Solution

Forces on x-axis

F_s, F_{mag}, mg_x, T, b

$$F_s = -k_1(x_1-d) - k_2(x_1-d)^3$$

Forces on y-axis

$$N - mg\cos\phi = 0$$

Total forces on x-axis

$$F_{mag} + mg\sin\phi - T - k_1(X-d) - k_2(X-d)^3 - b\dot{x} = m\ddot{x} \quad (1)$$

As we know that

$$M_T = T\cdot r = I\ddot{\theta}$$

$$= \frac{1}{2} m r^2 \ddot{\theta}$$

$$= \frac{1}{2} m\ddot{x} = T \qquad (\because \text{In terms of linear motion})$$

$$F_{mag} + mg\sin\phi - T - k_1(x-d) - k_2(x-d)^3 - b\dot{x} = \frac{3}{2} m\ddot{x}$$

As given

$$F_{mag} = \frac{c \cdot I^2}{y^2} \quad (2)$$

using KVL on the loop

$$V = IR + \dot{I}L$$

Integrate on b.s w.r.t y

$$\int V \cdot dy = IR\int 1\cdot dy + L\int \frac{di}{dy}\cdot dy$$

$$Vy = IRy + LI$$

$$Vy = I(Ry + L)$$

$$I = \frac{Vy}{(Ry+L)} \quad (3)$$

put eq. (3) in (2)

$$F_{mag} = \frac{c\,V^2y^2}{(Ry+L)^2 y^2} = \frac{c}{(Ry+L)^2} \times V^2 \quad (4)$$

put eq. (4) in eq. (1)

Model

$$\frac{c}{(Ry+L)^2}(V^2) + mg\sin\phi - b\dot{x} - k_1(x-d) - k_2(x-d)^3 = \frac{3}{2} m\ddot{x}$$

multiply the eq with $\frac{2}{3m}$

$$\frac{2c}{3(Ry+L)^2 m}(V^2) + \frac{2}{3}g\sin\phi - \frac{2}{3m}b\dot{x} - \frac{2k_1}{3m}(x-d) - \frac{2k_2}{3m}(x-d)^3 = \ddot{x}$$

$$x_1 = x$$
$$x_2 = \dot{x}_1 = \dot{x}$$
$$\dot{x}_2 = \ddot{x}_1 = \ddot{x}$$

$$\frac{2c}{3(Ry+L)^2 m}(V^2) + \frac{2}{3}g\sin\phi - \frac{2}{3}b x_2 - \frac{2k_1}{3m}(x_1-d) - \frac{2k_2}{3m}(x-d)^3 = \dot{x}_2$$

State space representation

$$\begin{cases} x_2 \\ \frac{2c}{3(Ry+L)^2 m}(V)^2 + \frac{2}{3}g\sin\phi - \frac{2}{3}b x_2 - \frac{2k_1}{3m}(x_1-d) - \frac{2k_2}{3m}(x_1-d)^3 \end{cases}$$

equillibrium

$$\dot{x} = f(x,v)$$

$\dot{x} = f(x,V)$ A pair (x^{eq}, v^{eq}) is an equillibrium point

$$\begin{cases} x_2 = 0 \\ \frac{2c}{3(Ry+L)^2 m}(v^{eq})^2 + \frac{2}{3}g\sin\phi - \frac{2}{3}b x_2^{eq} - \frac{2k_1}{3m}(x_1^{eq}-d) - \frac{2k_2}{3m}(x_1^{eq}-d)^3 = 0 \end{cases}$$

Linearization

$$\frac{2c}{3(Ry+L)^2 m}(V)^2 + \frac{2}{3}g\sin\phi - \frac{2}{3m}bx_2 - \frac{2k_1}{3m}(x_1-d) - \frac{2k_2}{3m}(x_1-d)^3 = \dot{x}_2$$

$$g(x_1) = (x_1 - d)^3$$

$$g'(x_1) = 3(x-d)^2$$

$$g'(x_1^{eq}) = 3(x_1^{eq} - d)^2$$

$$g(x_1^{eq}) = (x_1^{eq} - d)^3$$

By using Taylor Approximation Theorem

$$g(x_1) \approx g(x_1^{eq}) + g'(x_1^{eq})(x_1 - x_1^{eq})$$

By putting all the values

$(x_1-d)^3 = (x_1^{eq} - d)^3 + 3(x_1^{eq}-d)(\bar{x}_1)$ -- (5) ($\bar{x}_1$ is deviation Variable)

So,

$$\dot{x}_2 = \frac{2c}{3(Ry+L)^2 m}(V^2) + \frac{2}{\cancel{3}}\cancel{g}\sin\phi - \frac{2}{3m}bx_2 - \frac{2k_1}{3m}(x_1-d) - \frac{2k_2}{3m}(x_1-d)^3$$

(-)

$$0 = \frac{2c}{3(Ry+L)^2 m}(V^{eq})^2 + \frac{2}{\cancel{3}}\cancel{g}\sin\phi - \frac{2}{3m}bx_2^{eq} - k_1(x_1^{eq}-d) - k_2(x_1^{eq}-d)^3$$

$$= \frac{2}{3(Ry+L)^2 m}(V - V^{eq})^2 - \frac{2}{3m}b(x_2 - x_2^{eq}) - \frac{2}{3m}k_1(x_1 - x_1^{eq}) - \frac{k_2 2}{3m}\left[(x_1-d)^3 - (x_1^{eq}-d)^3\right]$$

By using eq (5)

$$= \frac{2}{3(Ry+L)^2 m}(\bar{V})^2 - \frac{2}{3m}b(\bar{x}_2) - \frac{2}{3m}k_1(\bar{x}_1) - \frac{2k_2}{3m}(3(x_1^{eq}-d)^2)\bar{x}$$

$$\frac{2c}{3m(R_y+L)^2}(\bar{V}^2) - \frac{2}{3m}(b)(\bar{X}_2) - \frac{2}{3m}\left[k_1 + k_2(3(X_1^{eq} - d)^2\right]\bar{X}_2$$

$$q = \frac{2c}{3m(R_y+L)^2}$$

$$u = \frac{2}{3m} b$$

$$w = \frac{2}{3m}\left[k_1 + k_2\, 3(X_1^{eq} - d)^2\right]$$

So,

$$\dot{X}_2 = q\bar{V}^2 - u\bar{X}_2 - \bar{X}_1 w$$

$$\bar{X} = ?$$

$$\dot{X}_1 = X_2$$

$$0 = X_2^{eq}$$

$$\dot{\bar{X}}_1 = \bar{X}_2$$

$$\dot{\bar{X}}_1 = \bar{X}_2$$

$$\dot{\bar{X}}_2 = q[\bar{V}^2] - u\bar{X}_2 - \bar{X}_1 w$$

So,

~~$\ddot{\bar{X}}_1 = q[\bar{V}^2] - u\dot{\bar{X}}_1 - \bar{X}_1 w$~~

Transfer function

taking Laplace on both sides

$$s\bar{X}_1 = \bar{X}_2$$

$$s\bar{X}_2 = q\bar{V}^2 - u\bar{X}_2 - \bar{X}_1 w$$

So,

$$s^2\bar{X}_1 = q\bar{V}^2 - u\bar{X}_1 s - \bar{X}_1 w$$

$$\bar{X}_1 = \frac{q\bar{V}^2}{s^2 + us + w}$$

$$\frac{X_1(s)}{\bar{V}^2(s)} = \frac{q}{s^2 + us + w}$$

Poles

Using quadratic formula

$$s = \frac{-u \pm \sqrt{u^2 - 4w}}{2}$$

$$\frac{s + u}{2} - \frac{\sqrt{u^2 - 4w}}{2} \quad ; \quad \frac{s + u}{2} + \frac{\sqrt{u^2 - 4w}}{2}$$

So

Transfer function = $\dfrac{v}{\left(\frac{s+\cancel{u}u}{2} - \frac{\sqrt{u^2-4w}}{2}\right)\left(\frac{s+u}{2} + \frac{\sqrt{u^2-4w}}{2}\right)}$

It has two poles ~~at~~

~~$s = u$~~

Applying Impulse function

$$X_1(s) = \frac{v}{s^2 + us + w} \quad (1)$$

So function will be same

$$X_2(s) = \frac{v}{s^2 + us + w}$$

condition at which system is oscilatory

As we know that when there is no damper the Impulse response of the transfer function will be oscillaroly. i.e. $b = 0$ or $u = 0$