Homework Assignment 2

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Personal Homework April 2, 2020

Contents

1 Theory

Starting from $P_t = \frac{1}{1+r}E_t[P_{t+1} + D_{t+1}]$ and $P_{t+1} = \frac{1}{1+r}E_{t+1}[P_{t+2} + D_{t+2}]$, we have the following price at time t:

$$\begin{aligned} P_t &= \frac{1}{1+r}E_t[(\frac{1}{1+r}E_{t+1}[P_{t+2} + D_{t+2}]) + D_{t+1}] \\ \iff P_t &= \frac{1}{1+r}E_t[D_{t+1}] + \frac{1}{(1+r)^2}E_t[E_{t+1}[P_{t+2} + D_{t+2}]] \\ \iff P_t &= \frac{1}{1+r}E_t[D_{t+1}] + \frac{1}{(1+r)^2}E_t[E_{t+1}[D_{t+2}]] + \frac{1}{(1+r)^3}E_tE_{t+1}E_{t+2}[D_{t+3} + P_{t+3}] \\ &\quad ... \end{aligned} \tag{1}$$

Applying the propriety $E_tE_{t+1}[X] = E_t[X]$ we have

$$P_t = \frac{E_t[D_{t+1}]}{1+r} + \frac{E_t[D_{t+2}]}{(1+r)^2} + ... + \frac{E_t[D_{t+n}]}{(1+r)^n} \tag{2}$$

Finally, if the $n \longrightarrow \infty$, we have

$$P_t = \sum_{i=0}^{\infty} \frac{E_t[D_{t+i}]}{(1+r)^i} \tag{3}$$

Assuming that the growth rate is zero and rate of return is constant, the price of a stock depends on the future expected dividends under the DDM. Principally, the price is determined by the present value of the dividend and the future expected dividends become more and more small during time.

The next step is assuming that the dividend process is driven by an AR(1) process. Starting from $D_{t+1} = (1+\mu)D_t + \epsilon_{t+1}$, we have

$$\begin{aligned} D_{t+2} &= (1+\mu)[D_t(1+\mu) + \epsilon_{t+1}] + \epsilon_{t+2} = (1+\mu)^2 D_t + (1+\mu)\epsilon_{t+1} + \epsilon_{t+2} \\ D_{t+3} &= (1+\mu)^3 D_t + (1+\mu)^2\epsilon_{t+1} + (1+\mu)\epsilon_{t+2} + \epsilon_{t+3} \\ &\quad ... \\ D_{t+i} &= (1+\mu)^i D_t + (1+\mu)^{i-1}\epsilon_{t+1} + ... + \epsilon_{t+i} \end{aligned} \tag{4}$$

Then, if we take the expected return of D_{t+i} knowing that $\epsilon_{t+i} \sim N(0,1)$ we have

$$E_t[D_{t+i}] = (1+\mu)^i E_t[D_t] \tag{5}$$

At time t the dividend is known $(E_t[E[D_t|D_t]])$. Then, we have

$$E_t[D_{t+i}] = (1+\mu)^i D_t \tag{6}$$

The last step now is to compute the link between the dividend paid at time t and the model's parameters (r and μ) and starting from $D_t = 1$, we need to rescale by T ($\sqrt{T}$). We know that

$$\begin{aligned}
P_t &= \sum_{t=1}^{\infty} \frac{E_t[D_{t+i}]}{(1+r)^i} \\
\iff E_t[D_{t+i}] &= (1+\mu)^i D_t \\
\iff P_t &= \sum_{t=0}^{\infty} \frac{(1+\mu)^i D_t}{(1+r)^i} \\
\iff P_t &= D_t \sum_{t=0}^{\infty} \frac{(1+\mu)^i}{(1+r)^i} \\
\iff D_t(\frac{1+\mu}{1+r}(\frac{1-(\frac{1+\mu}{1+r})^T}{1-\frac{1+\mu}{1+r}})) \\
\iff D_t(\frac{1+\mu}{1+r-\mu-1}) \\
\iff D_t(\frac{1+\mu}{r-\mu}) \\
\iff log(P_t) &= log(D_t) + log(\frac{1+\mu}{r-\mu}) \\
p_t &= d_t + log(\frac{1+\mu}{r-\mu})
\end{aligned} \tag{7}$$

Assuming that $log(\frac{1+\mu}{1+r}) = a$, which is the fundamental value of the stock price,we have:

$$\begin{aligned}
P_t &= d_t + a \\
P_t - d_t &= a
\end{aligned} \tag{8}$$

For the current value of the stock price,we have:

$$\begin{aligned}
P_t &= d_t + a + u_t \\
P_t - d_t &= a + u_t
\end{aligned} \tag{9}$$

Where $p_t - d_t \sim I(0)$.Then p_t and d_t must therefore have long run components that cancel out to produce u_t . p_t and d_t are then cointegrated.

2 Stationary test

2.1 Computing critical values

In order to test the stationarity of the stock price and dividend processes, we firstly computed a time series of stock price $p_t^{(i)} = p_{t-1}^{(i)} + \epsilon_t^{(i)}$, simulating also the path of T error terms ($\epsilon_t^{(i)}$) following a standardized normal distribution and using the following codes

In fact, the following graph shows the random walk paths of stock price, using the Monte Carlo simulation with N=100 number of replication and starting from the point $p_0^{(i)} = 0$.

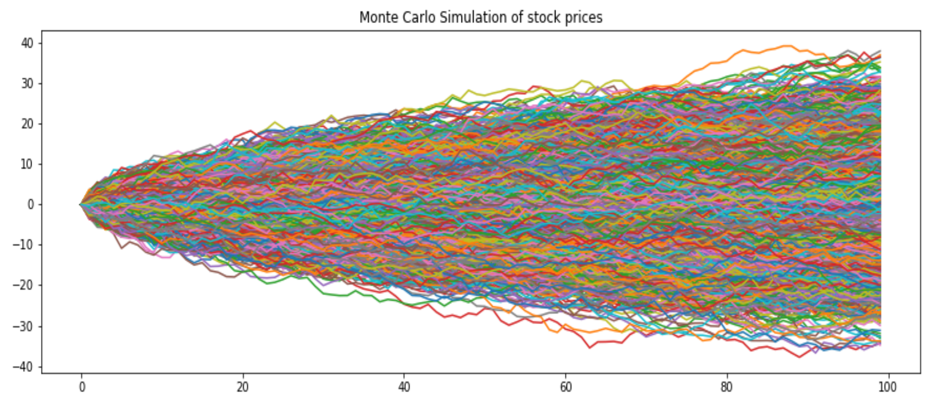

Figure 1: Monte Carlo simulation of stock prices

Furthermore, we have estimated the AR(1) model, assuming the $H_a : \Delta p_t^{(i)} = \alpha^{(i)} + \beta^{(i)} p_{t-1}^{(i)} + u_t^{(i)}$. On the contrary, the null hypothesis is $H_0 : \Delta p_t^{(i)} = u_t^{(i)}$. Finally, we have computed the t-stats for the different betas: $t(\beta^{(i)})$.
From a short summary of the values of $t(\beta^{(i)})$, it shows up that the values of the t-stats, which are equal to 10'000, have a mean of -1.5244 .The distribution of the t-stats are asymmetric to 0 because the 75% quantile is close to -1:

	Betas
count	10000.000000
mean	-1.524494
std	0.861763
min	-4.822214
25%	-2.086502
50%	-1.548926
75%	-0.994907
max	2.660512

Figure 2: Summary of $t(\beta^{(i)})$

From the graph below, you will find the vector containing all the N values of $t(\beta^{(i)})$ by increasing order, followed by the histogram of the value of betas around their increasing t-stat.

	Betas
t-stat 2632	-4.822214
t-stat 7445	-4.658143
t-stat 2178	-4.620994
t-stat 2272	-4.492922
t-stat 9520	-4.486264
...	...
t-stat 1330	1.726450
t-stat 9936	1.770856
t-stat 9721	1.787654
t-stat 6979	1.872534
t-stat 64	2.660512

Figure 3: Vector of $t(\beta^{(i)})$

When we observe the vector, we can also notice that the majority of the values tend to be negative and it's distribution is not far to be symmetric around it's mean as the skewness is close to 0.

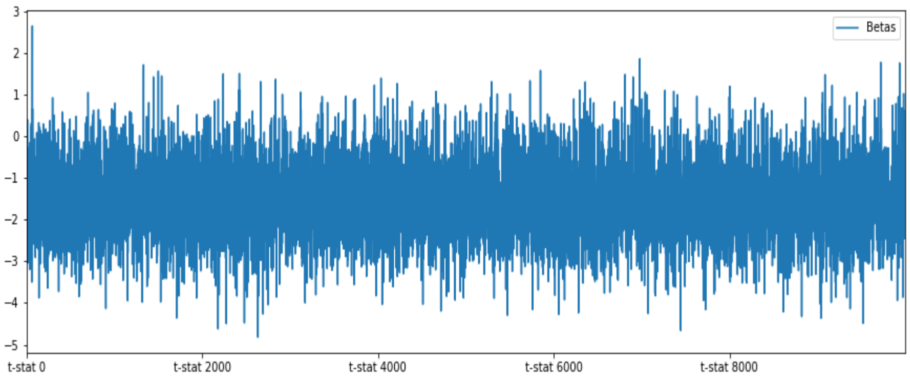

Figure 4: $t(\beta^{(i)})$ distribution

```
Critical Values:
        1%: -3.501
        5%: -2.893
        10%: -2.577
```

Figure 5: Critical Values

To conclude this paragraph, we have calculated the 10%, 5% and 1% critical values for the Dickey-Fuller test, which are respectively -2.577, -2.893 and -3.501.

2.2 Testing non-stationarity and Power of the test

Starting the analysis for the stocks, we have tested the null hypothesis using the critical values of 5%.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Price_differential   R-squared:                       0.008
Model:                            OLS   Adj. R-squared:                  0.005
Method:                 Least Squares   F-statistic:                     2.908
Date:                Wed, 01 Apr 2020   Prob (F-statistic):             0.0890
Time:                        14:23:29   Log-Likelihood:                 648.07
No. Observations:                 359   AIC:                            -1292.
Df Residuals:                     357   BIC:                            -1284.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0780      0.044      1.791      0.074      -0.008       0.164
Pt            -0.0091      0.005     -1.705      0.089      -0.020       0.001
==============================================================================
Omnibus:                       30.544   Durbin-Watson:                   1.912
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.975
Skew:                          -0.654   Prob(JB):                     3.44e-09
Kurtosis:                       3.946   Cond. No.                         171.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
--------------------------------------------------------------------------------
We fail to reject H0(B=0) whith a t-val of -1.705203579965758 using TOTMKUK(PI)'s stock- The
Time Series is Non-Stationary at an alpha of 5%
```

Figure 7: OLS regression results for a stock process (UK)

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Price_differential   R-squared:                       0.005
Model:                            OLS   Adj. R-squared:                  0.002
Method:                 Least Squares   F-statistic:                     1.632
Date:                Wed, 01 Apr 2020   Prob (F-statistic):              0.202
Time:                        14:23:29   Log-Likelihood:                 641.18
No. Observations:                 359   AIC:                            -1278.
Df Residuals:                     357   BIC:                            -1271.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0376      0.024      1.556      0.121      -0.010       0.085
Pt            -0.0044      0.003     -1.277      0.202      -0.011       0.002
==============================================================================
Omnibus:                       54.043   Durbin-Watson:                   1.914
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              100.500
Skew:                          -0.842   Prob(JB):                     1.50e-22
Kurtosis:                       4.971   Cond. No.                         79.8
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
--------------------------------------------------------------------------------
We fail to reject H0(B=0) whith a t-val of -1.277376630628786 using TOTMKUS(PI)'s stock- The
Time Series is Non-Stationary at an alpha of 5%
```

Figure 6: OLS regression results for a stock process (US)

With a t-value of -1.705 using TOTMKUK(PI)'s stocks. We then failed to reject the hypothesis H0 that the time series is non-stationary (-1.705 >to -2.893) at an alpha of 5%. For TOTMKUS(PI)'s stocks,we also failed to reject the hypothesis H0 of non-stationarity (-1.277 >-2.893) at an alpha of 5%.

Finally, the test for non-stationarity for the dividend process, using the $\Delta p_t = \alpha +$

$\beta p_{t-1} + u_t$ regression, had produced the same result of the stock price process.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:       div_differential   R-squared:                       0.003
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     1.158
Date:                Tue, 30 Jun 2020   Prob (F-statistic):              0.283
Time:                        13:24:55   Log-Likelihood:                 1079.7
No. Observations:                 359   AIC:                            -2155.
Df Residuals:                     357   BIC:                            -2148.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0044      0.001      4.825      0.000       0.003       0.006
Pt             0.0013      0.001      1.076      0.283      -0.001       0.004
==============================================================================
Omnibus:                      316.922   Durbin-Watson:                   1.730
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            11184.580
Skew:                          -3.479   Prob(JB):                         0.00
Kurtosis:                      29.444   Cond. No.                         2.57
==============================================================================
```

Figure 8: OLS regression result for a dividends process(US)

```
                            OLS Regression Results
==============================================================================
Dep. Variable:       div_differential   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.003
Method:                 Least Squares   F-statistic:                   0.07392
Date:                Tue, 30 Jun 2020   Prob (F-statistic):              0.786
Time:                        13:24:55   Log-Likelihood:                 872.25
No. Observations:                 359   AIC:                            -1740.
Df Residuals:                     357   BIC:                            -1733.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0058      0.008      0.702      0.483      -0.010       0.022
Pt            -0.0010      0.004     -0.272      0.786      -0.008       0.006
==============================================================================
Omnibus:                      299.706   Durbin-Watson:                   2.347
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7976.335
Skew:                          -3.300   Prob(JB):                         0.00
Kurtosis:                      25.128   Cond. No.                         19.9
==============================================================================
```

Figure 9: OLS regression result for a dividends process(UK)

According to a t-value of -0.2718 and 1.0759 for respectively TOTMKUK(DY) and TOTMKUS(DY). We failed to reject H0, the time series are stationary at an alpha of 5%. We can clearly see that the t-values of the dividend are lower than the t-values of the stock. They are further to the rejection area than the stocks(their computed Beta are lower).

Now, we want to analyze the power of the stationary test using the AR(1) process with an auto-regressive parameter of 0.96.

$$p_t = 0.96 p_{t-1} + \epsilon_t \tag{10}$$

We have simulated $T = 100$ series of the AR(1) above and the price evolution is given by the graph below Looking at the distribution of t-stats, we can say that it seems to be

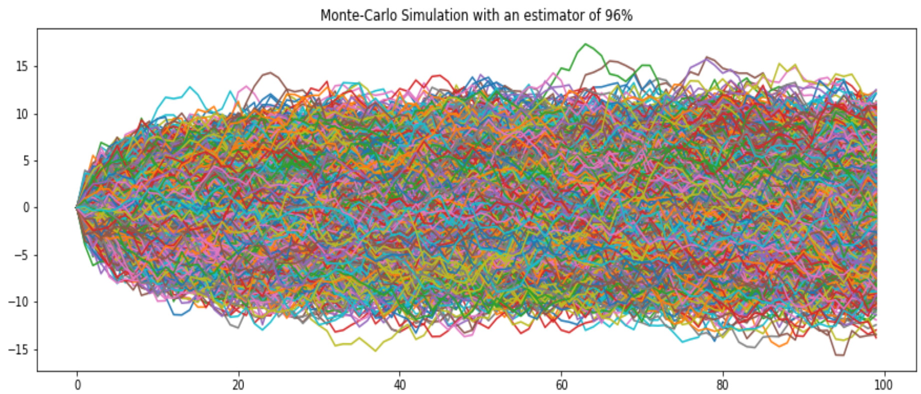

Figure 10: Monte Carlo simulation of $p_t = 0.96 p_{t-1} + \epsilon_t$

quite symmetric with a skewness very close to 0 . This leads to say that the process is stationary(as the auto-regressive parameter is equal to 0.96).

We can also check form the cumulative distribution function of the t-stats, which also seems to be a stationary process due to the several extreme values on the left of the CDF.

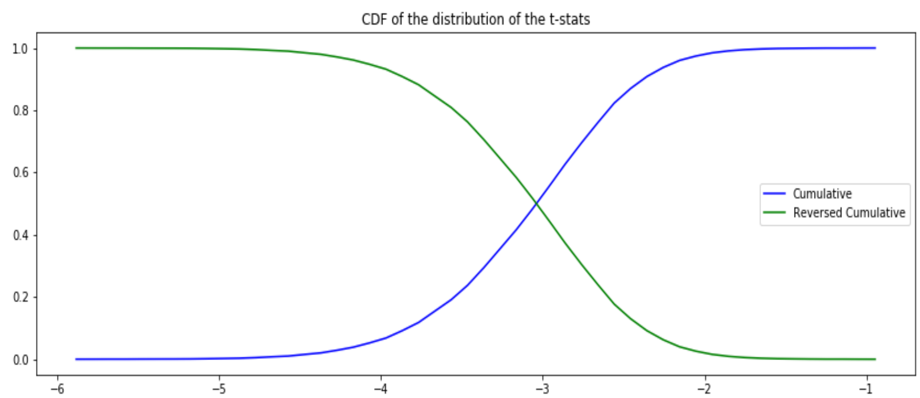

Figure 11: $t(\beta^{(i)})$ cumulative distribution with AR(1) process $p_t = 0.96 p_{t-1} + \epsilon_t$

We can say that the the power of the test decreases drastically if we change the autoregressive parameter of 0.80(As a powerful test has usually a power superior to 80%.): having then the stock price equal to:

$$p_t = 0.80 p_{t-1} + \epsilon_t \tag{11}$$

In this case we use an auto-regressive parameter of 0.80, the power of the test is equal to 11.5% . The power results to be far and away lower than using a higher auto-regressive coefficient(around 90% if using an an AR1 coefficeint of 0.96).

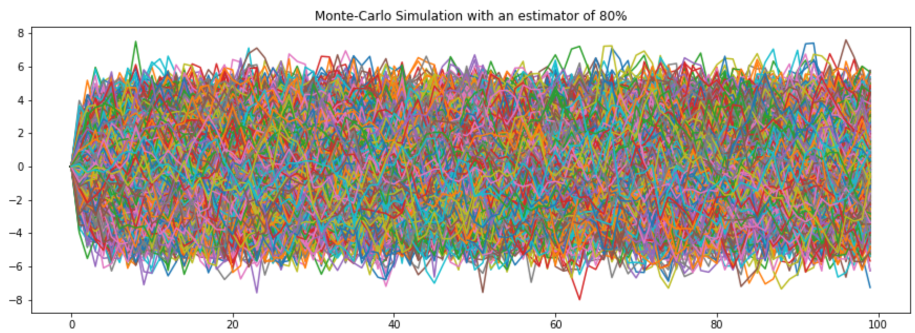

Figure 12: Monte Carlo simulation of $p_t = 0.8 p_{t-1} + \epsilon_t$

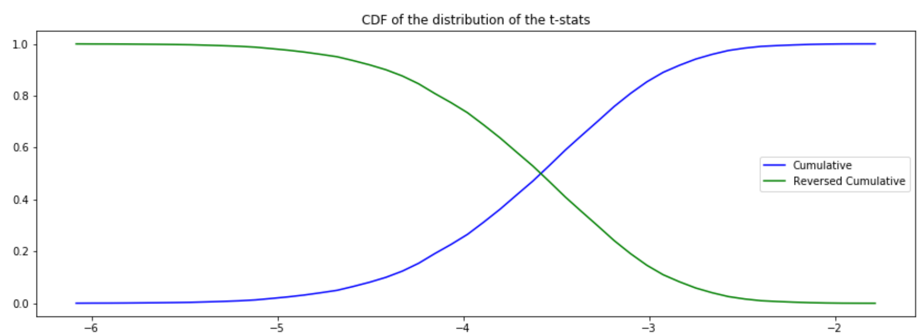

Figure 13: $t(\beta^{(i)})$ cumulative distribution with AR(1) process $p_t = 0.8 p_{t-1} + \epsilon_t$

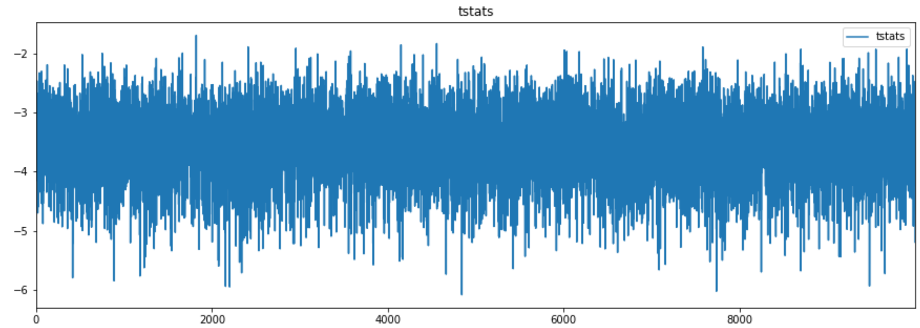

Figure 14: $t(\beta^{(i)})$ evolution with $p_t = 0.8p_{t-1} + \epsilon_t$

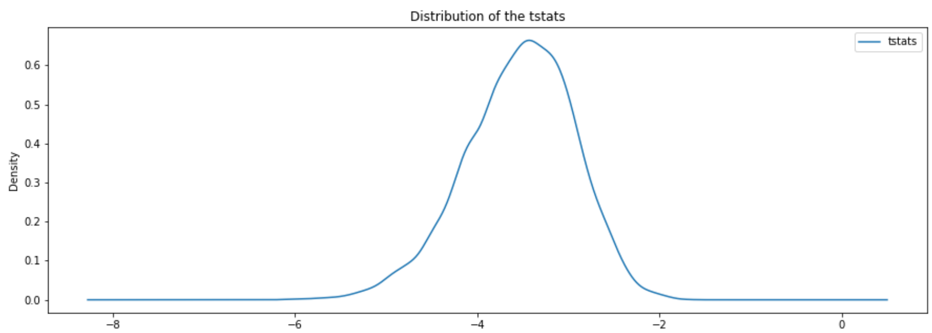

Figure 15: $t(\beta^{(i)})$ distribution with $p_t = 0.8p_{t-1} + \epsilon_t$

In conclusion, we can check the power of the stationary test changing the time horizon, let's say $T = 100$. Taking in consideration the analysis of the 96% as the auto-regressive coefficient with $T = 360$ and $T = 100$, we already can see from the two graph below that the process are stationary using both T. In the question 2b the hypothesis H1 was rejected because the Beta that we computed was small and our study on the power of the tests shows that smaller is the Beta less powerful is the test and then higher chances for the test to be actually biased, that could have happened to 2b.To fix that issue we should try to use other tests like the ad-fuller,the phillips perron test which add robustness to unspecified autocorrelation.

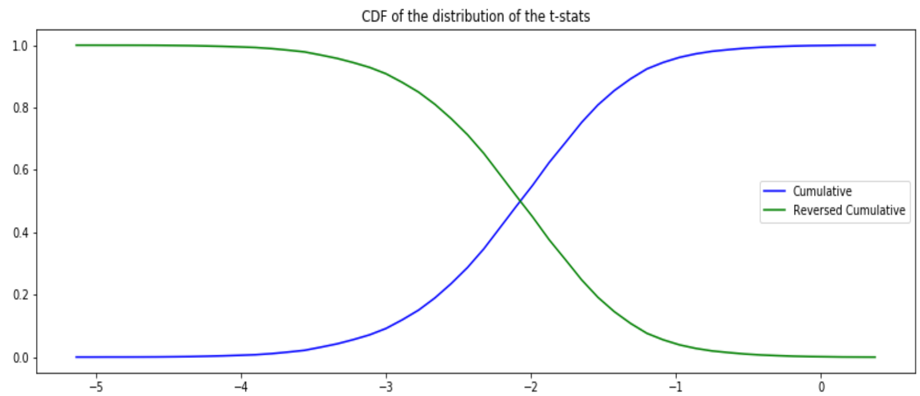

Figure 16: $t(\beta^{(i)})$ cumulative distribution with AR(1) process $p_t = 0.96p_{t-1} + \epsilon_t$ and $T = 100$

The result of the power of the test for the 0.96 auto-regressive parameter with $T = 100$ is equal to around 90%. Meanwhile, the power of the test with $T = 360$ parameter is around 44%. This means that stationarity of returns decreases with a lower dependency of p_t and the power increases if we decrease T. The auto-regressive process for Stocks and Dividend's yields was equal to 1. That can explain why the stationary hypothesis was rejected for both.

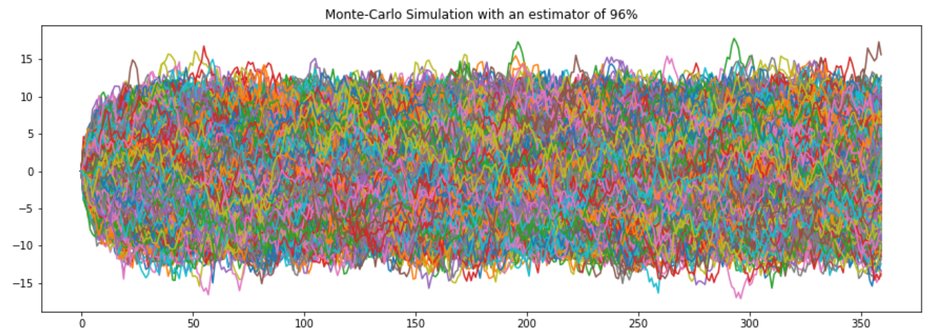

Figure 17: Monte Carlo simulation of $p_t = 0.96p_{t-1} + \epsilon_t$

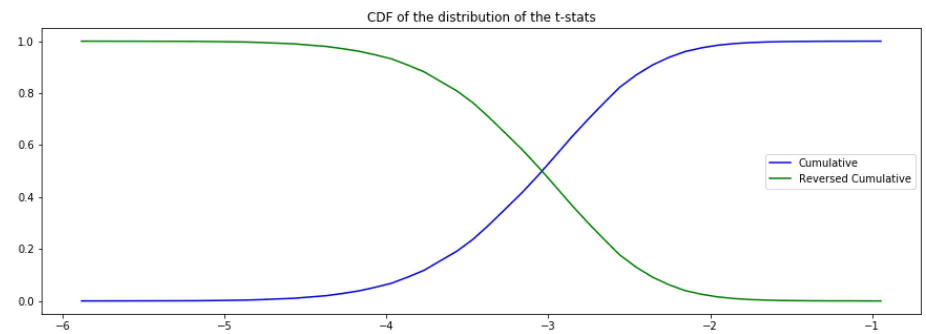

Figure 18: $t(\beta^{(i)})$ cumulative distribution with AR(1) process $p_t = 0.96p_{t-1} + \epsilon_t$ and $T = 360$

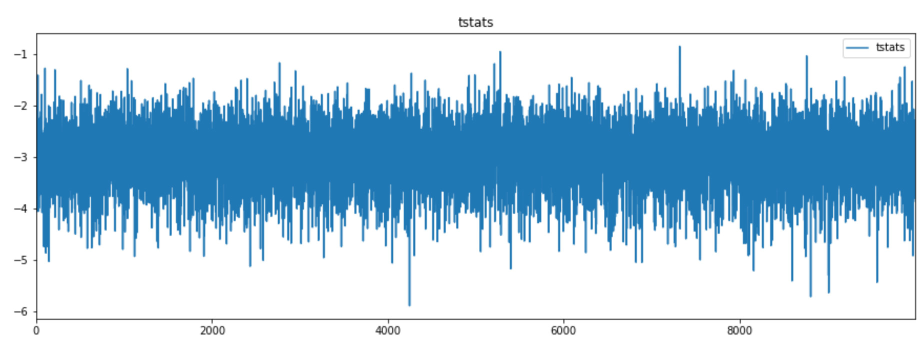

Figure 19: $t(\beta^{(i)})$ evolution with $p_t = 0.96p_{t-1} + \epsilon_t$

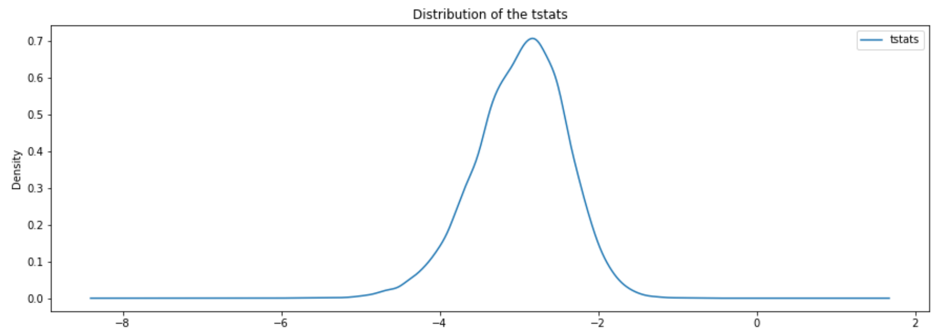

Figure 20: $t(\beta^{(i)})$ distribution with $p_t = 0.96p_{t-1} + \epsilon_t$

3 Co-integration test

3.1 Computing critical values

In order to test the co-integration between variables, we have firstly calculated simulated the two paths of stock price and dividend yield using, as before, the Monte Carlo simulation with N=100 and also in this section they start from $p_0^{(i)} = 0$ and $d_0^{(i)} = 0$.

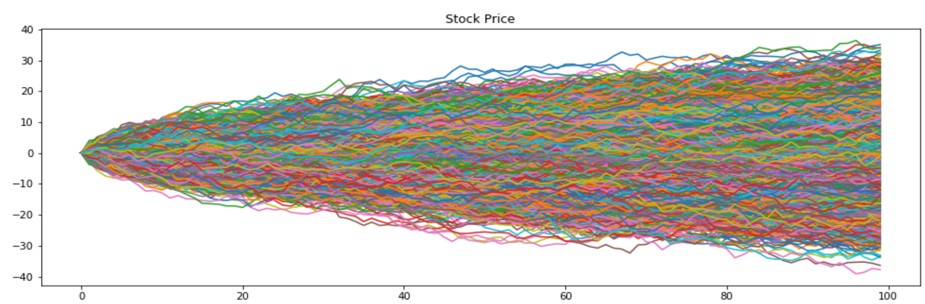

Figure 21: Monte Carlo simulation of stock prices $p_t^{(i)} = p_{t-1}^{(i)} + \epsilon_t^{(i)}$

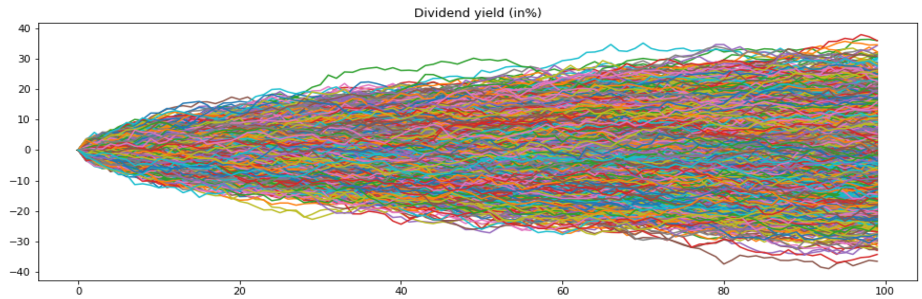

Figure 22: Monte Carlo simulation of dividend prices (in %) $d_t^{(i)} = d_{t-1}^{(i)} + \eta_t^{(i)}$

Where the ϵ_t and η_t follow a standardized normal distribution and simulated as a random walk process. The relation between the stock prices and dividends is defined as.

$$p_t^{(i)} = a + bd_t^{(i)} + z_t^{(i)} \tag{12}$$

	tstats
1328	-5.478431
5684	-5.172361
8508	-5.037713
858	-4.936077
273	-4.922248
...	...
2912	1.302591
5449	1.314408
5516	1.381303
9043	1.519456
156	2.355100

10000 rows × 1 cc

Figure 23: vector of $t(\beta^{(i)})$

On the vector we found we notice that we have more negative extreme values and less positive extreme values.

We found the following critical values for the Dickey Fuller test:

```
Critical Values:
        1%: -3.955
        5%: -3.348
        10%: -3.039
```

Figure 24: Critical values for the Dickey Fuller test for co-integration

The t-stat we found is equal to t=-2.034 which is superior to -2.890.We reject the hypothesis of non-stationarity for the residuals z_t.

3.2 Testing co-integration

As we are looking to know if the residuals are stationary we use the Dickey-Fuller test. As we have done in the sections before, we have estimated the AR(1) model for the residual under $H_a : \Delta\hat{z}_t^{(i)} = \alpha^{(i)} + \beta^{(i)}\hat{z}_{t-1}^{(i)} + u_t^{(i)}$.With an alpha of 5%,we failed to reject the hypothesis of H0 of non-stationary for the UK and US market because the Critical value (-3.348 at 5%) is inferior to the t-stats in absolute values which are equal to 3.48 and -0.591. The graphics below show the distribution of the t-stat residuals which seems to be symmetric and the t-stat residuals with T=10'000. In addition,we wanted to show that there is no the co-integration between the p_t and d_t(even if when we know that z_t is non stationary and then there is no cointegration). We decided to use the simple Durbin Watson test. The result demonstrates that there is no co-integration between the stock prices and dividend's yield ($0.1721 < 0.38$ which is the dw stat).

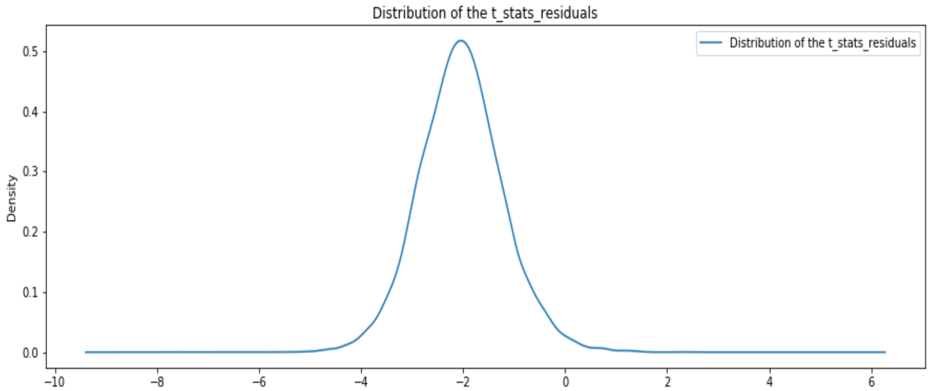

Figure 25: residuals distribution of $\Delta\hat{z}_t^{(i)} = \alpha^{(i)} + \beta^{(i)}\hat{z}_{t-1}^{(i)} + u_t^{(i)}$

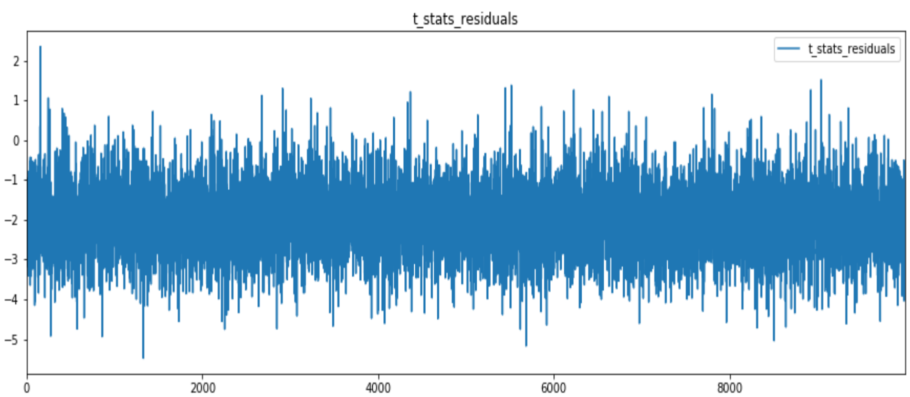

Figure 26: β for t stat residuals

The minimum t-stat is around the value of -6 and it's maximum is around 2. The distribution of $t(\beta^{(i)})$'s is around the 2. This lead to conclude that the residual process is non stationary. The quantiles at 10%,5%and 1% of the distribution of $t(\beta^{(i)})$ are -3.039, -3.348 and -3.955.

In order to use the Dickey-Fuller test, we have established the regression of $p_t = a + bd_t + z_t$ and consequently for the test the $\Delta\hat{z}_t^{(m)} = \alpha + \beta\hat{z}_{t-1}^{(m)} + u_t$.
We executed 2 OLS tests for each market(the first for the US one and the second for the

UK one)

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            Stock_Price   R-squared:                       0.853
Model:                            OLS   Adj. R-squared:                  0.853
Method:                 Least Squares   F-statistic:                     2083.
Date:                Tue, 30 Jun 2020   Prob (F-statistic):         2.76e-151
Time:                        13:58:11   Log-Likelihood:                 6.7723
No. Observations:                 360   AIC:                            -9.545
Df Residuals:                     358   BIC:                            -1.772
Df Model:                           1
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept          6.3439      0.018    353.651      0.000       6.309       6.379
Dividend_yield     1.0677      0.023     45.636      0.000       1.022       1.114
==============================================================================
Omnibus:                       10.735   Durbin-Watson:                   0.030
Prob(Omnibus):                  0.005   Jarque-Bera (JB):               10.956
Skew:                           0.424   Prob(JB):                      0.00418
Kurtosis:                       3.112   Cond. No.                         2.57
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
--------------------------------------------------------------------------------
Price and dividends are not Cointegrated Because the Durbin Watson stat 0.02971530380779572<0.38 at an alpha of 5%
```

Figure 27: OLS regression for US market of $p_t = a + bd_t + z_t$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            Stock_Price   R-squared:                       0.775
Model:                            OLS   Adj. R-squared:                  0.774
Method:                 Least Squares   F-statistic:                     1231.
Date:                Tue, 30 Jun 2020   Prob (F-statistic):         6.91e-118
Time:                        13:58:11   Log-Likelihood:                 93.110
No. Observations:                 360   AIC:                            -182.2
Df Residuals:                     358   BIC:                            -174.4
Df Model:                           1
Covariance Type:            nonrobust
==================================================================================
                     coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------
Intercept          5.6269      0.072     78.415      0.000       5.486       5.768
Dividend_yield     1.0914      0.031     35.081      0.000       1.030       1.153
==============================================================================
Omnibus:                        1.144   Durbin-Watson:                   0.063
Prob(Omnibus):                  0.564   Jarque-Bera (JB):                1.063
Skew:                          -0.133   Prob(JB):                        0.588
Kurtosis:                       3.009   Cond. No.                         19.9
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
--------------------------------------------------------------------------------
Price and dividends are not Cointegrated Because the Durbin Watson stat 0.06276432365328813<0.38 at an alpha of 5%
```

Figure 28: OLS regression for UK market of $p_t = a + bd_t + z_t$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            Differencial   R-squared:                       0.014
Model:                             OLS   Adj. R-squared:                  0.011
Method:                  Least Squares   F-statistic:                     5.114
Date:                 Tue, 30 Jun 2020   Prob (F-statistic):             0.0243
Time:                         13:58:11   Log-Likelihood:                 8.8235
No. Observations:                  359   AIC:                            -13.65
Df Residuals:                      357   BIC:                            -5.880
Df Model:                            1
Covariance Type:             nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept         0.0009      0.013      0.071      0.944      -0.024       0.025
Residualminus    -0.6898      0.305     -2.261      0.024      -1.290      -0.090
==============================================================================
Omnibus:                        11.265   Durbin-Watson:                   0.018
Prob(Omnibus):                   0.004   Jarque-Bera (JB):               11.570
Skew:                            0.437   Prob(JB):                      0.00307
Kurtosis:                        3.102   Cond. No.                         24.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

         Test on the residuals Time series for US:
We fail to reject H0(B=0) whith a t-val of -2.2613166552657202 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 10%
We fail to reject H0(B=0) whith a t-val of -2.2613166552657202 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 5%
We fail to reject H0(B=0) whith a t-val of -2.2613166552657202 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 1%
```

Figure 29: OLS regression (for the UK market) of $\Delta\hat{z}_t^{(m)} = \alpha + \beta\hat{z}_{t-1}^{(m)} + u_t$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            Differencial   R-squared:                       0.015
Model:                             OLS   Adj. R-squared:                  0.013
Method:                  Least Squares   F-statistic:                     5.603
Date:                 Tue, 30 Jun 2020   Prob (F-statistic):             0.0185
Time:                         13:58:11   Log-Likelihood:                 95.773
No. Observations:                  359   AIC:                            -187.5
Df Residuals:                      357   BIC:                            -179.8
Df Model:                            1
Covariance Type:             nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
Intercept         0.0006      0.010      0.057      0.955      -0.019       0.020
Residualminus    -0.4953      0.209     -2.367      0.018      -0.907      -0.084
==============================================================================
Omnibus:                         0.982   Durbin-Watson:                   0.031
Prob(Omnibus):                   0.612   Jarque-Bera (JB):                0.987
Skew:                           -0.126   Prob(JB):                        0.611
Kurtosis:                        2.951   Cond. No.                         21.3
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

         Test on the residuals Time series for UK:
We fail to reject H0(B=0) whith a t-val of -2.3671631250558547 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 10%
We fail to reject H0(B=0) whith a t-val of -2.3671631250558547 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 5%
We fail to reject H0(B=0) whith a t-val of -2.3671631250558547 using Residuals_tminus- The Time Series is Non-Stati
onary at an alpha of 1%
```

Figure 30: OLS regression (for the US market) of $\Delta\hat{z}_t^{(m)} = \alpha + \beta\hat{z}_{t-1}^{(m)} + u_t$

Finally, we can test the non stationarity of the residuals with the $H_0 : \beta = 0$ and using the different critical values (10%,5% and 1%).
For the UK/US markets,the $t - test$ results to not reject the null hypothesis: the time series is non-stationary
For the US market, the results are the same. The main differences between the two market

are the t-stats: -2.26 for the US and respectively -2.36 for the UK market. The residuals of the DDM are not co-integrated that's why we should use an OLS regression rather than a GLS regression. When we use DDM we can only explain the price process of a stock at 77.5% in the UK market against 85.3% in the US Market.

3.3 Error-correcting model

For the UK market, we want to estimate an error-correcting model with one lag of the form

$$\begin{aligned} \Delta p_t &= \mu_1 + \phi_{11}\Delta p_{t-1} + \phi_{12}\Delta d_{t-1} + \gamma_1 z_{t-1} + \epsilon_{1,t} \\ \Delta d_t &= \mu_2 + \phi_{21}\Delta p_{t-1} + \phi_{22}\Delta d_{t-1} + \gamma_2 z_{t-1} + \epsilon_{2,t} \end{aligned} \tag{13}$$

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Pdiff   R-squared:                       0.016
Model:                            OLS   Adj. R-squared:                  0.007
Method:                 Least Squares   F-statistic:                     1.887
Date:                Tue, 30 Jun 2020   Prob (F-statistic):              0.131
Time:                        13:58:11   Log-Likelihood:                 647.59
No. Observations:                 358   AIC:                            -1287.
Df Residuals:                     354   BIC:                            -1272.
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept       0.0420      0.039      1.082      0.280      -0.034       0.118
P_diff_minus    0.0513      0.053      0.964      0.336      -0.053       0.156
D_diff_minus   -0.0056      0.006     -0.986      0.325      -0.017       0.006
R_diff         -0.0118      0.016     -0.732      0.465      -0.043       0.020
==============================================================================
Omnibus:                       30.335   Durbin-Watson:                   1.996
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.917
Skew:                          -0.649   Prob(JB):                     3.54e-09
Kurtosis:                       3.962   Cond. No.                         177.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
----------------------------------------------------------------------------------------------
The sign of y is positive with y=-0.011757318699982715,it does correct the model if significant
----------------------------------------------------------------------------------------------
         Test on the residuals Time series for UK:
y is not significatif for an alpha of 10%
y is not significatif for an alpha of 5%
y is not significatif for an alpha of 1%
```

Figure 31: OLS regression for stocks differecials

Where z_{t-1} is the error-correcting terms. We are then interested in seeing the dynamic of the variable in the shock and in this case, the speed of adjustment from short equilibrium to the long one.

All the parameters are not significant in our model.The time series is stationary as the

Durbin Watson indicates us the presence of co integration for both models.(for the Dividend's yield differencial see the next page) In case of significance of y1 , the z_{t-1} would have corrected the model only for the stock prices as the coefficient is negative.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Ddiff   R-squared:                       0.008
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.9116
Date:                Tue, 30 Jun 2020   Prob (F-statistic):              0.435
Time:                        13:58:11   Log-Likelihood:                 870.98
No. Observations:                 358   AIC:                            -1734.
Df Residuals:                     354   BIC:                            -1718.
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept        0.0113      0.021      0.545      0.586      -0.030       0.052
P_diff_minus     0.0006      0.029      0.023      0.982      -0.055       0.057
D_diff_minus    -0.0011      0.003     -0.377      0.706      -0.007       0.005
R_diff           0.0120      0.009      1.393      0.165      -0.005       0.029
==============================================================================
Omnibus:                      291.158   Durbin-Watson:                   2.339
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7116.877
Skew:                          -3.203   Prob(JB):                         0.00
Kurtosis:                      23.883   Cond. No.                         177.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
----------------------------------------------------------------------------------------------------
The sign of y is negative with y=0.011992674725497938,it doesn't correct the model if significant
----------------------------------------------------------------------------------------------------
         Test on the residuals Time series for UK:
y is not significatif- The Time Series is Non-Stationary at an alpha of 10%
y is not significatif- The Time Series is Non-Stationary at an alpha of 5%
y is not significatif- The Time Series is Non-Stationary at an alpha of 1%
```

Figure 32: OLS regression summary for dividend's yields differencials