3336

Office Hour

11:00 am

start time

(meeting closes at 11:10 if no one shows)

Unmute to ask questions!

A (numerical) palindrome is a natural number that, when expressed in our standard digit system, reads the same forward as backward. For example, the number 12021 is a palindrome, as is 353. How many 12 digit palindromes are there?

1) explore (small / easy) examples

5-digit palindromes

11111, 22022, 55455

95759, ~~03830~~

not an example! only 4 digits!! (starts w/ 0)

2 3 5 3 2

only one choice

only one choice

_ _ _ _ _

9 10 10 1 1

mult. principle tells us

$9 \cdot 10 \cdot 10 = 9 \cdot 10^2 = 900$

$\boxed{\text{there are 900 5-digit palindromes}}$

How many 8-digit palindromes are there?

_ _ _ _ _ _ _ _

9 10 10 10 1 1 1 1

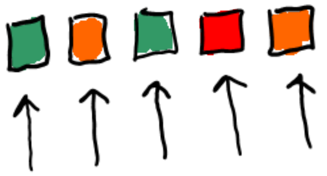
$9 \cdot 10^3 = 9000$ 8-digit palindromes

How many length-5 color bands are possible when we are only allowed to pick from 5 colors?

color band: • a list of squares

• each square is colored

• no two adjacent squares can have same color



$$\underbrace{5 \quad 4 \quad 4 \quad 4 \quad 4}$$

mult. princ. tells us

there are $5 \cdot 4^4$ length-5 color bands (5 colors)

How many 16-digit binary strings contain exactly 4 zeroes?

Q1 is it clear what a binary string is?

a binary string is a list of 0's and/or 1's

ex| length-4 binary string

0010, 1111, 1000, 0110, 1101, etc.

Q2 how many length-16 binary strings are there?

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}$$

2 choices for each digit a total of 2^{16} such strings

note: $2^{16} = 65{,}536$ that's a LOT of strings!!

you DONT want to write all of these out + then count the ones that have four 0's!

this would take too long!!!

we need a more time-efficient strategy to count the length-16 binary strings that contain exactly 4 zero's!

Smaller example: count the # of length-6 binary strings that contain exactly two 0's.

0 0 1 1 1 1   0 1 1 0 1 1   1 1 1 0 1 0

we have to place the two 0's somewhere → every other digit must be 1

we're really counting the number of places (two of them) for our 0's from a total of six

this is "six choose two" $= \binom{6}{2}$

originally, $\binom{n}{k}$ = # of size-k subsets of an n-element set $= \dfrac{n!}{k! \cdot (n-k)!}$

$\binom{6}{2}$ = # of size-2 subsets of a 6-element set $= \dfrac{6!}{2! \cdot 4!}$

— — — — — —

$$\binom{6}{2} = \frac{\overset{3}{\cancel{6}} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{2} \cdot 1 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}} = \boxed{15}$$

How many 16-digit binary strings contain exactly 4 zeroes?

$$\binom{16}{4} = \frac{16!}{4! \cdot 12!} = \frac{\overset{4}{\cancel{16}} \cdot \overset{5}{\cancel{15}} \cdot 13 \cdot \cancel{12!}}{\cancel{4} \cdot \cancel{3} \cdot 2 \cdot 1 \cdot \cancel{12!}}$$

$$= \frac{\overset{2}{\cancel{4}} \cdot 5 \cdot 13}{\cancel{2}} = \boxed{130}$$

<u>note</u> choosing places for 4 zeroes = choosing places for 12 ones

$$\Rightarrow \quad \binom{16}{4} = \binom{16}{12}$$

more generally: $\binom{n}{k} = \binom{n}{n-k}$

you can see this in the formula, too!

Dice Rolling Question

How many times do you need to roll a dice so that a number is repeated at least two times?

two times? three times? four times??

two times is not enough!!!

three is not enough!

first roll: shows 3

second roll: shows 5

third roll: shows 1

fourth roll: shows 2

fifth roll: shows 4

sixth roll: shows 6

Seventh roll: shows 1, 2, 3, 4, 5 or 6 repeat!!!!

How many rolls are required so that a value is repeated 3 times???

13 rolls (or more!!) are required

Seven rolls will require at least one value to be repeated twice but does <u>not</u> ensure a value is repeated 3-times!

2, 1, 6, 4, 3, 5, 1, 6, 4, 2, 3, 5, ___

13th roll