ELE476
Week-3
DSP with Python

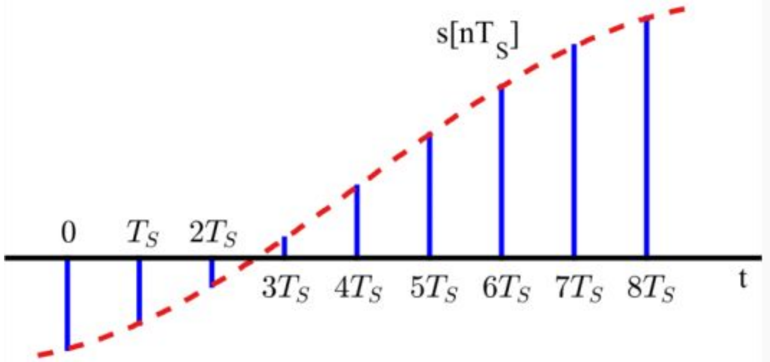
Murat Sever
ytregitim@gmail.com

Outline

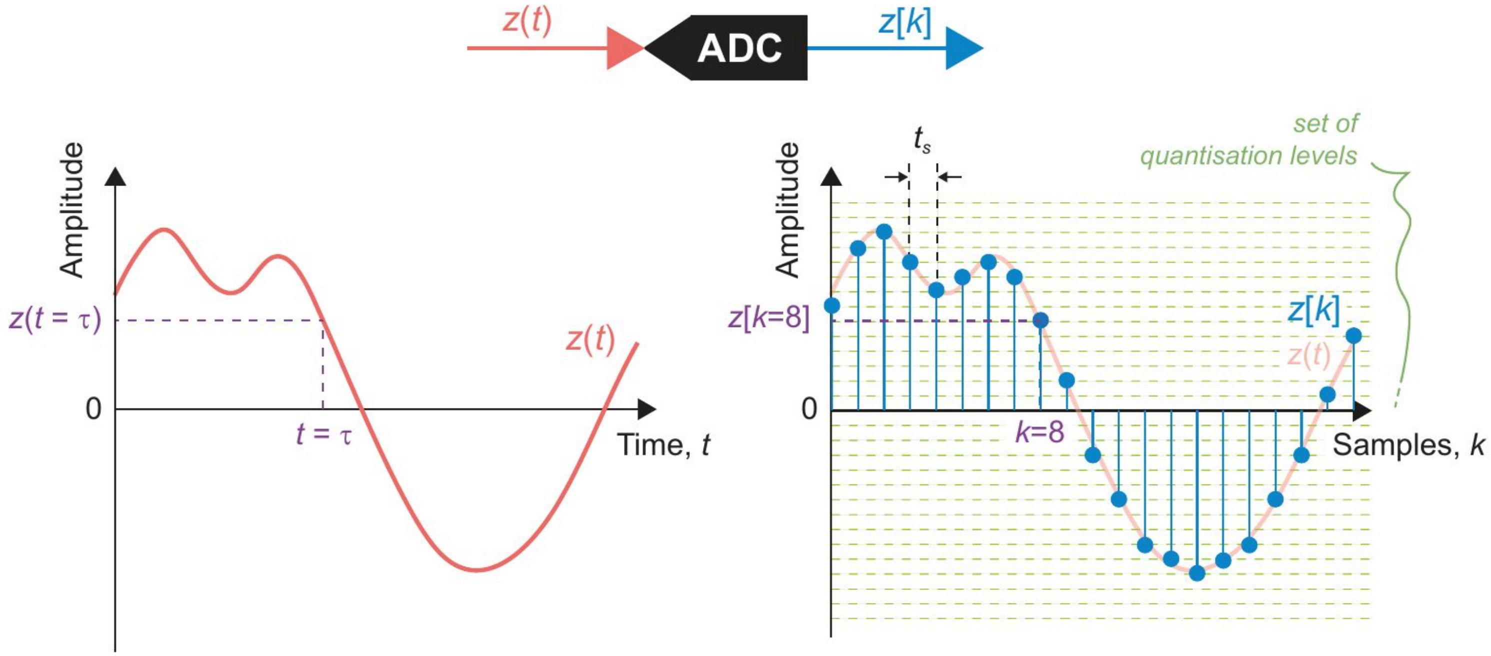
- Sampling
- Aliasing
- Nyquist Theorem
- Quantization
- ADC & DAC
- Linear and Time-Invariant Systems
- Convolution
- Digital Filtering
- FIR Filters
- Resampling and Multirate Signal Processing

Sampling

- Communication signals are continuous-time
- We (ADCs) take samples at regular times
- Ts is sampling period
- Fs is sampling frequency



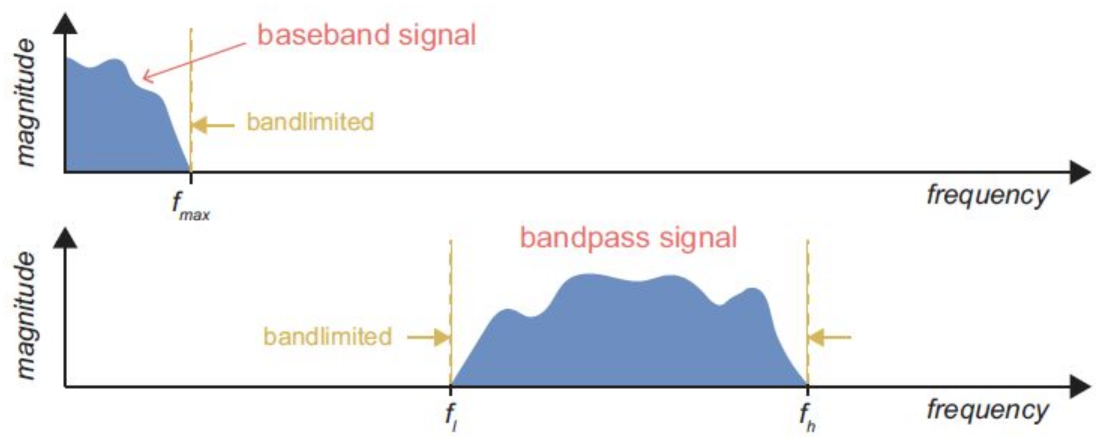
Sampling



z(t)
ADC
z[k]
Amplitude
z(t = τ)
0
t = τ
z(t)
Time, t
t_s
set of
quantisation levels
Amplitude
z[k=8]
0
k=8
z[k]
z(t)
Samples, k

Baseband & Bandpass

- Baseband: Information signal
- Bandpass: Communication signal

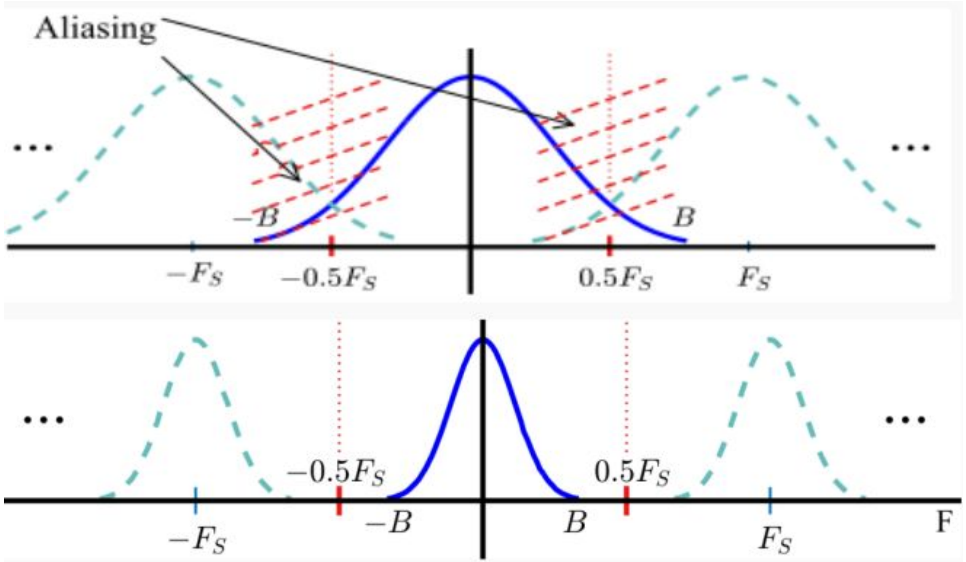


Nyquist Sampling Theorem

- The **Nyquist Sampling Theorem** states that a baseband, bandlimited signal must be sampled at **greater than twice the bandwidth** present in the signal, i.e.
  - fs > 2 * fmax
  - fs > 2 * (f_high - f_low)

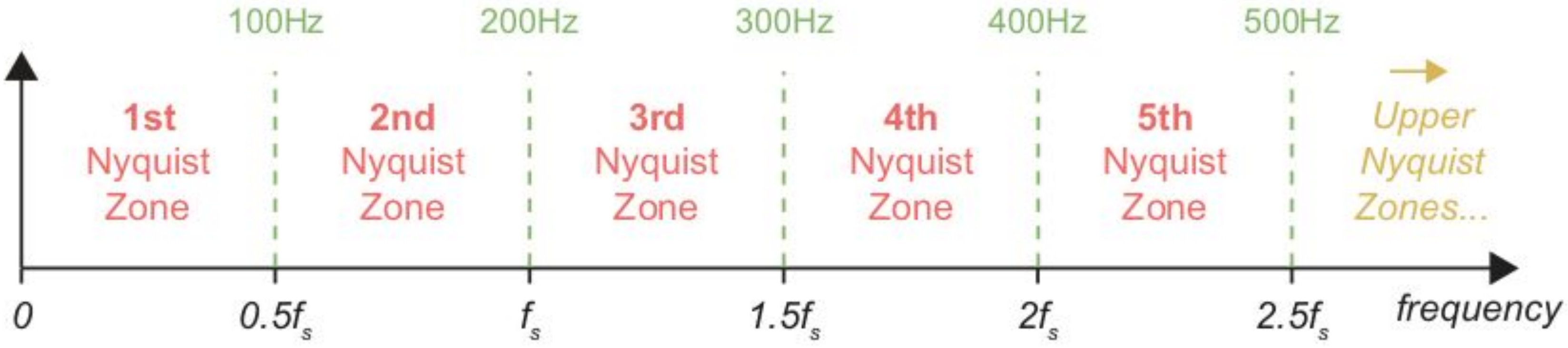
Aliasing

- Sampling produces aliases (spectral replicas)
- To prevent aliasing Fs must satisfy Fs > 2 * BW

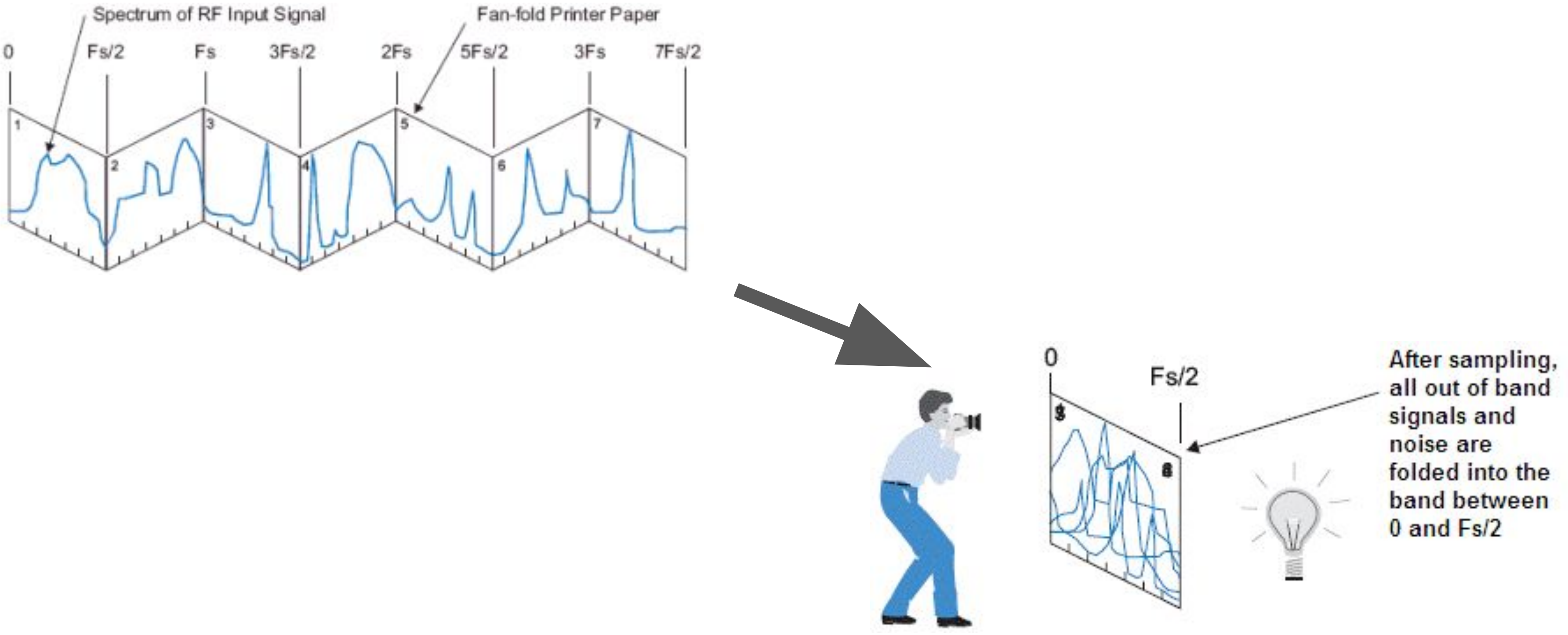


Nyquist Zones

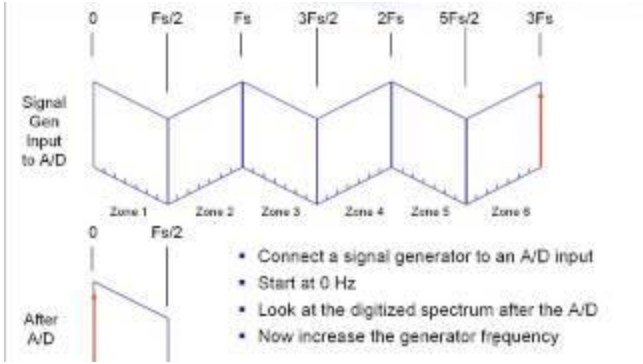
- Partitions of bandwidth 0.5f s in the frequency domain
- Any signal components present in higher Nyquist Zones are 'folded' down into the 1st Nyquist Zone as a result of aliasing



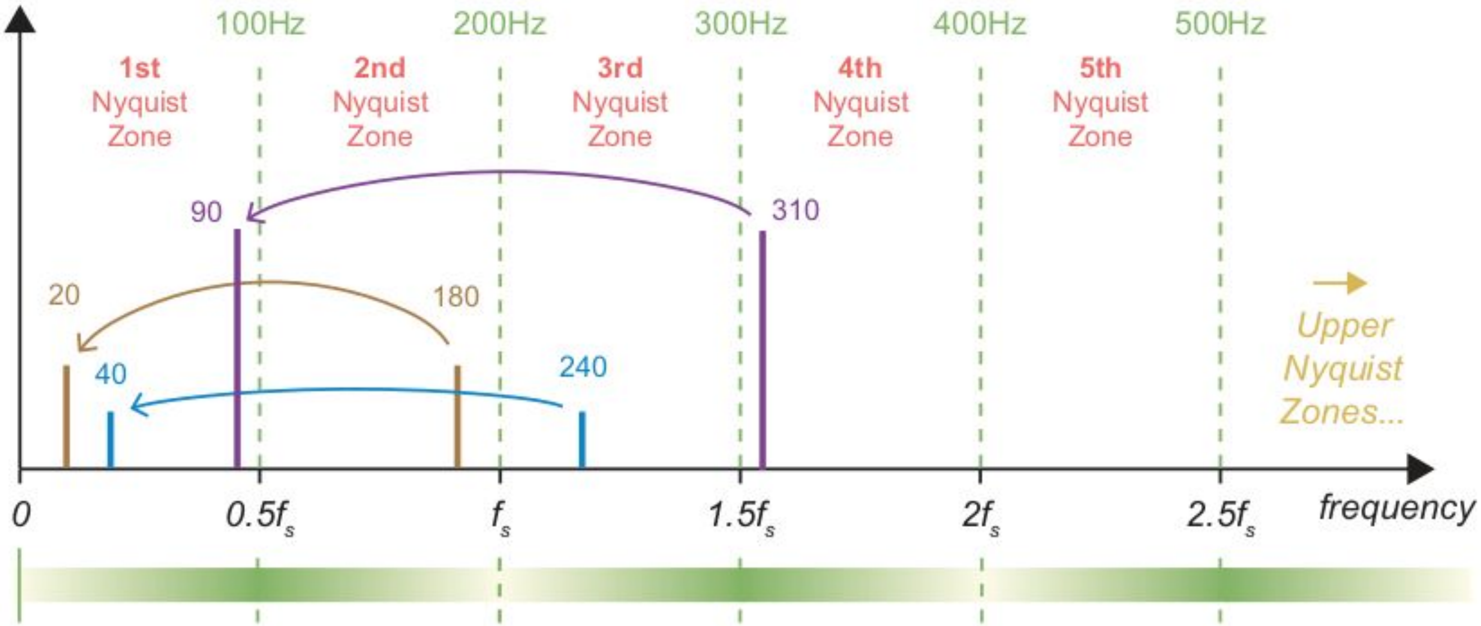
Folded Spectrum View



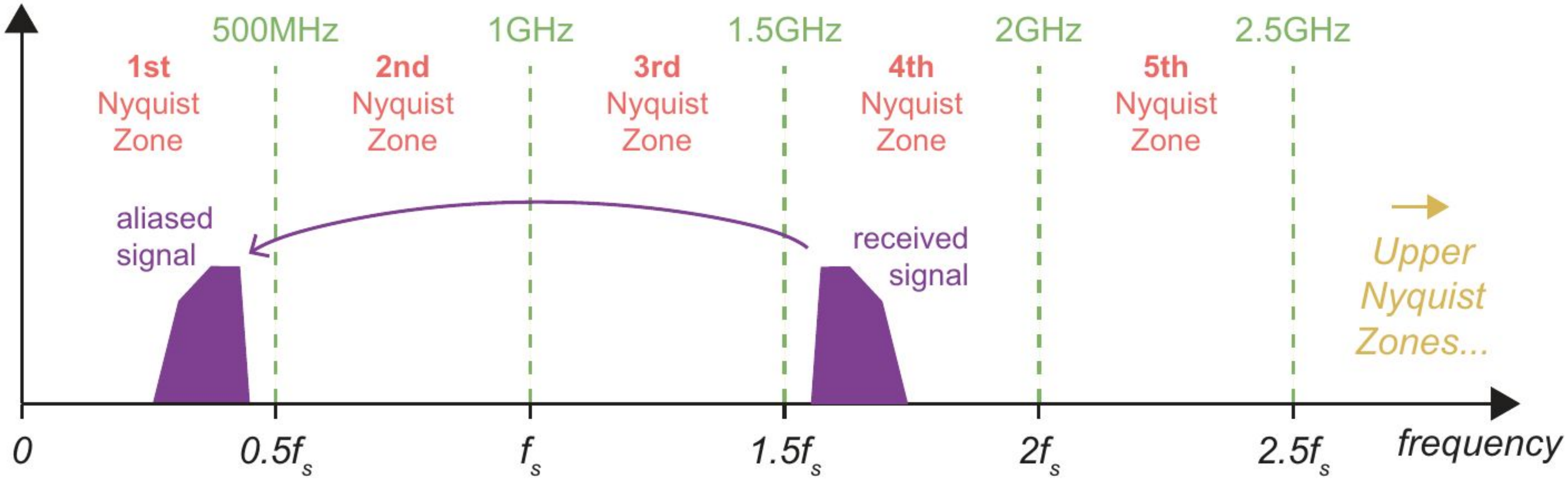
A/D and D/A Sampling Theory



Examples of aliasing with reference to Nyquist Zones

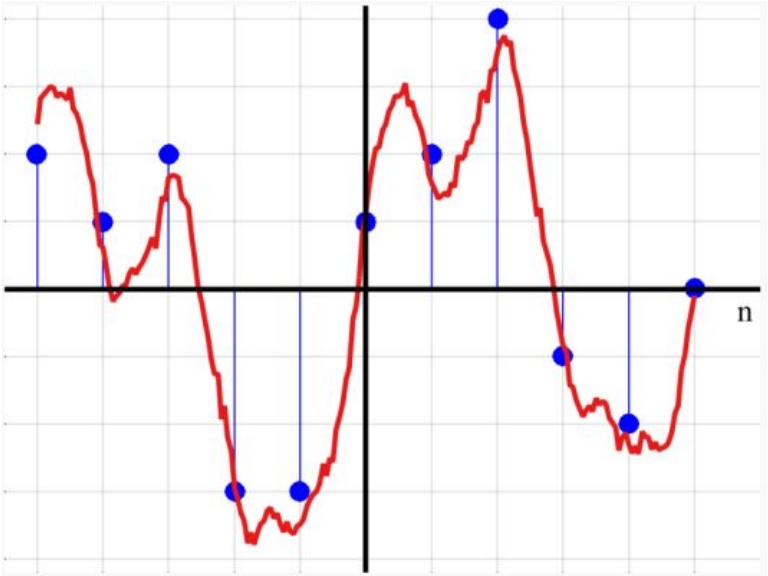


Advantage of Downconverting



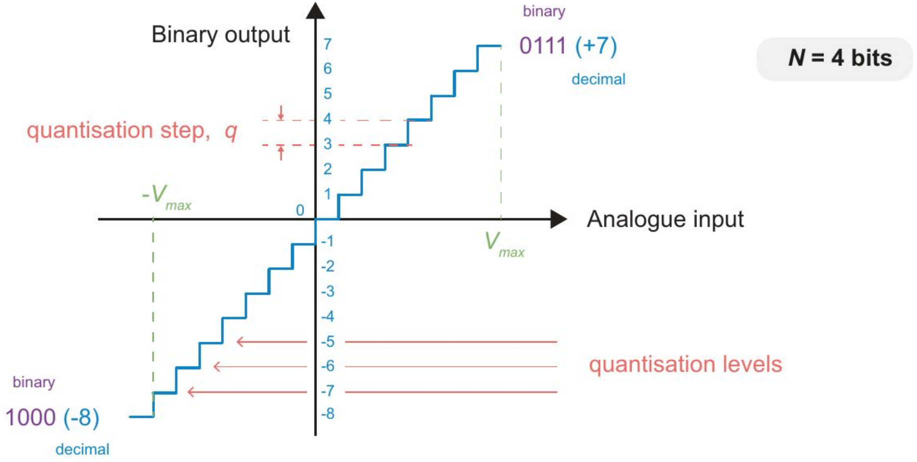
Digitization

- A discrete-time signal is sampled on time axis
- A digital signal is sampled at both time and amplitude axes



Quantization

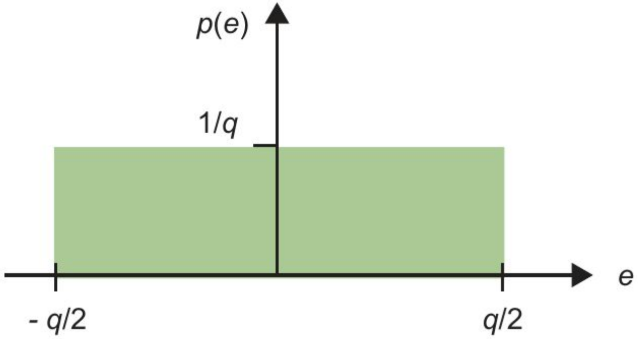
- Dynamic Range is ratio of largest to smallest represantable numbers
  - 20 log10(2**N)
  - 6.02*N



Quantization Error

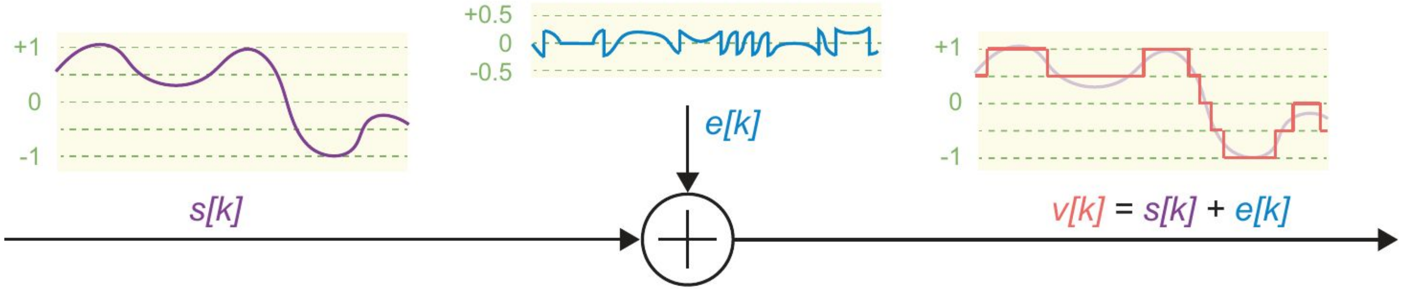
$$n_{ADC} = \int_{-\infty}^{\infty} e^2 p(e)de = \int_{-q/2}^{q/2} e^2 p(e)de\,.$$

$$n_{ADC} = \frac{1}{3q}e^3 \Big|_{-q/2}^{q/2} = \frac{q^2}{12}$$



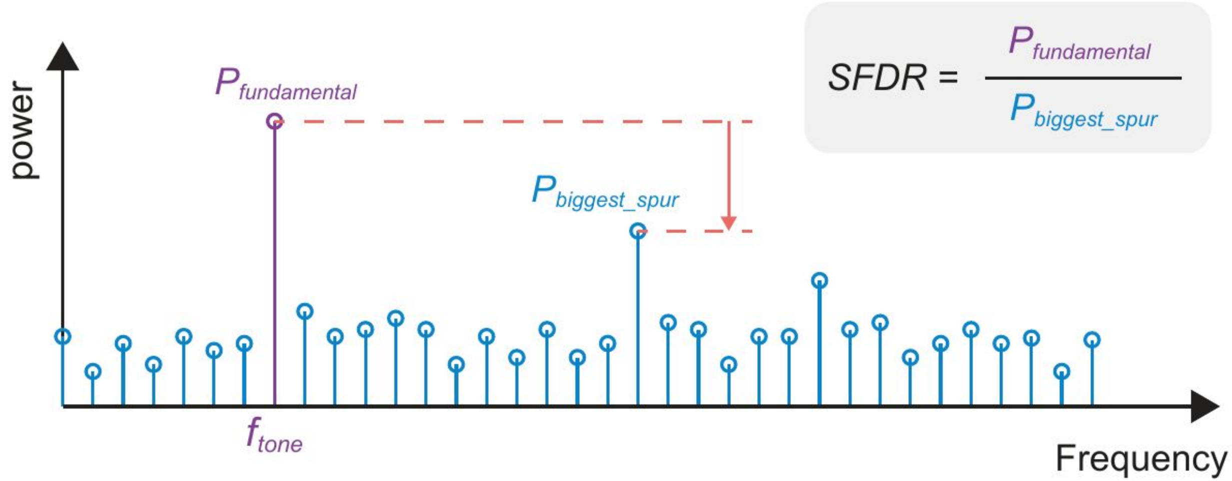
Frequency Spurs

- If an input signal is periodic, the sequence of quantisation errors follows a repeating pattern,
- Therefore, the quantisation error signal is also periodic
- Periodic components correspond to unwanted tones ie, spurs



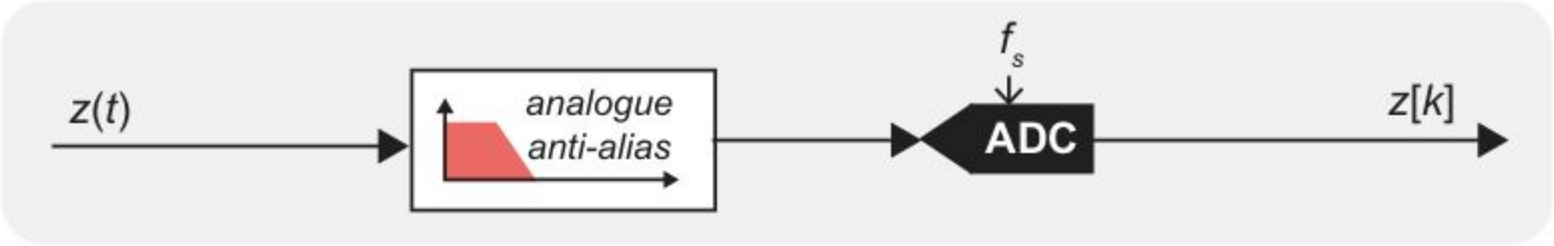
Spurious Free Dynamic Range (SFDR)

- Ratio between the fundamental component (e.g. the sine wave) and the most significant spur, expressed in dBs.

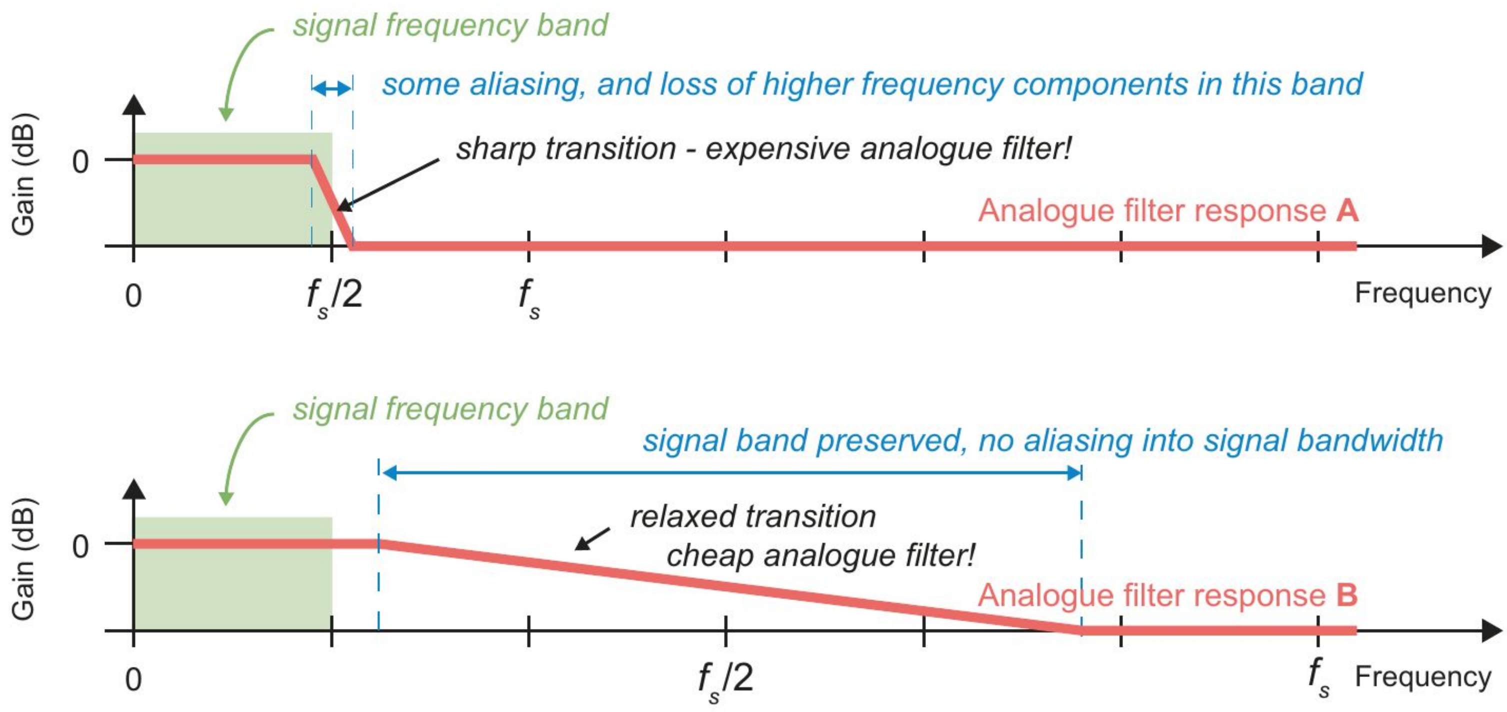


Analogue to Digital Conversion

A conventional ADC operating in Nyquist Zone 1 is preceded by an analogue low pass anti-alias filter, to retain only the frequency components in Nyquist Zone 1, and attenuate all higher frequency signal components that are present at the ADC input.

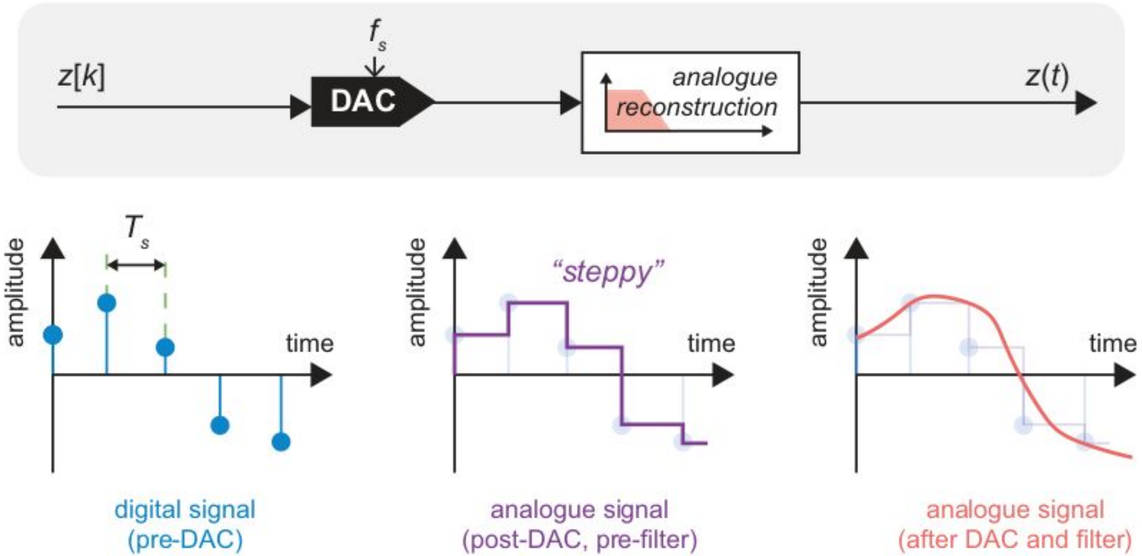


Oversampling at the ADC



Digital to Analogue Conversion

A low pass analogue reconstruction filter to attenuate the significant frequency components present in upper Nyquist Zones, and in doing so, smooths out the time domain signal to produce a more intuitively ‘analogue’ waveform.



Systems

- A device/algorithm that performs some prescribed operations on an input signal to generate an output signal
- Tx, wireless channel, and Rx are all systems

Linearity

$$\text{input} \quad s_1[n] + s_2[n] \longrightarrow r_1[n] + r_2[n] \quad \text{output}$$

then

$$\text{input} \quad \alpha_1 s_1[n] + \alpha_2 s_2[n] \longrightarrow \alpha_1 r_1[n] + \alpha_2 r_2[n] \quad \text{output}$$

Time Invariance

- If the input is shifted by a certain time, the output is also shifted by the same amount

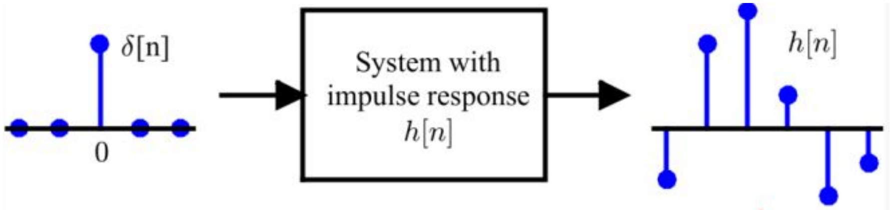
$$\text{input} \quad s[n - n_0] \longrightarrow r[n - n_0] \quad \text{output}$$

Effect of Time Shift

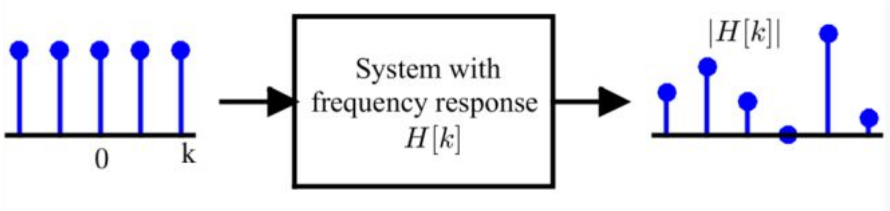
- Shift in one domain —> Multiplication by a complex sinusoid in the other domain

LTI Systems

- **Impulse response $h[n]$**: output response of a system to a unit impulse $\delta[n]$ as an input



- **Frequency response H[k]**: The DFT of the system impulse response



Convolution

For two signals $s[n]$ and $h[n]$, convolution is defined as

$$r[n] = \sum_{m=-\infty}^{+\infty} s[m]h[n-m]$$

Pseudo-code:

1. Select the time shift n
2. Compute the sample-by-sample product $s\ m\ h[n - m]$ and add for all m
3. Repeat for all n

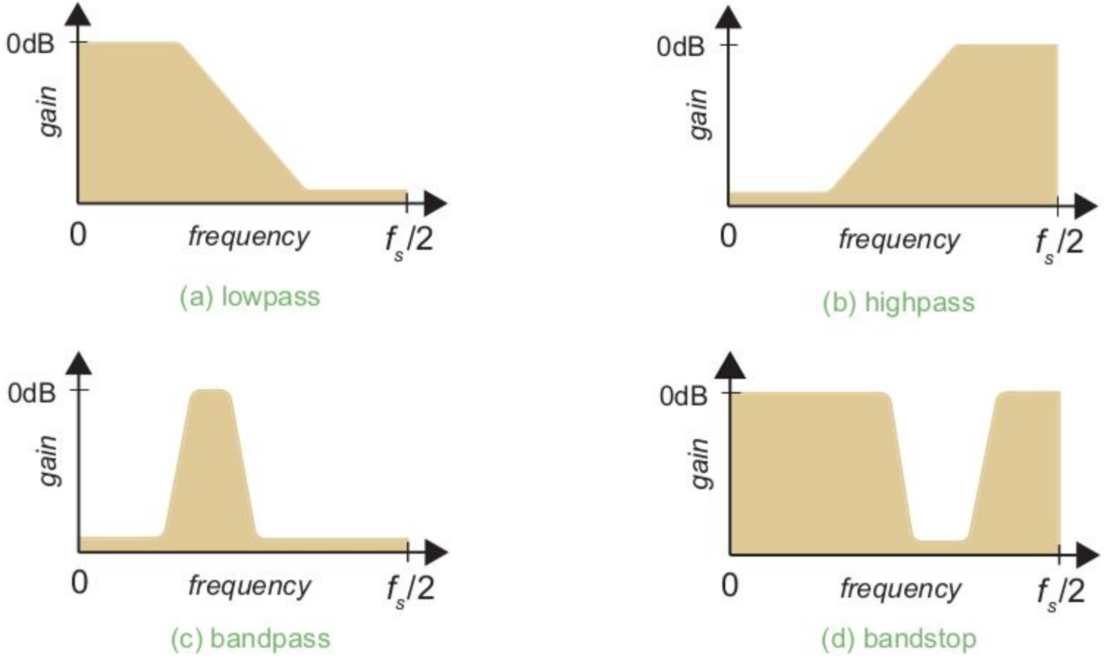
Convolution in Frequency Domain

- Convolution in one domain is equivalent to multiplication in the other domain

$$s[n] \circledast h[n] \xrightarrow{\mathrm{F}} S[k] \cdot H[k]$$

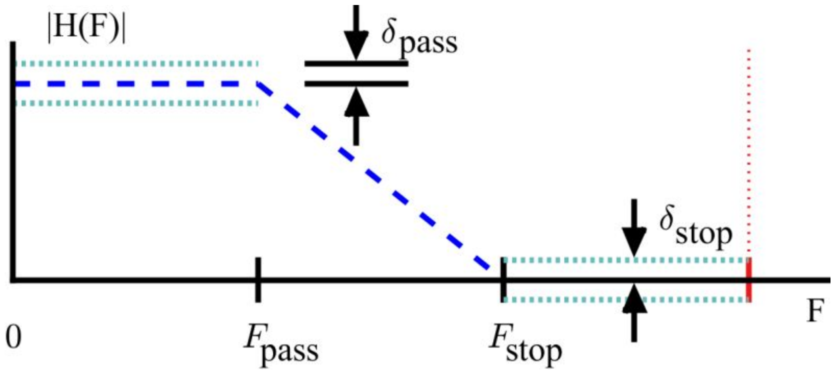
Digital Filters

- A filter modifies the frequency contents of an input signal
- Types
  - LPF
  - HPF
  - BPF
  - Notch



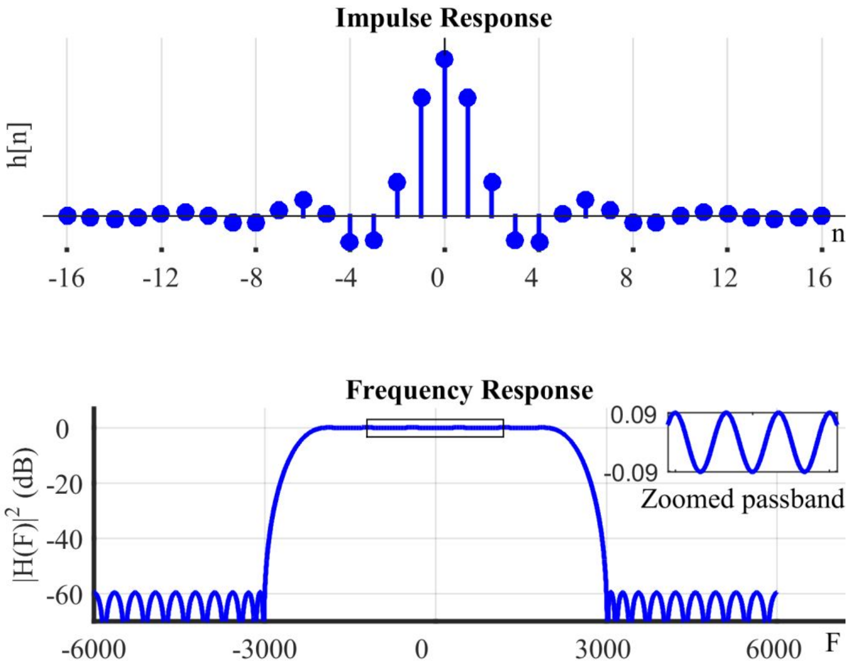
Filter Design

- Sample rate FS
- Passband frequency Fpass
- Stopband frequency Fstop
- Maximum passband ripple δpass
- Maximum stopband ripple δstop



FIR Design Example

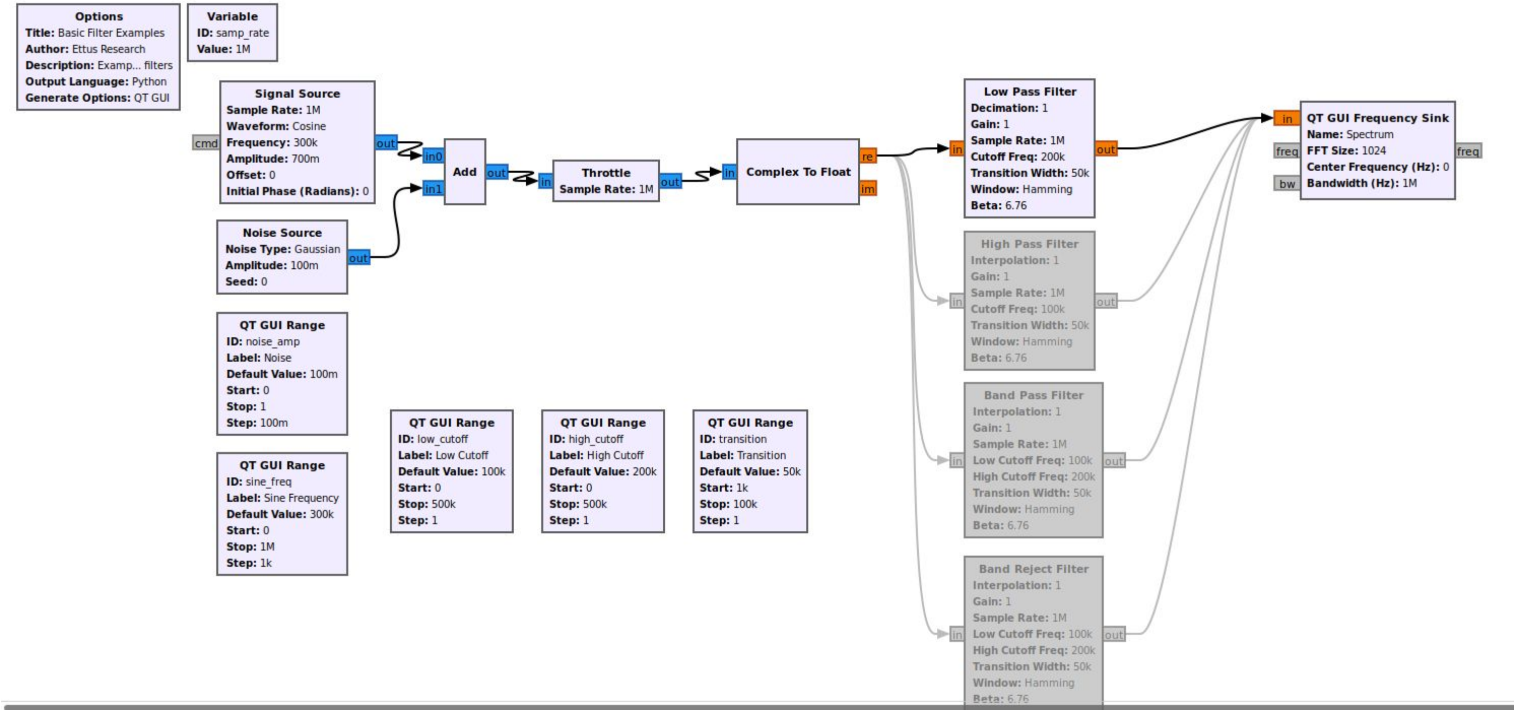
- Sample rate FS = 12 kHz
- Passband frequency Fpass = 2 kHz
- Stopband frequency Fstop = 3 kHz
- Passband ripple δpass,dB = 0.1 dB
- Stopband ripple δstop,dB = −60 dB



Filter Choices

- Finite Impulse Response (FIR)
  - FIR filters are intrinsically stable, as they have no feedback path
  - linear phase response: all frequencies passing through the filter are delayed by the same amount of time, which corresponds to a linearly increasing phase difference
- Infinite Impulse Response (IIR)
  - they can achieve the same magnitude response as an FIR filter, using fewer weights, and therefore they require less computation and are less costly to implement in hardware

Filters Using GNU Radio

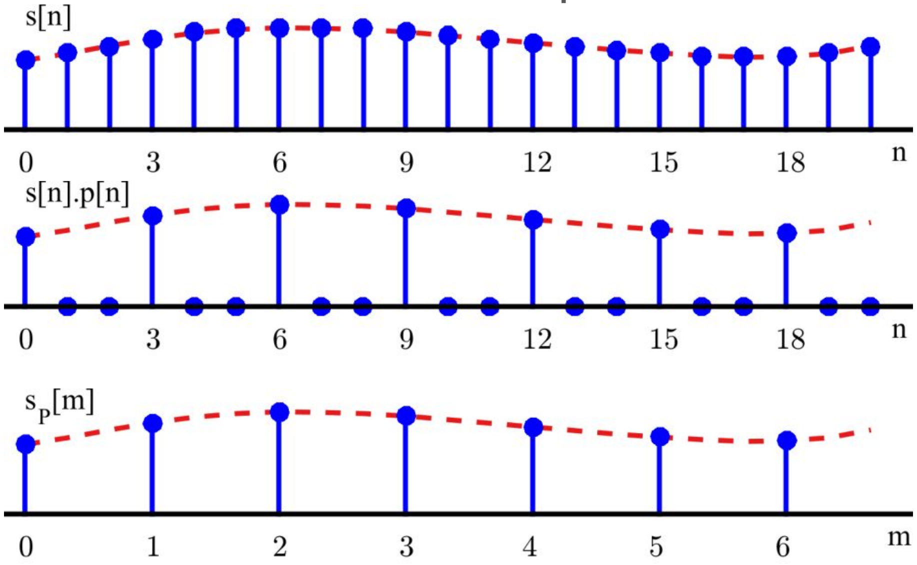


Multirate Signal Processing

- Multirate operations are required to change the sampling rate in a DSP system to optimise computational efficiency
- Some example scenarios
  - To match the sampling rates of two signal paths that will be combined
  - To adjust the sampling rate closer to Nyquist when the signal bandwidth changes
  - To match the sampling rate of an external interface, such as a DAC
  - To ease analogue anti-alias or image-rejection filter requirements

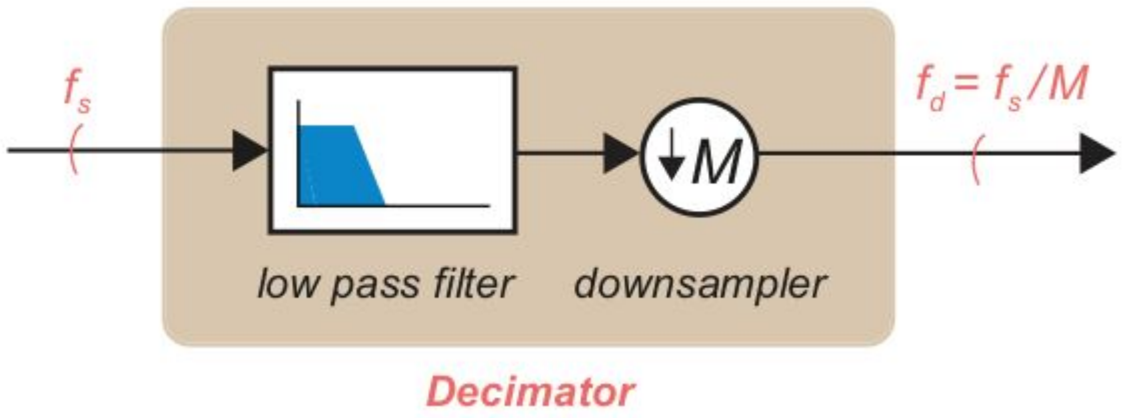
Decimation

- Reducing the sample rate by an integer factor
- Retain every Pth sample and discard the remaining samples
- The new slower sample rate is $1/P$ of the original faster sample rate



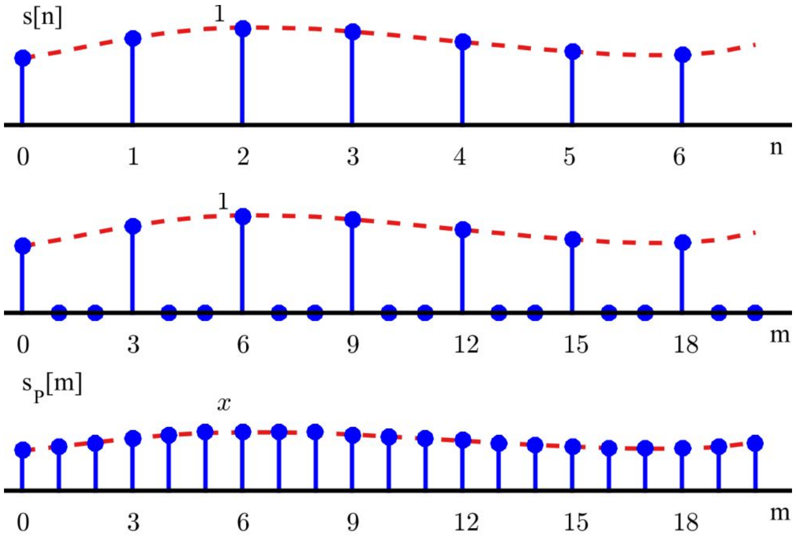
Decimation

- Decimation involves two processes:
  - anti-alias low pass filtering, followed by
  - downsampling



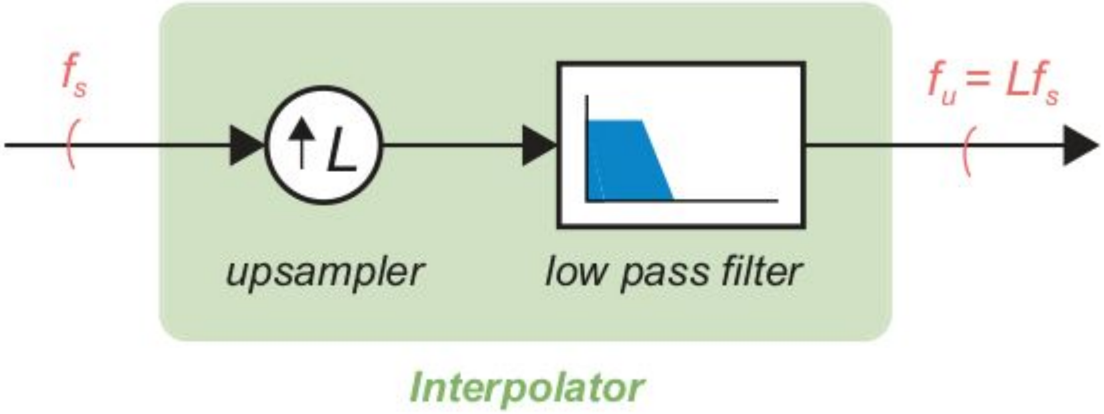
Interpolation

- Increasing the sample rate by an integer factor
- Insert $P - 1$ zeros between the original input samples and interpolate
- The new faster sample rate is P times the original slower sample rate



Interpolation

- An interpolator is composed of
  - an upsampling operation, followed by
  - a low pass image rejection filter

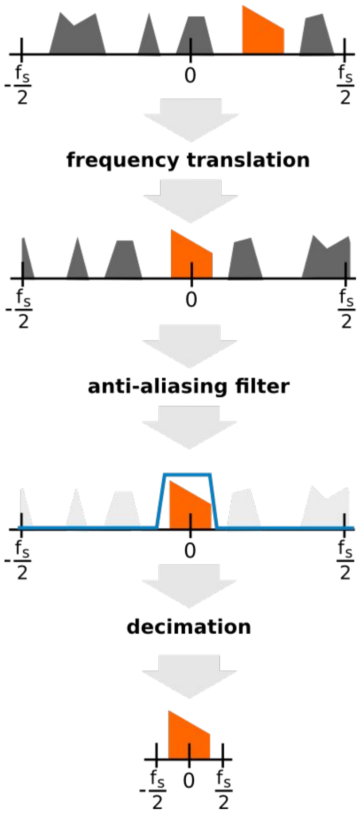


Other Multirate Operations

- There are other types of operation to be aware of, beyond simple decimation and interpolation by integer factors
- Resampling a signal by a **rational fraction**
  - If the sampling rate is to be changed by the ratio of two integers, e.g. a rate change from 100 MHz to 150 MHz could be expressed as $R = 3 / 2$. Rational fractional rate changes can be achieved using a **cascade** of an interpolator and decimator, e.g. $L = 3$ and $M = 2$ in this example. The resulting structure can be optimised using polyphase methods.
- Resampling a signal by an **irrational fraction**, or by a factor that changes over time
  - Where there is no convenient integer-based expression for the resampling ratio, or where it is dynamic, a different type of approach is required. Popular methods include highly oversampled polyphase filters, and Farrow structures.

Frequency Xlating FIR Filter

- Frequency Xlating FIR Filter is a block that:
  - performs **frequency translation** on the signal,
  - **downsamples** the signal by running a **decimating FIR filter** on it.
- It can be used as a **channelizer**:
  - it can select a narrow bandwidth channel from the wideband receiver input.



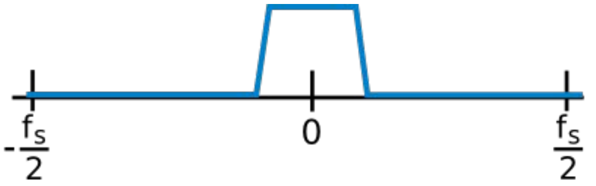
Suppose this is the stations in FM radio example!

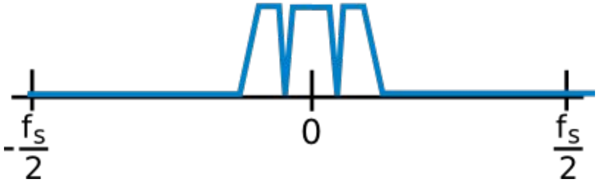
Our aim is to select only one channel

Frequency Xlating FIR Filter

- If you have Real taps, then your FIR filter will be symmetric in the frequency domain.

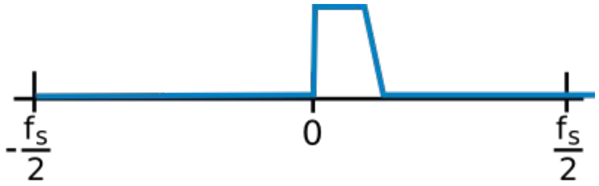
```
firdes.low_pass(1,samp_rate,samp_rate/(2*deci
mation), transition_bw)
```

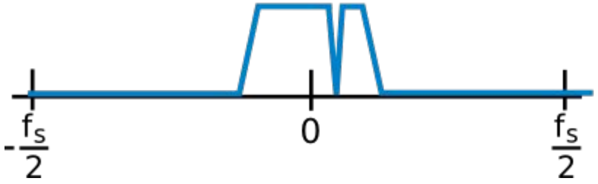




- If you have Complex taps, then your FIR filter will not have to be symmetric in the frequency domain.

```
firdes.complex_band_pass(1, samp_rate,
-samp_rate/(2*decimation),
samp_rate/(2*decimation), transition_bw)
```

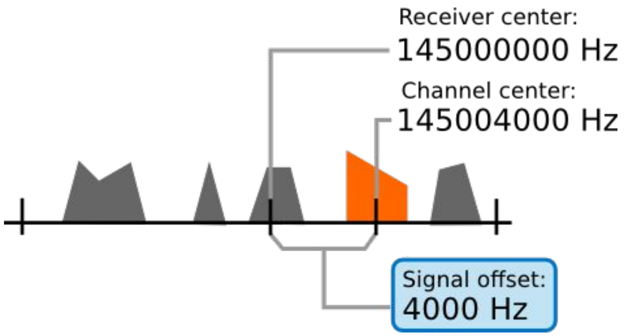




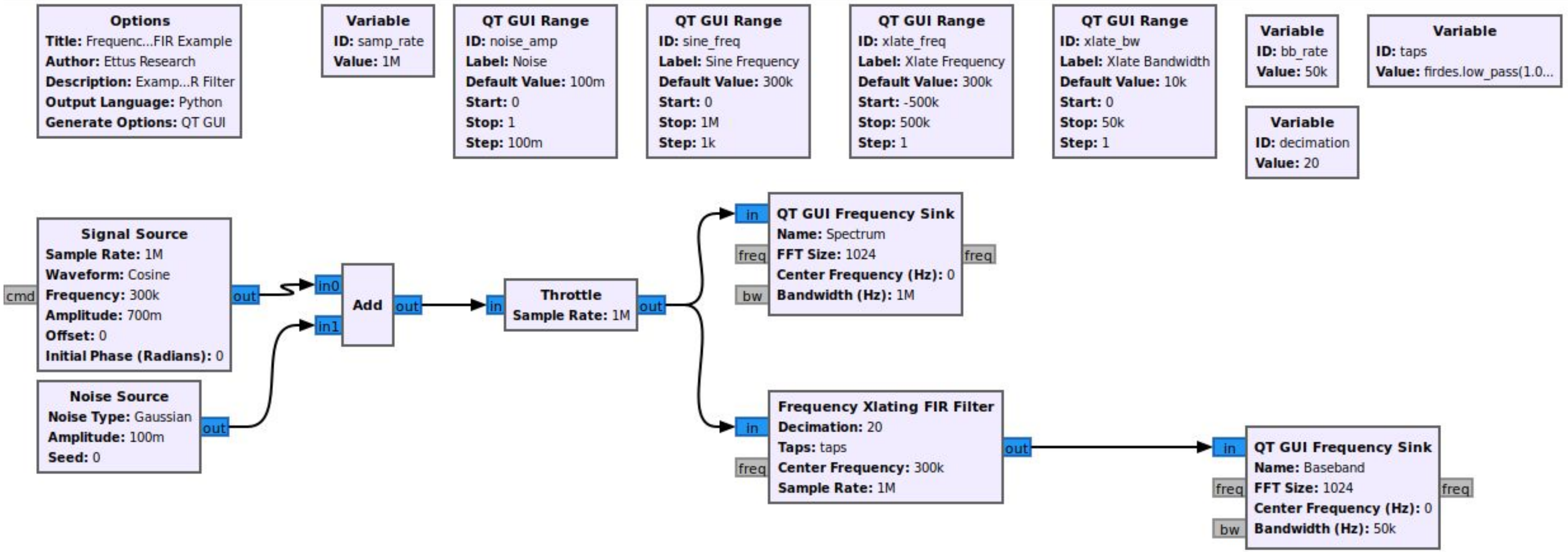
Frequency Xlating FIR Filter

- **Decimation**: the integer ratio between the input and the output signal's sampling rate.
- Example:
  - Input sample rate = 240000
  - Decimation factor = 5
  - Output sample rate = 240000 ÷ 5 = 48000

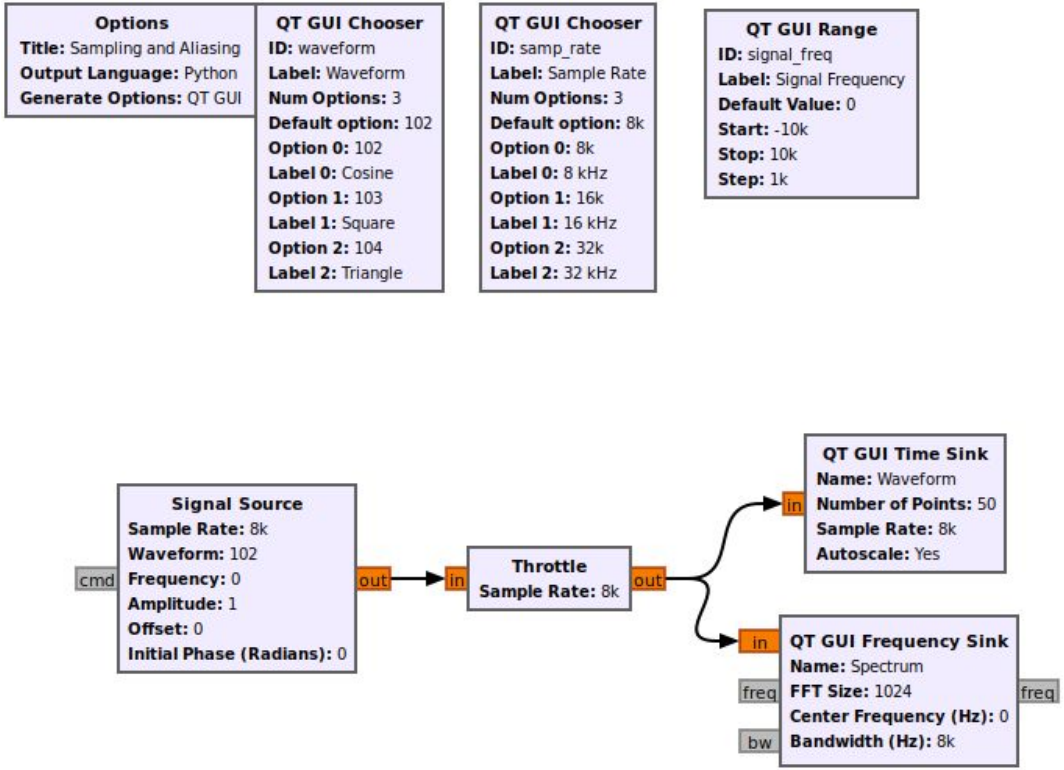
- **Center frequency**: the frequency translation offset frequency.
- In practice, it is the frequency offset of the signal if interest to be selected from the input.



Frequency Xlating FIR Filter



Sampling and Aliasing



Producing Sound and Aliasing

- **Sound card** is perfect for getting familiar to core DSP concepts.
- It is easily available and accessible.
- GNU Radio provides access to sound card via “audio source” and “audio sink” blocks.
- Create the following flowgraph
- Set the frequency to 48440, 48880, 47560, 51000, 52000, 96440 at a time
- And observe what you hear and explain your understanding!
  - ***Hint**: Remember the “folded spectrum view”*

Producing Sound and Aliasing