University of Houston

Midterm 3 Review

COSC 3320
Algorithms and Data Structures

Note

Read the Academic Honesty policy.

Exercise 1: Graph Traversal (20 Points)

Consider the graph G given below.

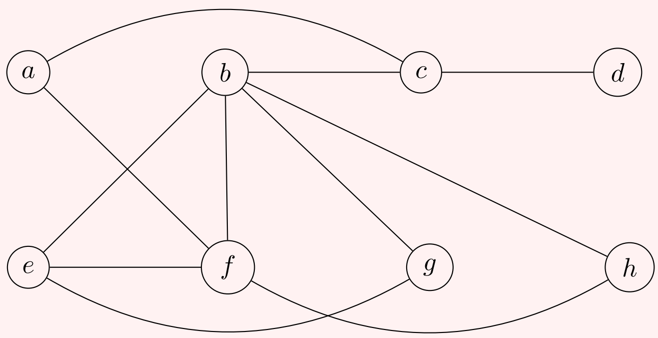

Figure 1: Undirected graph G.

1. Draw the DFS tree of G starting from vertex a. Label the nodes with their DFS numberings and `low` numberings. Draw back edges as dotted lines. Break ties alphabetically.
2. Draw the BFS tree of G starting from vertex a. Draw non-tree edges as dotted lines. Break ties alphabetically.
3. List the cut vertices of G.
4. List the biconnected components of G.

Exercise 2: Cut Edges (20 Points)

1. A cut-edge of a graph $G = (V, E)$ (also called a bridge) is an edge whose removal paritions increases the number of components of G. Give an $\mathcal{O}(m + n)$ time algorithm to find all the cut-edges of G. Give pseudocode.
2. Argue the correctness and runtime of your algorithm.
3. Run your algorithm on Figure 1. Show the main steps and output the cut edges of the graph.

Exercise 3: Cycle Finding (20 Points)

Given an undirected connected graph $G = (V, E)$ with n vertices and m edges:

1. Give an algorithm based on DFS that outputs a cycle in the graph if it contains one.
2. A cycle is *minimal* if no proper subset of its vertices forms a cycle. Assuming that the graph contains a cycle, give an algorithm that finds a minimal cycle in the graph.

Prove correctness and analyze the running time of your algorithms. Note that your algorithms should run in $O(m + n)$ time.

Exercise 4: Strongly Connected Components (20 Points)

Consider the directed graph G given below.

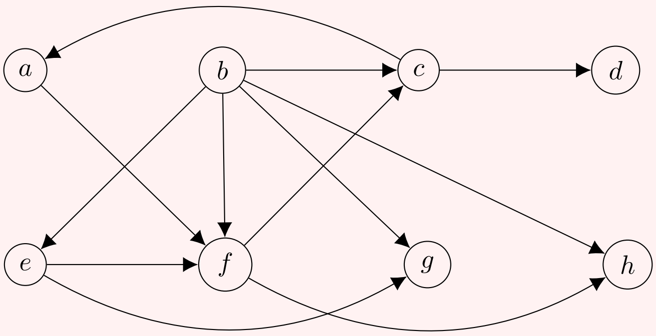

Figure 2: Digraph G.

1. Draw the DFS tree of G starting from vertex a. Label the nodes with their DFS numberings. Draw back-edges, forward-edges, and cross-edges as dotted lines. Break ties alphabetically.

2. Draw the *component* graph, i.e., the graph where nodes are the strongly connected components of G and there is an edge from component S_1 to S_2 if there exists an edge (u, v) with $u \in S_1$ and $v \in S_2$.

Exercise 5: Topological Sort (20 Points)

Give a topological ordering of the directed graph G given below.