# Math 3336
## Homework Assignment 4

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Instructions

- Record your answers to the following 10 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (Click this link for instructions on how to do this).
- Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.

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**1.** Is it possible to have a function $f : A \to B$ where the domain satisfies $|A| = 3$ and the codomain satisfies $|B| = 2$? If you think this is possible, provide an explicit example (make up your own sets $A$ and $B$, and make up your own function $f$). If you think this is impossible explain why.

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**2.** Consider the set $S = \{0, 1\}$. How many different functions $f : S \to S$ are there? List them out by making a diagram for each one.

**3.** Consider the function $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(x) = \lceil\, x \,\rceil - x$$

(a) Is there an element $a$ in the domain that satisfies $g(a) = 0.5$? If you think there is write down such an element and explain why it works. If you think there is not, explain why not.

(b) Is there an element $a$ in the domain that satisfies $g(a) = 1.5$? If you think there is write down such an element and explain why it works. If you think there is not, explain why not.

(c) Which, if any, inputs $a \in \mathbb{R}$ result in the output $g(a) = 0$?

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**4.** Consider the function $g : \mathbb{Q} \to B$ defined by the rule

$$g\left(\frac{a}{b}\right) = a$$

where the input $a/b \in \mathbb{Q}$ is written as a fully reduced fraction. What is the smallest codomain, $B$, one can use for this function?

**5.** Given two real numbers, $a \in \mathbb{R}$ and $r \in \mathbb{R}$, one can form the sequence

$$a_n = a\, r^n.$$

This type of sequence is called a **Geometric Sequence** (and the number $r$ is called the **common ratio**). Check that every Geometric Sequence satisfies the recurrence equation (and initial condition)

$$a_n = r \cdot a_{n-1} \text{ and } a_0 = a.$$

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**6.** Given two real numbers, $a \in \mathbb{R}$ and $d \in \mathbb{R}$, one can form the sequence

$$a_n = d \cdot n + a.$$

This type of sequence is called an **Arithmetic Sequence** (and the number $d$ is called the **common difference**). Check that every Arithmetic Sequence satisfies the recurrence equation (and initial condition)

$$a_n = d + a_{n-1} \text{ and } a_0 = a.$$

**7.** Consider the recursively defined sequence $a_n$ that satisfies the following recurrence equation:

$$a_n = \frac{4}{a_{n-1}}.$$

(a) Suppose we also use the initial condition $a_0 = 2$. Use the recurrence relation to write out the terms $a_1, a_2, a_3$ and $a_4$ (you can write out more if you like). What pattern do you notice about these terms? Do you think the pattern continues?

(b) Suppose we now use the initial condition $a_0 = 1$. Use the recurrence relation to write out the terms $a_1, a_2, a_3$ and $a_4$ (you can write out more if you like). What pattern do you notice about these terms? Do you think the pattern continues?

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**8.** Invent your own recursively defined sequence $a_n$, but make sure it satisfies the following properties:

- $a_n$ must depend on the *three* previous terms $a_{n-1}, a_{n-2}$ and $a_{n-3}$.
- You must use the initial conditions $a_0 = 1, a_1 = -1$, and $a_2 = 1/2$

Calculate the term $a_8$.

**9.** Recall the famous Fibonacci numbers $F_n$ defined by the recurrence relation and initial conditions

$$F_n = F_{n-1} + F_{n-2} \text{ and } F_0 = F_1 = 1.$$

If we use the recurrence relation to extend the sequence *backwards*, we can compute terms like $F_{-1}$ and $F_{-2}$, etc. Continuing this process, one would find that the value of $F_{-4}$ is

(a) $F_{-4} = -1$

(b) $F_{-4} = 1$

(c) $F_{-4} = -2$

(d) $F_{-4} = 2$

(e) $F_{-4} = 0$

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**10.** What did you learn (or re-learn) by working through this assignment? Which questions, if any, were particularly helpful? Which ones, if any, were unhelpful?