Math 3336
Homework Assignment 3

Instructions

- Record your answers to the following 9 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (Click this link for instructions on how to do this).
- Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.

---

1. Write out the resulting set in "set-builder notaiton" (that is, list out the elements in between squiggly braces { } If the set is inifinte, you can use "$\cdots$" after listing out enough of the elements). No work need be included for this question.

(a) $\mathbb{Q} \cap \{0, -1, -2, 1/2, 1/3, 1/4, \pi\} =$

(b) $(\mathbb{N} \cup \mathbb{Z}) \cap \left\{-4, -3, -2, -1, 0, 1, 2, \sqrt{2}, \pi\right\} =$

(c) $\mathbb{Z} - \mathbb{N} =$

(d) $\mathbb{Q} \cap (\mathbb{R} - \mathbb{Q}) =$

---

2. What does it mean for two sets A and B to satisfy $A \cap B = \emptyset$?

(a) $A = \emptyset$

(b) $B = \emptyset$

(c) This means the two sets have no elements in common; they are **disjoint**.

(d) $A \subseteq B$.

(e) $B \subseteq A$.

---

3. Is it possible for a set A to satisfy $|\mathcal{P}(A)| = 9$? If it is, give an example. If it is not possible, explain why.

---

4. Sketch a picture of the Cartesian product $\{-1, 0, 1\} \times \mathbb{R}$ as a subset of the usual "xy-plane $\mathbb{R}^2$."

5. Consider the set $A = \{-2, \heartsuit, 1/2\}$. Suppose we are told that

$$|A \times \mathcal{P}(B)| = 96$$

where B is some other finite set. What is the cardinality of set B? How do you know your answer is correct?

---

6. Suppose we have two sets S and T, each one defined by specifying conditions for its elements:

$$S = \{\forall\, x \in U : P(x)\} \text{ and } T = \{\forall\, x \in U : Q(x)\}.$$

Which of the following statements tells us $S = T$?

(a) $\forall\, x \in U,\ P(x) \Rightarrow Q(x)$

(b) $\exists\, x \in U,\ P(x) \iff Q(x)$.

(c) $\forall\, x \in U,\ P(x) \iff Q(x)$.

(d) $\exists\, x \in U,\ P(x) \Rightarrow Q(x)$.

Which of the following statements tells us $S \subseteq T$?

(a) $\forall\, x \in U,\ P(x) \Rightarrow Q(x)$

(b) $\exists\, x \in U,\ P(x) \iff Q(x)$.

(c) $\forall\, x \in U,\ P(x) \iff Q(x)$.

(d) $\exists\, x \in U,\ P(x) \Rightarrow Q(x)$.

7. Suppose we have two sets that satisfy $A \cup B = B$. Does it follow that $A \subseteq B$ or does it follow that $B \subseteq A$? Explain your answer.

8. One familiar set is missing from the "chain of inclusion" below. Fill it in! (No work need be included for this question.)

$$\emptyset \subseteq \{0\} \subseteq \mathbb{N} \subseteq \_\_\_\_ \subseteq \mathbb{Q} \subseteq \mathbb{R}.$$

9. What did you learn (or re-learn) by working through this assignment? Which questions, if any, were particularly helpful? Which ones, if any, were unhelpful?