PRINTABLE VERSION

Quiz 5

You scored 88.89 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

> Proposition. $P \Rightarrow Q$
>
> <u>Proof</u>. Suppose $\neg Q$.
> *Missing steps involving* $\neg Q$ *and* $\neg P$ *and any previously established facts*
> Therefore $\neg P$. $\square$

What type of proof was described in the outline?

a) A proof by contradiction is described in this outline.

b) A direct proof is described in this outline.

c) A proof by contrapositive is described in this outline.

d) Wait a minute... The proof described in this outline isn't a valid proof technique!

e) A proof by introspection is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $\neg P \Rightarrow Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg Q$

b) Suppose $\neg P \lor \neg Q$.

c) Suppose $\neg P \land \neg Q$.

d) Suppose $\neg P \land Q$.

e) Suppose $\neg P$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \notin B$.

b) First supposing $x \notin B$, and then showing $x \in A$.

c) First supposing $x \in A$, and then showing $x \in B$.

d) First supposing $x \notin A$, and then showing $x \in B$.

e) First supposing $x \in B$, and then showing $x \in A$.

Question 4

Your answer is INCORRECT.

Given two sets A and B one can prove $A \subseteq B$ by...

a) First supposing $x \notin A$, and then showing $x \in B$.

b) First supposing $x \notin B$, and then showing $x \in A$.

c) First supposing $x \notin B$, and then showing $x \notin A$.

d) First supposing $x \notin A$, and then showing $x \notin B$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If the product of two integers is even, then at least one of the integers is even.

Proof. Suppose $x, y \in \mathbb{Z}$ and xy is even, but that neither x nor y is even.
This means x and y are both odd, and so $x = 2n + 1$ and $y = 2m + 1$ for integers n, m.
It follows that $xy = (2n+1)(2m+1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$ which is odd since $(2nm + n + m)$
Therefore xy is both even and odd. $\Rightarrow\Leftarrow$

Determine the type of proof used.

a) A proof by contradiction was used.

b) A proof by contrapositive was used.

c) A direct proof was used.

d) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

e) A proof by indoctrination was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $15 \nmid a$ then $3 \nmid a$ or $5 \nmid a$.

Proof. (Contrapositive)

(1) Suppose the conclusion is false. That is, suppose $\neg\left(3 \nmid a \text{ or } 5 \nmid a.\right)$.
(2) This means $5|a$ and $3|a$, and we want to show $15|a$.
(3) By definition of divides this means $5 = ma$ and $3 = na$ for some $m, n \in \mathbb{Z}$,
and it also means we want to show $15 = ka$ for some $k \in \mathbb{Z}$.
(4) Since 5 is prime and $5 = ma$, it follows that $a = 5$ or $a = 1$.
(5) Since 3 is prime and $3 = na$, it follows that $a = 3$ or $a = 1$.
(6) The only possibility is for $a = 1$ and so we can use $k = 15$ to conclude $15 = ka = 15 \cdot 1$. $\square$

Identify the mistake, if any, in this proof.

a) There is a mistake in Line (4). 5 is not prime.

b) There is a mistake in Line (6). The value of a could also be $a = 0$.

c) There is a mistake in Line (2). The statement $(3 \nmid a \text{ or } 5 \nmid a)$ was not correctly negated.

d) There is a mistake in Line (5). The statement 3 is not prime.

e) There is a mistake in Line (3) where the definition of divides is incorrectly used.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. x is a multiple of 3 $\iff$ $(x+1)(x+2)-2$ is also a multiple of 3.

Proof. There are two parts or cases to prove.

($\Rightarrow$ Direct)
(1) Suppose $x = 3m$ for some $m \in \mathbb{Z}$.
(2) Since 3 is prime this means x is a multiple of a prime.
(3) It follows that $(x+1)(x+2)-2 = \left(x^2+3x+2\right)-2 = x^2+3x = (3m)^2+3\cdot 3m$
$= 9m^2+9m = 3\left(3m^2+3m\right)$, which is a multiple of 3.

($\Leftarrow$ By Contradiction)
(4) For a contradiction suppose $(x+1)(x+2)-2$ is multiple of 3, but that x is not a multiple of 3.
(5) Multiplying out this expression and combining like terms tells us $(x+1)(x+2)-2$
$= x^2+3x = 3b$ for some $b \in \mathbb{Z}$.
(6) From this equation we find $x^2 = 3b-3x = 3(b-x)$ and so x^2 is a multiple of 3.
(7) Since x is not a multiple of 3, it follows that x^2 is not a multiple of 3.
(8) Therefore x^2 is a multiple of 3 and x^2 is not a multiple of 3. $\Rightarrow\Leftarrow$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only line (2) is not needed. All other lines are needed.

b) All lines are needed.

c) Only line (3) is not needed. All other lines are needed.

d) Only line (4) is not needed. All other lines are needed.

e) Only line (5) is not needed. All other lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a direct proof of the proposition below:

$$\forall\, a, b \in R,\ (a+b)^2 \leq 2\,(a^2+b^2)\,.$$

Which of the following statements or properties do we need to use when composing this proof?

a) $2a^2+2b^2-(a+b)^2 = a^2-2ab+b^2 = (a-b)^2 \geq 0.$

b) $(a+b)^2 = a^2+b^2$

c) Let $a, b \in Q$.

d) $a\cdot(a^2+b^2) = a^3+ab^2$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\forall\, n \in N, \exists\, a \in N,\ a \geq n.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) Nothing can describe an accurate proof strategy since this proposition is false.

b) We would need to let $a \in N$ be an arbitrary natural number and *then* argue that there exists a natural number n that satisfies $a \geq n$.

c) We would need to let $n \in N$ be an arbitrary natural number and *then* argue that a natural number a satisfies $a \geq n$.