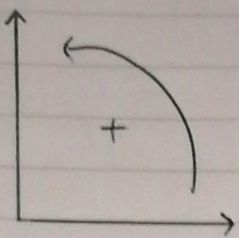
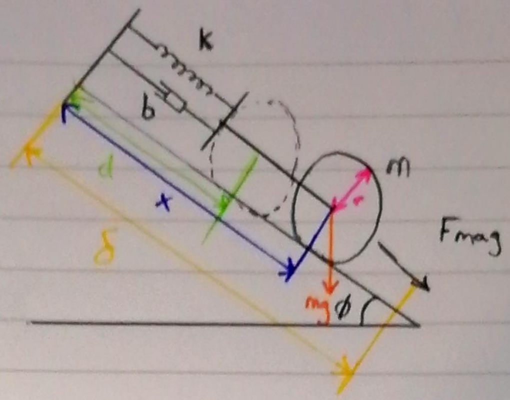
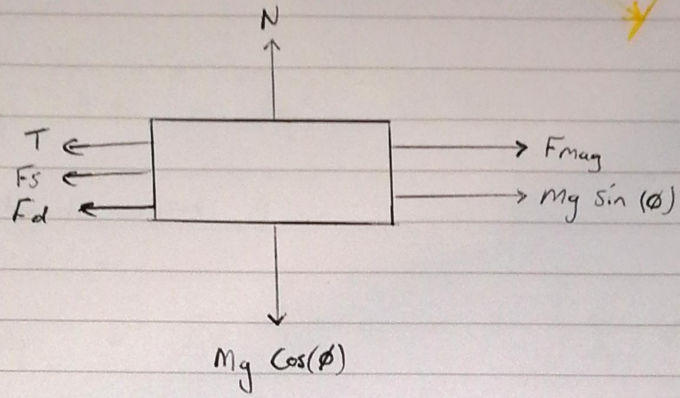
A1







Forces on x axis

$$F_{mag} + mg \sin(\phi) - T - F_{spring} - F_{damper} = m\ddot{x} \qquad (1)$$

Forces on y axis

$$N - Mg\cos(\phi) = 0$$

Moment of inertia: $i = 2/5\, mr^2$

Torque $M = -T \cdot r$ (-ve as rotating clockwise)

$$I\ddot{\theta} = -Tr \rightarrow \frac{1}{2}mr^2\ddot{\theta} = -T\cancel{r}$$

$$\boxed{T = -\frac{1}{2}mr\ddot{\theta}}$$

By Pizza slice theorem $\ddot{x} = \ddot{\theta} r$

$$T = -\frac{1}{2}m\ddot{x}$$

(1) -- $F_{mag} + mg\sin(\phi) - F_{spring} - F_{damper} = m\ddot{x} - \frac{1}{2}m\ddot{x}$

$F_{spring} \to k(x-d)$ $\qquad$ $F_{damper} \to b\dot{x}$

Finding F_{mag}

$$F_{mag} = c\frac{I^2}{y^2} \qquad \text{-- (2)}$$

Using KVL on loop $V = IR + \dot{I}L$

integrate wrt y

$$\int V\,dy = IR\int 1 \cdot dy + L\int \frac{di}{dy}\,dy$$

$$Vy = IRy + LI$$

$$Vy = I(Ry + L)$$

$$I = \frac{Vy}{(Ry + L)} \qquad \text{-- (3)}$$

Sub (3) into (2)

$$F_{mag} = \frac{c\,V^2\cancel{y^2}}{(Ry+L)^2\cancel{y^2}} = \frac{C}{(Ry+L)^2}V^2 \qquad \text{-- (4)}$$

Subbing (4) into (1)

$$\frac{CV^2}{(Ry+L)^2} + mg\sin(\phi) - k(x-d) - b\dot{x} = \frac{1}{2}m\ddot{x}$$

$$\Big\downarrow \times 2/m$$

$$\frac{2\,CV^2}{(Ry+L)^2\,m} + 2g\sin(\phi) - \frac{2k}{m}\cancel{x_0}\,x - \frac{2b}{m}\dot{x} = \ddot{x}$$

A2

$$x_1 = x$$
$$x_2 = \dot{x}_1 = \dot{x}$$
$$\dot{x}_2 = \ddot{x}$$

SSR

$$\underline{X} = \begin{cases} \dot{x} = x_2 \\ \frac{2 c v^2}{(R_4 + L)^2 m} + 2g \sin(\phi) - \frac{2k}{m}(x_1 - d) - \frac{2b}{m} x_2 = \dot{x}_2 \end{cases}$$

A3

$$\dot{x} = f(x, v)$$

$\dot{x} = f(x, v)$, a pair (x^{eq}, v^{eq}) is an equilibrium point if $f(x^{eq}, v^{eq}) = 0$

$$\begin{cases} x_2^{eq} = 0 \\ \frac{2 C v^{eq^2}}{(R_4 + L)^2 m} + 2g \sin(\phi) - \frac{2k}{m}(x_1^{eq} \cancel{- d}) - \frac{2b}{m} x_2^{eq} = \cancel{\dot{x}_2}\ 0 \end{cases}$$

at equilibrium $d = 0$

A4

$$\frac{2 c v^2}{(R_4 + L)^2 m} + 2g \sin(\phi) - \frac{2k}{m}(x_1 - d) - \frac{2b}{m} x_2 = \dot{x}_2$$

$$g(v) = v^2$$
$$g'(v) = 2v$$
$$g(v^{eq}) = (v^{eq})^2$$
$$g'(v^{eq}) = 2v^{eq}$$

By Taylors theorem

$$g(V) \simeq g(V^{eq}) + g'(V^{eq})(V - V^{eq})$$

$$V^2 \simeq (V^{eq})^2 + 2V^{eq}(\bar{V})$$

USING DEVIATION VARIABLES

$$V^2 \approx (V^{eq})^2 + 2(V^{eq})(\bar{V})$$

$\downarrow$ REARRANGING

$$V^2 - V^{eq^2} \approx 2(V^{eq})(\bar{V})$$

$$\dot{x}_2 - \dot{x}_2^{eq} = \dot{x}_2 - 0 = \dot{x}_2 = \dot{\bar{x}}_2$$

$\dot{x}_2 - \dot{x}_2^{eq}$:

$$\dot{x}_2 = \frac{2c}{m(R_4+L)^2} \overbrace{2V^{eq} \cdot \bar{V}}^{V^2 - V_{eq}^2} - \frac{2k}{m} \overbrace{(x_1 - x_1^{eq})}^{\bar{x}_1} - \frac{2b}{m} \overbrace{(x_2 - x_2^{eq})}^{\bar{x}_2}$$

$(x_1 - d) - (x_1^{eq} - d) = x_1 - x_1^{eq}$ (ds cancel)

USING Deviation Variables

$$\dot{\bar{x}}_2 = \underbrace{\left[\frac{2c}{m(R_4+L)^2} \cdot 2V^{eq}\right]}_{A} \bar{V} - \underbrace{\left[\frac{2k}{m}\right]}_{B} \bar{x}_1 - \underbrace{\left[\frac{2b}{m}\right]}_{C} \bar{x}_2$$

use capitals

$$\dot{\bar{x}}_2 = A\bar{V} - B\bar{x}_1 - C\bar{x}_2$$

A5 - Finding transfer function

$$x_2 = \dot{x}_1 \qquad x_2^{eq} = 0$$

$$x_2 - x_2^{eq} = \dot{x}_1 - 0$$

$$\bar{x}_2 = \dot{x}_1$$

$$\bar{x}_2 = \dot{\bar{x}}_1$$

$$\dot{\bar{x}}_2 = A\bar{V} - B\bar{x}_1 - C\dot{\bar{x}}_1$$

$$S \cdot \bar{x}_1 = \bar{x}_2 = \dot{\bar{x}}_1$$
$$S \cdot \bar{x}_2 = \dot{\bar{x}}_2$$
$$\therefore \quad S\bar{x}_2 = A\bar{v} - B\bar{x}_1 - C(S \cdot \bar{x}_1)$$
$$\downarrow$$
$$S\bar{x}_2 = S \cdot S \cdot \bar{x}_1$$

$$S^2\bar{x}_1 = A\bar{v} - B\bar{x}_1 - C(S\bar{x}_1)$$

$$S^2\bar{x}_1 + B\bar{x}_1 + C(S\bar{x}_1)$$

$$(S^2 + B + CS)\bar{x}_1 = A\bar{v}$$

Rearranging to get transfer function

$$\boxed{\frac{\bar{X}_1}{\bar{v}} = \frac{A}{S^2 + CS + B}}$$

TRANSFER function