ANALOG COMPUTER - Solving 1D Vertical Motion

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This project presents an analog computing approach to simulate the vertical motion of a mass-spring-damper system using op-amp circuits. By modeling the system's second-order differential equation through summing and integrator stages, the circuit converts acceleration into velocity and displacement in real time. The use of Howland current source integrators improves accuracy and simplifies initialization. This hardware-based simulation offers a practical way to visualise and analyse dynamic mechanical systems.

I. Objective:

To design an analog circuit that can simulate the motion of a block under gravity, damping, and spring force using op-amp-based circuits

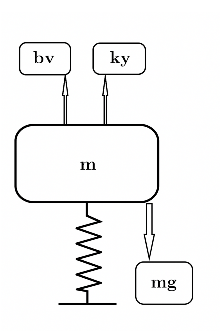

Figure 1: Mass-spring damper system

$$\boxed{F = ma = -mg - bv - ky}$$

m : mass of the block .

g : gravitational acceleration.

b : damping (drag) coefficient.

k : spring constant.

v : velocity.

y : displacement.

Substituting a and v in terms of displacement. We get the following equation.

$$\boxed{\frac{d^2y}{dt^2} = -g - \frac{b}{m}\frac{dy}{dt} - \frac{k}{m}y}$$

II. Components

- 2 10uF Capacitors in Howland Integrator
- Resistors, namely – 1kΩ ,5kΩ and 200kΩ
- 2 TL084 IC

III. Parts of the Circuit

- **Buffer Circuit:** Ensures no significant loading on the supply corresponding to gravity voltage.
- **Summation Circuit:** The system combines the input voltages corresponding to gravity, damping, and spring force, after individually amplifying each signal by its respective gain, to compute a unified output response.

$$\boxed{V_{\text{out}} = -\left(V_g + V_{\text{damping}} + V_{\text{spring}}\right)}$$

- **Integration Circuit:** An integration circuit takes an input voltage, typically representing a physical quantity such as acceleration, and outputs the integral time of that signal, such as velocity. It uses an op-amp with a capacitor in the feedback loop to continuously accumulate the input. This allows it to convert instantaneous rate-based signals into total accumulated values in real time.

$$\boxed{V_o = \frac{1}{\tau}\int_0^t V_i\,dt + V_o(0)}$$

- **Block Diagram and Design Requirements:** Assume that three voltages represent displacement, velocity, and acceleration proportionally.

$$V_a \propto a, \quad V_v \propto v, \quad V_y \propto y$$

Assume:

$a = 1\,\text{m/s}^2 \equiv V_a = 1\,\text{V}$

$v = 1\,\text{m/s} \equiv V_v = 1\,\text{V}$

$y = 1\,\text{m} \equiv V_y = 1\,\text{V}$

Initial Conditions:

$v(0) = 4.9\,\text{m/s} \Rightarrow V_v(0) = 4.9\,\text{V}$

$y(0) = 2\,\text{m} \Rightarrow V_y(0) = 2\,\text{V}$

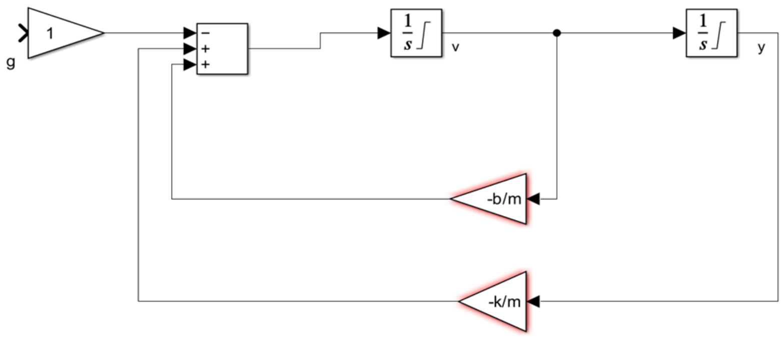

Figure 2: Block Diagram

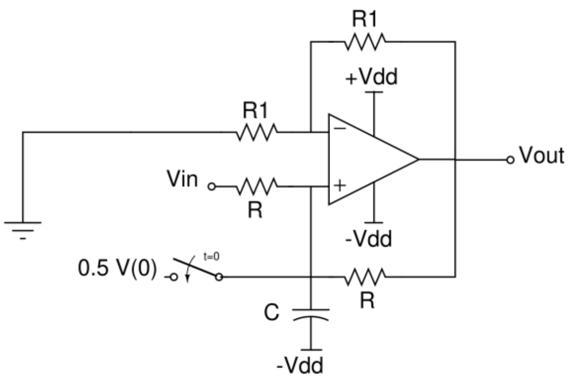

Figure 3: Howland Integrator

- **Drawbacks of Traditional Op-amp Integrator Circuits:**
  - **Difficulty in Setting Initial Conditions:** The capacitor in this configuration is floating, which means that setting an initial voltage requires direct manipulation of the op-amp output, which is impractical.
  - **Polarity Issues:** Since the voltage of the capacitor can change sign, an electrolytic capacitor cannot be used unless modifications are made.
- **Howland Current Source Integrator (Deboo integrator):**
  - This circuit improves traditional integrators by using a current source. This makes the integrator more accurate, more stable, and easier to setup with starting conditions.
  - Howland current source integrator overcomes these issues employing a grounded capacitor and a Howland current source for integration (Figure 3).

$$\boxed{\begin{aligned} V_o(t) &= 2V_+(t) \\ V_o(t) &= \frac{2}{RC}\int_0^t V_i(t)\,dt + 2V_+(0) \end{aligned}}$$

- **Advantages of the Howland Current Source Integrator:**
  - **Easier Initialization of Initial Conditions:** Since the capacitor is grounded, setting an initial condition is straightforward and does not require interfering with the op-amp output.
  - **Better Stability and Linearity:** The Howland current source ensures a regulated current flow, improving accuracy and minimizing drift errors.
- **Drawbacks of Traditional Op-amp Integrator Circuits:**
  - In the original setup, we cannot directly use an electrolytic capacitor because it might get reverse voltage.
  - However, by connecting the bottom end of the capacitor to Vdd instead of ground, we make sure the voltage across the capacitor always stays in the correct direction.

IV. Working of the Circuit

This circuit acts as an analog computer that simulates a second-order physical system—like a mass-spring-damper system—using op-amps to perform integration.

- **Input Signal (Acceleration Source):** The voltage source V3 (set at 9.8V) represents a constant acceleration (similar to gravity). The op-amp U1 buffers this signal, and U2 helps condition it further.
- **Acceleration to Velocity (First Integrator – U3):** The op-amp U3, along with capacitor C1 and resistors, acts as an integrator. It integrates the acceleration signal to produce velocity. The initial voltage on C1 sets the initial velocity.
- **Velocity to Distance (Second Integrator – U4):** The output of U3 is fed into another integrator (U4), which converts velocity into distance. The capacitor C2 stores the initial position value, defined using an initial condition.
- **Feedback and Damping Control (U5 and U6):** U5 and U6 form part of a feedback path, which adjusts the input based on current velocity and position. This introduces a damping effect, helping the system settle down like a real physical system would.

V. Spice simulation:

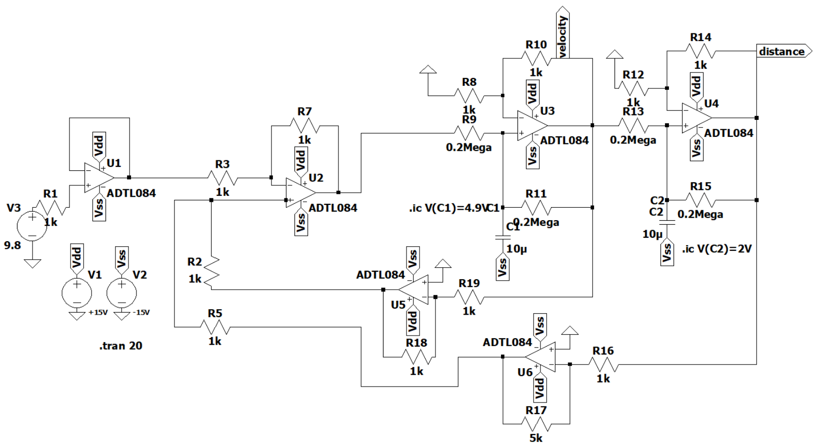

Figure 4: Analog computer simulation circuit for a spring-damper-mass system using ADTL084 op-amps

VI. Spice Output waveform:

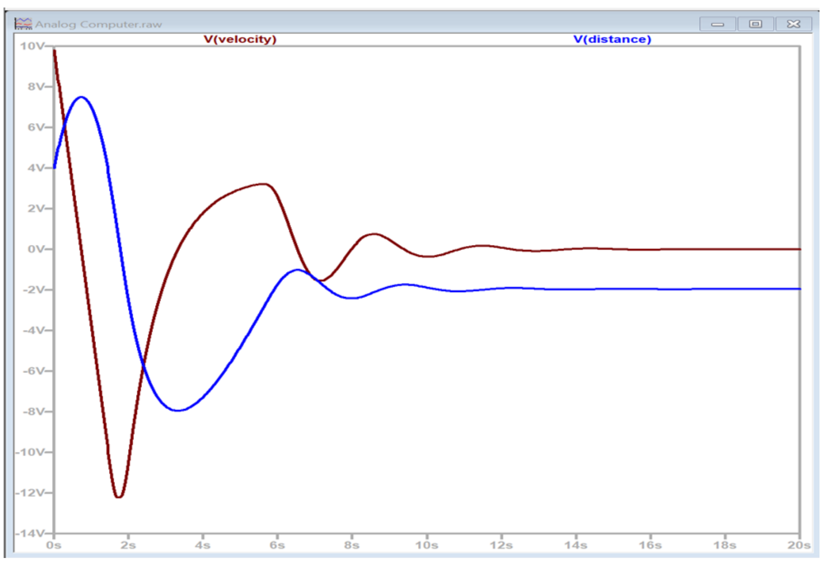

Figure 5: Output waveform from the simulation showing system response

Observations made from the graph:

- **Underdamped response:** The oscillations that gradually reduce in amplitude showing an underdamped second-order system.
- **Mean position shift:** The oscillation settles around −0.657 V, which shows a shifted equilibrium possibly due to the non-zero gravitational term (g = 3.0 m/s²).
- **Phase and frequency:** The signals exhibit comparable frequencies, with a slight phase difference showing how position and velocity waveforms behave in a damped second-order system.

VII. Summary

This work presents the successful design and implementation of an analog computing circuit capable of simulating 1D - vertical motion of a mass-spring-damper system under gravitational force. Using op-amp-based Howland current source integrators, the circuit models second-order differential equations with improved accuracy, stability, and ease of initial condition setup, overcoming common challenges faced in traditional integrator configurations.

Through a combination of signal buffering, summation, and dual-stage integration, the system effectively transforms an input acceleration signal into corresponding velocity and displacement outputs. Simulation results closely mirror the expected behaviour of a damped second-order system, demonstrating oscillatory motion that gradually stabilises around a steady-state value of about -0.657V . This validates the circuit's capability to represent dynamic mechanical systems in real time, offering valuable insights into analog computation for system modelling and control applications.

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