2 Proof a Sum formula using Induction

1. Prove the base case n=1

$$\frac{1}{1(1+1)} = 1 \cdot \frac{1}{1 \cdot 1} = \frac{1}{2} \leq \frac{1}{2}$$

2. Prove the Induction Step

Assume Statement is True for $n=m$
Prove for $n=m+1$

So for $n=m$ we have $\displaystyle\sum_{k=1}^{m} \frac{1}{m(m+1)} = 1 - \frac{1}{m+1}$

To check for $n=m+1$ we can take $1-\frac{1}{m+1}$ and add one more increment:

$$1 - \frac{1}{m+1} + \frac{1}{(m+1)((m+1)+1)}$$

$$= 1 + \frac{-1}{m+1} + \frac{1}{(m+1)(m+2)}$$

$$= 1 + \frac{-1(m+2)}{(m+1)(m+2)} + \frac{1}{(m+1)(m+2)}$$

$$= 1 + \frac{-m-2+1}{(m+1)(m+2)}$$

$$= 1 + \frac{-m-1}{(m+1)(m+2)} = 1 - \frac{m+1}{(m+1)(m+2)} = 1 - \frac{1}{(m+2)} \leq 1 - \frac{1}{(m+1)+1}$$