PRINTABLE VERSION

Quiz 5

You scored 88.89 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $\neg Q$.
Missing steps involving $\neg Q$ *and* $\neg P$ *and any previously established facts*
Therefore $\neg P$. $\square$

What type of proof was described in the outline?

a) A proof by contrapositive is described in this outline.

b) Wait a minute... The proof described in this outline isn't a valid proof technique!

c) A direct proof is described in this outline.

d) A proof by contradiction is described in this outline.

e) A proof by introspection is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $(P \wedge Q) \vee R$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P \vee \neg Q \vee \neg R$

b) Suppose $\neg P \vee (\neg Q \wedge \neg R)$

c) Suppose $(\neg P \wedge \neg Q) \vee \neg R$

d) Suppose $(\neg P \wedge \neg Q) \wedge \neg R$

e) Suppose $(\neg P \vee \neg Q) \wedge \neg R$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \in B$.

b) First supposing $x \in B$, and then showing $x \in A$.

c) First supposing $x \notin A$, and then showing $x \in B$.

d) First supposing $x \in A$, and then showing $x \notin B$.

e) First supposing $x \notin B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \notin B$, and then showing $x \notin A$.

b) First supposing $x \notin B$, and then showing $x \in A$.

c) First supposing $x \notin A$, and then showing $x \notin B$.

d) First supposing $x \in A$, and then showing $x \notin B$.

e) First supposing $x \notin A$, and then showing $x \in B$.

Question 5

Your answer is INCORRECT.

A lovely little proof is presented below:

Proposition. If the product of two integers is even, then at least one of the integers is even.

Proof. Suppose $x, y \in \mathbb{Z}$ and neither x nor y is even. (We will show that xy is not even.)
This means x and y are both odd so that $x = 2n + 1$ and $y = 2m + 1$ for integers n, m.
It follows that $xy = (2n+1)(2m+1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$ which is odd since $(2nm + n + m)$
Therefore xy is not even.$\square$

Determine the type of proof used.

a) A proof by contrapositive was used.

b) A proof by contradiction was used.

c) A proof by indoctrination was used.

d) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

e) A direct proof was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)
(1) Suppose $x, y \in \mathbb{Z}$ are integers.
(2) We can assume x is odd and that y is even.
(3) Since x is odd, it follows that $\exists\, y \in \mathbb{Z},\ x = 2y + 1$.
(4) Since y is even, it follows that $\exists\, m \in \mathbb{Z},\ y = 2m$.
(5)We now have $x + y = (2y + 1) + y = 3y + 1 = 3(2m) + 1 = 2(3m) + 1$.
(6) Because $x + y$ has the form of an odd number it is odd. $\square$

Identify the mistake, if any, in this proof.

a) There is an error in Line (2) since we do not know which integer is odd or even.

b) There is an error in Line (1) since we cannot simply assume $x, y \in Z$.

c) There is an error in Line (4) since where the definition of "even" is misapplied.

d) There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.

e) There is an algebraic mistake in Line (5).

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \cup B = B$, then $A \subseteq B$.

Proof. (Direct)

(1) Suppose $A \cup B = B$. To prove $A \subseteq B$ we also let $x \in A$ and will end the proof by showing $x \in B$.
(2) Because B is a set $\emptyset \subseteq B$.
(3) Since $A \subseteq A \cup B$ and $x \in A$ it follows that $x \in A \cup B$.
(4) Since, by assumption $A \cup B = B$ it also follows that $x \in B$.
(5) Because $A \cup B = B$ a Venn diagram shows that $A \subseteq B$.
(6) If $x \notin B$, then there would be a contradiction. □

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only lines (1) and (2) are not needed. All other lines are needed.

b) Only lines (3) and (4) are not needed. All other lines are needed.

c) Only lines (2) and (5) are not needed. All other lines are needed.

d) Only ;ines (1) and (5) are not needed. All other lines are needed.

e) All lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a proof by contradiction of the proposition below:

$$\forall a, b, c \in [0, \infty), \ (ab = c) \Rightarrow (a \leq \sqrt{c} \vee b \leq \sqrt{c}).$$

Which of the following statements or properties do we need to use when composing this proof?

a) The fact that for real numbers x, y, if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

b) The fact that for real numbers x, y $\sqrt{x} < \sqrt{y}$.

c) Suppose $ab = c$ and that both $a < \sqrt{c}$ and $b < \sqrt{c}$.

d) Suppose $ab = c$ and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

The recursively defined sequence $a_n = a_{n-1} - 1$ with initial conditions $a_0 = \pi$ has a term that is negative.

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) We need only check that the claim is true for one, single natural number.

b) We need to show the claim is true for an abitrary natural larger than 3, saying something like "Let $n \geq 4$. "

c) Nothing can describe an accurate proof strategy since this proposition is false.