Linear Algebra
Assignment 11 MATH 2318 (Fall 2022)

Deadline: Monday November 28th, 11:59pm.

Policy to turn in assignment:

- Assignment should be submitted via BlackBoard.
- Student needs to turn in their assignment as a single PDF file.
- No email or late submission will be accepted.
- Only three of the five problems will be graded.

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7 points

1. Let $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$. The eigenvalues of A are $\lambda_1 = 2$ with $a_{\lambda_1} = 2$, and $\lambda_2 = 1$, with $a_{\lambda_2} = 1$.

   a) Find a basis for the corresponding eigenspaces.

   b) Is A diagonalizable? Justify your answer. If it is diagonalizable, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

   c) Give a formula for A^k, for any positive integer k.

6 points

2. Let $\vec{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$.

   a) Show that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$.

   b) Write $\vec{v} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$.

4 points

3. Find the closest point to $\vec{v} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$ in $H = \text{Span}\{\vec{u}\}$, where $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$. What is the shortest distance from $\vec{v}$ to H?