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In GLMs are the Scale and Dispersion parameters the same?

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Given a data set and a genralized linear model I am asked to find the estimation of the scale parameter obtained with the Pearson statistic. But I am a bit confused: I know that

$$\mathrm{Var}(Y) = \phi \cdot V(\mu)$$

where $\mathrm{Var}(Y)$ is the variance and $V(\mu)$ is the Variance function. I thought that ϕ is the dispersion parameter, but I searched on internet and found some sources where it is referred as scale. Now I am confused, is it actually the same?
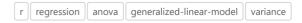
The output I get in R is:

```
Call:
glm(formula = H ~ Days, family = gaussian(link = "sqrt"), data = dd)

Deviance Residuals:
    Min       1Q    Median       3Q       Max
-9.4170   -1.7410    0.1247    2.0765    9.5232

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.559916   0.062123   25.11   <2e-16 ***
Days        0.037061   0.000629   58.92   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 7.659018)

    Null deviance: 35880.6  on 239  degrees of freedom
Residual deviance:  1822.8  on 238  degrees of freedom
AIC: 1173.7

Number of Fisher Scoring iterations: 5
```

I guess that Dispersion parameter for gaussian family... is the Dispersion parameter right? If the scale and the Dispersion parameter are not the same:

1- What is the difference?

2- How to find the scale in the summary?

Thanks in advance

r   regression   anova   generalized-linear-model   variance

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edited Jan 8 '19 at 7:04

asked Dec 26 '18 at 15:54

1 Answer

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Short Answer

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Yes, in generalized linear model (GLM) theory, you can assume that the "dispersion" parameter and the "scale" parameter are the same thing. Roughly speaking, "scale" is the older GLIM terminology and "dispersion" is the newer R terminology. However, the term "scale" has been used inconsistently in GLM terminology and so is best avoided (IMO).

A Bit of History

The original GLM article (Nelder and Wedderburn, 1972) defined the probability density function (pdf) of a GLM family to be

$$f(y; \theta, \phi) \propto \exp\{a(\phi)(y\theta - \kappa(\theta))\}$$

where $a()$ is a known function of the (possibly unknown) parameter ϕ. (They actually wrote z for y and g for κ.) This formulation leads to

$$\mathrm{var}(y) = \frac{1}{a(\phi)} V(\mu)$$

Nelder and Wedderburn called $a(\phi)$ the *scale factor* but didn't give ϕ a name.

In 1974, the GLIM software package was released by the Royal Statistical Society as the first software implementation of GLMs. The GLIM manual inverted the definition of the *scale factor* by assuming

$$f(y; \theta, \phi) \propto \exp\{\frac{1}{a(\phi)} (y\theta - \kappa(\theta))\}$$

and they further assumed

$$a(\phi) = \phi/w,$$

where w is a known *prior weight*, leading to

$$\mathrm{var}(y) = \frac{\phi}{w} V(\mu)$$

The GLIM manual called ϕ the *scale parameter* and the GLIM software had a `$scale` command by which ϕ could be optionally set to a constant.

The original GLM book (McCullagh and Nelder, 1989) used the same pdf as in the GLIM manual but called ϕ the *dispersion parameter*.

Here then is the source of the ambiguity. In the 1980s the definitive book reference on GLMs (McCullagh and Nelder) was calling ϕ the "dispersion parameter" while the definitive software (GLIM) was calling it the "scale parameter".

GLMs were introduced to the S language in 1991 (Chambers and Hastie, 1991) and they followed the McCullagh and Nelder (1989) terminology rather than that of GLIM. When R appeared around 1995, it followed the syntax and terminology of S, which is what you can see in your output above.

Gradually people have come to realize that *scale* is an over-used term in statistics and *dispersion parameter* has come to be more standard. Dunn and Smyth (2018), for example, define the pdf to be

$$f(y; \theta, \phi) \propto \exp\{\frac{w}{\phi}(y\theta - \kappa(\theta))\}$$

which leads to

$$\mathrm{var}(y) = \frac{\phi}{w} V(\mu)$$

and they call ϕ the *dispersion* parameter. The concept of a *scale factor* is no longer even mentioned, as not being of any independent value.

The *scale parameter* terminology is hard to kill off however. Although, GLIM is long gone, the terminology still appears in some longstanding programs like SPSS, SAS and Stata.

References

Nelder, JA, and Wedderburn, RWM (1972). Generalized linear models. *Journal of the Royal Statistical Society Series A*, 135(3), 370-384.

McCullagh, P, and Nelder, JA (1989). *Generalized Linear Models*, 2nd Edition. Chapman and Hall CRC, London.

Chambers, JM, and Hastie, TJ (1991). *Statistical Models in S*. Chapman and Hall CRC, New York.

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edited Jan 9 '19 at 7:53

answered Jan 8 '19 at 7:02

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