Exam 2 Practice — Linear Algebra MATH 2318 (Fall 2022)

1. Consider the following linear system

$$\begin{array}{rcrcrcrcr} x_1 & & & - & x_3 & + & x_4 & = & 3 \\ 2x_1 & - & x_2 & - & x_3 & & & = & -2 \\ -x_1 & + & x_2 & + & x_3 & + & x_4 & = & 4 \\ & & x_2 & + & x_3 & + & x_4 & = & -1 \end{array}$$

   a) Write the system as a matrix equation $A\vec{x} = \vec{b}$.
   b) Compute A^{-1} using any method from class.
   c) Use A^{-1} to solve the system.

2. Compute the determinant of $A = \begin{bmatrix} 3 & -1 & 2 & 1 \\ 4 & 3 & 0 & -2 \\ -1 & 0 & 2 & 3 \\ 6 & 2 & 5 & 2 \end{bmatrix}$ using any method from class.

3. Let A, B and C be $n \times n$ matrices such that $\det A = 2$, $\det B = -1$ and $\det C = 3$. Find $\det\left(2(A^{-1})^2 B^T C^3\right)$.

4. Let $A = \begin{bmatrix} a+1 & 0 \\ 1 & -1 \end{bmatrix}$. Find all the values of a so that the matrix $A^2 + 3A$ is singular.

5. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$. It is known that six of the cofactors of A are:

$$C_{11} = 1, \quad C_{12} = -1, \quad C_{13} = 1, \quad C_{21} = -1, \quad C_{22} = 1, \quad C_{23} = 1.$$

   a) Compute $\operatorname{adj}(A)$.
   b) Compute $\det A$.
   c) Find A^{-1}.

6. For each of the following, determine if the statement is true or false. Provide a short reasoning (one or two sentences).

   a) Cramer's rule can be applied to any kind of linear system.
   b) If $\det A \neq 0$, for some $n \times n$ matrix A, then the columns of A are linearly independent.
   c) If a square matrix has two identical columns, then its determinant is zero.
   d) If a square matrix A is not invertible, then the system $A\vec{x} = \vec{b}$ is inconsistent for all $\vec{b}$.
   e) If A and B are invertible, then so is $A + B$.
   f) If A and B are any two matrices such that $AB = I_n$, then A and B are both invertible.