Counting Subsets

$\binom{n}{k}$ = the # of size-k subsets of a size-n set

= the # of ways one can choose k elements from an n element set

= "n choose k"

ex) $n = 4$ $\quad X = \{\alpha, \beta, \delta, \gamma\}$

$\binom{4}{2}$ = # of ways to choose two elements from X

$\{\alpha, \beta\}, \{\alpha, \delta\}, \{\alpha, \gamma\}$ 3

$\{\beta, \delta\}, \{\beta, \gamma\}$ 2

$\{\delta, \gamma\}$ 1

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$\binom{4}{1}$ = # of 1-element subsets

$\{\alpha\}, \{\beta\}, \{\delta\}, \{\gamma\}$

$= 4$

$$\binom{4}{3} = \text{\# of 3-element subsets} = 4$$

$\{\beta, \delta, \gamma\}$, $\{\alpha, \delta, \gamma\}$, $\{\alpha, \beta, \gamma\}$, $\{\alpha, \beta, \delta\}$

$$\binom{4}{0} = \text{\# of 0-element subset} = 1$$

$\emptyset$

$$\binom{4}{4} = 1$$

A Generic Formula for $\binom{n}{k}$

$$\binom{n}{k} = \frac{1}{k!} P(n,k)$$

ex) $\binom{5}{3}$ = # 3-element subsets from a 5-element set

$P(5,3)$

$\underline{a}\,\underline{b}\,\underline{c} \to \underline{a}\,\underline{c}\,\underline{b}$, $\underline{b}\,\underline{a}\,\underline{c}$, $\underline{c}\,\underline{b}\,\underline{a}$, $\underline{b}\,\underline{c}\,\underline{a}$, $\underline{c}\,\underline{a}\,\underline{b}$

$X = \{a, b, c, d, e\}$

{a,b,c} {a,b,d} {a,b,e} {a,c,d} {a,c,e} {a,d,e} {b,c,d} {b,c,e} {b,d,e} {c,d,e}

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$$P(5,3) = 3! \cdot \binom{5}{3}$$

$$\frac{5!}{(5-3)!} = 3! \binom{5}{3}$$

$$\boxed{\frac{5!}{3! \cdot (5-3)!} = \binom{5}{3}}$$

$$\binom{n}{k} = \frac{1}{k!} P(n,k) = \frac{1}{k!} \frac{n!}{(n-k)!}$$

$$\boxed{\binom{n}{k} = \frac{n!}{k!\,(n-k)!}}$$

Example 3.16 How many 7-digit binary strings (0010100, 1101011, etc.) have an odd number of 1's?

observation: there will be one, three, five or seven 1's

_ _ _ _ _ _ _

$S_1 = \{$ 7-digit binary strings w/ one 1 $\}$ $\qquad \binom{7}{1} = 7$

_ _ _ _ _ _ _ ← where does the single 1 go?

$S_3 = \{$ " " w/ three 1's $\}$ $\qquad \binom{7}{3} = \frac{7!}{3!\,4!}$

_ _ _ _ _ _ _ ← where do the three 1's go?

$S_5 = \{$ " " five 1's $\}$ $\qquad \binom{7}{5} = \frac{7!}{5!\,2!}$

$S_7 = \{$ " " seven 1's $\}$ $\qquad \binom{7}{7} = 1$

1 1 1 1 1 1 1