Intro to Sets (Naive Set Theory)

Links: Math 3336

Lecture Video 9: Introduction to Sets, Textbook Section 1.1

Sets

All of modern mathematics is based on sets!

A **set**, S, is a collection of objects or things, the objects in a set are called *elements*.

> **notation**
> $x \in S$
> "x is an element of set S"

often times sets are described in terms of elements they contain using curly braces

> example:
> $\mathbb{N} =$ {1,2,3,4,...}
> $5 \in \mathbb{N}, 2022 \in \mathbb{N}, -5 \notin \mathbb{N}$

Two sets, A and B, are *equal* if an only if they have the exact same elements.

> **notaion:** $A = B$

another set notation example:
this set is described in terms of a condition

> $B = \{2n : n \in \mathbb{N}\}$
> pronounced: all things that look like *2n* **such that** *n* is a natural number

sets can have other sets as their elements

> **example**
> D={1}, E={{1}}
> $1 \in D$, $1 \notin E$, $D \in E$

> $S = \{\mathbb{N}\}$
> S has 1 element, $\mathbb{N}$ has infinite, but S only has 1.

The **cardinality** or *size* of a set S is the number of elements in S.

> **notation**
> $|S|$

The empty set, ϕ , is a set that contains no elements, its cardinality is 0 (i.e. $|\phi| = 0$).

Some familiar sets

- ϕ "empty set"
- $\mathbb{N}$ "natural numbers" {1,2,3,4,...}
- $\mathbb{Z}$ "integers" {...,-3,-2,-1,0,1,2,3,...}
- $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ "rational numbers" (all possible fractions of integers)
- $\mathbb{R}$ "real numbers" (all numbers on the number line)

Other math sets

- $M_2(\mathbb{R})$ = {all 2x2 matrices with real entries}
- $P_n(\mathbb{R})$ = {all degree *n* polynomials with real coefficients}

  > $7x^3 + 5x^2 - 1 \in P_3(\mathbb{R})$