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Engineering Project

VIBRATION ISOLATION SYSTEM
FOR A COMMERCIAL VEHICLE

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1 INTRODUCTION

In the automotive sector, with the increasing customer expectations, automotive companies have increasingly shifted towards more customer-oriented approaches. High customer expectations now extend beyond passenger vehicles, as commercial vehicles are also expected to provide a level of comfort comparable to passenger cars. This necessitates a focus on product development, particularly in improving comfort, by commercial vehicle manufacturers. The impact of noise, vibration, and motion during travel on the driver and passengers is defined as comfort, or more explicitly, "ride comfort." Comfort, in its literal sense, is defined as being free from discomfort, a state in which a person feels good.

Road irregularities, the unevenness of rotating vehicle components, engine vibrations, cruising movements, and vibrations affecting the driver and passengers are the main sources of ride comfort challenges.

For ride comfort, it is essential to minimize the transfer of energy generated by road roughness and shocks between the road and the wheels to the driver/passengers. Part of this energy is absorbed by shock absorbers and springs, and the remaining energy needs to be transmitted to the vehicle cabin at a frequency that can be easily tolerated. The term 'suspension' refers to all components placed between the transported passengers or cargo and the road surface. This term includes highly elastic springs, shock absorbers, tires, and seats, along with wheels and chassis that change shape due to load and impact effects.

Different suspension systems are employed at various points in vehicles to prevent or reduce the transmission of road-induced vibrations to the driver. These are generally applied at three points: between the vehicle body and the wheels, between the vehicle body and the driver's cabin, and between the vehicle body and the driver's seat.

In passenger vehicles, advanced models of suspension systems between the road and the body (chassis) are used, while lower-cost commercial vehicles commonly employ passive suspension systems between the road and the body. Therefore, improving the suspension system between the body and the seat in commercial vehicles and further reducing transmitted vibrations to the driver is seen as an economical application. Suspension systems placed between the driver's seat and the vehicle body, known as seat suspension systems, are effectively utilized due to their ability to significantly dampen all vibrations affecting the driver while not restricting the vehicle's mobility, thanks to their simple structures.

Suspension systems have two fundamental tasks: ensuring ride comfort and driving safety. Regarding ride comfort, the goal is to minimize the transmission of vibrations and the forces generated by road surface irregularities to the vehicle body. To achieve this, it is crucial to isolate the wheel from the vehicle body as much as possible. On the other hand, driving safety is defined as ensuring the continuous contact of the vehicle wheel with the road during motion. Enhancing driving safety requires a stiffer suspension mechanism, while ride comfort necessitates a softer suspension system. Therefore, the design of suspension systems depends significantly on the vehicle's task and operating conditions.

In high-end vehicles, active vehicle suspension systems are used to achieve vibration isolation. However, due to their high costs, these systems are not widely preferred in mid-range and entry-level vehicles. As a result, passive suspension systems are commonly used in mid-range and entry-level vehicles. Consequently, the use of next-generation suspension systems for vibration isolation in the driver's seat is considered suitable for dynamic comfort.

Customer expectations for vehicle seats can be categorized into functionality, safety, and comfort. Seats are crucial comfort elements for vehicle drivers and passengers, impacting their health significantly. Comfort is divided into static comfort and dynamic comfort. Static comfort depends on the user's sensations while the vehicle is stationary, and the pressure between the user and the seat determines this comfort variable. The ideal condition for static comfort is when the pressure on the user is minimized. Dynamic comfort, on the other hand, pertains to the user's comfort during driving and is influenced by parameters such as sound, vibration, and stiffness.

Seat suspension systems can generally be categorized as passive, semi-active, and active. Passive suspension systems consist of a fixed-rate damper and spring. Due to design simplicity and low costs, they are the most commonly used suspension systems. However, they prove insufficient in damping vibrations in the 1-5 Hz range experienced by the driver. Active suspension systems are the most effective in damping these vibrations but are not widely chosen due to their high costs and high power consumption during operation. Semi-active suspension systems offer a cost-effective solution, providing damping performance close to active suspension systems at a lower price.

Semi-active seat suspension systems can generate controllable damping forces without the need for moving mechanisms, utilizing an active damper and a simple spring. Generally, magnetorheological (MR) or electrorheological (ER) fluids are used in active damper structures. However, MR dampers developed by the Lord® company have gained widespread preference in commercial applications due to their high yield strength, stable operation at high temperatures, and low power consumption in recent years.

In this section, we analyze a vibration isolation system that improves the ride quality for a commercial vehicle such as a tractor-trailer used for long-distance transportation of freight. Travel over a rough road causes vibrations that are transmitted to the vehicle's cabin floor, and the floor vibrations, $z_0(t)$, are transmitted to the seat mass m1 and ultimately to the driver (mass m_2). A properly designed seat -suspension system will suppress the road vibrations transmitted to the driver. The seat suspension consists of a passive shock absorber, modeled by ideal (linear) damper and stiffness elements b_1 and k_1. Recall that the damping and stiffness of the seat cushion are modeled by ideal elements b_2 and k_2. The goal of this section is to analyze the dynamic response of the seat-suspension system.

2 GENERAL INFORMATION

As with all mechanical systems, the phenomenon of vibration in vehicles poses a challenge for both passengers and the components that make up the vehicle. Since the first moving vehicle in Mannheim in 1886 until today, various suspension systems have been designed and developed to mitigate this vibration problem. These vibrations, adversely affecting vehicle comfort, lead to the inability to meet increasing customer and comfort expectations. Given that commercial vehicles such as minibuses and buses spend nearly 70% of their day on the road, the importance of vehicle comfort for the driver becomes more evident.

In this study, the examination of the dynamic comfort of a commercial vehicle driver's seat is addressed. In this section, fundamental information will be provided first about vibration and its effects.

2.1 VIBRATION

Vibration is the oscillatory motion of a mass around a reference position. In other words, vibration can be expressed as the cyclical movement of a mass around a specific center. Vibration occurs when a mass undergoes oscillatory motion on an elastic element. This system, consisting of mass and an elastic element, is referred to as a vibration system. A simple vibration system is illustrated in Figure 2.1. In the vibration system depicted in Figure 2.1, the mass stores kinetic energy, while the spring stores potential energy. Vibration results from the energy transformation between potential and kinetic energy. During oscillation, a damping element, which absorbs energy from the system, slows down the motion and eventually brings it to a stop; this element is called a damper.

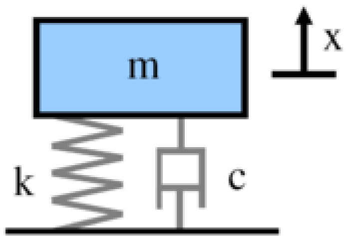


Figure 2.1: Vibration System

The fundamental elements in a vibration system can be defined as follows:

- Elastic Elements (Springs): Springs are elements that connect masses in vibration systems, enabling relative movements between masses.
- Inertial Elements: These elements store kinetic energy. Inertial elements can perform translational and rotational motions separately, or they can execute both translational and rotational motions simultaneously.
- Damping Elements: Damping elements provide energy dissipation in damped systems. Elements like shock absorbers achieve energy loss through fluid friction, exponentially reducing vibration amplitudes. In damping elements, mechanical energy is converted into thermal energy.

Vibrational motion can be classified into two categories: periodic and random vibrations. Periodic vibrational motion is characterized by the property of repeating exactly or partially over a certain period. On the other hand, random vibrational motion does not possess the ability to repeat over time.

In periodic vibrational motion, the period (T), which is the time for one complete cycle, and the frequency (f), representing the number of cycles per second, are key parameters. Mathematically, frequency is the reciprocal of the period and is calculated as follows:

$$f = T^{-1} = \frac{1}{T}$$

The unit of period is second (s), and the unit of frequency is Hertz (Hz). The frequency of the system during frictionless free vibration is called natural frequency. Resonance occurs if the applied external force frequency is equal to the system natural frequency.

Free vibration is the periodic motion that occurs when a system is displaced from its static equilibrium position and then released. The applied forces include the force of the spring, frictional force, and the weight of the mass. In the presence of friction, the vibration decreases over time. Typically, when external forces act on the system in the form of $F(t) = F_0 \sin\omega t$ or $F(t) = F_0 \cos\omega t$, the vibration motion becomes forced vibration. In forced vibration motions, the system is compelled to vibrate not only at its natural frequency but also at the frequency of the applied external force. In the presence of friction, the part of the motion that does not involve the applied sinusoidal external force gradually diminishes over time. Consequently, the system vibrates at the frequency of the applied external force, independent of the initial conditions and its natural frequency. The vibration resulting from the effects of external forces is referred to as the steady-state vibration or response.

2.2 EFFECTS OF VIBRATION ON HUMANS

Warnings from the road pass to the vehicle body through the wheels, springs, and damping elements. Typically, the bodies of vehicles, which are generally four-wheeled, undergo vertical, lateral, pitch, and yaw vibrations. These vibrations are transmitted to the person inside the vehicle through the seat system, which also consists of springs and damping elements. When the human body is considered as a vibrating system, it is known that different organs perceive and are affected by frequencies rather than the amplitudes of vibrations, as they have different natural frequencies. Research in this field indicates a frequency-dependent relationship between subjective perceptions and physical measurement values.

According to ISO 2631 standard; The effects of vibration movement on humans are evaluated in two frequency ranges:

Vibrations in the range of 0.5 Hz to 80 Hz have an impact on human health, comfort, and perceptions.

Vibrations in the range of 0.1 Hz to 0.5 Hz, on the other hand, can lead to motion sickness in humans.

According to the same standard, acceptable vibration amplitudes for comfort criteria are categorized as follows. However, while vibrational motion may be acceptable for some individuals, it can be annoying and uncomfortable for others. Therefore, making an accurate classification for the sensation of comfort is challenging due to its subjective nature.

For values less than 0.315 m/s^2 ⇒ not uncomfortable

For values between 0.315 m/s^2 and 0.63 m/s^2 ⇒ slightly uncomfortable

For values between 0.5 m/s^2 and 1 m/s^2 ⇒ almost uncomfortable

For values between 0.8 m/s^2 and 1.6 m/s^2 ⇒ uncomfortable

For values between 1.25 m/s^2 and 2.5 m/s^2 ⇒ very uncomfortable

For values greater than 2 m/s^2 ⇒ quite uncomfortable

Vibrations are transmitted to the human body through support surfaces, as seen in Figure 2.2 For vibrations with frequencies less than 1 Hz, the vertical movements of the body and the seat are nearly identical, resulting in a direct transmission of the vibration. As the frequency of vibration increases, the movements of the body become greater than the measured value on the seat. The transmission value peaks at one or more frequency values (resonance frequencies). At high frequencies, on the other hand, transmission decreases, meaning that the movement of the body is less than that measured on the seat. As observed, the frequencies at which transmission reaches its highest values vary depending on the direction of vibration and the individual's posture.

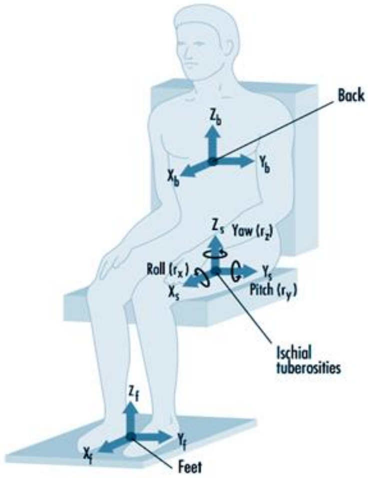


Figure 2.2: Transmission Surfaces Of Vibrations To The Human Body

2.3 SUSPENSION SYSTEMS

Different suspension systems are used in various locations in vehicles to prevent or reduce the transmission of vibrations from the road to the driver. Suspension systems have two fundamental and often conflicting objectives: ride comfort and ride dynamics. Ride comfort is determined by the effects of road irregularities on passengers and drivers while the vehicle is in motion. Ride dynamics, on the other hand, is primarily associated with the vehicle's maneuverability and performance during sudden maneuvers.

In general, ride comfort aims for a softer suspension system to prevent the transmission of effects from road irregularities to the driver and passengers. In contrast, ride dynamics seeks a stiffer suspension system to minimize the vehicle's sway and roll Figure 2.3 illustrates the elements responsible for the transmission and damping of vehicle vibrations. Among these suspension systems, seat suspension systems placed between the driver's seat and the vehicle body effectively dampen all vibrations affecting the driver. They are widely used due to their simple structures and the fact that they do not restrict the vehicle's mobility. Vibrations affecting the driver and passengers in the vehicle originate from road roughness, irregularities in rotating components of the vehicle, engine vibrations, and cruising movements.

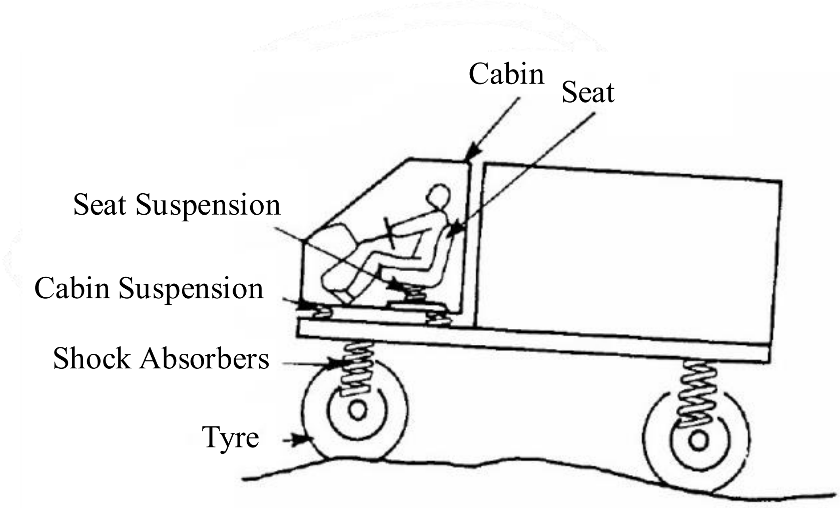


Figure 2.3: Vehicle Vibration Transmission / Damping Elements

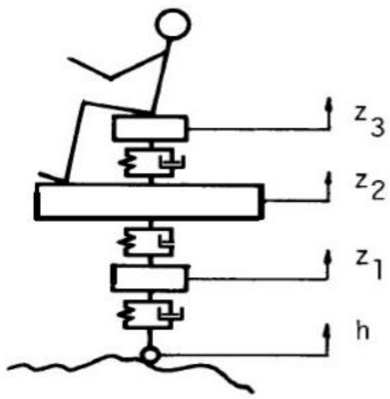


Figure 2.4: Vibrations Acting On The Driver

In Figure 2.4, the movements measured for comfort in land vehicles are depicted. It is important to note that it is the accelerations, not the actual movements, that are considered in these specified measurements.

2.3.1 Elements Of The Suspension System:

Suspension systems, in their most general form, consist of a spring and a damper (shock absorber) and, when considering transportation vehicles, they are mechanisms that bear the weight of the vehicle. Despite having various connection methods, these two elements generally work together in parallel to absorb and eliminate the effects from the road. If we examine these elements in a bit more detail:

Springs are elements that store energy on them. When a moving vehicle encounters bumps on the road, the shocks are transmitted to the springs as kinetic energy through the wheels in a very short period. The springs, by compressing in a short time, absorb this energy and store it as potential energy. After some time, they release this energy by converting it into kinetic energy through a slow oscillating motion, thereby smoothing out the shocks from the road. For example, the movement of a vehicle with a suspension system consisting only of springs passing over a bump in the road is illustrated in Figure 2.5.

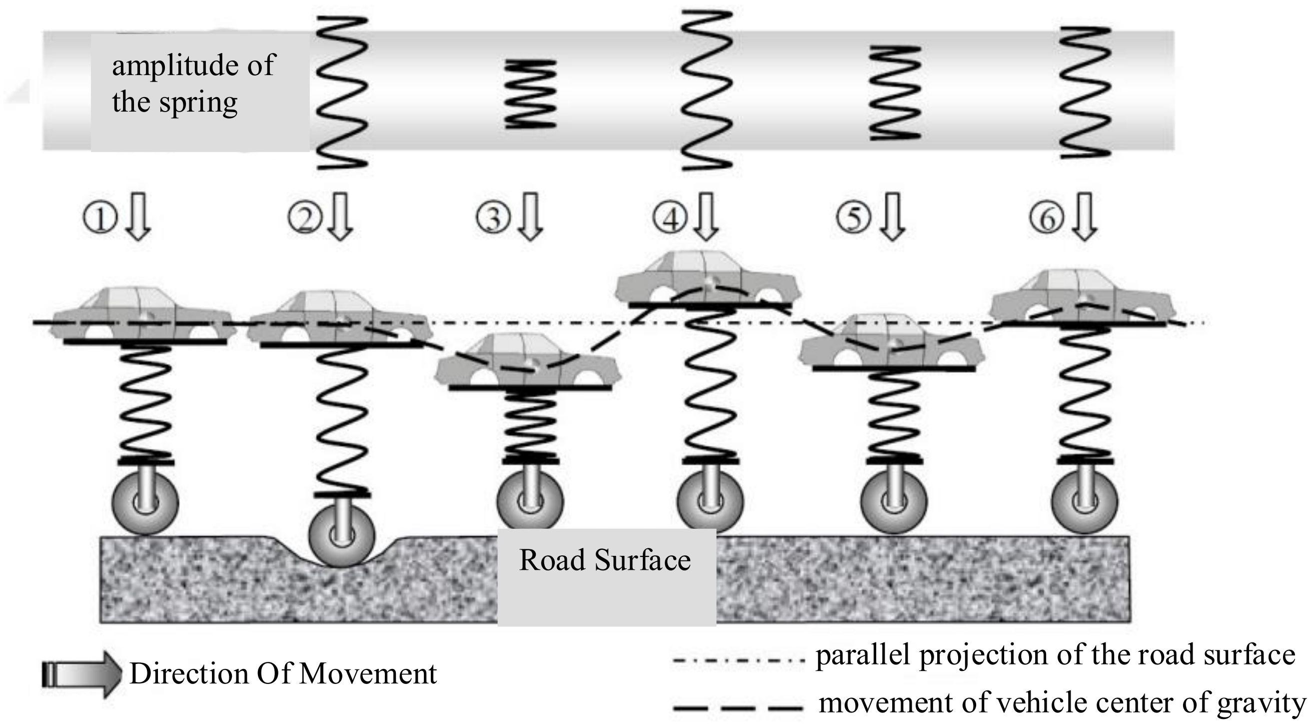


Figure 2.5: Behavior Of A Suspension System Consisting Only Of Springs

The movements of the body and the spring are separately indicated in the figure. In the first stage of the motion, the vehicle is in a stationary position, and in the second stage, the wheel group drops down due to a pit in the road. Subsequently, the vehicle body is forced into a downward motion, as seen in stage 3, with the spring compressing to store energy. This stored energy, during the opening motion in stage 4, can force the spring to open beyond its previous stationary position. The attempt of the vehicle to rise also assists in this movement, causing the spring to extend beyond its normal opening amount. In the next stage (stage 5), as the vehicle body undergoes a downward motion, the spring compresses again, and the resulting energy compresses the spring to dimensions below its normal load dimensions but less than in stage 3. This, in turn, leads to the spring attempting to open itself again, as seen in stage 6. The cycle continues in this way. To prevent this spontaneous oscillation, a component is needed to dissipate or expend the energy stored in the spring during compression. This component is the dampers, another element of the suspension systems.

Dampers correspond to the resistance element in electrical circuits. They dissipate the motion energy present in the system by converting it into heat energy through frictional forces on the elements that make up the structure. As a result, with decreasing motion energy, suspension systems suppress the oscillations on the vehicle. If we reconsider the movements of a vehicle equipped with only a spring suspension system, as discussed in the springs section, with the addition of a damper, it would be like this:

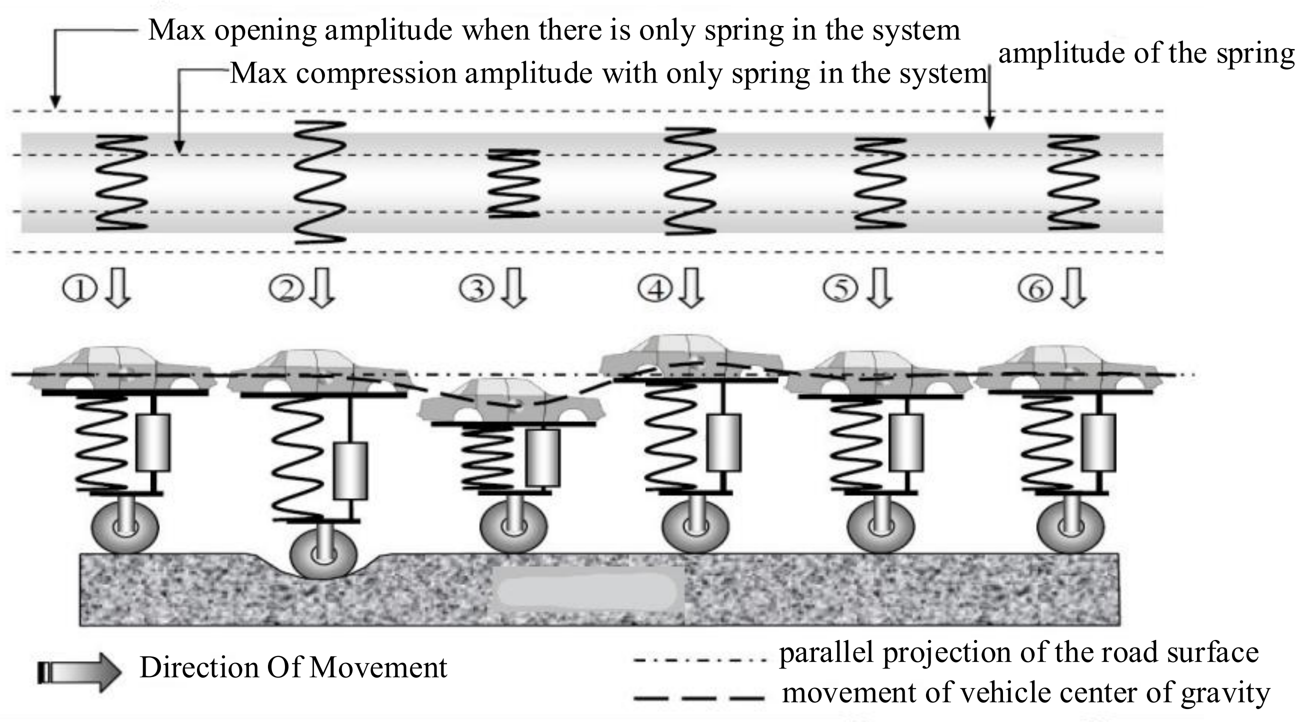


Figure 2.6: The bouncing motion of a vehicle equipped with a suspension system consisting of a damper and a spring when passing over a pothole.

Following the addition of the damper to the system, it is observed that the oscillatory motions and their amplitudes decrease. After the stationary start at position 1, the vehicle drops into the pothole at position 2, as seen in movement 3, the spring compresses, storing energy on it. However, this time, thanks to the frictional force created within the damper, a portion of its energy is dissipated from this moment onwards. In the subsequent opening and compressing movements at positions 4 and 5, the energy stored in the spring continues to be consumed, and eventually, as in movement 6, the vehicle and the suspension system return to the stationary starting position, i.e., the situation in position 1. In the system consisting only of the spring, the vehicle continued its bouncing motion in part 6.

To compare the oscillation performances of suspension systems consisting only of a spring and those with both a spring and a damper, it is possible to examine the vertical motion of both vehicle center of gravity by plotting them together. Accordingly:

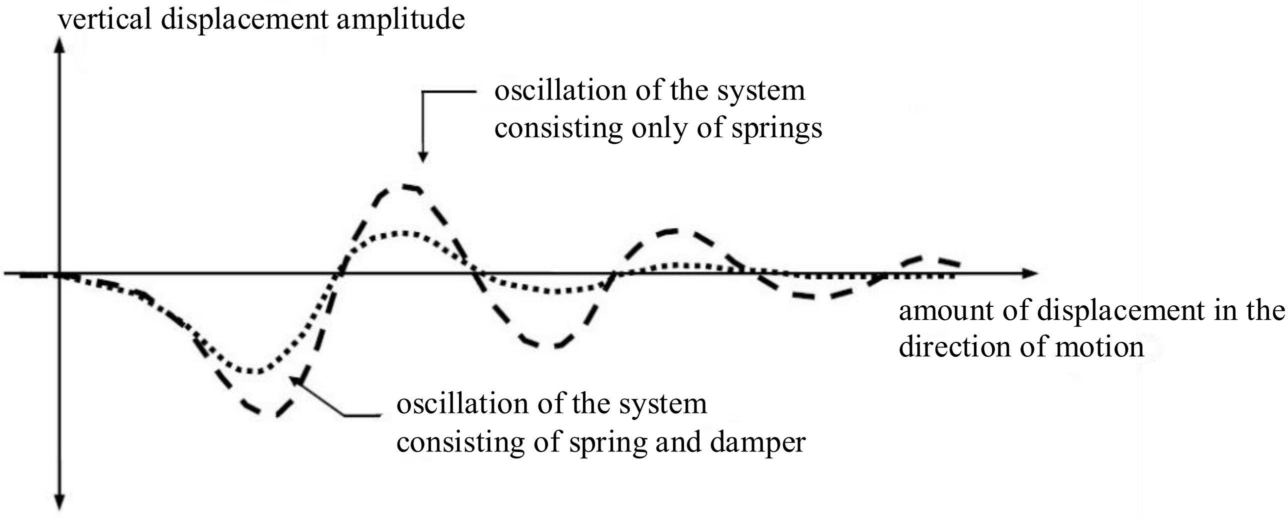


Figure 2.7: Comparison of the oscillatory motions between a suspension system consisting only of a spring and a suspension system comprising both a spring and a damper.

2.4 TYPES OF SUSPENSION SYSTEMS

Suspension systems that dampen vibrations from the road or driving conditions are classified into 3 main groups based on their damping characteristics:

i. Passive Suspension Systems

ii. Semi-Active Suspension Systems

iii. Active Suspension Systems

2.4.1 Passive Suspension Systems

Passive suspension systems are mechanisms consisting of conventional (conventional) springs and dampers. In other words, a passive suspension system is composed of elements (namely traditional springs and dampers) whose characteristic values are constant and do not change during operation. These characteristic values are determined by system designers in the design of the vehicle to achieve the desired objectives (comfort and safety) and are installed in the assembly location during manufacturing. After this point in passive suspension systems, the only way to change element values is to install new elements with the desired values into the system. The vibration damping capability of the system depends on the characteristics of these passive elements and the mechanism.

passive suspension system, as shown in Figure 3.20, has the ability to store energy in the spring and distribute this energy through the damper.

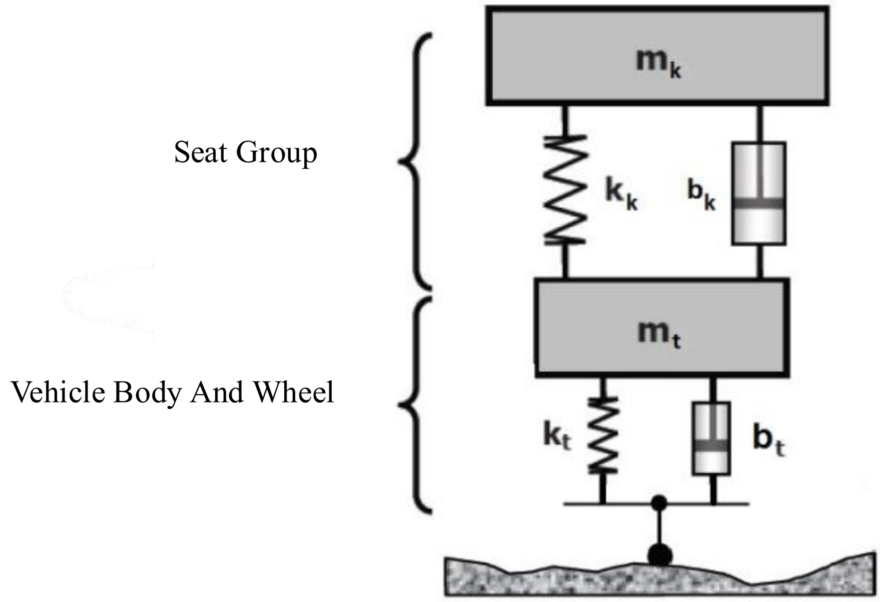


Figure 2.8: Passive Suspension Systems

The spring stiffness coefficients in the system are represented by k, and the damping coefficients are represented by b, and these parameter values cannot be changed during the ride. When creating the suspension system, once a spring is selected to carry all the load it can, the remaining task is to determine the damping coefficient that provides the desired damping effect.

For the system, if a small damping coefficient is chosen, resonance motion is observed when encountering a road irregularity with the natural frequencies of the vehicle or seat. However, good isolation is provided for high-frequency components coming from the road. When a large damping coefficient is chosen, on the other hand, there is a reduction in resonance motion, but less isolation is provided against high-frequency vibrations. In other words, the driver feels more vibration.

To maintain comfort and safety criteria under different road and driving conditions, the parameters in the suspension system need to be adjustable. As passive suspension systems do not allow for this, the use of semi-active or active suspension systems is becoming more common.

In this thesis, passive suspension system was examined.

2.4.2 Semi-Active Suspension Systems

While passive suspension systems do not allow for parameter changes, in semi-active suspension systems, the stiffness of the springs remains the same, but the damping coefficient of the damper can be adjusted. However, as there is no action like parameter change in passive suspension systems, there is no need for an additional energy source for this operation. In semi-active suspension systems, on the other hand, adjusting the damping coefficient and running the controller systems with sensors require an external energy source.

As seen in Figure 3.21, in the structure of the given system, semi-active suspension systems have an adjustable damping force, which is different from passive systems. Semi-active damping systems can be classified into two groups based on the range of change in the damping coefficient value.

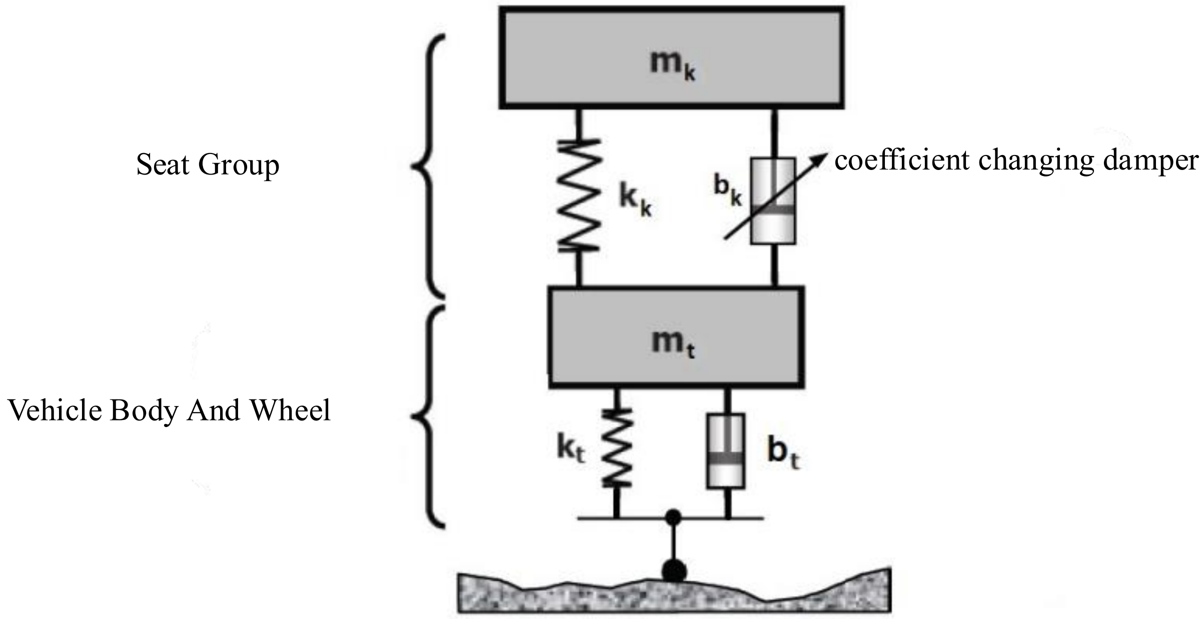


Figure 2.9: Semi-Active Suspension Systems

i. Open-closed semi-active suspension systems: According to criteria determined by the control algorithm, the damper switches between open and closed positions. In the open position, it has a stiff (high) damping coefficient. In the closed position, it takes a soft (low) damping coefficient.

ii. Continuously variable semi-active suspension systems: Similar to the open-closed structure, the damper switches between open and closed positions. However, in the open position, the structure of the damper is arranged to provide different damping coefficient values within the intervals permitted by its structure, according to the criteria determined by the control algorithm.

It is clear that the dampers used in these systems will be different from conventional dampers. In this context, the most suitable dampers that can be used are electro-rheological dampers with a damping coefficient that changes with electricity and magneto-rheological dampers with a damping coefficient that changes with a magnetic field.

2.4.3 Active Suspension Systems

Active suspension systems operate on the principle of action-reaction. In other words, the goal is to prevent vibration (and consequently acceleration) caused by the force coming from the road by applying a force in sync with it and of the same magnitude. For this purpose, active systems use hydraulic or electrical actuators. These systems require various sensors and a control mechanism to function.

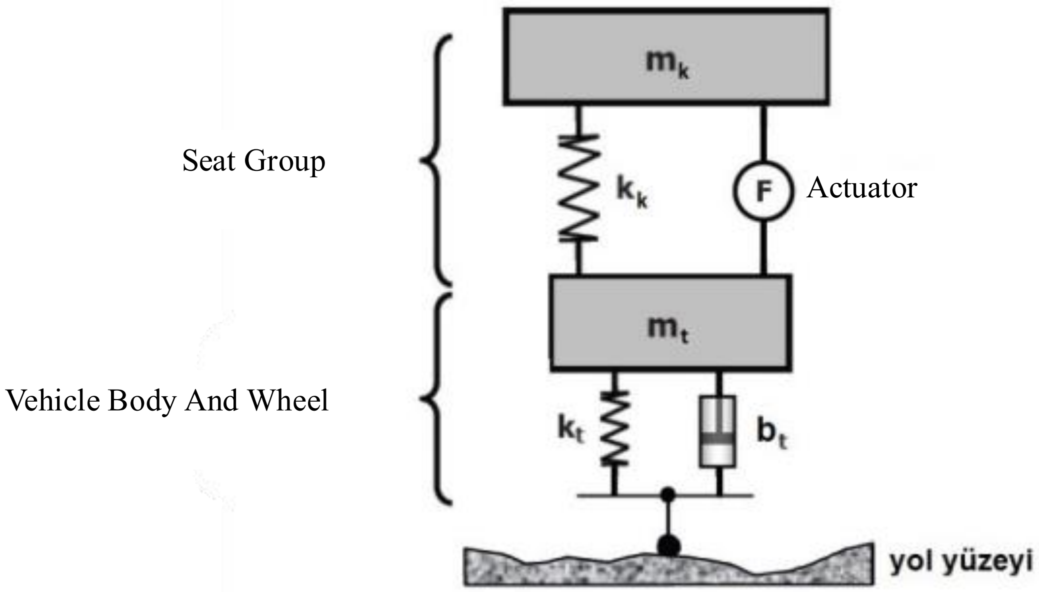


Figure 2.10: Active Suspension Systems

Active suspension systems, despite bringing performance improvements, can lead to additional cost increases and a complex structure due to their requirement for an external power source. However, with advancing technology, there has been a trend of cost reduction and simplification in structures.

When comparing these systems, the following points can be made:

- Semi-active suspension systems may not perform as well as active systems, but with good design, they can still significantly improve vibration levels. They do not require as high a cost as active systems, promising broader applications and adoption.

- Passive systems, while performing less effectively than active and semi-active systems, are cheaper and simpler in terms of cost and operating principles. A well-designed passive system can effectively meet the requirements, comparable to active and semi-active systems.
- In the literature, it is reported that active suspension systems can achieve around an 80% reduction in vibration amplitude, while semi-active systems can achieve around a 50% reduction when compared to passive systems.

3 DYNAMIC MODEL OF A SEAT WITH A PASSIVE SUSPENSION SYSTEM

In general, when modeling dynamic systems, mass, spring, and damping elements are used. In the system, objects that undergo temporary shape changes (with the ability to flex) in response to applied force are modeled as spring elements, objects with energy absorption (damping) capabilities are modeled as damping elements, and objects with non-negligible mass are modeled as mass elements, taking their weights into account in the analysis. In this study, the driver's seat, representing the suspension system of the seat, was also modeled using the same analogy. To represent the suspension system of the seat, the elevation mechanism (the part between the vehicle chassis and the seat) and the seating part (sponge and seat fabric) were modeled separately with mass-spring-damper groups. The model of this two-degree-of-freedom seat suspension system is shown in Figure 3.1.

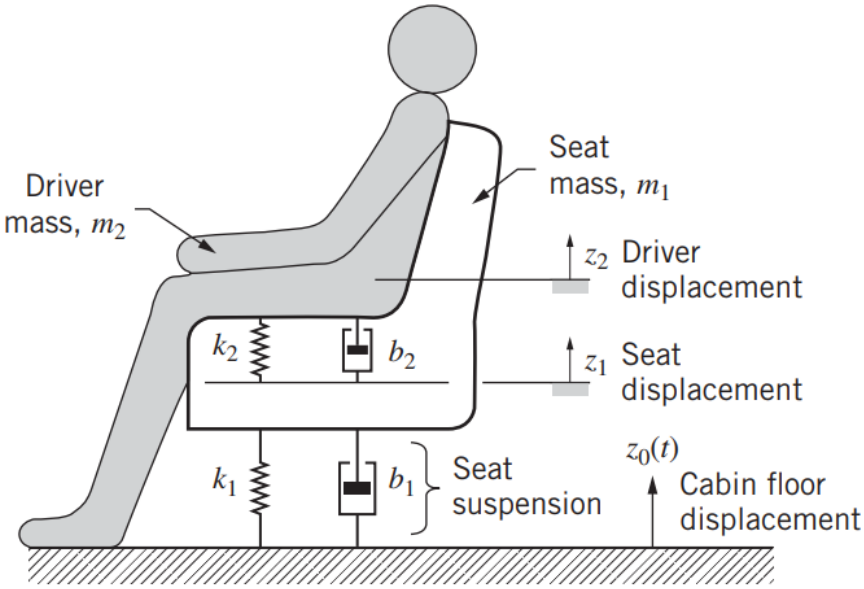


Figure 3.1: Schematic Diagram Of The Seat-Suspension System

Here, m_1 represents the total mass of the steel frame structure of the seat base and the parts in the elevation mechanism, m_2 represents the mass of the seat cushion, k_1 represents the total stiffness coefficient of the parts with flexural capabilities in the elevation mechanism, k_2 represents the stiffness of the cushion sponge, b1 represents the damping coefficient of the elevation mechanism, b_2 represents the damping coefficient of the cushion sponge, and z, x_1, and x_2 respectively represent the vertical displacements of the vehicle chassis, the elevation mechanism, and the cushion.

3.1 MATHEMATICAL MODEL

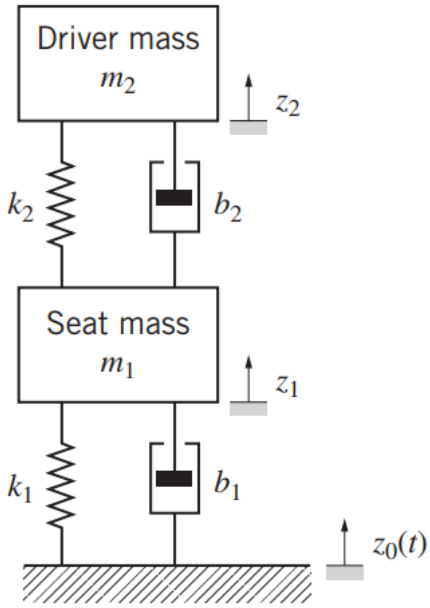


Figure 3.2: Mechanical model for the seat-suspension system.

Figure 3.2 shows the lumped mechanical model of the seat-suspension system. Mass m_1 is the total mass of the seat, while mass m_2 represents the mass of the driver. Ideal spring k_1 and viscous friction damper b_1 model a shock absorber connecting the seat to the vehicle's cabin floor. Spring constant k_2 and friction coefficient b_2 represent the stiffness and damping of the seat cushion, respectively. Finally, z_1 is the vertical displacement of the seat mass and z_2 is the displacement of the driver mass, and both are measured relative to their static equilibrium positions. The vertical displacement of the cabin floor (due to road vibrations) is $z_0(t)$ (note that upward is the positive sign convention for all displacements as defined by Reference 1). all design problems involving dynamic systems begin with the development of mathematical models, which are presented in this example.

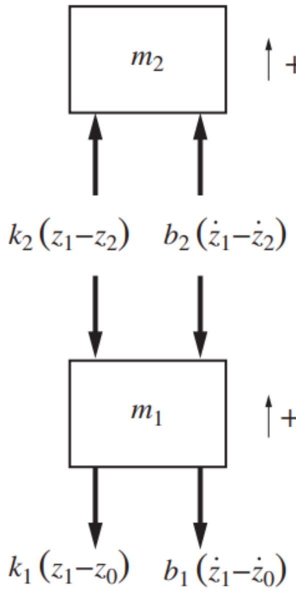


Figure 3.3: Free-Body Diagram For The Seat-Suspension System

Figure 3.3 shows the FBDs of the two-mass mechanical system. The positive (upward) conventions for displacements z_1 and z_2 are also presented. Clearly, all spring and damper forces depend on the relative displacements and velocities between the seat mass and cabin floor, and the driver and seat masses, respectively. If we assume that relative displacement $z_1 - z_0$ is positive, then the suspension spring k_1 is in tension and the reaction force acts downward on seat mass m1 as shown in Figure 3.3. Similarly, if we assume that relative displacement $z_1 - z_2$ is positive, then the seat cushion is compressed and the reaction force acts downward on seat mass m1 and upward on driver mass m2 as shown by the equal-and-opposite spring force $k_2(z_1 - z_2)$ in the FBD.

Friction forces depend on the relative velocities. If we assume that the relative velocity $\dot{z}_1 - \dot{z}_0$ is positive (i.e., the seat mass m1 is moving "away" from the cabin floor), then the reaction friction force $b_1(\dot{z}_1 - \dot{z}_0)$ on mass m_1 opposes the relative motion, as shown on the FBD. Similarly, if we assume that relative velocity $\dot{z}_1 - \dot{z}_2$ is positive (i.e., the seat mass m1 is "approaching" the driver mass m_2), then the seat cushion damping reaction force acts downward on seat mass m_1 and upward on driver mass m_2 as shown by the equal-and-opposite damper force $b_2(\dot{z}_1 - \dot{z}_2)$ in the FBD. The reader should see that the FBDs in Figure 3.3 remain valid if the assumed relative displacements and velocities are negative, in which case the force arrows are reversed. Finally, because displacements are referenced to the static equilibrium positions, the gravitational forces do not appear in the FBDs.

Summing all external forces with upward as the positive sign convention and applying Newton's second law, we obtain

$$Mass\ 1{:}\ +\uparrow \sum F = -k_2(z_1 - z_2) - b_2(\dot{z}_1 - \dot{z}_2) - k_1(z_1 - z_0) - b_1(\dot{z}_1 - \dot{z}_0) = m_1\ddot{z}_1$$

$$Mass\ 2{:}\ +\uparrow \sum F = k_2(z_1 - z_2) + b_2(\dot{z}_1 - \dot{z}_2) = m_2\ddot{z}_2$$

Rearranging these equations with the dynamic variables (z_1 and z_2) on the left-hand side and the input variable (z0) on the right-hand side, we have

$$m_1\ddot{z}_1 + b_1\dot{z}_1 + b_2(\dot{z}_1 - \dot{z}_2) + k_1 z_1 + k_2(z_1 - z_2) = b_1\dot{z}_0(t) + k_1 z_0(t) \quad (3.1a)$$

$$m_2\ddot{z}_2 + b_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) = 0 \quad (3.1b)$$

Equations (3.1a) and (3.1b) represent the mathematical model of the seat-suspension system. Because we have two inertia elements, the complete model consists of two second-order ODEs. Both ODEs are coupled, which means we cannot solve one ODE separately from the other. The model is linear because we have assumed linear stiffness and damper elements. Furthermore, the reader should note that all terms pertaining to the acceleration, velocity, and position of mass m_1 in Eq. (3.1a) have the same sign, that is, they are positive. Similarly, all terms associated with z_2 (and its derivatives) in Eq. (3.1b) have the same sign. In general, this condition holds for a system that is inherently stable. Intuition tells us that the seat-suspension system in Figure 3.2 is stable and will always return to static equilibrium when input vibrations $z_0(t)$ have ceased.

We assumed ideal damping and stiffness elements, the system is linear. A state-space representation (SSR) approach is likely best suited for system analysis as the two second-order linear ordinary differential equations (ODEs) are coupled, which presents a challenge to developing transfer functions.

The system variables are the vertical displacements of the seat mass (z_1) and driver mass (z_2), and that both are measured relative to their static equilibrium positions. The vertical displacement of the cabin floor (due to road vibrations) is $z_0(t)$, which is considered to be an input to the system. The remaining parameters are the seat and driver masses (m_1 and m_2), and the passive friction and stiffness coefficients. The two measurements associated with the test rig are driver displacement z_2 and driver acceleration $\ddot{z}_2$. Displacement is measured by a linear variable differential transformer (LVDT), an electromechanical device used to measure translational displacement. Acceleration is measured by an accelerometer.

The objective is to develop a complete SSR given the modeling equations (3.1a) and (3.1b). Clearly, this system is linear and fourth-order (n = 4). At first glance, displacements z1 and z2 and their derivatives are the obvious choices for state variables. However, if we choose $x_1 = z_1$ and $x_2 = \dot{z}_1$, we see from Eq. (3.1a) that the second state equation ($\dot{x}_2 = \ddot{z}_1$) will involve the term $\dot{z}_0(t)$, which is the derivative of the input $u = z_0(t)$. Our standard state equation does not contain a matrix-vector term involving $\dot{u}$, so we cannot utilize this choice of state variables along with a single input defined as $u = z_0(t)$.

In general, when the mathematical model involves derivatives of the input variables, such as Eq. (3.1a), the choice of state variables becomes more complicated and less intuitive. We show two solutions to this state-space problem when the time derivative of the input u appears in the system dynamics. The intuitive or easier solution is to simply define an additional input variable that is the derivative of z_0.

Hence, we define two input variables: $u_1 = z0(t)$ and $u_2 = \dot{z}_0(t)$. Now, we are able to define the following four state variables:

$$x_1 = z_1$$

$$x_2 = \dot{z}_2$$

$$x_3 = z_3$$

$$x_4 = \dot{z}_4$$

Taking the first time derivative of each state variable and substituting the system dynamics (3.1a) and (3.1b) for $\ddot{z}_1$ and $\ddot{z}_2$ yields

$$\dot{x}_1 = \dot{z}_1$$

$$\dot{x}_2 = \ddot{z}_1 = \frac{1}{m_1}[-b_1\dot{z}_1 - b_2(\dot{z}_1 - \dot{z}_2) - k_1 z_1 - k_2(z_1 - z_2) + b_1\dot{z}_o(t) + k_1 z_0(t)]$$

$$\dot{x}_3 = \dot{z}_2$$

$$\dot{x}_4 = \ddot{z}_2 = \frac{1}{m_2}[-b_2(\dot{z}_2 - \dot{z}_1) - k_2(z_2 - z_1)]$$

Next, we substitute for the physical variables using the definitions of the states ($x_1 = z_1$, $x_2 = \dot{z}_1$, $x_3 = z_2$, and $x_4 = \dot{z}_2$) and the two inputs ($u_1 = z_0(t)$, $u_2 = \dot{z}_0(t)$) to yield

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{-k_1 - k_2}{m_1}x_1 + \frac{-b_1 - b_2}{m_1}x_2 + \frac{k_2}{m_1}x_3 + \frac{b_2}{m_1}x_4 + \frac{k_1}{m_1}u_1 + \frac{b_1}{m_1}u_2$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{k_2}{m_2}x_1 + \frac{b_2}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b_2}{m_2}x_4$$

The complete state equation in matrix-vector format is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -(k_1+k_2)/m_1 & -(b_1+b_2)/m_2 & k_2/m_1 & b_2/m_1\\ 0 & 0 & 0 & 1\\ k_2/m_2 & b_2/m_2 & -k_2/m_2 & -b_2/m_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ k_1/m_1 & b_1/m_1\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \quad (3.2a)$$

Recall that the two system outputs or measurements are specified as $y_1 = z_2$ and $y_2 = \ddot{z}_2$. The first output equation is simply $y_1 = x_3$, and the second output equation is obtained from the fourth state equation

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ k_2/m_2 & b_2/m_2 & -k_2/m_2 & -b_2/m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (3.2b)$$

Equations (3.2a) and (3.2b) constitute the complete SSR. A strong word of caution is in order: we have greatly simplified the state-space derivation by defining two independent input variables $u_1 = z_0(t)$ and $u_2 = \dot{z}_0(t)$. In reality, there is only one independent system input, and it is $u = z_0(t)$. In our two-input SSR summarized by Eqs. (3.2a) and (3.2b), there is no constraint requiring that the second input u_2 is the time derivative of input u^1. Therefore, we must be careful to enforce the constraint $u_2 = \dot{u}_1$ when we analyze or simulate the dynamics of the two-input SSR derived in this part.

3.2 IMPULSE RESPONSE

In our examination of the homogeneous or free response of a linear system, we focused on understanding the system's intrinsic behavior, known as its natural dynamics. This can be ascertained by examining the roots of the characteristic equation. In the case of a system with a single transfer function, the free response is influenced by the poles of the transfer function. However, for a complex system like the seat-suspension system with multiple states, the state-space method proves to be the most suitable approach for analyzing system dynamics. Consequently, the natural dynamics of the seat-suspension system can be determined by assessing the eigenvalues of the state matrix A. It's worth noting that both the poles of a system's transfer function and the eigenvalues of its state matrix A are essentially equivalent to the roots of the characteristic equation.

Table 3.2.1 Parameters for the Seat-Suspension System

System Parameter	Value
Seat mass, m_1	20 kg
Driver mass, m_2	50 kg
Suspension stiffness, k_1	7410 N/m
Suspension friction coefficient, b_1	1430 N-s/m
Seat cushion stiffness, k_2	8230 N/m
Seat cushion friction coefficient, b_2	153 N-s/m

The state matrix of the seat-suspension system can be computed by using the numerical parameters in Table 3.2.1, and the result is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k_1+k_2)/m_1 & -(b_1+b_2)/m_2 & k_2/m_1 & b_2/m_1 \\ 0 & 0 & 0 & 1 \\ k_2/m_2 & b_2/m_2 & -k_2/m_2 & -b_2/m_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -782 & -79.15 & 411.5 & 7.65 \\ 0 & 0 & 0 & 1 \\ 164.6 & 3.06 & -164.6 & -3.06 \end{bmatrix}$$

The system eigenvalues are computed using the MATLAB command:

```
%  mechanical system parameters
m1 = 20;         %  seat mass, kg
m2 = 50;         %  driver mass, kg
k1 = 7410;       %  suspension stiffness, N/m
k2 = 8230;       %  seat cushion stiffness, N/m
b1 = 1430;       %  suspension friction, N-s/m
b2 = 153;        %  seat cushion friction, N-s/m

%  State-space representation:
%   x = [ z1 ; z1dot-b1*z0/m1 ; z2 ; z2dot ]'
%   u = z0(t) (floor displacement)
Arow1 = [ 0 1 0 0 ];
Arow2 = [(-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ];
Arow3 = [ 0 0 0 1 ];
Arow4 = [ k2/m2 b2/m2 -k2/m2 -b2/m2 ];
A = [ Arow1 ; Arow2 ; Arow3 ; Arow4 ];

eig(A)
```

```
ans =

 -67.5958 + 0.0000i
  -3.6733 +10.5189i
  -3.6733 -10.5189i
  -7.2675 + 0.0000i
```

Figure 3.4: Characteristic Roots

$$r_1 = -67.5958 \quad r_2 = -7.2675 \quad r_3 = -3.6733 + j10.5189 \quad r_4 = -3.6733 - j10.5189$$

Two roots are real and negative, and two roots are complex with negative real parts. Consequently, the free response will eventually decay to zero as the system reaches its steady-state value. The general form of the free response (for either output) is:

$$y_H(t) = c_1 e^{-67.5958t} + c_2 e^{-7.2675t} + c_3 e^{-3.6733t} + \cos(10.5189t + \emptyset)$$

The above equation shows that the free response consists of two damped exponential functions and a damped sinusoidal function.

The first exponential function $e^{-67.5958t} = e^{-t/0.3333}$ has a time constant $\tau_1 = 0.01479$s, so it "dies out" to zero in approximately $4\tau_1 = 0.06$s. The second exponential function $e^{-7.2675t} = e^{-t/0.13759}$ has a time constant $\tau_2 = 0.13759$ s, so it "dies out" to zero in approximately $4\tau_2 = 0.55$ s. the third exponential function $e^{-3.6733t}=e^{-t/0.2722}$ has a time constant $\tau_1 = 0.2722$s, so it "dies out" to zero in approximately $4\tau_1 = 1.09$s.

Root $r_1 = -67.5958$ is the "fastest" root, as its exponential function decays to zero in about 0.06 s, and therefore its contribution to the total response is extremely short-lived. Root $r_2 = -7.2675$ corresponds to an exponential function that decays to zero in about 0.55 s. The complex roots are the "slowest" roots because their exponential function decays to zero in about 1.09 s. Therefore, real root r_2 and complex roots r_3 and r_4 are the dominant roots.

We can construct the fourth-order characteristic equation from the four eigenvalues or roots:

$$(r + 67.5958)(r + 7.2675)(r + 3.6733 + j10.5189)(r + 3.6733 - j10.5189) = 0$$

Therefore, the underdamped part of the characteristic equation can be determined by multiplying the last two (or complex) terms of equation.

$$r^2 + 7.3467r + 124.1403 = 0$$

$$r^2 + 2\zeta\omega_n + \omega_n^2 = 0$$

The undamped natural frequency is:

$$\omega_n = \sqrt{124.1403} = 11.14\frac{rad}{s}$$

The damping ratio is:

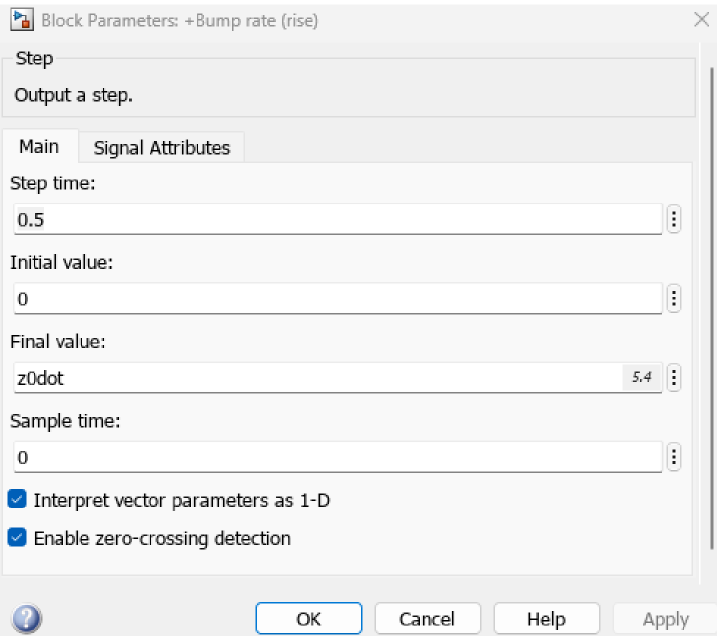
$$\zeta = \frac{7.3467}{2\omega_n} = \frac{7.3467}{2(11.14)} = 0.3297$$

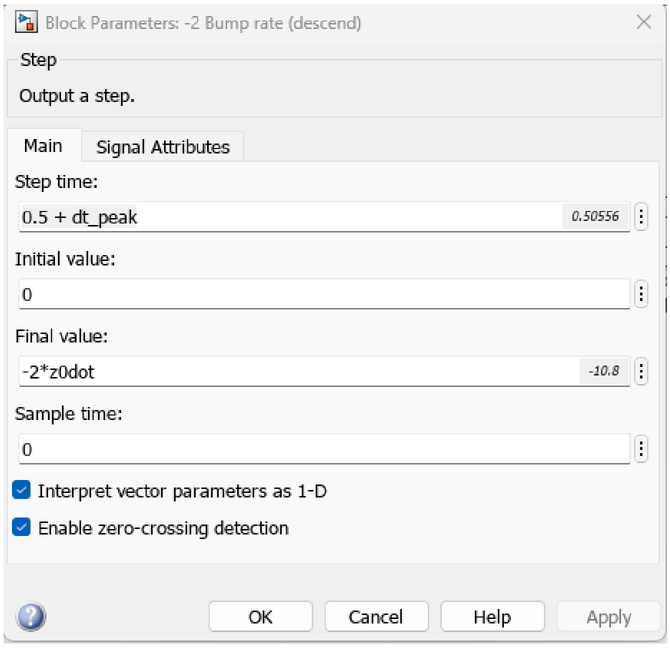
Because the complex roots are dominant roots, we expect the free response of the seat-suspension system to exhibit underdamped response characteristics.

3.2.1 Simulink Diagram

Subsequently, we employ **Simulink** to analyze how the seat-suspension system reacts to various inputs. Initially, we focus on a "spike" input representing the cabin floor displacement, denoted as $z_0(t)$. We hypothesize a scenario where the vehicle encounters a bump, leading to a sudden shift in the cabin floor. The model for the floor displacement, $z_0(t)$, adopts a "triangular pulse" shape, featuring a maximum displacement of 0.03 meters (3 cm) and a consistent vertical "bump rate" of $\dot{z}_0 = 5.4$ m/s (ascending).

The floor displacement is a symmetrical triangular pulse, and therefore the constant "bump rate" is $\dot{z}_0 = -5.4$ m/s (descending) after the pulse reaches its peak value of 0.03 m. Hence, the duration of half of the triangular pulse is $\Delta t = z_{0max}/\dot{z}_0 = 0.0056$ s or 5.6 ms. Because the total duration of the triangular pulse is about 11 ms, we can consider the "spike" input to essentially be an impulse input.





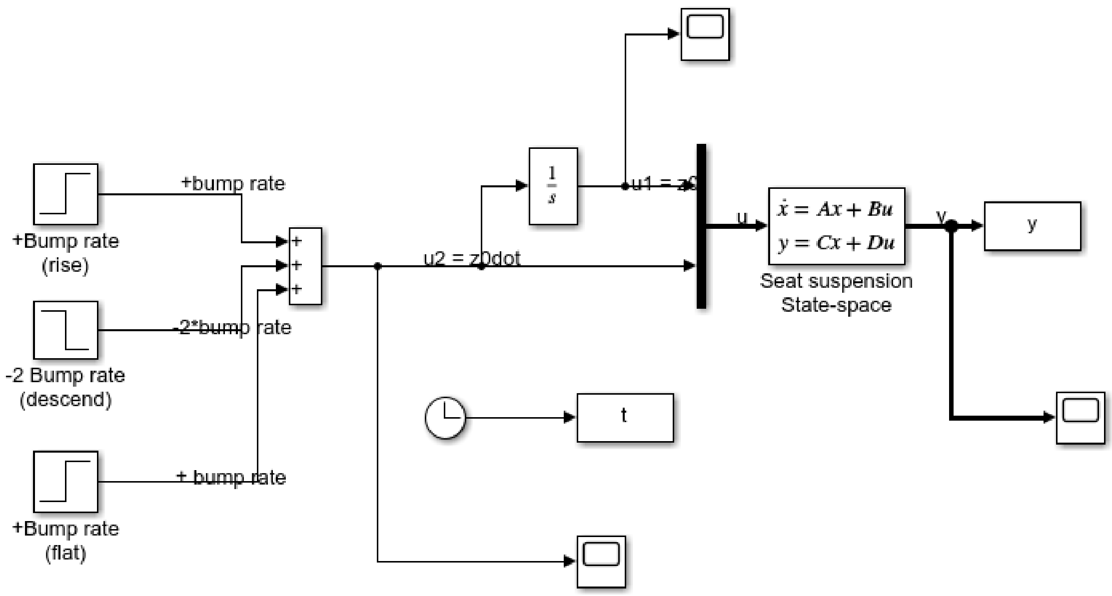


Figure 3.5: Simulink diagram for the seat-suspension system: triangular pulse input.

Figure 3.5 shows the Simulink model of the seat-suspension system with a triangular pulse input z0(t). The triangular pulse input is created by integrating a sequence of constant-velocity pulses. The initial positive "bump rate" $\dot{z}_0 = 5.4$ m/s is created by the first step function (step time = 0.5 s). The second step function has magnitude −10.8 m/s (step time = 0.5 + Δts) and is added to the first step input to create a negative velocity input of −5.4 m/s. The third step function has magnitude +5.4 m/s (step time = 0.5 + 2Δts), which, when added to the other two step functions, results in $\dot{z}_0 = 0$. Figure 11.3 shows zoomed-in views of the pulse input $\dot{z}_0(t)$ created by the three step functions, and the triangular pulse input z0(t) created by integrating the velocity pulses.

The SSR matrices A, B, C, and D that are required in the Simulink model are determined by Eqns. (3.2a) and (3.2b), and the system parameters found in Table 3.2.1. Finally, the initial state vector is set to a zero 4 × 1 column vector.

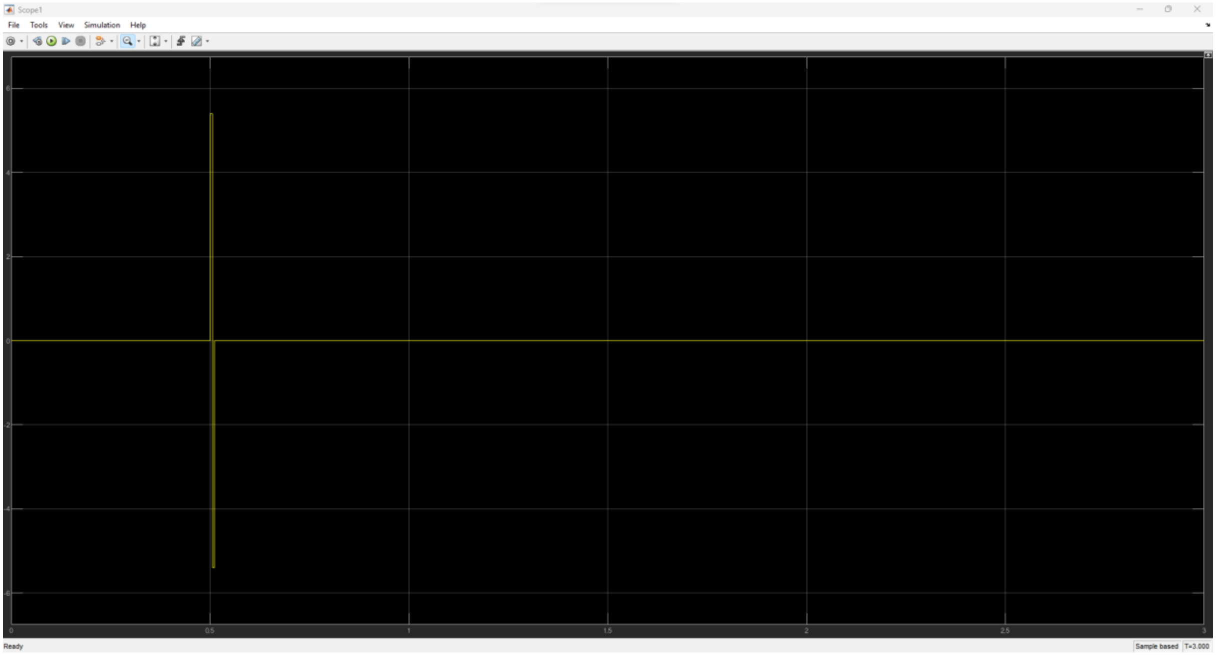


Figure 3.6: Seat-suspension system inputs: velocity-pulse input $u_2(t)$

Figure 3.7: Seat-suspension system inputs: triangular-pulse input $u_1(t)$.

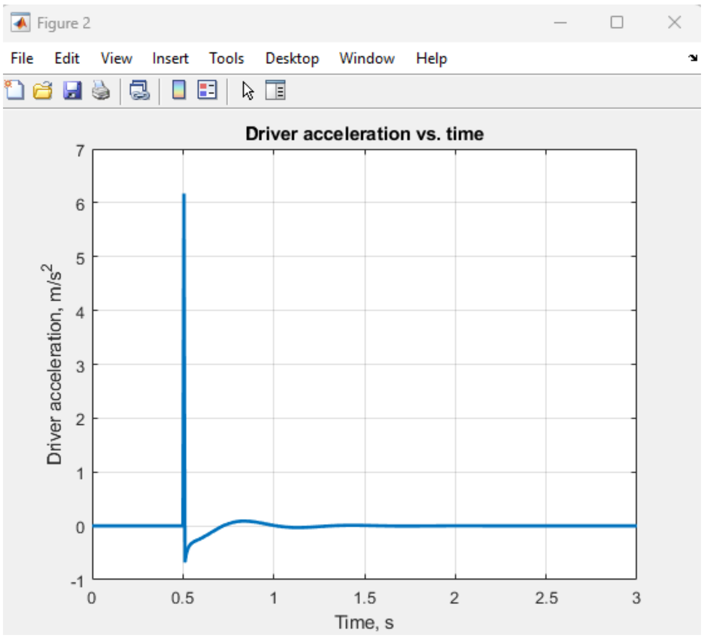


Figure 3.8: Impulse response of the seat-suspension system: driver acceleration

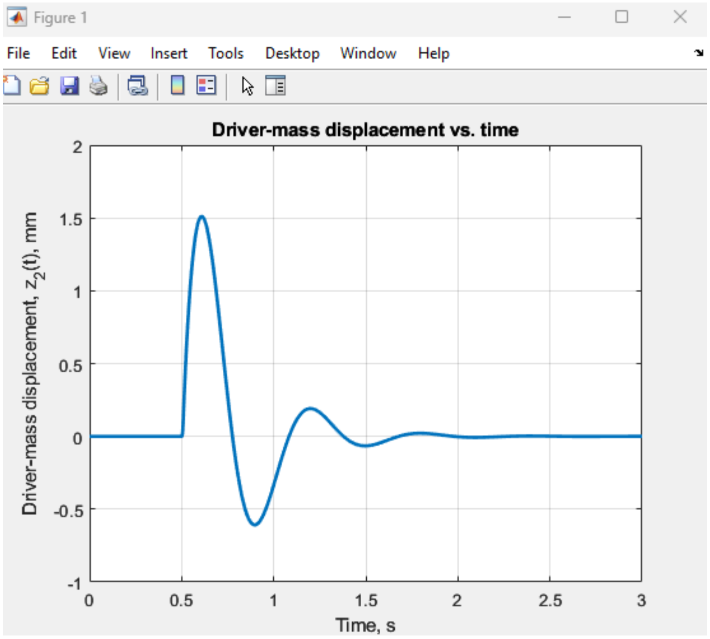


Figure 3.9: Impulse response of the seat-suspension system: driver displacement

3.3 FREQUENCY RESPONSE

The goal of the vibration isolation system is to minimize the movement of the vehicle cabin floor, denoted as $z_0(t)$, and prevent its transmission to the driver. In a prior scenario, it was demonstrated that the seat-suspension system effectively diminishes an impulsive input within approximately 1.1 seconds. The impulse response for the driver's displacement, $z_2(t)$, reveals two peaks during the transient response, each occurring with a period of around 0.6 seconds. Additionally, the impulse response for the driver's acceleration displays an initial sharp spike upon the application of the impulse, succeeded by a rapidly attenuating response that returns to zero.

Traveling on an uneven road introduces a repetitive (periodic) input. For instance, a vehicle moving at a constant speed over evenly spaced road bumps will encounter a periodic road displacement with a fixed frequency, and this can be represented as a sinusoidal input function. Consequently, we can employ frequency-response methods to assess the performance of the seat-suspension system. The periodic vibrations from the road will induce corresponding vibrations in the cabin floor, which can be expressed as $z_0(t) = a \sin(\omega t)$, where 'a' represents the amplitude of floor vibrations (in meters), and ω denotes the input frequency (in radians per second). As both the driving speed and the spacing between bumps impact the input frequency ω, our analysis involves studying the vibration isolation system's response across a range of frequencies.

3.3.1 Simulink Diagram

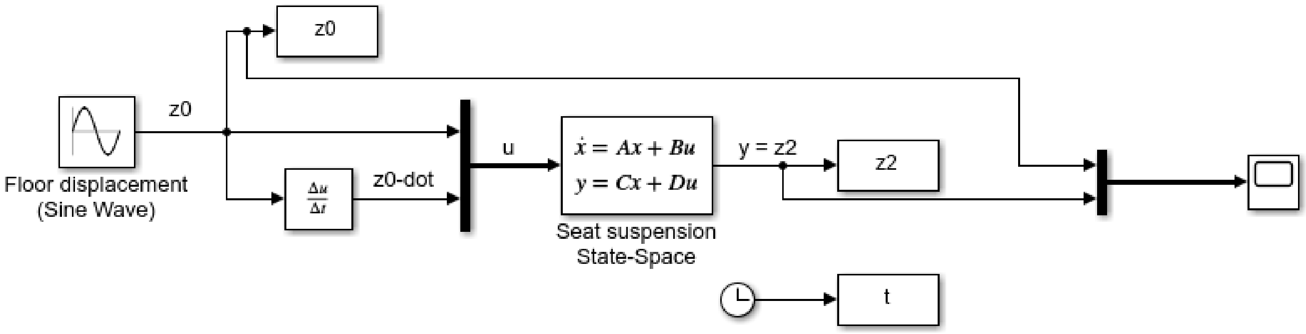
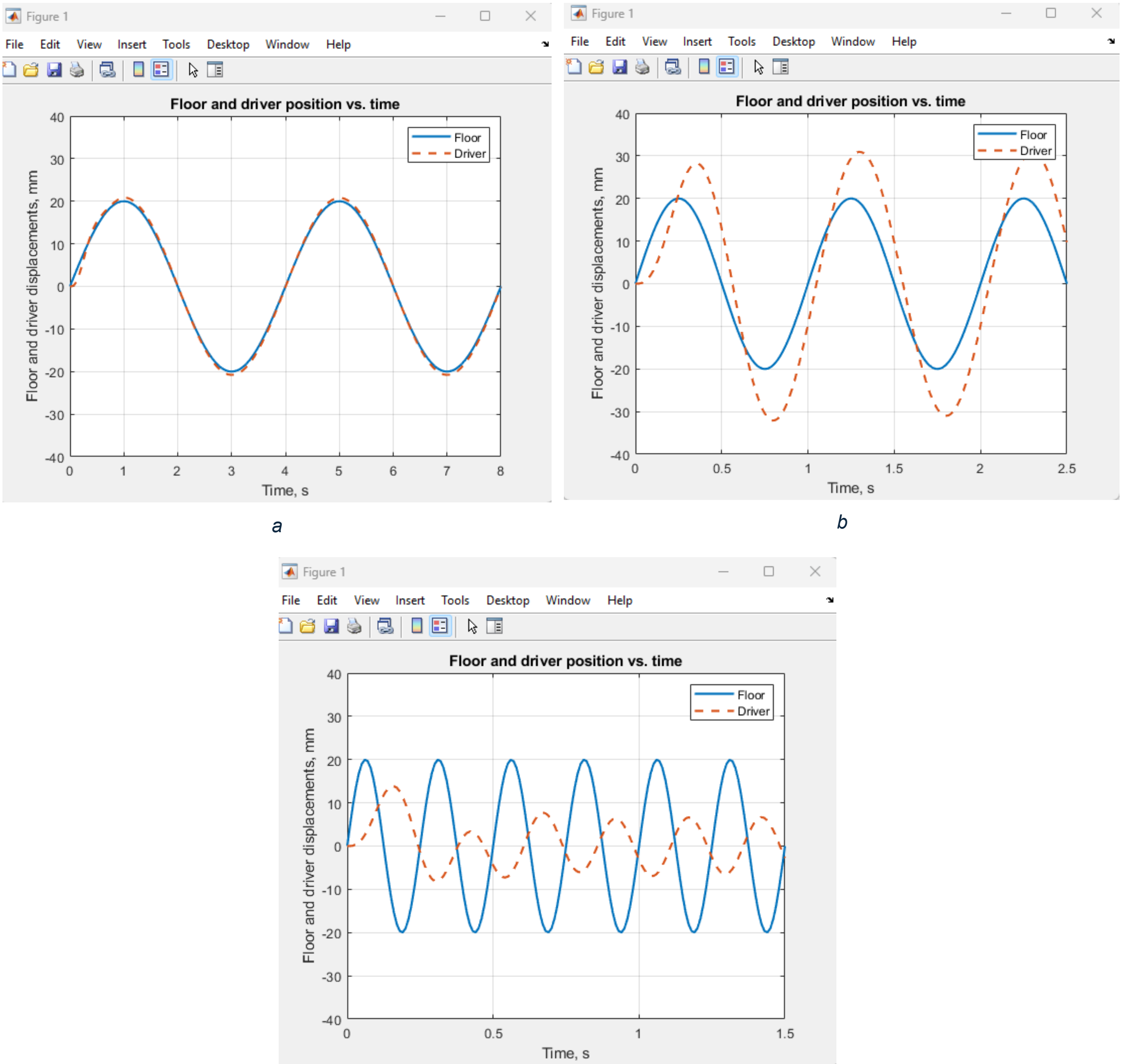


Figure 3.10: Simulink diagram for the seat-suspension system: sinusoidal input

Figure 3.10 shows the Simulink diagram for the seat-suspension system with a sinusoidal input for floor displacement $z_0(t) = a \sin\omega t$ (m). Note that the second input (floor velocity $\dot{z}_0$) is created by differentiating the displacement input (we could have inserted a second Sine Wave source with an amplitude of ωa and phase of $\pi/2$ to produce $\dot{z}_0(t) = \omega a \cos\omega t$). The amplitude of the floor displacement is fixed at a = 0.02 m (20 mm). Figures 3.11a–c show the frequency response of the driver-mass displacement z_2 from executing the Simulink model for three input frequencies of 0.25, 1, and 4 Hz.



a b c

Figure 3.11: Frequency response of driver-mass displacement z2: (a) input frequency = 0.25 Hz, (b) input frequency = 1 Hz, and (c) input frequency = 4 Hz.

Figure 3.11a shows that for the low-frequency case (0.25 Hz, or period= 4s), the driver and floor displacements are in phase with essentially the same amplitude (20 mm). Therefore, if we consider the transfer function G(s) that relates output z_2 (driver) to input $z_0(t)$ (floor), the magnitude of the corresponding sinusoidal transfer function $G(j\omega)$ is essentially unity when the input frequency is 0.25 Hz (or, $\omega = 1.57$ rad/s). Furthermore, the phase angle of $G(j1.57)$ is nearly zero (for 0.25 Hz), as there is no phase difference between the two sine waves in Fig. 3.11a Figure 3.11b shows that when the input frequency is 1 Hz (or, $\omega = 6.28$ rad/s), the amplitude ratio of the output/input sinusoids is about $31/20 = 1.55$, and the steady-state driver displacement is 55% greater than the floor displacement. Figure 3.11b also shows that the driver-mass sinusoid lags behind the input (floor) sinusoid. Figure 3.11c shows that when the input frequency is 4 Hz (or, $\omega = 25.13$ rad/s), the amplitude ratio of the output/input sinusoids is about $6.6/20 = 0.33$, and the steady-state driver displacement is 33% less than the floor displacement. The peaks of z_2 are nearly aligned with the valleys of z0. Hence, the phase lag between the two sinusoids is nearly 180°.

3.4 TRANSMISSIBILITY

Figure 3.12 illustrates the transmissibility $|z_2(t)|/|z_0(t)|$ for three different variations in seat-cushion stiffness: the nominal stiffness, denoted as k_2 = 8230 N/m, a 50% reduction in k_2, and a 50% increase in k_2. The input frequencies considered range from 0.1 to 5 Hz. Higher seat-cushion stiffness values result in the suppression of peak transmissibility and an elevation in the resonant frequency where the peak transmissibility occurs. Additionally, an increase in stiffness (k_2) leads to heightened transmitted vibrations at higher frequencies. Since vehicle vibrations primarily occur in the 2.5 Hz range, it is crucial to carefully select seat-cushion stiffness to achieve a balanced trade-off between minimizing peak transmissibility and maintaining low transmissibility around 2.5 Hz.

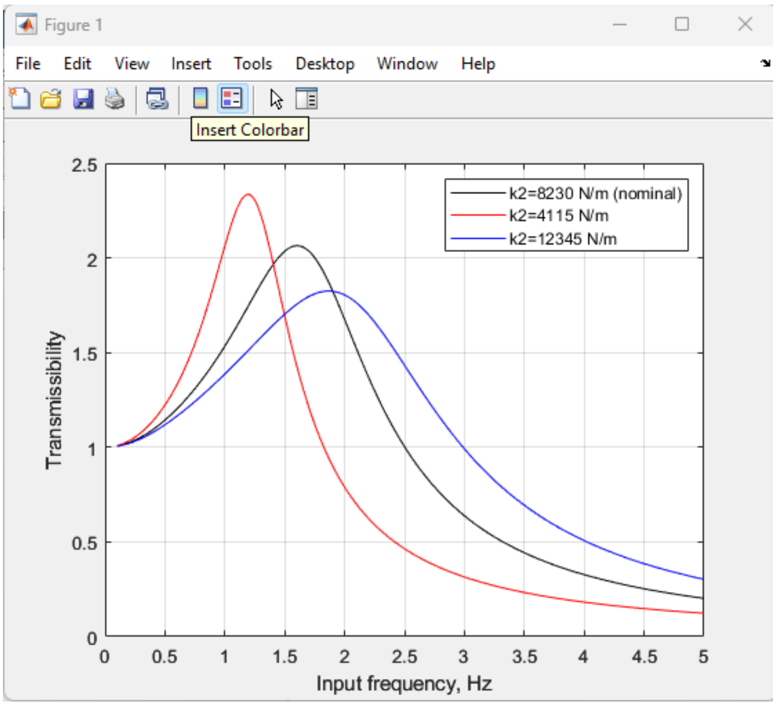


Figure 3.12: Transmissibility |z2|/|z0| for variations in seat-cushion stiffness k2

Considering these limitations, the nominal value of k_2 (8230 N/m) seems to yield satisfactory performance, given that the transmissibility at 2.5 Hz is approximately equal to one. If the priority is to minimize transmitted vibrations specifically at 2.5 Hz, rather than focusing on reducing peak transmissibility, opting for a reduction in seat-cushion stiffness appears to be a viable solution. This is supported by Figure 3.12, which indicates a high sensitivity of transmissibility at 2.5 Hz to changes in k_2.

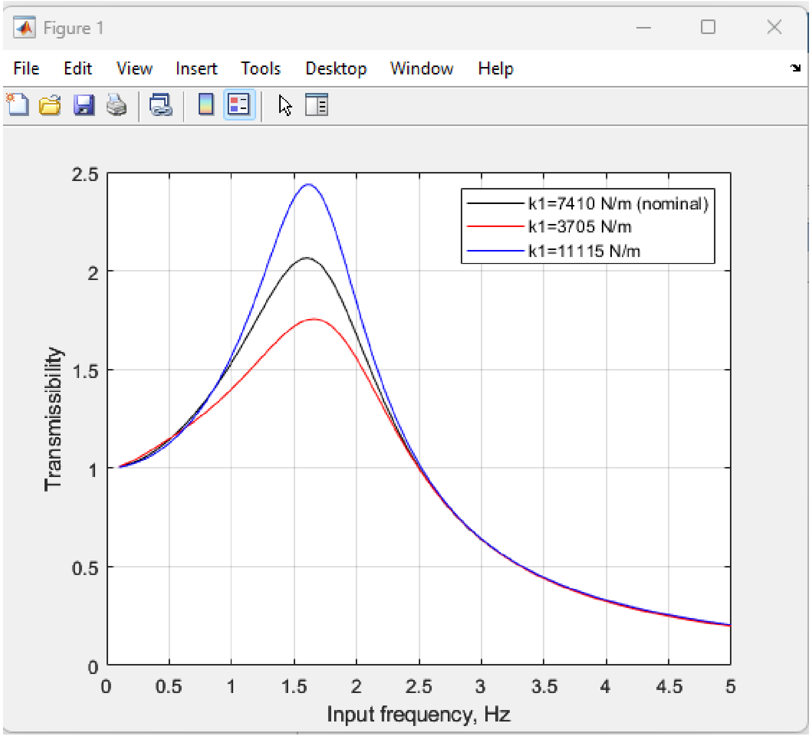


Figure 3.13: Transmissibility |z2|/|z0| for variations in suspension stiffness k1

The sensitivity to variations in suspension stiffness k1 was investigated. Figure 3.13 shows the transmissibility for three values of suspension stiffness: nominal k1 = 7410 N/m, a 50% reduction, and a 50% increase. Clearly, stiffer suspension springs increase the peak transmissibility, and varying k_1 has very little effect on the resonant frequency, or the transmissibility at higher frequencies. Varying suspension stiffness k_1 has almost no effect on transmissibility at 2.5 Hz, and therefore the lightest possible suspension spring k_1 will provide the best ride comfort for the driver.

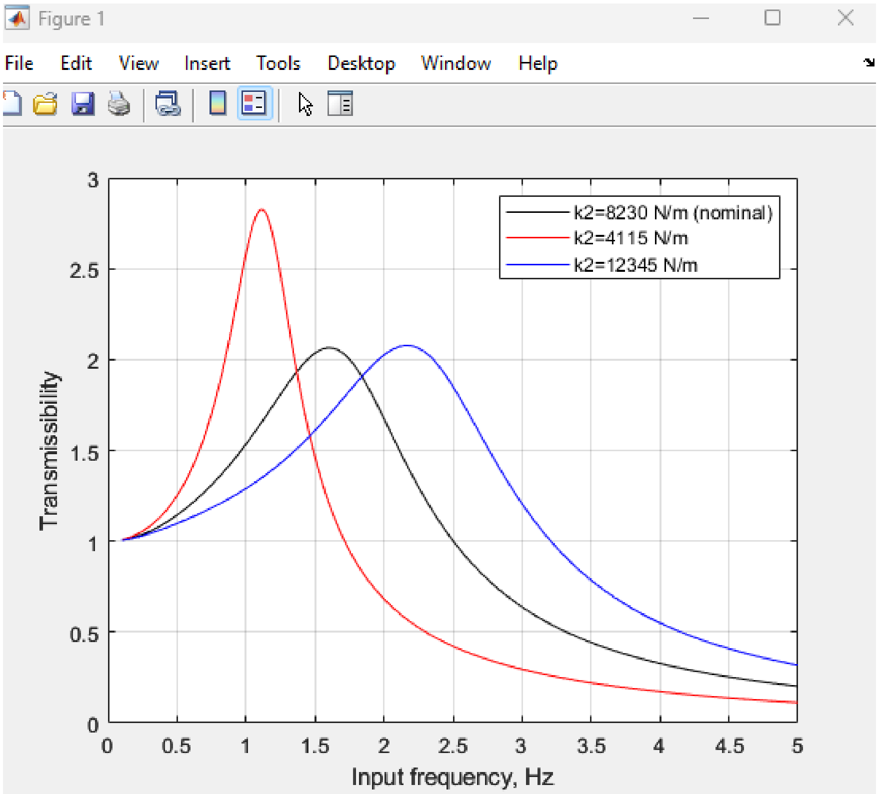


Figure 3.14: Transmissibility |z2|/|z0| for variations in suspension friction b1

Figure 3.14 displays the transmissibility for three different parametric changes in suspension friction coefficient: the nominal value, denoted as b_1 = 1430 N-s/m, a 50% reduction, and a 50% increase. Increased damping in suspensions results in the suppression of peak transmissibility and an elevation in the resonant frequency. An augmentation in friction coefficient b_1 results in an increase in transmitted vibrations at higher frequencies. Conversely, reducing b_1 below the nominal value has minimal impact on transmitted vibrations at higher frequencies. Figure 3.14 also indicates that the sensitivity of peak transmissibility to a decrease in suspension damping b_1 is more pronounced compared to the sensitivity observed in Figure 3.12 for variations in seat-cushion stiffness. Consequently, caution is necessary when contemplating options for decreasing suspension friction b_1, as the improvements in transmissibility at 2.5 Hz are relatively modest, and the corresponding rise in peak transmissibility at lower frequencies is accentuated.

4 MATLAB COMMAND

4.1 SINUSOIDAL INPUT

```
clear all
clc
%  run_seat_sine.m
%
%  M-file for setting up and simulating the seat-suspension system
%  with a sinusoidal input (frequency response)
%

%  mechanical system parameters
m1 = 20;        %  seat mass, kg
m2 = 50;        %  driver mass, kg
k1 = 7410;      %  suspension stiffness, N/m
k2 = 8230;      %  seat cushion stiffness, N/m
b1 = 1430;      %  suspension friction, N-s/m
b2 = 153;       %  seat cushion friction, N-s/m

%  SSR matrices
%  States:  x = [ z1 z1-dot z2 z2-dot ]'
%      x1 = z1 (seat mass position, m)
%      x2 = z1dot, m/s
%      x3 = z2 (driver mass position, m)
%      x4 = z2dot (driver mass velocity, m/s)
%  Inputs: u1 = z0
%          u2 = z0dot
A = [ 0 1 0 0 ; (-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ; 0 0 0 1 ; k2/m2 b2/m2 -k2/m2
-b2/m2 ];
B = [ 0 0 ; k1/m1 b1/m1 ; 0 0 ; 0 0 ];
C = [ 0 0 1 0 ];
D = zeros(1,2);

%  initial state (equilibrium)
x0 = [ 0 0 0 0 ]';

%  floor displacement amplitude, m
z0_amp = 0.02;          %  m

%  frequency
f = input('Enter input frequency in hertz (Hz) ');
w_rps = 2*pi*f;         %  input frequency in rad/s

%  end time
t_end = input(' Enter sim time ');

% run Simulink model
sim seat_sine

%  plot
figure(1)
plot(t,z0*1e3,'LineWidth',1.6)
hold on
plot(t,z2*1e3,'--','LineWidth',1.6)
title('Floor and driver position vs. time')
grid
axis([0 t_end -40 40 ])
```

```
  xlabel('Time, s')
ylabel('Floor and driver displacements, mm')
legend('Floor','Driver')
```

4.2 TRIANGULAR PULSE INPUT

```
%
%  run_seat_triangle_pulse.m
%
%  M-file for setting up and simulating the seat-suspension
%  system with a triangular pulse input
%

%  mechanical system parameters
m1 = 20;        %  seat mass, kg
m2 = 50;        %  driver mass, kg
k1 = 7410;      %  suspension stiffness, N/m
k2 = 8230;      %  seat cushion stiffness, N/m
b1 = 1430;      %  suspension friction, N-s/m
b2 = 153;       %  seat cushion friction, N-s/m


%  SSR matrices
%  States:  x = [ z1 z1-dot z2 z2-dot ]'
%      x1 = z1 (seat mass position, m)
%      x2 = z1dot, m/s
%      x3 = z2 (driver mass position, m)
%      x4 = z2dot (driver mass velocity, m/s)
%  Inputs: u1 = z0
%          u2 = z0dot ( = u1dot)
A = [ 0 1 0 0 ; (-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ; 0 0 0 1 ; k2/m2 b2/m2 -k2/m2
-b2/m2 ];
B = [ 0 0 ; k1/m1 b1/m1 ; 0 0 ; 0 0 ];

%  Output:  y1 = z2 = driver position, m
%           y2 = z2ddot = driver acceleration, m/s^2
C = [ 0 0 1 0 ; k2/m2 b2/m2 -k2/m2 -b2/m2 ];
D = zeros(2,2);

%  initial state (equilibrium)
x0 = [ 0 0 0 0 ]';

%  triangular pulse input
z0dot = 5.4;                %  vertical rate of triangular pulse, m/s
z0_max = 0.03;              %  max height of triangle pulse, m
dt_peak = z0_max/z0dot;     %  time to reach peak of triangle pulse, sec

%  end time
t_end = 3;

% run Simulink model
sim seat_triangle_pulse

%  Simulation output
y1 = y(:,1);
```

```
y2 = y(:,2);

%  plots
figure(1)
plot(t,y1*1e3,'LineWidth',2)
grid
title('Driver-mass displacement vs. time')
xlabel('Time, s')
ylabel('Driver-mass displacement, z_2(t), mm')

figure(2)
plot(t,y2,'LineWidth',2)
grid
title('Driver acceleration vs. time')
xlabel('Time, s')
ylabel('Driver acceleration, m/s^2')
```

4.3 TRANSMISSIBILITY

```
% M-file for computing transmissibility for
% the seat-suspension system
% mechanical system parameters
m1 = 20; % seat mass, kg
m2 = 50; % driver mass, kg
k1 = 7410; % suspension stiffness, N/m
k2 = 8230; % seat cushion stiffness, N/m
b1 = 1430; % suspension friction, N-s/m
b2 = 153; % seat cushion friction, N-s/m
% State-space representation:
% x = [ z1 ; z1dot-b1*z0/m1 ; z2 ; z2dot ]'
% u = z0(t) (floor displacement)
Arow1 = [ 0 1 0 0 ];
Arow2 = [(-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ];
Arow3 = [ 0 0 0 1 ];
Arow4 = [ k2/m2 b2/m2 -k2/m2 -b2/m2 ];
A = [ Arow1 ; Arow2 ; Arow3 ; Arow4 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;
% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 500;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz
for i=1:Npts
w_in = w_Hz(i)*2*pi; % input frequency in rad/s
[mag,phase] = bode(sys,w_in);
TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency
plot(w_Hz,TR,'black')
grid
xlabel('Input frequency, Hz')
ylabel('Transmissibility')
hold on
```

```
m1 = 20; % seat mass, kg
m2 = 50; % driver mass, kg
k1 = 7410; % suspension stiffness, N/m
k2 = 4115; % seat cushion stiffness, N/m
b1 = 715; % suspension friction, N-s/m
b2 = 153; % seat cushion friction, N-s/m
% State-space representation:
% x = [ z1 ; z1dot-b1*z0/m1 ; z2 ; z2dot ]'
% u = z0(t) (floor displacement)
Arow1 = [ 0 1 0 0 ];
Arow2 = [(-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ];
Arow3 = [ 0 0 0 1 ];
Arow4 = [ k2/m2 b2/m2 -k2/m2 -b2/m2 ];
A = [ Arow1 ; Arow2 ; Arow3 ; Arow4 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;
% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 500;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz
for i=1:Npts
w_in = w_Hz(i)*2*pi; % input frequency in rad/s
[mag,phase] = bode(sys,w_in);
TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency
plot(w_Hz,TR,'r')
grid
xlabel('Input frequency, Hz')
ylabel('Transmissibility')
hold on

m1 = 20; % seat mass, kg
m2 = 50; % driver mass, kg
k1 = 7410; % suspension stiffness, N/m
k2 = 12345; % seat cushion stiffness, N/m
b1 = 2145; % suspension friction, N-s/m
b2 = 153; % seat cushion friction, N-s/m
% State-space representation:
% x = [ z1 ; z1dot-b1*z0/m1 ; z2 ; z2dot ]'
% u = z0(t) (floor displacement)
Arow1 = [ 0 1 0 0 ];
Arow2 = [(-k1-k2)/m1 (-b1-b2)/m1 k2/m1 b2/m1 ];
Arow3 = [ 0 0 0 1 ];
Arow4 = [ k2/m2 b2/m2 -k2/m2 -b2/m2 ];
A = [ Arow1 ; Arow2 ; Arow3 ; Arow4 ];
B = [ b1/m1 ; (-b1*b1 - b1*b2 + k1*m1)/m1^2 ; 0 ; b1*b2/(m1*m2) ];
% output y = z2 = x3 (driver displacement)
C = [ 0 0 1 0 ];
D = 0;
% build SSR system
sys = ss(A,B,C,D);
% Loop for computing TR for range of input frequency
Npts = 500;
w_Hz = linspace(0.1,5,Npts); % range of frequency: 0.1 --> 5 Hz
```

```
  for i=1:Npts
w_in = w_Hz(i)*2*pi; % input frequency in rad/s
[mag,phase] = bode(sys,w_in);
TR(i) = mag; % transmissibility = |z2|/|z0|
end
% Plot TR vs input frequency
plot(w_Hz,TR,'blue')
grid
xlabel('Input frequency, Hz')
ylabel('Transmissibility')

hleng=legend('k2=8230 N/m (nominal)','k2=4115 N/m','k2=12345 N/m');
```

5 REFERENCES

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