

# Proof by induction by Thomas Barth

## 1.1 What is the base case?

The basis step case of induction is  $P(1)$ , in this case it would be  $3^1$  which is 3. So the base case would be that we should be able to use one measurement to find the heavier coin of the three.

## 1.2 What Is the induction hypothesis?

Given  $3^n$  coins, where one of the coins is heavier than the others, and a balancing scale we will always be able to find the heavier coin by doing  $n$  measurements on the scale.

## 1.3 Prove the inductive step (finish the proof)

Step 1, prove the base case:

- To prove the base case, we must prove we can find the heavier coin of three coins using one measurement on a balance scale. If we put any two of these coins on the scale, we get one of two scenarios. 1. The scale will be balanced and the coin not on the scale is the heavier, 2. The scale will tilt to one of the sides indicating which coin is the heavier.

Step 2, prove the induction step:

- We first assume that the hypothesis is true for  $n = k$  coins. We now need to show that it is also true for  $n = k + 1$ . When we have  $3^{k+1}$  coins this will be three times as many coins as for  $n = k$ . We now divide all the coins into three groups with  $3^k$  coins each, one of the groups will be heavier. We only need one weighing to find out which of the three groups is heavier since if we take any two groups on the scale we will get one of two scenarios. 1. The scale will be balanced and the group not on the scale will contain the heaviest coin, 2. The scale will tilt to one of the sides indicating which group is heaviest. Now that we are left with  $k$  and used one weighing, we are left with  $3^k$  coins and  $k$  weightings. We now continue this elimination until we are left with one coin which will be the heaviest.