Introduction To Sets

a **set**, S, is a collection of objects or things. the objects are called **elements**.

ex 1) S = the set of all students in this class

ex 2) A = the set of all possible dreams a human can have

notation $x \in S$ "x is an element of S"

Sarah $\in S$ from ex 1

more notation often times sets are described in terms of what they contain using "$\{ , \}$"

ex 3) $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ "natural numbers"

$5 \in \mathbb{N}$, $2022 \in \mathbb{N}$, $-5 \notin \mathbb{N}$

we say two sets, A and B, are <u>equal</u> if they have the same elements.

<u>notation</u> $A = B$

<u>ex 4</u> $A = \{\text{even natural numbers}\}$

$B = \{2n : n \in \mathbb{N}\}$

"such that"

(note: set B's elements are described in terms of a condition; the elements are not listed!)

$C = \{2, 4, 6, 8, 10, \ldots\}$

$A = B$, $B = C$, $A = C$ <u>all three are equal sets!</u>

<u>ex 5</u> $T = \{1\}$, $X = \{1, 2\}$

$T \neq X$

sets can have as elements weird things!

<u>ex 6</u> $D = \{1\}$, $E = \{\{1\}\}$

$1 \in D$ ✓ $\qquad 1 \notin E$ ✓

$\{1\} \in E$ ✓

$D \in E$

its useful to think of sets as "boxes" -- and you can have boxes that contain other boxes as elements! (we don't focus on these too much)

ex 7) $S = \{2, 3, 5, 7, 11, 13, 17, 19, 23, \mathbb{N}, \alpha, ⌂\}$

$2 \in S$

$3 \in S$

$\alpha \in S$

$⌂ \in S$

$\mathbb{N} \in S$

✓

$10 \in S$ ✗ false

S has infinitely many elements ✗

↳ S has 12 elements

the cardinality or size of a set S is the number of elements in S

notation: $|S|$ = cardinality of S

in ex 7, $|S| = 12$

note $|\mathbb{N}|$ is infinite

ex 8) the empty set, $\emptyset$

$\emptyset = \{ \quad \}$

= the set that contains no elements!

$|\emptyset| = 0$

consider $\{\emptyset\}$. Is this also empty?

If not, what is its size?

Some familiar sets from math courses

$\emptyset = \{\ \}$ empty set

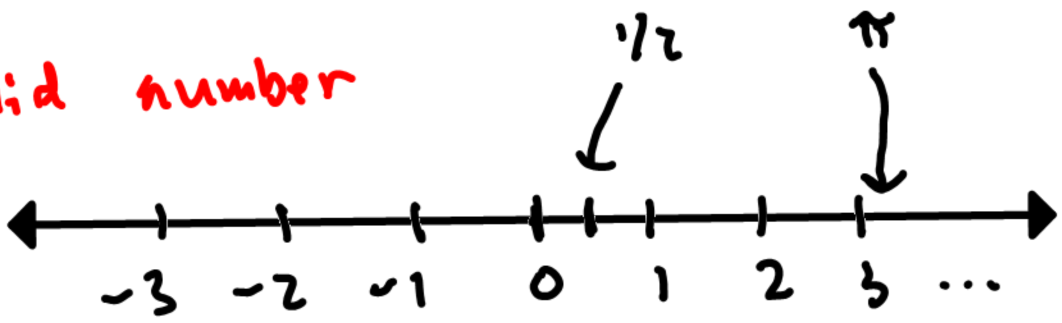
$\mathbb{N} = \{1, 2, 3, 4, \dots\}$ natural numbers

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ integers

$\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ rationals

$\mathbb{R} = \{\text{real numbers}\}$ real numbers

usually visualize this as a solid number line



other mathematical sets

$$M_2(\mathbb{R}) = \{\text{all } 2\times 2 \text{ matrices w/ real entries}\}$$

$$= \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

$P_n(\mathbb{R}) = \{$all degree n poly's w/ real coefficients$\}$

$7x^3 + 5x^2 - 1 \in P_3(\mathbb{R})$

All of modern math. is based on sets!

A rigorous theory of sets is actually harder / more technical than our version.

("Naive Set Theory" is a good introduction)