Linear Algebra

Assignment 10 MATH 2318 (Fall 2022)

Deadline: Saturday November 19th, 11:59pm.

Policy to turn in assignment:

- Assignment should be submitted via BlackBoard.
- Student needs to turn in their assignment as a single PDF file.
- No email or late submission will be accepted.

$\boxed{\text{4 points}}$ 1. Show that

$$B = \left\{ \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

is a basis for $M_{2\times 2}$. *Hint*: Recall that $\dim(M_{2\times 2}) = 4$, and note that B contains four matrices.

$\boxed{\text{9 points}}$ 2. Let $A = \begin{bmatrix} -4 & 0 & 2 \\ 2 & 4 & -8 \\ 2 & 0 & -4 \end{bmatrix}$.

a) Find the characteristic polynomial of A.

b) Find the eigenvalues of A and state their algebraic multiplicity.

c) Find a basis for each eigenspace of A.

$\boxed{\text{4 points}}$ 3. Let A be an $n \times n$ invertible matrix. Show that if λ is an eigenvalue of A with associated eigenvector $\vec{v}$, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1} with associated eigenvector $\vec{v}$. *Hint*: Consider $A\vec{v} = \lambda\vec{v}$ and multiply by A^{-1}.

$\boxed{\text{3 points}}$ 4. For each of the following, determine if the statement is true or false. Provide a short reasoning (one or two sentences).

a) If $\vec{v}$ is an eigenvector of a square matrix A with corresponding eigenvalue λ, then $2\vec{v}$ is also an eigenvector of a square matrix A with corresponding eigenvalue λ.

b) If the characteristic polynomial of a square matrix has a nonzero constant term, then the matrix is invertible.

c) If an eigenvalue of a square matrix has algebraic multiplicity 1, then the dimension of the associated eigenspace is 1.