3 Greatest common divisor and modulo

3.1 Find the multiplicative inverse of 34 mod 89 using the euclidean algorithm

34 mod 89

$89 = 34 \cdot 2 + 21$	$89 + 34(-2) = 21$ (7)
$34 = 21 \cdot 1 + 13$	$34 + 21(-1) = 13$ (6)
$21 = 13 \cdot 1 + 8$	$21 + 13(-1) = 8$ (5)
$13 = 8 \cdot 1 + 5$	$13 + 8(-1) = 5$ (4)
$8 = 5 \cdot 1 + 3$	$8 + 5(-1) = 3$ (3)
$5 = 3 \cdot 1 + 2$	$5 + 3(-1) = 2$ (2)
$3 = 2 \cdot 1 + 1$	$3 + 2(-1) = 1$ (1)

(1) $3 + (5 + 3(-1))(-1) = 1 = 3 + 5(-1) + 3 = 2(3) + 5(-1)$

(2) $2(8 + 5(-1)) + 5(-1) = 2(8) + 5(-2) + 5(-1) = 2(8) + 5(-3) = 8(2) + -3(5)$

(3) $-3(13 + 8(-1)) + 8(2) = 13(-3) + 8(3) + 8(2) = 13(-3) + 8(5)$

(4) $5(21 + 13(-1)) + 13(-3) = 21(5) + 13(-5) + 13(-3) = 5(21) + 13(-8)$

(5) $-8(34 + 21(-1)) + 21(5) = 34(-8) + 21(8) + 21(5) = 34(-8) + 21(13)$

(6) $13(89 + 34(-2)) + 34(-8) = 89(13) + 34(-26) + 34(-8) = 89(13) + 34(-34)$

$89(13) + 34(-34) = 1 \pmod{89}$

$89(13) + 34(55) = 1 \pmod{89}$

$34(55) = 1 \pmod{89} \mid : 34$

$55 = \frac{1}{34} \bmod 89 = 34^{-1} \bmod 89$

The multiplicative inverse of 34 mod 89 is 55

3.2 First we find the gcd, and since 89 is a Prime we know the gcd is 1. We also know now that its only one Solution

$34x+5 \equiv 0 \bmod(89)$

$= 34x \equiv -5 \bmod(89)$

$= 34x \equiv 84 \bmod(89) \mid \cdot 55$ (from last task)

$34(55)x \equiv (55)(84) \bmod(89)$

$x \equiv 4620 \bmod(89)$, $4620 \bmod(89) = 81 \bmod(89)$

$x \equiv 81 \bmod(89)$

$x = 87 + 89k$, where (k) is any integer.