PRINTABLE VERSION

Quiz 8

You scored 100 out of 100

Question 1

Your answer is CORRECT.

A clockmaker assigns to each clock produced a serial number consisting of 3 capital letters of the English alphabet followed by 5 numerals (0 through 9). Here is one example of such a serial number:

$$TTX31105$$

How many different serial numbers are possible if repetition of letters and digits is allowed?

a) ◉ $26^3 \cdot 10^5$

b) ○ $26^3 \cdot 9^5$

c) ○ $26^5 \cdot 9^3$

d) ○ $26^5 \cdot 10^3$

e) ○ $3 \cdot 5$

Question 2

Your answer is CORRECT.

Consider making lists from the symbols $T, U, V, W, X, Y, Z.$ How many length-4 lists are possible if repetition is not allowed and the list must contain a W (in any position)?

a) ○ $7 \cdot 6 \cdot 5 \cdot 4$

b) ◉ $4 \cdot (6 \cdot 5 \cdot 4)$

c) ○ $7^4 - 6^4 = 1105$

d) ○ 7^4

e) ○ $4!$

Question 3

Your answer is CORRECT.

Of the options provided below, which one best completes the sentence "The notation $n!$ ____________. "

a) ◉ refers to the number of non-repetitive length n lists that can be made from n symbols

b) ○ $= \dfrac{n!}{k!(n-k)!}$

c) ○ is very angry about natural numbers

d) ○ refers to the number of ways a non-repetitive length-k list may be formed using n symbols

e) ○ $= n^n$

Question 4

Your answer is CORRECT.

Suppose the set S has 0 elements. How many subsets of size 4 are there?

a) ○ 17

b) ○ 4

c) ○ 24

d) ◉ 0

Question 5

Your answer is CORRECT.

A (numerical) palindrome is a natural number that, when expressed in our standard digit system, reads the same forward as backward. For example, the number 12021 is a palindrome, as is 353. How many 3 digit palindromes are there?

a) ◉ $9 \cdot 10$

b) ○ 10^2

c) ○ $9 \cdot 10^2$

d) ○ 10^3

e) ○ $9^2 \cdot 10$

Question 6

Your answer is CORRECT.

This problem concerns lists of length 14 made from the (capital letters from the) English alphabet $A, B, C, \ldots, Y, Z$. How many lists will contain the word HOUSTON?

a) ○ 7^{13}

b) ◉ $8 \cdot 26^7$

c) ○ 26^7

d) ○ 7^{26}

e) ○ 26^{13}

Question 7

Your answer is CORRECT.

Of the options provided below, which one best explains why the following formula is true?

$$\binom{n}{k} = \binom{n}{n-k}$$

a) ◉ The expression on the left counts the number of k -element subsets of an n -element set. Because each such subset can be matched up with a subset of size $(n-k)$ (by taking its complement), this also counts the number of size-$(n-k)$ subsets, which is precisely what the expression on the right counts.

b) ○ The expression on the left counts the number of k -element subsets of an n -element set.

The expression on the right counts the number of $(n-k)$ -element subsets of an n -element set.

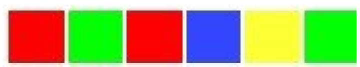
c) The equation follows from using the formulas $\binom{n}{k} = \frac{n!}{(n-k)!} = \frac{n!}{(n-(n-k))!} = \binom{n}{n-k}$.

d) No explanation can be given becuase this equation is not true.

Question 8

Your answer is CORRECT.

A length-n "color band" is a sequence of n squares arranged along a single row, where each square has been filled in with a particular color *and* the coloring obeys this one rule: *no two adjacent squares can have the same color.* An example of a length-6 color band is shown below:



How many length-3 color bands are possible when we are only allowed to pick from 7 colors?

a) 7^3

b) $7^1 \cdot 6^2$

c) $\frac{7!}{(7-3)!} = 210$

d) $7 \cdot 6^2$

e) $\frac{3!}{(3-7)!} = 0$

Question 9

Your answer is CORRECT.

How many 16-digit binary strings contain exactly 4 zeroes?

a) $\binom{16}{4} = 1820$

b) ○ $\binom{4}{16} = 0$

c) ○ $2^{16} - 2^4 = 65520$

d) ○ 2^{12}

Question 10

Your answer is CORRECT.

Thank you for working hard on this quiz! As a token of your instructor's appreciation, take just a few moments to enjoy answering this question: Which of the following most accurately summarizes the content of this quiz?

a) ○ Counting cards isn't illegal, but it can get you banned from casinos.

b) ○ The derivive of $\sin x$ is $\cos x$.

c) ◉ Counting strings (and related objects) uses the Multiplication Principle and often involves expressions like $n!$ or $\frac{n!}{(n-k)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)$.
Counting subsets (and related objects) is related to counting strings, but there are fewer subsets than strings since order doesn't matter; counting subsets (and related objects) often uses expressions like $\binom{n}{k} = \frac{n!}{n!(n-k)!}$.

d) ○ Counting is super easy! We learned about it when I was, like, six years old.

e) ○ None of the above.