1. Show that the set of polynomials in $\mathbb{P}_n(\mathbb{R})$ that satisfy $p(0) = 0$ is a subspace of $\mathbb{P}_n(\mathbb{R})$.

2. Show that the set $H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2} : c = b \right\}$ is a subspace of $M_{2\times 2}$.

3. Find a basis for Col$(A)$, Null$(A)$ and Row$(A)$. What is rank$A$?
$$A = \begin{bmatrix} 1 & 1 & -1 & -2 \\ -1 & -2 & 1 & 3 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

4. Show that $B = \{1+x^2, x+x^2, 1+2x+x^2\}$ is a basis for $\mathbb{P}_2$. *Hint*: Recall that $\dim(\mathbb{P}_2) = 3$ and note that $B$ has three vectors.

5. Let $\vec{v}$ be an eigenvector of $A \in M_{n\times n}$ with associated eigenvalue $\lambda$. Show that $k\vec{v}$, with $k$ a nonzero scalar, is an eigenvector of $A$ with associated eigenvalue $\lambda$.

6. Let $\vec{v}$ be an eigenvector of $A \in M_{n\times n}$ with associated eigenvalue $\lambda$. Show that $\vec{v}$ is also an eigenvector of $A^2$ and find the associated eigenvalue. *Hint*: Consider $A\vec{v} = \lambda\vec{v}$ and multiply by $A$.

7. Let $A = \begin{bmatrix} 6 & -3 & -3 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.

   a) Find the eigenvalues of $A$.
   b) Find a basis for each eigenspace of $A$.
   c) Is $A$ diagonalizable? Explain. If it is, diagonalize it and find a formula for $A^k$.

8. Find the shortest distance from $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$ to the line spanned by $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.

9. Let $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

   a) Show that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$.
   b) Write $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ as a linear combination of $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$.

10. Find the closest point to $\vec{v} = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$ in $H = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix} \right\}$. What is the shortest distance from $\vec{v}$ to $H$?

11. Let $\{\vec{u}, \vec{v}\}$ be an orthogonal set of vectors in $\mathbb{R}^n$, and let $\vec{x} \in \text{Span}\{\vec{u}\}$ and $\vec{y} \in \text{Span}\{\vec{v}\}$. Prove that $\vec{x}$ and $\vec{y}$ are orthogonal.

12. For each of the following, determine if the statement is true or false. Provide a short reasoning (one or two sentences).

    a) If $A$ is singular, then $A$ is not diagonalizable.

    b) If $A$ is invertible, then $A$ is diagonalizable.

    c) Let $A \in M_{n\times n}$. If $\dim(\text{Null}(A)) = 1$, then $A$ is not invertible.

    d) Any linearly independent set is an orthogonal set.

    e) Any orthogonal set of vectors is a linearly independent set.

    f) Let $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$ be an orthogonal set of vectors in $\mathbb{R}^n$, and let $t_1, t_2, \ldots, t_p$ be real numbers. The set $\{t_1\vec{v}_1, t_2\vec{v}_2, \ldots, t_p\vec{v}_p\}$ is orthogonal.