

## 3D Object Representation

Graphics scenes can contain trees, flowers , clouds rocks water, rubber, paper , bricks etc.

Polygon and quadratic surfaces provide precise descriptions for simple Euclidean objects such as polyhedrons ellipsoid.

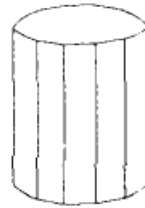
### Polygon Surfaces

3-D graphics object is a set of surface polygons that enclose the object interior

A polygon mesh is a set of connected polygonally bounded planar surfaces

Equation of plane is  $Ax + By + Cz + D = 0$

A polygon mesh is a collection of edges, vertices and polygons connected such that each edge is shared by at most two polygons.



### Polygon Tables

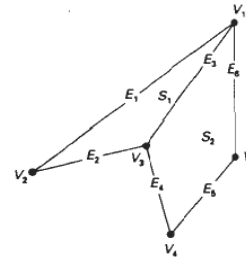
Polygon data tables can be organized into two groups: geometrical and attribute tables

Geometric data tables contain vertex coordinates and parameters to identify the spatial orientation of polygon surfaces

Attribute information for an object includes parameters specifying the degree of transparency of object and its surface reflectivity and texture characteristics.

A convenient organization for storing geometric data is to create three lists:

- i. a vertex table
- ii. an edge table
- iii. a polygon table



VERTEX TABLE	EDGE TABLE	POLYGON-SURFACE TABLE
$V_1: x_1, y_1, z_1$	$E_1: V_1, V_2$	$S_1: E_1, E_2, E_3$
$V_2: x_2, y_2, z_2$	$E_2: V_2, V_3$	$S_2: E_3, E_4, E_5, E_8$
$V_3: x_3, y_3, z_3$	$E_3: V_3, V_1$	
$V_4: x_4, y_4, z_4$	$E_4: V_3, V_4$	
$V_5: x_5, y_5, z_5$	$E_5: V_4, V_5$	
	$E_8: V_5, V_1$	

### Plane Equations

The equation for a plane surface can be expressed as

$$Ax + By + Cz + D = 0 \dots\dots(i)$$

Where,  $(x,y,z)$  is any point on the plane- coefficients ABCD are constants describing the spatial properties of the plane

For solving ABCD consider three successive polygon vertices  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$ ,  $(x_3, y_3, z_3)$

So equation (i) is modified to,

$$\frac{A}{D} x_k + \frac{B}{D} y_k + \frac{C}{D} z_k = -1 \dots\dots(ii) \quad k = 1,2,3$$

Using Cramers rule,

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \quad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix} \quad C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad D = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Expanding Determinants,

$$\begin{aligned} A &= y_1 (z_2 - z_3) + y_2 (z_3 - z_1) + y_3 (z_1 - z_2) \\ B &= z_1 (x_2 - x_3) + z_2 (x_3 - x_1) + z_3 (x_1 - x_2) \\ C &= x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \\ D &= -x_1 (y_2 z_3 - y_3 z_2) - x_2 (y_3 z_1 - y_1 z_3) - x_3 (y_1 z_2 - y_2 z_1) \end{aligned}$$

Plane equations are used to identify the position of spatial points relative to the plane surfaces of an object. For any point (x,y,z) not on plane with parameters ABCD we have

$$Ax + By + Cz + D = 0$$

We can identify the point as either inside or outside the plane surface according to the sign( + or -) of

$$Ax + By + Cz + D$$

If  $Ax + By + Cz + D < 0$  then the point (x,y,z) is inside the surface

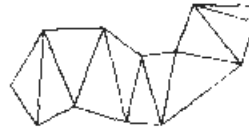
If  $Ax + By + Cz + D > 0$  then the point (x,y,z) is outside the surface

### Polygon Meshes

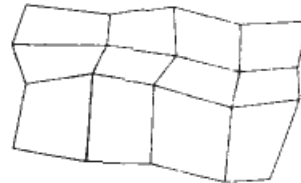
Some graphics packages (PHIGS programmer's Hierarchical Interactive Graphics Standard) provide several polygon functions for modeling objects.

One type of polygon mesh is the triangle strip

It produces  $n - 2$  connected triangles, given the coordinates for  $n$  vertices.



Another similar function is the quadrilateral mesh that generates a mesh of  $(n-1)(m-1)$  quadrilaterals, given the coordinates for an  $n$  by  $m$  array of vertices



When polygons are specified with more than 3 vertices, it is possible that the vertices may not all lie in one plane.

This can be due to numerical error or errors in selecting coordinate positions for the vertices

**Remedies:**

- i. Simply divide the polygons into triangles
- ii. Approximate the plane parameters ABC and project in same plane (i.e. approximate A to yz plane, B to xz plane, C to xy plane etc)

High quality graphics systems typically model objects with polygon meshes and set up a database of geometric and attribute information to facilitate processing of the polygon facets.

**Curved Surfaces and Lines**

Display of 3D curved lines and surfaces can be generated from an input set of mathematical functions (spheres, ellipsoid etc) defining the objects or from a set of user specified data points.

Spline representations are examples of generating curves and surfaces. These are commonly used to design new object shapes, to digitize drawings and to describe animation paths.

Curve fitting methods are also used to display graphs of data values by fitting specified curve functions to the discrete data set, using regression techniques such as the least square methods.