# Discrete Math

## Lecture 4

## Biconditional Statements

Recall: $P \iff Q$

"P if & only if Q"

$$(P \iff Q) \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

note: the "converse" of $P \Rightarrow Q$ is $Q \Rightarrow P$

$P \iff Q$ is T when P and Q have the same truth value

is F otherwise

**Example 4.1.**

(10) *The converse of "If a whole number is a multiple of 4, then it is even" is "If a whole number is even, then it is a multiple of 4."*

(11) *The converse of* $(1+2=3) \Rightarrow (\sin^2(\pi/7)+\cos^2(\pi/7)=2)$ *is* $(\sin^2(\pi/7)+\cos^2(\pi/7)=2) \Rightarrow (1+2=3)$

(12) *The converse of "If a shape is a square, then it is a rectangle" is "If* a shape is a rectangle, *then* it is a square."

(13) *The converse of "*$2^{13}+4$ *is even* $\Rightarrow (9^{-1})^{-1}=9$*" is* $(9^{-1})^{-1}=9 \Rightarrow 2^{13}+4$ is even

P(n): n is a multiple of 4

Q(n): n is even

converse: if Q(n), then P(n).

$P \Rightarrow Q$ ⟶ (converse) $Q \Rightarrow P$

**Example 4.4.** *Suppose $P$ and $Q$ are statements where $P \Rightarrow Q$ is vacuously true, and suppose we know that $P \iff Q$ is true. What is the truth value of statement $Q$?*

"vacuously true" means P is F

$P \iff Q$ is true means P & Q have the same truth value

it follows since P is F, Q is also F ✓

**Definition 1.5.** *A **bi-conditional statement** is a proposition of the form $P \iff Q$. It is true whenever $P$ and $Q$ have the same truth value, and it is false otherwise.*

| $P$ | $Q$ | $P \iff Q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ |

Truth Table for $\iff$

| Notation | English Phrasings |
|---|---|
| $P \iff Q$<br>P iff Q | $P$ if and only if $Q$<br>$P$ is necessary and sufficient for $Q$<br>If $P$ then $Q$, and conversely.<br>$P$ is logically equivalent to $Q$<br>$Q$ whenever $P$, and conversely |

ex| this video will be done if and only if its good enough.

this video will be done <=> its good enough.

note| alternate notation

P <=> Q

P <-> Q

P iff Q

P ≡ Q