Exercise: Lightest and Heaviest Stone

1/1 Points

2/4/2024

Attempt 1

Review Feedback
2/1/2024

Attempt 1 Score:
1/1

View Feedback

Anonymous Grading: **no**

Unlimited Attempts Allowed
1/19/2024 to 2/4/2024

Details

You are given $n \geq 2$ stones each having a distinct weight. You are also given a two-pan balance scale. Given two stones, the scale can determine which stone is *heavier* and which is *lighter*, but cannot determine the actual weight of the stones. Give a divide and conquer algorithm that uses only $\frac{3n}{2} - 2$ weighings to find the heaviest and lightest stones. Prove by induction that it uses $\frac{3n}{2} - 2$ weighings.

You may assume that n is a power of 2.

View Rubric

Lightest and Heaviest Stone

Criteria	Ratings		Pts
Correct DC algorithm (no points for iterative)	**0.5 pts** **Full Marks**	**0 pts** **No Marks**	/ 0.5 pts
Valid proof of number of weighings	**0.5 pts** **Full Marks**	**0 pts** **No Marks**	/ 0.5 pts
			Total Points: 0

Recursively find the lightest and heaviest stone:

For stones 0, 1, and 2, the solution is trivial. For stones greater than 2 (n $>$ 2), we will split the array in half $\left(\frac{n}{2}\right)$ and recursively find the heaviest and lightest stones from each subsets. We can call the heaviest and lightest stones from each subset as h_1, h_2, l_1, l_2 respectively. Then, we will compare which ones are the heaviest between h_1 and h_2. Same with picking the lightest stones between l_1 and l_2.

Let $T(n) =$ number of weights of n stones

Base case: n=2

$T(2) = \frac{3(2)}{2} - 2 = 3 - 2 = 1$

$T(2) = 1 \qquad T(n) = 2T\left(\frac{n}{2}\right) + 2$

By induction, we want to show that $T(n) = \frac{3n}{2} - 2$

Assume that $T(k) = \frac{3k}{2} - 2$ for $n > k \geq 2$

$$T(n) = 2T\left(\frac{n}{2}\right) + 2$$

$$= 2\left(\frac{3\left(\frac{n}{2}\right)}{2} - 2\right) + 2 = 2\left(\frac{3n}{4} - 2\right) + 2 = \frac{3n}{2} - 2.$$

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