Discrete Math

Lecture 2

Operations on Statements

Most important ones

$\neg$ "negation"

$\wedge$ "and"

$\vee$ "or"

} this video

$\Rightarrow$ "implies"

$\Leftrightarrow$ "biconditional"

} next video

$\oplus$ "xor" } this video

"operation" → a way of creating a new statement

ex] negating a statement creates a new one w/ opp. truth value & opp. meaning

$P \longrightarrow \neg P$ = "it is not true that" P

P: $\det\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 0$

$\neg P$: it is not true that $\det\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 0$

$\det\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \neq 0$

note: other books, courses notate "negation" w/ diff. symbols

$!$, $\sim$, $\neg$

Example 2.2. *Consider the statement Q : 35 is an odd number. Which of the following statements are correct ways of writing $\neg Q$?*

(1) *35 is an even number.* ✓
(2) *It is not true that 35 is an odd number.* ✓
(3) *35 is not odd.* ✓
(4) *35 is a number.*
(5) *There is no remainder when 35 is divided by 2.*

(old fact ; every whole number is either even or odd.)

And ($\wedge$)

$P \wedge Q$ is T when both are T

is F otherwise

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

ex) P : It is sunny.

Q : It is hot.

$P \wedge Q$ = " It is sunny AND it is hot."

Example 2.4. *Consider the statement $P : 5^2 < 10$. Can you come up with a statement Q so that $P \wedge Q$ is true? If you can, write one down. If not, explain why.*

note: P is false

($5^2 < 10$ is false)

$P \wedge Q \equiv (5^2 < 10) \wedge$ (1+3 = 0) ← we pick

$F \wedge T \equiv F$

$F \wedge F \equiv F$

because P is false, $P \wedge Q$ is false

no matter what Q is!

OR ($\vee$)

allows for both to be true

$P \vee Q$ is T when either P or Q is T

is F otherwise

ex| no one attends UH (F) OR the USA is a country (T).

so this entire statement is T.

ex| Casey will buy a new Lego set or a new video game.

(spoiler alert: both true, the statement is T)

ex| $(x-1)(x-2) = 0$

Solving this results in: $x=1$ or $x=2$

$x=1$ makes $(x-1)(x-2)=0$

or

$x=2$ makes $(x-1)(x-2)=0$

Truth Table for OR

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P is T, Q is T, both are T

→ neither is T

XOR ($\oplus$)

"X" = "exclusive"

$P \oplus Q$ will be T when only one part is T (not both)

will be F otherwise

ex| Casey will buy a new Lego set or a new video game.

(spoiler alert: both true, the statement is T)

P: Casey buys Lego — P v Q is T

Q: Casey buys video games — P ⊕ Q is F