3336

Office Hour

10:00 am

unmute to ask questions!

Ceiling

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = \lceil x \rceil$

$f\left(\frac{17}{10}\right) = \left\lceil \frac{17}{10} \right\rceil = 2$

Consider the ceiling function $f(x) = \lceil x \rceil$ with domain $\mathbb{R}$ and co-domain $\mathbb{R}$. Is there an input $c \in \mathbb{R}$ that f sends to the output $\frac{16}{11}$?

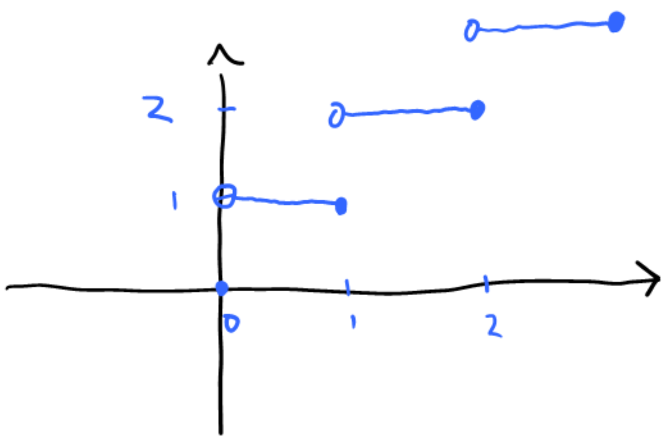
impossible

Consider the ceiling function $f(x) = \lceil x \rceil$ with domain $\mathbb{R}$ and co-domain $\mathbb{R}$. To which element in the co-domain does f send the input $x = \frac{17}{10}$?

a) There is no such element since the expression $17/10$ is not an integer, and f only outputs integers.
b) Because one cannot use $\mathbb{R}$ as the co-domain for the function f, this question has no answer.
c) The input is sent to the output 2.
d) The input is sent to the output 3.
e) The input is sent to the output 1.

a) is tempting if you confuse "domain" + "co-domain"

b) $\mathbb{R}$ is a valid co-domain



a direct proof of $P \Rightarrow Q$

1) assume P
2) use def's, previous results
3) conclude Q

1) assume Q
2) use def's, previous results
3) conclude P

this directly prove

$\underbrace{Q \Rightarrow P}_{\text{"converse"}}$

ex| If two numbers are even, then their sum is even.

P: two numbers are even

Q: their sum is even

compare to "$Q \Rightarrow P$"

If the sum of two numbers is even, then each number is even.

this is false: $3+3=6$

Note to prove $\boxed{P \Rightarrow Q}$ is not the same as proving $Q \Rightarrow P$.

However, it IS THE SAME as proving

$$\boxed{\sim Q \Rightarrow \sim P}$$

Contrapositive

trying to prove $P \Rightarrow Q$

1) assume $\sim Q$
2) use def's, previous results
3) conclude $\sim P$

note this is for an if-then

a proof by contradiction can be used for if-then & other statements

Proof by contradiction

trying to prove S

1) assume $\neg S$
2) use def's, previous results
3) find some contradiction

?!?!!! which one!
"$1=0$" or "2 is odd"

ex| $S: \sqrt{2} \notin \mathbb{Q}$

proof (by contradiction):

Assume $\sqrt{2} \in \mathbb{Q}$.

Suppose one wanted to prove the following proposition using a Proof by Contradiction

$$\text{If } x^2 - 4x + 3 \text{ is even, then } x \text{ is odd .}$$

Which of the following lines could be used as the first sentence in this proof?

a) Suppose x is odd.
b) Suppose x is even.
c) Suppose $x^2 - 4x + 3$ is odd.
d) Suppose $x^2 - 4x + 3$ is even and that x is even.
e) Suppose $x^2 - 4x + 3$ is even.

"Suppose $x^2 - 4x + 3$ is even $\Rightarrow$ x is odd is false."

"Suppose $x^2 - 4x + 3$ is even but x is even"

Proof by Induction

- used when you have a desired formula depending on $n \in \mathbb{N}$

$$\forall\, n \in \mathbb{N},\ P(n)$$

ex) $\forall\, n \in \mathbb{N},\ 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

- prove $P(0)$ is true "Base Case"
- suppose $P(k) \Rightarrow P(k+1)$ "Inductive Step"

Prove $\forall\, n \in \mathbb{N},\ 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$n=1$ ✓

Proof (by induction).

Base Case $(n=1)$

When $n=1$, the LHS is 1 and the RHS is $\frac{1\cdot(1+1)}{2}$

Since these are equal, our base case is checked.

Inductive Step

Suppose the statement is true for some value $n=k \in \mathbb{N}$.

(we want to show its true when $n=k+1$)

$$1+2+3+\cdots+k+k+1 = 1+2+3+\cdots+k + k+1$$

$$= \frac{k(k+1)}{2} + k+1$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{\boxed{k}(k+1) + \boxed{2}(k+1)}{2} = \frac{(k+1)(k+2)}{2} \qquad \square$$