Report.

Last time I reported to Zhou, I had the result
***B, K, S: OBM(skew) consistency theorem. Started on 23 July 2018 and I started working on its chainization, kernelization and betification

What has been done since 15th August

1. Kernelized our results. Theorem 6 on page 188 K: L2 consistency of diagonal kernel OBM. MSE tends to zero.

2. Chainization. Theorem 7, page 240. K: kernel MSE uniform convergence with chaining

3. Half of betification when beta is with the known moments. Conditions are not simplified. Theorem 8. Page 267

Need

1. Simplify the covariance conditions

2. Chainization of the betification.

3. Analyze the case when we build β in a diagonal way.

Question 1. On what conditions on g, $LRV(g(t,\xi))$ is L_{ip} in t?

It seems to me that if we just say, LRV is L_{ip} in t, is enough. If not, this question is valid.

Zhou:

The easiest is to leave it as is. You could try to assume something like SL about g to get something like Holder continuity on LRV. But it is fine to leave it.

Question 2.

Connection of δ and γ.

$$\delta_p(k) = \|g(t,\xi_0) - g(t,\xi_0^{*,k})\|_p$$

$$\gamma_k = \mathrm{Cov}(g(t,\xi_0), g(t,\xi_k))$$

The initial point of an answer is in supplement paper, page 11

Lemma 1. *Let $\{Z_j = G_n(t_j,\mathcal{F}_j)\}_{j=1}^n$ and $\{W_j = H(t_j,\mathcal{F}_j)\}_{j=1}^n$ be two sequences with $\max\|Z_j\| < \infty$ and $\max\|W_j\| < \infty$. Assume that*

$$\sum_{j=0}^{\infty}[\delta_Z(j,2) + \delta_W(j,2)] < \infty.$$

Then

$$|Cov[Z_i, W_j]| \leqslant \omega_{Z,W}(i,j,2),$$

where $\omega_{Z,W}(i,j,p) = \sum_{k=-\infty}^{\infty} \delta_Z(i-k,p)\delta_W(j-k,p)$.

And in the simple stationary case, it is described in notes 3, 5.6 redQuestions to Zhou on May 23, 2018,
from OneNote / LRV / communication / Questions to Zhou 22 May 2018 to

quest Zhou 2018 05 22

$$\gamma_k = \sum_{j=0}^{\infty} \mathbb{E}[P_{i-j}X_i P_{i-j}X_{i+k}] \leqslant \sum_{j=0}^{\infty} \|P_{i-j}X_i\|_2 \cdot \|P_{i-j}X_{i+k}\|_2 =$$

$$= \sum_{j=0}^{\infty} \theta_j(X) \cdot \theta_{k+j}(X) \leqslant \sum_{j=0}^{\infty} \delta_j(X) \cdot \delta_{k+j}(X)$$

We had the conditions

$$\begin{cases} 1) & \sum\limits_{l=0}^{\infty} \sup\limits_{t} \delta_{g(t,\xi_0),4}(l) < \infty \\ 2) & \sum\limits_{l=1}^{\infty} l|\gamma_l| < \infty \end{cases}$$

Can we simplify this conjunction? One includes the other?

Zhou:

You can leave them as is if you want to be general. They do not include each other. However, you could assume some power decline of δ to get 2) decline of γ.

Question 3.

For fixed k and t, for the following sequence $\left\{\beta'_{i,k}(t)\right\}_{i\geqslant 1}$ where

$$\beta'_{i,k}(t) = \beta_{i-k,k}(t) = \frac{1}{\gamma_0}[g(t,\xi_{i-k})g(t,\xi_i) - \rho_k(t)g(t,\xi_{i-k})^2],$$

Find out when

$$\sum_{j=1}^{\infty} j|j_{\beta'}(j)| < \infty \quad ?$$

It would be good to get an answer in terms of g function. Here

$$j_{\beta'}(j) = \operatorname{Cov}(\beta'_{k,k}(t), \beta'_{k+j,k}(t))$$

Zhou:

Again, you can formulate some sufficient condition in terms of δ of β. For example, imposing

$$\delta_{\beta',q}(l) = O(l^{-2}) \qquad (*)$$

will guarantee

$$\sum l \cdot j_{\beta'}(l) < \infty$$

And you can also try to transfer the assumption on $\delta(\beta)$ onto $\delta(X)$

Boris:

May be refresh V material. 5.10 Sum of lagged products operator.
5.13 Sum of lagged products operator continued.
supplement paper, page 11

S:

red point

find where we had the application of the quadratic operator. The inequality between γ and δ.

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