UNIVERSITETET I BERGEN

KANDIDAT

127

PRØVE

MNF130 0 Diskrete strukturer

Emnekode	MNF130
Vurderingsform	Skriftlig eksamen
Starttid	29.05.2024 09:00
Sluttid	29.05.2024 12:00
Sensurfrist	--
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Information

Oppgave	Tittel	Oppgavetype
i	General information	Informasjon eller ressurser

Compulsory assignments

Oppgave	Tittel	Oppgavetype
1	Compulsory assignments	Tekstfelt

Questions

Oppgave	Tittel	Oppgavetype
2	Multiple choice questions (12 points)	Flervalg
3	Logic and proofs (10 points)	Langsvar
4	Basic structures (8 points)	Langsvar
5	Mark the subset (2 points)	Feltvalg
6	Number theory and algorithms (8 points)	Langsvar
7	Mathematical induction (8 points)	Langsvar
8	Counting (8 points)	Langsvar
9	Graph theory (4 points)	Langsvar

1 Compulsory assignments

This is a placeholder to put the marks for the compulsory quizzes and mandatory assignments:

- points for compulsory passed quiz, maximum 10 points.
- the sum of points on the mandatory assignments, maximum 30 points.

You don't need to fill anything here.

Maks poeng: 40

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

3 3 6 6 8 4 7

2 Multiple choice questions (12 points)

a) What does the fundamental theorem of arithmetic say?
Select one alternative:

- every integer n greater than 1 can be uniquely written as a prime or as the product of two or more primes
- every integer n can be written as a prime
- every integer n can be uniquely written as a prime or as the product of two or more primes

b) We say that $f(n)$ is $\Theta(g(n))$ if there is a constant C such that $f(n) \leq C \cdot\ g(n)$ for all sufficiently large n.
Select one alternative

- Wrong definition
- Correct definition

c) Which statement is correct?
Select one alternative

- Quantifiers have lower precedence than all logical operators from propositional logic
- Quantifiers have equal precedence than all logical operators from propositional logic
- Quantifiers have higher precedence than all logical operators from propositional logic

d) Which statement is correct?
Select one alternative

- If f is one-to-one, then its inverse f^{-1} exists
- None of the above statements are correct
- If f is bijective, then its inverse f^{-1} exists
- If f is onto, then its inverse f^{-1} exists

e) How do you call these statements: $\neg\forall\ x\ P(x) \equiv \exists\ x\ \neg\ P(x)$ and $\neg\exists x P(x) \equiv \forall x \neg P(x)$?

Select one alternative

- the de Morgan laws for quantified statements
- the associative laws for quantified statements
- the idempotent laws for quantified statements

f) When do you use the following formula $P(n,r) = \frac{n!}{(n-r)!}$?

Select one alternative

- if repetitions are not allowed and the order of picked elements does not matter
- if repetitions are allowed and order of picked elements does not matter
- if repetitions are allowed and the order of picked elements matters
- if repetitions are not allowed and the order of the picked elements matters

g) If T is a fully binary tree, then

Select one alternative

- None of the above is correct
- the number of vertices is bounded by the height of the tree
- the number of vertices is bounded by the square product of the number of vertices

h) Given the corollary $\sum_{k=0}^{n} x^k \binom{n}{k} = (1+x)^n$. Choose the answer for which no special case corollary exists.

Select one alternative

- $x = y$
- $x = 1$
- $x = -1$

i) To proof $\sum_{k=0}^{n} b(k;n,p) = 1$, you need to apply

Select one alternative

- binomial theorem
- probability theorem
- pascal's identity theorem

j) Which of the following is not an element of the power set of {2,3}?

Select one alternative

- {{2,3}}
- The empty set
- {2,3}
- {2}

k) What is the probability that a fair coin lands heads four times out of five flips?

Select one alternative

- $\frac{5}{2^5}$
- 4/5
- 4!/5!
- $(\frac{1}{2})^4$

l) Let $\mathcal{G} = (V, E)$ be a graph with m edges. Then $2m = \sum_{v \in V} deg(v)$ holds for

Select one alternative

- bipartite graphs
- directed graphs

Maks poeng: 12

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

9 3 3 2 7 6 4

3 Logic and proofs (10 points)

Logic and proofs (10 points)

a) Fill out the truth table (3 points):

p	q	r	$p \to q$	$p \to r$	$q \lor r$	$(p \to q) \lor (p \to r)$	$p \to q \lor r$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

b) How many rows has a truth table with n propositional variables ? Explain briefly why. (1point)

c) Let p, q, r be propositions. Are the compound propositions $(p \to q) \lor (p \to r)$ and $p \to q \lor r$ logically equivalent? Explain why/why not. (2 points)

d) Let p, q be propositions. Use basic logical equivalences to prove that the compound proposition $(p \land q) \to (p \lor q)$ is a tautology. (2 points)

e) Let $P(m, n)$ be a predicate and let the domains for both m and n be the set of integers. Express the negations of $\exists n \forall m P(m, n)$ and $\forall n \exists m P(m, n)$ so that no negation precedes a quantifier. (2 points)

Fill in your answer here

∀∋

a)

p	q	r	p->q	p->r	qVr	(p->q)V(p->r)	p->qVr
t	t	t	t	t	t	t	t
t	t	f	t	f	t	t	t
t	f	t	f	t	t	t	t
t	f	f	f	f	f	f	f
f	t	t	t	t	t	t	t
f	t	f	t	t	t	t	t
f	f	t	t	t	t	t	t
f	f	f	t	t	f	t	t

b)

The amount of rows will be the amount of different combinations of truth values we can make. Meaning we must match every possible propositions T and F to every T and F of the other propositions, so for a table of n propositions we will have 2^n rows in the table when n is the amount of propositions.

c)

for two propositions to be logically equivalent they must always have the same truth values we can use the truth table above and compare.

we can se that they are logically equivalent since they have the same truth values for the whole table

d)

i will use = to express logical equivalence.

(p^q) -> (pVq) = ¬(p^q) v (pVq) = (¬pV¬q) V (pVq) = (pV¬p) V (qV¬q) = T V T = T

e)

i don't find the symbols in inspera for for the exists an so i will use ∋, if you meant express as in written, i wrote the translation in plain English also.

The first term will be:∀m∋n¬P(n,m) = for every m there exists an n so P(m,n) is false,

The second term will be: ∋m∀n¬P(n,m) = there exists an m so that for every n P(m,n) is false.

Ord: 277

Maks poeng: 10

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

5 5 4 8 4 6 8

Håndtegning 1 av 2

Fyll inn oppgavekode og emneinformasjon på alle skissearkene — Fill out question code and test information on every sheet

Oppgavekode / Question code	Dato / Date	Emnekode / Subject code	Kandidatnummer / Candidate number	Oppgavenummer / Question number	Sidetall / Page number
5548468	5/29/24	MNF130	127	4	1 av/of 5

Tegneområde — Drawing area

a) $(B-A)\cup(C-A) = (B\cap\bar{A})\cup(C\cap\bar{A}) = (B\cup C)\cap\bar{A}$

$= \underline{(B\cup C)-A}$

b) On INSPera instead!!!

~~- Injective means one-to-one, so every element in the domain must be connected to exactly one element in the co-domain here -1 and 1 would be connected to 1 so the function is not one-to-one and not injective~~

~~- Surjective means onto, meaning every element in the co-domain is connected to at least one element in the domain. So here the function is surjective since you can add any $\sqrt{y}$ and get y.~~

~~- to be bijective it must be both injective and surjective. Since it is not surjective it is not bijective. also if it was bijective it would have an inverse $f^{-1}(x)$ which it dose not.~~

c) $\cancel{(a_n+b_n)-5(a_{n-1}+b_{n-1})+6(a_{n-1}+b_{n-2}) = (5a_n-5a_{n-1}+6a_{n-2})+}$

$\cancel{5a_n}$

Håndtegning 2 av 2

Fyll inn oppgavekode og emneinformasjon på alle skissearkene Fill out question code and test information on every sheet

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
5548468	5/29/24	MNF130	127	4	2 av/of 2/5

Tegneområde Drawing area

c) (4)

① $(a_n + b_n) - 5(a_{n-1} + b_{n-1}) + 6(a_{n-2} + b_{n-2})$

$= (a_n - 5_{a_n} + 6_{a_{n-2}}) + b_n + 5_{b_{n-1}} + 6b_{n-2})$

$= \underline{\underline{0 + 0 = 0}}$

② I will call α for X

$Xa_n - 5Xa_{n-1} + 6\alpha a_{n-2} = X(a_n - 5a_{n-1} + 6a_{n-2}) = X(0) = \underline{0}$

4 Basic structures (8 points)

Basic structures (8 points)

a) Let A, B, C be sets. Prove using set identities that

$(B - A) \cup (C - A) = (B \cup C) - A.$ (2 points)

b) Let $f(x) = x^2$ be a function from the set of real numbers $\mathbb{R}$ to the set of non-negative real numbers $\mathbb{R}^+$. Is f injective? Surjective? Bijective? Explain why/why not. (3 points)

c) Let α be a constant. Let $a_n$and b_n be solutions to the recurrence relation

$$u_n - 5u_{n-1} + 6u_{n-2} = 0. \qquad (1)$$

Show that also $a_n + b_n$ and αa_n are solutions to (1). (3 points)

Fill in your answer here

a)
paper

b)
- injective means one-to-one, so every element in the domain must be connected to exactly one element in the co-domain. Here 1 and -1 in the domain would be both connected to 1 in the co-domain. So the function is not injective.
- surjective means onto, meaning every element in the co-domain must be connected to at least one element in the domain. Here we can get every value y in the co-domain by putting square root of y into f(x). So here the function is surjective.
- to be bijective it must be both injective and surjective. so this function is not bijective. To be bijective tells us that the function would have an inverse f^-1(x) witch this function dose not have.

c)
paper

Ord: 128

Maks poeng: 8

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

3 4 9 7 1 4 3

5 Mark the subset (2 points)

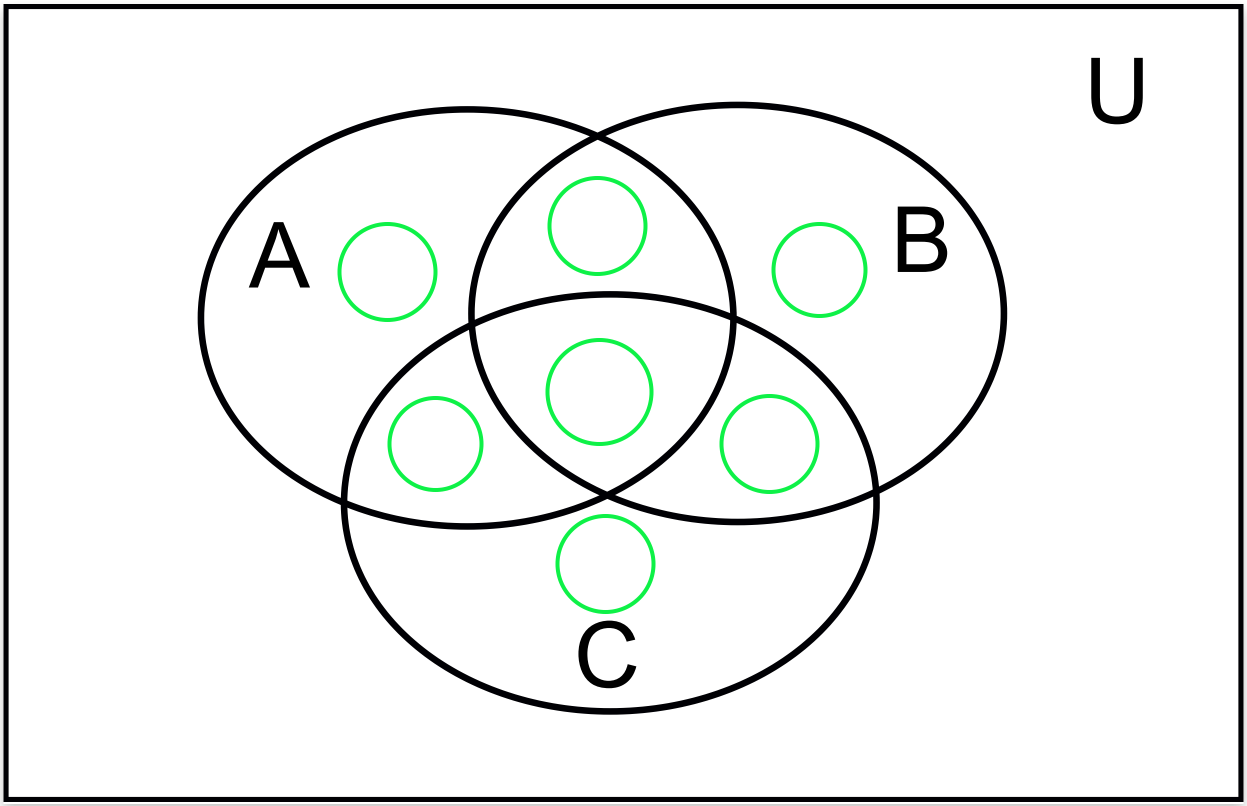
Mark all the hotspots in areas on the Venn diagram that make up $B \cap (A \cup C)$.

Note:

- You must **answer this question in Inspera**.
- You earn 2.0 points if you mark exactly the right combination of hotspots, 0 otherwise.

NB: If you have Inspera in the inverted/ high contrast mode, you must switch back to the "normal" viewing mode in order to see the Venn diagram.

Click the image



Maks poeng: 2

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

4 9 7 8 9 7 9

6 Number theory and algorithms (8 points)

Number theory and algorithms (8 points)

a) Translate the numbers 13 and 29 to binary and then add the two numbers, in binary. Finally, convert the resulting number back to decimal. (3 points)

b) What is the computational complexity in bit operations of adding two n-bit numbers the schoolbook way? Explain why the computational complexity of adding two numbers the schoolbook way is optimal. (2 points)

c) Does 23 have a multiplicative inverse in $\mathbb{Z}_{120}$? If so, then compute this inverse. (3 points)

Fill in your answer here

All on paper

Ord: 3

Maks poeng: 8

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

6 8 9 0 2 7 3

Håndtegning 1 av 1

Fyll inn oppgavekode og emneinformasjon på alle skissearkene | Fill out question code and test information on every sheet

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
6890273	5/29/24	MNF130	127	6	3 av/of 5

Tegneområde Drawing area

6

a) $13 = 2(6)+1 = \underline{1101}$, $\cancel{29 = 2(16)}$

$6 = 2(3)+0$

$3 = 2(1)+1$

$1 = 2(0)+1$

$29 = 2(14)+1 = \underline{11101}$

$14 = 2(7)+0$

$7 = 2(3)+1$

$3 = 2(1)+1$

$1 = 2(0)+1$

$$\begin{array}{r} 1\,1\,\,\,1\,\,\,\, \\ 11101 \\ +\ 1101 \\ \hline 101010 \end{array}$$

$\underset{5\,4\,3\,2\,1\,0}{101010} = 1\cdot 2^5 + 1\cdot 2^3 + 1\cdot 2^1 = 32+8+2 = \underline{\underline{42}}$

b) time complexity is $O(n)$. this is optimal since you have to visit every element to do a addition so it is impossible to add without, so the optimal time will be $O(n)$, to do it shorter would be like adding two numbers without looking at the whole number.

c)
$120 = 23(5)+5 \rightarrow 5 = 1(120) - 5(23)$

$23 = 5(4)+3 \rightarrow 3 = 1(23) - 4(5)$

$5 = 3(1)+2 \rightarrow 2 = 1(5) - 1(3)$

$3 = 2(1)+1 \rightarrow 1 = 1(3) - 1(2)$

$2 = 1(2)+0$

$1 = 1(3) - 1((5) - 1(3))$

$= 1(3) - 1(5) + 1(3)$

$= 2(3) - 1(5)$

$= 2(1(23) - 4(5)) - 1(5)$

$= 2(23) - 9(5)$

$= 2(23) - 9(1(120) - 5(23))$

$= 47(23) - 9(120)$

gcd(120, 23) = 1 so we have an inverse

The inverse is 47 using extended Euclid algorithm

7 Mathematical induction (8 points)

Mathematical induction (8 points)

Consider the Fibonacci numbers defined by $f_1 = 1, f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$, for $n \geq 3.$
Prove using induction that

$\sum_{i=1}^{n} f_i^2 = f_n f_{n+1},$

for $n \geq 1.$

Make sure to clearly divide up the proof into the base case (2 points), the induction hypothesis (2 points) and the induction step (4 points).

Fill in your answer here

all on paper

Ord: 3

Maks poeng: 8

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

8 5 0 1 1 9 7

Håndtegning 1 av 1

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
8501197	5/29/24	MNF130	127	7	4 av/of 5

Tegneområde Drawing area

(1) base case $n=1$

$\sum_{i=1}^{1} 1^2 = 1$, $f_n f_{n+1} = 1 \cdot 1 = 1$

base case is correct

(2) inductive step

(IH) if $f_n f_{n+1}$ is correct for f_n it must also be correct for f_{n+1}.

I must show that $\sum_{i=1}^{k} f_i^2 + f_{k+1}^2 = f_{(k+1)} f_{(k+1)+1}$

I assume $f_k f_{(k+1)}$ is correct for $\sum_{i=1}^{k} f_i^2$ and add $f_{(k+1)}^2$

$f_k f_{(k+1)} + f_{(k+1)}^2 = f_{(k+1)}(f_k + f_{(k+1)})$ $= f_{(k+1)} f_{k+2} = f_{k+1} f_{(k+1)+1}$

~~$= f_{(k+1)} f_k \; f_{k+1} f_{k+2}$~~

So the IH is proven

8 Counting (8 points)

Counting (8 points)

a) What is the coefficient of x^5 in the expansion $(2-x)^{13}$? It is OK to express the final answer as a product. (3 points)

b) Given a function $f: M \to N$, for finite sets M and N. Show that if $|M| > |N|$, then f is not an injective function. (3 points)

c) Prove, using an algebraic or combinatorial argument, that

$$\binom{N}{k} = \binom{N}{N-k},$$

where k, N are natural numbers such that $N \geq k$. (2 points)

Fill in your answer here

a)
We use the binomial theory to find our k.
$\sum_{k=0}^{n} b(k;n,m) = 1.$
So we can quickly see that k = meaning we must calculate 13 over 5 = $\frac{13!}{5!(13-5)!}$. The calculation part you can find written on a paper, but the answer is 13*11*3^2.

b)
Here we can use the pigeonhole principle. For f to be injective meaning that for every element in the domain is connected to an element in the co-domain once. So only one pigeon per hole but not every hole needs to have a pigeon. We see that since there are more elements in the domain than the co-domain, meaning there are more pigeons than holes. It is therefore impossible for f to be injective since it will be more than one pigeon per hole.

c)
algebraic argument on paper

Ord: 135

Maks poeng: 8

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

2 9 8 0 0 6 8

Håndtegning 1 av 1

Fyll inn oppgavekode og emneinformasjon på alle skissearkene — Fill out question code and test information on every sheet

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
2980068	5/29/24	MNF130	127	8	5 av/of 5

Tegneområde Drawing area

a) calculatin of $\binom{13}{5}$, arguing is on InSPera

$$\frac{13!}{5!(13-5)!} = \frac{13!}{5!\,8!} = \frac{13\cdot12\cdot11\cdot10\cdot9\cdot\cancel{8!}}{5!\,\cancel{8!}} = \frac{13\cdot12\cdot11\cdot\cancel{10}\cdot9}{\cancel{5}\cdot4\cdot3\cdot\cancel{2}}$$

$$= \frac{13\cdot\cancel{12}\cdot9}{\cancel{4}\,\cancel{3}} = 13\cdot11\cdot9 = \underline{\underline{13\cdot11\cdot3^3}}$$

c) $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

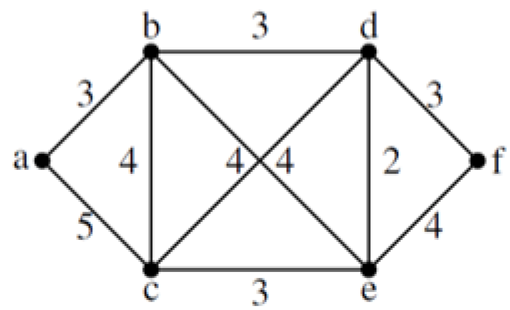
$$\binom{n}{n-k} = \frac{n!}{(n-k)!(n-(n-k))!} = \frac{n!}{(n-k)!\,k!} = \underline{\underline{\frac{n!}{k!(n-k)!}}}$$

Proving $\binom{n}{k} = \binom{n}{n-k}$

9 Graph theory (4 points)

Graph theory (4 points)

For the weighted graph shown, rank the vertices by their shortest-path length from a and give a shortest path from a to each vertex. (4 points)



Vertex	Shortest length	Shortest path

Fill in your answer here

Vertex	Shortest Length	Shortest path
a	0	{}
b	3	{a,b}
c	5	{a,c}
d	6	{a,b,d}
e	7	{a,b,e}
f	9	{a,b,d,f}

Ord: 23

Maks poeng: 4

Knytte håndtegninger til denne oppgaven?
Bruk følgende kode:

2 2 4 9 5 1 5