# PRINTABLE VERSION

## Quiz 9

## You scored 100 out of 100

**Question 1**

Your answer is CORRECT.

Use the Inclusion-Exclusion Principle to count how many 2-digit binary strings begin with 1 or end with 1

**a)** ◉ There are $2^1 + 2^1 - 2^0 = 3$ such binary strings.

**b)** ○ There are $2 \cdot 1^2 = 2$ such binary strings.

**c)** ○ There are $2^1 + 2^1 = .4$ such binary strings.

**d)** ○ There are $2^1 - 2^0 = 1$ such binary strings.

**e)** ○ There are $2^1 = 2$ such binary strings.

**Question 2**

Your answer is CORRECT.

Use the Pigeonhole Principle to answer the following question. What is the fewest number of times needed to roll a 6-sided dice so that $14$ or more of the rolls result in the same number? (Obviously you'll need to roll the dice at *least* $14$ times, but that may not be enough to guarantee the desired outcome... or will it!?)

**a)** ○ The dice would need to be rolled $84$ times

**b)** ○ No amount of rolls can guarantee $14$ or more of the rolls will result in the same number.

**c)** ○ The dice would need to be rolled $15$ times

**d)** ○ The dice would need to be rolled $14$ times

**e)** ◉ The dice would need to be rolled $79$ times

**Question 3**

Your answer is CORRECT.

At Squirrel University there are 1000 undergraduate students. 90 of them are math majors, 110 are computer science majors, and 70 are double-majoring in *both* math and CS! How many students are neither math nor

CS majors?

a) There are $910$ non-math and non-CS majors.

b) There are $1000$ non-math and non-CS majors.

c) There are $870$ non-math and non-CS majors.

d) There are $930$ non-math and non-CS majors.

e) There are $800$ non-math and non-CS majors.

**Question 4**

Your answer is CORRECT.

There are 30 different time periods during which classes at our university can be scheduled. If there are 720 different classes, what is the minimum number of different rooms that will be needed? (The Pigeonhole Principle should be useful in answering this question.)

a) A minimum of $30$ rooms will be needed.

b) A minimum of $721$ rooms will be needed.

c) A minimum of $23$ rooms will be needed.

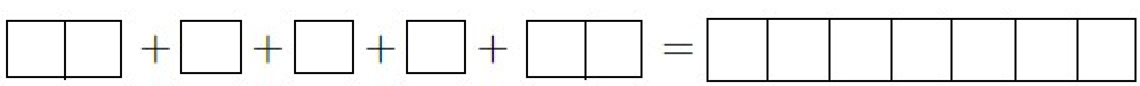
d) A minimum of $25$ rooms will be needed.

e) A minimum of $24$ rooms will be needed.

**Question 5**

Your answer is CORRECT.

A landscape architect is creating a straight path using combinations of $1 \times 1$ square tiles and $1 \times 2$ rectangular tiles. She accomplishes this by lying one tile down and then placing the subsequent tile next to the first one, moving from left to right. An example of this process is shown below where a $1 \times 7$ path is created using two rectangular and three square tiles.



Define the sequence $\{a_n\}$ to count the number of ways our landscape architecht can create a $1 \times n$ path; this means $a_1 =$ the number of ways a $1 \times 1$ path can be constructed, $a_2 =$ the number of ways a $1 \times 2$ path can be constructed, and so on. Make sure you understand why $a_1 = 1$ and $a_2 = 2$ . Explore the sequence $a_n$ some more until you discover a *recursive structure* or *recurrence relation* that can be used to determine the value of $a_9$.

a) $a_9 = 55$

**b)** ○ $a_9 = 89$

**c)** ○ $a_9 = 34$

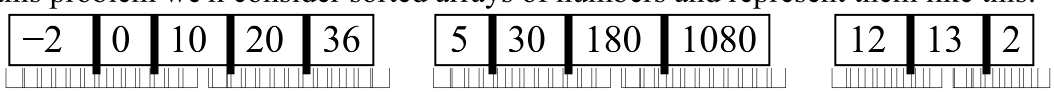
**d)** ○ $a_9 = 21$

**e)** ○ $a_9 = 9$

**Question 6**

Your answer is CORRECT.

A "sorted array" is a term you may have heard about in a Computer Science course, and its the name for a type of object or "data structure" that acts like a finite list or string. The elements in the list are sorted in some standard or specified order, and each element is placed at equally spaced addresses in computer memory. For this problem we'll consider sorted arrays of numbers and represent them like this:




length 5 sorted array     length 4 sorted array     not a sorted array

Suppose you want to determine where, in a given sorted array, a particular value $x$ may occur. One way Computer Scientists solve this problem is by using a "linear search" where the fixed number $x$ is compared to each array element, in order and one entry at a time. We will let $\{\ell_n\}$ denote the number of comparisons this "linear search" computes for a sorted array of length $n$ (make sure you understand why $\ell_1 = 1$ and explore other terms in this sequence). Which of the following is a true statement about the sequence $\{\ell_n\}$ ?

**a)** ○ The sequence satisfies the recurrence relation $\ell_{n+m} = \ell_n \cdot \ell_m$ for all n, m $> 1$.

**b)** ◉ The sequence satisfies the recurrence relation $\ell_{n+m} = \ell_n + m$ for all n, m $> 1$.

**c)** ○ The sequence satisfies the recurrence relation $\ell_{n+m} = \ell_n + 1$ for all n, m $> 1$.

**d)** ○ The sequence satisfies the recurrence relation $\ell_{n+m} = \ell_n \cdot m$ for all n, m $> 1$.

**e)** ○ The sequence satisfies the recurrence relation $\ell_{n+m} = \ell_n + \ell_m$ for all n, m $> 1$.

**Question 7**

Your answer is CORRECT.

The previous question was about searching through sorted arrays using a "linear search," and counting the number of steps involved in such a search gave us a sequence with recursive structure. There are *other* ways Computer Scientists search through sorted arrays, including one called a "binary search." This method gives rise to the more interesting sequence $\{b_n\}$ which satisfies the recurrence relation and initial conditions

$$b_n = b_{\lceil \frac{n}{2} \rceil} + 1$$

$$b_1 = 1$$

For example, this recurrence relation tells us that the second term
$b_2 = b_{\lceil \frac{2}{2} \rceil} + 1 = b_{\lceil 1 \rceil} + 1 = b_1 + 1 = 2.$ Determine the value of $b_{10}$.

a) $b_{10} = 2^{10}$

b) $b_{10} = \log_2(10)$

c) $b_{10} = 5$

d) $b_{10} = 3$

e) $b_{10} = 32$

**Question 8**

Your answer is CORRECT.

Lots of famous sequences are defined to *count* interesting things. One such example is the Tower-of-Hanoi Numbers, which is a sequence of numbers, $\{H_n\}$, that solve the recurrence equation and initial conditions

$$H_n = 2H_n + 1$$
$$H_1 = 1$$

Determine the value of the term $H_4$

a) $H_4 = 7.$

b) $H_4 = 7.$

c) $H_4 = 8.$

d) $H_4 = 15.$

**Question 9**

Your answer is CORRECT.

Consider the sequence $\{h_n\}$ that solves the recurrence relation and initial conditions

$$h_n = 2h_{n-1} + 1$$
$$h_1 = 7$$

If one uses the method of iteration to solve this recurrence relation, which closed-form expression for $h_n$ will they find?

a) $h_n = 2^{n-1} + 1$

b) $h_n = 8 \cdot 2^{n-1} + 1$

c) $h_n = 2^{n-1} - 1$

d) $h_n = 8 \cdot 2^{n-1} - 1$

e) ○ $h_n = 2(n-1) + 7$

**Question 10**

Your answer is CORRECT.

Consider the sequence $\{a_n\}$ that solves the recurrence relation and initial conditions

$$a_n = 17a_{n-1} - 66a_{n-2}$$
$$a_0 = 19, \ \ a_1 = 90$$

What is the Characteristic Equation for this sequence? What are the Characteristic roots?

a) ○ The charateristic equation is $x^2 - 19x + 90 = 0$ and the characteristic roots are $r_1 = 10, \ r_2 = 9$ .

b) ◉ The charateristic equation is $x^2 - 17x + 66 = 0$ and the characteristic roots are $r_1 = 6, \ r_2 = 11$ .

c) ○ The charateristic equation is $x^2 - 6x + 11 = 0$ and the characteristic roots are $r_1 = 17, \ r_2 = 66$ .

d) ○ The charateristic equation is $x^2 - 49x - 1122 = 0$ and the characteristic roots are $r_1 = -17, \ r_2 = 66$ .

e) ○ The charateristic equation is $x^2 - 10x + 9 = 0$ and the characteristic roots are $r_1 = 19, \ r_2 = 90$ .

**Question 11**

Your answer is CORRECT.

Consider the sequence $\{a_n\}$ that solves the recurrence relation and initial conditions

$$a_n = 20a_{n-1} - 99a_{n-2}$$
$$a_0 = 14, \ \ a_1 = 45$$

We know that a closed-form expression for this sequence is given by $a_n = \alpha(r_1)^n + \beta(r_2)^n$ where $r_1$ and $r_2$ are the characteristic roots. Determine the values of the constants $\alpha$ and $\beta$.

a) ○ $\alpha = \dfrac{-109}{81}, \ \beta = 1$

b) ○ $\alpha = \dfrac{2}{109}, \ \beta = \dfrac{2}{-81}$

c) ○ $\alpha = 14, \ \beta = 45$

d) $\alpha = \frac{109}{2},\ \beta = \frac{-81}{2}$

e) $\alpha = 20,\ \beta = 99$