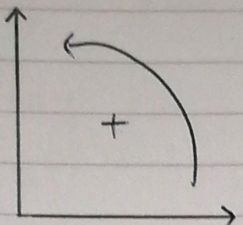
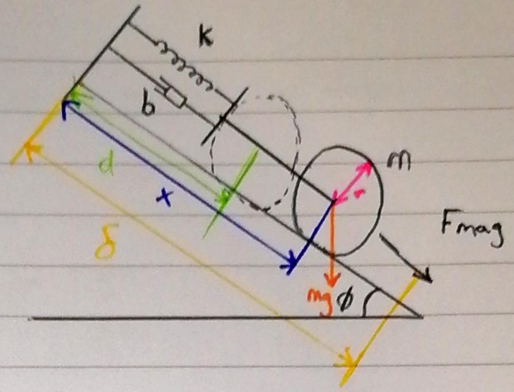
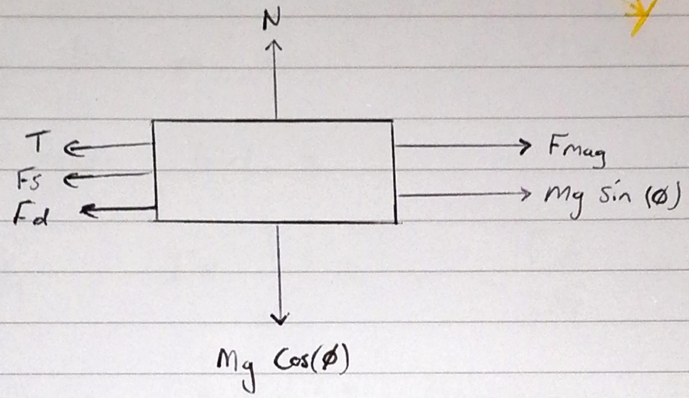
A1







Forces on x axis

$$F_{mag} + mg \sin(\phi) - T - F_{spring} - F_{damper} = m\ddot{x} \qquad (1)$$

Forces on y axis

$$N - Mg\cos(\phi) = 0$$

Moment of inertia : $i = 2/5\, mr^2$

Torque $M = -T \cdot r$ (-ve as rotating clockwise)

$$I\ddot{\theta} = -Tr \rightarrow \frac{1}{2} m r^{\cancel{2}} \ddot{\theta} = -T\cancel{r}$$

$$\boxed{T = -\frac{1}{2} m r \ddot{\theta}}$$

By Pizza slice theorem
$\ddot{x} = \ddot{\theta} r$

$$T = -\frac{1}{2} m \ddot{x}$$

$$(1) \;--\; F_{mag} + mg\sin(\phi) - \underset{\downarrow\, kx}{F_{spring}} - \underset{\downarrow\, b\dot{x}}{F_{damper}} = m\ddot{x} - \tfrac{1}{2}m\ddot{x}$$

Finding F_{mag}

$$F_{mag} = c\,\frac{I^2}{y^2} \quad --(2)$$

Using KVL on loop $\quad V = IR + \dot{I}L$

integrate wrt y

$$\int V\,dy = IR\int 2\,dy + L\int \frac{di}{dy}\,dy$$

$$Vy = IRy + LI$$

$$Vy = I(Ry + L)$$

$$I = \frac{Vy}{(Ry + L)} \quad --(3)$$

Sub (3) into (2)

$$F_{mag} = \frac{c\,V^2\cancel{y^2}}{(Ry+L)^2\cancel{y^2}} = \frac{c}{(Ry+L)^2} \times V^2 \quad --(4)$$

Subbing (4) into (1)

$$\frac{cV^2}{(Ry+L)^2} + mg\sin(\phi) - kx - b\dot{x} = \tfrac{1}{2}m\ddot{x}$$

$$\Big\downarrow \times 2m$$

$$\frac{2m\,cV^2}{(Ry+L)^2} + 2m^2g\sin(\phi) - 2mkx - 2mb\dot{x} = \ddot{x}$$

A2

$$x_1 = x$$
$$x_2 = \dot{x}_1 = \dot{x}$$
$$\dot{x}_2 = \ddot{x}$$

SSR

$$\underline{X} = \begin{cases} \dot{x}_1 = x_2 \\ \dfrac{2mcv^2}{(R_4+L)^2} + 2m^2 g \sin(\phi) - 2mk\,x_1 - 2mb\,x_2 = \dot{x}_2 \end{cases}$$

A3

$$\dot{x} = f(x, u)$$

$\dot{x} = f(x, u)$, a pair (x^{eq}, u^{eq}) is an equilibrium point

if $f(x^{eq}, u^{eq}) = 0$

$$\cancel{\begin{cases} x_2 = 0 \\ \dfrac{2mc(V_{eq})^2}{(R_4+L)^2} \end{cases}}$$

$$\begin{cases} x_2 = 0 \\ \dfrac{2mc(V_{eq})^2}{(R_4+L)^2} + 2m^2 g \sin(\phi) - 2mk\,x_1^{eq} - 2mb\,x_2^{eq} = 0 \end{cases}$$