Inferential Statistics, Test Statistics Manuel

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1 Test for $\mu = \mu_0$, w/σ^2 known

Assume that $X_i \sim N(\mu, \sigma^2)$ are i.i.d, then the test statistic is

$$T(X) = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

then under α significance level, we have the rejection region

$$R_\alpha(T) = (-\infty, z_{\frac{\alpha}{2}}) \cup (z_{1-\frac{\alpha}{2}}, \infty)$$

2 Test for $\mu = \mu_0$, w/σ^2 unknown

Assume that $X_i \sim N(\mu, \sigma^2)$ are i.i.d, then the test statistic is

$$T(X) = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$$

then under α significance level, we have the rejection region

$$R_\alpha(T) = (-\infty, t_{\frac{\alpha}{2}, df=n-1}) \cup (t_{1-\frac{\alpha}{2}, df=n-1}, \infty)$$

3 Test for $\sigma^2 = \sigma_0^2$

Assume that $X_i \sim N(\mu, \sigma^2)$ are i.i.d, then the test statistic is

$$T(X) = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2_{df=n-1}$$

and the α significance level rejection region is

$$R_\alpha(T) = (-\infty, \chi^2_{\frac{\alpha}{2}, df=n-1}) \cup (\chi^2_{1-\frac{\alpha}{2}, df=n-1}, \infty)$$

4 Likelihood Ratio Test

Define

$$\Lambda := \frac{max_{\theta \in \Omega_0}[L(\theta)]}{L(\hat{\theta})}$$

Denoting $p = \dim \Omega =$ number of free var in the whole space, and $d = \dim \Omega_0 =$ number of free var under our null hypothesis, we have

$$T(X) = -2 \ln \Lambda \xrightarrow{D} \chi^2_{df=p-d}$$

5 Equality of Variances $\sigma_x = \sigma_y$

If we have $X_1, ..., X_n \sim N(\mu_x, \sigma_x^2)$ and $Y_1, ..., Y_n \sim N(\mu_y, \sigma_y^2)$, then

$$T(X,Y) = \frac{S_x^2}{S_y^2} \sim F_{(n-1)(m-1)}$$

With α significance level, we then have the rejection region

$$R_\alpha(T) = \left(-\infty, F_{\frac{\alpha}{2}(n-1)(m-1)}\right) \cup \left(F_{\frac{1-\alpha}{2}(n-1)(m-1)}, \infty\right)$$

6 Equality of $\mu_x = \mu_y$, w/σ_x, σ_y known

If we have $\bar{X} \sim N\left(\mu_x, \frac{\sigma_x^2}{n}\right)$ and $\bar{Y} \sim N\left(\mu_y, \frac{\sigma_y^2}{n}\right)$, then

$$T(X,Y) = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}} \sim N(0,1)$$

7 Equality of $\mu_x = \mu_y$, w/$\sigma = \sigma_x = \sigma_y$ known

If this is the case, we can pull the $\sigma_x = \sigma_y = \sigma$ out from the above equation, we will have

$$T(X,Y) = \frac{\bar{X} - \bar{Y}}{\sigma\sqrt{(\frac{1}{n} + \frac{1}{m})}} \sim N(0,1)$$

8 Equality of $\mu_x = \mu_y$, w/$\sigma = \sigma_x = \sigma_y$ unknown

If we have $\bar{X} \sim N\left(\mu_x, \frac{\sigma_x^2}{n}\right)$ and $\bar{Y} \sim N\left(\mu_y, \frac{\sigma_y^2}{n}\right)$, then

$$T(X,Y) = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{(\frac{1}{n} + \frac{1}{m})}} \sim t_{n+m-2}$$

where, S_p is the polled sample variance, defined as

$$S_p := \frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}$$