Linear Algebra
Assignment 6 MATH 2318 (Fall 2022)

Deadline: Friday October 14th, 11:59pm.

Policy to turn in assignment:

- Assignment should be submitted via BlackBoard.
- Student needs to turn in their assignment as a single PDF file.
- No email or late submission will be accepted.

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4 points

1. Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$. Construct a 2×2 nonzero matrix B such that AB is the zero matrix. *Hint*: Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Compute AB and make it equal to the zero matrix.

5 points

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$.

   a) Find A^{-1} using the algorithm we saw in class.

   b) Use A^{-1} to solve the linear system $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix}$.

4 points

3. Let A and B be $n \times n$ invertible matrices such that

$$(A^T + I_n)^{-1} = (BA^{-1})^T.$$

   Find A^{-1}. *Note*: Your formula for A^{-1} should not depend on A or A^T. *Hint*: Try applying inverse and transpose on both sides. Then, manipulate the equation algebraically to obtain a matrix C such that $AC = I_n$.

3 points

4. For each of the following, determine if the statement is true or false. Provide a short reasoning (one or two sentences).

   a) Let A be an $n \times n$ matrix, and $\vec{b}$ be a vector in $\mathbb{R}^n$. If the system $A\vec{x} = \vec{b}$ has infinitely many solutions, then A is not invertible.

   b) Let A be an invertible square matrix. If $AB = AC$, then $B = C$.

   c) If A and B are square matrices such that $AB = BA$, then $A^{-1} = B$.