3336

Office Hour

11:05

Unmute to ask questions!

4. Explain why / prove the following:

If $f(n)$ is $O(\log_a(n))$ then $f(n)$ is also $O(\log_b(n))$

where a and b are real numbers larger than 1.

(Hint: you may want to use this old algebraic fact: $\boxed{\log_b x = (\log_a x)/(\log_a b)}$)

$\exists\ K, M,\ \forall n \geq K,\ |f(n)| \leq M \cdot \log_a n$

We want to show

$\exists\ K', M',\ \forall n \geq K',\ |f(n)| \leq M' \log_b(n)$

As proof-writers, which variables can we choose or substitute values for?

Because we want to show f is $O(\log_b(n))$, we need to choose / sub in values for K' and M'

The hint helps us choose $M' = \ldots$

want: $|f(n)| \leq M' \cdot \log_b(n)$

$= M' \cdot \log_a(n) / \log_a(b)$

choose $M' = M \cdot \log_a(b)$

$\underbrace{|f(n)| \leq M \cdot \log_a(n)}_{\text{if } n > K = K'} \cdot \cancel{\log_a(b) / \log_a(b)}$

1. B

look at some graphs to see if you think its TRUE or FALSE.

If you think its TRUE, then you use the definition of "big O".

If you think its FALSE, you can show the negated-definition holds.

$f(n)$ is $O(g(n))$ means

$$\exists M>0, \exists k>0, \forall n>k, |f(n)| < M \cdot g(n)$$

negation: $f(n)$ is NOT $O(g(n))$

$$\forall M>0, \forall k>0, \exists n>k, \underbrace{|f(n)| \geq M \cdot g(n)}_{2^{n+1} \geq M \cdot 2^n}$$

(Part B) Consider the claim that "2^{n+1} is $O(2^n)$." Is this true? if so, find "witnesses" k and C to explain why it is true. Is this claim false? If so, explain why.

A mathematician used the division algorithm to divide the number 9 by another number b. Their computation resulted in the facts that the quotient $q = 5$ and the remainder $r = -11$. Determine the value of b.

Our remainders are > 0

a) There must have been a mistake, as there is no value of b that makes this possible.

right answer

$0 \leq r < |b|$

b) $b = 36$

c) $b = 9/4$

d) $b = 4$

e) $b = 34$

$$9 = 5 \cdot b - 11$$

$$b = 4 \checkmark$$