PRINTABLE VERSION

Quiz 5

You scored 100 out of 100

Question 1
Your answer is CORRECT.
An outline for a proof of an implication $P\Rightarrow Q$ is provided below:
a) A proof by contradiction is described in this outline.
b) A direct proof is described in this outline.
c) A proof by contrapositive is described in this outline.
d) Wait a minute The proof described in this outline isn't a valid proof technique!
e) A proof by introspection is described in this outline.
Question 2
Your answer is CORRECT.
Suppose a mathematician wants to prove a statement of the form P . However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?
a) \bigcirc Suppose $\neg P \lor \neg Q$.
b) \bigcirc Suppose $\neg P \land \neg Q$.
c) Suppose ¬Q
d) \odot Suppose $\neg P$.
e) \bigcirc Suppose $\neg P \land Q$.
Question 3
Your answer is CORRECT.
Given two sets A and B one can prove $B \subseteq A$ by
a) \bigcirc First supposing $x \in A$, and then showing $x \in B$.
b) \odot First supposing $x \in B$, and then showing $x \in A$.
c) \bigcirc First supposing $x \notin A$, and then showing $x \in B$.
d) \bigcirc First supposing $x \in A$, and then showing $x \notin B$.

e) \bigcirc First supposing $x \notin B$, and then showing $x \in A$.
Question 4
Your answer is CORRECT.
Given two sets A and B one can prove $B \subseteq A$ by
a) \bigcirc First supposing $x \notin A$, and then showing $x \in B$.
b) \odot First supposing $x \notin A$, and then showing $x \notin B$.
c) \bigcirc First supposing $x \notin B$, and then showing $x \in A$.
d) \bigcirc First supposing $x \notin B$, and then showing $x \notin A$.
e) \bigcirc First supposing $x \in A$, and then showing $x \notin B$.
Question 5
Your answer is CORRECT.
A lovely little proof is presented below:
Proposition. If $a + 7$ is even, then a is odd.
Proof. Suppose a is odd. (We will show $a+7$ is even.) It follows that $a=2m-1$ for some $m\in\mathbb{Z}$. By adding 7 to both sides of this equation we find $a+7=2m-1+7=2m+6=2(m+3)$. This completes the proof as we have shown $a+7$ is even. \square
Determine the type of proof used.
a) O A proof by contrapositive was used.
b) Wait a minute This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.
c) A proof by indoctrination was used.
d) A direct proof was used.
e) A proof by contradiction was used.
Question 6
Your answer is CORRECT.
A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)

- (1) Suppose $x, y \in \mathbb{Z}$ are integers.
- (2) We can assume x is odd and that y is even.
- (3) Since x is odd, it follows that $\exists y \in \mathbb{Z}, x = 2y + 1$.
- (4) Since y is even, it follows that $\exists m \in \mathbb{Z}, y = 2m$.
- (5) We now have x + y = (2y + 1) + y = 3y + 1 = 3(2m) + 1 = 2(3m) + 1.
- (6) Because x + y has the form of an odd number it is odd. \square

Identify the mistake, if any, in this proof.

- a) \bigcirc There is an error in Line (1) since we cannot simply assume $x, y \in Z$.
- b) There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.
- c) There is an error in Line (2) since we do not know which integer is odd or even.
- d) There is an algebraic mistake in Line (5).
- e) There is an error in Line (4) since where the definition of "even" is misapplied.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \cup B = B$, then $A \subseteq B$.

Proof. (Direct)

- (1) Suppose $A \cup B = B$. To prove $A \subseteq B$ we also let $x \in A$ and will end the proof by showing $x \in B$.
- (2) Because B is a set $\emptyset \subseteq B$.
- (3) Since $A \subseteq A \cup B$ and $x \in A$ it follows that $x \in A \cup B$.
- (4) Since, by assumption $A \cup B = B$ it also follows that $x \in B$.
- (5) Because $A \cup B = B$ a Venn diagram shows that $A \subseteq B$.
- (6) If $x \notin B$, then there would be a contradiction. \square

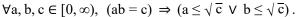
One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

- a) Only lines (1) and (2) are not needed. All other lines are needed.
- **b)** Only lines (3) and (4) are not needed. All other lines are needed.
- c) Only lines (2) and (5) are not needed. All other lines are needed.
- d) Only ;ines (1) and (5) are not needed. All other lines are needed.
- e) All lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a proof by contradiction of the proposition below:



Which of the following statements or properties do we need to use when composing this proof?

- a) \bigcirc The fact that for real numbers x, y, if x > y then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.
- **b)** The fact that for real numbers $x, y, \sqrt{x} < \sqrt{y}$.
- c) Suppose ab = c and that both $a < \sqrt{c}$ and $b < \sqrt{c}$.
- d) Suppose ab = c and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true. The recursively defined sequence $a_n = a_{n-1} - 1$ with initial conditions $a_0 = \pi$ has a term that is negative. Of the following options, which one best describes what needs to be done in order to prove this claim?

- a) We need only check that the claim is true for one, single natural number.
- b) \bigcirc We need to show the claim is true for an abitrary natural larger than 3, saying something like "Let $n \ge 4$."
- c) Onothing can describe an accurate proof strategy since this proposition is false.