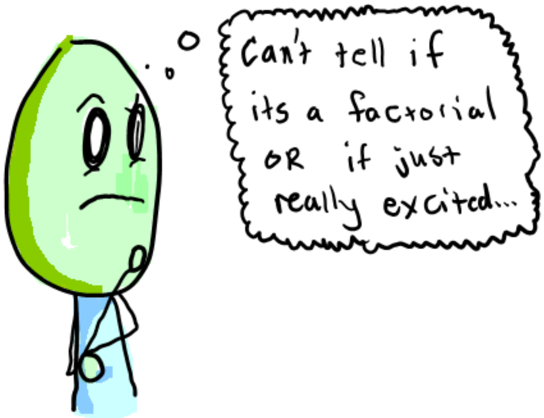
3336

Office Hour

10:00 am



$n!$

$n \cdot (n-1) \cdots 2 \cdot 1$

$0! = 1$

this week: counting techniques

lots of / most of chap. 3 from Book of Proof

Week 8: Book of Proof

- Chapter 3, sections 3.1, 3.2, 3.3, 3.4, 3.5 (this is for the first half of this week)
  - There are videos on each of these sections that go into depth; these were recorded from Spring 2022; link to these long-ish videos + links to their video notes
    - If this material is new to you, then you probably will need to use these videos and the reading from Book of Proof
  - Here is a comprehensive video that summarizes ALL of these sections
    - This video is about one hour and goes through defining examples and defines key concepts
- Chapter 3, sections 3.7, 3.9
  - There are videos on each of these two sections that go into depth; these were recorded from Spring 2022. Link to these long-ish videos + links to their video notes
    - If this material is new to you, then you probably will need to use these videos and the reading from Book of Proof
  - Here is a comprehensive video that summarizes these sections
  - There will be another video discussing the topic of **solving recurrence equations** + here are some written notes on this topic of recurrence equations

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two quizzes + 1 HW

due at end of this week ?

OR due at end of spring break

we will go with this one

$\boxed{\text{Please don't wait to start these!!}}$

Unmute to ask questions!

Reminder: two exams + one final

5. (10 points) One special type of graph is called **a cycle graph**. It is common to notate a Cycle Graph with n vertices as C_n. Which, if any, of the facts about Cycle Graphs would be *most* useful or relevant in the "inductive" portion of an induction proof?

relate next case to previous one

(a) C_n has n edges.

(b) Every vertex of C_n is joined to exactly two other vertices in C_n.

(c) C_n can be drawn as a regular n-gon.

(d) $C_n = C_{n-1}$ plus one additional vertex and one additional edge.

(e) $C_n = C_{n-1}$ plus one additional vertex.

one of these

ind step will need us to relate C_n to C_{n-1} (C_{k+1} to C_k)

examples help show this!!

1) explore examples

C_2:

should/does this count as a cycle graph?

C_3:

C_4:

linked definition

examples

C_5:

In graph theory, a **cycle graph** or **circular graph** is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3, if the graph is simple) connected in a closed chain. The cycle graph with n vertices is called

6. (10 points) One special type of graph is called **a complete graph**. It is common to notate a Complete Graph with n vertices as K_n, and this type of graph is defined via the following property: every pair of distinct vertices in K_n are joined by one edge. Which, if any, of the following claims about Complete Graphs is both true and informative about "recursive structure?"

relate "next ones" to "previous ones"

K_n has n vertices AND every pair of vertices has an edge

(a) $\boxed{K_n = K_{n-1} \text{ plus one additional vertex and } n-1 \text{ additional edges.}}$

(b) $K_3 = C_3$

(c) $K_n = K_{n-1}$ plus one additional edge and n additioinal vertices

(d) $K_n = C_n$

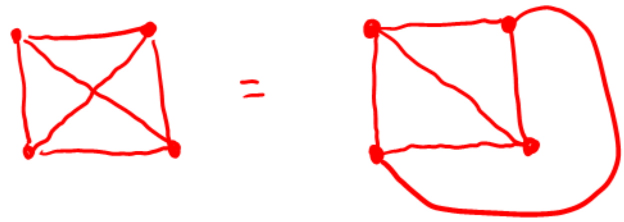
(e) None of the above

1) explore examples

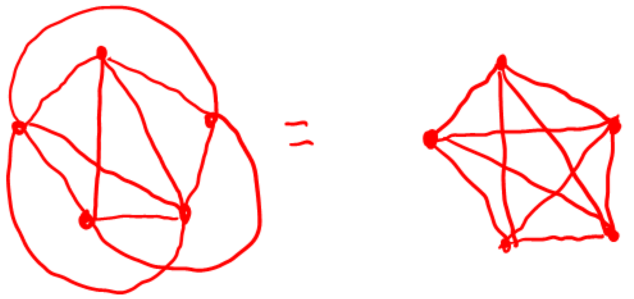
K_2:

K_3:

K_4:



K_5:



Note complete graphs can be used to motivate the "counting" material we are now exploring during week 8 !

K_3:

$n = 3$

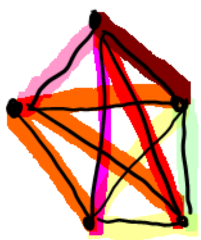
$e = 3$

K_4:

$n = 4$

$e = 6$

K_5:



$n = 5$

$e = 10$

K_{2023}

$n = 2023$

$e = 2,045,253$

K_n

$n = n$

$$\boxed{e = \frac{n(n-1)}{2}}$$

we can create sequences that help us count

a_n = # edges in K_n

$a_3 = 3$ $a_4 = 6$ $a_5 = 10$ $a_6 = 15$ $a_7 = 21$

cool idea: is there recursive structure in this sequence?

is a_n related to a_{n-1} ??

$\boxed{\text{Recursive structure: } a_n = n-1 + a_{n-1}}$

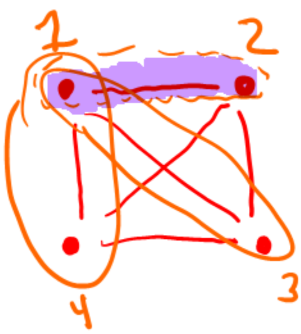
K_n has $\frac{n(n-1)}{2}$ edges

why??

every vertex is connected to every other vertex

every pair of distinct vertices has an edge

how many pairs of vertices are there?



$n \cdot (n-1)$ distinct pairs

double count!

(1,2), (1,3), (1,4)
(2,1), (2,3), (2,4)
(3,1), (3,2), (3,4)
(4,1), (4,2), (4,3)

So to fix the double count, we divide our formula by 2

# of distinct pairs of vertices in K_n = $\boxed{\frac{n(n-1)}{2} = e}$