EXAMPLE TO SOLVE QUESTION 1B FROM ASSIGNMENT – CH3

Sally can save \$100 a month currently and has \$1000 in her bank account. She is committed to increasing her savings every month by a small amount. For example, if this extra amount was \$2, her monthly saving would be:

\$100, \$102, \$104, \$106 …

(1) What is the bank balance after N months if the increment is \$Q?
(2) What is the needed increment for her to have a total of \$5000 at the end of 2 years?

Question (1): What is the bank balance after N months if the increment is \$Q?

The savings with increment would be in each month:

- Month 1: 100 + 0Q > 100 + (1-1)Q
- Month 2: 100 + 1Q > 100 + (2-1)Q
- Month 3: 100 + 2Q > 100 + (3-1)Q
- Month 4: 100 + 3Q > 100 + (4-1)Q
- …
- Month N: > 100 + (N-1)Q

Therefore, the equation for the bank balance after N months with increment \$Q would be:

Total = Initial Deposit + Monthly Increments

- Total = 1000 + (100 + 100+Q + 100+2Q + 100+3Q + … + 100+(N-1)Q)
- Total = 1000 + 100N + (Q + 2Q + 3Q + … + (N-1)Q)
- Total = 1000 + 100N + Q(1 + 2 + 3 + … + (N-1))
- Total = 1000 + 100N + Q((N-1) * (N/2))
- **Total = 1000 + 100N + QN(N-1)/2**

Question (2): What is the needed increment for her to have a total of \$5000 at the end of 2 years?

In this question, we have the total (\$5000) and months (2 years = 24 months). We want to find the monthly increment Q. Therefore, we can shift the question from Question (1) to get the answer.

- Total = 1000 + 100N + QN(N-1)/2
- Total – 1000 – 100N = QN(N-1)/2
- QN(N-1) = 2(Total – 1000 – 100N)
- Q = 2(Total – 1000 – 100N) / (N(N-1))
- Q = 2(5000 – 1000 – 100(24)) / (24(24-1))
- Q = 2(4000 – 2400) / (24(23))
- Q = 2(1600) / 552
- Q = 3200 / 552
- Q = 5.79710…

Therefore, for Sally to have a balance of \$5000 at the end of 2 years, she needs to increment the monthly payments by \$5.80.

Assignment for Chapter 3

Question 1a

- Population of the US (total_p) = 330 million > **total_p** = 330
- Number of people already vaccinated (vacc_p) = 32 million > **vacc_p** = 32
- Current vaccinated rate (rate) = 1.2 million per day > **rate** = 1.2

In the program, we will ask the user to provide the target time period information:

- Target time period (period) > **period** will be provided by the user

According to the example, consider the user input 7 weeks (or 49 days) as **period**.

With above information, we can formulate the percentage of population vaccinated (**perc_vac**) by the end of target time period.

- Percentage of population vaccinated = (Total population vaccinated / Total US population) * 100
- Percentage of population vaccinated = ((Number of people already vaccinated + Current vaccination rate * Target time period) / Total US population) * 100
- Formula for Python code: **perc_vac = ((vacc_p + rate * period) / total_p) * 100**

If we insert the number in each variable, we have the following:

- **perc_vac** = ((32 + 1.2 * 49) / 330) * 100= ((32 + 58.8) / 330) * 100 = 90.8 / 330 * 100 = 27.51515151…

Therefore, 27.51% of the US population would be vaccinated at the end of 7-week period

In case we need to find the vaccine rate (needed_rate) in order to reach the herd immunity (80% of US population), all we need is to revise the formula for Python code from above:

- **perc_vac = ((vacc_p + needed_rate*period) / total_p) * 100**
- perc_vac = ((vacc_p + needed_rate*period) / total_p) * 100
- (vacc_p + needed _rate*period) / total_p = perc_vac/100
- vacc_p + needed _rate*period = perc_vac*total_p/100
- needed_rate*period = perc_vac*total_p/100 – vacc_p
- **needed_rate = (perc_vac*total_p/100 – vacc_p) / period**

If we insert the variables perc_vac (80%), total_p (330 million), vacc_p (32 million) and period (7 weeks or 49 days), we have:

- **needed_rate** = (80*330/100 – 32) / 49 = 4.734694….

Therefore, for US population to reach herd immunity (80%), the daily vaccination rate (needed_rate) must be 4.73%

Question 1b

Unlike the previous question 1a, the daily vaccination rate is no longer a constant but increase by a certain percentage every day.

- Population of the US (**total_p**) = 330 million > **total_p** = 330
- Number of people already vaccinated (**vacc_p**) = 32 million > **vacc_p** = 32
- Current vaccinated rate (**rate**) = 1.2 million per day > **rate** = 1.2

In the program, we will ask the user to provide the target time period information:

- Target time period (**period**) > **period** will be provided by the user
- Daily vaccination increase rate (**inc_rate**) > **inc_rate** will be provided by the user

According to the example, consider the user input 7 weeks (or 49 days) as **period** and 10% as **inc_rate**.

Due to the daily vaccination rate no longer being the constant, we cannot use the formula from question 1b but to modify based on the example question (Sally depositing question).

From the formula, what we need to modify is the vaccinated US population:

- Percentage of population vaccinated = **((Number of people already vaccinated + Current vaccination rate * Target time period)** / Total US population) * 100
- When vaccination rate remains constant: **Final vaccinated US population = Number of people already vaccinated + Current vaccinated * Target time period**

Using the sample question from above,

Consider the daily increase rate (**inc_rate**) is 10% (or 0.1), the daily vaccination rate would be:

- Day 1: **rate** + **rate***inc_rate***0 **rate** + **rate***inc_rate***(1-1)
- Day 2: **rate** + **rate***inc_rate** *1 **rate** + **rate***inc_rate** *(2-1)
- Day 3: **rate** + **rate***inc_rate** *2 **rate** + **rate***inc_rate** *(3-1)
- Day 4: **rate** + **rate***inc_rate** *3 **rate** + **rate***inc_rate** *(4-1)
- …
- Day N: **rate** + **rate***inc_rate** *(N-1) **rate** + **rate***inc_rate** *(N-1)

Therefore, the equation for the Final vaccinated US population after N days with increment **rate***inc_rate** million per day would be:

- **Final vaccinated US population = Number of people already vaccinated (vacc_p) + Current vaccinated * Target time period**
- Final = **vacc_p** + (**rate** + (**rate** + **rate***inc_rate**) + (**rate** + **rate***inc_rate***2) + (**rate** + **rate***inc_rate***3) + … + (**rate** + **rate***inc_rate***(**N**-1)))
- Final = **vacc_p** + **rate*N** + (**rate*inc_rate** + **rate*inc_rate***2 + **rate*inc_rate***3 + … + **rate*inc_rate***(**N**-1))
- Final = **vacc_p** + **rate*N** + **rate*inc_rate***(1 + 2 + 3 + … + (**N**-1))
- Final = **vacc_p** + **rate*N** + **rate*inc_rate***((**N**-1) * (**N**/2))
- **Final = vacc_p + rate*N + rate*inc_rate*N(N-1)/2**

Using the above, we can finalize the formula in Python code

- Initial Formula for Python code: **perc_vac = ((vacc_p + rate * period) / total_p) * 100**
- Final Formula for Python code: **perc_vac = ((vacc_p + rate*period + rate*inc_rate*period*(period-1)/2)/total_p * 100**

Using the above variables, we have

- perc_vac = (32 + 1.2*49 + 1.2*0.1*49*(49-1)/2) / 330 * 100
- perc_vac = (32 + 58.8 + 2.94*(49-1)) / 330 * 100
- perc_vac = (90.8 + 2.94*48) / 330 * 100
- perc_vac = (90.8 + 141.12) / 330 * 100
- perc_vac = 231.92 / 330 * 100
- **perc_vac** = 70.27878787878…

Therefore, the total percentage of vaccinated US population (perc_vac) at the end of 7 weeks with the initial vaccination rate of 1.2 million per day and daily vaccination increase rate of 10% is 70.28%.

In case we need to find the daily increase vaccination rate (needed_inc_rate) in order to reach the herd immunity (80% of US population), all we need is to revise the formula for Python code from above:

- **perc_vac = ((vacc_p + rate*period + rate*needed_inc_rate*period*(period-1)/2)/total_p * 100**
- vacc_p + rate*period + rate*needed_inc_rate*period*(period-1)/2 = perc_vac*total_p/100
- rate*needed_inc_rate*period*(period-1)/2 = perc_vac*total_p/100 – vacc_p – rate*period
- **needed_inc_rate = 2 * (perc_vac*total_p/100 – vacc_p – rate*period) / (rate*period*(period-1))**

If we insert the variables perc_vac (80%), total_p (330 million), vacc_p (32 million), rate (1.2 million per day) and period (7 weeks or 49 days), we have:

- **needed_inc_rate** = 2 * (80*330/100 – 32 – 1.2*49) / (1.2*49*(49-1))
- **needed_inc_rate** = 2 * (264 – 32 – 58.8) / (1.2*49*48)
- **needed_inc_rate** = 2 * 173.2 / 2882.4
- **needed_inc_rate = 0.122732…**

Therefore, for US population to reach herd immunity (80%), the daily increase rate (needed_inc_rate) must be 12.27%