



**MARMARA UNIVERSITY
FACULTY OF ENGINEERING**



DESIGN AND OPTIMIZATION OF AN ACTIVE SUSPENSION SYSTEM

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**Design and Optimization of An Active Suspension
System**

by

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ABSTRACT

Design and Optimization of An Active Suspension System

The active suspension system is an electronic system that was first developed in the eighties and then perfected by Williams in 1992, by predicting the obstacles on the runway via the computer and informing the driver.

Nowadays, suspension systems are standard on vehicles with high prices and equipment. When the vehicle enters any pit and heights, people inside the vehicle can travel comfortable. The fact that the active suspension system is a new technology than other vehicle control systems and that it is developed in a short time has been an important factor in the selection of active suspension systems in this thesis. Furthermore, the fact that the suspension system is one of the most important factors in the safety of vehicles emphasize the necessity of this system.

In this study, the design and optimization of active suspension system of a quarter vehicle were carried out. In order to reach optimal values, firstly passive suspension system and active suspension system were compared. Active suspension system was selected cause of results of active suspension is more suitable than passive suspension system. The values have been obtained from comparing of passive and active suspension system and only these values of active suspension system were used. Then, by comparing of two different active suspension systems, values which has been obtained has been optimized. Active suspension system has been considered to reduce the vibration of the vehicle. The main idea is based on the LQR parameter. By comparing of the LQR parameters, more efficient results were obtained. In order to examine the effectiveness of this approach, the results of the optimization were examined in different simulations and the results were compared with those obtained from similar quarter car model simulation with an active suspension system. The results show that the active suspension system with the LQR system provides an improvement than the other active suspension system.

SYMBOLS

c_b	: coefficient of the damper [Ns/m]
$c_b(t)$: damping force [N]
f_b	: actuator force [N]
k_b	: the spring constant of body [N/m]
k_w	: the spring constant of wheel [N/m]
m_b	: the body mass of quarter car [kg]
m_w	: the mass of tire [kg]
y_0	: road input

ABBREVIATIONS

ABC	: Active Body Control
MBC	: Magic Body Control
LQR	: Linear Quadratic Regulator
LQI	: Linear Quadratic Integral
MPC	: Model Predictive Control
PID	: Proportional Integral Derivative
ANFIS	: Adaptive Neuro Fuzzy Inference System
ISO	: International Organization Standardization
LQG	: Linear Quadratic Gaussian
NLQC	: Nonlinear Quarter Car
VDV	: Vibration Dose Value
RMS	: Root Mean Square

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1. INTRODUCTION

Nowadays, with the development of technology, the suspension systems of the vehicles are also developing. Mechanical systems tended to leave their places in electromechanical systems. Especially, developments in the field of computer and electronics have led to the rapid development of sensor and actuator technologies. The automotive sector is one of the most rapidly affected sectors. The electromechanical systems used in the vehicles are generally used in order to improve safety, performance, energy saving and comfort. In recent years, great improvements have been made in the systems for providing and maintaining driving comfort and driving safety, which are called the smart suspension system and interfere with the vehicle suspension systems either directly or indirectly. Researches that started in 1980s have completed the transition from theory to practice by using Mitsubishi's electronically controlled semi-active suspension system in Galant models produced in 1987. Intelligent suspension systems have been developing rapidly since the 90s. Infiniti, Toyota and Mitsubishi companies have used the active suspension systems in mass production and become the leading companies in the sector [\[1\]](#).

Vehicle suspension systems have many tasks. They can be used to carry the vehicle, increase the control skill of the vehicle with steering, increase the friction between the wheel and the road, reduce the vibrations caused by the rough road surface and provide driving comfort.

There are different models for suspension systems. Also, suspension systems are examined under 3 headings. Passive, semi-active and active suspension systems. In this study, we will work on active suspension systems in the quarter vehicle model.

2. VEHICLE MODELS

Currently, there are three different vehicle models to describe the vertical dynamic behavior of the vehicle and for each model there is another interest of motion.

2.1. Quarter Vehicle Model

The vertical dynamic primary degrees of freedom of a vehicle are the heave, roll and pitch movements of the chassis. The quarter vehicle model provides a lot of information in terms of seeing the efficiency of the designed suspension and control system without going into full vehicle model. Typically, a passive quarter vehicle model contains the sprung mass, unsprung mass, the suspension system (in order to set up the sprung and unsprung masses together), a tire model, which includes parallel spring and damper configuration. The sprung mass represents the mass of the chassis with passengers and loading. The unsprung mass represents the tire, wheel, brake and mass of wheel carrier and suspension system. In the semi-active suspension, the damping $c_b = c_b(t)$ (Figure 3.3) is adjustable. Also, in the active suspension system, a force f_b (Figure 3.4) can be applied by an actuator between the sprung and the unsprung mass [\[2\]](#).

2.2. Half Vehicle Model

In principle, half vehicle model unites two quarter vehicle models (front and rear) joined by a rigid frame. The half-vehicle model refers to the movements in the vertical direction as well as the tapping movements. It consists of an upper mass representing the vehicle body and the sub-masses representing the axle weight and the suspension system between these masses.

2.3. Fully Vehicle Model

The full vehicle model is a detailed model in which the motion movements of the vehicle in deceleration and acceleration can be examined, the wobble movement that may occur during cornering and the bounce movements in the vertical axis. There are four suspension systems connected to the main body. Acceptances for full vehicle model;

- The kinematic effects resulting from the suspension geometry have been neglected. The suspension system only transmits forces in the vertical direction to the vehicle body. It is assumed that there is no cornering beam.
- Equilibrium bar plays an important role in heavy vehicles exposed to tipping torque.
- The vehicle body plane is assumed to be parallel with the road surface. Tools are generally inclined forward to reduce aerodynamic resistance.

3. VEHICLE SUSPENSION SYSTEMS

Suspension system includes the combined functions of dampers, suspension springs and stabilizers.

In vehicles, suspension systems are mainly responsible for the following tasks, which are very significant for the overall function of the chassis:

- Comfort and safety,
- Driving behavior,
- Roll, pitch and yaw inclinations of the vehicle.

In general, the suspensions are divided into two groups as dependent or independent. The reason why the first group is called dependent is because the front wheels are physically connected to each other by the suspension systems.

On the contrary, the suspension system used for each wheel independently operates independently of each other. The selection of the suspension type depends on many parameters. These include speed range, load to axle, current area, cost, weight, kinematic properties etc. Suspension can be examined in three classes as passive, semi-active and active suspension. Passive suspension consists of dampers and fixed flow dampers. Suspensions in which control algorithms are generally adaptable are semi-active and active suspensions. The use of shock absorber and coil springs in the passive suspension system, despite the softness and flexibility of the system has been insufficient to prevent oscillations.

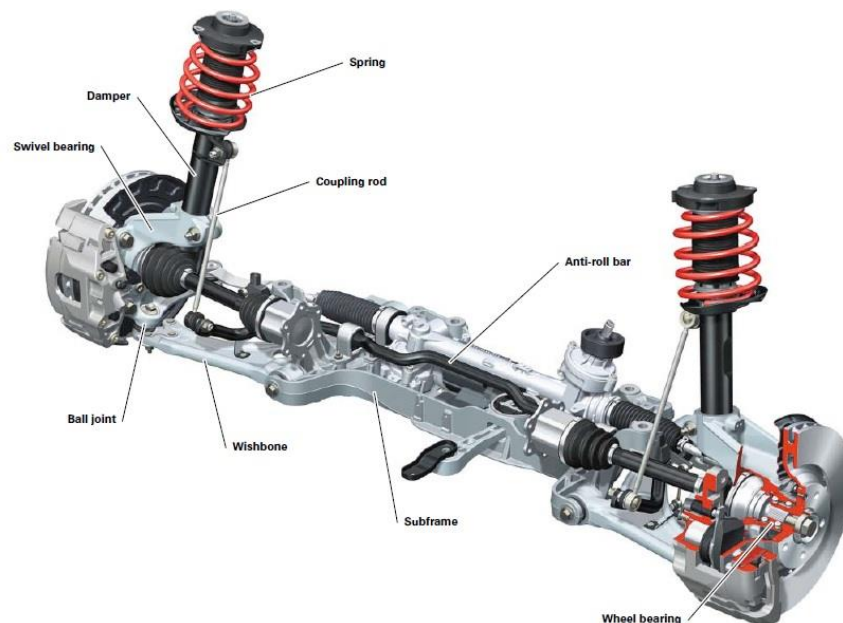


Figure 3.1. Vehicle Suspension System Sample

Through these tasks, the vehicle structure and the passengers as well are protected by the suspension system from the effect of disturbing vertical displacements, yaw and pitch fluctuations. Furthermore, the suspension system assists the wheels to sustain contact with the road. This is the basis for an effective force transfer, required for power transfer, road holding, acceleration, braking and eventually [\[3\]](#).

The suspension system consists of the vehicle body and the axle and the wheels, which dampen the vibrations and movements caused by the vehicle dynamics and the structure of the road. The suspension system is an indirectly required system in terms of driving comfort and safety, directly in terms of performance and energy. The tasks of the suspension system can be summarized as follows:

1. It provides to isolate unwanted forces caused by the structure of the road and vehicle dynamics together with the wheels while driving and to increase the comfort characteristics of the vehicle. These comfort characteristics are generally associated with the body structure of the vehicle and the suspension system reduces vibration and oscillations, control the head and wobble movements, and damps sudden shocks.
2. If the vehicle is in the bad road conditions; provides handling properties during acceleration, deceleration or braking. These traction characteristics are caused by the deformation of the wheel by the effect of vertical forces. The suspension system reduces the impact of that forces on the wheel and ensures high road holding even during acceleration, braking and turning.
3. Compensates the static weight of the vehicle on uneven curved roads and corners. This feature relates to the impact of the suspension. [\[4\]](#).

3.1. Passive Suspension Systems

The suspension system reduces the impact of different forces on the wheel and ensures high road holding even during acceleration, braking and rotation. Although the technology has advanced and new suspension systems have been developed, passive suspension systems which are capable of providing up to a certain level of acceptance of the three basic properties of the suspension are preferred [\[4\]](#). These systems, which are more economical, consist of spring and damper systems whose features cannot be changed. The system does not contain any actuators or sensor elements that play an active role in the system. The diagram of the system is shown in Figure 3.2.

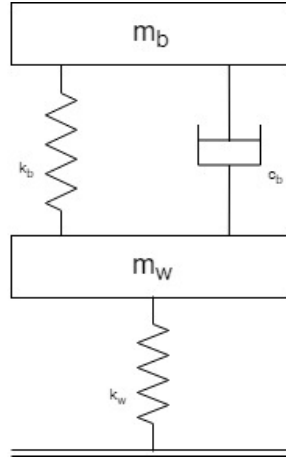


Figure 3.2. Diagram of Passive Suspension System

3.2. Semi-Active Suspension Systems

Semi-active suspension systems do not apply an external force or energy to the system so they are not preferred for cheap, complex design and are highly preferred suspension systems. While passive suspension systems are not allowed to change parameters in semi-active suspension systems. In semi-active suspension systems, the hardness of the springs remains the same but the damping coefficient of the damper can change. In semi-active suspension systems, the modulus of the spring coefficient is almost never used. Usually a much simpler, replaceable damper stiffness method is used [\[2\]](#).

When modeling the semi-active suspension system (Figure 3.3), it will be adopted by the changeable damper coefficient method.

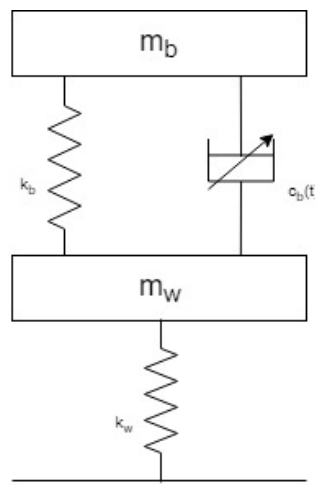


Figure 3.3. Diagram of Semi-Active Suspension System

3.3. Active Suspension Systems

Active suspension systems consist of sensors, controllers and actuators that are mounted on each wheel in the suspension system, and are capable of actively applying force and energy. According to the condition of the road and vehicle in semi-active suspension systems, the control of damper hardness can carry the performance up to certain levels and since it is not possible to apply an external inverse force to the system, suspension performance criteria can be provided up to certain points. In active suspension systems, it is possible to apply control forces to the suspension system in some cases by applying direct force to the system.

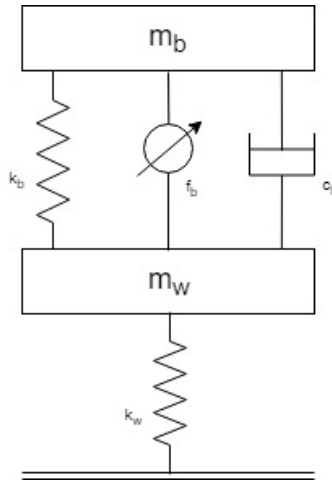


Figure 3.4. Diagram of Active Suspension System

The quarter vehicle model, based on the vertical forces for active suspension system modeling and with two simple degrees of freedom, will be sufficient to examine the ride comfort, road holding and static balance performances. Unlike passive suspension, a force control will be integrated into the system, except for the entry of the road.

Active suspension systems include actuators and these actuators supply additional forces. These forces are determined by a feedback control law using data from sensors. Various control strategies such as adaptive control presented by Nugroho et al. [5], fuzzy control in Ranjbar-Sabrine et al. [6] and optimal control developed by Paschedag et al. [7] have been proposed in the past years to control of active suspension system.

4. PREVIOUS WORK

In this section, we have obtained the works of companies and scientist on active suspension systems. This literature review showed us the uses of these systems and in what areas they differed.

4.1. Works of Industry

Mercedes-Benz currently uses 2 types of suspension systems in the market. These systems are called “Active Body Control” and “Magic Body Control”. When the vehicle encounters a rugged road, the suspension and shock absorbers work as absorbents and absorb some of the force but also transfer some of them to us. Due to this situation, the company continues to work for new active suspension systems.

First of all, Mercedes-Benz was using AirMatic air suspension system in its vehicles. This system is a semi-active suspension system used by the company for a long time. The system is used frequently in trucks and buses as well as in many other vehicles. This system is seen in many recent luxury cars, especially the Mercedes-Benz AirMatic. Adaptive vehicle classes include SUVs. The air suspension system, which is standard in the premium 4x4 vehicles of brands such as Range Rover, Toyota, provides height and balance in off road conditions. In order to raise the vehicle, the air delivered to the air bellows must be increased. The compressed air produced by the compressor is stored in tanks for use at any time.

Active Body Control system not only absorbs the force but also shows a reaction to the effect. ABC completely balances the vehicle body and tries to reduce this effect to zero. To do this, there are 5 different sensors on the vehicle. These sensors detect the agglomeration movements of the vehicle and try to maintain the balance. The system is built on a quarter vehicle model and has an advanced suspension arm on each wheel. Spring on the arm, shock absorber and active hydraulic cylinder. There is also a sensor on each wheel, which measures the amount of compression in the suspension. The 2 processors in the system process the data received by the sensors and according to these data, the hydraulic cylinder provides fluid flow from the existing pump and the suspension length is adjusted. Thus, the vehicle continues to maintain its balance. According to the study, the response time of the system was measured as 10 milliseconds. In Magic Body Control system, there are extra stereo cameras unlike the ABC system. These cameras detect obstacles on the road and help the processor to send data to the suspension system as they approach the obstacle. MBC, which can work efficiently even at high speeds, is available in the premium class vehicles of Mercedes-Benz [\[8\]](#).

The company BOSE has been working on a concept for a high bandwidth active suspension system since 1980 [\[9\]](#). This system, which works with the magnetic field produced by the electromagnetic motors which are found independently on each wheel, keeps the control of the vehicle suspension with electricity by the motors. The system is managed by a processor and each motor can be intervened separately.

A reaction mass absorber is attached to each wheel in order to reduce the resonance peak at the natural frequency of the unsprung mass without transferring the reaction forces directly to the chassis. This processor controls every suspension in milliseconds, so that the vehicle is kept in equilibrium at all levels. The disadvantages of the system are that it is heavy and expensive [10].

4.2. Academic Works

Altun [11] worked on comparison of LQR and LQI controllers for quarter vehicle active suspension system. In this study designed LQR controller with status feedback to ensure priority driving safety and passenger comfort. Bharali and Bragohain [12] worked on 3 different active control systems for the active suspension system. In these studies, Fuzzy LQR control scheme gives more better results than the other control methods. They also observed that the active suspension system simulations in the Matlab & Simulink program yielded more successful results than the passive suspension system. Durmaz et al. [13] observed that LQR and Model Predictive Control (MPC) control systems yield similar results, but the LQR controller consumed less power. They also saw that the LQR controller gave fewer errors than the other controllers. Kaleemullah et al. [14] LQR performs better than Robust, Fuzzy and H infinity in body control and body acceleration but also Robust has the best settling time. Gandhi [15] tested and developed PID, LQR, FUZZY and ANFIS controllers using a random path profile (ISO 8608) and analyzed the results. ANFIS controller performs better than others but has a higher cost. Therefore, it is not preferred.

Different nonlinear models have been formed in active suspension systems and many different studies have been conducted. Nonlinear suspension models are developed by Čorić et al. [16], Al-Holou et al. [17] and Nguyen et al. [18]. Also, Basari [19] investigate the performance of a quarter car nonlinear active suspension system with a backstepping control approach.

Yoshimura [20] worked on the construction of a pneumatic active suspension system for a one-wheel car model using a disturbance observer. Ansari [21] also conducted similar studies on this subject. Additionally, Sadati et al. [22] defined an optimal control of an 8-DOF vehicle active suspension system using Kalman observer. Maden [23], observed that the damping time is longer than the standard time in LQR.

Zuo and Nayfeh [24] studied 8 DOF full-car model and optimized it within the framework of structured H_2 optimal control with disturbance delays. Also, they found that significant performance losses occur if the time delays are ignored.

Chai and Sun [25] designed an active suspension with Linear Quadratic Gaussian (LQG) controller based on full vehicle model with 7 DOF and they observed LQG controller has a good stability when suspension damping and stiffness vary.

Sam and Ghani [26] in this research, passive and active suspension analyzed the direction of the systems and focused on improving the quality of the ride. According to the simulation result; the active suspension system performs better for ride quality than passive suspension.

Darus and Enzai [\[27\]](#) observed LQR and PID control strategy performance due to changes in road surface. LQR and PID have worked successfully. However, a limitation was observed when the LQR control technique was used whereas LQR gives better results in vehicle handling and ride quality than PID.

Nagarkar et al. [\[28\]](#) worked on optimization of LQR and PID controller parameters by considering Vibration Dose Value (VDV) at head and Root Mean Square (RMS) acceleration. They used an 8 DOF nonlinear quarter car (NLQC) model including head mass, upper torso mass, lower torso mass and pelvis mass for human body.

5. ANALYSIS OF ACTIVE SUSPENSION SYSTEMS

In the Matlab / Simulink environment, we compared the model we created with the parameters of the active suspension control with 2 different articles that we determined and evaluated the graphical results. Thus, we have proved the accuracy of our model. The model we created in the Simulink environment is as in Figure 5.3. With this model, we have determined our road input, actuator and state. The input disturbance of our system is road disturbance as shown in Figure 5.1. The following performances were observed in the figures on the active suspension systems with LQR.

5.1 LQR Systems

Passenger driving comfort, suspension movements and handling are the most important features when designing vehicle suspension. Passenger driving comfort is associated with acceleration. The suspension movement is associated with the distance between the opening mass and the spring mass. Road holding is associated with displacement. In this project, we have tried to determine the optimal controller in the values.

The various states considered are body mass displacement and velocity, wheel mass displacement and velocity. Hence the state vector matrix is $x^T = [x_1 \ x_2 \ x_3 \ x_4]$. Equation (7) is an unchanging linear system. The quadratic optimal control provides a calculating the state feedback control. State feedback control for active suspension is an important control for design. In the control design, it is assumed that all values in the situations are present and can be measured. But this is not always the case. Especially in active control systems, it may be difficult and complicated to measure the data compared other systems.

Linear Quadratic Regulator (LQR) is used for optimization of linear systems. LQR controlling can used on linear systems with the non-disturbances or the disturbances. In disturbance systems, LQR can requires measurable data and disturbances. The suspension system consists of disturbances, i.e. the LQR control is provided with measurable disturbances.

5.2 Design Steps of LQR Controller

LQR is used by coding the control system in Matlab. With the LQR command, you can obtain the K gain matrix as shown below. Therefore, the LQR command can be used as P controller.

$$[K,S,e] = \text{LQR}(A,B,Q,R) \quad (1)$$

The design stages of the LQR control system are as follows:

1. In this study, 2 DOF models were designed and investigated.
2. The next step after designing the 2 DOF model system is to verify that the system is controllable. Rank of controllability matrix is 4 for 2 DOF which is equal to rank of the system. For the 2 DOF system, the matrix size must be equal to 4. So, it has 0 uncontrollable states. Thus, both models can be controlled.

3. The generated matrices were created using MATLAB / Simulink.
4. The next stage is the creation of Q and R parameters from the system. In the LQR control system, the Q matrix is a 4x4 diagonal matrix. R matrix is 1x1 matrix. As a result of validations, non-fixed parameters are changed in the Q matrix according to the need. At the same time, if needed, in order to obtain more accurate data, R matrix can be replaced with Q. $[K, S, e] = \text{LQR}(A, B, Q, R)$ command is used to calculate the LQR optimal gain k_b in Matlab and disturbance gain k_w is calculated using LQR matrix control equations.

It turns out that in any case of the values of Q and R, the cost work encompasses a special least that can be gotten by solving the linear quadratic regulation.

The parameters Q and R can be utilized as design parameters to penalize the state factors and the control signals. The bigger these values are, the more you penalize these signals. Fundamentally, choosing a huge value for R implies you attempt to stabilize the system with less energy(weighted). This is more often than not called costly control procedure. On the other hand, (cheap control procedure) choosing a little value for R implies you don't need to penalize the control signal.

When selecting the performance measure, the person tries to make a mathematical definition which indicates that the system is working as desired when minimized according to the icon state. Therefore, when choosing a performance criterion, it is important to convert the system schema to mathematical terms. As a result, the performance data cannot perform mathematically and physically similar comparisons.

The matrices Q and R are utilized for forming the compromise between keeping the state mistakes and the control rectifications, separately little amid the complete mission.

For cost work; Q based on with the state energy, R is based on with the energy of the controlled inputs. In LQR, one looks for a controller that minimizes both energies; in any case, diminishing the energy of the states will require a huge control signal and a little control signal will lead to huge states deviation.

In generally, In the Linear Quadratic Regulation solving system, the Q matrix shows the weights in statues. The R matrix represents the weight in the control input. The aim of this study is; design and optimization of active suspension system. Therefore, Q, R, N values were chosen for the purpose of the project. The Q matrix performs minimization in statues, which helps to optimize the system. R matrix provides optimization of the force given to the system. The N matrix is used for similar purposes, like the others. The matrix N minimizes the power in the system. The N matrix is associated with power consumption optimization.

5.3 Mathematical Model of Active Suspension System

The active suspension system is modeled with 2 DOF. Two degree of model is so simple according to another complex systems but the state variables are obtainable in this model. Quarter car model contain only vertical displacements.

This model is shown Figure 3.4. In this figure, m_b is symbolized body mass and m_w is symbolized wheel mass. The spring, damper and actuator is located between body and wheel masses. Additionally, k_w is determined wheel spring constant and f_b is the force generator (actuator) of the system.

Differential equation of quarter car is shown below (1) (2).

$$m_w \ddot{y}_1 = c_b(\dot{y}_2 - \dot{y}_1) + k_b(y_2 - y_1) - k_w(y_1 - y_0) - f \quad (2)$$

$$m_b \ddot{y}_2 = -c_b(\dot{y}_2 - \dot{y}_1) - k_b(y_2 - y_1) + f \quad (3)$$

The matrix form of these two equations is;

$$\begin{bmatrix} m_w & 0 \\ 0 & m_b \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} c_b & -c_b \\ -c_b & c_b \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} - \begin{bmatrix} (k_b + k_w) & -k_b \\ -k_b & k_b \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} f - \begin{bmatrix} -k_w \\ 0 \end{bmatrix} y_0 \quad (4)$$

In state form equations of active suspension system are showing which parameters are considered and optimized, in this work there are four state are determined. x_1 is the velocity of quarter car body, x_2 is suspension deflection, x_3 is wheel deflection and x_4 is the velocity of the wheel.

In this model, the movement of the surface of the y_0 vehicle wheel when it touches the ground, the movement of the center of gravity of the wheels y_1 , the mass of the wheels of the m_w vehicle, the spring constants of the springs modeling the flexure of the k_w wheels, the spring constants of the coil springs in the k_b suspension system, the constants of the shock absorbers in the c_b suspension system, the f_b suspension system represents the force exerted by the actuator.

State variables are determined as;

$$\dot{x}_1 = \ddot{y}_2 = \frac{c_b}{m_b}(x_1 - x_4) - \frac{k_b}{m_b}x_2 + \frac{f}{m_b} ; \quad (5)$$

$$\dot{x}_2 = \dot{y}_2 - \dot{y}_1 = x_1 - x_4 ; \quad (6)$$

$$\dot{x}_3 = \dot{y}_1 - \dot{y}_0 = x_4 - \dot{y}_0 ; \quad (7)$$

$$\dot{x}_4 = \ddot{y}_1 = \frac{c_b}{m_w}(x_1 - x_4) + \frac{k_b}{m_w}x_2 - \frac{k_w}{m_w}x_3 - \frac{f}{m_w} ; \quad (8)$$

Steady state model of the system;

$$\dot{x} = Ax + Bf + L\dot{y}_0 \quad (9)$$

In steady state equation (8) , \dot{y}_0 is the derivative of the road disturbance.

$$A = \begin{bmatrix} \frac{-c_b}{m_b} & \frac{-k_b}{m_b} & 0 & \frac{-c_b}{m_b} \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{c_b}{m_w} & \frac{k_b}{m_w} & \frac{-k_w}{m_w} & \frac{-c_b}{m_w} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{m_b} \\ 0 \\ 0 \\ -\frac{1}{m_b} \end{bmatrix} \quad L = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

5.4 Matlab/Simulink Modeling

To observe that the model works correctly and benefits against passive suspension system, LQR controller was used in MATLAB and outputs of the states was compared. To obtain these datas, Simulink models were works together in same workspace. All design parameters could be same for accurate comparison.

PARAMETER	SYMBOL	VALUE	UNIT
Mass for The Car Wheel	m_w	59	kg
Mass for Car Body	m_b	290	kg
Stiffness of The Car Body Spring	k_b	16812	N/m
Stiffness of The Car Tire	k_w	200000	N/m
Damping Ratio	c_b	5000	N*s/m

Table 5.1. Vehicle Parameters

The main factor to consider when choosing these values was body velocity because it is of great importance for human health and driving comfort. However, while focusing on this, other parameters, such as suspension deflection, wheel deflection and wheel velocity, should be carefully controlled for driving safety. This is because deformation in the suspension or wheel can cause many other accident factors, especially crashing the vehicle body floor to the road.

In order to do all this work, a road input had to be chosen and the sudden response of the vehicle to a 40 cm bump was wanted to be observed. The input we selected for this was in the shape of triangle.

[0 0.4 0 0 0] m in time vector [0 0.125 0.25 4 20] second

In this application vector of road profile that was used is;

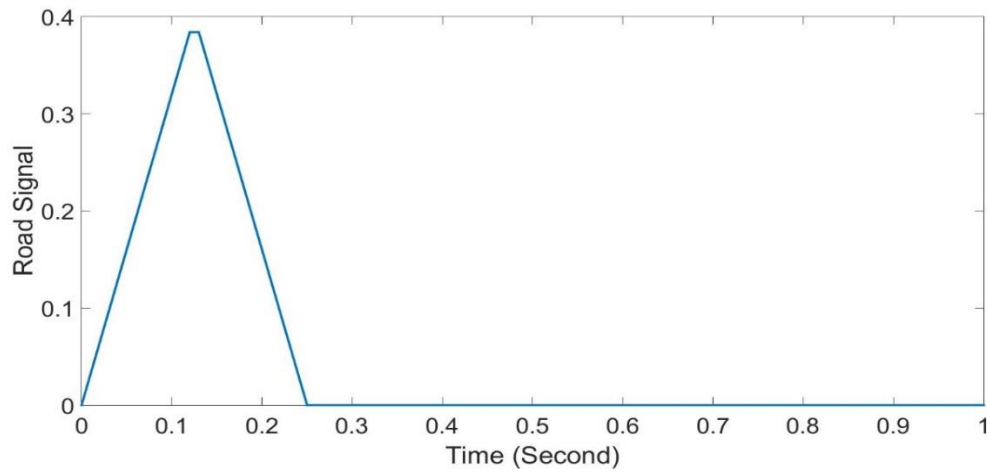


Figure 5.1. Road Profile

The Simulink model of the passive suspension system is;

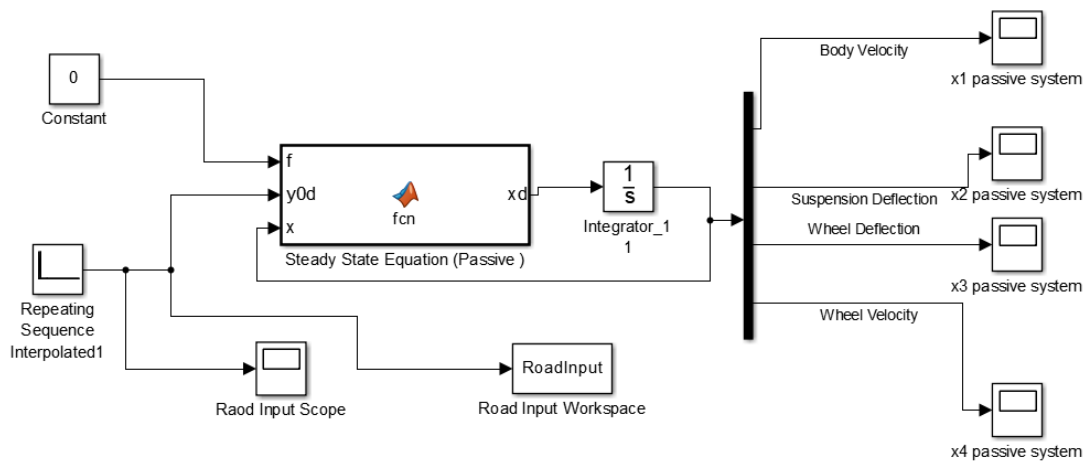


Figure 5.2. Passive Suspension System

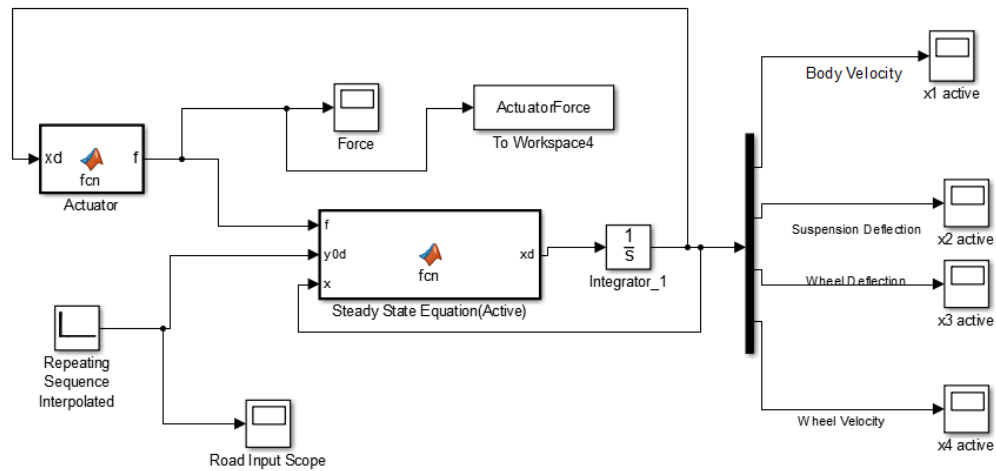


Figure 5.3. Active Suspension System

The steady state function in passive and active subsystems are same;

```
function xd = fcn(f,y0d,x)
```

```
mw=59; %mass for the car wheel - kg
```

```
mb=290; %mass for car body - kg
```

```
kb=16812; %stiffness of the car body spring - N/m
```

```
kw=200000; %stiffness of the car tyre - N/m
```

```
cb=5000; %damping - N*s/m
```

```
A=[-cb/mb -kb/mb 0 cb/mb;1 0 0 -1;0 0 0 1;cb/mw kb/mw -kw/mw -cb/mw];
```

```
B=[1/mb;0;0;-1/mw];
```

```
L=[0;0;-1;0];
```

```
% Steady State Formulation%
```

```
xd=A*x+B*f+L*y0d;
```

The using the LQR command in Matlab ;

```
mw=59; %mass for the car wheel - kg
```

```
mb=290; %mass for car body - kg
```

```
kb=16812; %stiffness of the car body spring - N/m
```

```
kw=200000; %stiffness of the car tyre - N/m
```

```
cb=5000; %damping - N*s/m
```

```
A=[(-cb/mb) (-kb/mb) 0 (cb/mb);1 0 0 -1;0 0 0 1;(cb/mw) (kb/mw) (-kw/mw) (-cb/mw)];
```

```
B=[1/mb;0;0;-1/mw];
```

```
L=[0;0;-1;0];
```

```
Q=diag(q1,q2,q3,q4)
```

```
N=zeros(1);
```

```
R=[r1];
```

```
%Linear Quadratic Regulator Matlab Function%  
[K] = lqr(A,B,Q,R,N)
```

The actuator subfunction in Matlab ;

```
function f = fcn(xd)  
K=[k1 k2 k3 k4];  
f = -K*xd;
```

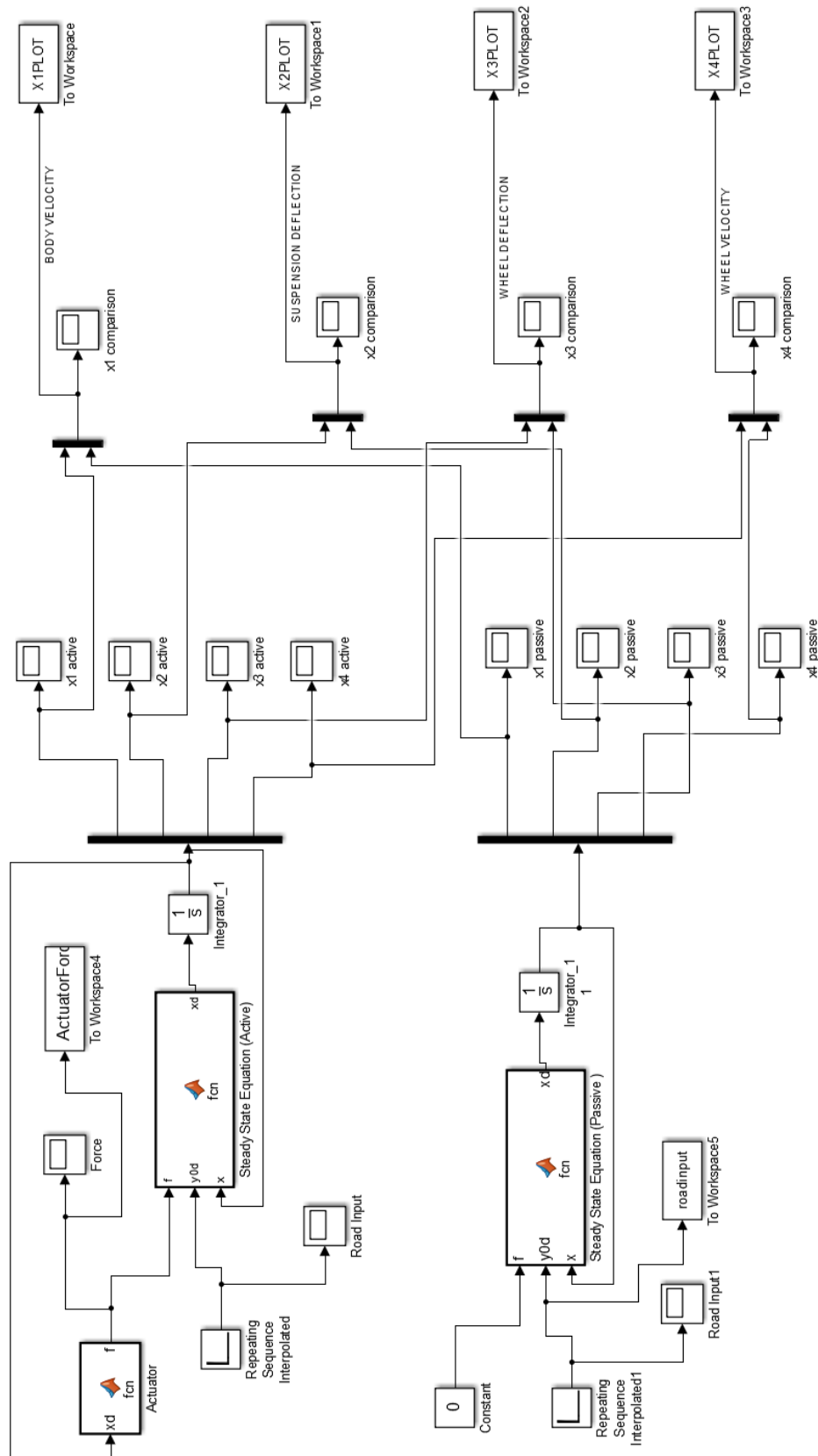



Figure 5.4. Comparison of Models

5.5 Academic Studies and Confirmation Simulation

In order to see that our model works correctly and to observe the effects of LQR parameters, a mean of the values used in this literature was compared with the passive system.

These LQR parameters are ;

$Q = \text{diag}(10^5, 2 \cdot 10^5, 10, 100);$

$N = \text{zeros}(1);$

$R = [0.0005];$

The full-state feedback matrix;

$K = [2228, 9315, -2423.5, 35]$

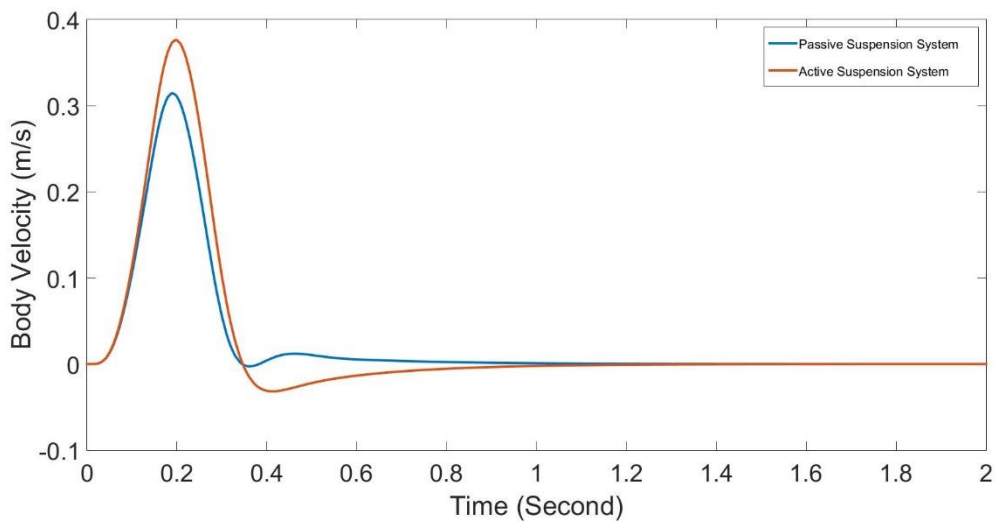


Figure 5.5. Body Velocity-Time

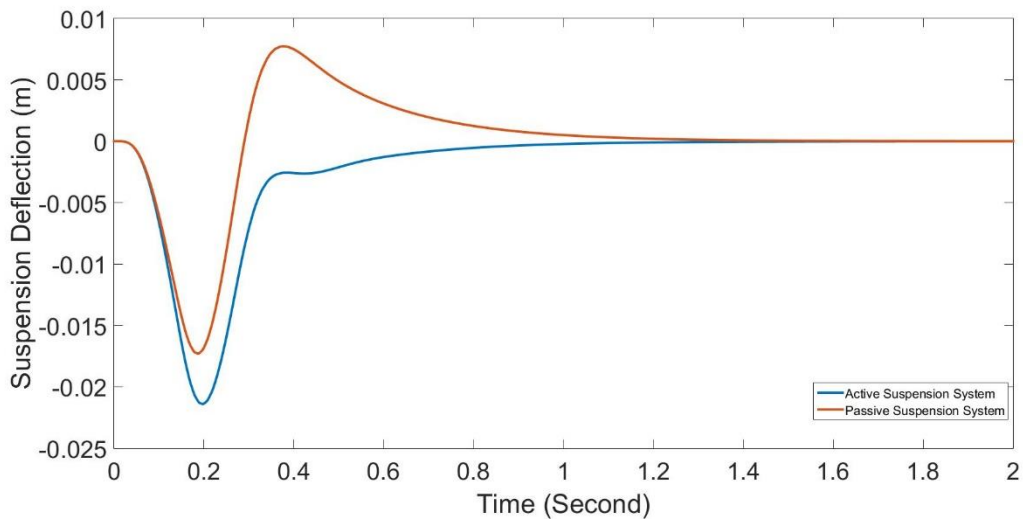


Figure 5.6. Suspension Deflection-Time

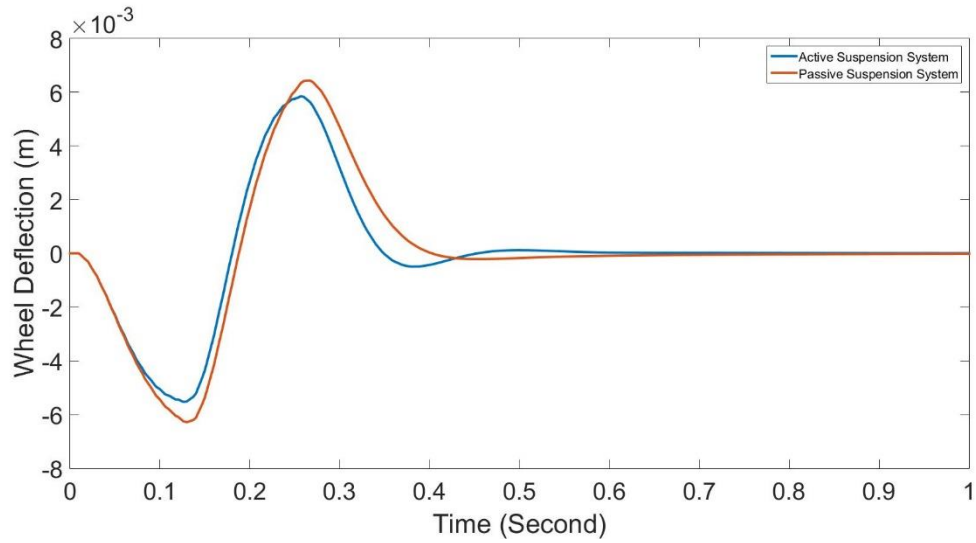


Figure 5.7. Wheel Deflection-Time

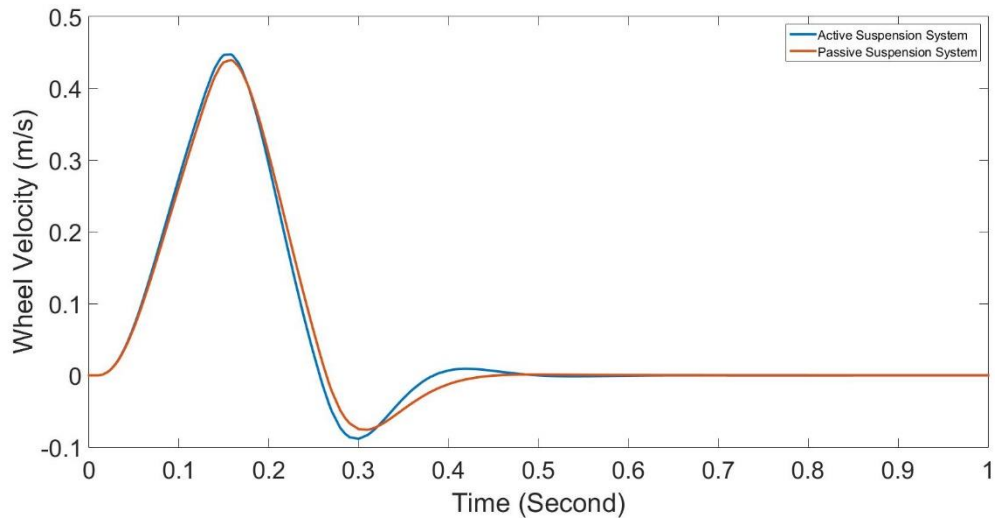


Figure 5.8. Wheel Velocity-Time

The Q value basically represents the states in which you want to influence more, while the R value is the magnitude of the force applied to the system and the N value is the magnitude that affects speed and force at the same time.

5.6 System Constraints

In addition to improvements in body speed, attention must be paid to parameters that restrict the system. These are actuator-generated force, suspension deformation and wheel deformation. Actuator force quantity indicates whether the system is applicable and the amount of suspension deflection and wheel deflection indicates safe driving.

As seen in research on electrical actuators could generate 3 kN continuous force and 8 kN maximum force values in active suspension systems.

At the same time, the suspension and wheel deflection, which is one of our statues, was aimed to examine the operation within safe limits and to give correct responses.

In order to determine these values correctly and within the safe limits, the negative effects were also encountered. The suspension of the graphics and the wheel system could not work properly under the force and deformed. This showed that these parameters cannot be used.

```
Q= diag(10^5,2*10^5,10,100);  
N=zeros(1);  
R=[0.000005];
```

These results;

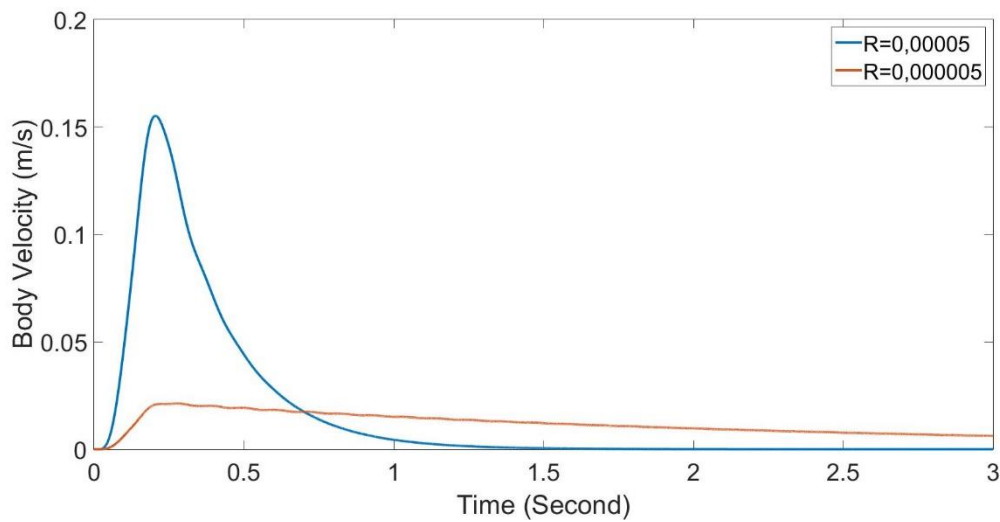


Figure 5.9. Body Velocity-Time

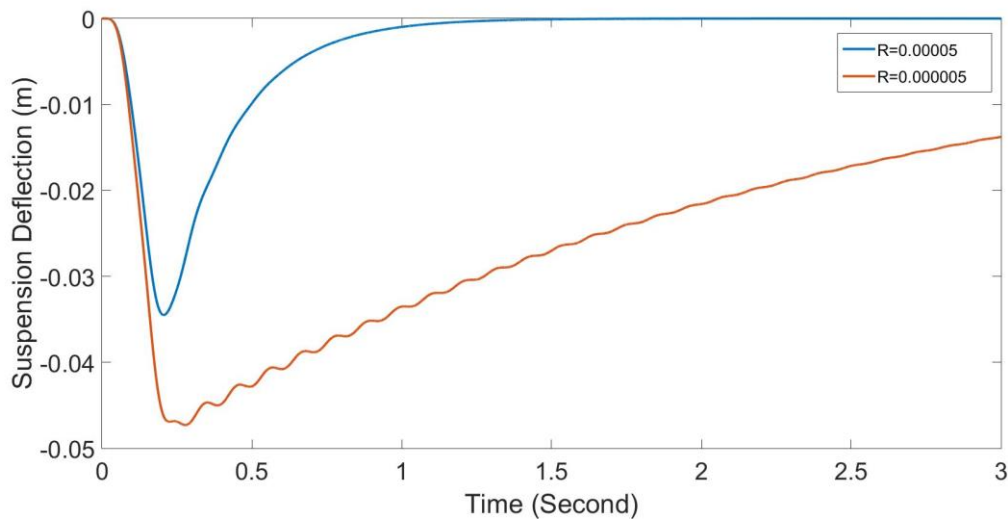


Figure 5.10. Suspension Deflection-Time

6. RESULTS

Our goal is to pass the vehicle through a bump to 40 cm, body velocity should not exceed 0.15 m/s.

Considering all the researches and our studies, we have chosen LQR parameters;

```
Q=diag(10^6,2*10^6,0.1,0.1);
```

```
N=zeros(1);
```

```
R=[0.00005];
```

The full state feedback matrix;

```
K=[138120,615870,-55530,3170]
```

The results were compared with the passive and previous studies;

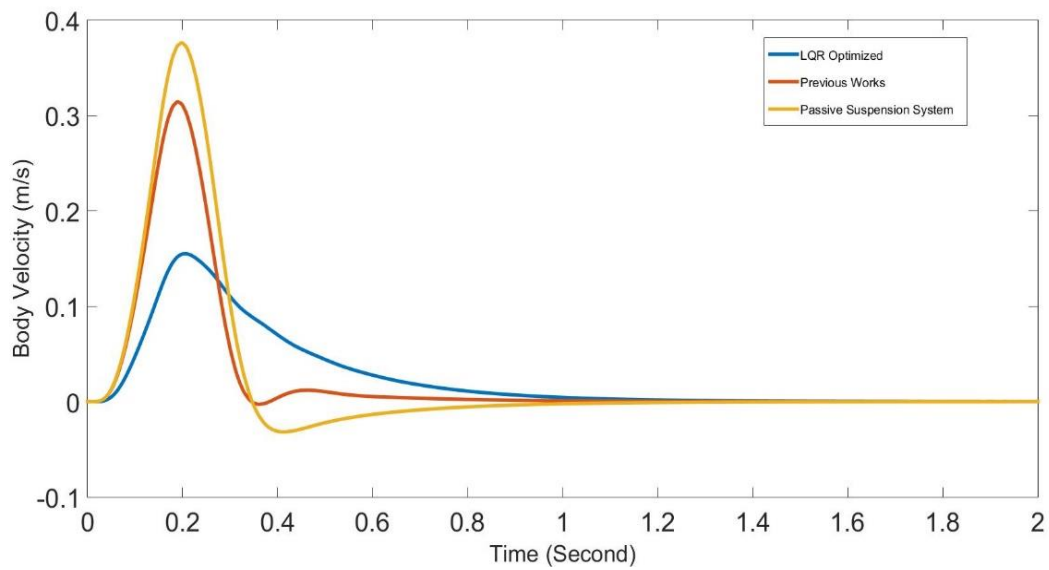


Figure 6.1. Body Velocity-Time

The peak value of body velocity is less than 0.15 m/s.

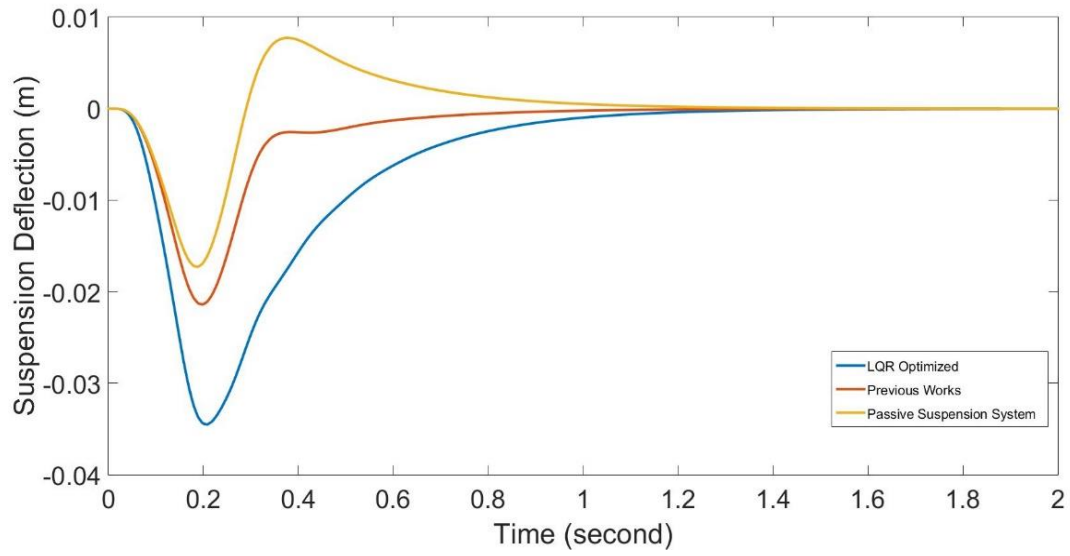


Figure 6.2. Suspension Deflection-Time

There was no deformation in the suspension that threatened driving safety.

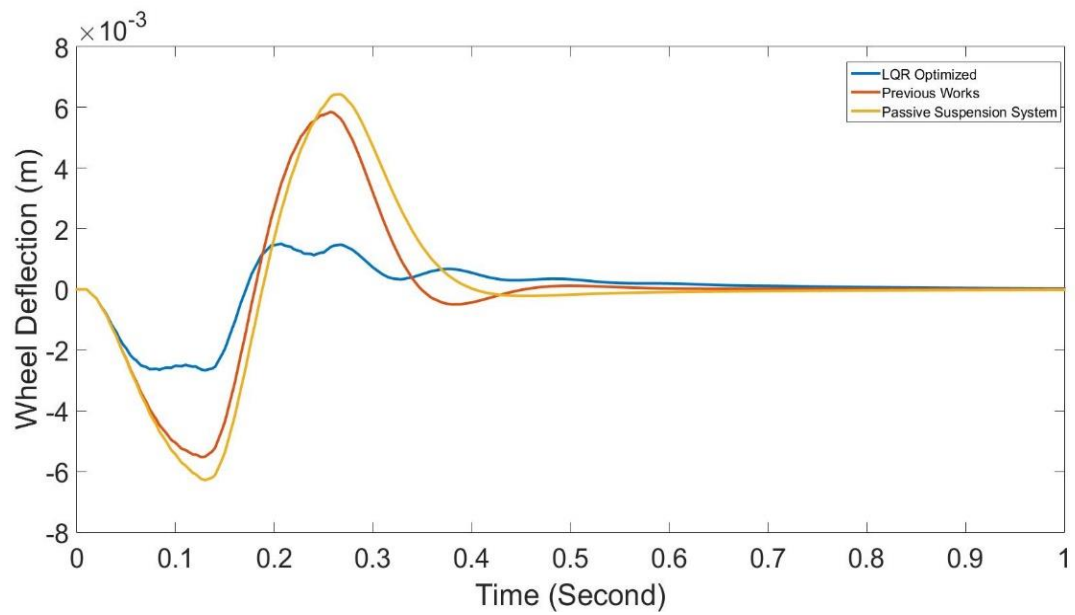


Figure 6.3. Wheel Deflection-Time

The wheel deflection is better than result of passive suspension value.

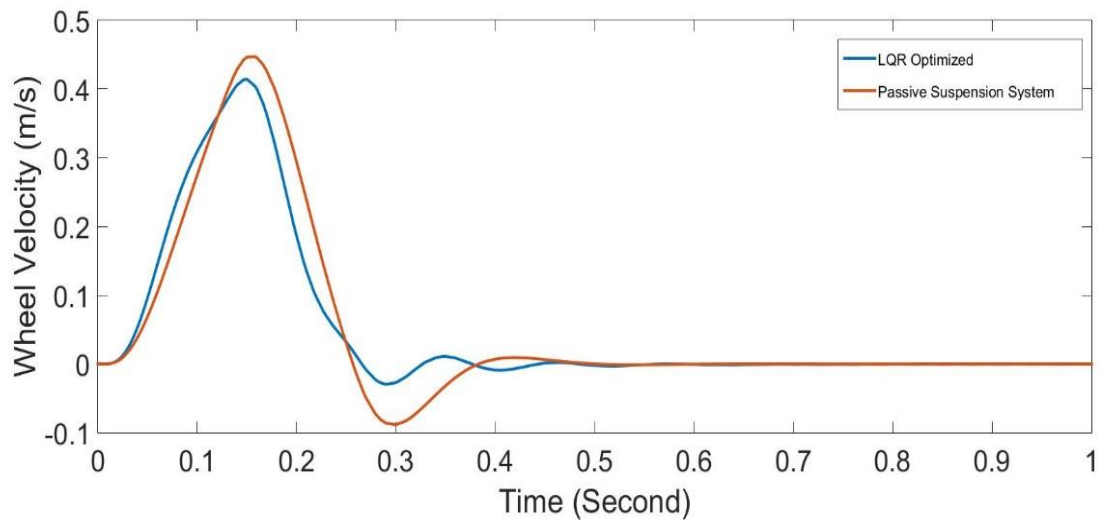


Figure 6.4. Wheel Velocity-Time

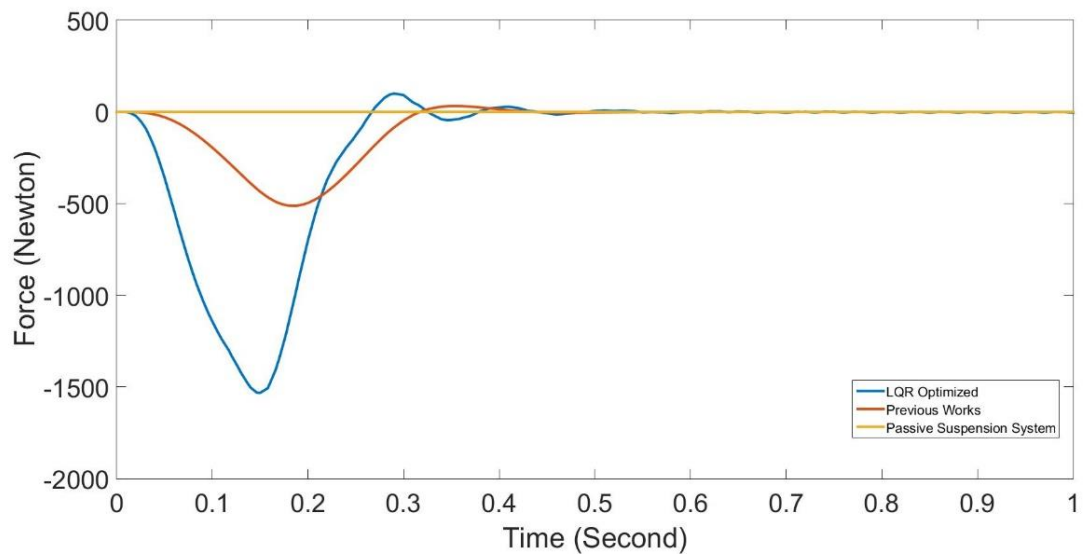


Figure 6.4. Actuator Force

7. CONCLUSION

In automobile industry, currently using passive components only utilizing spring and damper. The systems appraised by their capacities to provide good road handling and develop passenger comfort. Passive suspension systems just offer compromise between these two criteria.

Firstly, a road profile was defined and passive suspension system has been created at Matlab/Simulink. The road profile was given the system as input and the results were examined. After that study, the mathematical model of the quarter vehicle model was determined. In this study, quarter car model contains only vertical displacements. In final of the determining of the differential equations of this quarter car model, state variables were determined; x_1 (velocity of body), x_2 (deflection of suspension), x_3 (deflection of wheel), x_4 (velocity of unsprung mass). Comfort parameter was observed with graph of velocity of body. As this parameter, lifetime of spring corresponds to deflection of suspension, handling corresponds to deflection of wheel, dynamic of driving corresponds to velocity of wheel mass. Purpose of this thesis is getting the optimum results from that parameters. For this, after designing of passive suspension system, the active suspension system is modeled with 2 DOF. The state feedback LQR controller with 2 different parameters -Q and R- sets of the active suspension system was designed for a quarter car model and compared with the passive suspension system by observing the change in the performance values of the system. Comparison between these two systems are performed by using same road profile. Finally, effects of suspension systems on states and effects of active suspension system were observed.

In conclusion, active suspension system more effective than passive system. With depends on road profile and LQR controller comfort, stability and driveability increase in active systems. This situation causes its use to become widespread.

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