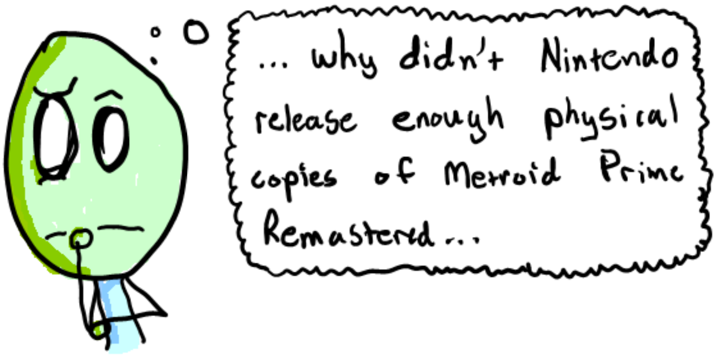
3336

Office Hour

11:00



unmute to ask QUESTIONS

A lovely little proof is presented below:

P Q

Proposition. If $a+7$ is even, then a is odd.

Q

Proof. Suppose a is odd. (We will show $a+7$ is even.) supposed Q !

It follows that $a = 2m-1$ for some $m \in \mathbb{Z}$.

By adding 7 to both sides of this equation we find $a+7 = 2m-1+7 = 2m+6 = 2(m+3)$.

This completes the proof as we have shown $a+7$ is even. $\square$

P

concluded P !

Determine the type of proof used.

a) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

b) A proof by indoctrination was used.

c) A proof by contrapositive was used.

d) A proof by contradiction was used.

e) A direct proof was used.

this "proof" showed $Q \Rightarrow P$ ooops!!

Your answer is CORRECT.

A mathematician wishes to prove the following proposition using a Proof by Induction: 3^{2n-1} is divisible by 8 for $n \geq 1$.

Which of the following can be used as the Base Case part of her proof?

a) $3^2 - 1 = 8$ is divisible by 8.

b) $3^4 - 1 = 80$ is divisible by 8.

c) $3^{k+1} - 1$ is divisible by 8.

d) Suppose the proposition is true for some value, say $n = k$. We now prove it is true when $n = k + 1$.

e) None of the above.

typo

$3^{2n} - 1$ is div. by 8

plug in $n=1$

$3^2 - 1 = 8$ is div. by 8 ✓

Question 3, Quiz 6

Using induction to prove summation formulas

idea: $P(n)$ says $\sum_{i=1}^{n} (\text{stuff}) = \text{formula}$

Base case

$P(n=1) = P(1)$ ← this is easy to check

$\sum_{i=1}^{1} (\text{stuff}) = \text{one term} = \text{formula}$ ✓

Inductive Step

$P(k) \Rightarrow P(k+1)$

$\sum_{i=1}^{k} \text{stuff} = \text{formula}$

$\sum_{i=1}^{k+1} \text{stuff} = \sum_{i=1}^{k} \text{stuff} + (k+1)\text{-term}$

$= \underbrace{\text{formula} + (k+1)\text{-term}}_{\text{combine using algebra}}$

$= \boxed{\text{formula when } n = (k+1)}$

ex) $$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof (by induction)

Base Case ($n = 1$)

$$\sum_{i=1}^{1} i^2 = 1^2 = 1 = \frac{1\cdot(1+1)(2+1)}{6} \checkmark$$

Inductive Step Suppose the statement is true when $n = k \in \mathbb{N}$.

(We want to show the statement is true when $n = k+1$)

This means
$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$
and we want to

show
$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}.$$

It follows that
$$\sum_{i=1}^{k+1} i^2 = \underbrace{1^2 + 2^2 + \cdots + k^2}_{} + (k+1)^2$$
$$= \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

by our inductive hypothesis. We can use "old algebra" to rewrite this as

$$= \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

Note its easy / natural to relate $P(k+1)$ & $P(k)$ when we're dealing with summations:

$$\sum^{k+1} = \sum^{k} + \text{(last term)}$$

If $P(k)$ does not involve a sum, relating $P(k+1)$ & $P(k)$ can require creativity!

5. Use induction to prove the "extended De Morgan's law"

$$\text{For all } n \geq 1, \ \overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \cdots \cup \overline{A_n}$$

Here each A_i is *some* set, and there is *some* arbitrary universal set, U, that contains them all as subsets.

Proof (by induction)

Base Case $(n=1)$

$\overline{A_1} = \overline{A_1}$ ✓

(note: a base case of $n=2$ is our original De Morgan laws: $\overline{A_1 \cap A_2} = \overline{A_1} \cup \overline{A_2}$)