

5 Algorithms

5.1 describe how to find out who has the pen asking $O(n^2)$ questions.

To create an algorithm that uses $O(n^2)$ questions, we can think of writing a program that uses two for-loops where one of the loops is inside the other. We select a student which we call student i and we ask them about student j . student i starts at student 0 and we start asking them if student j has the pen which also starts at student 0 (The first student can both have and know where the pen is). If the student says no, we set student j as the next student (student $j + 1$). If we get to the last student j , we start asking the next student ($i+1$) questions. We repeat this process until we have found a student i (the one who knows where the pen is) and a student j (the one who has the key). This will take $O(n^2)$ questions.

5.2 now assume that the student who has the pen knows it. Describe how to find the pen asking $O(n)$ questions.

To create an algorithm that uses $O(n)$ questions when student who has the pen knows is, we can simply start at the first student and ask: do you have the pen? If that student says no, we go to the next student. We continue this process until we have gotten to the student who has the pen. This will take $O(n)$ questions.

5.3 now assume all the students knows who has the pen. Describe how to find the pen asking $O(\log n)$ questions.

To create an algorithm that uses $O(\log n)$ questions we need to half the number of students each time we ask a question. We divide the students into two groups, which we can explain to the student we are asking without asking a question and ask them if the pen is in their group. If they say yes, we have eliminated the other group, if they say no, we eliminate the group of the student we asked. We have now halved the amount of students, and we continue the process until we are left with one student which will have the pen. This will take $O(\log n)$ questions.