PRINTABLE VERSION

Quiz 5

You scored 100 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose P.
Missing steps involving P and Q and any previously established facts
Therefore Q. $\square$

What type of proof was described in the outline?

a) A proof by introspection is described in this outline.

b) Wait a minute... The proof described in this outline isn't a valid proof technique!

c) A proof by contradiction is described in this outline.

d) A proof by contrapositive is described in this outline.

e) A direct proof is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P \lor Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P$.

b) Suppose $\neg P \land Q$.

c) Suppose $\neg P \land \neg Q$.

d) Suppose $\neg P \lor \neg Q$.

e) Suppose $\neg Q$

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $A \subseteq B$ by...

a) First supposing $x \notin B$, and then showing $x \in A$.

b) First supposing $x \notin A$, and then showing $x \in B$.

c) First supposing $x \in A$, and then showing $x \notin B$.

d) First supposing $x \in B$, and then showing $x \in A$.

e) First supposing $x \in A$, and then showing $x \in B$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \notin B$, and then showing $x \in A$.

b) First supposing $x \notin A$, and then showing $x \in B$.

c) First supposing $x \notin A$, and then showing $x \notin B$.

d) First supposing $x \notin B$, and then showing $x \notin A$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. The empty set is a subset of every set.

Proof. Suppose the proposition is false.
This means $\exists\, S,\ \emptyset \not\subseteq S$.
By (negating the) defintition of subset it follows that $\exists\, x \in \emptyset,\ x \notin S$.
However, this implies that the empty set contains an element x.
Therefore $x \in \emptyset$ and, by definition of $\emptyset$, $x \notin \emptyset$. $\Rightarrow\Leftarrow$

Determine the type of proof used.

a) A direct proof was used.

b) A proof by contrapositive was used.

c) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

d) A proof by contradiction was used.

e) A proof by indoctrination was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)

(1) Suppose $x, y \in \mathbb{Z}$ are integers.
(2) We can assume x is odd and that y is even.
(3) Since x is odd, it follows that $\exists\, y \in \mathbb{Z},\ x = 2y + 1$.
(4) Since y is even, it follows that $\exists\, m \in \mathbb{Z},\ y = 2m$.
(5)We now have $x + y = (2y+1) + y = 3y + 1 = 3(2m) + 1 = 2(3m) + 1$.
(6) Because $x + y$ has the form of an odd number it is odd. □

Identify the mistake, if any, in this proof.

a) There is an error in Line (1) since we cannot simply assume x, y ∈ Z .

b) There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.

c) There is an error in Line (2) since we do not know which integer is odd or even.

d) There is an algebraic mistake in Line (5).

e) There is an error in Line (4) since where the definition of "even" is misapplied.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. $\forall\, n \in \mathbb{N},\ n(n+1)$ is even.

Proof. (Direct)

(1) Let $n \in \mathbb{N}$. We will use cases to prove the proposition.

Case 1: n is even.
(2) In this case we have $n = 2m$ for some $m \in \mathbb{N}$.
(3) Because n is even, it follows that when n is divided by 2, there is no remainder.
(4) n being even implies $n + 1$ is odd.
(5) It now follows that $n(n+1) = (2m)(2m+1) = 2 \cdot \big(m(2m+1)\big)$ which has the form of an even number.
(6) Therefore $n(n+1)$ is even, proving the proposition in this case.

Case 2: n is odd.
(7) In this case we have $n = 2\ell + 1$ for some $\ell \in \mathbb{Z}$.
(8) If n is not odd then it is even and Case 1 applies.
(9) It follows that $n(n+1) = (2\ell+1)(2\ell+1+1) = (2\ell+1)(2\ell+2) = 2(2\ell+1)(\ell+1)$
(10) Because the expression above has the form of an even number, $n(n+1)$ is even.
(11) If $n(n+1)$ is odd, then there is a contradiction.
(12) This completes the proof. □

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only lines (3),(4),(8), and (11) are not needed. All other lines are needed.

b) Only line (8) is not needed. All other lines are needed.

c) Only lines (4) and (11) are not needed. All other lines are needed.

d) All lines are needed.

e) Only lines (3) and (11) are not needed. All other lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a contrapositive proof of the proposition below:

$$\forall x \in Z,\ \text{ if } x^2 - 6x + 5 \text{ is even, then } x \text{ is odd.}$$

Which of the following statements or properties do we need to use when composing this proof?

a) $x^2 - 6x + 5 \leq x^2 - 6x + \pi.$

b) $(2a)^2 - 6(2a) + 5 = 10a^2 - 12a + 5 = 2\,(5a^2 - 6a + 2)\,.$

c) $(2a)^2 - 6(2a) + 5 = 10a^2 - 12a + 5 = 2\,(5a^2 - 6a + 2) + 1.$

d) Therefore $x = 2a + 1$ for some $a \in Z$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\forall n \in N, \exists a \in N,\ a \geq n.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) We would need to let $n \in N$ be an arbitrary natural number and *then* argue that a natural number a satisfies $a \geq n$.

b) We would need to let $a \in N$ be an arbitrary natural number and *then* argue that there exists a natural number n that satisfies $a \geq n$.

c) This can only be proved by paying someone else to do it for me.