PHL245 Modern Symbolic Logic

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Contents

1 Arguments

1.1 Validity

We say a deductive argument is valid iff it is not invalid. This means that we can find out if a argument is valid or not by assessing the possiblity of the case where the premise is true and conclusion is false.

1.2 Soundness

We say a deductive argument is sound iff it is

- It is valid
- All the premises are TRUE

2 Semantics in Sentential Logic

2.1 Syntax

Sentential Logic(SL) Complex (compound) statements are all built up by joining statements together using LOGICAL CONNECTIONS. we have $\wedge, \vee, \rightarrow, \leftarrow, \sim$. Where $\sim$ is the only unitary connector and others are binary.

Atomic vs Molecular A **statement** is Atomic if it has no logical connector and is molecular otherwise. We use **P-Z letters** to represent **atomic statments** Here is an example: You can have fries or salad. $\equiv P \vee Q$. Then, P is "You can have fires" and Q is "You can have salad". Notice that we are folowing the definition that P and Q are **statments**.

Informal Notation Hierachy [1] We shall see this through some examples:

- In official notation, we need $(P \vee Q)$ while it is safe to drop the parenthesis.

We can check if a statment is official or informal if it has the same number of brackets as binary connectors in the statement. If you see a bracket around a unitary connector, the sentence is not well-formed. **Right Most Rule** We say the rightmost connector in a sentence with connectors of the same level the main connector.

[1] We use this Informal notation because the formal one is cumbersome. But in order to use the informal one, we have some conventions to follow.

3 Truth Tables

3.1 Summary

First we remark that we are here talking about binary operators, so they will have and only have two operands. We have the following to consider:

- **Logical OR** ($\vee$) is true when either one of the operands is true. It evaluates to false otherwise.
- **Logical AND** ($\wedge$) is true when both of the operands are true and false otherwise.
- **Implication** ($\implies$) is true in two cases. The first case is when the first operand is false and second case is when both operands are true.
- **Double implication, Iff** ($\iff$) is true when either both operands are true or when both operands are false.
- **Negation** ($\neg$[2]) is true when the operand is false and false otherwise.

3.2 Sementic Properties in Truth Tables

- Tautology if always true
- Contradiction if always false
- Contingent if mixed
- Consistent if there exists a row where the conclusion is all true
- Inconsistent if all rows are not all true, negation of the consistent
- Logically equivalent if two sentences have the same truth table

3.3 Full truth tables

Example: [3] evaluate $(P \wedge \neg Q) \vee R$. $\neg R \vee Q$. $\therefore \neg P \implies Q$. Notice that this is equivalent to evaluating

$$(((P \wedge \neg Q) \vee R) \wedge (\neg R \vee Q)) \implies \neg P \implies Q$$

[2]Or in this course, we may see $\sim$

[3]This example was adapted from Scharer 4.4 EG3.