Geography 4203 / 5203

GIS Modeling

Class (Block) 8: Spatial Estimation
-Interpolation Techniques-

Some Updates

- Handouts online at:

http://www.colorado.edu/geography/class_homepages/geog_4203_s08/

Last Lecture

- So we finished with **Terrain Analysis** that took us to to discussions about **geomorphometry**, **hydrological functions** and **viewsheds**
- You have seen the basic **concepts** and **implementations** of the most used indices for Terrain Analysis
- You hopefully could make use of this information to understand more if you are modeling on a daily basis
- We discussed the influence of **resolution**, **scale** and **accuracy** to Terrain Analysis

Today‘s Outline

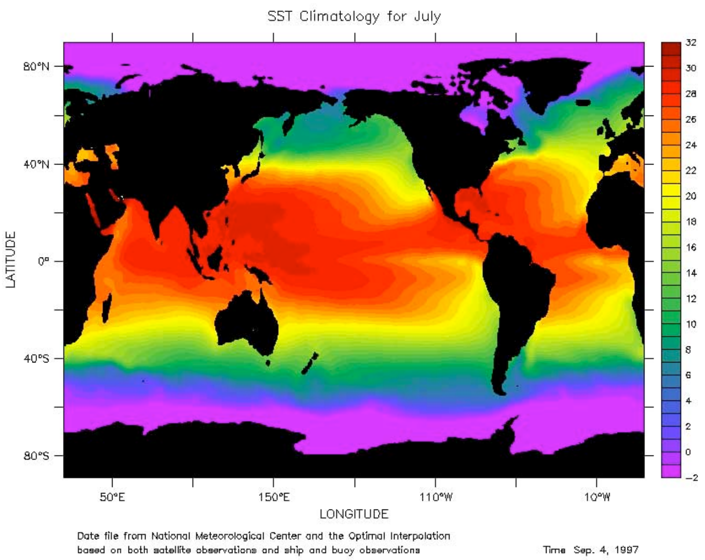
- We will start with an introduction into **spatial interpolation** methods
- We will talk about some **conceptual** basics and **preassumptions** for interpolation
- We will discuss some **sampling strategies**, and the **methodological** ideas of **global** and **local** interpolation

Learning Objectives

- You will understand where interpolation makes sense and what the **basic** idea is behind it
- You will understand what **autocorrelation** means and how it **influences** estimation
- You will see the first aproaches in a **mathematical** formulation and hear about **global** and **local** interpolators

Introduction

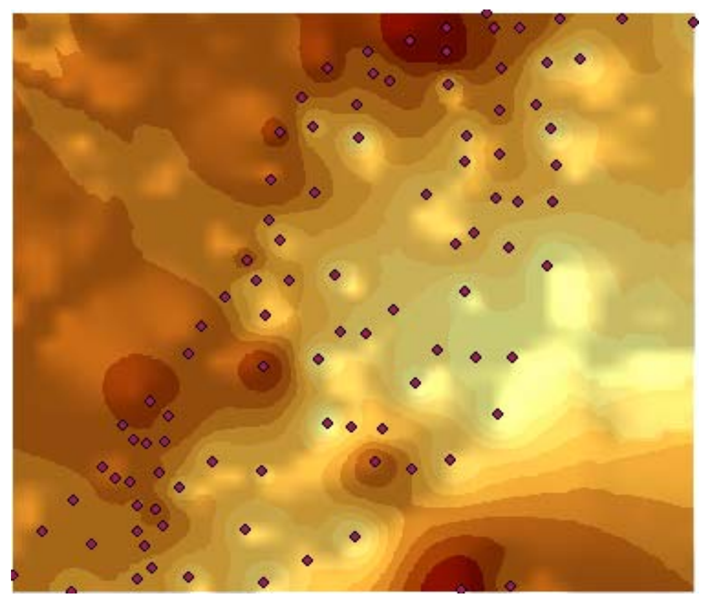
- Methods for **spatial prediction** to estimate values at **unsampled point locations** which could be of interest
- **Time** and **money** are limited; safety and accessibility restrictions
- =>Small **subsets** of object, points or raster cells for estimating the total population
- **Loss** of parts of collected samples (recovery) or **unsuitability**, obvious **outlier** points or just for closing “**gaps**”



from http://oceanworld.tamu.edu/resources/ocng_textbook/chapter06/chapter06_03.htm

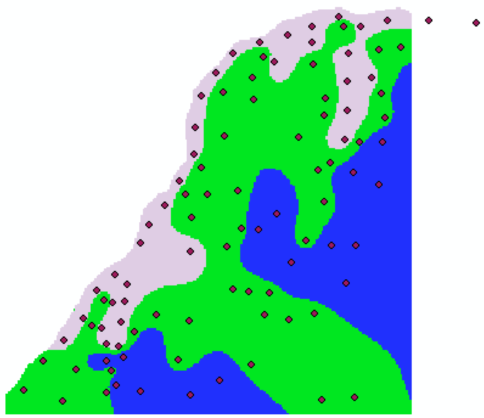
Intro: Spatial Interpolation

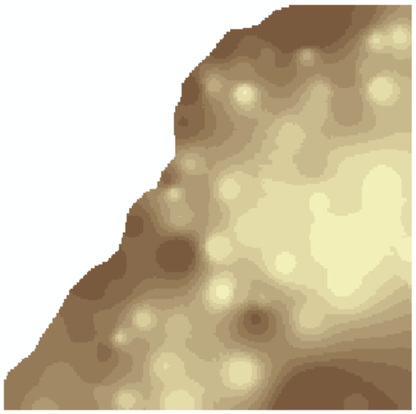
- **Prediction** of variables at **unmeasured** locations based on a **sample** at **known locations**
- Creation of **surfaces** of **continuous** values
- Examples: Temperature, productivity, elevation, population density



Intro: Spatial Prediction

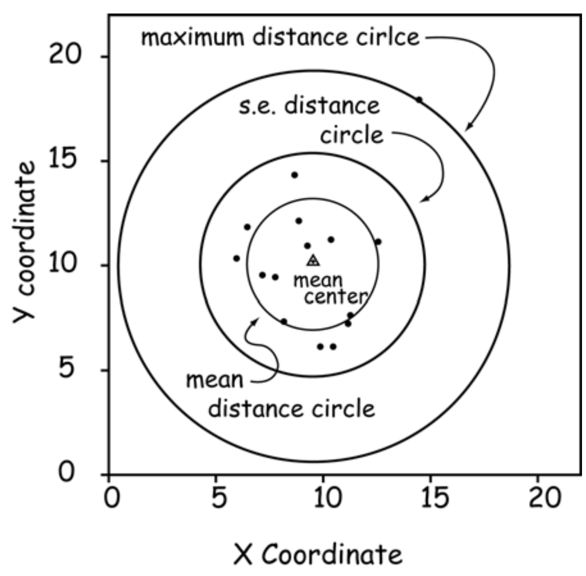
- Involves the estimation of variables at **unsampled** locations
- **BUT**: Estimates are based at least in part on **other variables**
- Examples: Elevation to better estimate temperature due to known influences





Intro: Concept of the Core Area

- Area that characterizes **high values** for a variable / event (high use, density, probability of occurrence
- Based on **samples** (derived from sample points or observations)
- Predicting the **frequency** or **likelihood** of **occurrence** of an object / event (not the value of that variable)
- For example: centers of criminal activity, home ranges,…



Spatially Estimating/Predicting

- Translating from lower to the **same or higher spatial dimensions** (points, lines, areas from points)
- Extending the collected information and thus **improving the quality** of the data
- Translating into the opposite direction: Point value estimation from higher order data (lines, aggregated area data)
- E.g., MAUP - modifiable areal unit problem
- Point data from transect (aggregated) data

Sampling Basics

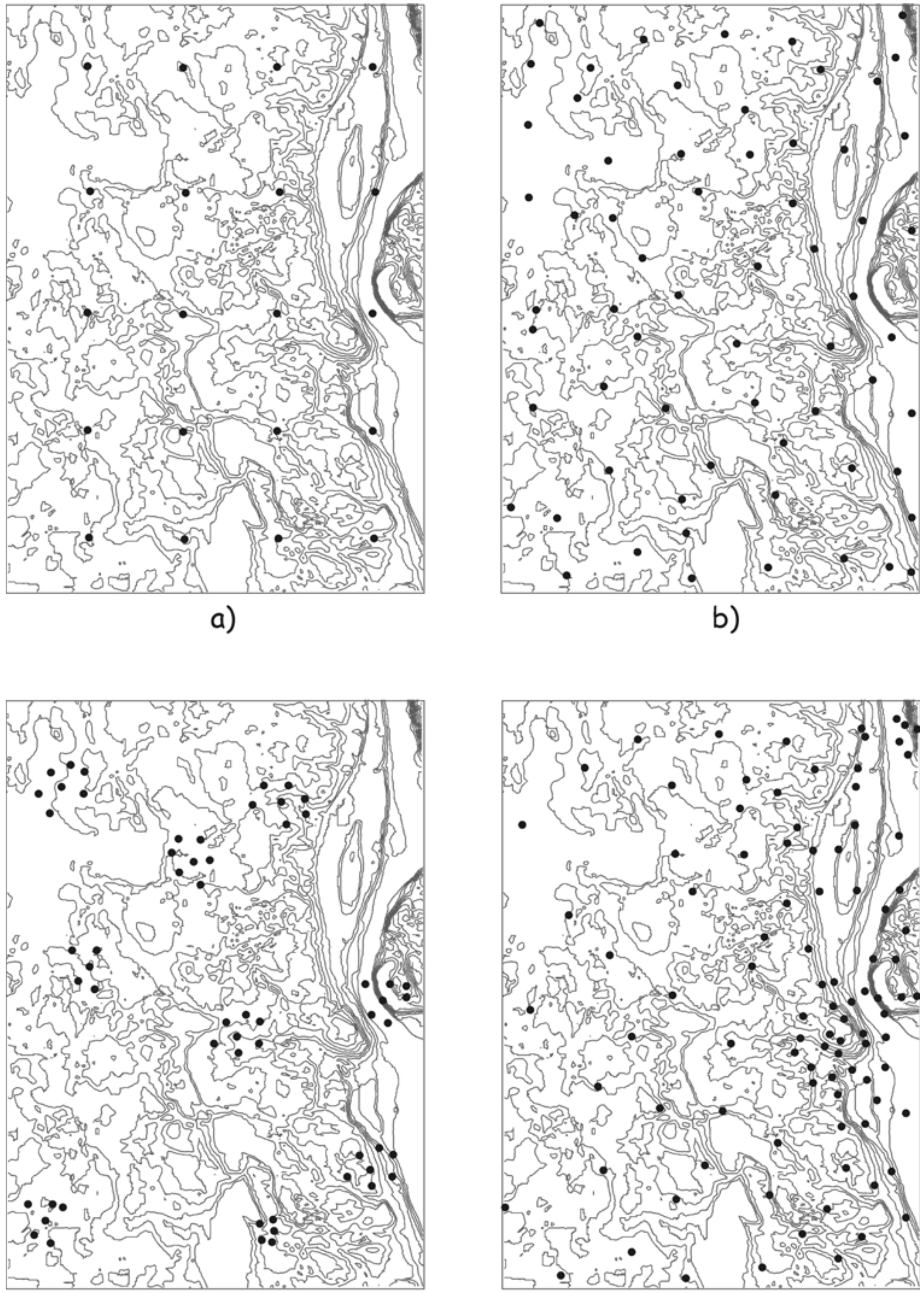
- Aim of estimation is to find values for a variable at **unknown locations** based on values measured at **sampled** locations
- Planning important to make the sampling more efficient / accurate
- **Control** taken over locations of sample points (**patterns/dispersion**) and **sample size**
- Sometimes neither can be controlled (**diseases** within a population)

Control of Sample Size

- Law of **diminishing returns**: Situation where further sample points add relatively little **additional information** or **gain** in **accuracy** for substantially **increased costs**
- The rule is: most surfaces from interpolation are **undersampled** (funds as limiting factor)
- Difficult to determine the optimal sample size for interpolation methods

Control of Sampling Patterns

- Sample locations **spread** across our working area - so it's important **how to collect** sample point data
- **Patterns** we choose affect the quality of the interpolation carried out (and have effect on **sample sizes** needed)
- Wrong **distributions** increase estimation errors and simply cost money…
- **Systematic**, **random**, **cluster**, **adaptive/ stratified, transect and contour** sampling
- Remember sampling from photogrammetric data sources: regular, progressive, selective, composite



a) b)

c) d)

Spatial Interpolation

Basics of Spatial Interpolation I

- "… procedure of **predicting** the value of attributes at **unsampled** sites from **measurements** made at **point** locations within the same area" (Burrough & McDonnell 1998)
- Combining sampled values and positions to estimate values at unmeasured locations
- Based on **mathematical functions** incorporating the **distance** between interpolation points and sample points and **values** at sample points
- What is **extrapolation**??? This has to do with the convex hull…!

Basics of Spatial Interpolation II

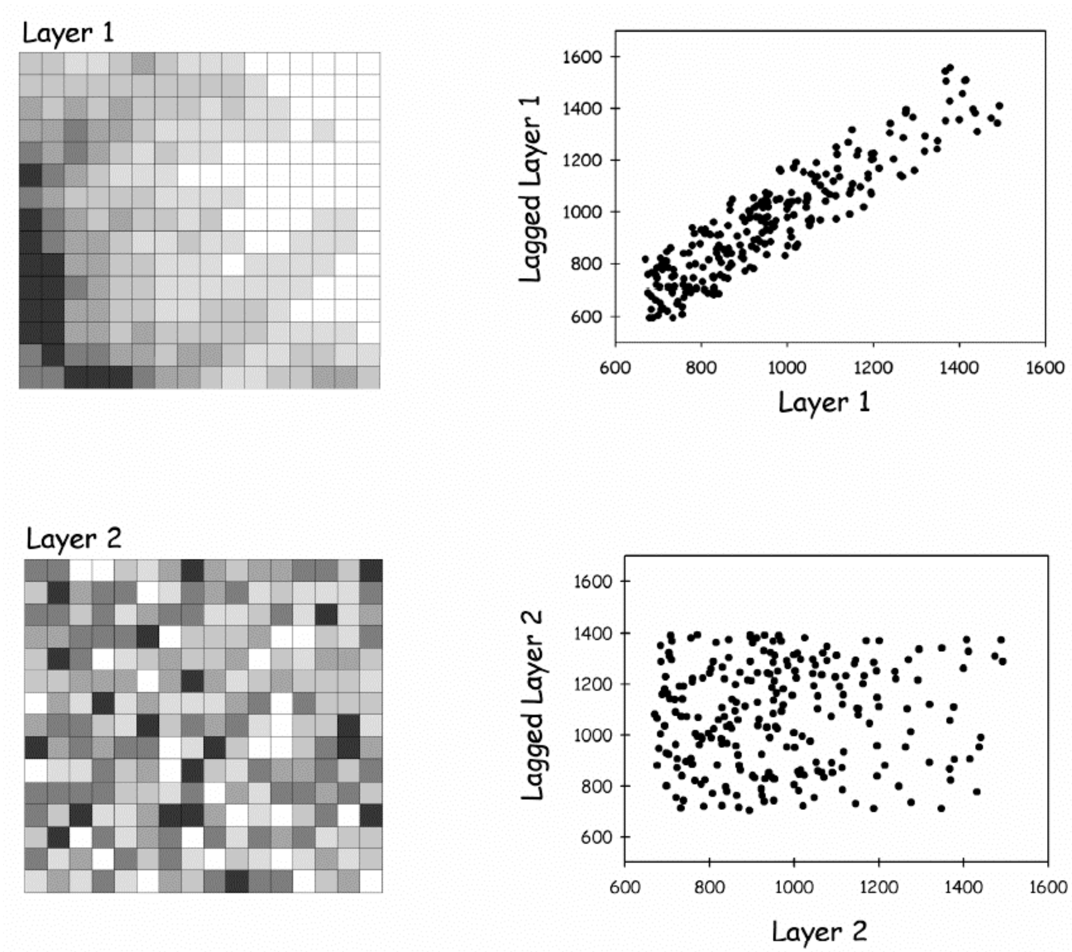
- Values are usually estimated for a **continuous raster layer** (sometimes for contour lines / isolines)
- Mathematical functions are used to **weight** the observations (distance!)
- Different interpolators demand different **numbers of observations** (this is critical in reality)
- No dominant interpolation method… the optimal method depends on different aspects and the specific purpose

Some Definitions (Burr. & McDo.)

- **Exact interpolator** – An interpolator which shows the exact values of the data points (as opposed to an **approximate interpolator**)
- **Global interpolator** – Method where all the data points are used to estimate a field
- **Local interpolator** – Methods which use some subset of data points to locally estimate a field
- **Continuous** – smoothly varying field (with continuous values and, potentially, derivatives)
- **Abrupt** – a field whose values are discontinuous, or whose derivatives are discontinuous
- **Support** - area/volume on which a measurement is made

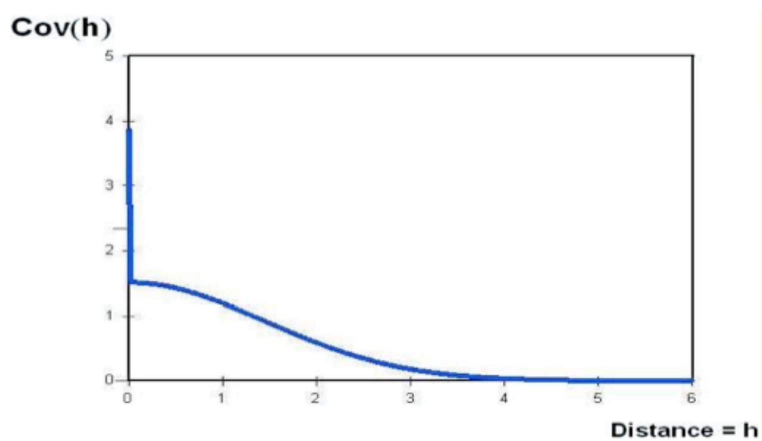
Near Things in Space…

- Back to **Tobler**: "… everything is related to everything else, but **near things** are more related than distant things…"
- Or: Many things measured out there are **spatially dependent / spatially autocorrelated**



Autocorrelation

- A measures' **correlation** with itself relative to **proximity/location**
- When values $Z(s_i)$ and $Z(s_j)$ in **close** proximity to one another $|s_i - s_j| < h$ are more **alike** than values located at **further** distance $|s_i - s_j| > h$
- Or: as **h increases** between two observations, the **correlation** between attributes Z **decreases**



A Rationale for Interpolation

- Thus interpolation is useful where the variable of consideration has some **autocorrelation** (which means that we will have a better estimation than solely based on their distribution)
- If the distribution of values is a **random** one the estimation can be made using the **distribution** (not important **where** the values are)

When will we need interpolation?

… **-** if resolution, cell size or orientation is different than required (convolution)

- if a field is represented by a different data model than required (TIN2Grid)

- if the study or analysis area is not completely covered by measurements

Methods:

-Deterministic:

Local: Nearest Neighbor (Thiessen), Fixed Radius, Inverse Distance Weighting (IDW), Splines

Global: Classifications, trend surfaces, regressions

-Geostatistics: Kriging (optimal weighting interpolation), Co-Kriging

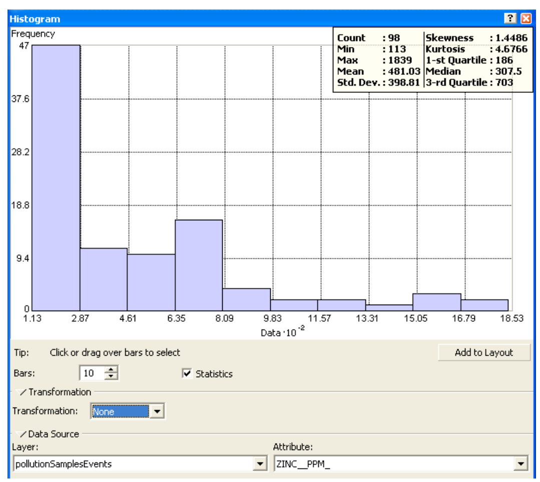
Interpolation Error / Accuracy

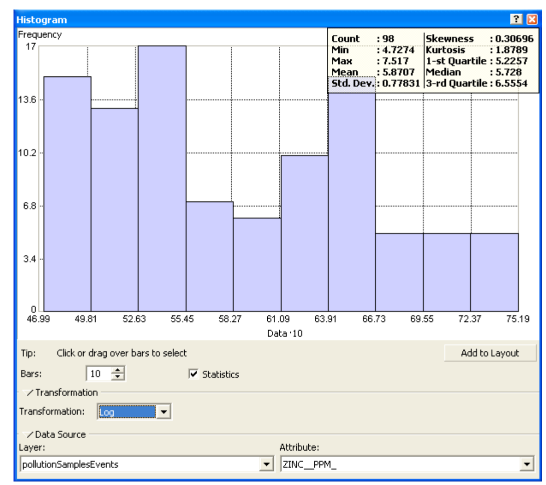
- **Accuracy** assessment: Difference between the measured and interpolated values using a “withheld” or **validation** sample (not used for interpolation but compared against the resulting surface)
- Or: using **resampling** approaches (withholding one data point and measuring the error; point replaced, new point selected and the same done iteratively): **Cross-validation**

$$\operatorname{MSE}(\overline{X}) = \operatorname{Var}\left[\overline{X}\right] = \operatorname{Var}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n^2}\left(\sum_{i=1}^{n}\sigma^2\right) = \frac{\sigma^2}{n}$$

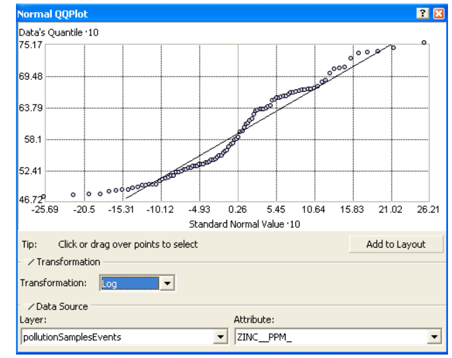
Data Exploration

- Before creating any surface deriving some knowledge of:
- Data distribution, outliers,global trends, spatial autocorrelation, covariation among different variables
- **Tools**:
- Histogram:

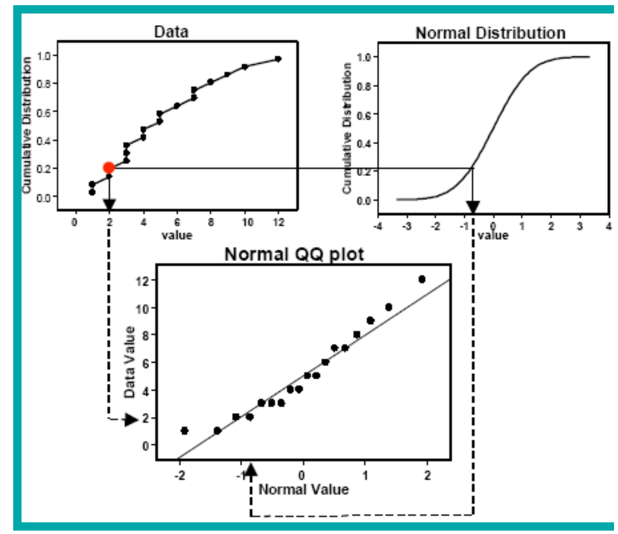


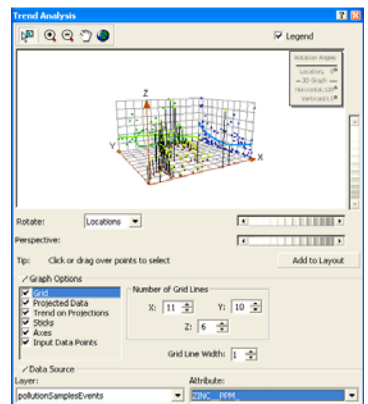


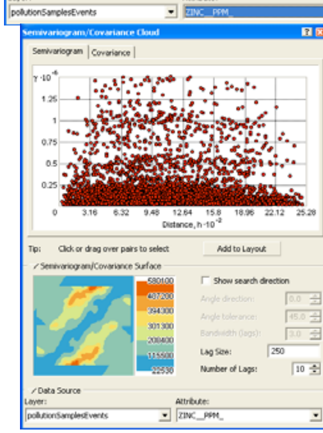
Data Exploration

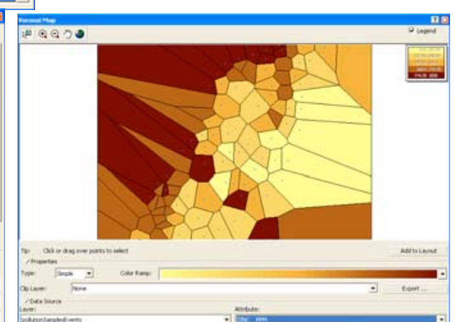


- Plotting quantiles of two distributions
- Mapping/removing trends (global)
- Voronoi maps
- Similarity/spread using semivarogram and covariance







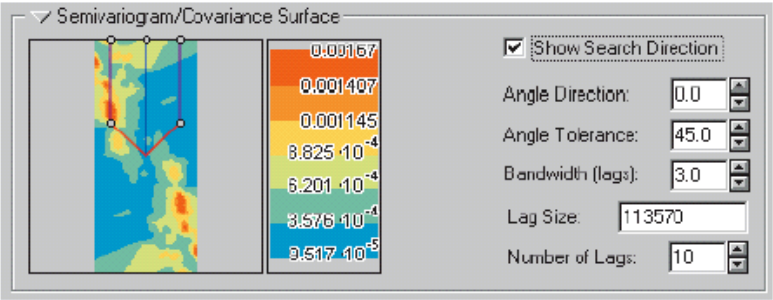


Global Interpolation

- Global interpolators are used to identify effects of **global variation** by taking into account **all data points**
- Trends in surfaces: "**trend surface analysis**"
- E.g. a decreasing mean annual temperature from south to north within within Europe
- Classifications, Regressions (Trend surfaces, transfer functions)

Global Trend and Anisotropy

- **Global trend** as an overriding process is tried to be presented as mathematical formula (**polynomials**)
- **Anisotropy** is a random process which shows different degrees of autocorrelation in different directions ("**directional autocorrelation**")
- **Directional influence** using different search directions (which pairs of data are plotted)
- Lag size - lag distance, and number of lags



Global Interpolation: Spatial Regression

- Using observations of **dependent** variables AND further "**independent**" variables and sample coordinates to develop prediction equations (This is **prediction**!!)
- **Mathematical relationships** between dependent and independent variables (e.g. temperature is influenced by elevation, latitude and longitude)
- General functions: $Z_i = f(x_i, y_i, \alpha_i, \beta_i)$

Trend Surfaces / Simple Spatial Regression

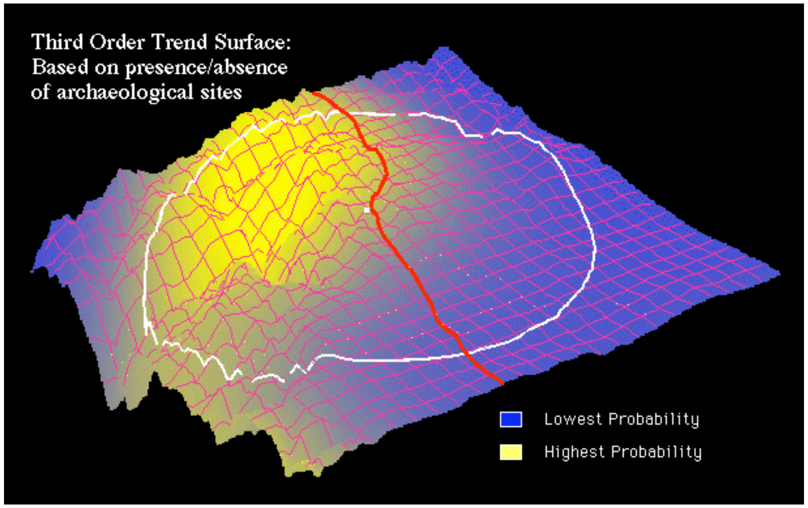
- Spatial regression involving fitting a statistical model (a function f(x,y)) through the measured points
- Surface is a **polynomial** in the X and Y coordinate system derived using least squares
- Local residuals are the difference between the data points and the surface
- Linear: $z_i = b_0 + b_1 x_i + b_2 y_i + \varepsilon_i$

b_0 = offset of surface at origin

b_1 = gradient in x-direction

b_2 = gradient in y-direction

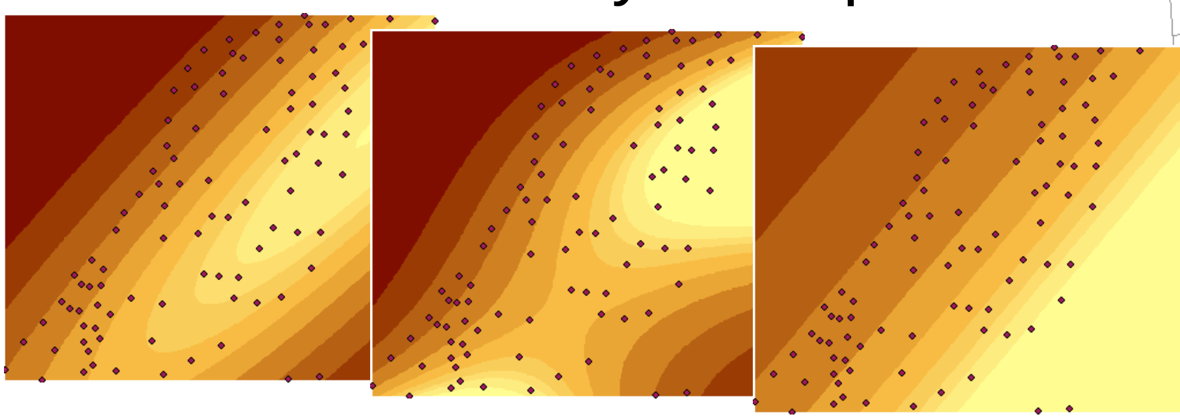
ε_i = residual at data point i

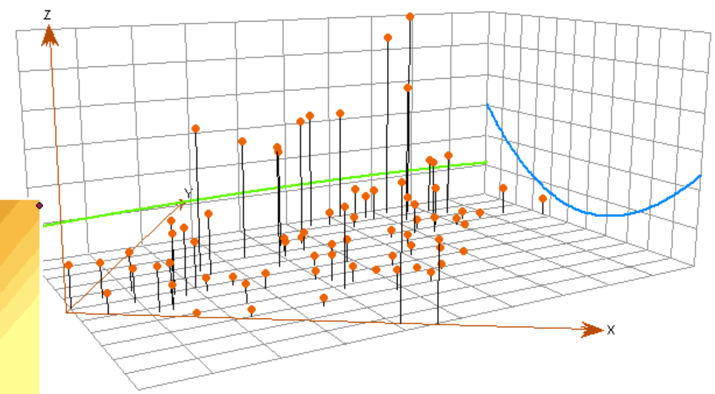


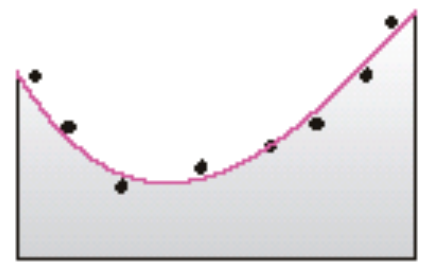
From http://www.casa.arizona.edu/MPP/hrs1_report/hrs1.html

Trend Surfaces

- Caution with **higher order** polynomials
- **Inexact** (approximate) interpolator
- Higher-order polynomials **fit** better but have larger **deviations**
- As a first analysis step





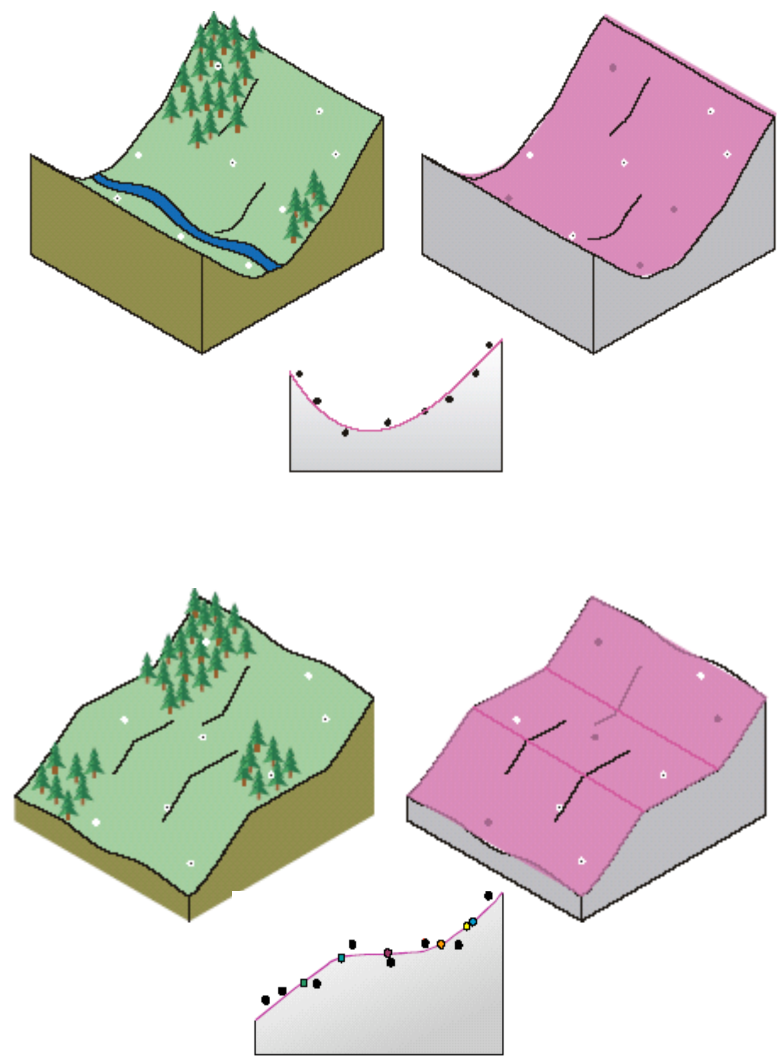


$$Z(x_i, y_i) = \beta_0 + \beta_1 x_i + \beta_2 y_i + \beta_3 x_i^2 + \beta_4 y_i^2 + \beta_5 x_i y_i + \beta_6 x_i^3 + \beta_7 y_i^3 + \beta_8 x_i^2 y_i + \beta_9 x_i y_i^2 + \varepsilon(x_i, y_i),$$

Third order

Global and Local Polynomials

- **Global**: fitting a polynomial to the **entire surface**
- **Local**: Polynomials fits a **short-range variation** (in addition to long-range);
- Sensitive to **neighborhood size**

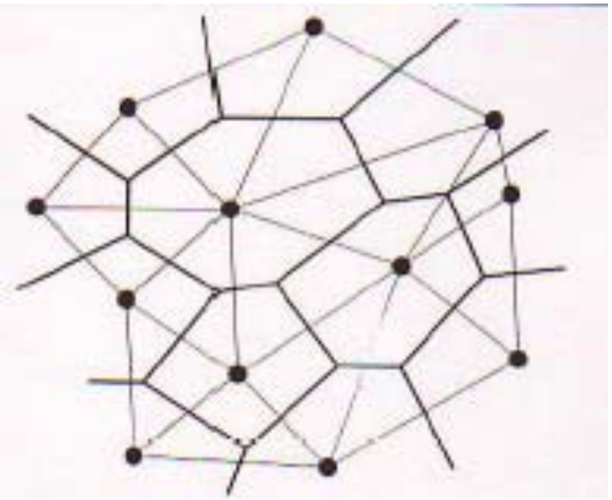


Local Interpolators

- **Local variations** become important where data values are expected to be **similar** to **closely** located points (**implicitly** based on Tobler's Law)
- **Smoothing** values based on information from their **neighborhood**
- What is **near**? (back to **neighborhood**, its **size**, **shape** and **orientation**)
- **How many** and how to find the **data points** within?
- What is the **mathematical function** to represent the variation over this subset of points?
- What is the distribution of the data points?
- Any external information (trends,…)?

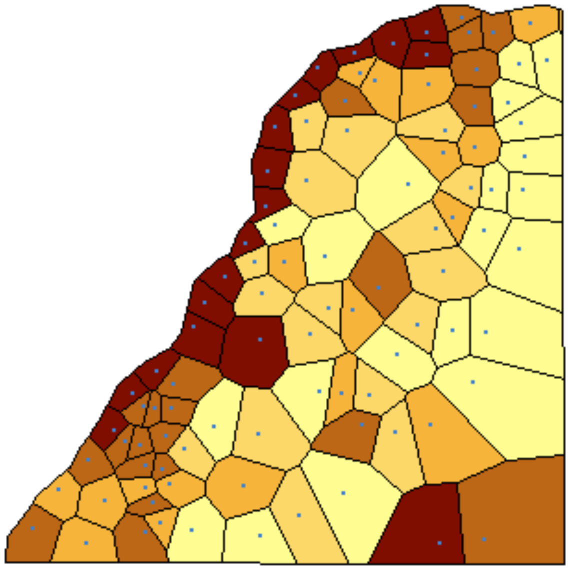
Nearest Neighbor - Thiessen Polygons

- "**Thiessen (Voronoi) polygon**" are created by the set of locations nearest to the data point
- Conceptually the simplest method
- Mathematical function: **Equality function**
- Only one point (the nearest one to the unmeasured location) used for value assignment
- Thus **homogeneous** regions



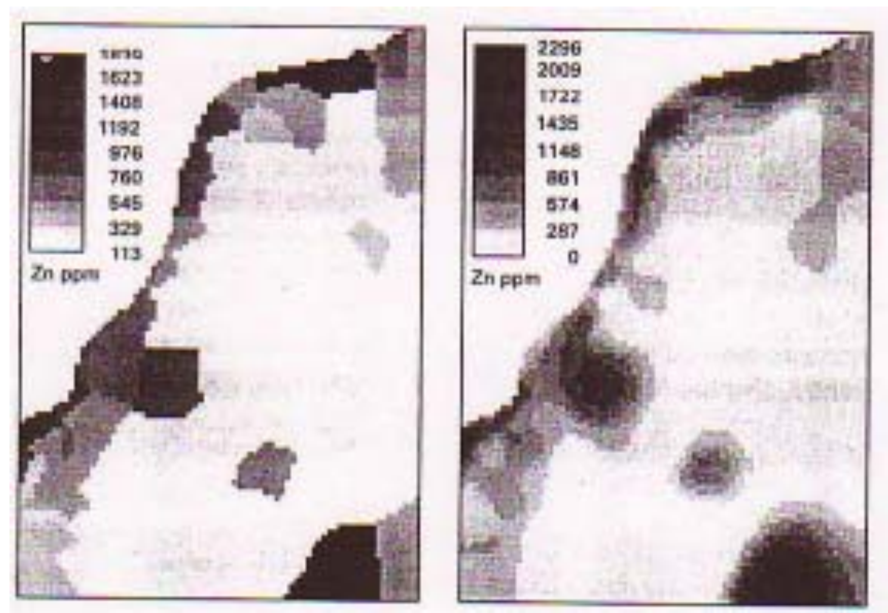
How to… Nearest Neighbor

- Defining a set of polygons within which all locations have an **identical Z-value** (**exact** interpolator)
- Thus **polygons** define a region around each sample point that have a value **equal** to the sample point value
- **Abrupt transitions** between polygons
- Lines joining the data points results in the Delaunay triangulation



Pycnophylactic Interpolation

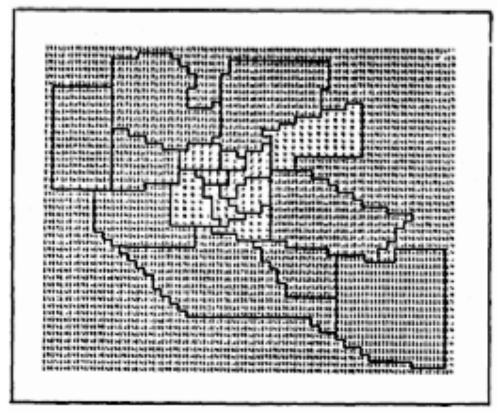
- Voronoi polygons assume **homogeneity** within and **abrupt** changes between the polygons which is inappropriate for **locally varying** phenomena (precipitation, density, concentrations)
- One alternative is **pycnophylactic interpolation by Tobler**

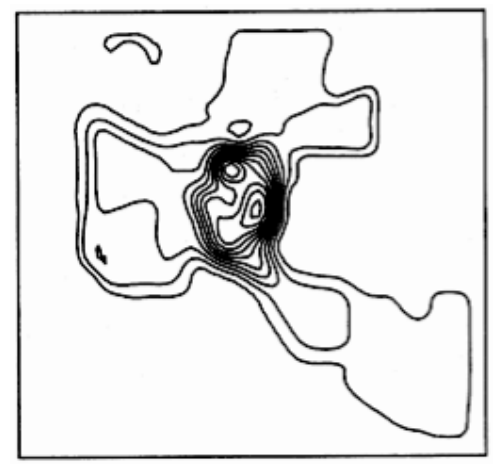


from Burrough & McDonnell, 1998

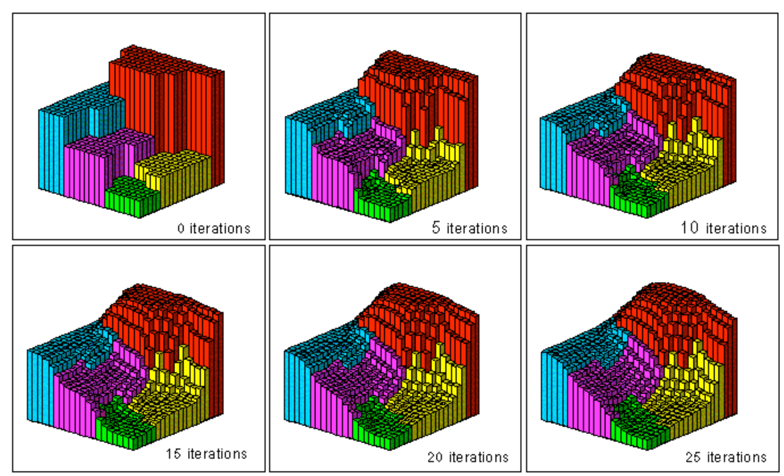
Pycnophylactic Interpolation

- Here, values are reassigned by **mass preserving reallocation** to remove abrupt changes
- **Volume** of the attribute within a region **remains the same but varies smoothly at boundaries**
- It is assumed that a better representation of the variation is a **smooth surface**



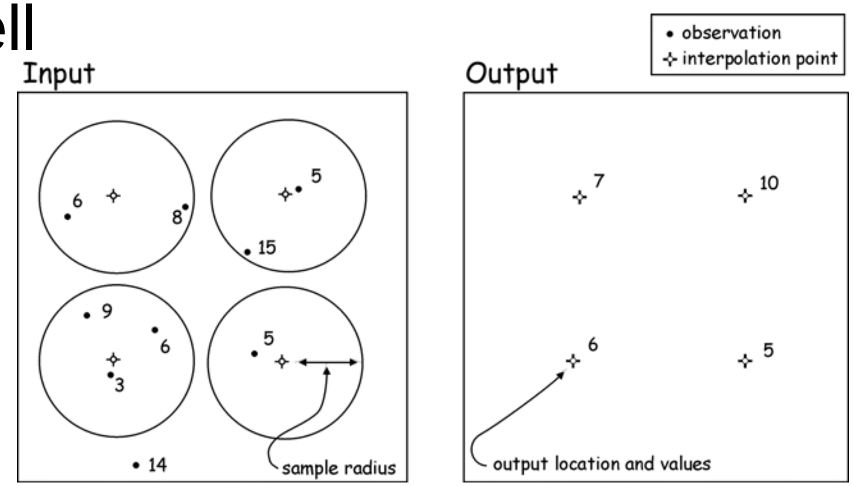


from Tobler, 1979



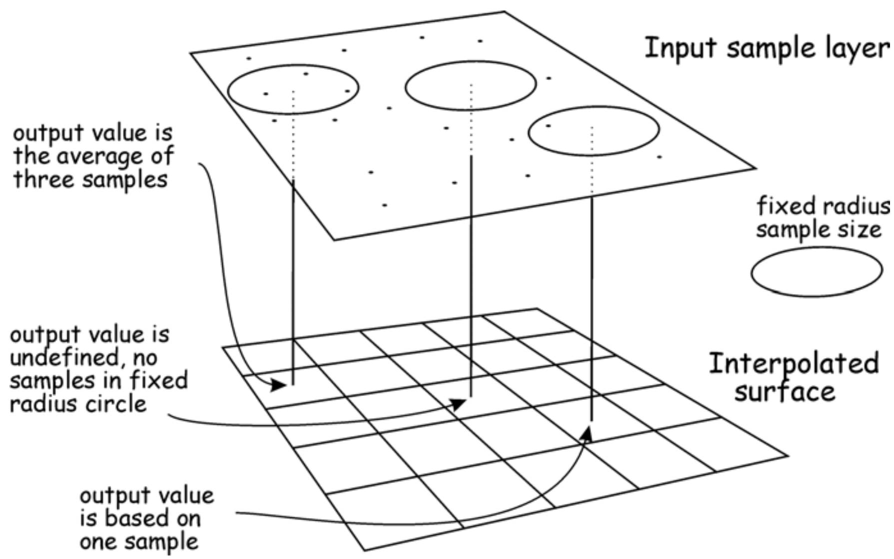
“Fixed Radius” - Local Averaging

- Cell values of a raster grid are estimated based on the average of nearby sample points
- A **“search radius”** is used to determine the “**nearby**” sample points
- Size of a **circle** centered on each cell
- Sample points within this circle are **averaged** to **interpolate** the value for that cell



“Fixed Radius” - Local Averaging

- Zero or **no data values** where no sample points fall within the circle
- Circles (larger than cell width) **overlap** for adjacent cells - **What is the consequence??**
- **Smoothing** effect
- **Non-exact** interpolator ()
- **Circle size** influences the occurrence of empty cells, smoothing, extreme preservation
- What is this function like in GIS?



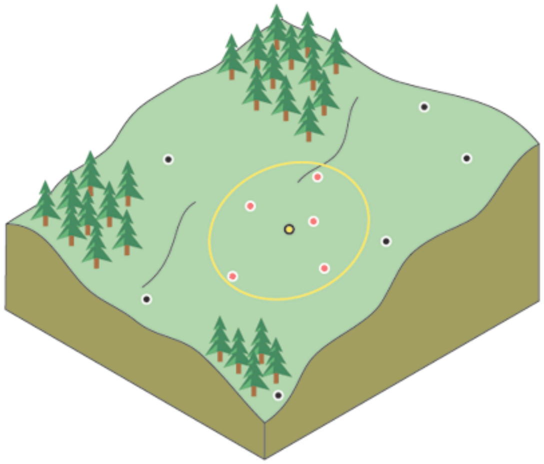
Inverse Distance Weighted Interpolation (IDW)

- **Accounting for "vicinity/nearness"** by

  (1) selecting points within a Kernel radius or

  (2) a fixed number of "near" points (known points)
- "Contribution of a point is the more **decreased** the more **distant** it is from the unmeasured location"
- **Weight** of each sample point is the **inverse** proportion to the **distance**
- **This is an exact interpolator**

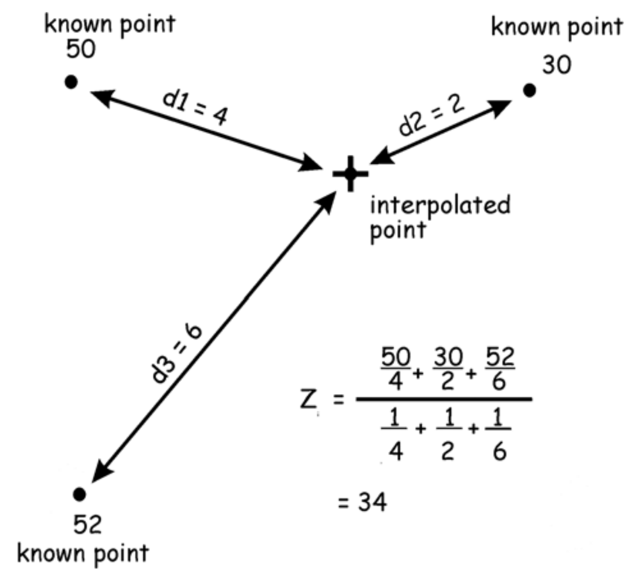
  where d = 0 surface takes the value of the data point



IDW Computation

$$Z_j = \frac{\sum_i \frac{Z_i}{d_{ij}^n}}{\sum_i \frac{1}{d_{ij}^n}}$$

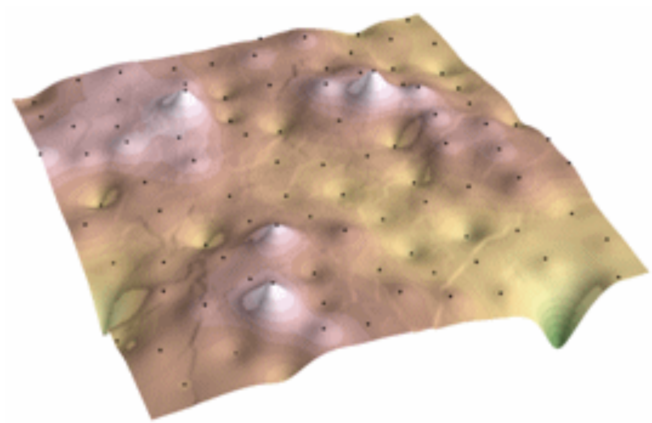
- Z_j - estimated value for the unknown point at location *j*
- d_{ij} - distance between known point *i* and unknown point *j*
- Z_i - is the value at known point *i*
- *n* - user-defined exponent for weighting
- Fixed number of points normally



Characteristics of IDW

- **Exact** interpolator
- Interpolated values equal sample point values at the sample locations
- Reduction of the formula at sample point locations
- **Smoothed** surfaces (no value jumps) but “bulls-eyes”

$$Z_j = \frac{\dfrac{Z_i}{d_{ij}^n}}{\dfrac{1}{d_{ij}^n}} = Z_i$$

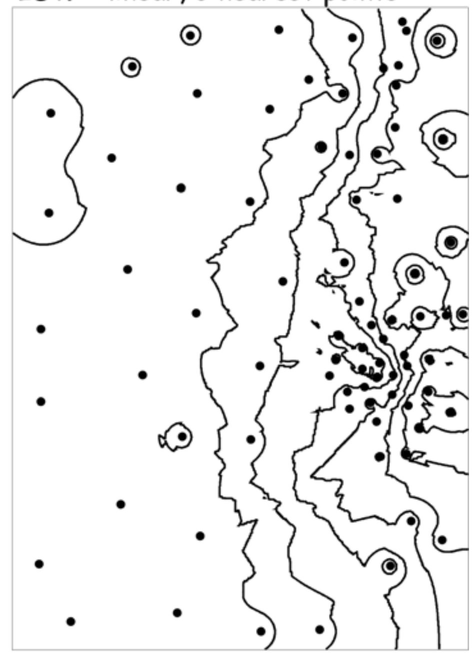


Influences to IDW

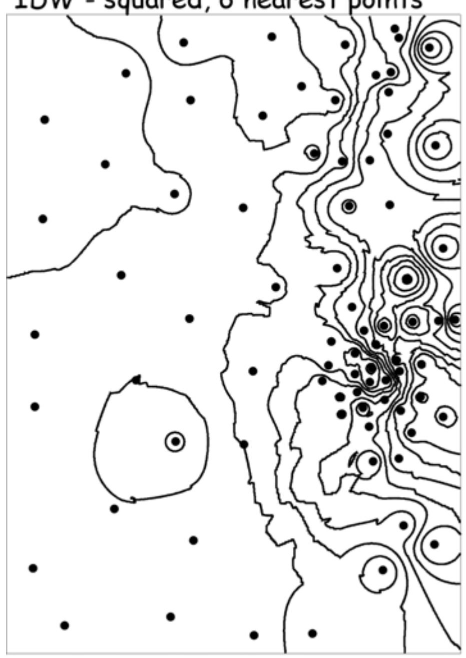
- Effects of changing ***n* and *r*** to be examined using **validation sample** and different combinations within *n* and *r* ranges
- *n* affects the **surface shape** (**closer points** become more influential with larger *n*)
- **higher exponents** create **higher peaks** with steeper gradients close to sample points, and **lower valleys** in the surface ("**bulls-eyes**")
- The higher the number of points *i* the **smoother** the surface

Original surface

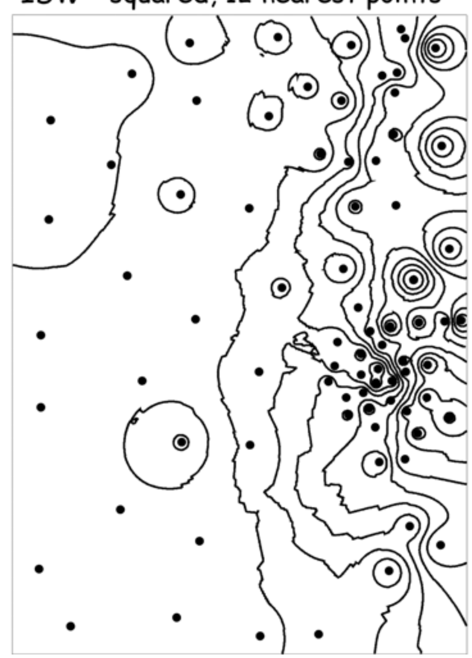
IDW - linear, 6 nearest points



IDW - squared, 6 nearest points

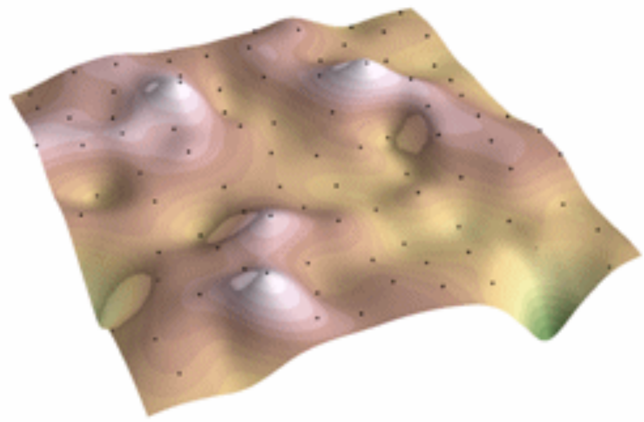


IDW - squared, 12 nearest points



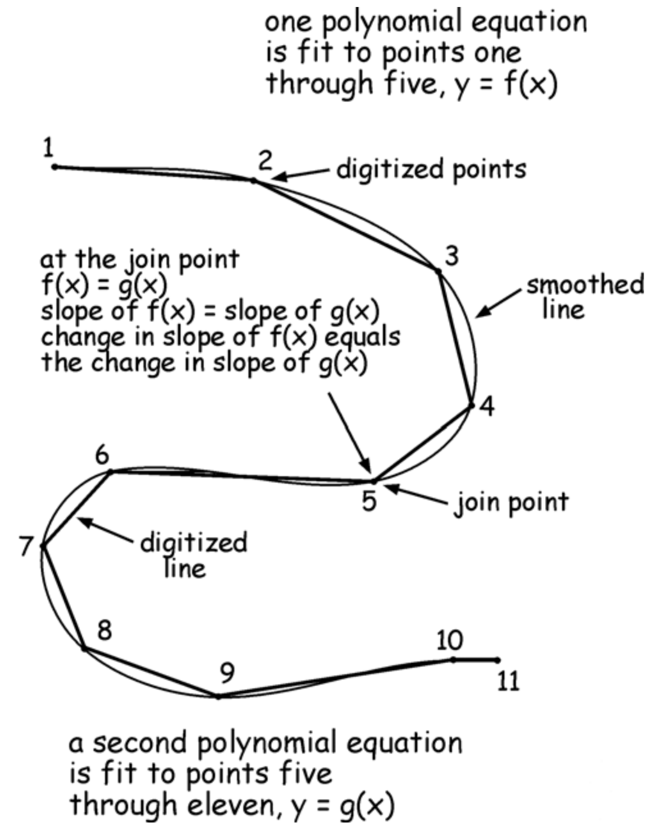
Splines

- The basic idea of **locally fitting** smooth curves (boat building)
- Functions to interpolate along a **smooth** curve using data points
- Smooth lines are enforced to **pass through** this set of points (“guide points”)
- Can be used for **lines** or **surfaces**



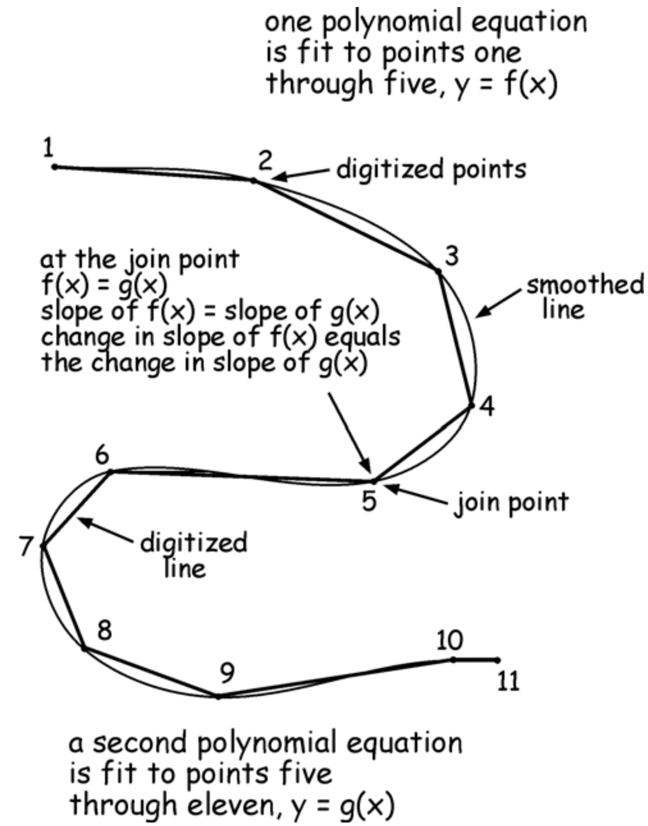
Computing Splines

- Spline functions are constructed from a set of **joined polynomial functions**
- Polynomial functions are fit to **short segments** (**piecewise** polynomials)
- **Exact** or least-squares method to fit the lines through the points in each segment
- Normally **first**, **second** or **third** order polynomials



Characteristics of Splines

- Minimizing curvature…
- Segments meet at ***knots*** or ***join points***
- Constraints: **slope** of a line and **change** in slopes of the line have to be equal **across segments** on either side of the join point
- **Exact interpolation** and **smooth transitions**



Computing Splines

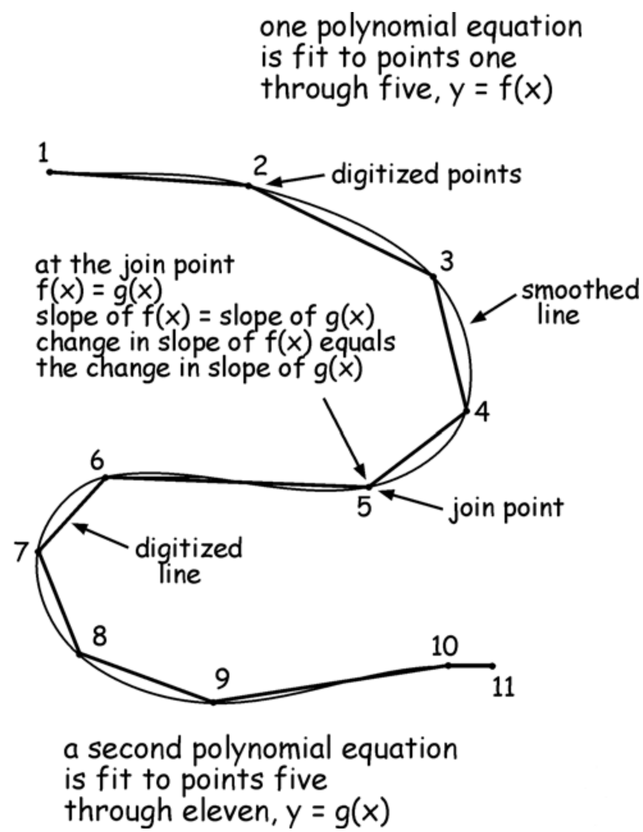
- For the interval *[x,x_i]* this is a **cubic spline** (for a line - for surfaces we would take bi-cubic splines):

  $$s_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$

  for i = 1, 2, …, n-1

  where

  $s'(x_i) = s_{i+1}'(x_{i+1})$ and $s''(x_i) = s_{i+1}''(x_{i+1})$

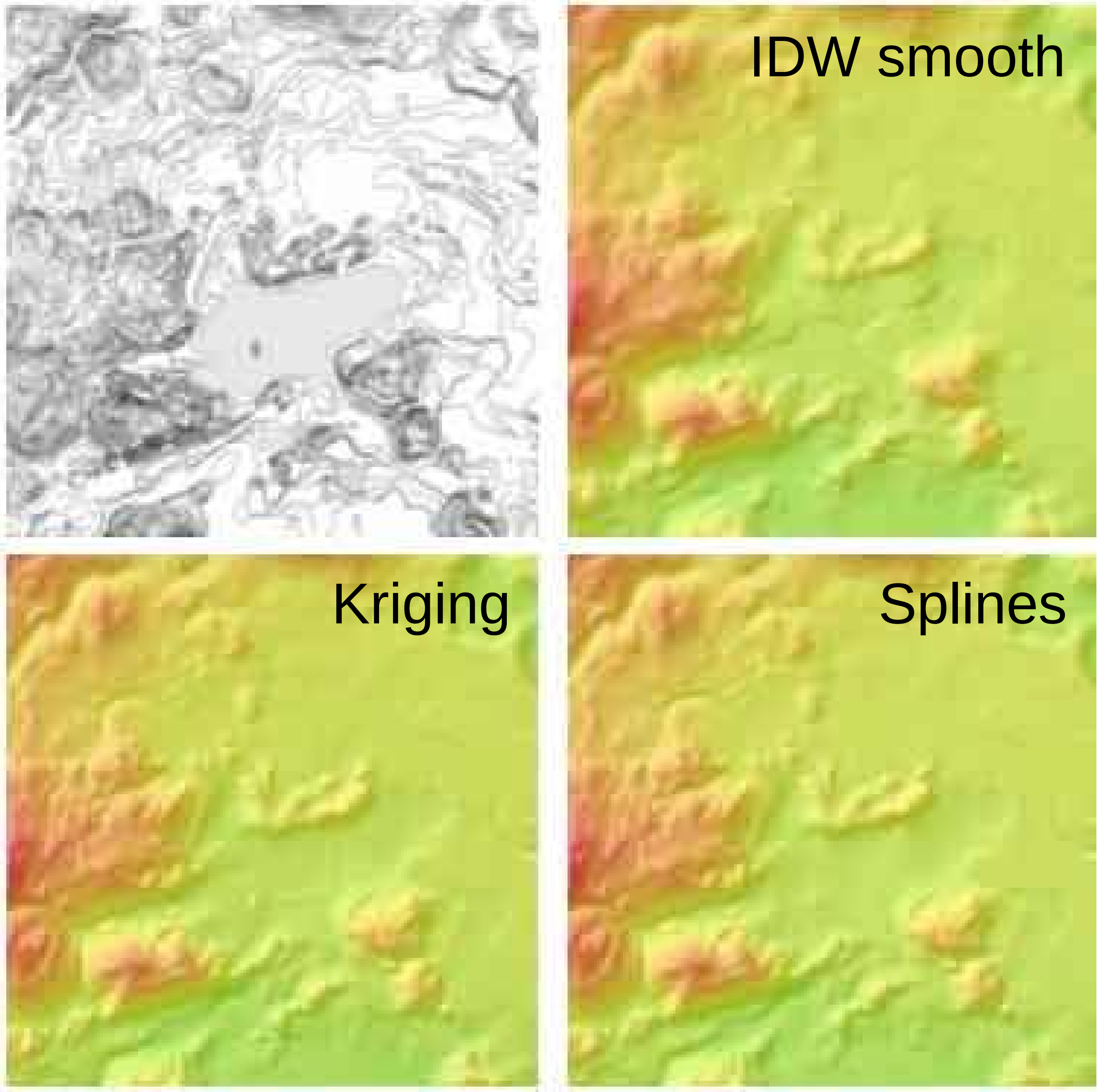


Splines - Some Comments

- **Visually** very **smooth**
- Quick to compute and **recalculate** after modifying (**piece-wise**)
- But: **Judgement** beforehand, if our data can be represented that smooth?
- Will we see maxima and minima if they far exceed our attributes?

Spatial Interpolation - Which is the Best

- How **smooth** do you want the surface to be, what is appropriate?
- Do you need the data points to be **preserved** at their corresponding location
- Can you guess any **dependencies** between data as a function of **distance**
- **Global trends**
- How can you **validate**, can you at all?
- Any data that could **improve estimations**?



From www.gisdevelopment.net/technology/tm/tm003pf.htm

How to Improve?

- Even if interpolators give satisfying results, parameters are often arbitrary
- Radius?, # neighbors?, Weighting?
- How to make theory-based choices and parameter-settings?
- This is where Geostatistical methods come into the picture to help us with this decision-making process

Summary

- We have seen some introduction into **spatial interpolation** and why we would like to do it
- You got some insights into the **methodological keys** of different approaches
- You know (more or less) what **spatial autocorrelation** means
- You know how **Nearest Neighbor**, **Mass preservation**, **IDW** and **Splines** work