MATH3336 Extra Credit (+5)

Dr Douglas

April 22, 2023

The following is an extra credit assignment. Show all your work for full credit. Please make sure to attempt the assignment. I appreciate originality. Copied work from other students will result in 0. The deadline is **May 1st, 2023**. Please email me your work at **sarahsyed56@gmail.com**

1 Use the Principle of Mathematical Induction to prove for all natural numbers

$$\sum_{i=1}^{n}(3i-2)=\frac{n}{2}(3n-1)$$

Base Case

$n=1$

$$\sum_{i=1}^{1}(3i-2)=\frac{1}{2}(3\cdot 1-1)$$

$$3-2=\frac{1}{2}(2)$$

$$1=1 \checkmark$$

The proposition is true for the base case.

Inductive Step

Suppose the proposition is true for $n=k\in\mathbb{N}$

this means that $\sum_{i=1}^{k}(3i-2)=\frac{k(3k-1)}{2}$

(We want to show that $\sum_{i=1}^{k+1}(3i-2)=\frac{(k+1)}{2}(3(k+1)-1)=\frac{(k+1)(3k+2)}{2}$)

We know that $\sum_{i=1}^{k+1}(3i-2)=\sum_{i=1}^{k}(3i-2)+(3(k+1)-2)$

By our inductive hypothesis it follows that

$$\sum_{i=1}^{k}(3i-2)+(3(k+1)-2)=\frac{k(3k-1)}{2}+(3k+1)$$

$$=\frac{k(3k-1)+2(3k+1)}{2}=\frac{3k^2-k+6k+2}{2}=\frac{3k^2+5k+2}{2}$$

$$=\frac{(k+1)(3k+2)}{2}. \quad \square$$