Exercise: Big-O

2/4/2024

1/1 Points

Attempt 2

Review Feedback
1/31/2024

Attempt 2 Score:
1/1

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Anonymous Grading: **no**

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1/19/2024 to 2/4/2024

Details

Rank the listed functions by order of growth.

- n^2
- $\frac{n}{\log n}$
- $n \log n$
- $(1.001)^n$
- $\frac{1}{n^2}$
- $\log^{100} n$
- $n!$
- $n^{\lg \lg n}$

That is, find an arrangement f_1, f_2, ..., f_8 of the functions satisfying

$$\begin{aligned} f_1 &= \mathcal{O}(f_2) \\ f_2 &= \mathcal{O}(f_3) \\ &\vdots \\ f_7 &= \mathcal{O}(f_8) \end{aligned}$$

Justify your ordering.

> Note that $\log^k n$ is the usual way of writing $(\log n)^k$ and that $\log n = \log_2 n$.

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Figurine

Criteria	Ratings		Pts
Correct ordering	**0.3 pts** **Full Marks**	**0 pts** **No Marks**	0.3 / 0.3 pts
Correct justification (0.1 points each)	**0.7 to >0 pts** **Full Marks**	**0 pts** **No Marks**	0.7 / 0.7 pts

Figurine

Criteria	Ratings	Pts
		Total Points: 1

$\frac{1}{n^2}, \log^{100} n, \frac{n}{\log n}, n \log n, n^2, n^{\log(\log n)}, (1.001)^n, n!$

1.

() is decreasing and () is increasing. Therefore, $(\frac{1}{n^2})$ is O$(\log^{100} n)$.

2.

$\lim_{n\to\infty} \frac{(\log^{100} n)}{\left(\frac{n}{\log n}\right)} = \lim_{n\to\infty} \frac{\log^{101} n}{n} = 0$ by theorem showed in class.

Therefore, $(\log^{100} n)$ is O$(\frac{n}{\log n})$.

3.

$\lim_{n\to\infty} \frac{\left(\frac{n}{\log n}\right)}{n \log n} = \lim_{n\to\infty} \frac{n}{n \log^2 n} = \lim_{n\to\infty} \frac{1}{\log^2 n} = 0$

Therefore, $(\frac{n}{\log n})$ is O$(n \log n)$.

4.

$\lim_{n\to\infty} \frac{n \log n}{n^2} = \lim_{n\to\infty} \frac{\log n}{n} = 0$

Therefore, $(n \log n)$ is O(n^2).

5.

$\lim_{n\to\infty} \frac{n^2}{n^{\log(\log n)}} = \lim_{n\to\infty} \frac{1}{n^{\log(\log n)-2}} = 0$

Therefore, (n^2) is O$(n^{\log(\log n)})$.

6.

$\lim_{n\to\infty} \frac{n^{\log(\log n)}}{(1.001)^n} = \lim_{n\to\infty} \frac{\log\left(n^{\log(\log n)}\right)}{\log(1.001^n)} = \lim_{n\to\infty} \frac{\log(\log n)\cdot\log n}{n\cdot\log(1.001)} = 0$ by logpol.

Therefore, $(n^{\log(\log n)})$ is O$((1.001)^n)$

7.

Proceed by mathematical induction on n.

When n = 2, $(1.001)^2 \leq 2!$

Assume when n = k, $(1.001)^k \leq k!$

(We want to show this is true for n = k + 1, where $(1.001)^{k+1} \leq (k+1)!$)

Based on the inductive hypothesis ($(1.001)^n \leq n!$) , it follows

$(1.001)^{k+1} = 1.001^k \cdot 1.001$

$1.001 \cdot k!$ by inductive hypothesis

$(k+1) \cdot k!$ since n 0.001

$= (k+1)!$

Therefore, $(1.001)^n$ is O$(n!)$

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