1. Solve the following linear system

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 4 \\ 2x_1 & - & 3x_2 & - & x_3 & + & 4x_4 & = & 7 \\ -2x_1 & + & 4x_2 & + & x_3 & - & 2x_4 & = & 1 \\ 5x_1 & - & x_2 & + & 2x_3 & + & x_4 & = & -1 \end{array}$$

2. Given the reduced echelon form of an augmented matrix of a linear system, identify the pivots, basic variables, free variables (if any) and write the solution of the linear system in parametric form.

$$\left[\begin{array}{cccc|r} 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 1 & 2 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

3. Let $A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ -2 & -6 & -1 & 0 \\ 1 & 3 & 2 & 3 \end{bmatrix}$.

   a) Solve the homogeneous system $A\vec{x} = \vec{0}$.

   b) **Without performing any row operations**, solve the system $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{bmatrix} -4 \\ 10 \\ -2 \end{bmatrix}$.

4. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ h \\ 4 \end{bmatrix}$. Find the values of h so that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

5. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$, and define $T : \mathbb{R}^3 \to \mathbb{R}^3$ as $T(\vec{x}) = A\vec{x}$, for all $\vec{x} \in \mathbb{R}^3$. Find an $\vec{x}$ in $\mathbb{R}^3$ whose image under T is $\vec{b} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$. Is there more than one $\vec{x}$ in $\mathbb{R}^3$ whose image under T is $\vec{b}$?

6. Let $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \vec{y} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \vec{z} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

   a) Write $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ as a linear combination of $\vec{x}, \vec{y}, \vec{z}$.

   b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(\vec{x}) = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}, T(\vec{y}) = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix}, T(\vec{z}) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. Find $T(\vec{b})$.

7. For each of the following, determine if the statement is true or false. Provide a short reasoning (one or two sentences).

   a) If the equation $A\vec{x} = \vec{0}$ only has the trivial solution, then the equation $A\vec{x} = \vec{b}$ has a unique solution for any $\vec{b}$.

   b) Let T be a linear transformation. If $T(\vec{x}) = T(\vec{y})$, then $\vec{x} = \vec{y}$.

   c) Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$ be vectors in $\mathbb{R}^n$. If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent, then each of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ can be expressed as a linear combination of the other two.

   d) If $\vec{x}$ is in Span$\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$, then $\vec{x}$ is in Span$\{\vec{v_1}, \vec{v_2}, t\vec{v_3}\}$, where t is any real number.