PRINTABLE VERSION

Quiz 5

You scored 100 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $\neg Q$.
Missing steps involving $\neg Q$ *and* $\neg P$ *and any previously established facts*
Therefore $\neg P$. $\square$

What type of proof was described in the outline?

a) A proof by contradiction is described in this outline.

b) A direct proof is described in this outline.

c) A proof by contrapositive is described in this outline.

d) Wait a minute... The proof described in this outline isn't a valid proof technique!

e) A proof by introspection is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form P. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P \lor \neg Q$.

b) Suppose $\neg P \land \neg Q$.

c) Suppose $\neg Q$

d) Suppose $\neg P$.

e) Suppose $\neg P \land Q$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \in B$.

b) First supposing $x \in B$, and then showing $x \in A$.

c) First supposing $x \notin A$, and then showing $x \in B$.

d) First supposing $x \in A$, and then showing $x \notin B$.

e) First supposing $x \notin B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \notin A$, and then showing $x \in B$.

b) First supposing $x \notin A$, and then showing $x \notin B$.

c) First supposing $x \notin B$, and then showing $x \in A$.

d) First supposing $x \notin B$, and then showing $x \notin A$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

> Proposition. If $a + 7$ is even, then a is odd.
>
> Proof. Suppose a is odd. (We will show $a + 7$ is even.)
> It follows that $a = 2m - 1$ for some $m \in \mathbb{Z}$.
> By adding 7 to both sides of this equation we find $a + 7 = 2m - 1 + 7 = 2m + 6 = 2(m + 3)$.
> This completes the proof as we have shown $a + 7$ is even. $\square$

Determine the type of proof used.

a) A proof by contrapositive was used.

b) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

c) A proof by indoctrination was used.

d) A direct proof was used.

e) A proof by contradiction was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)

(1) Suppose $x, y \in \mathbb{Z}$ are integers.
(2) We can assume x is odd and that y is even.
(3) Since x is odd, it follows that $\exists y \in \mathbb{Z},\ x = 2y + 1$.
(4) Since y is even, it follows that $\exists m \in \mathbb{Z},\ y = 2m$.
(5)We now have $x + y = (2y + 1) + y = 3y + 1 = 3(2m) + 1 = 2(3m) + 1$.
(6) Because $x + y$ has the form of an odd number it is odd. $\square$

Identify the mistake, if any, in this proof.

a) ○ There is an error in Line (1) since we cannot simply assume x, y ∈ Z .

b) ◉ There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.

c) ○ There is an error in Line (2) since we do not know which integer is odd or even.

d) ○ There is an algebraic mistake in Line (5).

e) ○ There is an error in Line (4) since where the definition of "even" is misapplied.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \cup B = B$, then $A \subseteq B$.

Proof. (Direct)

(1) Suppose $A \cup B = B$. To prove $A \subseteq B$ we also let $x \in A$ and will end the proof by showing $x \in B$.
(2) Because B is a set $\emptyset \subseteq B$.
(3) Since $A \subseteq A \cup B$ and $x \in A$ it follows that $x \in A \cup B$.
(4) Since, by assumption $A \cup B = B$ it also follows that $x \in B$.
(5) Because $A \cup B = B$ a Venn diagram shows that $A \subseteq B$.
(6) If $x \notin B$, then there would be a contradiction. $\square$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) ○ Only lines (1) and (2) are not needed. All other lines are needed.

b) ○ Only lines (3) and (4) are not needed. All other lines are needed.

c) ◉ Only lines (2) and (5) are not needed. All other lines are needed.

d) ○ Only ;ines (1) and (5) are not needed. All other lines are needed.

e) ○ All lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a proof by contradiction of the proposition below:

$$\forall a, b, c \in [0, \infty),\ (ab = c) \Rightarrow (a \leq \sqrt{c} \lor b \leq \sqrt{c}).$$

Which of the following statements or properties do we need to use when composing this proof?

a) ◯ The fact that for real numbers x, y , if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

b) ◯ The fact that for real numbers x, y $\sqrt{x} < \sqrt{y}$.

c) ◯ Suppose $ab = c$ and that both $a < \sqrt{c}$ and $b < \sqrt{c}$.

d) ◉ Suppose $ab = c$ and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

The recursively defined sequence $a_n = a_{n-1} - 1$ with initial conditions $a_0 = \pi$ has a term that is negative.

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) ◉ We need only check that the claim is true for one, single natural number.

b) ◯ We need to show the claim is true for an abitrary natural larger than 3, saying something like "Let $n \geq 4$. "

c) ◯ Nothing can describe an accurate proof strategy since this proposition is false.