Math 3336
Homework Assignment 12

Instructions

- Record your answers to the following 10 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (Click this link for instructions on how to do this).
- Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.
- I understand that if any of the questions from this assignment (or future ones) are shared in ways that violate our Academic Honesty Policy, then the syllabus will change. Specificlaly, Homework and Quizzes will be worth zero points.

Name:

Signature:

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1. Consider the psuedo-code for a famous kind of algorithm called the "Bubble Sort" algorithm.

```
1   sorted = False
2   n = length(list)
3   while (not sorted)
4   {
5       sorted = True
6       for element in list[0: n-1]
7       {
8           if element > next_element:
9           {
10              swap(element, next_element)
11              sorted = False
12          }
13      }
14      n = n − 1
15  }
```

Figure 1: Bubble Sort

This algorithm takes a length-n list of integers and sorts them into ascending order. Which, if any, of the following accurately describes why the worst-case run-time is $O\left(n^2\right)$?

(a) In a worst-case scenario, the original list may be arranged in *descending* order. This algorithm will have to loop n times to compare and swap elements to get an element in position and repeat this for every element.

(b) In the worst-case scenario, the algorithm will need to run a loop n^2 times, each time comparing n elements.

(c) In the worst-case scenario, the algorithm will compare each of the n elements to n^2 elements, and so the run-time is $O\left(n^2\right)$.

(d) There is no such algorithm that works like this, and so the run-time is actually infinite.

(e) None of the above.

Note: here is a fun, brief and helpful article on this Bubble Sort array.

2. Which, if any, of the following functions are $O(n)$?

I. $n^2 - n + 1$ II. $\log n$ III. $2022n + 1300$

(a) I and III only.

(b) III only.

(c) II and III only.

(d) I and II only.

(e) I, II and III.

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3. Suppose we know that the congruence equation

$$ax \equiv 18 \bmod 24$$

has four distinct solutions from the set $\{0, 1, 2, 3, 4, 5, \ldots, 22, 23\}$. Of the options provided below, which could be the value of a?

(a) $a = 2$

(b) $a = 3$

(c) $a = 6$

(d) $a = 12$

(e) There are no values of a that can make this true.

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4. Two integers $a, b \in \mathbb{Z}$ are relatively prime...

(a) if the only positive divisor they have in common is 1

(b) if the Euclidean-Algorithm returns the value 1

(c) if Bezout's identity claims that $\exists\, x, y \in \mathbb{Z}$, $ax + by = 1$

(d) if $gcd(a, b) = 1$

(e) ALL of the above

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5. Suppose there are 5 different types of ice cream that you like. How many random samples of ice cream must be eaten to guarantee that you have had at least 7 samples of one type?

(a) 31

(b) 26

(c) 7

(d) 5

(e) 12

6. The faces of a cube are to be painted, and only one rule must be followed: no pair of opposite faces can have the same color.

Figure 2: Coloring a cube using 6 colors

If there are six colors to use, how many different ways are there to paint the the cube?

(a) 6^6

(b) 6^3

(c) $6^3 \cdot 5^3$

(d) $6 \cdot 5^5$

(e) None of the above

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7. How many size-5 subsets of a size-15 set are there?

(a) $\binom{15}{5}$

(b) 2^5

(c) 2^{15}

(d) 5!

(e) None of the above

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8. Which of the following facts is useful to keep in mind when proving a formula that involves *summation* by induction?

(a) The inductive step uses $n = 0$.

(b) One can relate a summation that stops at $k+1$ to the same summation that stops at k by "peeling off the last term."

(c) One can relate a summation that stops at $k+1$ to the same summation that stops at k by multiplying the summations.

(d) This is a trick question. Induction cannot be used on formulas involving summation.

9. The second-order, homogeneous recurrence equation

$$a_n = 5a_{n-1} + 3a_{n-2}$$

has which of the following as its characteristic quadratic?

(a) $3x^2 + 5x - 1$

(b) $x^2 + 5x + 3$

(c) $x^2 - 5x - 3$

(d) This is a trick question. You need to know the equation's initial conditions in order to write down its characteristic quadratic.

(e) None of the above

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10. A (hopefully familiar) truth-table for a logical operator is shown below (the operator is labelled with the random symbol $\star$).

P	Q	$P \star Q$
T	T	T
T	F	F
F	T	T
F	F	T

Which of the following is true?

(a) $P \star Q = P \vee Q$

(b) $P \star Q = P \oplus Q$

(c) $P \star Q = P \wedge Q$

(d) $P \star Q = P \Rightarrow Q$

(e) None of the above