Fluid mechanics

$$\boxed{\rho = \rho_{water}\,(20°C) \cdot S.G.}$$

pg. 136 $$\boxed{\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V^2}{2} = \frac{\dot{W}_s}{\dot{m}} - \mathcal{F}}$$ ✱ g, ρ = constant

$\dot{Q} = VA$

$A = \pi r^2$

Reynolds number (Re):

Re < 2000 laminar

Re: 2000–4000 transition

Re > 4000 turbulent

pg. 177 $$\boxed{Re = \frac{DV\rho}{\mu} = \frac{DV}{\gamma}}$$

use this v

$$\boxed{\mathcal{F} = \frac{4fL}{D} \cdot \frac{v^2}{2}}$$

$$\boxed{f = 0.001375\left[1 + \left(20{,}000\,\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right]}$$

if fitting (globe valve)

$$\mathcal{F} = \frac{v^2}{2}\left(\frac{4fL}{D} + K_{gv}\right)$$

K entrance from Big Tank to small pipe

K exit from small pipe to Big Tank = 1.

ex. globe valve, angle valve, entrance from Big to small

$$\mathcal{F} = \frac{v^2}{2}\left(\frac{4fL}{D} + K_{gv} + K_{AV} + K_{ent}\right)$$

tol 0.00001

Iterations

$Re = \frac{DV}{\gamma}$ · known D & γ

· unknown Re & V

Steps #

① ② ③ ④ ⑤

until below tolerance

① solve $\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta V^2}{2} = -\mathcal{F}$ for v^2 and f

② guess f ($f = 0.025$)

③ solve eq. 1 for v^2

④ solve $Re = \frac{DV\rho}{\mu}$

⑤ plug Re into $f = 0.001375[\ldots]$ and solve for f

⑥ use f calc again in #2

in density are unimportant, rather than of an incompressible fluid. As a general rule, all steady flows of liquids and most steady flows of gases at low velocities (see Sec. 5.6) may be considered incompressible, whereas some unsteady flows of liquids (see Sec. 7.4) and all steady flows of gases at high velocities may not be considered incompressible. We will consider the flow of gases at high velocities in Chap. 8, where we will see that the same terms that appear in B.E. will reappear in different combinations. Therefore, we will apply B.E. only to incompressible flows and use only the incompressible-flow meaning of $\Delta u - dQ/dm$, that is, friction heating per unit mass.

To save writing, we now introduce a new symbol for the friction heating per unit mass,

$$\Delta u - \frac{dQ}{dm} = \mathcal{F} = \begin{pmatrix}\text{friction heating}\\ \text{per unit mass}\end{pmatrix} \qquad \begin{bmatrix}\text{constant-density}\\ \text{flow}\end{bmatrix} \tag{5.4}$$

Here we use $\mathcal{F}$ to avoid confusion with F for force. Most civil engineering texts call this quantity gh_f or gh_L, where g is the acceleration of gravity and h_f or h_L stands for *friction head loss* (Sec. 5.4). Some thermodynamics textbooks introduce the idea of the lost work in explaining the second law of thermodynamics. It can be shown that for a constant-density fluid at the heat reservoir temperature the friction heating per unit mass is exactly equal to the lost work per unit mass, so some texts call this term *LW*. Other texts call it $(-\Delta P/\rho)_{\text{friction}}$, since for the most common pipe friction problem, steady flow in horizontal, constant-area pipes, $\mathcal{F} = (-\Delta P/\rho)_{\text{friction}}$.

Substituting the definition of $\mathcal{F}$ into Eq. 5.1 changes it to the final working form of B.E.,

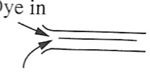
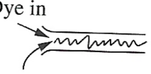
$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{\text{n.f.}}}{dm} - \mathcal{F} \tag{5.5}$$

One may show as a consequence of the second law of thermodynamics that $\mathcal{F}$ is zero for frictionless flows and positive for all real flows. One sometimes calculates flows in which $\mathcal{F}$ is negative. This indicates that the assumed direction of the flow is incorrect; for the assumed conditions at the inlet and outlet locations the flow is thermodynamically possible only in the opposite direction. On the other hand, frictionless flows are reversible; any flow described by B.E. in which $\mathcal{F}$ is zero could be reversed in direction without any change in magnitude of the velocities, pressures, elevations, etc.

Since for all real flows $\mathcal{F}$ is positive, the effect in Eq. 5.5 with a minus sign before $\mathcal{F}$ is to indicate that friction causes a decrease in pressure or a decrease in elevation or a decrease in velocity or a decrease in the work that can be extracted by a turbine or an increase in the work that must be put in by a pump or some combination of these effects.

In Eq. 5.5 we now have only terms that can be measured mechanically; we have eliminated the Q and u terms, which require thermal measurements. Therefore, this equation, the working form of B.E., is often referred to as the *mechanical-energy balance*. Mechanical energy is conserved only if we include an "energy destruction" term, $\mathcal{F}$. This equation has the same restrictions as Eq. 5.1 and, in addition, the restriction that the effects of changes in density are negligible.

TABLE 6.1
Comparison of laminar, transition, and turbulent flows

	Type of flow		
	Laminar	Transition	Turbulent
Behavior of dye streak	Dye in  Flow	Oscillates between laminar and turbulent	Dye in  Flow
Pressure drop proportional to	$Q^{1.0}$	Oscillates from one value to another; very difficult to measure	$Q^{1.8}$ (very smooth pipes) to $Q^{2.0}$ (very rough pipes)
Reynolds number	<2000	≈2000 to 4000	>4000

The transition region on Fig. 6.2 corresponds to Reynolds numbers between about 2000 and 4000. For Reynolds numbers above about 4000, the flow is stably turbulent. For flows other than pipe flow, some other appropriate length is substituted for the pipe diameter in the Reynolds number, producing a different Reynolds number, as will be discussed later. All Reynolds numbers are (some length · velocity · density / viscosity).

The difference between laminar and turbulent flows is one of the most important differences in fluid mechanics. The equations in this book for laminar flow do not describe turbulent flow, nor do the turbulent flow equations describe laminar flow. If you learn nothing else in this chapter, learn that. In pipe flow, the boundary between laminar and turbulent flow is the region from Reynolds number ≈2000 to ≈4000. This means that almost all flows of gases and liquids like water in ordinary-sized pipes are turbulent. The only exceptions to that statement are flows of fluids much more viscous than water, such as asphalt, maple syrup, or polymer solutions. (The fluid used as an example to make up Fig. 6.2 is 50 times as viscous as water; if that figure had been made for water, the laminar region would have practically disappeared into the left axis!) However, in very small tubes or other flow passages the flow is normally laminar. The flow in the heart and the major arteries near it in our bodies and those of most animals our size are turbulent. The rest of the blood flow in our bodies is laminar, as is the flow of fluids in filters, in groundwater, and in oil fields. (These latter are not exactly pipe flow, but as shown in Chap. 11, the flow passages between the solid particles in filters and in the ground behave as irregular-shaped pipes.) River flows are mostly turbulent, and the main flows of the atmosphere are turbulent, but in low-wind situations and in the stratosphere the atmosphere can be laminar. Both laminar and turbulent flows are important; you could not read this statement without the turbulent flow near your heart or the laminar flow of blood to your brain and eyes.

The results of Reynolds' experiments are summarized on Table 6.1.

Figure 6.1 Moody plot, roughness

6.3 LAMINAR FLOW

Laminar flow is the simplest flow, so we discuss it first. Consider a steady laminar flow of an incompressible Newtonian fluid in a horizontal circular tube or pipe. A section of the tube Δx long with inside radius r_0 is shown in Fig. 6.4. We arbitrarily select a rod-shaped

TABLE 6.7
Equivalent lengths and K values for various kinds of fitting*

Type of fitting	Equivalent length, L/D, dimensionless	Constant, K, in Eq. 6.25, dimensionless
Globe valve, wide open	350	6.3
Angle valve, wide open	170	3.0
Gate valve, wide open	7	0.13
Check valve, swing type	110	2.0
90° standard elbow	32	0.74
45° standard elbow	15	0.3
90° long-radius elbow	20	0.46
Standard tee, flow-through run	20	0.4
Standard tee, flow-through branch	60	1.3
Coupling	2	0.04
Union	2	0.04

*Source: Reference 10.

Example 6.11. Rework Example 6.4 on the assumption that, in addition to the 3000 ft of 3-in pipe, the line contains two globe valves, a swing check valve, and nine 90° standard elbows.

Using the constants in Table 6.7, we can calculate the equivalent length of 3-in pipe that would have the same friction effect as these fittings. This is:

$$\sum L/D = 2 \cdot 350 + 1 \cdot 110 + 9 \cdot 32 = 1098 \qquad \text{(6.AG)}$$

From Eq. 6.26 we see that this is the number of pipe diameters needed to have the same friction loss as the fittings. Thus, the equivalent length is $1098 \cdot [(3.068/12)\text{ ft}] = 281$ ft. Therefore, the adjusted length of the pipe is

$$\begin{pmatrix}\text{Adjusted}\\\text{length}\end{pmatrix} = \begin{pmatrix}\text{actual pipe}\\\text{length}\end{pmatrix} + \begin{pmatrix}\text{equivalent length}\\\text{for fittings}\end{pmatrix}$$
$$= 3000 + 281 = 3281 \text{ ft} \qquad \text{(6.AH)}$$

The total pressure drop is

$$-\Delta P_{\text{total}} = 484 \text{ psi} \cdot \frac{3281 \text{ ft}}{3000 \text{ ft}} = 529 \text{ psi} \qquad \text{(6.AI)}$$

and that due to the valves and fittings

$$-\Delta P_{\text{valves and fittings}} = 529 \text{ psi} - 484 \text{ psi} = 45 \text{ psi} = 310 \text{ kPa} \qquad \text{(6.AJ)}$$

■

The second way to represent the same experimental data for the friction losses in valves or fittings is to assign a value of K in Eq. 6.25 to each kind of fitting. Those values, based on friction-loss experiments, are also shown in Table 6.7.

Example 6.12. Repeat Example 6.11, using the K values in Table 6.7.

Using those values, we compute that

$$\sum K_{\text{valves and fittings}} = 3 \cdot 6.3 + 1 \cdot 2.0 + 9 \cdot 0.74 = 27.56 \qquad \text{(6.AK)}$$

TABLE 6.2
Values of surface roughnesses for various materials,* to be used with Fig. 6.10

	Surface roughness	
	ε, ft	ε, in
Drawn tubing (brass, lead, glass, etc.)	0.000005	0.00006
Commercial steel or wrought iron	0.00015	0.0018
Asphalted cast iron	0.0004	0.0048
Galvanized iron	0.0005	0.006
Cast iron	0.00085	0.010
Wood stave	0.0006–0.003	0.0072–0.036
Concrete	0.001–0.01	0.012–0.12
Riveted steel	0.003–0.03	0.036–0.36

*From Moody [4].

The existence of the two values means that, whenever engineers plan to use a chart like Fig. 6.10 or an equation with f in it, they must check to see on which of the two f values the chart or equation is based. Throughout this book we will use the value of f_{chem} defined by Eq. 6.18. It is often called the *Fanning friction factor,* while the one 4 times as large is called the *Darcy* or *Darcy-Weisbach friction factor:* $f_{\text{Fanning}} = \tau / (\rho V^2 / 2)$; $f_{\text{Darcy-Weisbach}} = 4\tau / (\rho V^2 / 2)$.

There is really not much point in having a curve for laminar flow on a friction factor plot, since laminar flow in a pipe can be solved analytically. Poiseuille's equation (Eq. 6.8) can be rewritten (Prob. 6.12) as

Laminar only

$$f = \frac{16}{\mathcal{R}} \tag{6.20}$$

Plotting any equation this simple is unnecessary. However, the laminar-flow line usually is included in friction factor plots, as it is in Fig. 6.10. Furthermore, the turbulent and transition region curves on Fig. 6.10 can be represented with very good accuracy by [7]

$$f = 0.001375 \cdot \left[1 + \left(20{,}000\,\frac{\varepsilon}{D} + \frac{10^6}{\mathcal{R}}\right)^{1/3}\right] \tag{6.21}$$

which has no theoretical basis, but reproduces the turbulent region in Fig. 6.10 well (see Prob. 6.39).

Example 6.3. Read the value of the friction factor from Fig. 6.10 for $\mathcal{R} = 10^5$ and $(\varepsilon / D) = 0.0002$, and compare that value to the value from Eq. 6.21.

From Fig. 6.10, as closely as I can read it, $f = 0.00475$. From Eq. 6.21,

$$f = 0.001375 \cdot \left[1 + \left(20{,}000 \cdot 0.0002 + \frac{10^6}{10^5}\right)^{1/3}\right] = 0.0047 \tag{6.J}$$

The difference between the two values is less than our ability to read Fig. 6.10. ■

Figure 6.1 moody plot, roughness

TABLE 6.4
The equations to be solved in all pipe-flow-with-friction problems

Bernoulli's equation	$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = \frac{dW_{\text{n.f.}}}{dm} - \mathcal{F}$
Friction heating term in B.E.	$\mathcal{F} = 4f\frac{\Delta x}{D}\cdot\frac{V^2}{2}$
Reynolds number	$\mathcal{R} = \frac{DV\rho}{\mu} = \frac{DV}{\nu} = \frac{4Q}{\pi D\nu}$
Friction factor, laminar flow, if $\mathcal{R} < 2000$ or	$f = \frac{16}{\mathcal{R}}$
Friction factor, turbulent flow, if $\mathcal{R} > 4000$	$f = 0.001375\cdot\left[1 + \left(20{,}000\frac{\varepsilon}{D} + \frac{10^6}{\mathcal{R}}\right)^{1/3}\right]$
Volumetric flow rate as function of velocity, some problems	$Q = \frac{\pi}{4}D^2V_{\text{avg}}$

To use Fig. 6.10 or Eq. 6.21, we need a value of ε / D. Reading the value for commercial steel pipe from Table 6.2, we have

$$\frac{\varepsilon}{D} = \frac{0.0018\text{ in}}{3.068\text{ in}} = 0.0006 \tag{6.L}$$

Then we enter Fig. 6.10 at the right at $\varepsilon / D = 0.0006$ and follow that curve to the left to $\mathcal{R} = 6192$, finding (as best we can read that crowded part of the chart) $f = 0.009$. We may check that value from Eq. 6.21, finding 0.00905. We will discuss later the uncertainties in friction factor values, so for now we accept 0.0091 as a good estimate of f.

The B.E. analysis is the same as in Example 6.1, leading to Eq. 6.B. Combining that with Eqs. 6.17 and 6.21, we find

$$\begin{aligned}\Delta P &= 4f\frac{\Delta x}{D}\rho\frac{V^2}{2}\\ &= 4\cdot 0.0091\frac{3000\text{ ft}}{(3.068 / 12)\text{ ft}}\cdot 62.3\frac{\text{lbm}}{\text{ft}^3}\cdot\frac{(13.0\text{ ft / s})^2}{2}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\cdot\frac{\text{ft}^2}{144\text{ in}^2}\\ &= 484\text{ psi} = 3340\text{ kPa}\end{aligned} \tag{6.M}$$

This corresponds to 16.1 psi / 100 ft, which is the value shown on the turbulent flow line in Fig. 6.2, which in turn was made by repeating this calculation in a spreadsheet for various values of Q. It is $(484 / 23.1) \approx 21$ times the value in Example 6.1. If the flow had remained laminar, it would be 6 times the value in Example 6.1. ■

In all such problems (and the ones that follow) it is necessary to convert the volumetric flow rate (gal / min or ft^3 / s or m^3 / s) into linear velocity (in this case, 300 gal / min in a 3-in pipe = 13.0 ft / s). This routine calculation can be simplified by the use of App. A.2, which shows the volumetric flow rate in gal / min corresponding to a velocity of 1 ft / s for all schedule 40 standard U.S. pipe sizes. In the foregoing example we could have looked up the value of 23.00 (gal / min) / (ft / s) for

Figure 6.12