Mälardalen University

Exam- Advanced signal processing- ELA412, 2023-03-22

Please read these instructions carefully.

The exam consists of 12 tasks (questions) with subtasks each of which is marked with its worth in points (P). The total number of points is 55.

The exam consists of two parts: hand calculations and MATLAB. A students needs to pass both parts of the exam to pass the exam. The first part contains 36 points and the second group contains 19 points. To pass the exam a student need to collect at least **18 points from the first part**, and at least **9 points from the second part**. Final grade is calculated from the total number of points: at least 36 for 4 and at least 45 for 5.

The hand calculations should be written neatly and according to well-established practice in engineering education. The hand calculations need to contain the whole calculation procedure. When you have finished, you need to take pictures of your paper solutions, make a single pdf of it and upload it to Canvas. Please order appropriately the pages in the created pdf. You are allowed to write on a tablet or other writing device.

The second group of question is to be solved in MATLAB (which might also require some calculation by hand).

Your solution for each MATLAB task should be uploaded as a separate .m or .mlx file named Task[number].m or task[number].mlx. Please be careful to upload correct MATLAB file in the corresponding question in the Canvas.

Both English and Swedish answers are acceptable for the questions requiring text answers.

Please make sure that you have enough time to create the pdf and upload it to Canvas, as well as to upload correctly the MATLAB files. When you are ready to submit all your solutions, press "lämna in quiz" at the bottom of the exam. It is not possible to upload the exam after the time for the exam has expired.

If there are any questions during the exam:

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1 Paper calculation:

1.1 Task 1

(2P) Develop the formula that relates continuous-time and discrete-time frequencies: $\omega = \Omega T$.

1.2 Task 2

Let $x[n]$ and $h[n]$ denote two finite length sequences, both of length N.

a) (1.5P) What is the maximum possible length of the linear convolution of $x[n]$ with $h[n]$?
b) (1.5P) What is the maximum possible length of the N-point circular convolution of $x[n]$ with $h[n]$?

Justify (explain why) your answers. Without justification, each task is worth only 0.5P.

1.3 Task 3

a) (1P) Draw a figure and describe the general system for filtering analog signals with D/C (C/D) conversion and digital filters.
b) (1P) In the previous system (point a), what is the relationship between the digital filter frequency characteristic and the effective frequency characteristic of the whole system. Write the formula for that relationship and name the variables in it.
c) (1P) Calculate the system function for the system described by the following differential equation:

$$\frac{d^2y(t)}{dt} + 0.2\frac{dy(t)}{dt} + 0.16y(t) = \frac{dx(t)}{dt} + 0.1x(t)$$

d) (1P) Sketch the magnate of the frequency characteristics for the system from c.
e) (1P) Impulse invariance method of filter design: Write the two formulas that describe sampling of the impulse response of an analog filter: one for time domain and one for the frequency domain.
f) (1P) What is the main advantage of the impulse invariance method compared to the bilinear transform?

1.4 Task 4

a) (2P) Mathematically describe the window method of designing FIR filters.
b) (2P) Mathematically describe why and how windowing introduces distortion in the obtained filter frequency characteristics.

1.5 Task 5

a) (1P) Write 2 differences between PCA and ICA.
b) (3P) Calculate the singular value decomposition of the matrix:

$$X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

c) (1P) What is the meaning of the content of the 3 matrices in the result.

1.6 Task 6

a) (1P) By using effective widths of signals in time and frequency domain, mathematically state the Heisenberg's uncertainty principle and describe what it means.
b) (1P) For the continuous wavelet transform (CWT), how does the effective with of analysis in frequency domain depend on the scaling factor of the transform? (write the formula)
c) (1P) For the short time Fourier transform (STFT), how does the effective with of analysis in frequency domain depend on the effective width of the window used? (write the formula)
d) (2P) Describe the difference between STFT and CWT based on the properties of their effective widths stated in b and c.

1.7 Task 7

a) (3P) Mathematically state the Einstein-Wiener-Khinchin theorem, define the variables involved and describe the theorem.

1.8 Task 8

Suppose $x[n]$ is a white process with mean value of 0 ($\mu_x = 0$) and autocorrelation function $R_{xx}[m] = K\delta[m]$, with $K > 0$. Let

$$y[n] = A + x[n],$$

where A is a zero-mean random variable with variance $\sigma_A^2 > 0$ and uncorrelated with $x[\cdot]$.

a) (0.5P) Calculate the mean value μ_y of the process $y[n]$.
b) (1P) Calculate the autocorrelation function $R_{yy}[m]$ of the process.
c) (0.5P) Calculate the autocovariance function $C_{yy}[m]$ of the process.
d) (0.5P) Calculate the power spectral density (PSD) of the process.
e) (0.5P) Calculate the fluctuation spectral density (FSD) of the process.
f) (2P) Is $y[n]$ ergodic in mean and why?

1.9 Task 9

Suppose we have available the measured process:

$$x[n] = y[n] + v[n]$$

where the zero-mean WSS process $y[n]$ that we wish to estimate from our measurements $x[n]$, is the output of a causal and stable LTI state-space system driven by an unknown white process $w[\cdot]$ of known variance $\sigma_w^2 > 0$. The measured signal $x[n]$ is the result of an unknown additive white-noise process $v[n]$ of known variance $\sigma_v^2 > 0$, and uncorrelated with $w[\cdot]$, corrupting the signal $y[n]$.

a) [1P] Write the reachable and observable state-space model for the $x[n]$.
b) [1P] For the system from a), write the observer for which observer gain can be chosen such that this observer has precisely the transfer function of the causal Wiener filter for $y[n]$.
c) [1P] The observer output is the LMMSE estimate of $y[n]$. Why?

2 MATLAB

2.1 Task 10

The file Task10.m contains solution to a part of the exercise in lecture 4 Blind source separation techniques. Questions for you are marked with TODO. Please answer each question directly below the question. Save the file with your answers as Task10.m and upload it as the solution for this task.

Data.zip contains data files that are needed if you want to run the code in Task10.m

2.2 Task 11

File Task11.mlx contains MATLAB code for extracting breathing rate from ECG as we did that in the exercises. File s.mat contains the ECG.

Questions for you are marked with TODO in the file. Please answer each question directly below the question. Save the file with your answers as Task11.mlx and upload it as the solution for this task.

2.3 Task 12

In Lecture 7: State-space models and Kalman filtering, we developed a state-space model for an RLC circuit. The MATLAB code of the related exercise is provided in Task12.mlx with questions for you marked with TODO. Please answer each question directly below the question. Save the file with your answers as Task12.mlx and upload it as the solution for this task.