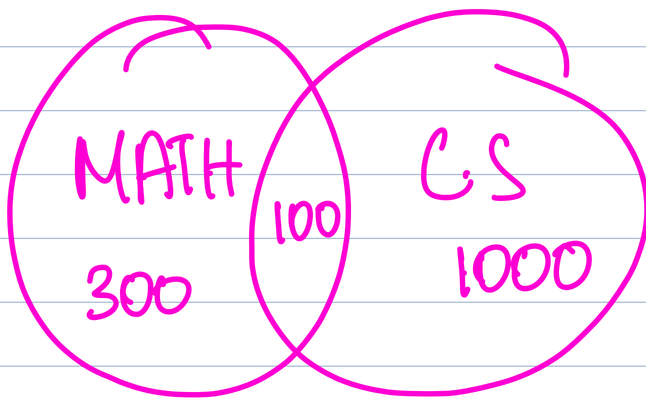
If you have 50,000 students;
300 of them are math majors, 1000 of them are C.S majors and 100 of them are double majors (both C.S & math).
How many students are neither math or C.S majors?

MATH OR C.S = 1000 + 300 = 1300

MATH AND C.S = 100

= 1300 − 100 = 1200



= total # of students ?

Total − 1200 = 50000 − 1200 = $\boxed{48800}$

Find the GCD of 270 and 192 using the Euclidean algorithm: → memorize the definition!

GCD (A, B) and if A = 0 then GCD(A, B) = B

$270 = 192(q) + r$

$270 = 192(1) + 78$

$192 = 78(q) + r$

$192 = 78(2) + 36$

$78 = 36(q) + r$

$78 = 36(2) + 6$

$36 = 6(q) + r$

$36 = 6(6) + 0$

$\frac{270}{192}$

$\begin{array}{r} 270 \\ -192 \\ \hline 78 \end{array}$

$\begin{array}{r} 78 \\ 2 \\ \hline 156 \end{array}$

$\begin{array}{r} 192 \\ -156 \\ \hline 36 \end{array}$

GCD(270, 192) = 6

(20 pts)

** (b) Use your answer from above to verify bezout's identity for the integers 270 and 192.

Suppose that the congruence equation $4x \equiv b \bmod 8$ has a solution $x=2$. WOTF statements about b is true?

1. It is possible that $b=-39$
2. There are no values for b that allow for the solution $x=2$
3. It's possible $b=5$
4. It's possible $b=3$
5. NOTA ✓

Use induction to prove $4^n - 1$ is divisible by 3. $n \in \mathbb{N}$
$n=1$

Base case: $n=1$

$4^{(1)}-1 = 3 = 3(1)$, hence $4^{(1)}-1$ is divisible by 3

Inductive case:

~~**~~ let $A = \{n \in \mathbb{N} : 4^n - 1 \text{ is divisible by } 3\}$

let $n \in A$, so there exists $k \in \mathbb{N}$ such that

$4^n - 1 = 3k$; $4^n = 3k+1$

Hence

$$\begin{aligned} 4^{(n+1)} - 1 &= 4^n \cdot 4^1 - 1 = 4 \cdot (3k+1) - 1 \\ &= 12k + 4 - 1 \\ &= 12k + 3 \\ &= 3(4k+1) \end{aligned}$$

Since A is closed under multiplication/addition, $(4k+1) \in \mathbb{N}$ and hence $4^{n+1}-1$ is divisibly by 3!

$$4^{(n+1)} - 1 = 3(4k+1)$$

1, 2, 4, 8

$1 \times 2 = 2$

$1 \times 4 = 4$

$2 \times 4 = 8$

Use induction to prove if 8 divides $5^{2n}-1$, $n \in \mathbb{N}, n=1$

Base case: $(n=1)$

$5^{2(1)}-1 = 5^2-1 = 25-1 = 24 = 8(3)$

hence $5^{2(1)}-1$ is divisible by 8

inductive case $(n+1)$

let $A = \{n \in \mathbb{N} ; 8 \text{ divides } 5^{2n}-1\}$

Assume there exists a $k \in A$ such that

$5^{2n}-1 = 8k \rightsquigarrow 5^{2n} = 8k+1$

Hence,

$$5^{2(n+1)}-1 = 5^{2n} \cdot 5^2 - 1 = 5^2(8k+1)-1$$
$$= 25(8k+1)-1$$
$$= 200k+25-1$$
$$= 200k+24$$
$$= 8(25k+3)$$

So $5^{2(n+1)}-1 = 8(25k+3)$ and therefore $5^{2(n+1)}-1$ is divisible by 8.

Consider the recursively defined set $S \subseteq \mathbb{Z}$ given by two rules

1. $40, 80 \in S$
2. $x, y \in S \rightarrow ux + vy \in S$, where $u, v \in \mathbb{Z}$

If you want to prove $S \subseteq \{40m : m \in \mathbb{Z}\}$ using structural induction. Which of the following would be recursive step

① Suppose $x, y \in \mathbb{Z}$ and they are multiples of 40 and let $u, v \in \mathbb{Z}$ be arbitrary. Show that $ux + vy$ is also a multiple of 40

② No structural induction

③ Suppose $x, y \in \mathbb{Z}$ and multiples of 40 and let $u, v \in \mathbb{Z}$ be arbitrary. Show $x+1$ and $y+1$ are both multiples of 40.

④ NOTA

Revise structural induction

Suppose the following inequality:

$$(n+1)! > 2^{n+3} \quad \text{for all } n \geq 5$$

which of the following would describe the inductive step.

(1) let $k \in N$ such that $(k+1)! > 2^{k+3}$

(2) Prove for $n = 0$

(3) Suppose $k \in N$ and assume $(k+1)! > 2^{k+3}$ holds. Use this hypothesis to show $((k+1)+1)! > 2^{k+3}$

(4) None of the above

$$((k+1)+1)! > 2^{(k+1)+3}$$

Q#) Assume $q = 6$ and remainder $r = 2$. Use the division algorithm to find the value of b if b divides 16.

a

Division algorithm:

$$a = b \cdot q + r$$

Memorize!

$$16 = b \cdot (6) + (2)$$

$$16 - 2 = 6b$$

$$14 = 6b$$

$$\boxed{b = \frac{14}{6} = \frac{7}{3}}$$