MARMARA UNIVERSITY
FACULTY OF ENGINEERING

Optimal Control of Active Suspension Systems

Sergen Cepdibi

GRADUATION PROJECT REPORT

Department of Mechanical Engineering

Supervisor

Assoc. Prof. İbrahim Sina KUSEYRİ

ISTANBUL, 2024

MARMARA UNIVERSITY
FACULTY OF ENGINEERING

Optimal Control of Active Suspension Systems

by

Sergen Cepdibi

June 2024 , Istanbul

SUMBITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF

BACHELOR OF SCIENCE

AT

MARMARA UNIVERSITY

Signature of Author(s)

Department of Mechanical Engineering

Certified By ……………………………………………………………………………

Project Supervisor, Department of Mechanical Engineering

Accepted By ……………………………………………………………………………

Head of the Department of Mechanical Engineering

ACKNOWLEDGEMENT

First of all, I would like to thank my supervisor **Assoc. Prof. Dr. İbrahim Sina KUSEYRİ** for the valuable guidance and advice on preparing this thesis and giving me moral and material support.

June 2024 Sergen Cepdibi

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ABSTRACT

Vehicle suspension systems are pivotal in balancing road handling and passenger comfort, yet passive suspensions inherently entail a compromise between these aspects. Active suspensions address this trade-off by directly controlling suspension force actuators. However, designing an active suspension system poses a complex control challenge. This study aims to tackle and compare this issue through different control techniques, particularly focusing on Linear Quadratic Regulator (LQR) techniques, while analyzing a quarter-car model. It is essential to note that real-world suspension systems are more intricate than simple spring-mass-damper systems, involving interactions between four wheels and potentially varying actuator dynamics. Our objective is to provide comprehensive insights by employing various techniques and conducting MATLAB and Simulink simulations. Our approach involves formulating the equations of motion for a vehicle's suspension system and defining performance metrics for passenger comfort(body acceleration) and road holding(suspension travel). Based on this mathematical framework, we create a Simulink model of the vehicle suspension system and simulate its behavior under various road conditions. To establish a baseline, we conduct an open-loop passive suspension system simulation. Subsequently, we develop and implement an LQR (Linear Quadratic Regulator) controller to optimize the trade-off between passenger comfort and road holding. We then perform simulations for both the open-loop passive system and the LQR-controlled system, analyzing and comparing their performance by evaluating body acceleration, suspension travel. Graphs illustrating body acceleration, suspension deflection, body deflection and road disturbances at the same time to get a comprehensive look. Finally, we assess the effectiveness of the LQR control strategy in achieving the desired balance between passenger comfort and road holding.

SYMBOLS

m_b : Quarter Car Body Mass

m_w : Wheel Mass

c_s Damping Coefficient of the Suspension System

k_s : Spring Constant of Suspension System

k_w : Spring Constant of the Wheel

x(t) : State vector

u(t): Control Vector

A : State Matrix

B : Input Matrix

C : Output Matrix

D : Feedforward Matrix

J : Cost function of the LQR Control

K: Gain Matrix:

Q:Symmetric Positive Semidefinite Matrix – State Weight/Penalty

R: Symmetric Positive Semidefinite Matrix- Control Weighth/Penalty

ABBREVIATIONS

LQR : Linear Quadratic Regulator

LTI :Linear Time Invariant

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1. INTRODUCTION

Vehicle suspension systems are crucial components in ensuring the balance between road handling and passenger comfort. Achieving an optimal balance between passenger comfort and road holding is a critical challenge, and traditional passive suspension systems often struggle to provide an optimal balance between these two conflicting objectives. Passenger comfort is primarily influenced by body acceleration, while road holding is determined by the vehicle's ability to maintain tire contact with the road surface, which is largely influenced by suspension travel. Passive suspensions, though widely used, require a compromise between these two aspects. This compromise is due to the inherent limitations of passive suspension systems, which cannot actively adapt to changing road conditions or adjust to meet specific operational requirements. To overcome these limitations, active suspension systems have emerged as a potential solution. These active suspension systems use actuators to directly control the suspension forces, allowing for real-time adjustments and improved performance in terms of both handling and comfort. Active suspension systems have garnered significant interest in the field of vehicle dynamics, as they offer the potential for enhanced ride quality, improved vehicle stability, and increased road holding capabilities.

A critical method for optimizing active suspension systems is the use of Linear Quadratic Regulator (LQR) control. LQR is an optimal control strategy that aims to minimize a cost function, which is typically a weighted sum of the states and control inputs of the system. By carefully selecting the weights, LQR can achieve a desirable trade-off between ride comfort (minimizing body acceleration) and road holding (maintaining tire contact with the road). The application of LQR in suspension systems allows for precise control over the suspension dynamics, resulting in significant improvements in both stability and comfort compared to passive systems. Even though this study focuses on a quarter-car analysis, the use of LQR techniques can effectively handle varying actuator dynamics and interactions between multiple wheels in real-world suspension systems. The LQR approach is particularly beneficial for managing the complex dynamics of multi-wheel suspension systems, offering a pathway towards more advanced and responsive vehicle suspension designs. Our study can be considered a steppingstone towards understanding and designing these real-world suspension systems. The objective of this study is to apply LQR techniques on a quarter-car model to analyze and optimize the performance of vehicle suspension systems, thereby enhancing both passenger comfort and road holding capabilities.

2. LITERATURE REVIEW

2.1. Vehicle Suspension Systems

Vehicle suspension systems are crucial for ensuring passenger comfort and vehicle stability by isolating the vehicle body from road irregularities. These systems can be passive or active, with active systems offering more advanced control capabilities to enhance stability in various driving conditions. The design and optimization of suspension systems involve methodologies such as using optimization algorithms like genetic algorithms for multi-objective optimization, sliding mode control for efficiency improvement, and fuzzy logic control for mimicking active systems in passive designs. Additionally, Linear Quadratic Regulator (LQR) control is used for its optimal control strategy to balance ride comfort and handling stability.

Passive suspension systems, which consist of fixed mechanical components like springs and dampers, are simple and cost-effective but often require a compromise between ride comfort and road handling. In contrast, active suspension systems employ actuators that adjust the suspension characteristics in real-time based on feedback from various sensors. This allows active systems to provide superior performance by continuously adapting to changing road conditions and driving dynamics.

LQR control is a powerful technique in active suspension systems due to its ability to systematically balance multiple performance objectives. The LQR controller minimizes a cost function, typically a combination of state variables (such as body acceleration and suspension deflection) and control inputs (such as actuator forces). By adjusting the weights in the cost function, the LQR controller can achieve a desired trade-off between ride comfort and road handling, making it highly effective in optimizing suspension performance.

Overall, the field of vehicle suspension systems is continuously evolving, driven by the need for better ride comfort, stability, and performance. The use of advanced control strategies like LQR, along with optimization algorithms and integration with other vehicle systems, is paving the way for more advanced and efficient suspension systems in modern vehicles.

2.2.Linear Quadratic Regulator(LQR)

The Linear Quadratic Regulator (LQR) is an optimal control strategy designed to regulate the behavior of a dynamic system to achieve the best performance according to a predefined criterion. The LQR method aims to minimize a cost function, typically a quadratic function of the system states and control inputs. This cost function is carefully designed to balance performance metrics such as passenger comfort and road holding in vehicle suspension systems.

LQR controller is effectively a full state feedback controller where the gain matrix K is computed in a very particular fashion. The gain matrix K is derived by solving the Riccati equation, which ensures that the control inputs are optimal in minimizing the predefined cost function. This results in a controller that provides the best possible trade-off between different performance metrics, such as minimizing body acceleration for comfort and maintaining tire contact for road holding.

One of the significant advantages of the LQR controller is its ability to address some of the practical implementation issues encountered with full state feedback controllers. These issues include control saturation, where the control inputs exceed their physical limits, and the inability to measure all the states of the system directly. By incorporating these considerations into the design process, LQR controllers can be effectively implemented in real-world applications, providing robust and optimal performance.

In vehicle suspension systems, the LQR controller can adapt to varying road conditions and dynamically adjust the suspension forces to enhance both comfort and stability. The controller achieves this by continuously monitoring the system states and applying the optimal control inputs derived from the gain matrix K. This results in a system that is not only responsive but also capable of maintaining optimal performance across a wide range of operating conditions.

3. Linear Quadratic Regulator Controller Design

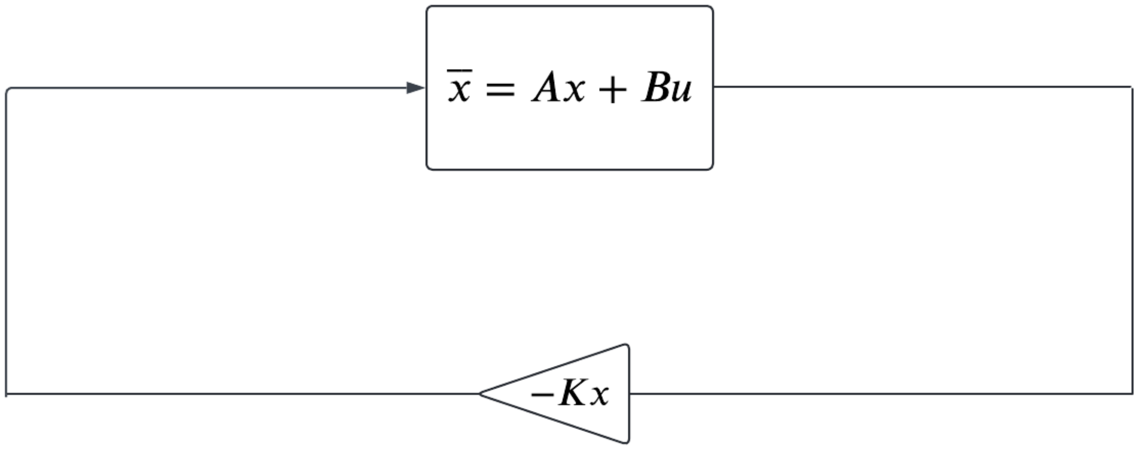

Figure 1- Block Diagram for LQR Controller

Since our system is controllable-the rank of the controllability matrix is 4-we can design a full state feedback controller $u = -Kx$-to place the eigenvalues of the system wherever desired. The control law is defined as:

$$u = -Kx$$

Thus, the system dynamics become:

$$\dot{x} = Ax + Bu$$

$$\dot{x} = (A - BK)x$$

By applying this control law, we can stabilize our system. However, the challenge lies in determining the optimal placement of the poles, or equivalently, the best eigenvalues for the system. Identifying the optimal eigenvalues through trial and error can be time-consuming and inefficient.

An exceptionally powerful tool in control theory that addresses this challenge is the Linear Quadratic Regulator (LQR). LQR controller provides a systematic approach to determine the optimal eigenvalues by minimizing a cost function. The idea is to formulate a cost function that quantifies the performance of the system, considering both the state and the control input.

The cost function is defined as:

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]\, dt$$

where Q is a symmetric positive semi-definite matrix that penalizes deviations of the state $x(t)$ from the desired trajectory, and R is a symmetric positive definite matrix that penalizes the control effort $u(t)$. In our case, Q is a 4x4 matrix.

The term $x^T Qx(t)$ represents the penalty on the state deviation, and $u^T Ru(t)$ represents the penalty on the control effort. By appropriately choosing the weights in the Q matrix, we can specify the relative importance of different states. For instance, increasing the weight of a particular state in Q will place a higher penalty on deviations of that state from its desired value. Similarly, the weights in the R matrix determine the penalty on the control effort. Reducing the corresponding elements in Rwill allow for more aggressive control actions.

The LQR method then computes the optimal gain matrix Kthat minimizes the cost function J.This optimal K not only stabilizes the system but also ensures that the state x converges to zero in an optimal manner. The resulting control law minimizes the trade-off between state deviations and control effort, providing an efficient and effective means of stabilizing the system.

3.1. Solving the Optimization Problem

Algebraic Ricatti equation is a type of nonlinear equation that frequently arises in the context of infinite-horizon optimal control problems, whether in continuous time or discrete time. This equation plays a crucial role in determining the optimal state feedback control law for linear-quadratic regulator (LQR) problems. The Ricatti equation characterizes the relationship between the state variables and the control inputs in a way that minimizes a specified cost function over an infinite time horizon.

In continuous time, the algebraic Riccati equation is typically formulated as:

$$A^T S + SA - SBR^{-1}B^T S + Q = 0$$

where S is the symmetric positive definite solution matrix, is the symmetric positive definite solution matrix, A, and B are the system matrices, Q is the state weighting matrix, and R is the control weighting matrix.

$$K = R^{-1}B^T S$$

where K is the optimal gain matrix.

We know that:

$$A_{cl} = A - BK$$

After substituting the values of K derived from the algebraic Riccati equation into the closed-loop system equation, we achieve a configuration where the eigenvalues of the closed-loop system matrix $A - BK$ are located in the left half of the complex plane. This ensures that all the eigenvalues have negative real parts, which is a fundamental criterion for system stability.

3.2.Procedure for LQR

We will proceed through the following systematic steps in the design of our Linear Quadratic Regulator (LQR) controller:

1-Given A and B matrix.

2-Choose weighting/penalty matrices Q and R

3- Solve Algebraic Riccati Equation for S

4- Compute K

5- Choose the K solution that yields a stable system.

4. MATHEMATICAL MODELING OF A VEHICLE SUSPENSION SYSTEM

Representing real physical systems as mathematical models is advantageous because it allows for systematic design, analysis, and simulation of these systems. Different types of mathematical models can be chosen such as differential equations, state space representations, transfer functions, each capturing information in a manner that is useful for various situations. This flexibility enables researchers to select the most appropriate modeling approach to address specific aspects of their study, thereby enhancing the understanding and optimization of complex systems. In this paper, we will look at the quarter car suspension system, and use 3 of these approaches; differential equations, state space and transfer functions in Laplace domain.

4.1.Linear Hydraulic Motor

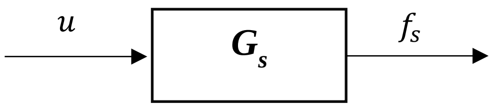

Figure 1: For the Linear Motor Dynamics

A linear motor, often used in electromagnetic suspension systems, can be modeled as a linear actuator. The transfer function of a linear motor typically includes aspects such the mechanical dynamics of the moving part. For the sake of simplicity, we can consider a standard model for a linear motor used in control systems. You can approximate the system as a first-order transfer function,

$$G(s) \approx \frac{K}{\tau s + 1}$$

where K is the gain, tau is the time constant

With the mass of 10 kg and damping of 600 Ns/m and normalized gain of 1:

Note that: These values are consistent with parameters used in research and design of automotive suspension systems. For more detailed information, you can refer to sources such as the IOSR Journal of Mechanical and Civil Engineering (IOSR Journals)

$$K = 1$$

$$\tau = \frac{m}{c} = \frac{1}{600} = 0.0167s$$

$$G(s) \approx \frac{1}{(0.0167)s + 1}$$

The system's response, which is a simple, stable, exponential decay with a time constant of 0.02 seconds. In essence, this approach reduces complexity, making the system easier to understand and control, while still providing a realistic approximation of its behavior.

4.2.Modeling of a Quarter Car

Designing an automotive suspension system presents a complex control challenge. To simplify the problem, engineers often use a quarter-car model, representing one of the vehicle's four wheels, as a 1-D multiple spring-damper system. Figure 1 illustrates this simplified model, which serves as the foundation for our design and analysis. This model pertains to an active suspension system, where an actuator is integrated to generate a control force U, allowing precise control of the vehicle body's motion.

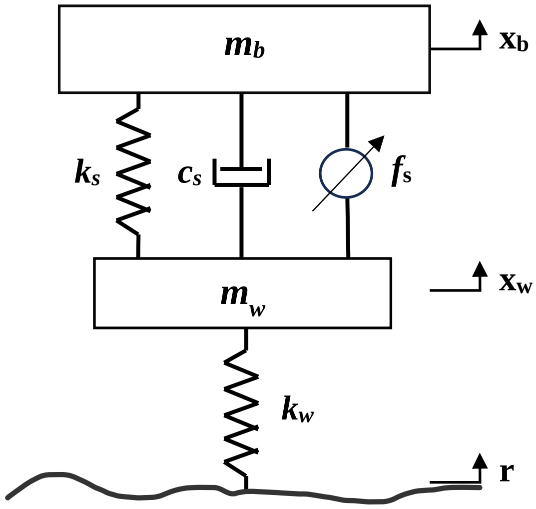

Figure 2: Quarter Car Active Suspension System Model

The key difference between the passive and active suspension model is control input existence.

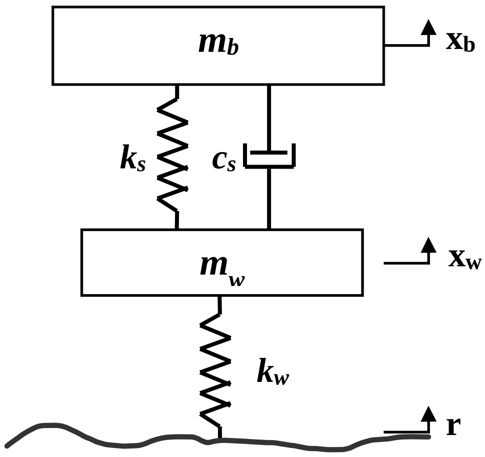

Figure 3: Quarter Car Passive Suspension Model

The primary distinction between active and passive suspension models lies in their adaptive capabilities: active suspension systems employ actuators and an electronic control unit to continuously monitor and dynamically adjust the suspension settings in real-time, optimizing for varying road conditions and driving scenarios, thereby enhancing both comfort and handling; in contrast, passive suspension systems rely on fixed mechanical components such as springs and dampers, which provide a static balance between ride comfort and vehicle control, lacking the ability to adapt to changing conditions. In short, the key difference between passive and active suspensions is control input: active suspensions dynamically adjust via actuators and an electronic control unit, while passive suspensions rely on fixed mechanical components.

Here is the system parameters and their values which we are going to use in this study, can be seen the table below. The data provided for the quarter car model parameters are representative values for a BMW 530i.

Table 1: System Parameters and Their Values

Description	**Symbols**	**Values**	**Deviation**	**Unit**
Quarter Car Body Mass	m_b	395.3	-42.77, +75.38	kg
Wheel Mass	m_w	48.3	0	kg
Damping Coefficient of the Suspension System	c_s	1200	-550, + 250	N·s/m
Spring Constant of Suspension System	k_s	30,000	0	N/m
Spring Constant of the Wheel	k_w	340,000	±30000	N/m

4.3.Free Body Diagrams and Equation of Motion

Taking advantage of the Superposition Theorem which states the effect of the forces is the sum of the individual effects of the forces considered separately and the Newton's Second Law of Motion which states the acceleration of an object depends on the mass of the object and the amount of force applied we can analyze our system and get equation of motions.

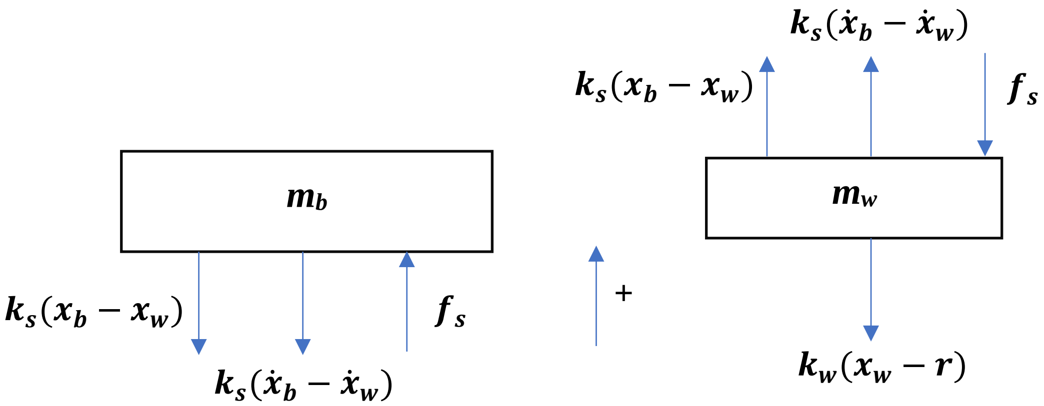

Figure 4: Free Body Diagram of the Active Suspension Model

$$\sum F = m\ddot{x} \tag{1}$$

$$m_b\ddot{x}_b = -k_s(x_b - x_w) - c_s(\dot{x}_b - \dot{x}_w) + f_s \tag{2}$$

$$m_b\ddot{x}_b = -k_s x_b - c_s\dot{x}_b + k_s x_w + c_s\dot{x}_w + f_s \tag{3}$$

$$\boxed{\ddot{x}_b = -\frac{k_s}{m_b}x_b - \frac{c_s}{m_b}\dot{x}_b + \frac{k_s}{m_b}x_w + \frac{c_s}{m_b}\dot{x}_w + \frac{f_s}{m_b}} \tag{4}$$

$$m_w\ddot{x}_w = k_s(x_b - x_w) + c_s(\dot{x}_b - \dot{x}_w) - k_w(x_w - r) - f_s \tag{5}$$

$$m_w\ddot{x}_w = k_s x_b - k_s x_w + c_s\dot{x}_b - c_s\dot{x}_w - k_w x_w + k_w r - f_s \tag{6}$$

$$m_w\ddot{x}_w = k_s x_b + c_s\dot{x}_b + (-k_s - k_w)x_w - c_s\dot{x}_w + k_w r - f_s \tag{7}$$

$$\boxed{\ddot{x}_w = \frac{k_s}{m_w}x_b + \frac{c_s}{m_w}\dot{x}_b + \frac{(-k_s - k_w)}{m_w}x_w - \frac{c_s}{m_w}\dot{x}_w + \frac{k_w}{m_w}r - \frac{f_s}{m_w}} \tag{8}$$

4.4.State Space Model for LQR

Let us define the state variables as follows:

$$x_1 = x_b \tag{9}$$

$$x_2 = \dot{x}_b \tag{10}$$

$$x_3 = x_w \tag{11}$$

$$x_4 = \dot{x}_w \tag{12}$$

Hence,

$$\dot{x}_1 = x_2 \tag{13}$$

$$\dot{x}_3 = x_4 \tag{14}$$

Then, the equations(4)(8) become:

$$\dot{x}_2 = -\frac{k_s}{m_b}x_1 - \frac{c_s}{m_b}x_2 + \frac{k_s}{m_b}x_3 + \frac{c_s}{m_b}x_4 + \frac{f_s}{m_b} \tag{15}$$

$$\dot{x}_4 = \frac{k_s}{m_w}x_1 + \frac{c_s}{m_w}x_2 + \frac{(-k_s - k_w)}{m_w}x_3 - \frac{c_s}{m_w}x_4 + \frac{k_w}{m_w}r - \frac{f_s}{m_w} \tag{16}$$

Our state space model consists of:

$$\dot{x} = Ax + Bu + Hd \tag{17}$$

$$y = Cx + Du \tag{18}$$

where:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{19}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \tag{20}$$

and our inputs and outputs:

$$u = [f_S] \tag{21}$$

$$d = [r] \tag{22}$$

$$y = \begin{bmatrix} x_b \\ \dot{x}_b \\ x_w \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{23}$$

and corresponding state matrices,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_b} & -\frac{k_s}{m_b} & \frac{k_s}{m_b} & \frac{c_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{c_s}{m_w} & \frac{(-k_s - k_w)}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} \tag{24}$$

$$B = \begin{bmatrix} 0 \\ \frac{10^3}{m_b} \\ 0 \\ -\frac{10^3}{m_w} \end{bmatrix} \tag{25}$$

$$H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_w}{m_w} \end{bmatrix} \tag{26}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

,our state space model will be:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_b} & -\frac{k_s}{m_b} & \frac{k_s}{m_b} & \frac{c_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{c_s}{m_w} & \frac{(-k_s - k_w)}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{10^3}{m_b} \\ 0 \\ -\frac{10^3}{m_w} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_w}{m_w} \end{bmatrix} d \tag{29}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \tag{30}$$

4.5.LQR Controller Design

If the follow the procedure, mentioned in the Section 3.2:

1- Given A and B matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_b} & -\frac{k_s}{m_b} & \frac{k_s}{m_b} & \frac{c_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{c_s}{m_w} & \frac{(-k_s-k_w)}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} \qquad (31)$$

$$B = \begin{bmatrix} 0 \\ \frac{10^3}{m_b} \\ 0 \\ -\frac{10^3}{m_w} \end{bmatrix} \qquad (32)$$

2- Choose weighting/penalty matrices Q and R

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (33)$$

$$R = [1] \qquad (34)$$

3-4-5 Solve Algebraic Riccati Equation for S, compute K, and choose the K solution that yields a stable system.

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \qquad (35)$$

Best value of S:

$$S = \begin{bmatrix} 107.1380 & 0.5374 & -9.7676 & -0.0576 \\ 0.5374 & 1.5869 & -5.8166 & 0.0227 \\ -9.7676 & 1.1769 & 157.0150 & -0.0691 \\ -0.0576 & 0.0227 & -0.0691 & 0.0181 \end{bmatrix} \qquad (36)$$

$$K = R^{-1}B^T S \qquad (37)$$

Corresponding best value of K that yields stability:

$$K = [0.1662 \quad 3.5450 \quad -13.2843 \quad -0.3172] \qquad (38)$$

Here is the eigenvalues for the $A_{cl} = A - BK$ *:*

$$E = \begin{bmatrix} -5.4157 + 6.4392i \\ -5.4157 - 6.4392i \\ -16.2918 + 85.5725i \\ -16.2918 - 85.5725i \end{bmatrix} \tag{39}$$

4.6.Simulink Models

The Simulink modeling process involves creating a graphical representation of the vehicle suspension system based on its mathematical equations of motion. This process begins with defining the system components, such as masses, springs, and dampers, and then connecting them using Simulink blocks to reflect their dynamic interactions. Input parameters, such as road profiles and external forces, are incorporated to simulate real-world conditions. Control algorithms, including the LQR controller, are implemented within the model to regulate the system's response. The model is then configured to run simulations, allowing for the analysis of system behavior under various scenarios and the assessment of performance metrics such as body acceleration, suspension travel. This visual and interactive approach facilitates the understanding, testing, and optimization of complex dynamic systems.

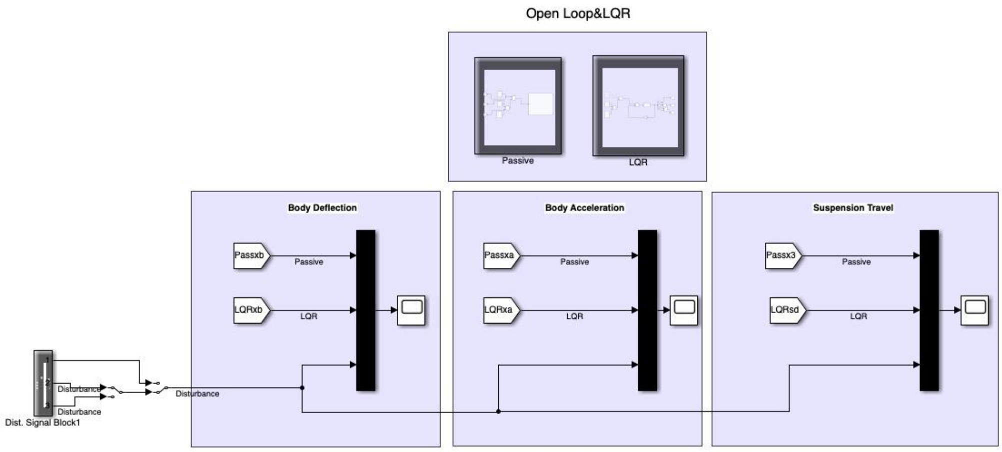

Figure 5: Main Frame of the Simulink Models

4.6.1. Passive System

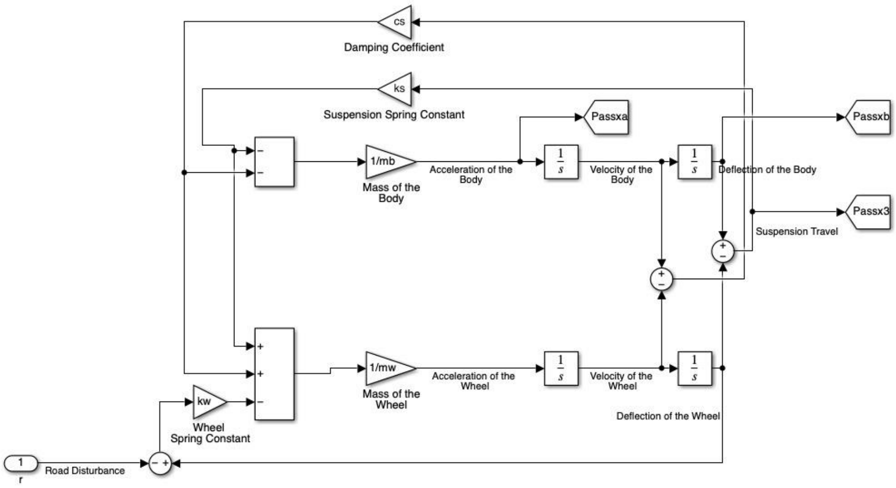

Figure 6 :Passive Suspension Quarter Car Block

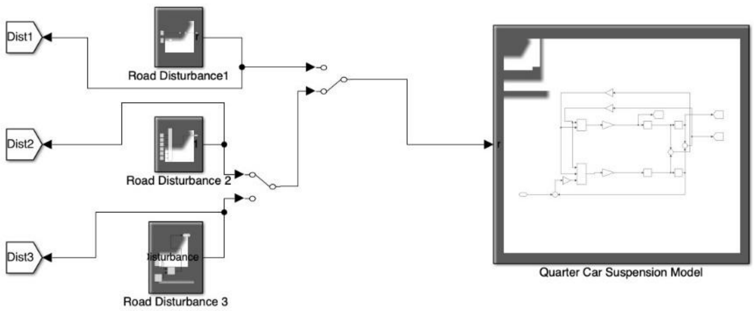

Figure: Passive Suspension Main Block

4.6.2. Active System

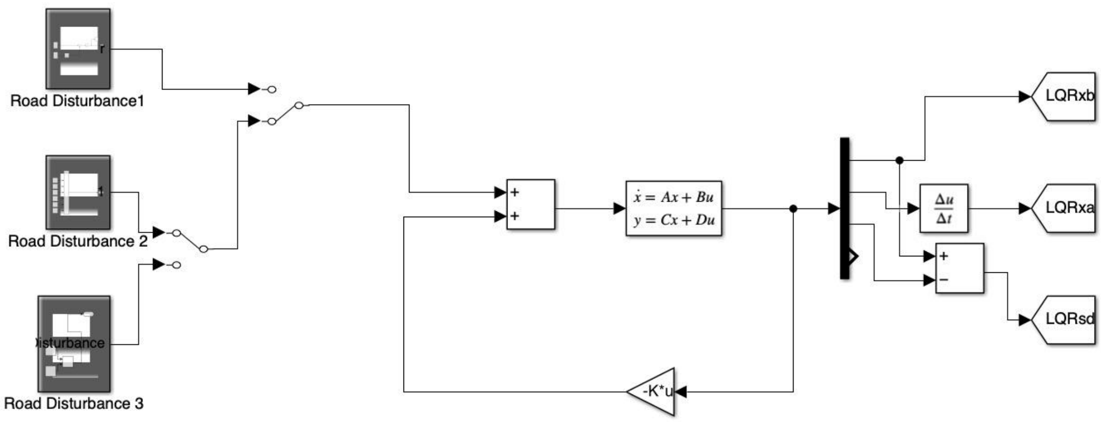

Figure 7: LQR Controller Design Block

5. SIMULATION & RESULTS

5.1.Road Profiles

Road profiles refer to the variations and irregularities in the surface of a road, including bumps, potholes, and undulations. These profiles significantly impact the performance and comfort of a vehicle's suspension system, as they introduce dynamic disturbances that the suspension must absorb and mitigate to ensure a smooth ride and maintain vehicle stability. Accurate modeling of road profiles is essential for designing and testing effective suspension systems.

For the simulation, we consider three distinct types of road disturbances to comprehensively evaluate the performance of the suspension system.

5.1.1. Road Profile 1

$$d_1(t) = \begin{cases} 0.05(1 - cos(8\pi t)) & 0.25 \leq t \leq 0.5 \\ 0 & otherwise \end{cases}$$

where time in seconds, and d in meters.

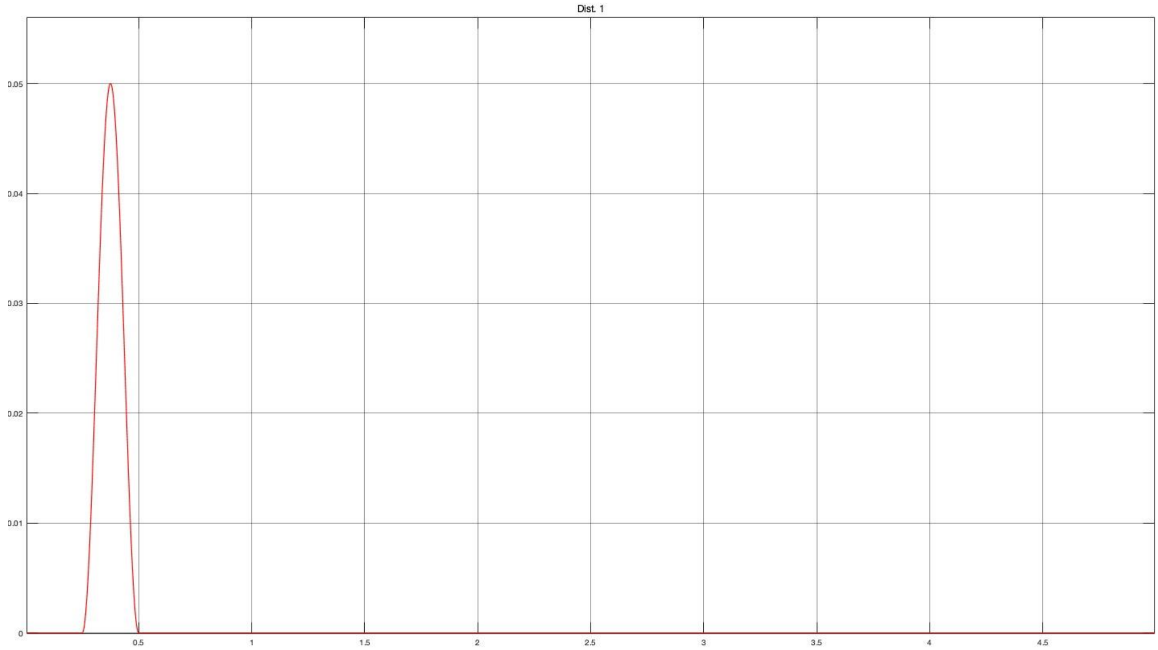

Figure 8: Road Profile 1 – Elevation(m) versus Time(s) Graph

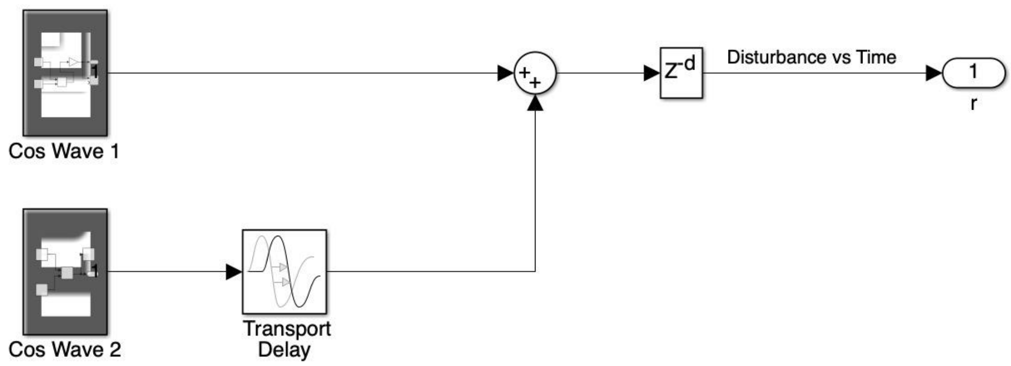

Figure 9: Road Profile 1 Subblock in Simulink

5.1.2. Road Profile 2

$$d_2(t) = \begin{cases} 0.08 & 0.5 \leq t \leq 0.6 \\ 0.06 & 3.5 \leq t \leq 3.6 \\ 0.12 & 7 \leq t \leq 7.6 \\ 0 & otherwise \end{cases}$$

where time in seconds, and d in meters

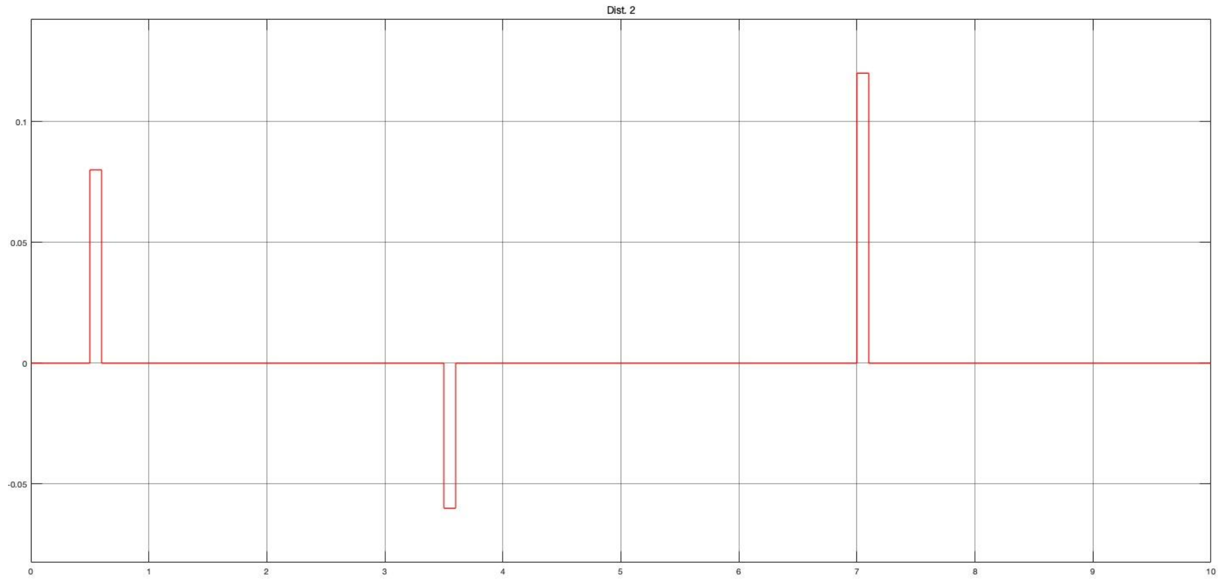

Figure 10: Road Profile 2 – Elevation(m) versus Time(s) Graph

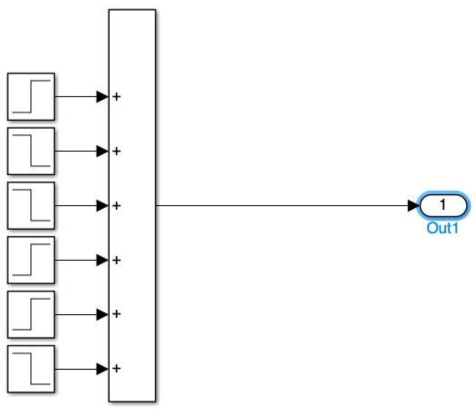

Figure 11: Road Profile 2 Subblock in Simulink

5.1.3. Road Profile 3

$$d_3(t) = \begin{Bmatrix} 0.10 & 1 \leq t \leq 1.2 \\ 0 & otherwise \end{Bmatrix}$$

where time in seconds, and d in meters

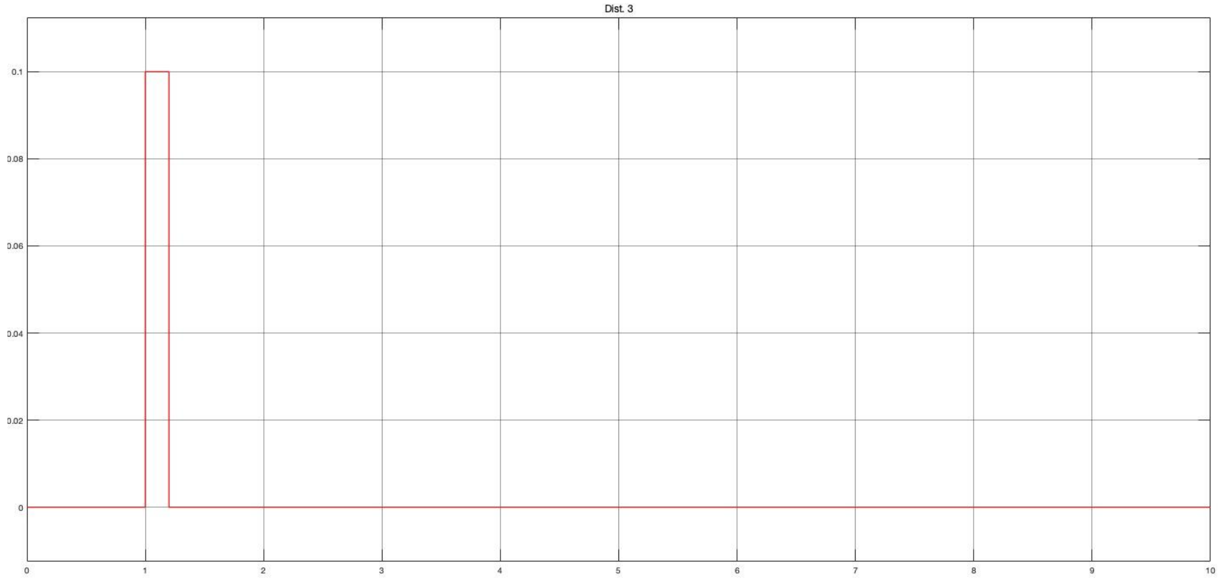

Figure 12: Road Profile 3 – Elevation(m) versus Time(s) Graph

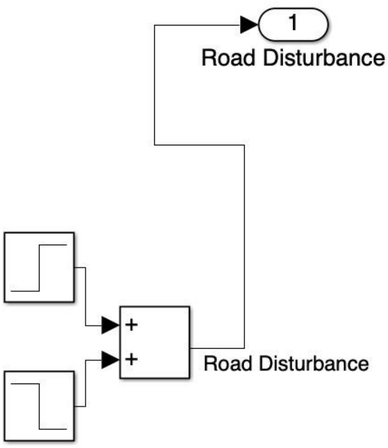

Figure 13: Road Profile 3 Subblock in Simulink

5.2.Simulation Graphs

5.2.1. Body Deflection versus Time Graph wrt Road Disturbance 1

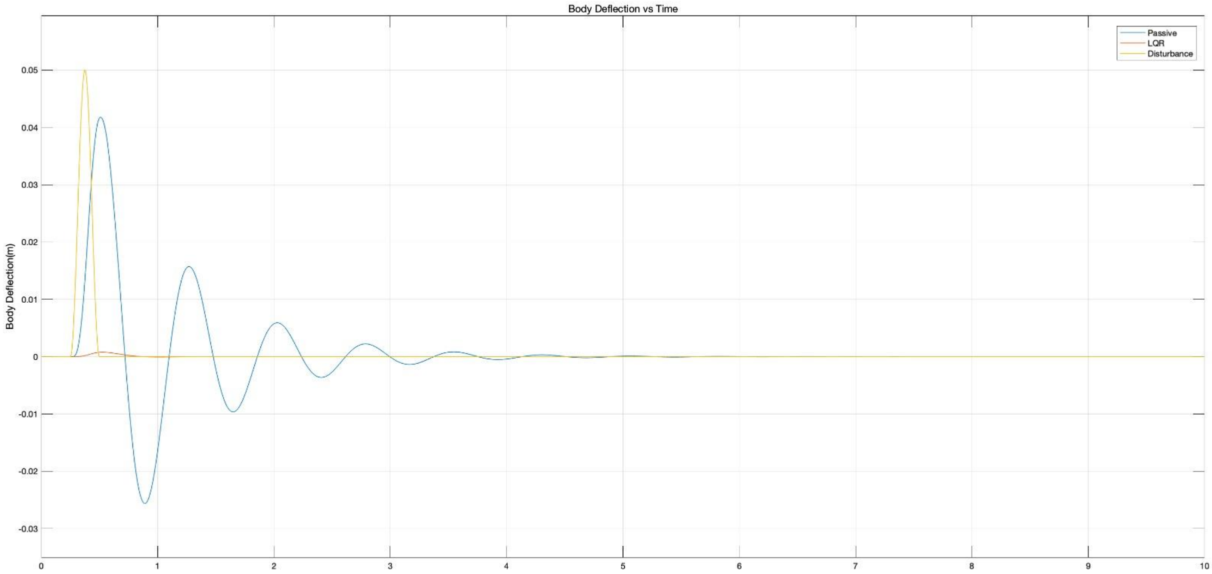

Figure 14: Body Deflection(m) versus Time(s) Graph for Road Profile 1

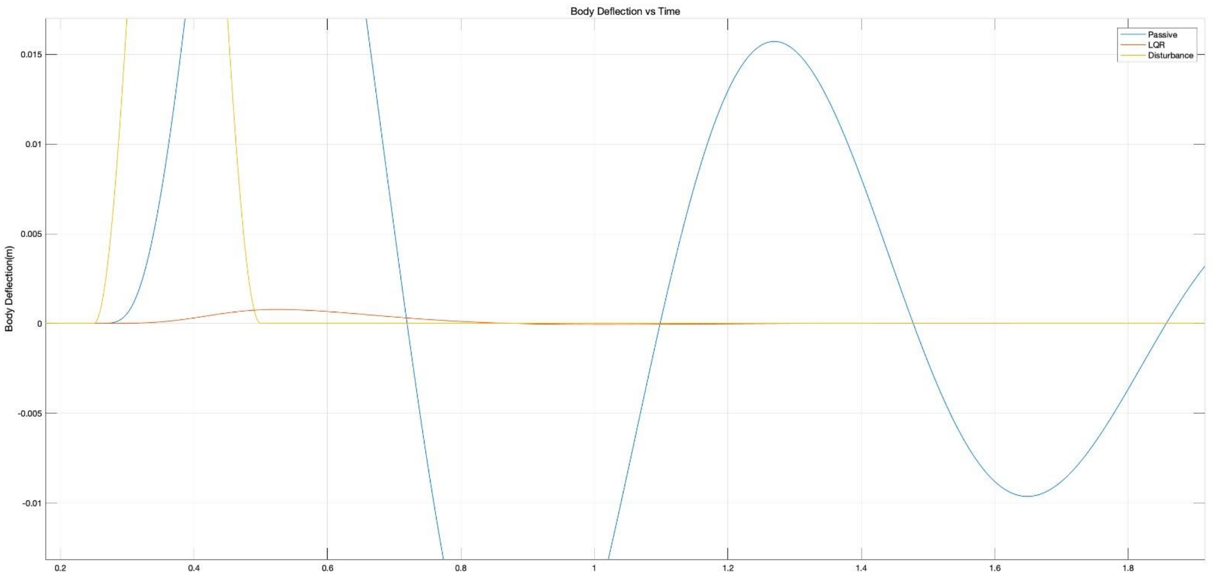

Figure 15: Closer Look on Body Deflection(m) versus Time(s) Graph for Road Profile 1

5.2.2. Body Acceleration versus Time Graph wrt Road Disturbance 1

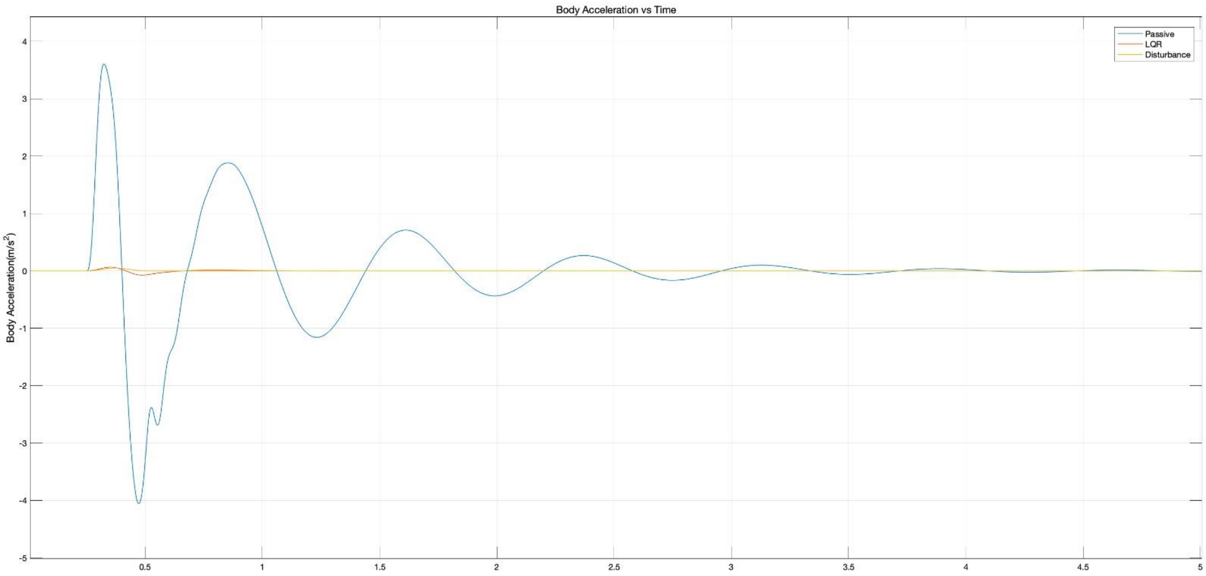

Figure 16: Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 1

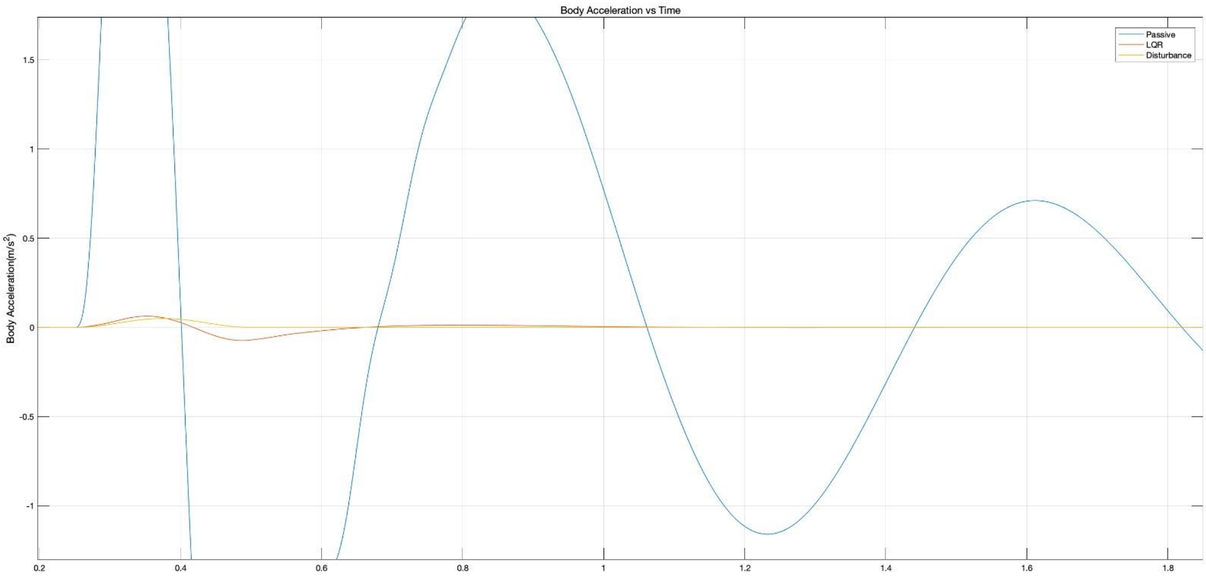

Figure 17: Closer Look on Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 1

5.2.3. Suspension Travel versus Time Graph wrt Road Disturbance 1

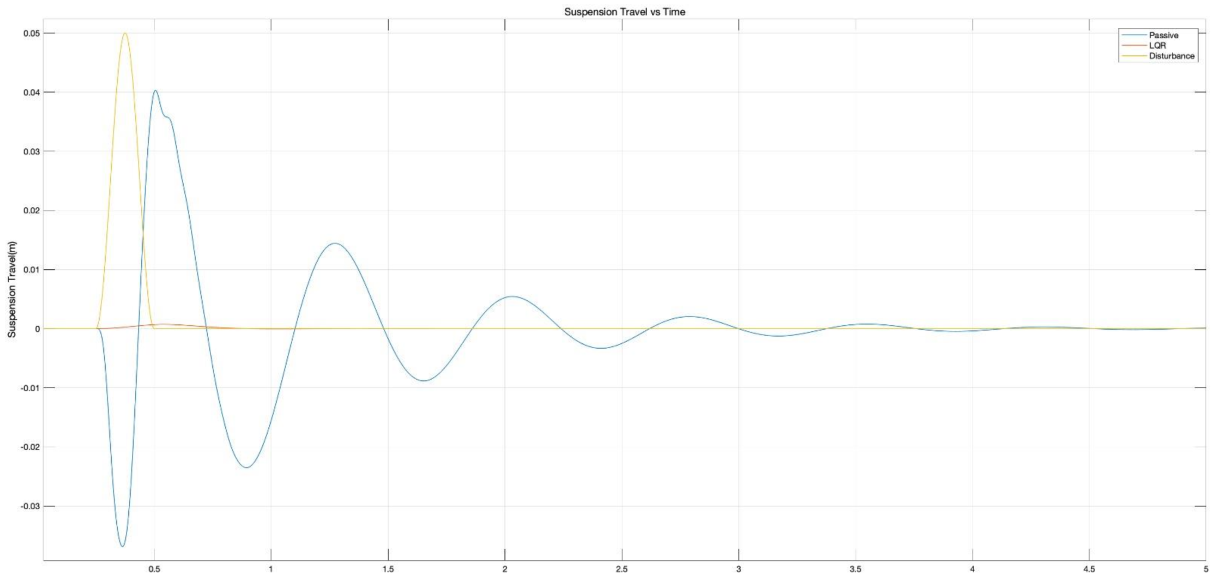

Figure 18: Suspension Travel(m) versus Time(s) Graph for Road Profile 1

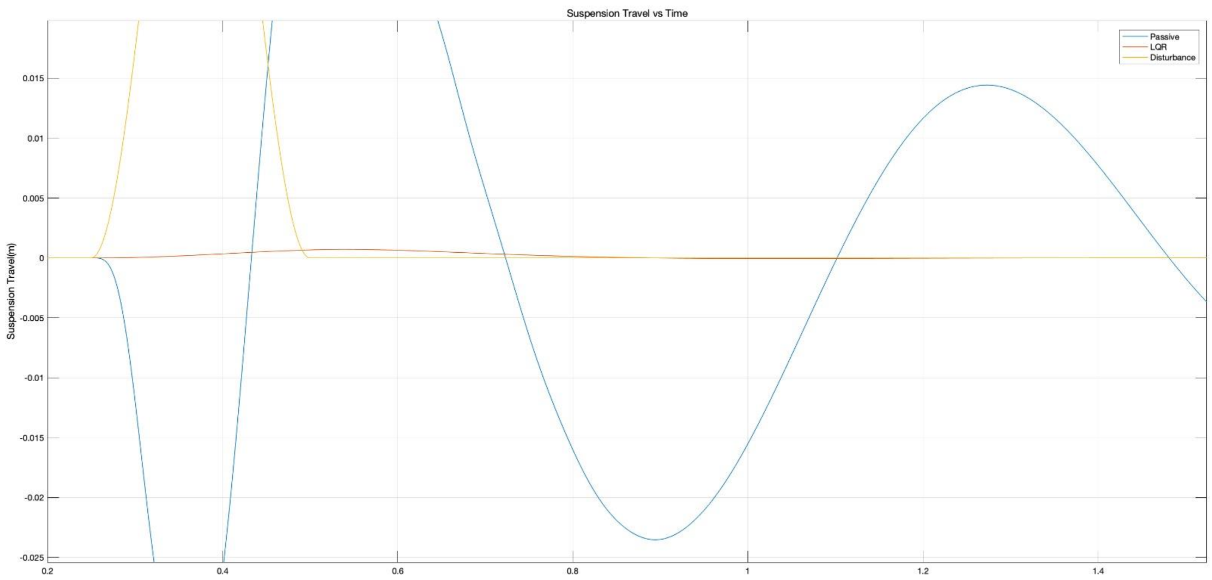

Figure 19: Closer Look on Suspension Travel(m) versus Time(s) Graph for Road Profile 1

5.2.4. Body Deflection versus Time Graph wrt Road Disturbance 2

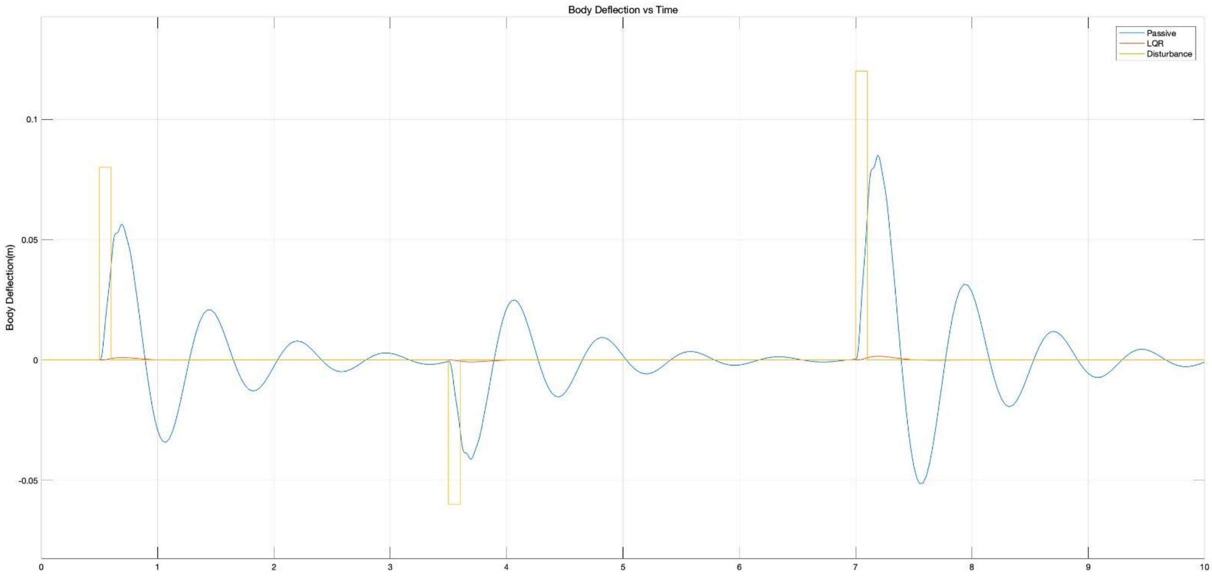

Figure 20: Body Deflection(m) versus Time(s) Graph for Road Profile 2

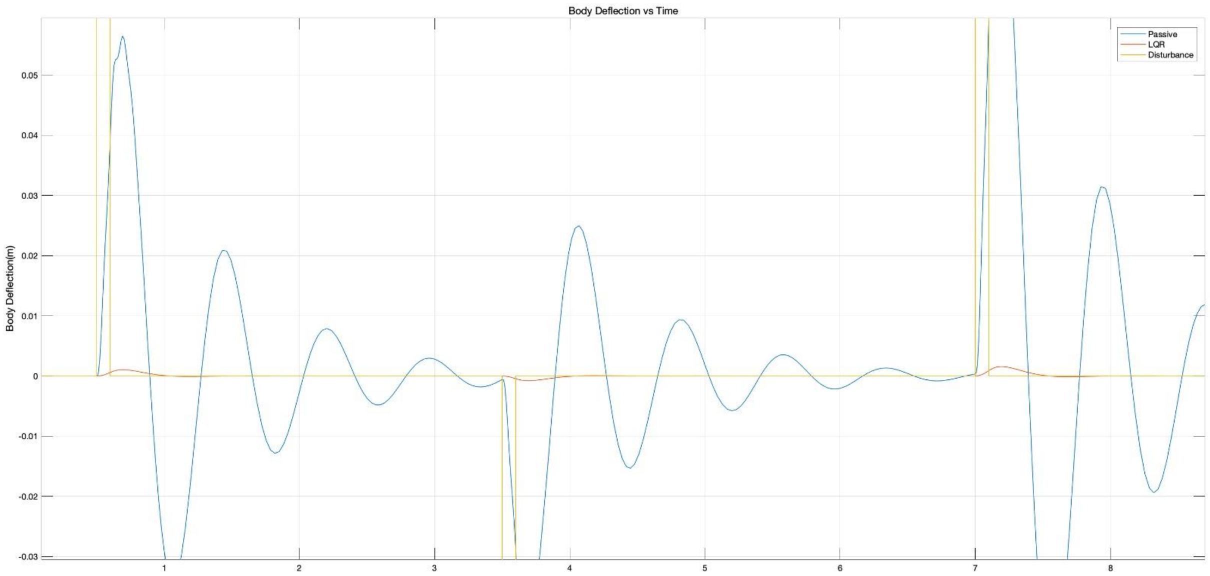

Figure 21: Closer Look on Body Deflection(m) versus Time(s) Graph for Road Profile 2

5.2.5. Body Acceleration versus Time Graph wrt Road Disturbance 2

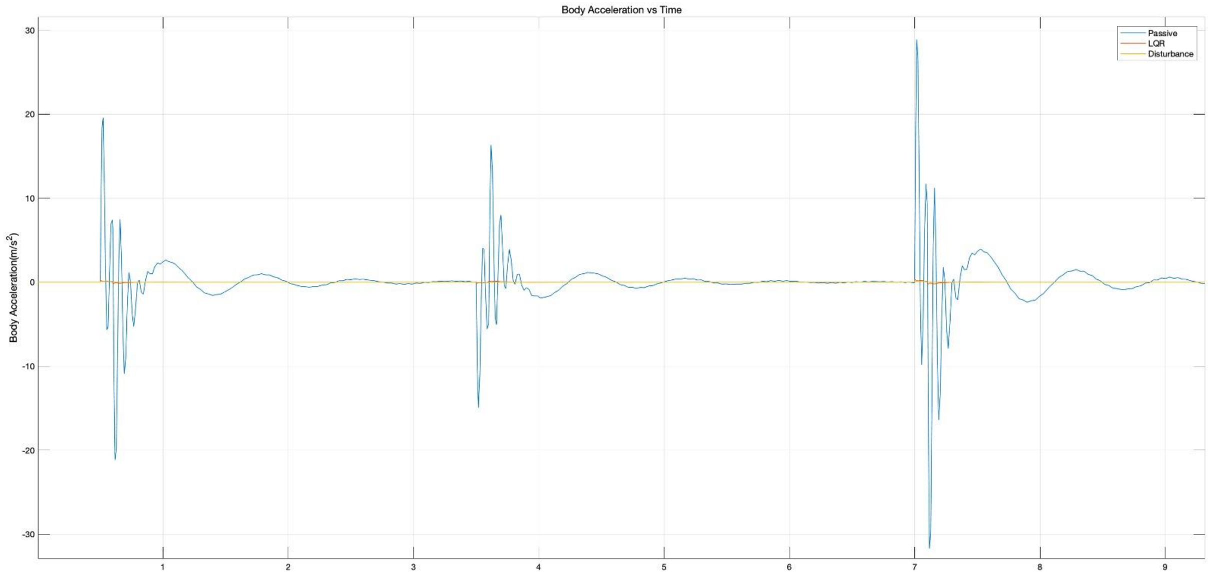

Figure 22: Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 2

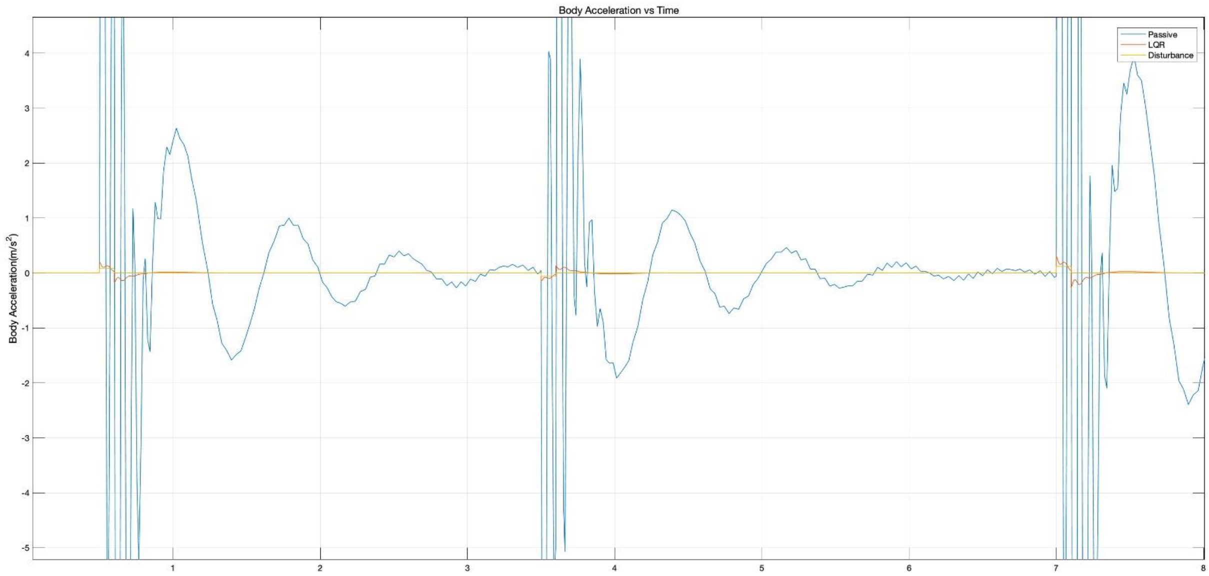

Figure 23: Closer Look on Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 2

5.2.6. Suspension Travel versus Time Graph wrt Road Disturbance 2

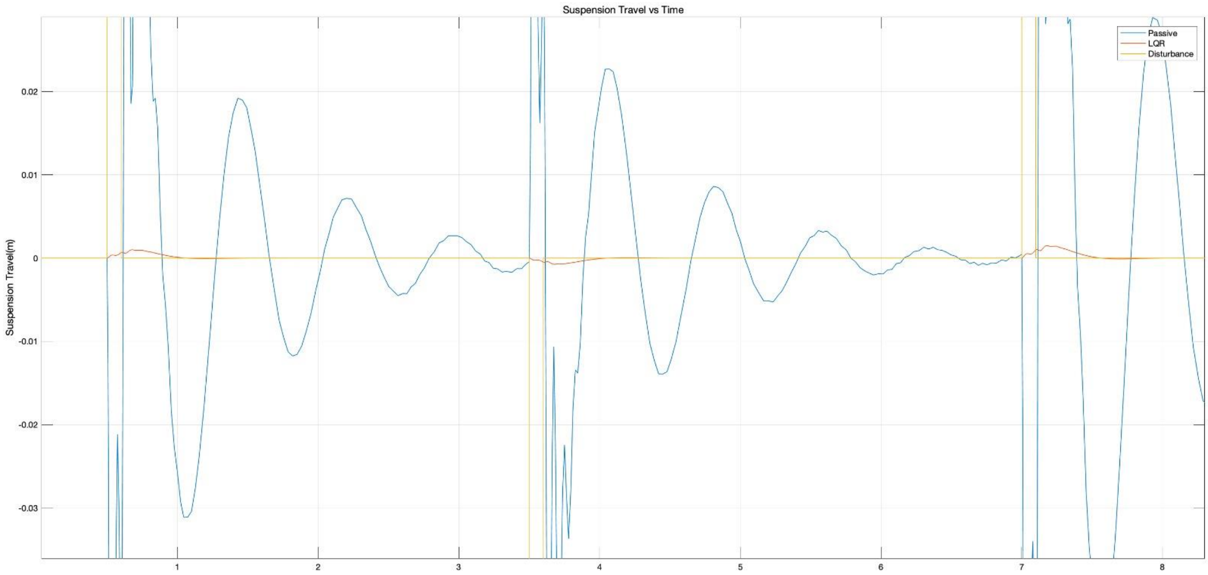

Figure 24: Suspension Travel(m) versus Time(*s*) Graph for Road Profile 2

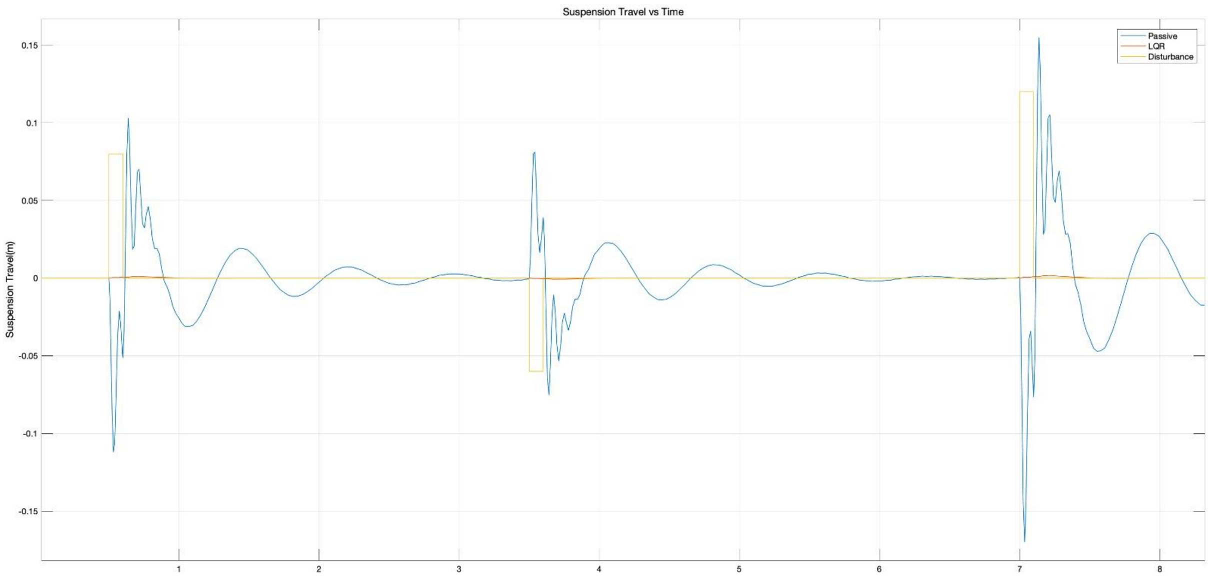

Figure 25: Closer Look on Suspension Travel(m) versus Time(*s*) Graph for Road Profile 2

5.2.7. Body Deflection versus Time Graph wrt Road Disturbance 3

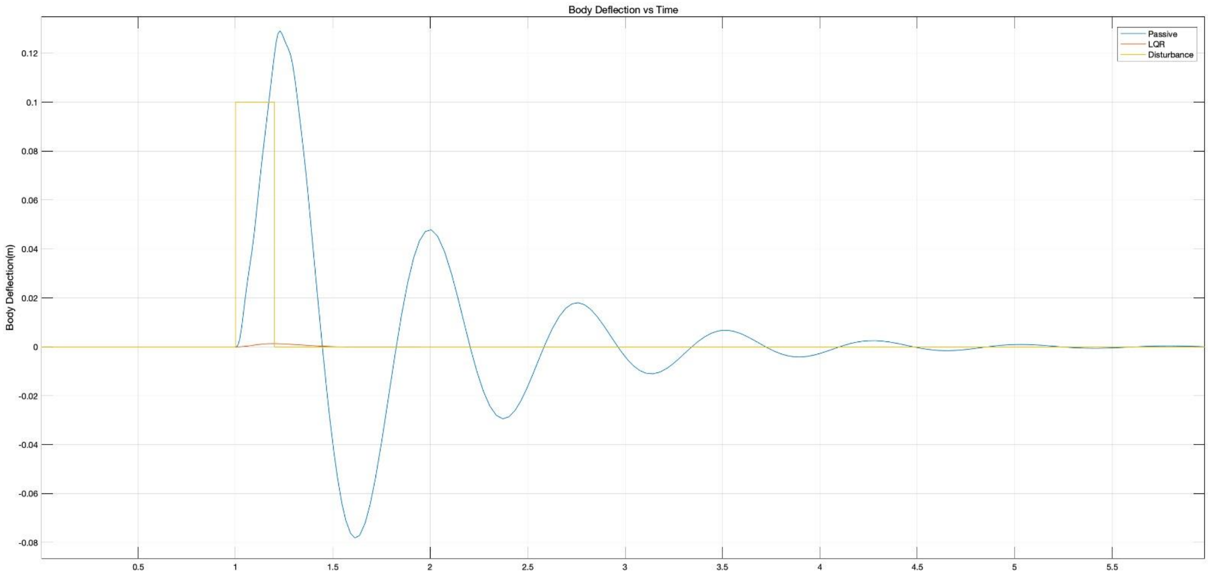

Figure 26: Body Deflection(m) versus Time(s) Graph for Road Profile 3

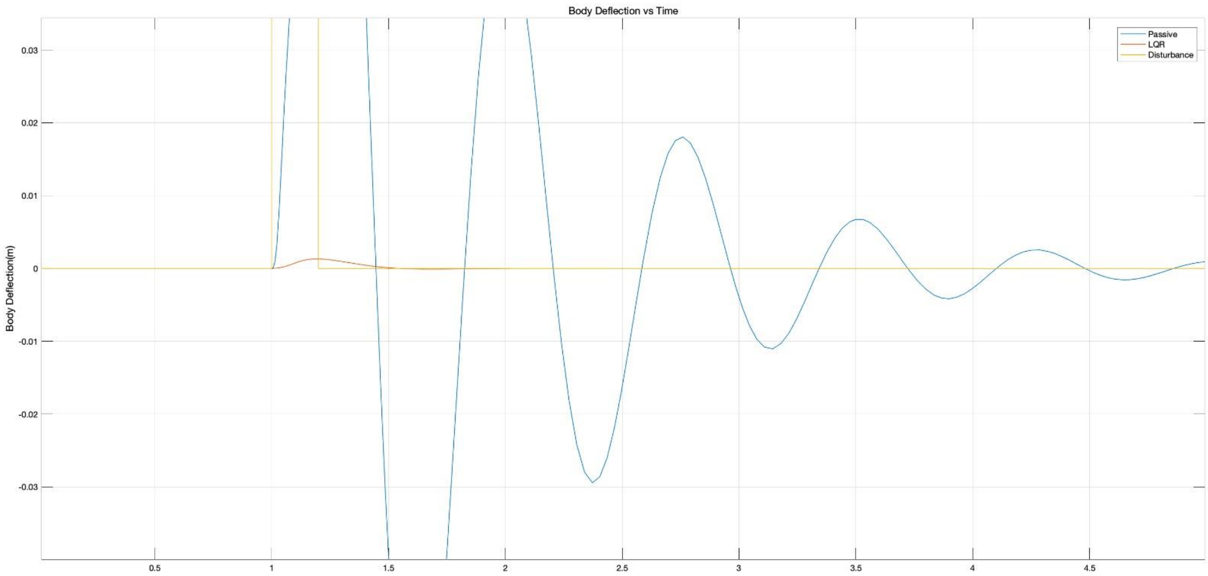

Figure 27: Closer Look on Body Deflection(m) versus Time(s) Graph for Road Profile 3

5.2.8. Body Acceleration versus Time Graph wrt Road Disturbance 3

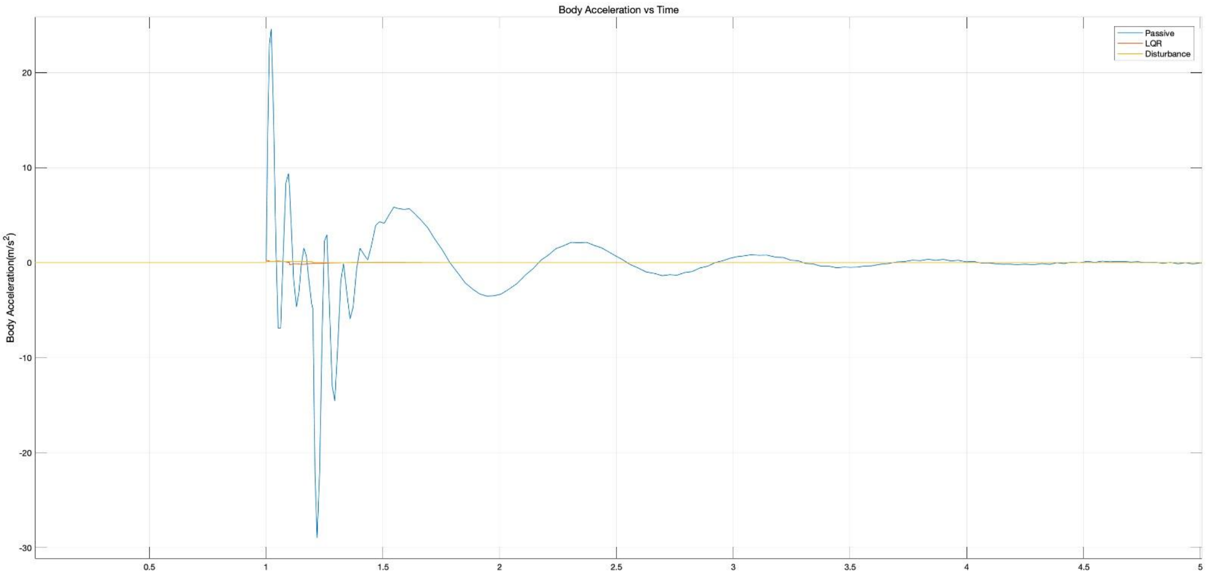

Figure 28: Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 3

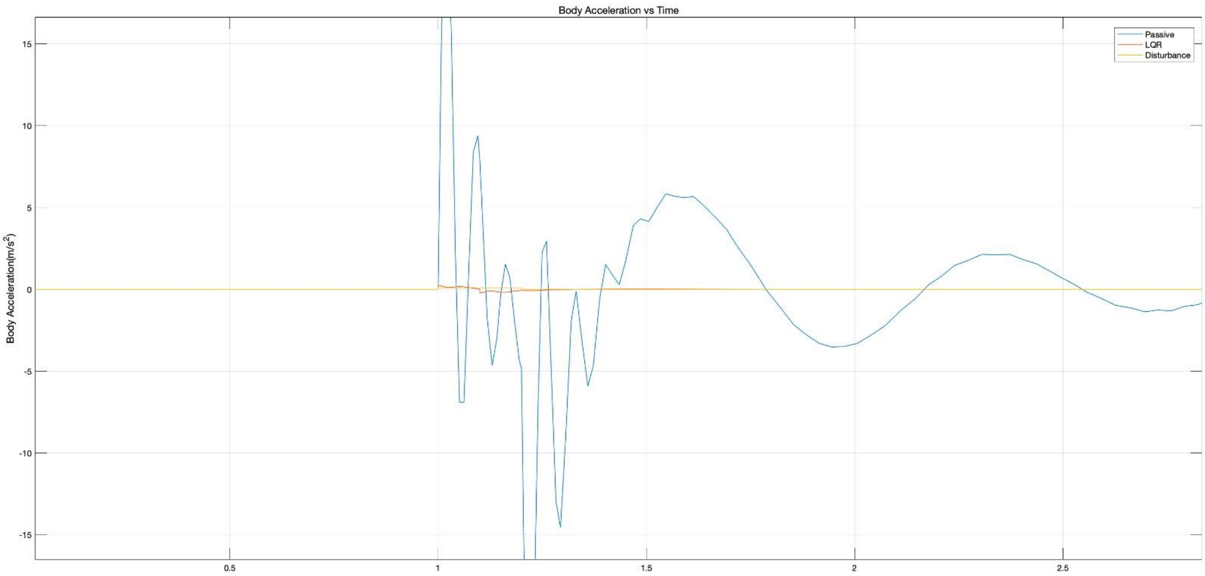

Figure 29: Closer Look on Body Acceleration($\frac{m}{s^2}$) versus Time(s) Graph for Road Profile 3

5.2.9. Suspension Travel versus Time Graph wrt Road Disturbance 3

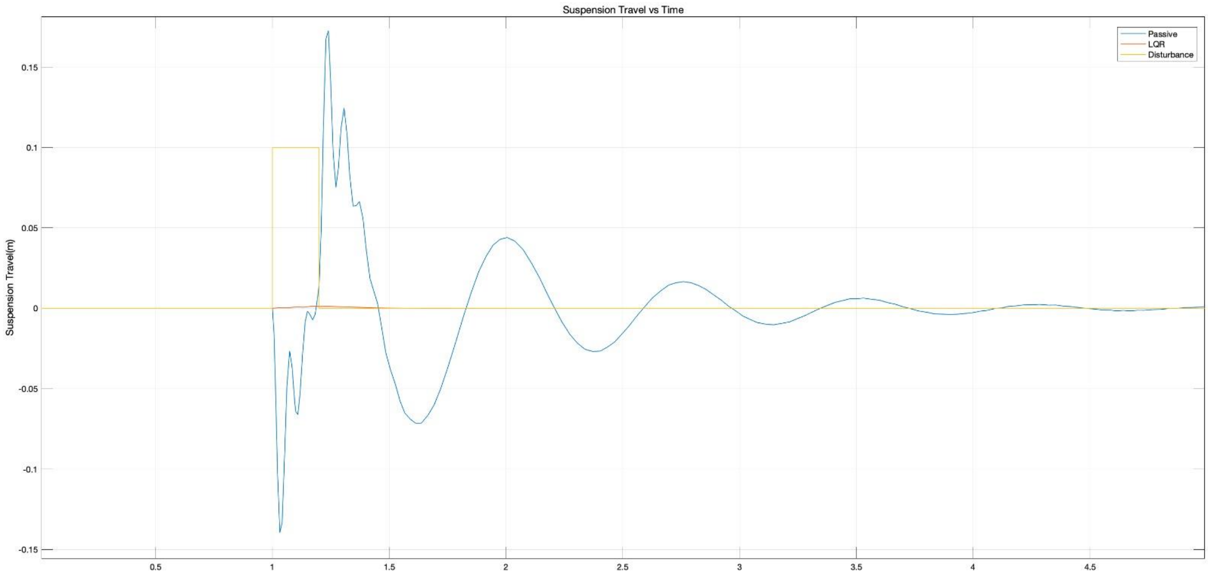

Figure 30: Suspension Travel(m) versus Time(*s*) Graph for Road Profile 3

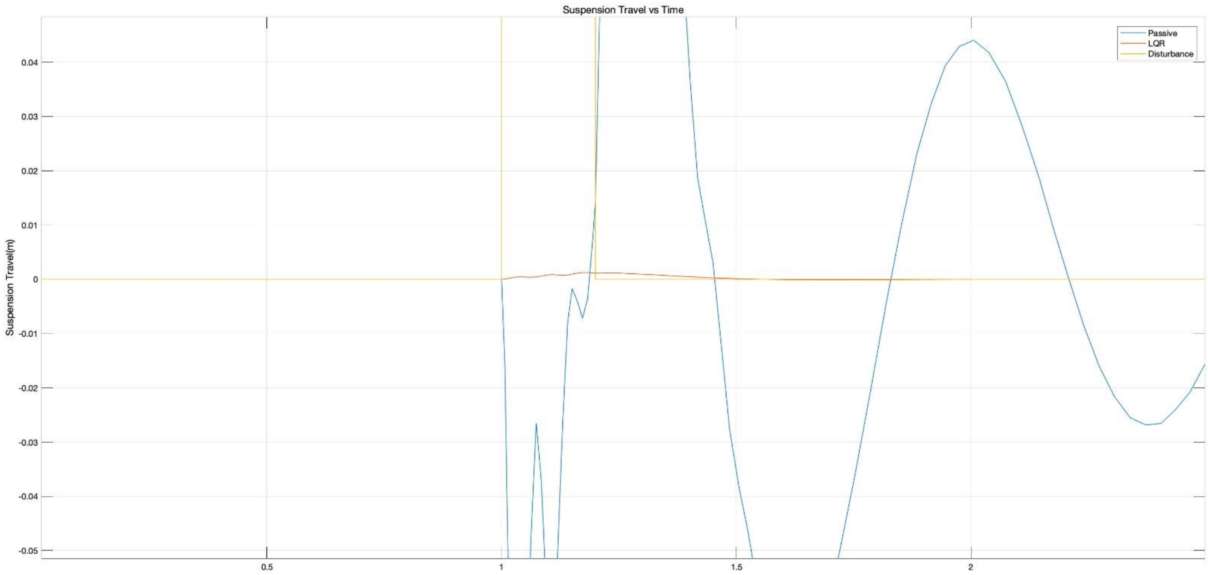

Figure 31: Closer Look on Suspension Travel(m) versus Time(*s*) Graph for Road Profile 3

5.3.Discussion

The simulation results show a significant improvement in ride comfort and vibration suppression when using the LQR control scheme compared to a passive suspension system for all road profiles. LQR-controlled system performs incredibly better in key areas such as reduced settling time and lower percentage overshoot in body deflection, suspension travel, and body acceleration. These results highlight the potential of the LQR approach in improving suspension system performance. We clearly see that implementing the LQR control scheme in modern vehicle designs can lead to better ride comfort and handling, making driving more enjoyable and safer.

Passenger comfort is closely linked to body acceleration; high levels of acceleration can cause discomfort and fatigue. The LQR-based control scheme achieves a faster response and reduced amplitude in body deflection, suspension travel, and body acceleration, demonstrating its ability to maintain vehicle stability and passenger comfort effectively. The reduced settling time means the LQR controller helps the suspension system return to equilibrium more quickly after road disturbances, enhancing ride quality. Additionally, the lower percentage overshoot shows the controller’s efficiency in minimizing excessive oscillations, which contributes to a smoother and more comfortable ride.

Compared to passive suspension system, the LQR-based control scheme is more responsive and effective. The LQR controller can adapt dynamically to changing road conditions, optimizing the balance between comfort and stability. The fast-settling time is especially beneficial as it reduces the duration of passenger discomfort following a disturbance. Moreover, the LQR controller’s ability to lower the amplitude of oscillations in body and suspension movements results in a more stable and comfortable ride.

6. CONCLUSION & FUTURE WORK

In conclusion, the implementation of the Linear Quadratic Regulator (LQR) control scheme in vehicle suspension systems offers significant improvements in ride comfort and stability compared to passive suspension systems. Our simulations demonstrate that the LQR-controlled system excels in reducing body deflection, suspension travel, and body acceleration, leading to enhanced passenger comfort and better handling stability. The key benefits of the LQR approach include faster response times, reduced settling times, and lower percentage overshoot, all of which contribute to a smoother and more comfortable ride experience.

The LQR controller's ability to dynamically adapt to varying road conditions allows it to maintain an optimal balance between ride comfort and vehicle stability. This dynamic adaptability is crucial for minimizing the discomfort associated with road disturbances and ensuring that the vehicle remains stable and responsive under different driving scenarios. Additionally, the fast-settling time provided by the LQR control reduces the duration of passenger discomfort following a disturbance, further enhancing the overall ride quality.

These findings underscore the potential of the LQR control scheme as a superior alternative to traditional passive suspension systems. By effectively managing the trade-offs between comfort and stability, the LQR approach paves the way for advancements in vehicle suspension technology. Incorporating LQR controllers into modern vehicle designs can lead to significant improvements in passenger comfort, vehicle performance, and safety, ultimately contributing to a more enjoyable and secure driving experience.

For future research, enhancing LQR-controlled vehicle suspension systems could involve integrating advanced sensors and actuators to improve real-time adaptability and accuracy. Extending LQR control from quarter-car models to full-vehicle dynamics would address complexities like vehicle roll, pitch, and yaw interactions. Real-world testing is essential to validate LQR system performance across diverse road conditions. Hybrid control strategies, combining LQR with adaptive or machine learning techniques, could better handle disturbances. Additionally, optimizing energy efficiency and conducting cost-benefit analyses would assess the economic viability of LQR implementation in production vehicles.

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1. Electromagnetic suspension system for vehicle, European Patent Application, EP 1 445 131 A2, (2004)

2. Suspensıon system for a vehıcle ıncludıng an electromagnetıc actuator, United States Patent, US 8,682,530 b2 ,(2014)

8. APPENDICES

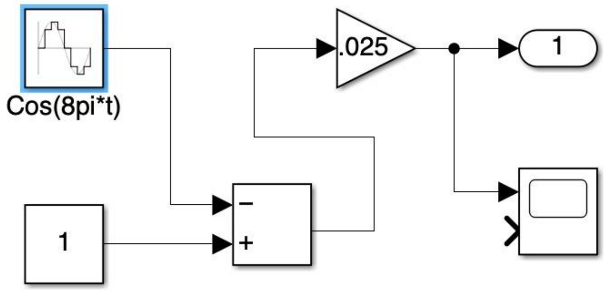

Figure 32: Road Disturbance 1 Modeling - Cosine Signal Subblock 1

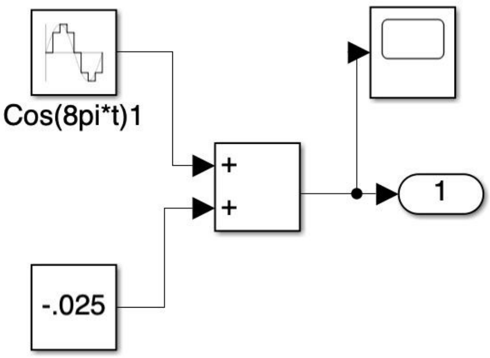

Figure 33: Road Disturbance 1 Modeling - Cosine Signal Subblock 2