

PRINTABLE VERSION

Quiz 11

You scored 90 out of 100

Question 1

Your answer is CORRECT.

The congruence equation " $-41 \equiv -105 \pmod{16}$ " means

- a) ☐ $-41 \mid (16 - (-105))$
- b) ☐ $(-105) \mid (-41 - 16)$
- c) ☐ $(-41 - (-105)) \mid 16$
- d) ☒ $16 \mid (-41 - (-105))$

Question 2

Your answer is CORRECT.

The integers 93 and 38 are congruent mod n for which value of n ?

- a) ☐ $n = 93$
- b) ☐ There are no values of n for which these two integers are congruent (except $n = 1$).
- c) ☐ $n = 38$
- d) ☐ $n = 6$
- e) ☒ $n = 5$

Question 3

Your answer is CORRECT.

Consider the following proposition:

Proposition. If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.

If you were writing a direct proof of this proposition, which of the following statements could be used as your last line?

- a) ☐ Therefore remainders exist.
- b) ☐ Therefore n is a multiple of $a^2 - b^2$.
- c) ☐ Therefore b^2 is a multiple of $a^2 - n$.
- d) ☐ Therefore a^2 is a multiple of $n - b^2$.
- e) ☒ Therefore $a^2 - b^2$ is a multiple of n .

Question 4

Your answer is CORRECT.

Is the following statement true or false?

$$\exists x, y, a, b \in \mathbb{Z}, n \in \mathbb{N}^*, (x \equiv a \pmod{n} \wedge y \equiv b \pmod{n}) \Rightarrow xy \equiv ab \pmod{n}.$$

(Note: for this problem \mathbb{N}^* refers to the positive natural numbers

$$\mathbb{N}^* = \mathbb{N} - \{0\} = \{1, 2, 3, \dots\} \quad .)$$

- a) ☒ This statement is false.
- b) ☐ This statement is true.

Question 5

Your answer is CORRECT.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Undefined control sequence \square

- a) ☒ Technically no proposition was proven true since there is an algebraic mistake in Line (3).
- b) ☐ If you add up six consecutive integers, then the result is equivalent to 1 mod 6.
- c) ☐ If $x \in \mathbb{Z}$ then $\sum_{i=0}^5 x + i \equiv 0 \pmod{6}$.
- d) ☐ The sum of 6 consecutive integers is never congruent to 0 mod 6.

Question 6

Your answer is CORRECT.

Use the Euclidean Algorithm to find a solution to the congruence equation $-18x \equiv 1 \pmod{49}$ (if a solution exists).

- a) ☒ $x = 19$ is a solution.
- b) ☐ $x = -49/18$ is a solution.
- c) ☐ $x = -1/18$ is a solution.
- d) ☐ There are no solutions because $\gcd(-18, 49) \neq 1$.
- e) ☐ $x = 49$ is a solution.

Question 7

Your answer is **INCORRECT**.

Of the options provided below, determine the one that best completes this sentence: "The modular equation $35x \equiv -27 \pmod{24}$ _____"

- a) ☒ has multiple solutions.
- b) ☐ has exactly one solution.
- c) ☐ has no solutions.

Question 8

Your answer is **CORRECT**.

Which steps should one take when solving a congruence equation $ax \equiv b \pmod{n}$? A helpful summary is presented below, only one step is missing:

Steps for solving $ax \equiv b \pmod{n}$.

Step 1.

Step 2. If $\gcd(a, n) \mid b$, then proceed to step 3, otherwise there are no solutions.

Step 3. Use work from Step 1 to calculate one solution $x_0 \in \mathbb{Z}$.

Step 4. Add $\frac{n}{\gcd(a, n)}$ to x_0 to create other solutions.

Of the following options, which could be used for the missing Step 1?

- a) ☐ Step 1. Multiply both sides by $1/a$.
- b) ☐ Step 1. Divide n by b .

- c) ☒ Step 1. Use the Euclidean Algorithm to compute $\gcd(a, n)$.
- d) ☐ Step 1. Use the Euclidean Algorithm to compute $\gcd(b, n)$.
- e) ☐ Step 1. Use the Euclidean Algorithm to compute $\gcd(b, n)$.

Question 9

Your answer is CORRECT.

Find a solution to the congruence equation $17x \equiv -15 \pmod{5}$.

- a) ☒ $x = 25$ is a solution.
- b) ☐ $x = 5/17$ is a solution.
- c) ☐ $x = 6$ is a solution.
- d) ☐ $x = 15/17$ is a solution.
- e) ☐ $x = 14$ is a solution.

Question 10

Your answer is CORRECT.

Find a solution to the congruence equation $-18x \equiv 5 \pmod{23}$.

- a) ☐ $x = 71$ is a solution.
- b) ☐ $x = 0$ is a solution.
- c) ☒ $x = 70$ is a solution.
- d) ☐ $x = 69$ is a solution.
- e) ☐ There are no solutions.