Math 3336 : Test 1 Review

Instructions

- This is not an assignment. Neither work nor answers are to be submitted.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.

---

1. Write down

(a) an example of a true statement.

(b) an example of a false statement.

(c) an example of an open sentence.

(d) an example of a non-statement (that is also *not* an open sentence).

---

2. suppose we have two finite sets, A and B, that satisfy

$$|A| = 5 \text{ and } |B| = 7$$

(a) Is it possible for $|A \cup B| = 5$?

(b) Is it possible for $|A \cup B| = 12$?

(c) Is it possible for $|A \cup B| = 4$?

(d) Compute $|\mathcal{P}(A)|$

(e) Compute $|A \times B|$

(f) Compute $|\mathcal{P}(A) \times B|$

(g) Is it possible for $|A \cap B| = 0$?

3. Suppose P, Q and R are statements. Is it possible that the following is true?

$$\begin{aligned} (P \Rightarrow Q) \iff R &\text{ is true} \\ P \vee R &\text{ is false} \end{aligned}$$

4. Fill in the following truth-tables:

P	Q	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \iff Q$	$P \oplus Q$

5. Is $\neg(P \wedge Q) \vee P = \neg P \vee (\neg Q \vee P)$? Explain your answer using a truth table.

6. Determine which, if any, of the following statements is true when using the universal set $U = \{2n : n \in \mathbb{Z}\}$.

(a) $\exists!\, x \in U,\ x^4 = 16.$

(b) $\forall\, t \in U,\ 3t$ is odd.

(c) $\exists\, y \in U,\ y^3 = 27.$

(d) $\forall\, s \in U,\ \cos(\pi s) = 1.$

(e) None of the other statements are true.

---

7. Write down a non-empty universal set, U, that makes the following statement true:

$$\forall\, m \in U,\ 3|m \ \wedge\ 2|m.$$

---

8. Consider the following claim:

$$\forall\, (x, y) \in \mathbb{R}^2,\ \exists\, (u, v) \in \mathbb{R}^2,\ (x, y) + (u, v) = (x, y).$$

If one wanted to prove this using the Contrapositive method, then what could be used as the first line of the proof? What could be used as the last line?

9. A sequence of real numbers, $\{a_n\}$, satisfies the recurrence equation and initial condition

$$a_n = a_{n-1} + 5 \text{ and } a_0 = 3.$$

Write the first four terms of this sequence. Do *any* of the terms in the sequence equal 33? If so, which one(s)?

10. Consider the statement

$$P : \exists\, q \in \mathbb{Q}, \forall p \in \mathbb{Z},\ q \cdot p = 1.$$

Use logical symbols to write $\neg P$. Which statement is true, P or $\neg P$? (You do not need to write a proof.)

11. Consider the floor function $f(x) = \lfloor x \rfloor$ with domain and co-domain equal to $\mathbb{R}$.

(a) Where does f send the input $x = 5/3$?

(b) Is there an input $a \in \mathbb{R}$ that satisfies $f(a) = 5/3$? If so, identify the element(s) a that make this true. If not, explain why.

---

12. (HW 2 repeat) A new logical operator, $\blacksquare$, is partially defined by the following truth table information:

P	Q	$P \blacksquare Q$	$\neg(P \blacksquare Q) \wedge P$	$\neg(P \blacksquare Q) \vee Q$
T	T	T		
T	F		F	
F	T	F		
F	F			T

Complete this truth table. Is $P \blacksquare Q$ a contradiction? A tautology? Neither?

---

13. Write a proof of the following proposition:

Proposition. Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.