Exercise: Motorbike



Attornat 2		Review Feedback
Attempt 3		2/4/2024

Attempt 3 Score:

1/1

Fiv View Feedback

Anonymous Grading: no

Unlimited Attempts Allowed

1/19/2024 to 2/4/2024

∨ Details

There are n adults in town A and they all need to go to town B. There is only a single motorbike available which is owned by two children. The motorbike can be driven by either:

- exactly one adult, or
- exactly one child, or
- both children (i.e., the children drive together)

In particular, an adult and a child cannot ride together. Give an algorithm to move all adults from town A to town B while, at the end, leaving the motorbike with the two children in town A.

Prove the correctness of your algorithm by using mathematical induction. Analyze the number of trips and prove by induction that your algorithm requires that many trips.

► Hint

∨ View Rubric

Motorbike

Criteria	Ratings		Pts
Correct procedure	0.5 pts Full Marks	0 pts No Marks	/ 0.5 pts
Proof of Correctness	0.25 pts Full Marks	0 pts No Marks	/ 0.25 pts
Proof of Number of 0.25 pts Trips Full Marks	•	0 pts No Marks	/ 0.25 pts
			Total Points: 0

The procedure is as follows (see below diagram for reference): Both boys take the motorbike to town B. One boy comes back to town A with the motorbike. One adult goes to town B with the motorbike. The one boy who stayed in town B takes the motorbike back to town A.

$$b = boys$$

$$a = adults$$

$$b\,b \longrightarrow$$

$$\leftarrow$$
 b

$$i \longrightarrow k$$

Town A B

Proceed by Induction on n

he algorithm base case (n=0) sends all adults with both boys in town A in 4n=0 steps.

Assume the algorithm correctly sends all adults from town A to town B in 4n trips for some $n \geq 0$

(We want to show this algorithm sends n + 1 adults to town B in 4(n + 1) trips)

For n + 1 adults, the process sends one adult to town B in 4 trips. Now, there are n adults in town A.

By our Inductive Hypothesis, this process sends all n adults to town B in 4n trips.

Altogether, this is 4n+4=4(n+1) trips.

