Oppgave 9) a)

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \sim \begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & (x_2-x_1) & (x_2-x_1)(x_2+x_1) \\ 0 & (x_3-x_1) & (x_3-x_1)(x_3+x_1) \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2+x_1 \\ 0 & 1 & x_3+x_1 \end{bmatrix} \sim \begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2+x_1 \\ 0 & 0 & (x_3+x_1)-(x_2+x_1) \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2+x_1 \\ 0 & 0 & x_3-x_2 \end{bmatrix}$$

Since we divided row 2 with (x_2-x_1) and row 3 with (x_3-x_1) we have to factor it back

$$\sim \begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & (x_2-x_1) & (x_2+x_1)(x_2-x_1) \\ 0 & 0 & (x_3-x_2)(x_3-x_1) \end{bmatrix}$$

Determinant of triangular matrix is to multiply the diagnol.

$$\text{Det}(V) = 1 \cdot (x_2-x_1) \cdot (x_3-x_2)(x_3-x_1)$$

$$= \underline{\underline{(x_2-x_1)(x_3-x_2)(x_3-x_1)}}$$

9)

b)

Since Det $\neq 0$, we know this since $x_1 < x_2 < x_3$ and Det $(x_2 - x_1)$, $(x_3 - x_1)$ and $(x_3 - x_2)$ are all none zero.

V is invertible

c)

$$V \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Since V is invertible we can multiply with V^{-1} and since V^{-1} exists

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = V^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

= a, b, c exist uniquely

a)

d)

$$\begin{bmatrix} 1 & 1 & 1^2 \\ 1 & 2 & 2^2 \\ 1 & 4 & 4^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ 7 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 11 \\ 1 & 4 & 16 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 6 & -18 \end{array}\right]$$

$6c = -18 \Rightarrow c = -3$

$b + 3(-3) = 7 \Rightarrow b = 16$

$a + b + c = 4 = a + 16 - 3 = 4 \Rightarrow a = -9$

$\underline{\underline{P(x) = -9 + 16x - 3x^2}}$