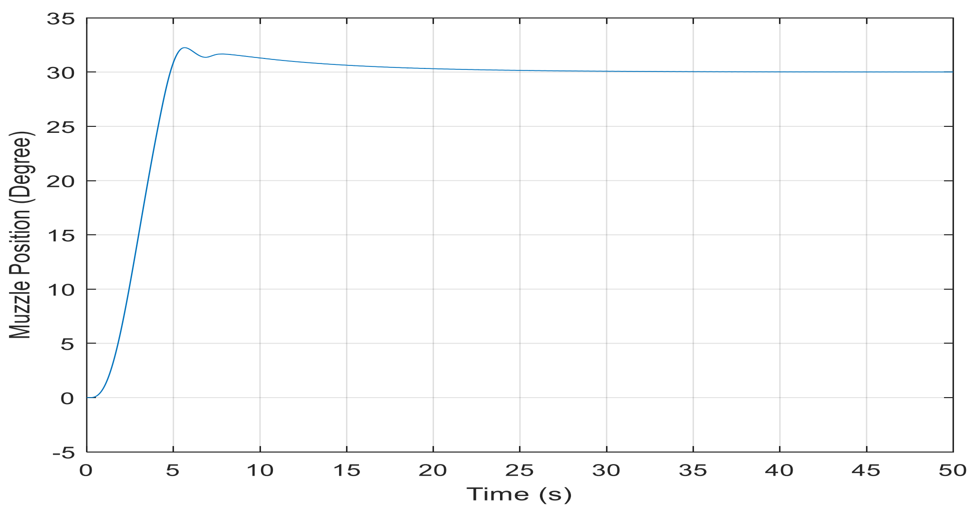

Figure 5.3. Muzzle Position Without Disturbances
Characteristic Responses
Overshoot=% 0.05
Settling time =16s

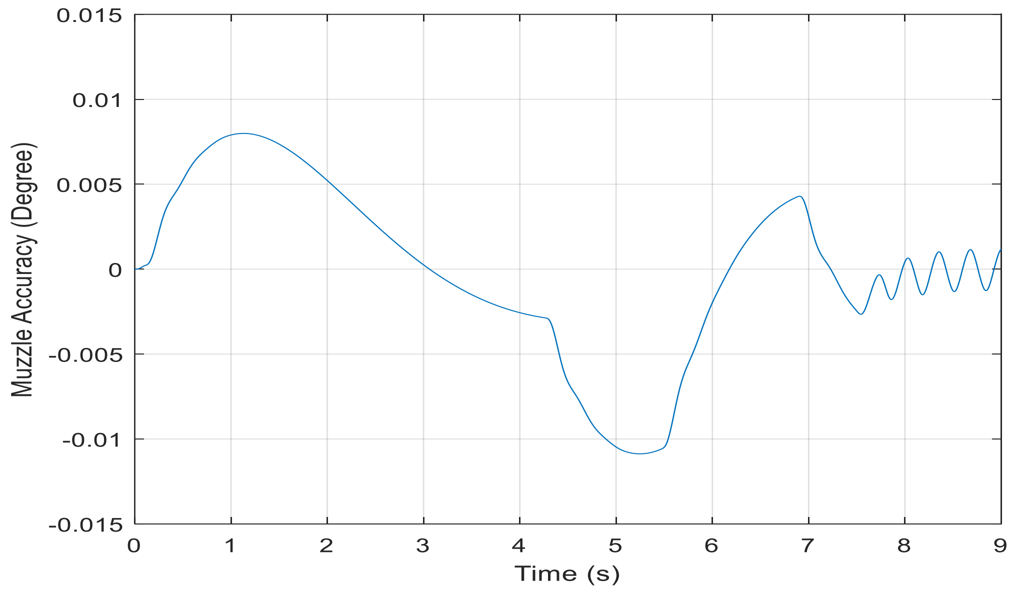

Figure 5.4. Muzzle Accuracy Without Disturbances

Muzzle Accuracy is acceptable level.Tolerance was 0.00025 degree .

Now disturbances disturbaneces will be given to the system.

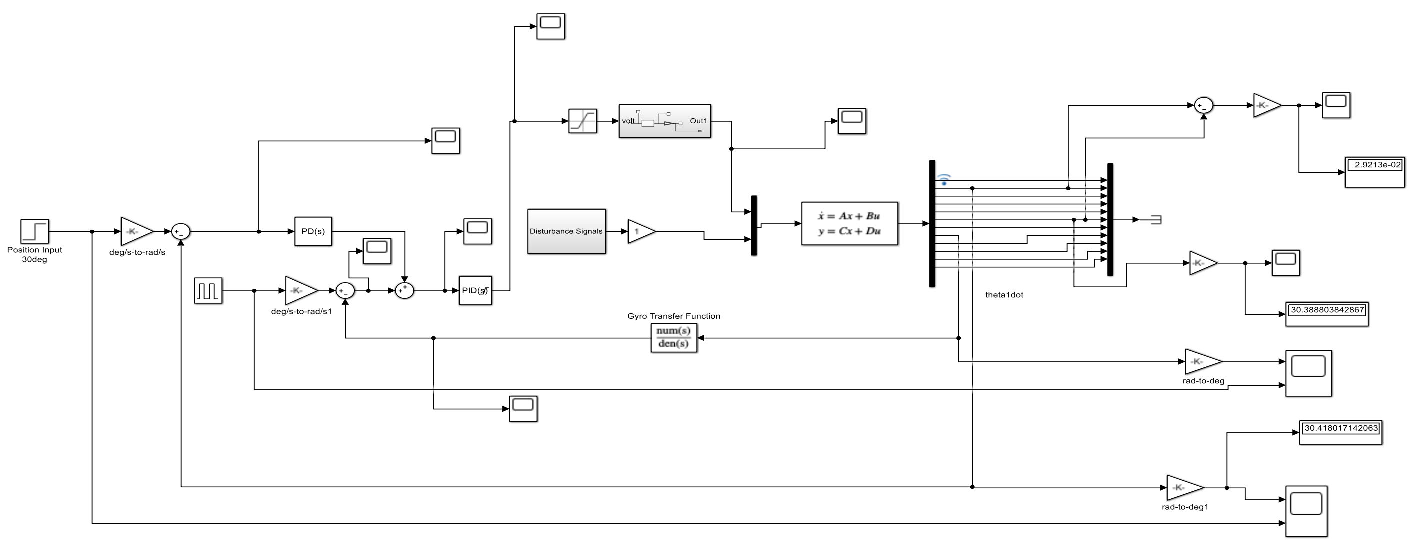

Figure 5.5. Simulink design with controller (disturbances active)

As it seems, Muzzle position is 30.388 degree and finally breech position is 30.418 degree.When disturbances are activated new muzzle accuracy equal 0.0292.

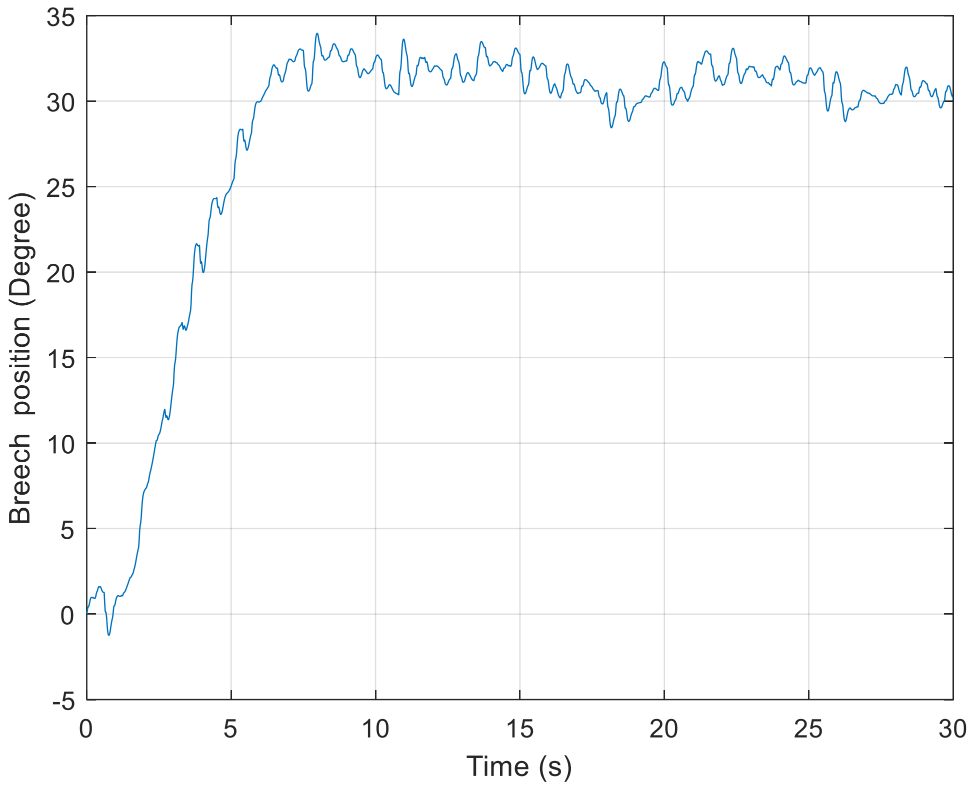

Figure 5.6. Breech Position with Disturbances

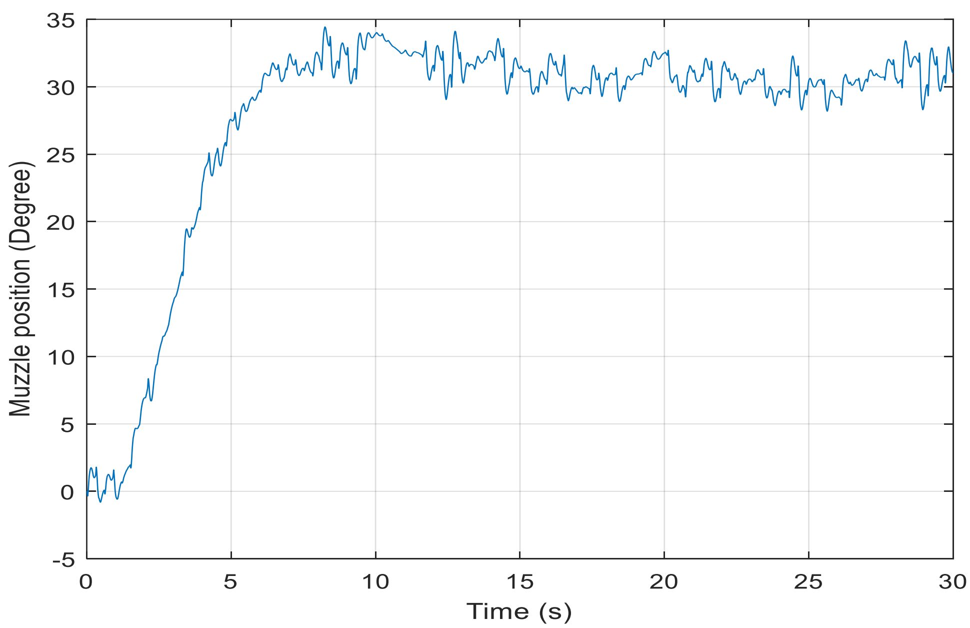

Figure 5.7. Muzzle Position with Disturbances

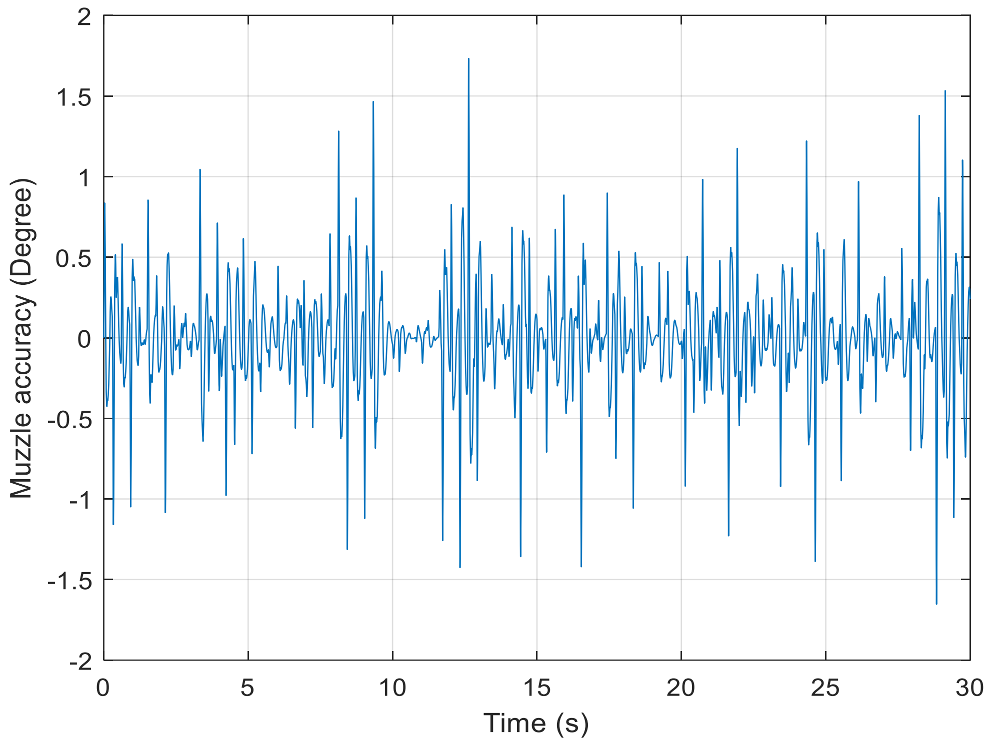

Figure 5.8. Muzzle Accuracy with Disturbances

In this part controller designed and reference tracking done succesfully. Looking at the results, muzzle and breech reached steady status close to the desired values and muzzle accuarcy remained within the limits of the tolerance threshold.

6. BASED OBSERVER DESIGN

In this section,cause that the sensor cannot be attached to the end of the tank barrel and is costly to inspect,observer-based design will be applied to stabilized muzzle position . An observer for muzzle will be studied and the results will be analyzed

6.1. Based observer Model

The purpose of the observer is to generate an estimate of the state (x) based on measurements of the system output (y) and the system input (u). The input and output signals are assumed to be exactly measurable no noise or other interference. The observer uses a mathematical model of the state space realization of the system. Therefore, the [A, B, C, D] matrices are assumed to be known exactly.

The possibility of modeling errors is not included in the derivation of the observer. The observer is an n th-order linear dynamic system, where n is the number of state variables in the system.For elevation system n equal to 12.

The observer being considered in these notes is a deterministic system. It assumes that there is no measurement noise or unmeasured disturbances acting on the system. If there are disturbances and measurement noise acting on the system, then the Kalman filter should be implemented since it uses knowledge of the statistical properties of the system in its design.In this project disturbance observer will not be used.

The approach in these notes is to model the observer state equations as a model of the actual system plus a correction term based on the measured output and the estimate of what that output is expected to be. With the actual system described by

$$\dot{x}=Ax+Bu$$
$$y=Cx+Du$$

Equaition 4.1.1.

the observer is modeled as

$$\hat{z} = A\,z + Bu + L\,[\bar{y} - y\,]\,,$$

$$\bar{y} = Cz + Du$$

Equaition 4.1.2.

where L is the n × m gain matrix for the observer. The state equation in Equation 4.1.2 is seen to model the actual state equation, with the true state x replaced by the estimate $\hat{z}$, and a correction term which is the difference between the actual measured output y and its estimate $\bar{y}$. The output equation in (2) is also seen to be a model of the system's output equation, with x replaced by its estimate.

Substituting the expression for $\bar{y}$ into the observer's state equation yields the following alternative forms for the model of the observer.

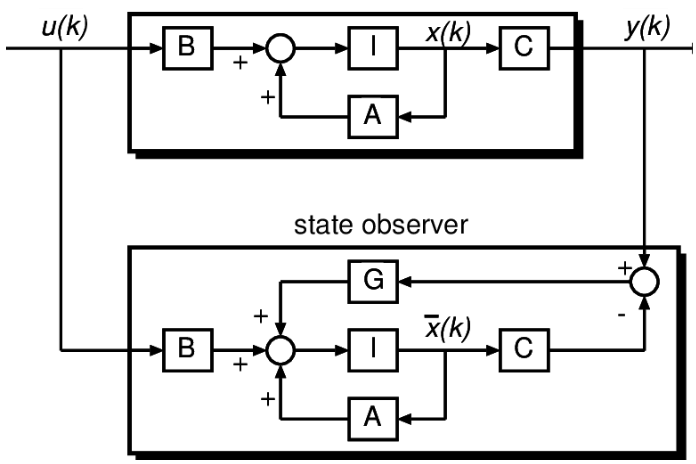

Figure 6.1. State Observer Modelling

6.2. Estimation Error

The purpose of the observer is to produce an estimate of the true state x. It is reasonable to assume that there will be some error in the estimate at the initial time, but it is hoped that the error would decrease over time. The estimation error will be defined as

$$\mathbf{e} = \mathbf{x} - \mathbf{z}$$
$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \hat{\mathbf{z}}$$

Using the system model and the observer ,

$$\dot{e} = Ax + Bu - Az - Bu - L(y - Cz)$$

$$\dot{e} = [A - LC](x - z)$$

Thus, the state equation for the estimation error is a homogeneous differential equation governed by the n × n matrix A −LC. If the gain matrix L is chosen so that the eigenvalues of A−LC are strictly in the left-half of the complex plane, then the error equation is asymptotically stable, and the estimation error will decay to zero over time. If the system (A, C) is completely observable, then L can be chosen to place the eigenvalues of A − LC at arbitrary locations in the plane, under the restriction that complex eigenvalues must appear in complex conjugate pairs. As long as (A, C) is at least detectable, then A − LC can be made asymptotically stable by choice of L.

6.3. Observability

A system is said to be observable if, for any possible evolution of state and control vectors, the current state can be estimated using only the information from outputs (physically, this generally corresponds to information obtained by sensors). In other words, one can determine the behavior of the entire system from the system's outputs. On the other hand, if the system is not observable, there are state trajectories that are not distinguishable by only measuring the outputs.

If the row rank of the observability matrix, defined as

O=[C CA^2 CA^3 … CA^n-1];

is equal to n then the system is observable.

Due to the limited number of significant digits in the matlab, a numerical error occurs, and the system looks like unobservable. But this problem can be solved by using Multiprecision Computing Toolbox is a app in Matlab. As a result, the ranks of controllability and observability matrices were found eqaul and observer-based design can be done. Outputs in matrix C are selected only as muzzle and breech and re-created.

6.4. Observer based simulink design

One of the important points that ,when designed observer state ,feedback will be taken theta 5 which is muzzle position to better stabilized.

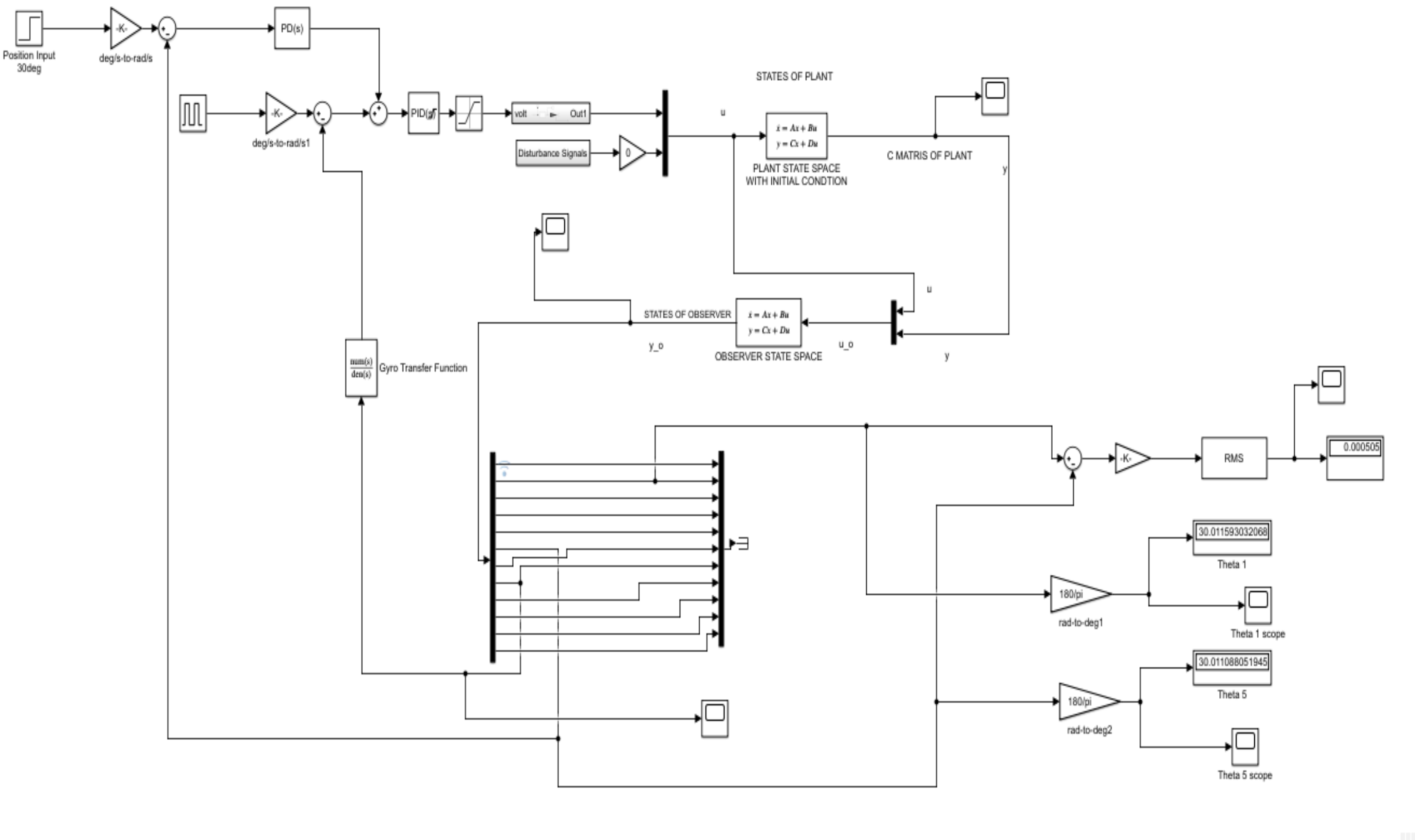

Figure 6.2. Based Observer Model Without Disturbances

With this method,breech reach 30.01108 degree and muzzle reach 30.01159 degree. It is seen that the angle of muzzle and breech are closer to 30 degrees.Also, the difference between these two angles has decreased by 2.5 times, making it more stabilized.

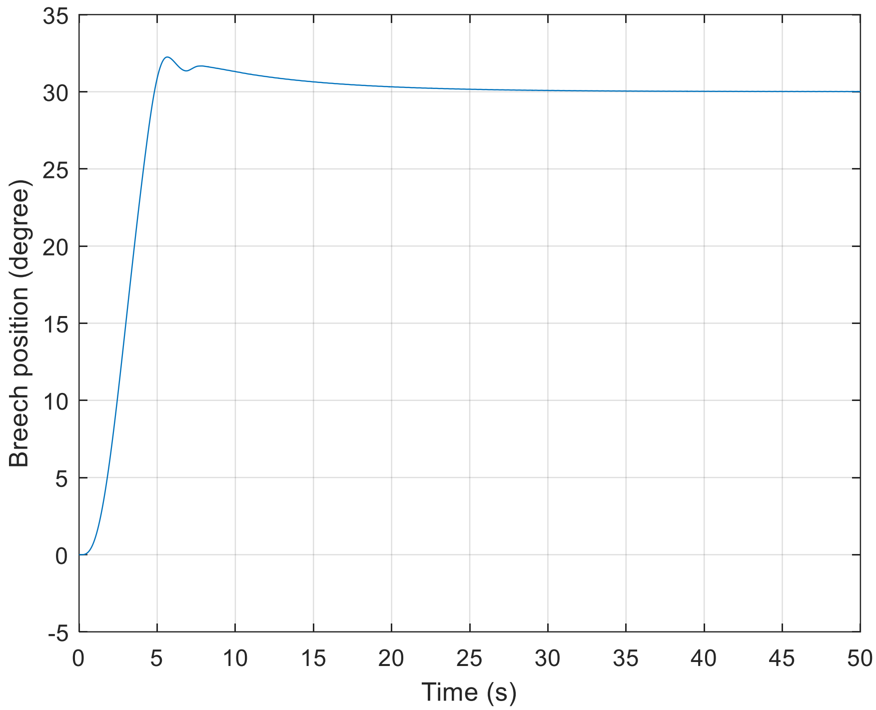

Figure 6.3. Breech Position Without Disturbances

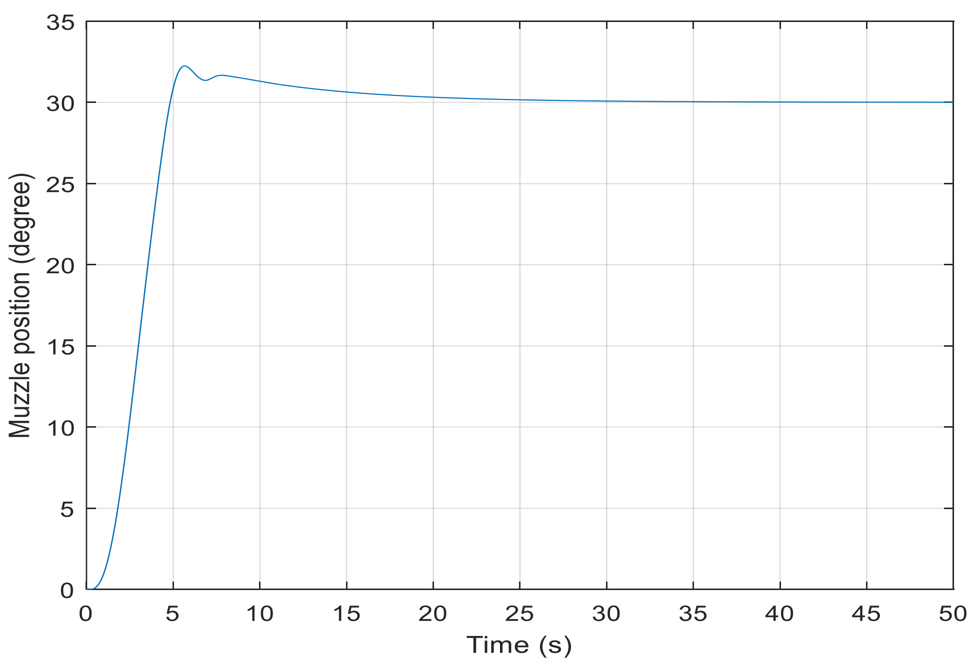

Figure 6.4. Muzzle Position Without Disturbances

Now disturbances are activated.New design like below,

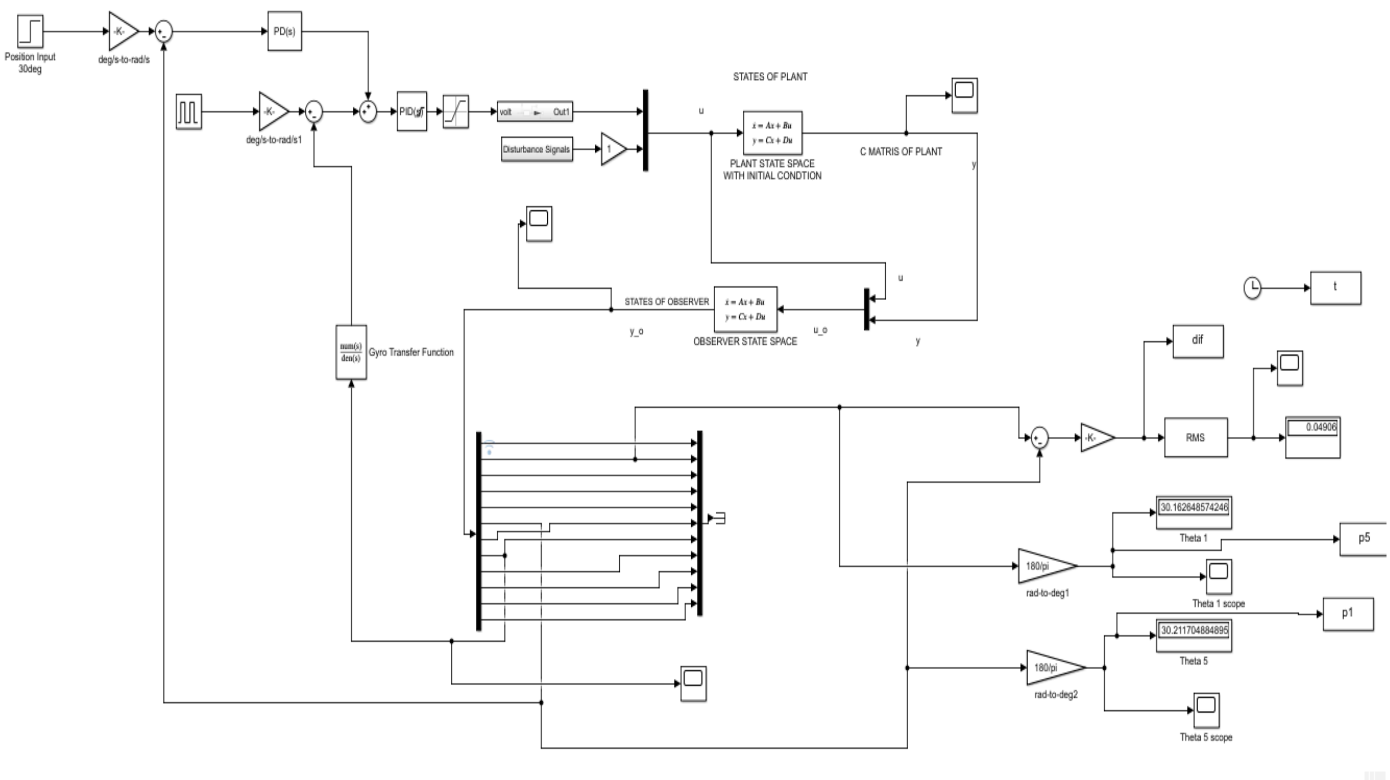

Figure 6.5. Based Observer Model with Disturbances

Add disturbances to the system, albeit a small amount of change.New breech position is 30.211 degree and new muzzle position is 30.162 degree.Difference with these positions is 0.049.

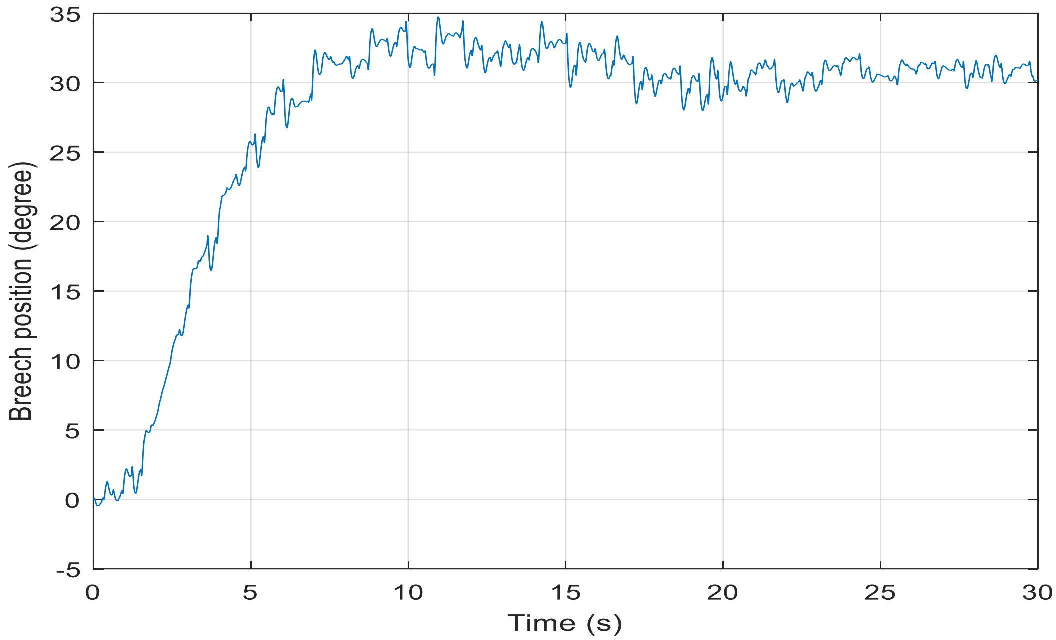

Figure 6.6. Breech Position with Disturbances

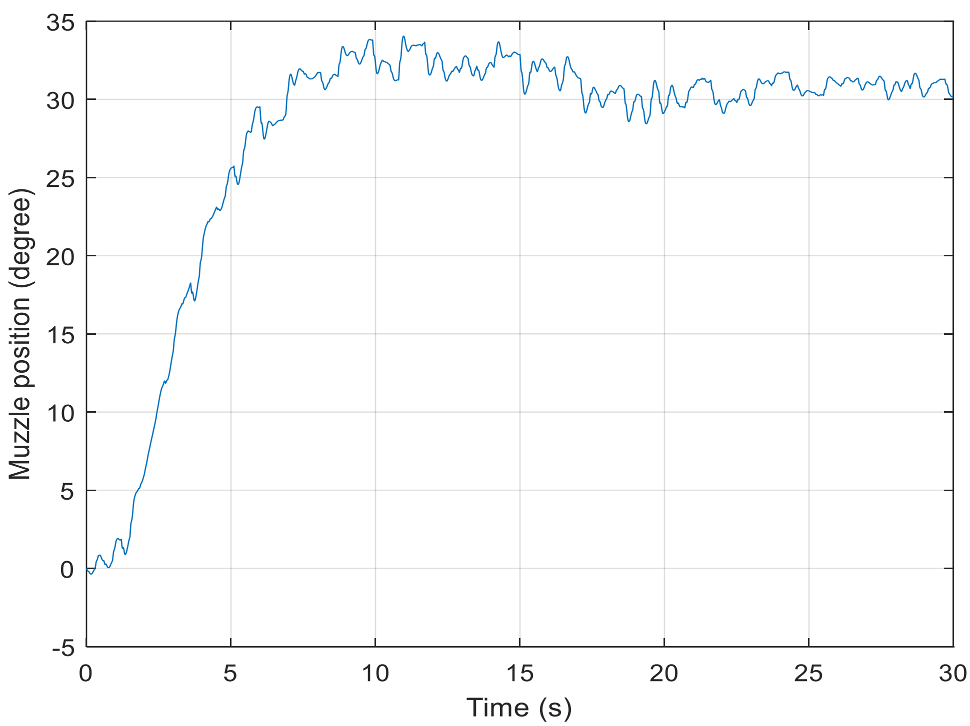

Figure 6.7. Muzzle Position with Disturbances

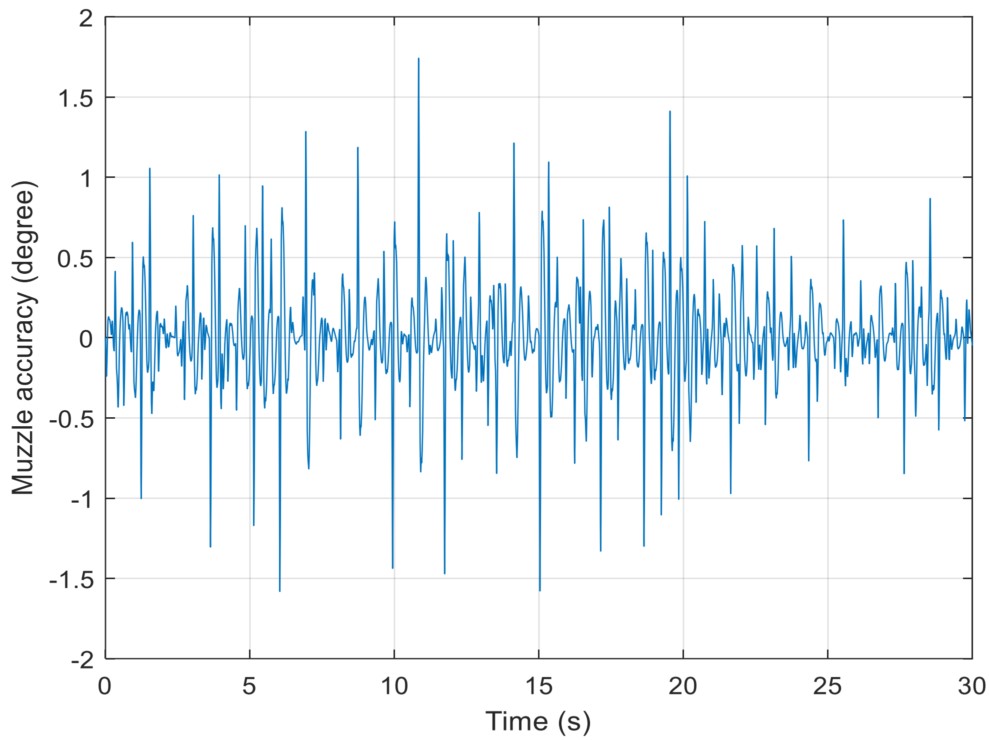

Figure 6.8. Muzzle Accuracy with Disturbances

	Breech position(deg)	Muzzle position(deg)	Muzzle accuracy(deg)
Without based observer	30.0823	30.0814	0.00118
Based observer	30.011	30.010	0.00050

Table 6.1. Observer Effect without Disturbances

	Breech position(deg)	Muzzle position(deg)	Muzzle accuracy(deg)
Without based observer	30.418	30.388	0.0292
Based observer	30.211	30.162	0.0215

Table 6.2. Observer Effect with Disturbances

It seems to be that ,both without distrubances and with disturbances , two part of barrel’s position approached 30 degree and also muzzle accuracy improved. By applying observer based to the system, the barrel has been made more stablilized and a system that is difficult to examine has been designed comfortably.

7. CONCLUSION

In this project, the deviation of the gun-barrel target caused by the barrel flexibility in the main battle tank was improved by improving the based observer system. The barrel system with 6 degrees of freedom was modeled for the controller design. The 7-degree tower rotation system was also modeled to observe the status of muzzle and breech against the disturbing effect. Controllers are designed based on parameters such as oscillation and settling time to create a reference tracking system. In addition, dc motor was designed and added to the system to solve the source problem in the controller design.In this part feed back taken from breech position. Before applying observer method, it was provided to come to the steady state position with the tolerance value determined by adhering to the given input value.When designing based obser, muzzle and breech positions were chosen to have outputs of the matrix C. The observability of the system was examined, and the solution of some numerical problems caused by matlab that affect this situation was explained. After all the conditions were met, the state observer was applied and this time the feedback was taken from the muzzle and the system was re-examined. Then, all these processes were repeated with the disturbance effect included in the system.

As a result, it was seen that the 30-degree input value given for both muzzle and breech is closer than the previous design. Finally, it was also shown that the angle difference between the tank's breech and muzzle has decreased and the system has been successfully stabilized. The benefits of the based observer method were seen at the end of this project when the use of sensors is difficult, or the system is costly to implement.

8. FUTURE WORKS

When designing this system, the degree of freedom was chosen 5 for the barrel. However, a barrel with a higher degree of freedom will give more precise results in terms of design. Also, with the theoretical knowledge, doing some practical experiments and observing the reaction of the system will increase the quality of the study.

In addition, some situations affecting the system in this project are not included, for example, vibrations or some additional forces. Including and re-examining these parameters will increase the realism of the project.

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[16] MATLAB 2018b Product Help Documentions

10. APPENDICES

10.1 Appendix A1 – MATLAB® m-file for Elevation Axis

```
%***********PARAMETERS OF 6-DOF MBT GUN-BARREL AND DRIVE LINE************

Le1 = 1;          % Length between trunnion and Gun part 1 (m)
Le2 = 1;          % Length between Gun part 1 and 2 (m)
Le3 = 1;          % Length between Gun part 2 and 3 (m)
Le4 = 1;          % Length between Gun part 3 and 4 (m)
Le5 = 1;          % Length between Gun part 4 and 5 (m)
n1 = 0.5;         % Distance between trunnion to CG of Gun Part 1 (m)
n2 = 0.5;       % Distance between CG of Gun Part 2 to intersection point of Gun Part 1 and Gun Part 2 (m)
n3 = 0.5;       % Distance between CG of Gun Part 3 to intersection point of Gun Part 2 and Gun Part 3 (m)
n4 = 0.5;       % Distance between CG of Gun Part 4 to intersection point of Gun Part 3 and Gun Part 4 (m)
n5 = 0.5;       % Distance between CG of Gun Part 5 to intersection point of Gun Part 4 and Gun Part 5 (m)
Xt = 1;           % Distance between CG of Hull and trunnion (m)
Xtp = 0.9;        % Distance between trunnion and center of pinion (m)
m1 = 2500;        % Mass of Gun part 1 (kg)(Includes gun Breech)
m2 = 125;         % Mass of Gun part 2 (kg)
m3 = 150;         % Mass of Gun part 3 (kg)
m4 = 125;         % Mass of Gun part 4 (kg)
m5 = 150;         % Mass of Gun part 5 (kg)
Ide = 0.5;        % Inertia of Elevation drive(kgm^2)
I1 = 1000;        % Inertia of Gun part 1 (kgm^2)
I2 = 9.5;         % Inertia of Gun part 2 (kgm^2)
I3 = 9.5;         % Inertia of Gun part 3 (kgm^2)
I4 = 9.5;         % Inertia of Gun part 4 (kgm^2)
I5 = 9.5;         % Inertia of Gun part 5 (kgm^2)
Rpe = 0.04;       % The radius of pinion (m)
Cde = 1.5e3;      % Elevation drive viscous friction (Nms/rad)
Kde = 6e6;        % Elevation driveline stiffness (N/m)
C12 = 2e3;        % Torsional viscous friction between GP 1 and GP 2 (N*m*s/rad)
C23 = 2e3;        % Torsional viscous friction between GP 2 and GP 3 (N*m*s/rad)
C34 = 2e3;        % Torsional viscous friction between GP 3 and GP 4 (N*m*s/rad)
C45 = 2e3;        % Torsional viscous friction between GP 4 and GP 5 (N*m*s/rad)
```

```
K12 = 4e6;      % Torsional stiffness between GP 1 and GP 2 (N*m/rad)
K23 = 4e6;      % Torsional stiffness between GP 2 and GP 3 (N*m/rad)
K34 = 4e6;      % Torsional stiffness between GP 3 and GP 4 (N*m/rad)
K45 = 4e6;      % Torsional stiffness between GP 4 and GP 5 (N*m/rad)
C1p = 9e4;      % Trunnion viscous friction (N*m*s/rad)
Kt = 1;         % Coefficient of volt

%*********SYSTEM DYNAMICS IN MULTIVARIABLE MATRIX FORM***********

%M(thetaddot)+D(thetadot)+K(theta) = Fu
%theta = [theta_de; theta_1; theta_2; theta_3; theta_4; theta_5]
%u= [v_i; (y_t)ddot; (theta_p)dot; theta_p]


     M = [ Ide  ,0      ,0   ,0     ,0   ,0;
          0    ,I1+m1*n1^2+Le1^2*(m2+m3+m4+m5)   ,m2*n2*Le1+Le1*Le2*(m3+m4+m5)
,m3*n3*Le1+(m4+m5)*Le1*Le3   ,m4*n4*Le1+m5*Le4*Le1   ,m5*n5*Le1;
          0    ,m2*Le1*n2+(m3+m4+m5)*Le1*Le2     ,I2+m2*n2^2+Le2^2*(m3+m4+m5)
,m3*n3*Le2+(m4+m5)*Le2*Le3   ,m4*n4*Le2+m5*Le4*Le2   ,m5*n5*Le2;
          0    ,m3*Le1*n3+(m4+m5)*Le1*Le3        ,m3*n3*Le2+Le2*Le3*(m4+m5)
,I3+m3*n3^2+Le3^2*(m4+m5)    ,m5*Le4*Le3+m4*n4*Le3   ,m5*n5*Le3;
          0    ,m4*Le1*n4+m5*Le1*n4              ,m4*Le2*n4+m5*n4*Le2            ,m4*n4*Le3+m5*n4*Le3
,I4+m4*n4^2+m5*Le4^2    ,m5*n5*Le4;
          0    ,m5*n5*Le1                        ,m5*n5*Le2                      ,m5*n5*Le3
,m5*n5*Le4              ,I5+m5*n5^2];


     D = [Cde    ,0               ,0          ,0            ,0          ,0     ;
          0      ,C1p+C12         ,-C12       ,0            ,0          ,0     ;
          0      ,-C12            ,C12+C23    ,-C23         ,0          ,0     ;
          0      ,0               ,-C23       ,C23+C34      ,-C34       ,0     ;
          0      ,0               ,0          ,-C34         ,C34+C45    ,-C45  ;
          0      ,0               ,0          ,0            ,-C45       ,C45   ];


     K = [Kde*Rpe^2         ,-Kde*Rpe*Xtp   ,0           ,0           ,0            ,0     ;
          -Kde*Rpe*Xtp      ,K12+Kde*Xtp^2  ,-K12        ,0           ,0            ,0     ;
```

```
        0                  ,-K12             ,K12+K23    ,-K23       ,0            ,0     ;
        0                  ,0                ,-K23       ,K23+K34    ,-K34         ,0     ;
        0                  ,0                ,0          ,-K34       ,K34+K45      ,-K45  ;
        0                  ,0                ,0          , 0         ,-K45         ,K45   ];


   F = [Kt    ,0                           ,0         ,-Kde*Rpe*(Xtp)  ;
        0     ,(m1*n1+Le1*(m2+m3+m4+m5))   ,C1p       ,Kde*Xtp^2       ;
        0     ,-(m2*n2+Le2*(m3+m4+m5))     ,0         ,0               ;
        0     ,-(m3*n3+Le3*(m4+m5))        ,0         ,0               ;
        0     ,-(m4*n4+Le4*m5)             ,0         ,0               ;
        0     ,-m5*n5                      ,0         ,0               ];


%*********STATE SPACE EQUATION OF 5-DOF MBT GUN-BARREL ELAVATION DRIVE****

%xidot=A1*xi+B1*u1 and y=C1*xi+D*u1
%xi is denoted by ?
%xi=[theta_de; theta_1; theta_2; theta_3; theta_4; theta_5; thetadot_de; thetadot_1; thetadot_2
thetadot_3; thetadot_4; thetadot_5]
%u1=[v_i; yt_ddot; (theta_p)dot; theta_p]

    A1 = [zeros(6)        ,eye(6);           %define state matrix
          -inv(M)*K       ,-inv(M)*D];

    B1 = [zeros(6,4);                        %define input matrix
          inv(M)*F];

    C1 = eye(12);                            %define output matrix

    D1 = zeros(12,4);                        %define direct-link matrix
```

10.2 Appendix A2 – MATLAB® m-file for Traverse Axis

```
Ida=25;       %Azimuth Drive Inertia (kg.m^2)
It=45000;     %Turret Inertia (kg.m^2)
m1=2500;      %Mass of Gun Part 1 (kg)(Includes Gun Breech)
m2=125;       %Mass of Gun Part 2 (kg)
m3=150;       %Mass of Gun Part 3 (kg)
m4=125;       %Mass of Gun Part 4 (kg)
m5=100;       %Mass of Gun Part 5 (kg) (This is the Gun Muzzle)
L=1;          %Length of each gun part except Part 1 (m)
n1 = 0.5;         % Distance between trunnion to CG of Gun Part 1 (m)
I1=1000;      %Inertia of Gun Part 1 (kg.m^2)
I2=9.5;       %Inertia of Gun Part 2 (kg.m^2)
I3=9.5;       %Inertia of Gun Part 3 (kg.m^2)
I4=9.5;       %Inertia of Gun Part 4 (kg.m^2)
I5=9.5;       %Inertia of Gun Part 5 (kg.m^2)
cda=150;       %Drive viscous friction (N*m*s/rad)
ct=9e4;       %Turret viscous friction (N*m*s/rad)
ctg=1e4;      %Turret to gun(m1) viscous friction (N*m*s/rad)
ktg=4.5e8;    %Turret to gun(m1) stiffness (N*m/rad)
kd=2e6;       %Drive stiffness (N*m/rad)
cb=2e3;       %Gun parts joint viscous friction (N*m*s/rad) (Between m1,m2,m3,m4,m5)
kb=4e6;       %Gun parts joint stiffnesses (N*m/rad) (Between m1,m2,m3,m4,m5)
Rp=0.08;      %Pinion Pitch Circle Radius (m)
Rg=0.9;       %Turret rotation center to Turret-Gun_m1 Joint Distance
Rt=1.1;       %Turret Ring Gear Pitch Circle Radius (m)
Ia=It+(m1+m2+m3+m4+m5)*Rg^2; %Total azimuth inertia (turret + gun) (kg.m^2)
eta=0.5;      %Trunnion to CG of breech (m1) part (m)
```

```
%**********SYSTEM DYNAMICS IN MULTIVARIABLE MATRIX FORM************

 M = [ Ida   ,0                             ,0                                    ,0
,0                       ,0                     ,0;
              0    ,It+Rg^2*(m1+m2+m3+m4+m5)      ,Rg*(-m1*n1+L*(0.5*m2+m3+m4+m5))
,L*Rg*(0.5*m2+m3+m4+m5)       ,L*Rg*(0.5*m3+m4+m5)      ,L*Rg*(0.5*m4+m5)      ,0.5*L*Rg*m5 ;
              0    ,Rg*(-m1*n1+L*(m2+m3+m4+m5))  ,I1+m1*n1^2+L^2*(m2+m3+m4+m5)
,L^2*(0.5*m2+m3+m4+m5)        ,L^2*(0.5*m3+m4+m5)       ,L^2*(0.5*m4+m5)       ,0.5*L^2*m5 ;
              0    ,L*Rg*(0.5*m2+m3+m4+m5)        ,0.5*m2+m3+m4+m5
,I2+L^2*(0.25*m2+m3+m4+m5)    ,L^2*(0.5*m3+m4+m5)       ,L^2*(0.5*m4+m5)       ,0.5*L^2*m5 ;
              0    ,L*Rg*(0.5*m3+m4+m5)           ,0.5*m3+m4+m5
,L^2*(0.5*m3+m4+m5)           ,I3+L^2*(0.25*m3+m4+m5)   ,L^2*(0.5*m4+m5)       ,0.5*L^2*m5;
              0    ,L*Rg*(0.5*m4+m5)              ,L^2*(0.5*m4+m5)                ,L^2*(0.5*m4+m5)
,L^2*(0.5*m4+m5)           ,I4+L^2*(0.25*m4+m5)  ,0.5*L^2*m5 ;
              0    ,0.5*L*m5*Rg                   ,0.5*L^2*m5                     ,0.5*L^2*m5
,0.5*L^2*m5                ,0.5*L^2*m5           ,I5+0.25*L^2*m5 ];

       D = [ cda     ,0     ,0     ,0      ,0      ,0     ,0 ;
              0      ,ct    ,0     ,0      ,0      ,0     ,0 ;
              0      ,0     ,cb    ,-cb    ,0      ,0     ,0 ;
              0      ,0     ,-cb   ,2*cb   ,-cb    ,0     ,0 ;
              0      ,0     ,0     ,-cb    ,2*cb   ,-cb   ,0 ;
              0      ,0     ,0     ,0      ,-cb    ,2*cb  ,-cb;
              0      ,0     ,0     ,0      ,0      ,-cb   ,cb];

       K = [ kd            ,-(kd*Rt)/Rp        ,0           ,0      ,0      ,0     ,0 ;
            -(kd*Rt)/Rp   ,ktg-(kd*Rt^2)/Rp    ,-ktg        ,0      ,0      ,0     ,0 ;
              0             ,-ktg                ,ktg+kb      ,-kb    ,0      ,0     ,0 ;
              0             ,0                   ,-kb         ,2*kb   ,-kb    ,0     ,0 ;
              0             ,0                   ,0           ,-kb    ,2*kb   ,-kb   ,0 ;
              0             ,0                   ,0           ,0      ,-kb    ,2*kb  ,-kb;
              0             ,0                   ,0           ,0      ,0      ,-kb   ,cb];

       F = [  1     ,1     ,0;
              0     ,ct    ,Ia+Rg^2*(m1+m2+m3+m4+m5);
              0     ,0     ,Rg*(-m1*n1+L*(m2+m3+m4+m5));
              0     ,0     ,L*Rg*(0.5*m2+m3+m4+m5);
              0     ,0     ,L*Rg*(0.5*m3+m4+m5);
```

```
          0      ,0      ,L*Rg*(0.5*m4+m5);
          0      ,0      ,0.5*L*m5*Rg];


A1 = [zeros(7)        ,eye(7);          %define state matrix
      -inv(M)*K       ,-inv(M)*D];

B1 = [zeros(7,3);                       %define input matrix
      inv(M)*F];

C1 = eye(14);                           %define output matrix

D1 = zeros(14,4);                       %define direct-link matrix
```

10.3 Appendix A3 – MATLAB® m-file of State Observer

```
%%****************Observer*********

    U_u=eig(A1);
    C2=zeros(12,12);
    C2(8,8)=1;
    C2(2,2)=1;
    P=[-300 -44 -0.05 -5  -7 -0.8 -9 -100 -340 -115 -1070 -1300];
    L_1=place(A1',C2',P);
    L_1=L_1';
    A_o=A1-L_1*C2;
    B_o=[B1 L_1];
    C_o=eye(12);
    D_o=0;
    Ec2=eig(A_o);
```