MARMARA UNIVERSITY
FACULTY OF ENGINEERING

DESIGN OF A TRUCK CAB SUSPENSION SYSTEM

BERKE UĞUR ÖZDEN, NUMAN KARATAŞ

GRADUATION PROJECT REPORT

Department of Mechanical Engineering

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Design of a Truck Cab Suspension System

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ABSTRACT

The reason for working on cabin suspensions is usually to increase comfort. In this project, studies will be carried out to increase comfort. The examinations will first start with the extraction of the mathematical model, and then modeling will be done using MATLAB Simulink. In this process, cabin suspension will be examined over the spring and damper, and the modeling will be done specifically for the spring and damper. When creating the model, the system to be used is half car vehicle model. A 6 degree of freedom system will be modeled according to the half-car vehicle model. After modeling on Simulink, the required parameters will be selected and entered, and the result will be obtained.

ÖZET

Kabin süspansiyonları konusunda yapılan çalışmaların amacı genellikle konforu arttırmaktır. Bu projede konforu arttırmaya yönelik çalışmalar yapılacaktır. İncelemeler öncelikle matematiksel modelin çıkarılması ile başlanacak, sonrasında MATLAB Simulink kullanılarak modelleme yapılacaktır. Bu süreçte kabin süspansiyonu, yay ve sönüm elemanı üzerinden incelenecek ve modelleme bu elemanlar esas alınarak yapılacak. Model oluşturulurken kullanılacak sistem yarım araba modelidir. Yarım araba modeline göre 6 serbestlik dereceli sistem modellenecektir. Simulink üzerinde modelleme yapıldıktan sonra gerekli parametreler seçilip girilecek ve sonuca ulaşılacaktır.

SYMBOLS

$\mathbf{c_{cf}}$, $\mathbf{c_{cr}}$: damper parameters of cabin

$\mathbf{c_{sf}}$, $\mathbf{c_{sr}}$: damper parameters of chassis

$\mathbf{c_{wf}}$, $\mathbf{c_{wr}}$: damper parameters of wheels

d : the longitudinal position of front chassis suspension to the center of gravity of cab

$\mathbf{I_c}$: Inertia moment of cab

$\mathbf{I_s}$: Inertia moment of chassis

$\mathbf{k_{cf}}$, $\mathbf{k_{cr}}$: stiffness parameters of cabin

$\mathbf{k_{sf}}$, $\mathbf{k_{sr}}$: stiffness parameters of chassis

$\mathbf{k_{wf}}$, $\mathbf{k_{wr}}$: stiffness parameters of wheels

$\mathbf{m_c}$: mass of cabin

$\mathbf{m_s}$: mass of chassis

$\mathbf{m_{wf}}$: mass of front wear and axle

$\mathbf{m_{wr}}$: mass of rear wear and axle

p : distance of rear cabin suspension elements from the cabin's center of gravity

r : distance of front cabin suspension elements from the cabin's center of gravity

s : distance of front suspension elements of the chassis from the chassis' center of gravity

t : distance of rear suspension elements of the chassis from the chassis' center of gravity

$\mathbf{z_c}$: vertical displacement of cabin

$\mathbf{z_{rf}}$, $\mathbf{z_{rr}}$: profile of the treadmill on which the truck is moving

$\mathbf{z_s}$: vertical displacement of chassis

$\mathbf{z_{wf}}$, $\mathbf{z_{wr}}$: vertical displacement of front/rear wheel

$\boldsymbol{\beta}$: pitch of chassis

$\boldsymbol{\theta}$: pitch of cabin

ABBREVIATIONS

SSR	**:** Steady State Representation
RMS	**:** Root Mean Square

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1. INTRODUCTION

In our world, most of the vehicles are using suspension systems. Suspension systems are connecting the frame and wheels with each other. Therefore, the main function of suspension systems is transmitting the loads from the road to the chassis. Suspension systems perform load transfer for two main purposes. Suspension systems are invented for increasing the travel safety and the comfort for driver (Özmen & Topaç, 2022).

In a world where technology has advanced so much, a suspension system can be designed that will not make you feel that you are passing through a huge pit. However, increasing comfort this much is a danger for driver. In long trips, if driver will not feel anything on the road it makes him sleepy. When designing a suspension system, this criterion should be considered (Alexandru & Alexandru, 2011).

When considering the suspension systems, first thing that comes to mind is vehicle suspensions. In the 21st century vehicles, suspension systems are used to decrease the vibrations induced on driver by road. On the truck, there are three suspension systems that effects vibration. These are, the vehicle suspensions that connect the wheels and chassis, the cab suspensions between the chassis and the cab and the suspension of the driver's seat. Wheel to frame suspensions can adjust the vibration very well but design process is complex, and it is expensive. Usually, seat suspensions are used because design process is easy, and it is not expensive. However, it adjusts only the vertical vibration. Because of this ability of isolation is restricted. On the other hand, the cab suspension system has a complex structure, but it is cheaper than other suspensions. Also, it effectively isolates vibration and noise (Özmen & Topaç, 2022). Cab suspensions are mostly used on trucks. Truck cab suspensions consist of two important parts. These are spring and damper. Spring is absorbing the road oscillation and stores the energy. Damper is the element that dissipates energy of suspension vibration (Mohammed & Alktranee, 2021).

Modeling the car suspension systems are done in three ways. These models are quarter car vehicle model, half car vehicle model and full car vehicle model. The quarter car vehicle model is the easiest method for analysis, but it is an inadequate model because two movements are missing (pitching and rolling). Half car vehicle model is a more complex model for analysis but rolling motion is still not considered. Full car vehicle model takes into account all possible motions. However, complexity makes parameter identification difficult (Desai, Guha, & Seshu, 2021). As previously mentioned, quarter car model is not capable of considering pitching motion. Therefore, truck cab suspension systems cannot be modeled in general with quarter car vehicle model. For most practical applications half car vehicle models are suitable for modeling truck cab suspension (Stan & Iozsa, 2019).

Half car vehicle model will be used for designing truck cab suspension system. Design criteria is maximizing the comfort in safe zone for driver. With this criteria cabin stiffness and damper parameters will be selected (Sharma, Pare, Chouksey, & Rawal, 2016). After the parameters selection is done, system will be modeling in the Simulink environment and will be analyzed (Kluever, 2015).

2. MODELING OF TRUCK CAB SUSPENSION

2.1. Half Car Truck Model

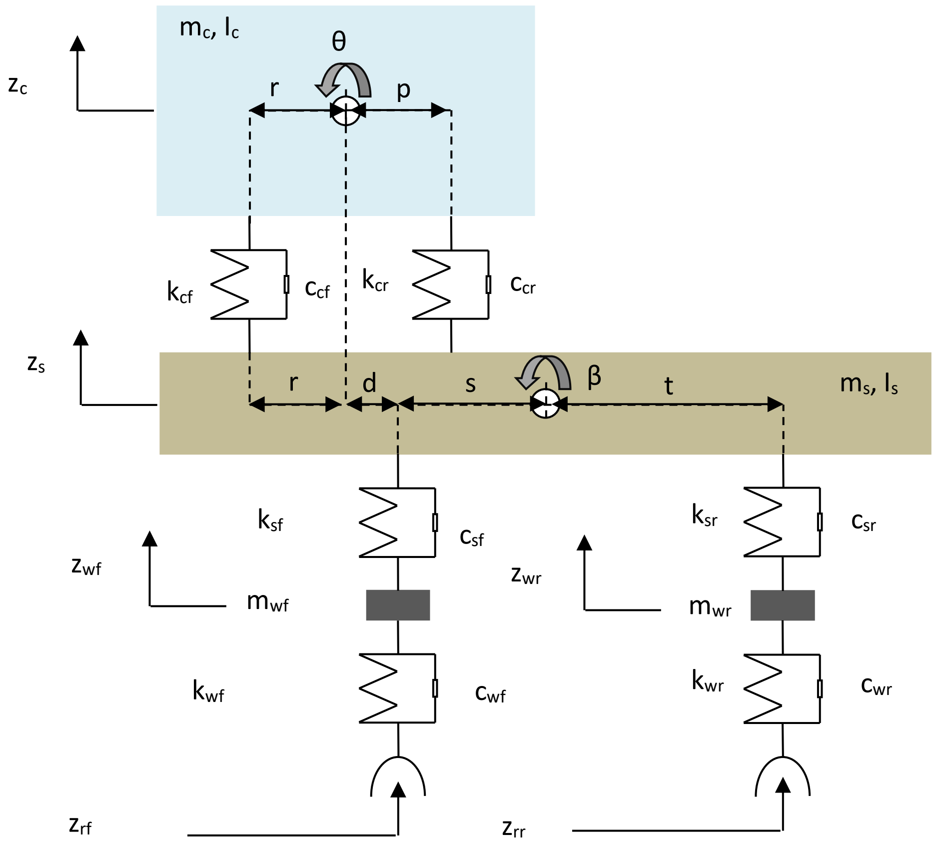

Figure 1. Model of the Half Truck

The half car truck model with 6 degree of freedom(6-DoF) is shown in the figure 1. This model covers the vertical movement of the cab, chassis, front wheels, and rear wheels. Also, it covers the pitch motion of cab and chassis. Wheels, cab, and chassis, structurally work with spring and damper. The inertia moments respectively about y axes of the tires are taken as I_c for cab and I_s for chassis. The letters r and p indicate the distance of the front and rear cabin suspension elements from the cabin's center of gravity. While the letters s and t indicate the distance between the front and rear suspension elements of the chassis and the center of gravity of the chassis. The longitudinal position of front chassis suspension to the center of gravity of cab was defined by d (Stan & Iozsa, 2019).

2.2. Mathematical Model of the Half Truck

The modeling of the half truck can be done with Lagrange's Form of Newton's Equations of Motion. The equation for Lagrange's Form of Newton's Equations of Motion shown in Equation (1) (Jazar, 2017).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} = F_r \; where, r = 1,2, \dots, n \tag{1}$$

When using Equation (1) for Half Truck Model, potential energy and dissipation function should also be used, since it is contained in the damper and spring. Considering to the Figure (1), 6 degree of freedom are observed. From the 6 degree of freedom, Equation (1) transforms to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial D}{\partial \dot{q}_r} + \frac{\partial V}{\partial q_r} = 0 \; where, q_r = \; z_c, z_s, \theta, \beta, z_{wf}, z_{wr} \tag{2}$$

Kinetic energy, potential energy and dissipation function equations are shown in Equation (3), (4) and (5) (Jazar, 2017).

$$K = \frac{1}{2}\sum_{i=1}^{n} m_i \dot{x}_i^2 \tag{3}$$

$$V = \frac{1}{2}\sum_{i=1}^{n} k_i x_i^2 \tag{4}$$

$$D = \frac{1}{2}\sum_{i=1}^{n} c_i \dot{x}_i^2 \tag{5}$$

As mentioned before, applying these 6 degree of freedom to the equations (3), (4) and (5) these equations are shown below.

$$K = \frac{1}{2} m_c \dot{z}_c^2 + \frac{1}{2} I_c \dot{\theta}^2 + \frac{1}{2} m_s \dot{z}_s^2 + \frac{1}{2} I_s \dot{\beta}^2 + \frac{1}{2} m_{wf} \dot{z}_{wf}^2 + \frac{1}{2} m_{wr} \dot{z}_{wr}^2 \tag{6}$$

$$V = \frac{1}{2} k_{cf}[z_c - z_s - \theta * r + \beta(r + d + s)]^2 + \frac{1}{2} k_{cr}[z_c - z_s + \theta * p + \beta(d + s - p)]^2 + \frac{1}{2} k_{sf}\left(z_s - z_{wf} - \beta * s\right)^2 + \frac{1}{2} k_{sr}(z_s - z_{wr} + \beta * t)^2 + \frac{1}{2} k_{wf}\left(z_{wf} - z_{rf}\right)^2 + \frac{1}{2} k_{wr}(z_{wr} - z_{rr})^2 \tag{7}$$

$$D = \frac{1}{2} c_{cf}\left[\dot{z}_c - \dot{z}_s - \dot{\theta} * r + \dot{\beta}(r + d + s)\right]^2 + \frac{1}{2} c_{cr}\left[\dot{z}_c - \dot{z}_s + \dot{\theta} * p + \dot{\beta}(d + s - p)\right]^2 + \frac{1}{2} c_{sf}\left(\dot{z}_s - \dot{z}_{wf} - \dot{\beta} * s\right)^2 + \frac{1}{2} c_{sr}\left(\dot{z}_s - \dot{z}_{wr} + \dot{\beta} * t\right)^2 + \frac{1}{2} c_{wf}\left(\dot{z}_{wf} - \dot{z}_{rf}\right)^2 + \frac{1}{2} c_{wr}(\dot{z}_{wr} - \dot{z}_{rr})^2 \tag{8}$$

The six equation of motion for the half truck model is obtained with solving the Equation (2) with the obtained kinetic energy, potential energy, and dissipation function according the 6 degree of freedom.

For the cab, the first equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}_c}\right)-\frac{\partial K}{\partial z_c}+\frac{\partial D}{\partial \dot{z}_c}+\frac{\partial V}{\partial z_c}=0 \tag{9}$$

Equation (10) is obtained by solving Equation (9).

$$m_c\ddot{z}_c+c_{cf}\left[\dot{z}_c-\dot{z}_s-\dot{\theta}*r+\dot{\beta}(r+d+s)\right]+c_{cr}\left[\dot{z}_c-\dot{z}_s+\dot{\theta}*p+\dot{\beta}(d+s-p)\right]+ k_{cf}[z_c-z_s-\theta*r+\beta(r+d+s)]+k_{cr}[z_c-z_s+\theta*p+\beta(d+s-p)]=0 \tag{10}$$

For the chassis, the second equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}_s}\right)-\frac{\partial K}{\partial z_s}+\frac{\partial D}{\partial \dot{z}_s}+\frac{\partial V}{\partial z_s}=0 \tag{11}$$

Equation (12) is obtained by solving Equation (11).

$$m_s\ddot{z}_s-c_{cf}\left[\dot{z}_c-\dot{z}_s-\dot{\theta}*r+\dot{\beta}(r+d+s)\right]-c_{cr}\left[\dot{z}_c-\dot{z}_s+\dot{\theta}*p+\dot{\beta}(d+s-p)\right]+ c_{sf}\left(\dot{z}_s-\dot{z}_{wf}-\dot{\beta}*s\right)+c_{sr}\left(\dot{z}_s-\dot{z}_{wr}+\dot{\beta}*t\right)-k_{cf}[z_c-z_s-\theta*r+\beta(r+d+ s)]-k_{cr}[z_c-z_s+\theta*p+\beta(d+s-p)]+k_{sf}\left(z_s-z_{wf}-\beta*s\right)+k_{sr}(z_s-z_{wr}+ \beta*t)=0 \tag{12}$$

For the front wheel, the third equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}_{wf}}\right)-\frac{\partial K}{\partial z_{wf}}+\frac{\partial D}{\partial \dot{z}_{wf}}+\frac{\partial V}{\partial z_{wf}}=0 \tag{13}$$

Equation (14) is obtained by solving Equation (13).

$$m_{wf}\ddot{z}_{wf}-c_{sf}\left(\dot{z}_s-\dot{z}_{wf}-\dot{\beta}*s\right)+c_{wf}\left(\dot{z}_{wf}-\dot{z}_{rf}\right)-k_{sf}\left(z_s-z_{wf}-\beta*s\right)+ k_{wf}\left(z_{wf}-z_{rf}\right)=0 \tag{14}$$

For the rear wheel, the fourth equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}_{wr}}\right)-\frac{\partial K}{\partial z_{wr}}+\frac{\partial D}{\partial \dot{z}_{wr}}+\frac{\partial V}{\partial z_{wr}}=0 \tag{15}$$

Equation (16) is obtained by solving Equation (15).

$$m_{wr}\ddot{z}_{wr} - c_{sr}\left(\dot{z}_s - \dot{z}_{wr} + \dot{\beta} * t\right) + c_{wr}(\dot{z}_{wr} - \dot{z}_{rr}) - k_{sr}(z_s - z_{wr} + \beta * t) + k_{wr}(z_{wr} - z_{rr}) = 0 \tag{16}$$

For the rotation angle of cab, the fifth equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}}\right) - \frac{\partial K}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} + \frac{\partial V}{\partial \theta} = 0 \tag{17}$$

Equation (18) is obtained by solving Equation (17).

$$I_c\ddot{\theta} - c_{cf}\left[\dot{z}_c - \dot{z}_s - \dot{\theta} * r + \dot{\beta}(r + d + s)\right] + c_{cr}\left[\dot{z}_c - \dot{z}_s + \dot{\theta} * p + \dot{\beta}(d + s - p)\right] - k_{cf}[z_c - z_s - \theta * r + \beta(r + d + s)] + k_{cr}[z_c - z_s + \theta * p + \beta(d + s - p)] = 0 \tag{18}$$

For the rotation angle of chassis, the sixth equation of the six degree of freedom half truck cab system is obtained according to Equation (2).

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\beta}}\right) - \frac{\partial K}{\partial \beta} + \frac{\partial D}{\partial \dot{\beta}} + \frac{\partial V}{\partial \beta} = 0 \tag{19}$$

Equation (20) is obtained by solving Equation (19).

$$I_s\ddot{\beta} + c_{cf}\left[\dot{z}_c - \dot{z}_s - \dot{\theta} * r + \dot{\beta}(r + d + s)\right] + c_{cr}\left[\dot{z}_c - \dot{z}_s + \dot{\theta} * p + \dot{\beta}(d + s - p)\right] - c_{sf}\left(\dot{z}_s - \dot{z}_{wf} - \dot{\beta} * s\right) + c_{sr}\left(\dot{z}_s - \dot{z}_{wr} + \dot{\beta} * t\right) + k_{cf}[z_c - z_s - \theta * r + \beta(r + d + s)] + k_{cr}[z_c - z_s + \theta * p + \beta(d + s - p)] - k_{sf}\left(z_s - z_{wf} - \beta * s\right) + k_{sr}(z_s - z_{wr} + \beta * t) = 0 \tag{20}$$

These set of equations may be rearranged in a matrix form like this (Jazar, 2017):

$$[M][\ddot{x}] + [C][\dot{x}] + [K][x] = [U] \tag{21}$$

Where:

x: column vector of displacement

$\dot{x}$: column vector of velocity

$\ddot{x}$: column vector of acceleration

M: Generalized mass (or inertia) matrix

C: Generalized damping matrix

K: Generalized stiffness matrix

U: Column vector of generalized external forces

The x matrix is the displacement matrix which is:

$$[x] = \begin{bmatrix} z_c \\ z_s \\ \theta \\ \beta \\ z_{wf} \\ z_{wr} \end{bmatrix} \tag{22}$$

The M matrix is the mass (or inertia) matrix which is:

$$[M] = \begin{bmatrix} m_c & 0 & 0 & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 & 0 & 0 \\ 0 & 0 & I_c & 0 & 0 & 0 \\ 0 & 0 & 0 & I_s & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{wf} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{wr} \end{bmatrix} \tag{23}$$

The C matrix is the damping matrix which is:

$$[C] = \begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & C_{1,6} \\ C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} & C_{2,5} & C_{2,6} \\ C_{3,1} & C_{3,2} & C_{3,3} & C_{3,4} & C_{3,5} & C_{3,6} \\ C_{4,1} & C_{4,2} & C_{4,3} & C_{4,4} & C_{4,5} & C_{4,6} \\ C_{5,1} & C_{5,2} & C_{5,3} & C_{5,4} & C_{5,5} & C_{5,6} \\ C_{6,1} & C_{6,2} & C_{6,3} & C_{6,4} & C_{6,5} & C_{6,6} \end{bmatrix} \tag{24}$$

For the C matrix, the elements of first row are:

$$C_{1,1} = c_{cf} + c_{cr} \tag{25}$$
$$C_{1,2} = -c_{cf} - c_{cr} \tag{26}$$
$$C_{1,3} = -r * c_{cf} + p * c_{cr} \tag{27}$$
$$C_{1,4} = (r + d + s) * c_{cf} + (d + s - p) * c_{cr} \tag{28}$$
$$C_{1,5} = \mathrm{C}_{1,6} = 0 \tag{29}$$

For the C matrix, the elements of second row are:

$$C_{2,1} = -c_{cf} - c_{cr} \tag{30}$$
$$C_{2,2} = +c_{cf} + c_{cr} + c_{sf} + c_{sr} \tag{31}$$
$$C_{2,3} = r * c_{cf} - p * c_{cr} \tag{32}$$
$$C_{2,4} = (r + d + s) * \left(-c_{cf}\right) + (d + s - p) * (-c_{cr}) + s * \left(-c_{sf}\right) + t * c_{sr} \tag{33}$$
$$C_{2,5} = -c_{sf} \tag{34}$$
$$C_{2,6} = -c_{sr} \tag{35}$$

For the C matrix, the elements of third row are:

$$C_{3,1} = -c_{cf} + c_{cr} \tag{36}$$
$$C_{3,2} = c_{cf} - c_{cr} \tag{37}$$
$$C_{3,3} = r * c_{cf} + p * c_{cr} \tag{38}$$
$$C_{3,4} = (r + d + s) * (-c_{cf}) + (d + s - p) * c_{cr} \tag{39}$$

$$C_{3,5} = C_{3,6} = 0 \tag{40}$$

For the C matrix, the elements of fourth row are:

$$C_{4,1} = c_{cf} + c_{cr} \tag{41}$$
$$C_{4,2} = -c_{cf} - c_{cr} - c_{sf} + c_{sr} \tag{42}$$
$$C_{4,3} = -r * c_{cf} + p * c_{cr} \tag{43}$$
$$C_{4,4} = (r + d + s) * c_{cf} + (d + s - p) * c_{cr} + s * c_{sf} + t * c_{sr} \tag{44}$$
$$C_{4,5} = c_{sf} \tag{45}$$
$$C_{4,6} = -c_{sr} \tag{46}$$

For the C matrix, the elements of fifth row are:

$$C_{5,2} = -c_{sf} \tag{47}$$
$$C_{5,4} = s * c_{sf} \tag{48}$$
$$C_{5,5} = c_{sf} + c_{wf} \tag{49}$$
$$C_{5,1} = C_{5,3} = C_{5,6} = 0 \tag{50}$$

For the C matrix, the elements of sixth row are:

$$C_{6,2} = -c_{sr} \tag{51}$$
$$C_{6,4} = t * (-c_{sr}) \tag{52}$$
$$C_{6,6} = c_{sr} + c_{wr} \tag{53}$$
$$C_{6,1} = C_{6,3} = C_{6,5} = 0 \tag{54}$$

The K matrix is the stiffness matrix which is:

$$[K] = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} & K_{1,6} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} & K_{2,5} & K_{2,6} \\ K_{3,1} & K_{3,2} & K_{3,3} & K_{3,4} & K_{3,5} & K_{3,6} \\ K_{4,1} & K_{4,2} & K_{4,3} & K_{4,4} & K_{4,5} & K_{4,6} \\ K_{5,1} & K_{5,2} & K_{5,3} & K_{5,4} & K_{5,5} & K_{5,6} \\ K_{6,1} & K_{6,2} & K_{6,3} & K_{6,4} & K_{6,5} & K_{6,6} \end{bmatrix} \tag{55}$$

For the K matrix, the elements of first row are:

$$K_{1,1} = k_{cf} + k_{cr} \tag{56}$$
$$K_{1,2} = -k_{cf} - k_{cr} \tag{57}$$
$$K_{1,3} = -r * k_{cf} + p * k_{cr} \tag{58}$$
$$K_{1,4} = (r + d + s) * k_{cf} + (d + s - p) * k_{cr} \tag{59}$$
$$K_{1,5} = K_{1,6} = 0 \tag{60}$$

For the K matrix, the elements of second row are:

$$K_{2,1} = -k_{cf} - k_{cr} \tag{61}$$
$$K_{2,2} = +k_{cf} + k_{cr} + k_{sf} + k_{sr} \tag{62}$$
$$K_{2,3} = r * k_{cf} - p * k_{cr} \tag{63}$$
$$K_{2,4} = (r + d + s) * \left(-k_{cf}\right) + (d + s - p) * (-k_{cr}) + s * \left(-k_{sf}\right) + t * k_{sr} \tag{64}$$
$$K_{2,5} = -k_{sf} \tag{65}$$
$$K_{2,6} = -k_{sr} \tag{66}$$

For the K matrix, the elements of third row are:

$$K_{3,1} = -k_{cf} + k_{cr} \tag{67}$$
$$K_{3,2} = k_{cf} - k_{cr} \tag{68}$$
$$K_{3,3} = r * k_{cf} + p * k_{cr} \tag{69}$$
$$K_{3,4} = (r + d + s) * (-k_{cf}) + (d + s - p) * k_{cr} \tag{70}$$
$$K_{3,5} = \mathrm{K}_{3,6} = 0 \tag{71}$$

For the K matrix, the elements of fourth row are:

$$K_{4,1} = k_{cf} + k_{cr} \tag{72}$$
$$K_{4,2} = -k_{cf} - k_{cr} - k_{sf} + k_{sr} \tag{73}$$
$$K_{4,3} = -r * k_{cf} + p * k_{cr} \tag{74}$$
$$K_{4,4} = (r + d + s) * k_{cf} + (d + s - p) * k_{cr} + s * k_{sf} + t * k_{sr} \tag{75}$$
$$K_{4,5} = k_{sf} \tag{76}$$
$$K_{4,6} = -k_{sr} \tag{77}$$

For the K matrix, the elements of fifth row are:

$$K_{5,2} = -k_{sf} \tag{78}$$
$$K_{5,4} = s * k_{sf} \tag{79}$$
$$K_{5,5} = k_{sf} + k_{wf} \tag{80}$$
$$K_{5,1} = \mathrm{K}_{5,3} = K_{5,6} = 0 \tag{81}$$

For the K matrix, the elements of sixth row are:

$$K_{6,2} = -k_{sr} \tag{82}$$
$$K_{6,4} = t * (-k_{sr}) \tag{83}$$
$$K_{6,6} = k_{sr} + k_{wr} \tag{84}$$
$$K_{6,1} = K_{6,3} = K_{6,5} = 0 \tag{85}$$

The U matrix is the external forces matrix which is:

$$[U] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c_{wf}\dot{z}_{rf} + k_{wf}z_{rf} \\ c_{wr}\dot{z}_{rr} + k_{wr}z_{rr} \end{bmatrix} \tag{86}$$

2.3. Steady State Representation of the System

Steady state representation is most appropriate method in the analysis of suspension elements in second order linear ordinary differential equations. The formulation of steady state representation is shown in below (Kluever, 2015).

$$[\dot{x}] = [A][x] + [B][u] \quad (87)$$
$$[\dot{y}] = [C][x] + [D][u] \quad (88)$$

where:

x: state vector

y: output vector

u: input (or control) vector

A: State (or system) matrix

B: Input matrix

C: Output matrix

D: Feedforward matrix

Input vertical displacements and vertical velocities of the model are z_{rr}, z_{rf} and $\dot{z}_{rr}$, $\dot{z}_{rf}$. Those are written as input vector in "u". The output vertical displacements and vertical velocities of the model are z_c, z_s, z_{wf}, z_{wr}, θ, β and $\dot{z}_c$, $\dot{z}_s$, $\dot{z}_{wf}$, $\dot{z}_{wr}$, $\dot{\theta}$, $\dot{\beta}$. Those are written as state vector in "x". The required output vertical displacements and vertical accelerations of the model are z_c, z_s and $\ddot{z}_c$, $\ddot{z}_s$ for analyzing the system. So, those variables are used in y vector as output vector.

Every output vertical displacement and velocity variables are written as below:

$$x_1 = z_c \quad (89)$$
$$x_2 = \dot{z}_c \quad (90)$$

$$x_3 = z_s \quad (91)$$
$$x_4 = \dot{z}_s \quad (92)$$

$$x_5 = z_{wf} \quad (93)$$
$$x_6 = \dot{z}_{wf} \quad (94)$$

$$x_7 = z_{wr} \quad (95)$$
$$x_8 = \dot{z}_{wr} \quad (96)$$

$$x_9 = \theta \quad (97)$$
$$x_{10} = \dot{\theta} \quad (98)$$

$$x_{11} = \beta \tag{99}$$
$$x_{12} = \dot{\beta} \tag{100}$$

Every input vertical displacement and velocity variables are written as below:

$$u_1 = z_{rf} \tag{101}$$
$$u_2 = \dot{z}_{rf} \tag{102}$$

$$u_3 = z_{rr} \tag{103}$$
$$u_4 = \dot{z}_{rr} \tag{104}$$

Every required output vertical displacement and velocity variables are written as below:

$$y_1 = z_c = x_1 \tag{105}$$
$$y_2 = \ddot{z}_c = \dot{x}_2 \tag{106}$$

$$y_3 = z_s = x_3 \tag{107}$$
$$y_4 = \ddot{z}_s = \dot{x}_4 \tag{108}$$

Identification of the variables are done in the x, y, and u vector. Now, ordinary differential equations that written with the Lagrange's Form of Newton's Equations of Motion are written with the new format as shown. In new format, state vector values are put in place of the displacement and velocity variables in the equation.

For the cab, the first equation of the steady state variable obtained according to Equation (89) and (90).

$$\dot{x}_1 = \dot{z}_c = x_2 \tag{109}$$

For the cab, the second equation of the steady state variable obtained according to Equation (10).

$$\dot{x}_2 = \ddot{z}_c = -\frac{c_{cf}[x_2-x_4-x_{10}*r+x_{12}(r+d+s)]}{m_c} - \frac{c_{cr}[x_2-x_4+x_{10}*p+x_{12}(d+s-p)]}{m_c} - \frac{k_{cf}[x_1-x_3-x_9*r+x_{11}(r+d+s)]}{m_c} - \frac{k_{cr}[x_1-x_3+x_9*p+x_{11}(d+s-p)]}{m_c} \tag{110}$$

For the chassis, the first equation of the steady state variable obtained according to Equation (91) and (92).

$$\dot{x}_3 = \dot{z}_s = x_4 \tag{111}$$

For the chassis, the second equation of the steady state variable obtained according to

Equation (12).

$$\dot{x}_4=\ddot{z}_s=\frac{c_{cf}[x_2-x_4-x_{10}*r+x_{12}(r+d+s)]}{m_s}+\frac{c_{cr}[x_2-x_4+x_{10}*p+x_{12}(d+s-p)]}{m_s}-\frac{c_{sf}(x_4-x_6-x_{12}*s)}{m_s}-\frac{c_{sr}(x_4-x_8+x_{12}*t)}{m_s}+\frac{k_{cf}[x_1-x_3-x_9*r+x_{11}(r+d+s)]}{m_s}+\frac{k_{cr}[x_1-x_3+x_9*p+x_{11}(d+s-p)]}{m_s}-\frac{k_{sf}(x_3-x_5-x_{11}*s)}{m_s}-\frac{k_{sr}(x_3-x_7+x_{11}*t)}{m_s} \tag{112}$$

For the front wheel, the first equation of the steady state variable obtained according to Equation (93) and (94).

$$\dot{x}_5=\dot{z}_{wf}=x_6 \tag{113}$$

For the front wheel, the second equation of the steady state variable obtained according to Equation (14).

$$\dot{x}_6=\ddot{z}_{wf}=\frac{c_{sf}(x_4-x_6-x_{12}*s)}{m_{wf}}-\frac{c_{wf}(x_6-u_2)}{m_{wf}}+\frac{k_{sf}(x_3-x_5-x_{11}*s)}{m_{wf}}-\frac{k_{wf}(x_5-u_1)}{m_{wf}} \tag{114}$$

For the rear wheel, the first equation of the steady state variable obtained according to Equation (95) and (96).

$$\dot{x}_7=\dot{z}_{wr}=x_8 \tag{115}$$

For the rear wheel, the second equation of the steady state variable obtained according to Equation (16).

$$\dot{x}_8=\ddot{z}_{wr}=\frac{c_{sr}(x_4-x_8+x_{12}*t)}{m_{wr}}-\frac{c_{wr}(x_8-u_4)}{m_{wr}}+\frac{k_{sr}(x_3-x_7+x_{11}*t)}{m_{wr}}-\frac{k_{wr}(x_7-u_3)}{m_{wr}} \tag{116}$$

For the rotation angle of the cab, the first equation of the steady state variable obtained according to Equation (97) and (98).

$$\dot{x}_9=\dot{\theta}=x_{10} \tag{117}$$

For the rotation angle of the cab, the second equation of the steady state variable obtained according to Equation (18).

$$\dot{x}_{10}=\ddot{\theta}=\frac{c_{cf}[x_2-x_4-x_{10}*r+x_{12}(r+d+s)]}{I_c}-\frac{c_{cr}[x_2-x_4+x_{10}*p+x_{12}(d+s-p)]}{I_c}+\frac{k_{cf}[x_1-x_3-x_9*r+x_{11}(r+d+s)]}{I_c}-\frac{k_{cr}[x_1-x_3+x_9*p+x_{11}(d+s-p)]}{I_c} \tag{118}$$

For the rotation angle of the chassis, the first equation of the steady state variable obtained according to Equation (99) and (100).

$$\dot{x}_{11} = \dot{\beta} = x_{12} \tag{119}$$

For the rotation angle of the cab, the second equation of the steady state variable obtained according to Equation (20).

$$\dot{x}_{12} = \ddot{\beta} = -\frac{c_{cf}[x_2 - x_4 - x_{10} * r + x_{12}(r + d + s)]}{I_s} - \frac{c_{cr}[x_2 - x_4 + x_{10} * p + x_{12}(d + s - p)]}{I_s} + \frac{c_{sf}(x_4 - x_6 - x_{12} * s)}{I_s} - \frac{c_{sr}(x_4 - x_8 + x_{12} * t)}{I_s} - \frac{k_{cf}[x_1 - x_3 - x_9 * r + x_{11}(r + d + s)]}{I_s} - \frac{k_{cr}[x_1 - x_3 + x_9 * p + x_{11}(d + s - p)]}{I_s} + \frac{k_{sf}(x_3 - x_5 - x_{11} * s)}{I_s} - \frac{k_{sr}(x_3 - x_7 + x_{11} * t)}{I_s} \tag{120}$$

These set of equations may be rearranged in a matrix form like this:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} & A_{1,5} & A_{1,6} & A_{1,7} & A_{1,8} & A_{1,9} & A_{1,10} & A_{1,11} & A_{1,12} \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} & A_{2,5} & A_{2,6} & A_{2,7} & A_{2,8} & A_{2,9} & A_{2,10} & A_{2,11} & A_{2,12} \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} & A_{3,5} & A_{3,6} & A_{3,7} & A_{3,8} & A_{3,9} & A_{3,10} & A_{3,11} & A_{3,12} \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} & A_{4,5} & A_{4,6} & A_{4,7} & A_{4,8} & A_{4,9} & A_{4,10} & A_{4,11} & A_{4,12} \\ A_{5,1} & A_{5,2} & A_{5,3} & A_{5,4} & A_{5,5} & A_{5,6} & A_{5,7} & A_{5,8} & A_{5,9} & A_{5,10} & A_{5,11} & A_{5,12} \\ A_{6,1} & A_{6,2} & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & A_{6,7} & A_{6,8} & A_{6,9} & A_{6,10} & A_{6,11} & A_{6,12} \\ A_{7,1} & A_{7,2} & A_{7,3} & A_{7,4} & A_{7,5} & A_{7,6} & A_{7,7} & A_{7,8} & A_{7,9} & A_{7,10} & A_{7,11} & A_{7,12} \\ A_{8,1} & A_{8,2} & A_{8,3} & A_{8,4} & A_{8,5} & A_{8,6} & A_{8,7} & A_{8,8} & A_{8,9} & A_{8,10} & A_{8,11} & A_{8,12} \\ A_{9,1} & A_{9,2} & A_{9,3} & A_{9,4} & A_{9,5} & A_{9,6} & A_{9,7} & A_{9,8} & A_{9,9} & A_{9,10} & A_{9,11} & A_{9,12} \\ A_{10,1} & A_{10,2} & A_{10,3} & A_{10,4} & A_{10,5} & A_{10,6} & A_{10,7} & A_{10,8} & A_{10,9} & A_{10,10} & A_{10,11} & A_{10,12} \\ A_{11,1} & A_{11,2} & A_{11,3} & A_{11,4} & A_{11,5} & A_{11,6} & A_{11,7} & A_{11,8} & A_{11,9} & A_{11,10} & A_{11,11} & A_{11,12} \\ A_{12,1} & A_{12,2} & A_{12,3} & A_{12,4} & A_{12,5} & A_{12,6} & A_{12,7} & A_{12,8} & A_{12,9} & A_{12,10} & A_{12,11} & A_{12,12} \end{bmatrix}}_{[A]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} B_{1,1} & B_{1,2} & B_{1,3} & B_{1,4} \\ B_{2,1} & B_{2,2} & B_{2,3} & B_{2,4} \\ B_{3,1} & B_{3,2} & B_{3,3} & B_{3,4} \\ B_{4,1} & B_{4,2} & B_{4,3} & B_{4,4} \\ B_{5,1} & B_{5,2} & B_{5,3} & B_{5,4} \\ B_{6,1} & B_{6,2} & B_{6,3} & B_{6,4} \\ B_{7,1} & B_{7,2} & B_{7,3} & B_{7,4} \\ B_{8,1} & B_{8,2} & B_{8,3} & B_{8,4} \\ B_{9,1} & B_{9,2} & B_{9,3} & B_{9,4} \\ B_{10,1} & B_{10,2} & B_{10,3} & B_{10,4} \\ B_{11,1} & B_{11,2} & B_{11,3} & B_{11,4} \\ B_{12,1} & B_{12,2} & B_{12,3} & B_{12,4} \end{bmatrix}}_{[B]} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad (121)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & C_{1,5} & C_{1,6} & C_{1,7} & C_{1,8} & C_{1,9} & C_{1,10} & C_{1,11} & C_{1,12} \\ C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} & C_{2,5} & C_{2,6} & C_{2,7} & C_{2,8} & C_{2,9} & C_{2,10} & C_{2,11} & C_{2,12} \\ C_{3,1} & C_{3,2} & C_{3,3} & C_{3,4} & C_{3,5} & C_{3,6} & C_{3,7} & C_{3,8} & C_{3,9} & C_{3,10} & C_{3,11} & C_{3,12} \\ C_{4,1} & C_{4,2} & C_{4,3} & C_{4,4} & C_{4,5} & C_{4,6} & C_{4,7} & C_{4,8} & C_{4,9} & C_{4,10} & C_{4,11} & C_{4,12} \end{bmatrix}}_{[C]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} +$$

$$\underbrace{\begin{bmatrix} D_{1,1} & D_{1,2} & D_{1,3} & D_{1,4} \\ D_{2,1} & D_{2,2} & D_{2,3} & D_{2,4} \\ D_{3,1} & D_{3,2} & D_{3,3} & D_{3,4} \\ D_{4,1} & D_{4,2} & D_{4,3} & D_{4,4} \end{bmatrix}}_{[D]} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{122}$$

For the A matrix, the elements of first row are:

$$A_{1,1} = A_{1,3} = A_{1,4} = A_{1,5} = A_{1,6} = A_{1,7} = A_{1,8} = A_{1,9} = A_{1,10} = A_{1,11} = A_{1,12} = 0 \tag{123}$$

$$A_{1,2} = 1 \tag{124}$$

For the A matrix, the elements of second row are:

$$A_{2,1} = \frac{-k_{cf} - k_{cr}}{m_c} \tag{125}$$

$$A_{2,2} = \frac{-c_{cf} - c_{cr}}{m_c} \tag{126}$$

$$A_{2,3} = \frac{k_{cf} + k_{cr}}{m_c} \tag{127}$$

$$A_{2,4} = \frac{c_{cf} + c_{cr}}{m_c} \tag{128}$$

$$A_{2,9} = \frac{k_{cf} * r - k_{cr} * p}{m_c} \tag{129}$$

$$A_{2,10} = \frac{c_{cf} * r - c_{cr} * p}{m_c} \tag{130}$$

$$A_{2,11} = \frac{-k_{cf}(r + d + s) - k_{cr} * (d + s - p)}{m_c} \tag{131}$$

$$A_{2,12} = \frac{-c_{cf} * (r + d + s) - c_{cr} * (d + s - p)}{m_c} \tag{132}$$

$$A_{2,5} = A_{2,6} = A_{2,7} = A_{2,8} = 0 \tag{133}$$

For the A matrix, the elements of third row are:

$$A_{3,1} = A_{3,2} = A_{3,3} = A_{3,5} = A_{3,6} = A_{3,7} = A_{3,8} = A_{3,9} = A_{3,10} = A_{3,11} = A_{3,12} = 0 \quad (134)$$

$$A_{3,4} = 1 \quad (135)$$

For the A matrix, the elements of fourth row are:

$$A_{4,1} = \frac{k_{cf}+k_{cr}}{m_s} \quad (136)$$

$$A_{4,2} = \frac{c_{cf}+c_{cr}}{m_s} \quad (137)$$

$$A_{4,3} = \frac{-k_{cf}-k_{cr}-k_{sf}-k_{sr}}{m_s} \quad (138)$$

$$A_{4,4} = \frac{-c_{cf}-c_{cr}-c_{sf}-c_{sr}}{m_s} \quad (139)$$

$$A_{4,5} = \frac{k_{sf}}{m_s} \quad (140)$$

$$A_{4,6} = \frac{c_{sf}}{m_s} \quad (141)$$

$$A_{4,7} = \frac{k_{sr}}{m_s} \quad (142)$$

$$A_{4,8} = \frac{c_{sr}}{m_s} \quad (143)$$

$$A_{4,9} = \frac{-k_{cf}*r+k_{cr}*p}{m_s} \quad (144)$$

$$A_{4,10} = \frac{-c_{cf}*r+c_{cr}*p}{m_s} \quad (145)$$

$$A_{4,1} = \frac{k_{cf}*(r+d+s)+k_{cr}*(d+s-p)+k_{sf}*s-k_{sr}*t}{m_s} \quad (146)$$

$$A_{4,12} = \frac{c_{cf}*(r+d+s)+c_{cr}*(d+s-p)+c_{sf}*s-c_{sr}*t}{m_s} \quad (147)$$

For the A matrix, the elements of fifth row are:

$$A_{5,1} = A_{5,2} = A_{5,3} = A_{5,4} = A_{5,5} = A_{5,7} = A_{5,8} = A_{5,9} = A_{5,10} = A_{5,11} = A_{5,12} = 0 \quad (148)$$

$$A_{5,6} = 1 \quad (149)$$

For the A matrix, the elements of sixth row are:

$$A_{6,3} = \frac{k_{sf}}{m_{wf}} \quad (150)$$

$$A_{6,4} = \frac{c_{sf}}{m_{wf}} \quad (151)$$

$$A_{6,5} = \frac{-k_{sf}-k_{wf}}{m_{wf}} \quad (152)$$

$$A_{6,6} = \frac{-c_{sf}-c_{wf}}{m_{wf}} \quad (153)$$

$$A_{6,11} = \frac{-k_{sf}*s}{m_{wf}} \quad (154)$$

$$A_{6,12} = \frac{-c_{sf}*s}{m_{wf}} \quad (155)$$

$$A_{6,1} = A_{6,2} = A_{6,7} = A_{6,8} = A_{6,9} = A_{6,10} = 0 \quad (156)$$

For the A matrix, the elements of seventh row are:

$$A_{7,1} = A_{7,2} = A_{7,3} = A_{7,4} = A_{7,5} = A_{7,6} = A_{7,7} = A_{7,9} = A_{7,10} = A_{7,11} = A_{7,12} = 0 \tag{157}$$

$$A_{7,8} = 1 \tag{158}$$

For the A matrix, the elements of eighth row are:

$$A_{8,3} = \frac{k_{sr}}{m_{wr}} \tag{159}$$

$$A_{8,4} = \frac{c_{sr}}{m_{wr}} \tag{160}$$

$$A_{8,7} = \frac{-k_{sr}-k_{wr}}{m_{wr}} \tag{161}$$

$$A_{8,8} = \frac{-c_{sr}-c_{wr}}{m_{wr}} \tag{162}$$

$$A_{8,11} = \frac{k_{sr}*t}{m_{wr}} \tag{163}$$

$$A_{8,12} = \frac{c_{sr}*t}{m_{wr}} \tag{164}$$

$$A_{8,1} = A_{8,2} = A_{8,5} = A_{8,6} = A_{8,9} = A_{8,10} = 0 \tag{165}$$

For the A matrix, the elements of ninth row are:

$$A_{9,1} = A_{9,2} = A_{9,3} = A_{9,4} = A_{9,5} = A_{9,6} = A_{9,7} = A_{9,8} = A_{9,9} = A_{9,11} = A_{9,12} = 0 \tag{166}$$

$$A_{9,10} = 1 \tag{167}$$

For the A matrix, the elements of tenth row are:

$$A_{10,1} = \frac{k_{cf}-k_{cr}}{I_c} \tag{168}$$

$$A_{10,2} = \frac{c_{cf}-c_{cr}}{I_c} \tag{169}$$

$$A_{10,3} = \frac{-k_{cf}+k_{cr}}{I_c} \tag{170}$$

$$A_{10,4} = \frac{-c_{cf}+c_{cr}}{I_c} \tag{171}$$

$$A_{10,9} = \frac{-k_{cf}*r-k_{cr}*p}{I_c} \tag{172}$$

$$A_{10,10} = \frac{-c_{cf}*r-c_{cr}*p}{I_c} \tag{173}$$

$$A_{10,11} = \frac{k_{cf}(r+d+s)-k_{cr}*(d+s-p)}{I_c} \tag{174}$$

$$A_{10,12} = \frac{c_{cf}*(r+d+s)-c_{cr}*(d+s-p)}{I_c} \tag{175}$$

For the A matrix, the elements of eleventh row are:

$$A_{11,1} = A_{11,2} = A_{11,3} = A_{11,4} = A_{11,5} = A_{11,6} = A_{11,7} = A_{11,8} = A_{11,9} = A_{11,10} = A_{11,11} = 0 \tag{176}$$

$$A_{11,12} = 1 \tag{177}$$

For the A matrix, the elements of twelfth row are:

$$A_{12,1} = \frac{-k_{cf}-k_{cr}}{I_s} \tag{178}$$
$$A_{12,2} = \frac{-c_{cf}-c_{cr}}{I_s} \tag{179}$$
$$A_{12,3} = \frac{k_{cf}+k_{cr}+k_{sf}-k_{sr}}{I_s} \tag{180}$$
$$A_{12,4} = \frac{c_{cf}+c_{cr}+c_{sf}-c_{sr}}{I_s} \tag{181}$$
$$A_{12,5} = -\frac{k_{sf}}{I_s} \tag{182}$$
$$A_{12,6} = -\frac{c_{sf}}{I_s} \tag{183}$$
$$A_{12,7} = \frac{k_{sr}}{I_s} \tag{184}$$
$$A_{12,8} = \frac{c_{sr}}{I_s} \tag{185}$$
$$A_{12,9} = \frac{k_{cf}*r-k_{cr}*p}{I_s} \tag{186}$$
$$A_{12,10} = \frac{c_{cf}*r-c_{cr}*p}{I_s} \tag{187}$$
$$A_{12,11} = \frac{-k_{cf}*(r+d+s)-k_{cr}*(d+s-p)-k_{sf}*s-k_{sr}*t}{I_s} \tag{188}$$
$$A_{12,12} = \frac{-c_{cf}*(r+d+s)-c_{cr}*(d+s-p)-c_{sf}*s-c_{sr}*t}{I_s} \tag{189}$$

For the B matrix, the four elements are shown in below:

$$B_{6,1} = \frac{k_{wf}}{m_{wf}} \tag{190}$$
$$B_{6,2} = \frac{c_{wf}}{m_{wf}} \tag{191}$$
$$B_{8,3} = \frac{k_{wr}}{m_{wr}} \tag{192}$$
$$B_{8,4} = \frac{c_{wr}}{m_{wr}} \tag{193}$$

For the B matrix, rest of the element are zero.

$$B_{1,1} = B_{1,2} = B_{1,3} = B_{1,4} = 0 \tag{194}$$
$$B_{2,1} = B_{2,2} = B_{2,3} = B_{2,4} = 0 \tag{195}$$
$$B_{3,1} = B_{3,2} = B_{3,3} = B_{3,4} = 0 \tag{196}$$
$$B_{4,1} = B_{4,2} = B_{4,3} = B_{4,4} = 0 \tag{197}$$
$$B_{5,1} = B_{5,2} = B_{5,3} = B_{5,4} = 0 \tag{198}$$
$$B_{6,3} = B_{6,4} = 0 \tag{199}$$
$$B_{7,1} = B_{7,2} = B_{7,3} = B_{7,4} = 0 \tag{200}$$
$$B_{8,1} = B_{8,2} = 0 \tag{201}$$
$$B_{9,1} = B_{9,2} = B_{9,3} = B_{9,4} = 0 \tag{202}$$
$$B_{10,1} = B_{10,2} = B_{10,3} = B_{10,4} = 0 \tag{203}$$
$$B_{11,1} = B_{11,2} = B_{11,3} = B_{11,4} = 0 \tag{204}$$
$$B_{12,1} = B_{12,2} = B_{12,3} = B_{12,4} = 0 \tag{205}$$

For the C matrix, the elements of first row are:

$$C_{1,2} = C_{1,3} = C_{1,4} = C_{1,5} = C_{1,6} = C_{1,7} = C_{1,8} = C_{1,9} = C_{1,10} = C_{1,11} = C_{1,12} = 0 \quad (206)$$
$$C_{1,1} = 1 \quad (207)$$

For the C matrix, the elements of second row are:

$$C_{2,1} = \frac{-k_{cf}-k_{cr}}{m_c} \quad (208)$$
$$C_{2,2} = \frac{-c_{cf}-c_{cr}}{m_c} \quad (209)$$
$$C_{2,3} = \frac{k_{cf}+k_{cr}}{m_c} \quad (210)$$
$$C_{2,4} = \frac{c_{cf}+c_{cr}}{m_c} \quad (211)$$
$$C_{2,9} = \frac{k_{cf}*r-k_{cr}*p}{m_c} \quad (212)$$
$$C_{2,10} = \frac{c_{cf}*r-c_{cr}*p}{m_c} \quad (213)$$
$$C_{2,11} = \frac{-k_{cf}(r+d+s)-k_{cr}*(d+s-p)}{m_c} \quad (214)$$
$$C_{2,12} = \frac{-c_{cf}*(r+d+s)-c_{cr}*(d+s-p)}{m_c} \quad (215)$$
$$C_{2,5} = C_{2,6} = C_{2,7} = C_{2,8} = 0 \quad (216)$$

For the C matrix, the elements of third row are:

$$C_{3,1} = C_{3,2} = C_{3,4} = C_{3,5} = C_{3,6} = C_{3,7} = C_{3,8} = C_{3,9} = C_{3,10} = C_{3,11} = C_{3,12} = 0 \quad (217)$$
$$C_{3,3} = 1 \quad (218)$$

For the C matrix, the elements of fourth row are:

$$C_{4,1} = \frac{k_{cf}+k_{cr}}{m_s} \quad (219)$$
$$C_{4,2} = \frac{c_{cf}+c_{cr}}{m_s} \quad (220)$$
$$C_{4,3} = \frac{-k_{cf}-k_{cr}-k_{sf}-k_{sr}}{m_s} \quad (221)$$
$$C_{4,4} = \frac{-c_{cf}-c_{cr}-c_{sf}-c_{sr}}{m_s} \quad (222)$$
$$C_{4,5} = \frac{k_{sf}}{m_s} \quad (223)$$
$$C_{4,6} = \frac{c_{sf}}{m_s} \quad (224)$$
$$C_{4,7} = \frac{k_{sr}}{m_s} \quad (225)$$
$$C_{4,8} = \frac{c_{sr}}{m_s} \quad (226)$$
$$C_{4,9} = \frac{-k_{cf}*r+k_{cr}*p}{m_s} \quad (227)$$
$$C_{4,10} = \frac{-c_{cf}*r+c_{cr}*p}{m_s} \quad (228)$$
$$C_{4,11} = \frac{k_{cf}*(r+d+s)+k_{cr}*(d+s-p)+k_{sf}*s-k_{sr}*t}{m_s} \quad (229)$$

$$C_{4,12} = \frac{c_{cf}*(r+d+s)+c_{cr}*(d+s-p)+c_{sf}*s-c_{sr}*t}{m_s} \tag{230}$$

For the D matrix, all elements are zero:

$$D_{1,1} = D_{1,2} = D_{1,3} = D_{1,4} = 0 \tag{231}$$
$$D_{2,1} = D_{2,2} = D_{2,3} = D_{2,4} = 0 \tag{232}$$
$$D_{3,1} = D_{3,2} = D_{3,3} = D_{3,4} = 0 \tag{233}$$
$$D_{4,1} = D_{4,2} = D_{4,3} = D_{4,4} = 0 \tag{234}$$

2.4. Modeling of the System in Simulink

The definitions of A, B, C and D matrices are done. Now simulation of the system can be done with the Simulink as shown in below system (Kluever, 2015).

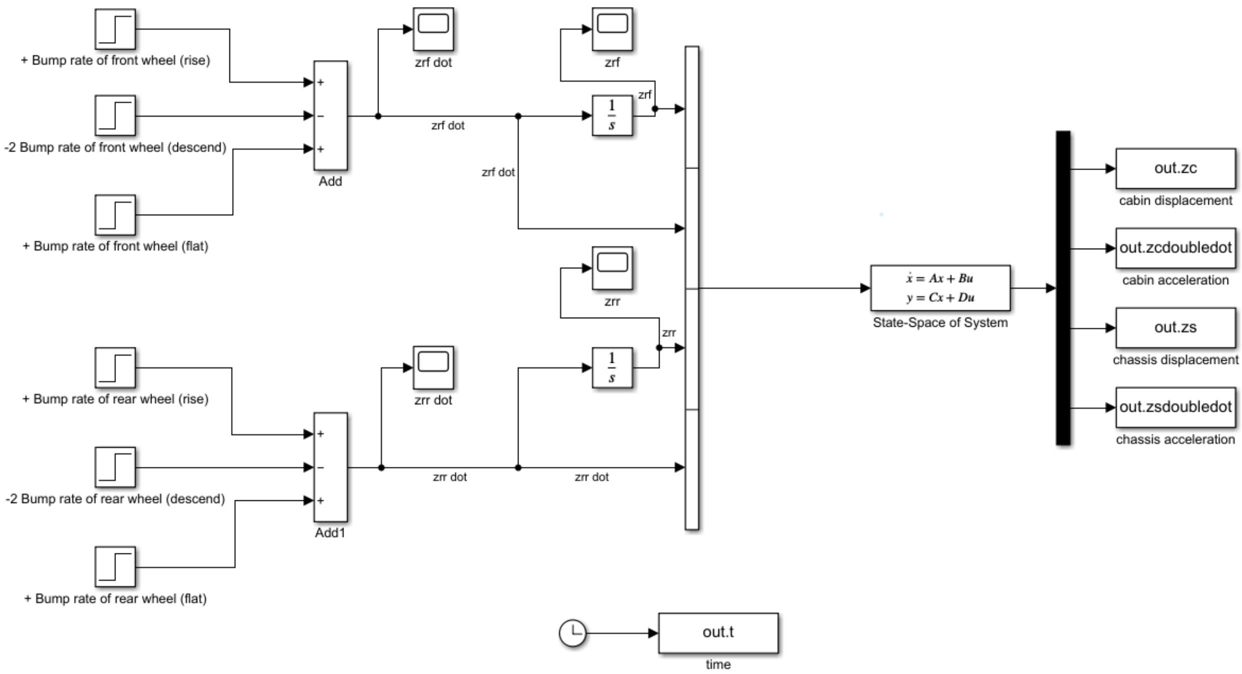

Figure 2. Simulink Model of the System

Parameters for the truck cab suspension system are shown in below table (Stan & Iozsa, 2019).

Table 1. Parameters of the System

Truck Cab Suspension System Model Fixed Parameters			
Parameter	Value	Parameter	Value
m_c	1300 (kg)	$c_{wf} = c_{wr}$	$2*10^3$ (N/m)
m_s	2050 (kg)	I_c	1100 (kg*m^2)
$m_{wf} = m_{wr}$	500 (kg)	I_s	7000 (kg*m^2)
k_{sf}	$17*10^4$ (N/m)	p	1.04 (m)
k_{sr}	$15*10^4$ (N/m)	r	1.05 (m)
c_{sf}	$75*10^2$ (N*s/m)	s	1.46 (m)
c_{sr}	$45*10^2$ (N*s/m)	t	2.34 (m)
k_{wf}	$1.2*10^6$ (N/m)	d	0.15 (m)
k_{wr}	$1.1*10^6$ (N/m)		

Analysis of the system can be done with the bump input. Velocity of the truck can be assumed as 10 km/h (almost 2.777 m/s). First front wheels will be affected from the bump and after a while rear wheels will be affected. The necessary time between the front and rear wheels with the 2.777 m/s can be calculated as shown:

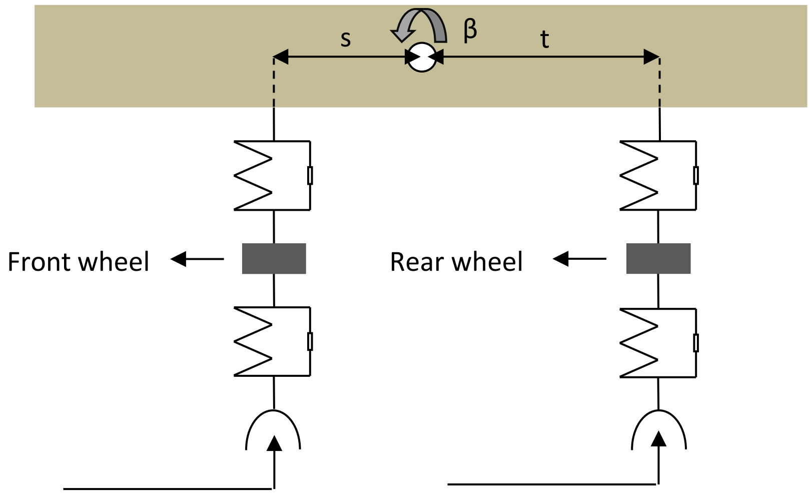

Figure 3. Model of the Half Truck

$$s + t = 1.46 + 2.34 = 3.8 \text{ m} \quad (235)$$

$$\Delta t = \frac{3.8 \text{ m}}{2.777 \text{ m}/\text{s}} = 1.368 \text{ s} \quad (236)$$

The necessary time between the front and rear wheels with the 2.777 m/s is 1.368 s. There are two input in the truck cab suspension system. These are front wheel and rear wheel. Triangular pulse input will be used for each of the front and rear wheel. In the triangular pulse input, first input will be considered as constant vertical rising input until it reaches its peak value. Second input will be considered as constant vertical descending input after the pulse reaches its peak value. In the third input, it is assumed that the effects of the bump have passed. So, the total input should be zero after the bump effect passes (Kluever, 2015).

It is assumed that the truck encounters a raised surface, leading to an instantaneous change in vertical displacement of its wheels. The instantaneous change is caused by the vertical displacement input with 20 cm (0.2 m) and the vertical velocity input with 10 km/h (2.777 m/s) with a starting time of 0.5 s. The necessary time for the total displacement of the wheels are zero again can be calculated with below equation.

$$\Delta T = \frac{z_{max}}{\dot{z_0}} = \frac{0.2 \text{ m}}{2.777 \text{ m}/\text{s}} = 0.072 \text{ s} \tag{237}$$

Now, three input of the triangular pulse input can be given. As mentioned before, first input is rising input, second input is descending input and third input is fixed input.
First input of the front wheel should be applied with an initial positive bump rate of 2.777 m/s by 0.5 s step time. Incremental input is given with this way.

Table 2. First Input of the Front Wheel

Step time	0.5
Final value	2.777

Second input of the front wheel should be applied with an initial negative bump rate of 5.554 m/s by 0.572 s step time. Bump rate is doubled with the negative sign because 2.777 m/s incremental input should be converted to the 2.777 m/s decremental input. And step time is increased with a magnitude of 0.072 s because it starts to rising at 0.5 s and it will start to descending after 0.072 s. Decremental input is given with this way.

Table 3. Second Input of the Front Wheel

Step time	0.572
Final value	5.444

In the third input, total input should be equal to zero when the bump affects are passes. Bump affects are passes when descending is done. So, step time will be increased with a magnitude of 0.072 s. Because of this, third input of the front wheel should be applied with an initial positive bump rate of 2.777 m/s by 0.644 s step time. Fixed input is given with this way.

Table 4. Third Input of the Front Wheel

Step time	0.644
Final value	2.777

First input of the rear wheel should be applied with an initial positive bump rate of 2.777 m/s by 1.868 s step time. Step time is increased with a magnitude of 1.368 s because necessary time between the front and rear wheels is 1.368 s. Starting time was 0.5 s, so if we sum 0.5 s and 1.368 s, step time is going to be 1.868 s. Incremental input is given with this way.

Table 5. First Input of the Rear Wheel

Step time	1.868
Final value	2.777

Second input of the rear wheel should be applied with an initial negative bump rate of 5.554 m/s by 1.94 step time. Bump rate is doubled with the negative sign because 2.777 m/s incremental input should be converted to the 2.777 m/s decremental input. And step time is increased with a magnitude of 0.072 s because it starts to rising at 1.868 s and it will start to descending after 0.072 s. Decremental input is given with this way.

Table 6. Second Input of the Rear Wheel

Step time	1.94
Final value	5.444

In the third input, total input should be equal to zero when the bump affects are passes. Bump affects are passes when descending is done. So, step time will be increased with a magnitude of 0.072 s. Because of this, third input of the rear wheel should be applied with an initial positive bump rate of 2.777 m/s by 2.012 step time. Fixed input is given with this way.

Table 7. Third Input of the Rear Wheel

Step time	2.012
Final value	2.777

Simulation is almost ready to run with the truck cab suspension system fixed parameters that given in the table. Only four variables are missing and these are k_{cf}, k_{cr}, c_{cf}, c_{cr}. If k_{cf}, k_{cr}, c_{cf} and c_{cr} are determined, the simulation will be ready to run.

The main goal of this project is maximizing the comfort in a safe zone for the driver. As mentioned before, increasing comfort so much is a danger for driver. In long trips, if driver will not feel anything on the road it makes him sleepy. The comfort level of the driver is associated with these four missing variables (k_{cf}, k_{cr}, c_{cf}, c_{cr}). After some research, minimum and maximum stiffness, and damper parameters in safe zone for driver are founded for the truck cab suspension system. The iteration process will be done with evenly spaced points for each stiffness and damper parameters. Values that maximize the comfort level are selected in this region. Comfort is maximized for driver when the root mean square (rms) acceleration difference between the cabin and chassis is maximum (Sharma, Pare, Chouksey, & Rawal, 2016). Rms is known as quadratic mean of a variable set and calculated with the below formula. It is easy to calculate rms value of a variable set on MATLAB with just one "rms" command (Chai & Draxler, 2014).

$$x_{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} \qquad (238)$$

The range of the stiffness and damper parameters are shown in below.

Table 8. Range of the Stiffness and Damper Parameters

	Minimum	Number of Variable	Maximum
$k_{cf} = k_{cr}$	30000	20	120000
$c_{cf} = c_{cr}$	1500	20	12000

3. RESULT AND DISCUSSION

The aim of this study is to increase the comfort in the truck cab by trying values for the truck cab suspension elements. The performance of the suspension elements was analyzed by modeling in Simulink environment, considering that they pass over the bump. After modeling in Simulink, iteration process for possible suspension parameters done by MATLAB code that given in the appendix part A. This code calculates the root mean square (RMS) acceleration in cabin and chassis and difference between the cabin and chassis. RMS acceleration values, stiffness parameters and damper parameters are plotted.

The RMS acceleration values for cabin from the iteration is shown in Figure 4.

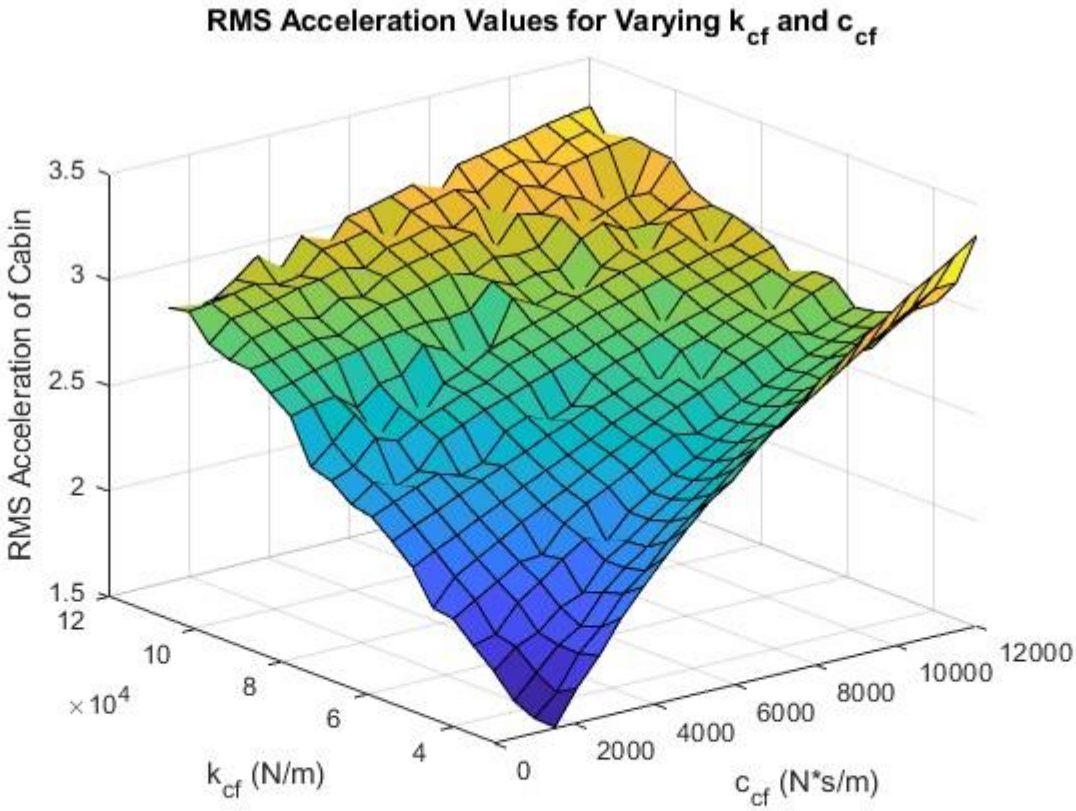

Figure 4. RMS Acceleration Values of Cabin for Varying k_{cf} and c_{cf}

The RMS acceleration values for chassis from the iteration is shown in Figure 5.

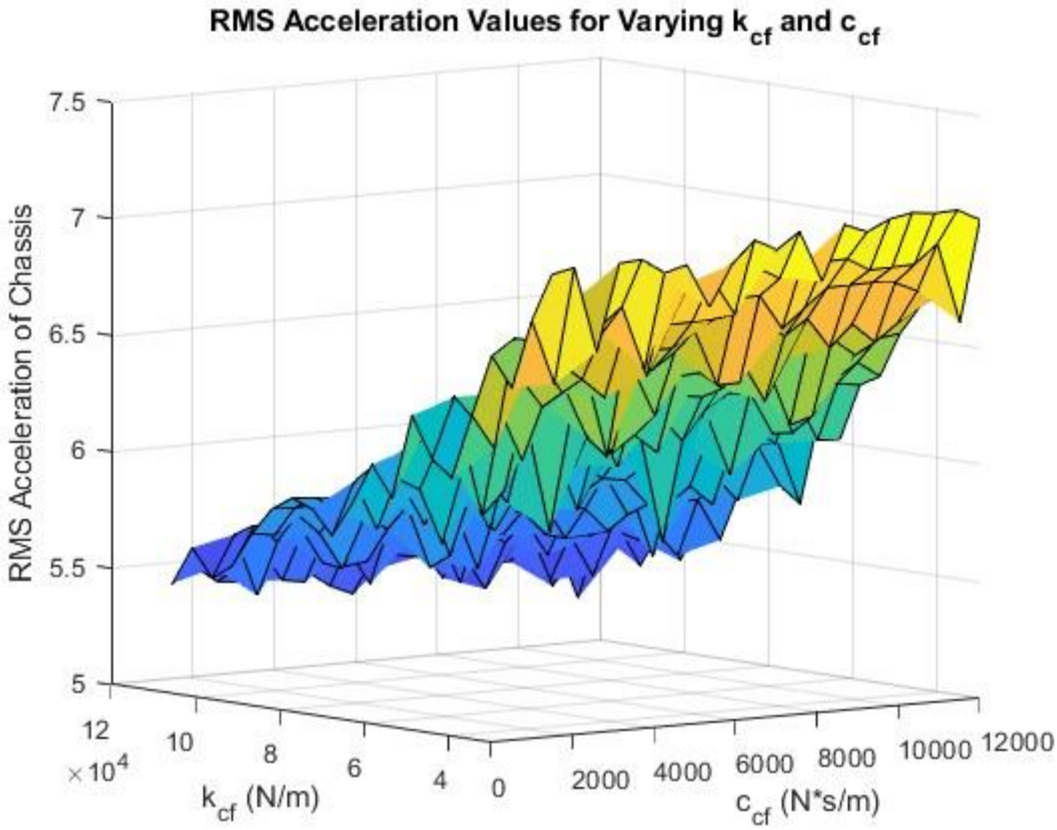

Figure 5. RMS Acceleration Values of Chassis for Varying k_{cf} and c_{cf}

The RMS acceleration difference between cabin and chassis from the iteration is shown in Figure 6.

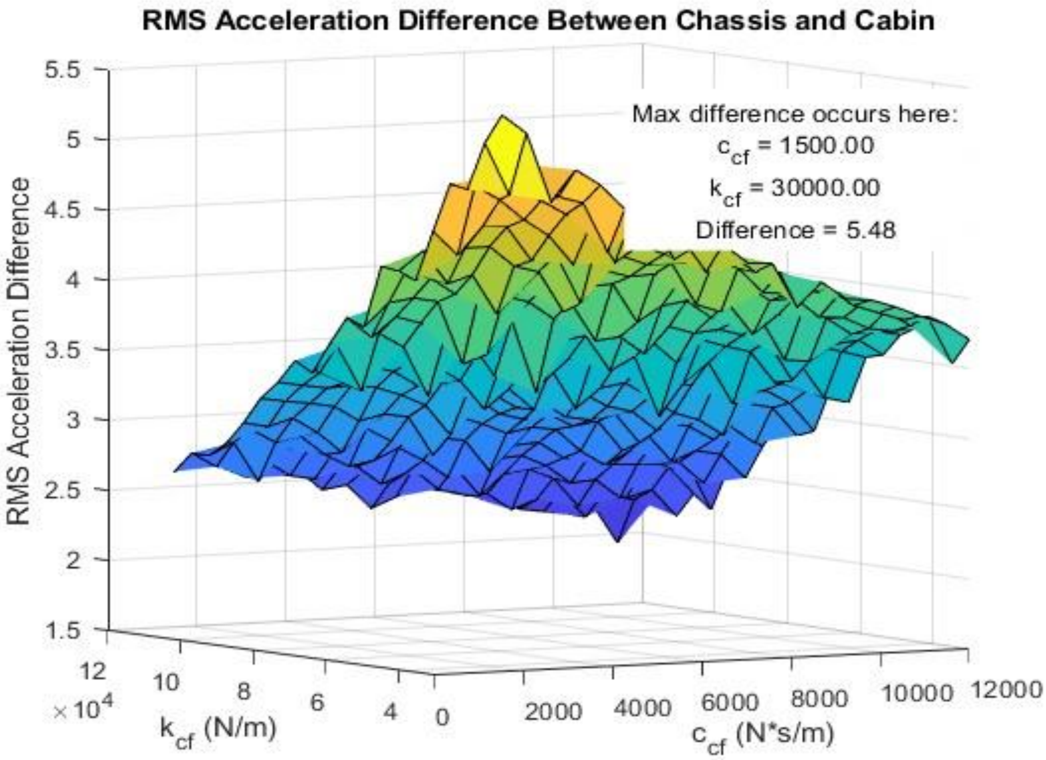

Figure 6. RMS Acceleration Difference Between the Cabin and Chassis for Varying k_{cf} and c_{cf}

From this graph, it can be seen that the maximum rms acceleration difference between the cabin and chassis is 5.48 m/s^2 with the stiffness parameter of 30000 N/m and damper parameter of 1500 N*s/m.

So, every parameter is known for the simulation of the system now. Simulation can be started with these parameters. Displacement vs time and acceleration vs time graphs for each cabin and chassis can be plotted with the MATLAB code that given in the appendices B. Results are shown in Figure 7.

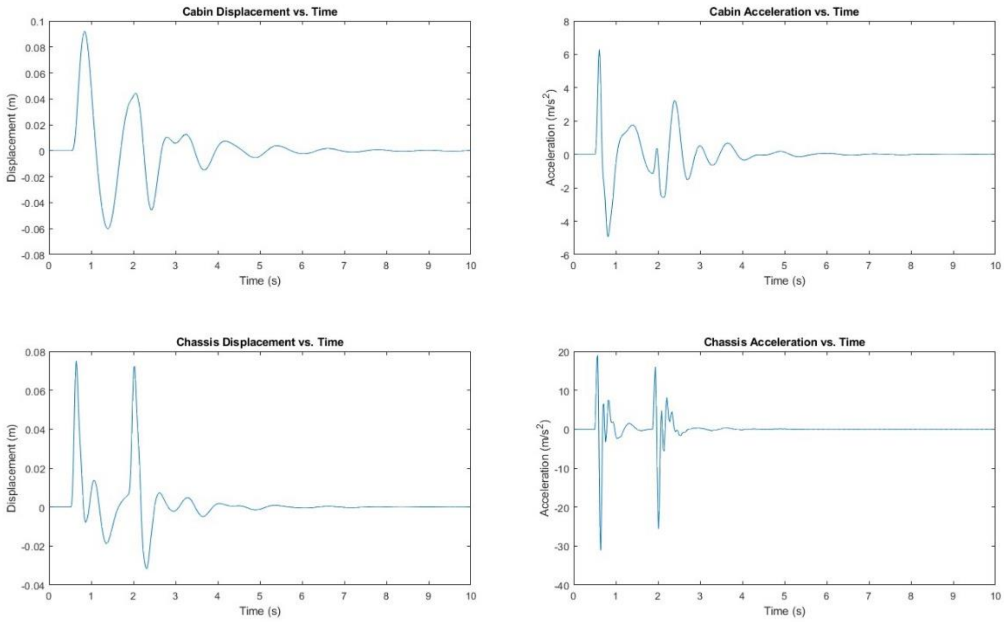

Figure 7. Displacement and Acceleration vs. Time Graphs for Cabin and Chassis

Acceleration graphs were combined into a single graph as shown in Figure 8 for comparison purposes.

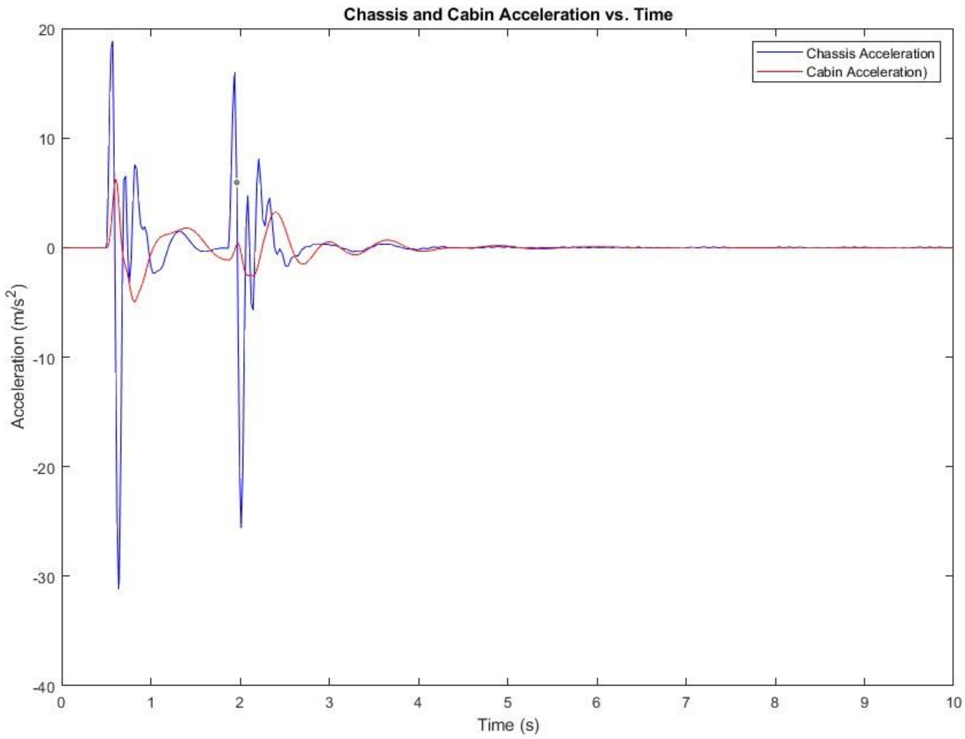

Figure 8. Acceleration vs. Time Comparison Graph between Cabin and Chassis

The aim of this study is to increase comfort by maximizing this difference. As seen in the graphs, the vertical acceleration for the cabin is lower than the vertical acceleration for the chassis. Therefore, it was predicted that comfort is increased. As a result of the studies, it was seen that minimum values in possible range of cabin and stiffness parameter should be taken in order to maximize the difference in the range given for the cabin suspension elements.

4. CONCLUSION

The aim of the project was to examine the cabin suspension elements to increase comfort at a safe zone. Design criteria was determined for increasing the comfort of the driver. This design criterion is based on maximizing the RMS acceleration difference between cabin and chassis. For this purpose, firstly, the system is modeled using the Rayleigh dissipation function in 6 degrees of freedom in Lagrange's form. Separate equations for each degree of freedom were obtained and converted to steady state representation(SSR) format and modeled in Simulink environment. After the modeling was completed, parameters were determined in the safe zone to analyze how the cabin suspension parameters would affect comfort. For all combinations of values in this range, iteration was done on MATLAB and the output data were examined. As a result of these examinations, the cabin parameters were determined by predicting that the values that maximize the RMS acceleration difference are the values that keep the comfort at the highest level. After all the values were determined, the acceleration graphs were compared and it was observed how much the driver comfort increased due to the cabin suspension parameters. In conclusion, it has been observed that the driver comfort is maximized by choosing the minimum values of the suspension elements in the safe zone.

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APPENDICES

A. Stiffness and Damper Parameters Determination Code

```
    clear all;
    clc;
    mc = 1300;  %mass of cabin (kg)
    ms = 2050;  %mass of chassis (kg)
    Ic = 1100;  %inertia moment of cab (kg*m^2)
    Is = 7000;  %inertia moment of chassis (kgm^2)
    mwf = 500;  %mass of front wear and axle (kg)
    mwr = 500;  %mass of rear wear and axle (kg)
    r = 1.05;   %distance of front cabin suspension elements from the cabin's
center of gravity (m)
    p = 1.04;   %distance of rear cabin suspension elements from the cabin's
center of gravity (m)
    d = 0.15;   %the longitudinal position of front chassis suspension to the
center of gravity of cab (m)
    s = 1.46;   %distance of front suspension elements of the chassis from the
chassis  center of gravity (m)
    t = 2.34;   %distance of rear suspension elements of the chassis from the
chassis  center of gravity (m)
    %%kcf = ...........; %front stiffness parameter of cabin (N/m)
    %%kcr = ...........;  %rear stiffness parameter of cabin (N/m)
    ksf = 17*10^4;  %front stiffness parameter of chassis (N/m)
    ksr = 15*10^4;  %rear stiffness parameter of chassis (N/m)
    kwf = 1.2*10^6; %front stiffness parameter of wheel (N/m)
    kwr = 1.1*10^6; %rear stiffness parameter of wheel (N/m)
    %%ccf = ..........;    %front damper parameter of cabin (N*s/m)
    %%ccr = ..........;  %rear damper parameter of cabin (N*s/m)
    csf = 75*10^2;  %front damper parameter of chassis (N*s/m)
    csr = 45*10^2;  %rear damper parameter of chassis (N*s/m)
    cwf = 2*10^3;   %front damper parameter of wheel (N*s/m)
    cwr = 2*10^3;   %rear damper parameter of wheel (N*s/m)
    %Range of the stiffness and damper parameters
    kcf_trial = linspace(30000,120000,20);
    ccf_trial = linspace(1500,12000,20);
    %Vector identification of rms
    rms_cabin = zeros(length(kcf_trial), length(ccf_trial));
    rms_chassis = zeros(length(kcf_trial), length(ccf_trial));
    %Iteration Part
    for i = 1:length(kcf_trial)
        for j = 1:length(ccf_trial)
            kcf = kcf_trial(i);
            ccf = ccf_trial(j);
            kcr = kcf;
            ccr = ccf;
            A = [0 1 0 0 0 0 0 0 0 0 0 0;
                (-kcf-kcr)/mc (-ccf-ccr)/mc (kcf+kcr)/mc (ccf+ccr)/mc 0 0 0 0
(kcf*r-kcr*p)/mc (ccf*r-ccr*p)/mc (-kcf*(r+d+s)-kcr*(d+s-p))/mc (-ccf*(r+d+s)-
ccr*(d+s-p))/mc;
                0 0 0 1 0 0 0 0 0 0 0 0;
                (kcf+kcr)/ms (ccf+ccr)/ms (-kcf-kcr-ksf-ksr)/ms (-ccf-ccr-csf-
csr)/ms ksf/ms csf/ms ksr/ms csr/ms (-kcf*r+kcr*p)/ms (-ccf*r+ccr*p)/ms
(kcf*(r+d+s)+kcr*(d+s-p)+ksf*s-ksr*t)/ms  (ccf*(r+d+s)+ccr*(d+s-p)+csf*s-
csr*t)/ms;
                0 0 0 0 0 1 0 0 0 0 0 0;
                0 0 ksf/mwf csf/mwf (-ksf-kwf)/mwf (-csf-cwf)/mwf 0 0 0 0 (-
ksf*s)/mwf (-csf*s)/mwf;
```

```
                0 0 0 0 0 0 0 1 0 0 0 0;
                0 0 ksr/mwr csr/mwr 0 0 (-ksr-kwr)/mwr (-csr-cwr)/mwr 0 0
(ksr*t)/mwr (csr*t)/mwr;
                0 0 0 0 0 0 0 0 0 1 0 0;
                (kcf-kcr)/Ic (ccf-ccr)/Ic (-kcf+kcr)/Ic (-ccf+ccr)/Ic 0 0 0 0
(-kcf*r-kcr*p)/Ic (-ccf*r-ccr*p)/Ic (kcf*(r+d+s)-kcr*(d+s-p))/Ic (ccf*(r+d+s)-
ccr*(d+s-p))/Ic;
                0 0 0 0 0 0 0 0 0 0 0 1;
                (-kcf-kcr)/Is (-ccf-ccr)/Is (kcf+kcr+ksf-ksr)/Is (ccf+ccr+csf-
csr)/Is -ksf/Is -csf/Is ksr/Is csr/Is (kcf*r-kcr*p)/Is (ccf*r-ccr*p)/Is (-
kcf*(r+d+s)-kcr*(d+s-p)-ksf*s-ksr*t)/Is (-ccf*(r+d+s)-ccr*(d+s-p)-csf*s-
csr*t)/Is]
            B = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0; kwf/mwf cwf/mwf 0
0; 0 0 0 0; 0 0 kwr/mwr cwr/mwr; 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]
            C = [1 0 0 0 0 0 0 0 0 0 0 0;
                (-kcf-kcr)/mc (-ccf-ccr)/mc (kcf+kcr)/mc (ccf+ccr)/mc 0 0 0 0
(kcf*r-kcr*p)/mc (ccf*r-ccr*p)/mc (-kcf*(r+d+s)-kcr*(d+s-p))/mc (-ccf*(r+d+s)-
ccr*(d+s-p))/mc;
                0 0 1 0 0 0 0 0 0 0 0 0;
                (kcf+kcr)/ms (ccf+ccr)/ms (-kcf-kcr-ksf-ksr)/ms (-ccf-ccr-csf-
csr)/ms ksf/ms csf/ms ksr/ms csr/ms (-kcf*r+kcr*p)/ms (-ccf*r+ccr*p)/ms
(kcf*(r+d+s)+kcr*(d+s-p)+ksf*s-ksr*t)/ms  (ccf*(r+d+s)+ccr*(d+s-p)+csf*s-
csr*t)/ms]
            D = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]
            out = sim("Truck_suspension_system_modeling.slx")
            time = out.t.get.Data;
            zcdoubledot = out.zcdoubledot.get.Data;
            zsdoubledot = out.zsdoubledot.get.Data;
            rms_cabin(i,j) = rms(zcdoubledot);
            rms_chassis(i,j) = rms(zsdoubledot)
        end
    end

figure;
surf(ccf_trial,kcf_trial,rms_cabin');
xlabel('c_c_f (N*s/m)');
ylabel('k_c_f (N/m)');
zlabel('RMS Acceleration of Cabin');
title('RMS Acceleration Values for Varying k_c_f and c_c_f');

figure;
surf(ccf_trial,kcf_trial,rms_chassis');
xlabel('c_c_f (N*s/m)');
ylabel('k_c_f (N/m)');
zlabel('RMS Acceleration of Chassis');
title('RMS Acceleration Values for Varying k_c_f and c_c_f');

difference = (rms_chassis - rms_cabin);
% find the maximum value and its indices
[max_diff, max_ind] = max(difference(:));
[ccf_ind, kcf_ind] = ind2sub(size(difference), max_ind);

figure;
surf(ccf_trial,kcf_trial,difference');
xlabel('c_c_f (N*s/m)');
ylabel('k_c_f (N/m)');
zlabel('RMS Acceleration Difference');
title('RMS Acceleration Difference Between Chassis and Cabin');
```

```
% add text to the figure
text(ccf_trial(ccf_ind), kcf_trial(kcf_ind), max_diff, sprintf('Max difference 
occurs here:\nc_c_f = %.2f\nk_c_f = %.2f\nDifference = 
%.2f',ccf_trial(ccf_ind), kcf_trial(kcf_ind), max_diff), 'HorizontalAlignment', 
'center', 'BackgroundColor', 'w','Units', 'normalized', 'Position', [0.8 0.8 
0]);
```

B. Output of the Modeling Code

```
    clear all;
    clc;
    mc = 1300;  %mass of cabin (kg)
    ms = 2050;  %mass of chassis (kg)
    Ic = 1100;  %inertia moment of cab (kg*m^2)
    Is = 7000;  %inertia moment of chassis (kg*m^2)
    mwf = 500;  %mass of fro    nt wear and axle (kg)
    mwr = 500;  %mass of rear wear and axle (kg)
    r = 1.05;   %distance of front cabin suspension elements from the cabin's 
center of gravity (m)
    p = 1.04;   %distance of rear cabin suspension elements from the cabin's 
center of gravity (m)
    d = 0.15;   %the longitudinal position of front chassis suspension to the 
center of gravity of cab (m)
    s = 1.46;   %distance of front suspension elements of the chassis from the 
chassis  center of gravity (m)
    t = 2.34;   %distance of rear suspension elements of the chassis from the 
chassis  center of gravity (m)
    kcf = 30000; %front stiffness parameter of cabin (N/m)
    kcr = 30000;  %rear stiffness parameter of cabin (N/m)
    ksf = 17*10^4;  %front stiffness parameter of chassis (N/m)
    ksr = 15*10^4;  %rear stiffness parameter of chassis (N/m)
    kwf = 1.2*10^6; %front stiffness parameter of wheel (N/m)
    kwr = 1.1*10^6; %rear stiffness parameter of wheel (N/m)
    ccf = 1500;    %front damper parameter of cabin (N*s/m)
    ccr = 1500;  %rear damper parameter of cabin (N*s/m)
    csf = 75*10^2;  %front damper parameter of chassis (N*s/m)
    csr = 45*10^2;  %rear damper parameter of chassis (N*s/m)
    cwf = 2*10^3;   %front damper parameter of wheel (N*s/m)
    cwr = 2*10^3;   %rear damper parameter of wheel (N*s/m)

A = [0 1 0 0 0 0 0 0 0 0 0 0;
    (-kcf-kcr)/mc (-ccf-ccr)/mc (kcf+kcr)/mc (ccf+ccr)/mc 0 0 0 0 (kcf*r-
kcr*p)/mc (ccf*r-ccr*p)/mc (-kcf*(r+d+s)-kcr*(d+s-p))/mc (-ccf*(r+d+s)-
ccr*(d+s-p))/mc;
    0 0 0 1 0 0 0 0 0 0 0 0;
    (kcf+kcr)/ms (ccf+ccr)/ms (-kcf-kcr-ksf-ksr)/ms (-ccf-ccr-csf-csr)/ms 
ksf/ms csf/ms ksr/ms csr/ms (-kcf*r+kcr*p)/ms (-ccf*r+ccr*p)/ms 
(kcf*(r+d+s)+kcr*(d+s-p)+ksf*s-ksr*t)/ms  (ccf*(r+d+s)+ccr*(d+s-p)+csf*s-
csr*t)/ms;
    0 0 0 0 0 1 0 0 0 0 0 0;
    0 0 ksf/mwf csf/mwf (-ksf-kwf)/mwf (-csf-cwf)/mwf 0 0 0 0 (-ksf*s)/mwf (-
csf*s)/mwf;
    0 0 0 0 0 0 0 1 0 0 0 0;
    0 0 ksr/mwr csr/mwr 0 0 (-ksr-kwr)/mwr (-csr-cwr)/mwr 0 0 (ksr*t)/mwr 
(csr*t)/mwr;
    0 0 0 0 0 0 0 0 0 1 0 0;
```

```
    (kcf-kcr)/Ic (ccf-ccr)/Ic (-kcf+kcr)/Ic (-ccf+ccr)/Ic 0 0 0 0 (-kcf*r-
kcr*p)/Ic (-ccf*r-ccr*p)/Ic (kcf*(r+d+s)-kcr*(d+s-p))/Ic (ccf*(r+d+s)-ccr*(d+s-
p))/Ic;
    0 0 0 0 0 0 0 0 0 0 0 1;
    (-kcf-kcr)/Is (-ccf-ccr)/Is (kcf+kcr+ksf-ksr)/Is (ccf+ccr+csf-csr)/Is -
ksf/Is -csf/Is ksr/Is csr/Is (kcf*r-kcr*p)/Is (ccf*r-ccr*p)/Is (-kcf*(r+d+s)-
kcr*(d+s-p)-ksf*s-ksr*t)/Is (-ccf*(r+d+s)-ccr*(d+s-p)-csf*s-csr*t)/Is]
B = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0; kwf/mwf cwf/mwf 0 0; 0 0 0 0;
0 0 kwr/mwr cwr/mwr; 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]
C = [1 0 0 0 0 0 0 0 0 0 0 0;
    (-kcf-kcr)/mc (-ccf-ccr)/mc (kcf+kcr)/mc (ccf+ccr)/mc 0 0 0 0 (kcf*r-
kcr*p)/mc (ccf*r-ccr*p)/mc (-kcf*(r+d+s)-kcr*(d+s-p))/mc (-ccf*(r+d+s)-
ccr*(d+s-p))/mc;
    0 0 1 0 0 0 0 0 0 0 0 0;
    (kcf+kcr)/ms (ccf+ccr)/ms (-kcf-kcr-ksf-ksr)/ms (-ccf-ccr-csf-csr)/ms
ksf/ms csf/ms ksr/ms csr/ms (-kcf*r+kcr*p)/ms (-ccf*r+ccr*p)/ms
(kcf*(r+d+s)+kcr*(d+s-p)+ksf*s-ksr*t)/ms  (ccf*(r+d+s)+ccr*(d+s-p)+csf*s-
csr*t)/ms]
D = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]
    out = sim("Truck_suspension_system_modeling.slx")
    time = out.t.get.Data;
zc = out.zc.get.Data;
zcdoubledot = out.zcdoubledot.get.Data;
zs = out.zs.get.Data;
zsdoubledot = out.zsdoubledot.get.Data;
f = figure;
f.Position = [250 75 1500 900];
t = tiledlayout(2,2);
plot1 = nexttile;
plot(plot1,time,zc)
title(plot1,'Cabin Displacement vs. Time')
xlabel(plot1,'Time (s)')
ylabel(plot1,'Displacement (m)')
plot2 = nexttile;
plot(plot2,time,zcdoubledot)
a = rms(zcdoubledot);
hold on
plot(plot2,time,a)
title(plot2,'Cabin Acceleration vs. Time')
xlabel(plot2,'Time (s)')
ylabel(plot2,'Acceleration (m/s^2)')
plot3 = nexttile;
plot(plot3,time,zs)
title(plot3,'Chassis Displacement vs. Time')
xlabel(plot3,'Time (s)')
ylabel(plot3,'Displacement (m)')
plot4 = nexttile;
plot(plot4,time,zsdoubledot)
title(plot4,'Chassis Acceleration vs. Time')
xlabel(plot4,'Time (s)')
ylabel(plot4,'Acceleration (m/s^2)')
hold off

plot5 = figure;
plot5.Position = [500 200 1000 700];
plot(time, zsdoubledot, 'b');
title('Chassis and Cabin Acceleration vs. Time');
xlabel('Time (s)');
ylabel('Acceleration (m/s^2)');
```

```
hold on
plot(time, zcdoubledot, 'r');
legend('Chassis Acceleration','Cabin Acceleration)')
```