Discrete Math

Lecture 1

Statements

Recall: a statement is a sentence that is true or false (not both)

ex 1 | $3+1=4$ (true sentence) ✓

$\frac{100}{5}=10$ is a statement (false) ✓

ex 2 | Watch this video. ← not T or F
non-statement

What time is it? ← non-statement

Example 1.1. *The sentence "Houston is a city in Texas." is an example of a statement because it is* true.
The sentence "Every dog has brown eyes." is an example of a statement because it is false.
The sentence "$3+2=32$" is an example of a statement because it is F.

Example 1.4. *Determine whether the statements P, Q, R and S (given below) are true or false.*

P : $\tan(\pi/4) = 1$ true

Q : *the derivative of a polynomial is a polynomial.* true

R : $\pi \leq e$ false

False → S : *Every even whole number is a multiple of 4.*

0, 2, 4, 6, 8, 10, 12, ...

↓

$6 \neq 4$. (number)

6 is not a multiple of 4

Example 1.5. *The open sentences below are non-statements, each containing one variable. Find a value you can assign this variable that makes the open sentence a true statement.*

$$P(\theta) : \tan(\theta) = 1$$
$$Q(x) : x^2 = 0$$
$$R(t) : \int_0^t 2x\,dx = 9$$

Recall: an open sentence is a special kind of non-statement

- not true, not false UNTIL you Sub. values for its variables

$$R(t) : \int_0^t 2x\,dx = 9$$

false !

plug in $t = 0$: $R(0) = \int_0^0 2x\,dx = 9$

plug in $t = 1$: $R(1) : \int_0^1 2x\,dx = 9$

$= [x^2]_0^1$

$= 1^2 - 0^2 = 1$

false

Spoiler: $R(3) : \int_0^3 2x\,dx = 9$ ← true!

example of a weird + "math-breaking" non-statement

LP : This sentence is false.

If LP is a statement, it must be T or F.
(it must have a single "truth value")

1) what happens if LP is true?
LP must be false! → its both T + F !!

2) what happens if LP is false?
LP is falsely describing itself as false! → oh, so LP is true → its both F + T!!

oh, so LP cannot be treated as a statement

In this section you read about **statements (propositions), non-statements, open sentences (predicates)**.

Definition 1.1. *A **statement** or **proposition** is a sentence that is either true or false (but not both). We say that statements are sentences that "have a truth value." We use capital letters like P, Q, R, etc. to denote these.*

Definition 1.2. *An **open sentence** or **predicate** is a non-statement that contains variables, and when those variables are replaced the sentence becomes a statement. We use expressions like $P(x), Q(s,t)$, etc., to denote open sentences and their variables.*

Statements (Propositions)	Non-Statements
sentences that are true or false (not both)	Open Sentences (variables need subst.) Commands Questions Self-refuting sentences