# Discrete Math

## Lecture 15

### Functions

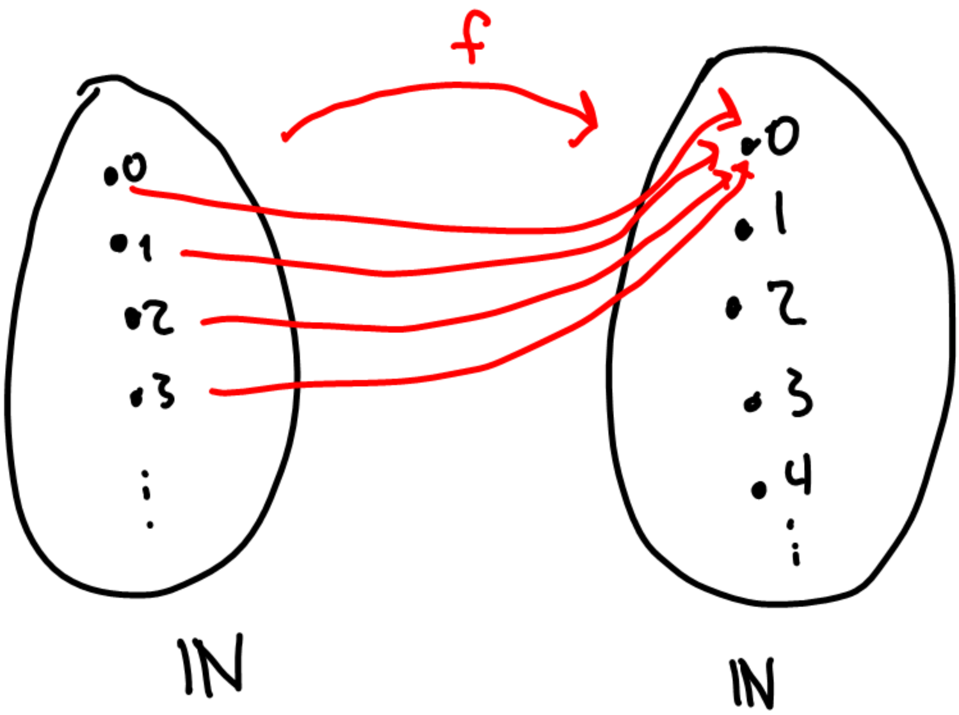
$$f : A \longrightarrow B$$

domain        codomain

we need to be able to follow the "rule"

ex] $f : \mathbb{N} \longrightarrow \mathbb{N}$

$$f(n) = 0$$



ex] $f: \mathbb{N} \to \mathbb{N}$

$f(n) = n^3$

ordered pairs of (inputs, outputs)

| | | |
|---|---|---|
| $f(0) = 0$ | $0 \mapsto 0$ | $(0,0)$ |
| $f(1) = 1$ | $1 \mapsto 1$ | $(1,1)$ |
| $f(2) = 8$ | $2 \mapsto 8$ | $(2,8)$ |
| $f(3) = 27$ | $3 \mapsto 27$ | $(3,27)$ |

non-ex] $f(x)$ = solution to

$f: [0,\infty) \longrightarrow \mathbb{R}$ $\quad (f(x))^2 = x$

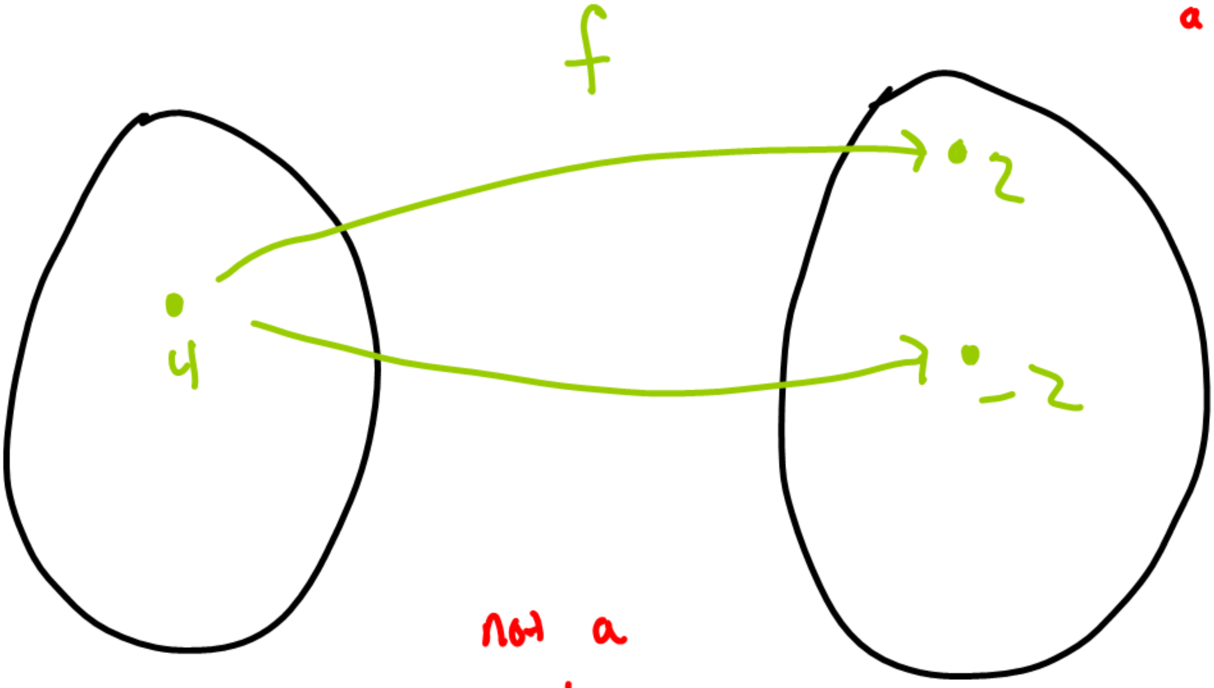
this does NOT define a function because we can't follow this "rule"

ex] $x = 4$ $\quad f(x) = f(4) = ?$

$$(f(4))^2 = 4 \begin{cases} f(4) = 2 \\ f(4) = -2 \end{cases}$$

one input $\longmapsto$ more than one output X

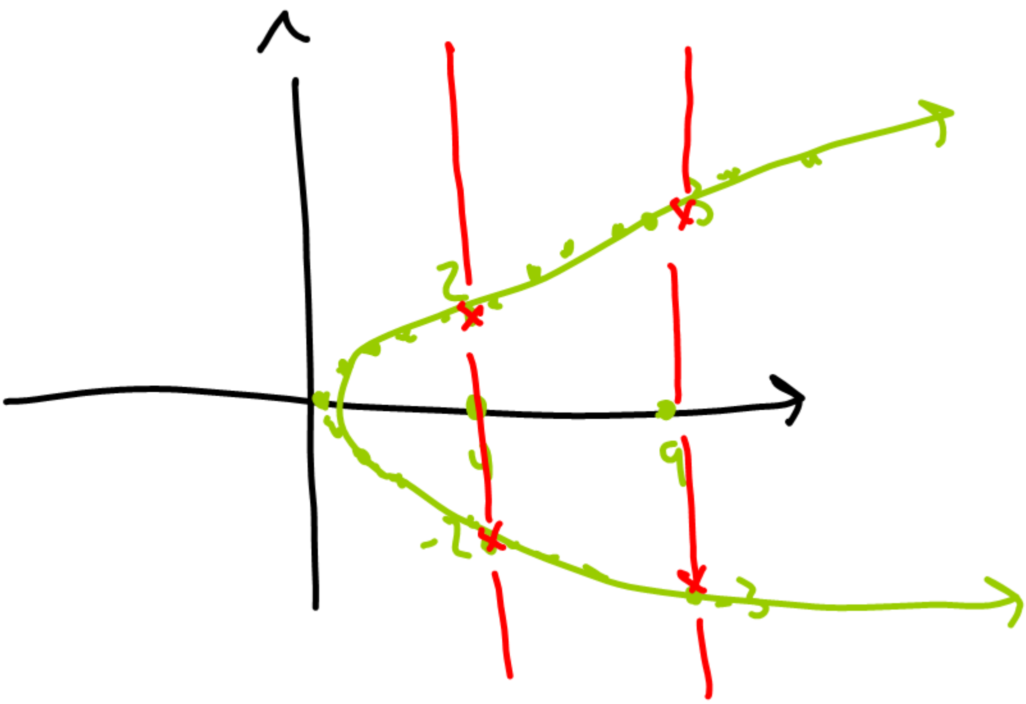
not a function




$A = [0, \infty)$

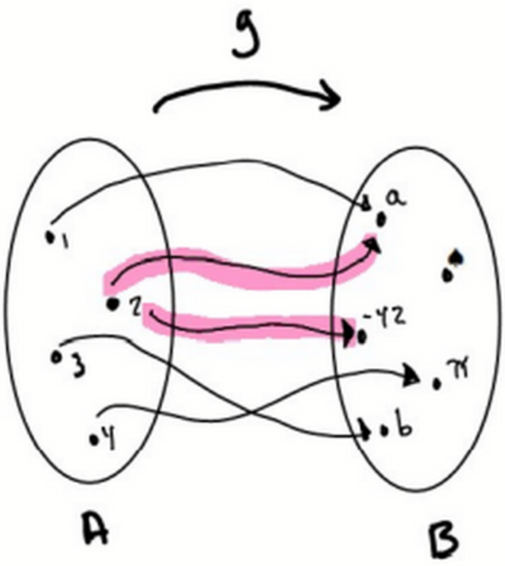
not a rule we can follow

$B = \mathbb{R}$




VLT fails

**Example 4.4.** *Consider the diagram below:*




*This diagram is presented and labelled in a way that suggestions $g : A \to B$ is a function, however* ***this is not a function!*** *The issue is highlighted in red: $g$ attempts to send the element $2 \in A$ to two different outputs in the codomain $B$. According to the diagram, $g(2) = a$ and $g(2) = -42$ – which output do we assign? We are being asked to follow to conflicting rules, and so there is no single rule.*

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Calc 1 $\quad f : \mathbb{R} \longrightarrow \mathbb{R}$

Calc III $\quad c : \mathbb{R} \longrightarrow \mathbb{R}^2 \qquad c(t) = (1+t,\ t^2)$

$d : \mathbb{R} \longrightarrow \mathbb{R}^3 \qquad d(t) = (5,\ \cos(t),\ e^t)$

$F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$G : \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \qquad G(u,v) = (u^2 - v^2,\ 2uv,\ u)$

$H : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

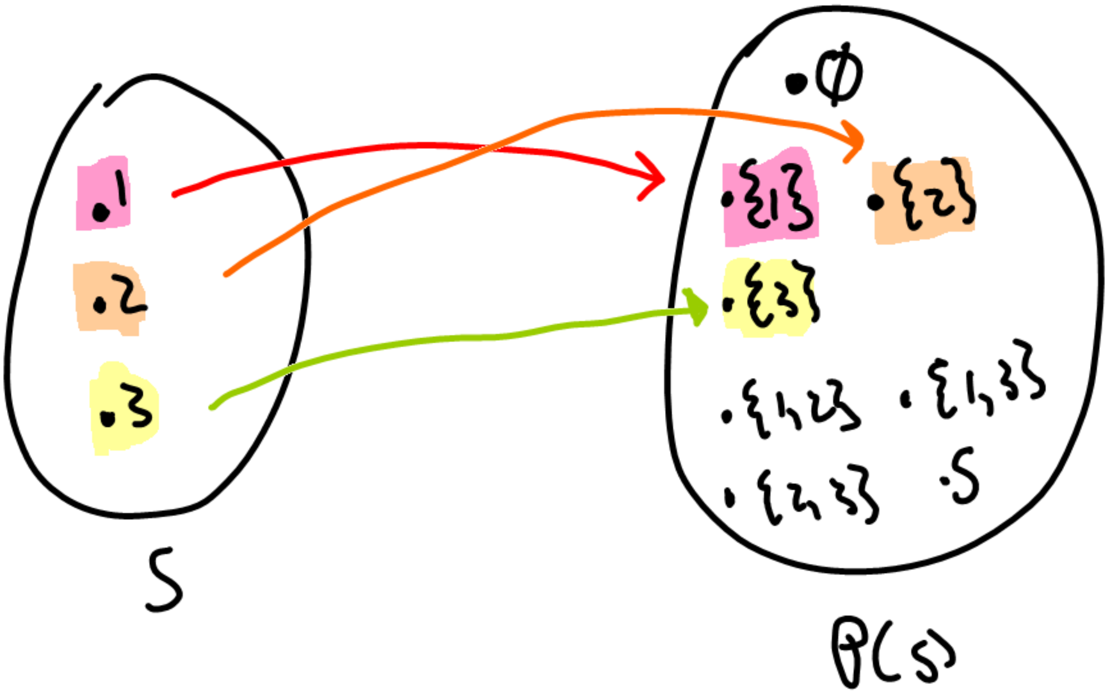
$J : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

ex| $S = \{1, 2, 3\}$

$$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$$f : S \to P(S)$$

$$f(x) = \{x\}$$




Note: not all elements in codomain were used!

ex: $\forall x \in S,\ f(x) \neq \emptyset$

"$\emptyset$ was not <u>hit</u> or <u>used</u>"

ex] $g(\theta) = \cos\theta$

$g : \mathbb{R} \longrightarrow \mathbb{R}$

$g(\theta) = \cos\theta$

$g(\theta) \in [-1, 1]$

note all elements in the domain have to be used!

ex] $f(x) = 1/x$

$f : \mathbb{R} \rightarrow \mathbb{R}$

×

we can't plug $x = 0$ into $f$ !

$f : (\mathbb{R} - \{0\}) \longrightarrow \mathbb{R}$ ✓

note: functions are kind of everywhere!

$$\text{at UH} \quad \{\text{students}\} \xrightarrow{h} \mathbb{N}$$

$$h(\text{student}) = \text{PSID}$$

"look up tables"

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Two functions you will care about:

the "floor function"

$$\lfloor \ \rfloor : \mathbb{R} \longrightarrow \mathbb{Z}$$

$\lfloor x \rfloor =$ the integer you get by "rounding $x$ down"

$$\lfloor 3.5 \rfloor = 3 \qquad \lfloor -7 \rfloor = -7$$

the "ceiling function"

$\lceil\ \rceil : \mathbb{R} \longrightarrow \mathbb{Z}$ "rounds up"

$\lceil 3.5 \rceil = 4$ $\lceil 3.0001 \rceil = 4$

given any $y \in \mathbb{Z}$, you can always solve

$\lceil x \rceil = y$ , $\lfloor x \rfloor = y$

$$\forall y \in \mathbb{Z}, \exists x \in \mathbb{R}, \lceil x \rceil = y$$

ex) $1000 \in \mathbb{Z}$

$\lceil x \rceil = 1000$ ?

$x = 1000$ $x = 999.7$

note we could have said

$$\lceil\ \rceil : \mathbb{R} \longrightarrow \mathbb{R}$$

but this is not the smallest possible codomain for $\lceil\ \rceil$

also: $\lceil\ \rceil : \mathbb{R} \longrightarrow \mathbb{Q}$

also: $\lceil\ \rceil : \mathbb{R} \longrightarrow \mathbb{Z}$

small as possible