# PRINTABLE VERSION

**Quiz 5**

## You scored 88.89 out of 100

**Question 1**

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $\neg P$.
*Missing steps involving* $\neg P$ *and* $\neg Q$ *and any previously established facts*
Therefore $\neg Q$. □

What type of proof was described in the outline?

**a)** Wait a minute... The proof described in this outline isn't a valid proof technique!

**b)** A direct proof is described in this outline.

**c)** A proof by introspection is described in this outline.

**d)** A proof by contradiction is described in this outline.

**e)** A proof by contrapositive is described in this outline.

**Question 2**

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

**a)** Suppose $\neg P \wedge Q$.

**b)** Suppose $\neg Q$

**c)** Suppose $\neg P \vee \neg Q$.

**d)** Suppose $\neg P \wedge \neg Q$.

**e)** Suppose $\neg P$.

**Question 3**

Your answer is CORRECT.

Given two sets $A$ and $B$ one can prove $B \subseteq A$ by...

**a)** First supposing $x \in A$, and then showing $x \notin B$.

**b)** First supposing $x \notin B$, and then showing $x \in A$.

**c)** First supposing $x \in A$, and then showing $x \in B$.

**d)** First supposing $x \notin A$, and then showing $x \in B$.

**e)** First supposing $x \in B$, and then showing $x \in A$.

**Question 4**

Your answer is CORRECT.

Given two sets $A$ and $B$ one can prove $B \subseteq A$ by...

**a)** First supposing $x \notin A$, and then showing $x \in B$.

**b)** ◉ First supposing $x \notin A$, and then showing $x \notin B$.

**c)** ○ First supposing $x \notin B$, and then showing $x \in A$.

**d)** ○ First supposing $x \notin B$, and then showing $x \notin A$.

**e)** ○ First supposing $x \in A$, and then showing $x \notin B$.

**Question 5**

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If the product of two integers is even, then at least one of the integers is even.

Proof. Suppose $x, y \in \mathbb{Z}$ and $xy$ is even, but that neither $x$ nor $y$ is even.
This means $x$ and $y$ are both odd, and so $x = 2n + 1$ and $y = 2m + 1$ for integers $n, m$.
It follows that $xy = (2n+1)(2m+1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$ which is odd since $(2nm + n + m)$
Therefore $xy$ is both even and odd. $\Rightarrow\Leftarrow$

Determine the type of proof used.

**a)** ○ A direct proof was used.

**b)** ○ A proof by indoctrination was used.

**c)** ◉ A proof by contradiction was used.

**d)** ○ A proof by contrapositive was used.

**e)** ○ Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

**Question 6**

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $15 \nmid a$ then $3 \nmid a$ or $5 \nmid a$.

Proof. (Contrapositive)

(1) Suppose the conclusion is false. That is, suppose $\neg\left(3 \nmid a \text{ or } 5 \nmid a.\right)$.
(2) This means $5|a$ and $3|a$, and we want to show $15|a$.
(3) By definition of divides this means $5 = ma$ and $3 = na$ for some $m, n \in \mathbb{Z}$,
and it also means we want to show $15 = ka$ for some $k \in \mathbb{Z}$.
(4) Since 5 is prime and $5 = ma$, it follows that $a = 5$ or $a = 1$.
(5) Since 3 is prime and $3 = na$, it follows that $a = 3$ or $a = 1$.
(6) The only possibility is for $a = 1$ and so we can use $k = 15$ to conclude $15 = ka = 15 \cdot 1$. □

Identify the mistake, if any, in this proof.

**a)** ○ There is a mistake in Line (4). 5 is not prime.

**b)** ○ There is a mistake in Line (6). The value of $a$ could also be $a = 0$.

**c)** ○ There is a mistake in Line (2). The statement $(3 \nmid a \text{ or } 5 \nmid a)$ was not correctly negated.

**d)** ○ There is a mistake in Line (5). The statement 3 is not prime.

**e)** ◉ There is a mistake in Line (3) where the definition of divides is incorrectly used.

**Question 7**

Your answer is INCORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. $\forall n \in \mathbb{N}$, $n(n+1)$ is even.

Proof. (Direct)

(1) Let $n \in \mathbb{N}$. We will use cases to prove the proposition.

Case 1: $n$ is even.

(2) In this case we have $n = 2m$ for some $m \in \mathbb{N}$.

(3) Because $n$ is even, it follows that when $n$ is divided by 2, there is no remainder.

(4) $n$ being even implies $n+1$ is odd.

(5) It now follows that $n(n+1) = (2m)(2m+1) = 2 \cdot \left(m(2m+1)\right)$ which has the form of an even number.

(6) Therefore $n(n+1)$ is even, proving the proposition in this case.

Case 2: $n$ is odd.

(7) In this case we have $n = 2\ell + 1$ for some $\ell \in \mathbb{Z}$.

(8) If $n$ is not odd then it is even and Case 1 applies.

(9) It follows that $n(n+1) = (2\ell+1)(2\ell+1+1) = (2\ell+1)(2\ell+2) = 2(2\ell+1)(\ell+1)$

(10) Because the expression above has the form of an even number, $n(n+1)$ is even.

(11) If $n(n+1)$ is odd, then there is a contradiction.

(12) This completes the proof. $\square$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

**a)** Only lines (3),(4),(8), and (11) are not needed. All other lines are needed.

**b)** Only line (8) is not needed. All other lines are needed.

**c)** Only lines (4) and (11) are not needed. All other lines are needed.

**d)** All lines are needed.

**e)** Only lines (3) and (11) are not needed. All other lines are needed.

**Question 8**

Your answer is CORRECT.

Suppose we want to write a direct proof of the proposition below:

$$\forall x \in Z,\ x^3 - x \text{ is a multiple of } 3.$$

Which of the following statements or properties do we need to use when composing this proof?

**a)** A case where $x \in R - Z$.

**b)** A case where $x = 2k$ is even, and $x = 2k$ is plugged into $x^3 - x$.

**c)** The definition of rational number.

**d)** The definition of rational number.

**Question 9**

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

The recursively defined sequence $a_n = a_{n-1} - 1$ with initial conditions $a_0 = \pi$ has negative terms for all $n \geq 4$.

Of the following options, which one best describes what needs to be done in order to prove this claim?

**a)** This can only be proved by paying someone else to do it for me.

**b)** ○ We need only check that the claim is true for one, single natural number.

**c)** ◉ We need to show the claim is true for an abitrary natural larger than 3, saying something like "Let $n \geq 4.$ "