# Exercise: Motorbike

**2/4/2024**

**1/1** Points

Attempt 3

Review Feedback
**2/4/2024**

Attempt 3 Score:
**1/1**

View Feedback

Anonymous Grading: **no**

**Unlimited Attempts Allowed**
1/19/2024 to 2/4/2024

**Details**

There are $n$ adults in town $A$ and they all need to go to town $B$. There is only a single motorbike available which is owned by two children. The motorbike can be driven by either:

- *exactly* one adult, or
- *exactly* one child, or
- both children (i.e., the children drive together)

In particular, an adult and a child cannot ride together. Give an algorithm to move all adults from town $A$ to town $B$ while, at the end, leaving the motorbike with the two children in town $A$.

Prove the correctness of your algorithm by using mathematical induction. Analyze the number of trips and prove by induction that your algorithm requires that many trips.

▶ Hint

**View Rubric**

**Motorbike**

| Criteria | Ratings | | Pts |
|---|---|---|---|
| Correct procedure | **0.5 pts** **Full Marks** | **0 pts** **No Marks** | / 0.5 pts |
| Proof of Correctness | **0.25 pts** **Full Marks** | **0 pts** **No Marks** | / 0.25 pts |
| Proof of Number of Trips | **0.25 pts** **Full Marks** | **0 pts** **No Marks** | / 0.25 pts |
| | | | Total Points: 0 |

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The procedure is as follows (see below diagram for reference): Both boys take the motorbike to town B. One boy comes back to town A with the motorbike. One adult goes to town B with the motorbike. The one boy who stayed in town B takes the motorbike back to town A.

$b$ = boys
$a$ = adults

$$b\,b \longrightarrow$$

$$\longleftarrow b$$

$a \longrightarrow$

$\longleftarrow b$

Town A B

Proceed by Induction on n

he algorithm base case (n=0) sends all adults with both boys in town A in 4n = 0 steps.

Assume the algorithm correctly sends all adults from town A to town B in 4n trips for some $n \geq 0$

(We want to show this algorithm sends n + 1 adults to town B in 4(n + 1) trips)

For n + 1 adults, the process sends one adult to town B in 4 trips. Now, there are n adults in town A.

By our Inductive Hypothesis, this process sends all n adults to town B in 4n trips.

Altogether, this is $4n + 4 = 4(n + 1)$ trips.

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