3336

Office Hours

11:05

Note your lowest exam score can't will be replaced by your score on the final (if its higher)

Final

15 MC Questions ⟶ 80 pts

1 FR Question ⟶ 20 pts

110 minutes

Given two integers $a, b \in \mathbb{Z}$, the Euclidean Algorithm...

~~a)~~ ○ fails if either a or b is prime.

~~b)~~ ○ can be used to determine whether or not $a^{(1/b)} \in \mathbb{Z}$.

$\boxed{\text{c)}}$ ○ can be used to determine if a and b are relatively prime.

~~d)~~ ○ tells us whether or not a or b is prime.

~~e)~~ ○ tells us that the prime number 11 either divides a or divides b.

$$a = bq_1 + r_1$$

$$b = r_1 q_2 + r_2$$

$$r_1 = r_2 q_3 + r_3$$

$$\vdots$$

stop at 0

remainder above = gcd(a,b)

$\gcd(a,b) = 1 \iff a \,\&\, b$ are relatively prime

Let S_3 denote the subset of length-7 binary strings with the following property: the number of 1's in each string is a multiple of 3. In other words

$$S_3 = \{\text{ all length-7 binary strings whose total number of 1's equals a multiple of 3}\}$$

Which, if any, of the following statements is true?

$$S_3 = \{\text{zero } 1\text{'s}\} \cup \{\text{three } 1\text{'s}\} \cup \{\text{six } 1\text{'s}\}$$

only 1: 0000000 ; $\binom{7}{3}$; $\binom{7}{6}$

$$1 + \binom{7}{3} + \binom{7}{6}$$

Suppose the congruence equation $4x \equiv b \bmod 8$ has a solution $x = 10$. Which of the following statements about b is possibly true?

a) ◯ It is possible that $b = 28$
b) ◯ There are no possible values for b that allow for the solution $x = 10$
c) ◯ It is possible that $b = 9$
d) ◯ It is possible that $b = 39$
e) ◯ It is possible that $b = 24$

$4x \equiv b \bmod 8$

$40 \equiv b \bmod 8$

which of these values is $\equiv 40 \bmod 8$?

$40 - b \equiv 0 \bmod 8$

$40 - b$ is a mult. of 8

$40 - 24 = 16 = 2 \cdot 8$

40 & b have same remainder when divided by 8

28 has $r = 4$
9 has $r = 1$
39 has $r = 7$
$\boxed{\begin{array}{l}24 \text{ has } r = 0 \\ 40 \text{ has } r = 0\end{array}}$

A six-sided die is rolled 37 times. Which of the following statements is true?

a) ◯ At least 37 rolls produced the same number.
b) ◯ At least 12 rolls produced the same number.
c) ◯ At least 6 rolls produced the same number.
d) ◯ At least 7 rolls produced the same number.
e) ◯ Exactly 4 rolls produced the same number.
f) ◯ None of the above.

Six possible numbers: $\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}$

37 rolls

$\underbrace{1\ 2\ 3\ 4\ 5\ 6}_{\text{six rolls}}$ ← could have diff. numbers

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{six more rolls}}$ ← could have diff. numbers

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{six rolls}}$ ← could have diff. numbers

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{six more rolls}}$ ← could have diff. numbers

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{six rolls}}$ ← could have diff. numbers

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{six more rolls}}$ ← could have diff. numbers

one more roll

PH

one number repeated 7 times

Given the set $X = \{0, 1, 2, \cdots, 7\}$ we define the subset $T \subseteq \mathcal{P}(X)$ by

1 2 3 8

$$T = \left\{ S \in \mathcal{P}(X) : |S| = 5 \right\}$$

Which, if any, of the following statement is true?

a) $\boxed{|T| = 56}$
b) $|T| = 0$
c) $|T| = 128$
d) $|T| = 2520$
e) $|T| = 21$

$$\binom{8}{5} = \frac{8!}{5! \cdot 3!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2} \cdot \cancel{3 \cdot 2}} = 56$$

it was easy to get this question wrong by miscounting $|X|$

$$|T| = \binom{8}{5}$$

note: $|X| = 8$ $(|X| \neq 7)$

For this problem P, Q, and R are statements that satisfy the following:

$$(P \iff Q) \wedge R \text{ is false, and } Q \text{ is true.}$$

If possible, use this information to determine the truth value of P.

a) The truth value of P cannot be determined by the given information.

b) P is a true statement.

c) P is a false statement.

$$\boxed{(P \Leftrightarrow Q) \wedge R \text{ is } F}$$

either $P \Leftrightarrow Q$ is F or R is F

$\boxed{Q \text{ is true}}$

We can't conclude truth value of P

one possibility	another poss.	other possibilities...
P is true	P is false	
Q is true	Q is true	
R is false	R is true	

For this problem P, Q, and R are statements that satisfy the following:

$$(P \iff Q) \wedge R \text{ is true} \quad \text{and } Q \text{ is true.}$$

If possible, use this information to determine the truth value of P.

a) The truth value of P cannot be determined by the given information.

b) P is a true statement.

c) P is a false statement.

$(P \Leftrightarrow Q) \wedge R$ is True

Both $\underbrace{P \Leftrightarrow Q \text{ is True}}_{\text{P + Q have the same truth value}}$ and R is True

P + Q have the same truth value

Q is True

$\boxed{\therefore P \text{ is True}}$

Consider the following proposition:

Proposition. $\forall n \in \mathbb{N}$, at least one of the consecutive natural numbers, $n, n+1$, and $n+2$, is divisible by 3

Suppose a mathematician wishes to prove this proposition using a Proof by ("Regular" or "Weak") Induction. Which of the following statements best describes or summarizes the inductive step she could use in her proof?

a) Let $k \in \mathbb{N}$ be arbitrary, and show that one of the numbers $k, k+1$, or $k+2$ is divisible by 3. and then what?

b) Let $k \in \mathbb{N}$ be arbitrary, and suppose the number k is divisible by 3. Use this Inductive Hypothesis to show that this implies $k+1$ is divisible by 3.

c) Let $k \in \mathbb{N}$ be arbitrary, and suppose one of the numbers $k, k+1$, or $k+2$ is divisible by 3. Use this Inductive Hypothesis to show that this implies one of the numbers $k+1, k+2$, or $k+3$ is divisible by 3.

d) This is a trick question. A proof by Induction cannot be used for this proposition since it involves the natural numbers.

e) Show that the proposition is true when $n = 0$.

a base case

wrong inductive hyp.