

# PRINTABLE VERSION

## Quiz 11

You scored 80 out of 100

### Question 1

Your answer is CORRECT.

The congruence equation " $17 \equiv -73 \pmod{30}$ " means

- a) ☒ 17 and  $-73$  have the same remainder when they are divided by 30.
- b) ☐ 17 and 30 have the same remainder when they are divided by  $-73$ .
- c) ☐  $-73$  and 30 have the same remainder when they are divided by 17.
- d) ☐ 17 and  $-73$  have the same quotient when they are divided by 30.

### Question 2

Your answer is CORRECT.

The integers 92 and  $-28$  are congruent mod  $n$  for which value of  $n$ ?

- a) ☐  $n = 92$
- b) ☐  $n = -28$
- c) ☐  $n = 16$
- d) ☐ There are no values of  $n$  for which these two integers are congruent (except  $n = 1$ ).
- e) ☒  $n = 15$

### Question 3

Your answer is CORRECT.

Consider the following proposition:

Proposition. If  $a \equiv b \pmod{n}$ , then  $a^2 \equiv b^2 \pmod{n}$ .

If you were writing a direct proof of this proposition, which of the following statements could be used as your first line?

- a) ☐ Suppose  $(a - b) \mid n$ .

- b) ☒ Suppose  $n|(a - b)$  .
- c) ☐ Suppose  $a|n$  and  $a|b$  .
- d) ☐ Suppose  $n$  divides  $a$  and  $b$  .
- e) ☐ Suppose  $n|a$  and  $b|a$  .

#### Question 4

Your answer is **INCORRECT**.

Is the following statement true or false?

$$\forall x, y, a, b \in \mathbb{Z}, n \in \mathbb{N}^*, (x \equiv a \pmod{n} \wedge y \equiv b \pmod{n}) \Rightarrow (x + y) \equiv (a + b) \pmod{n}.$$

(Note: for this problem  $\mathbb{N}^*$  refers to the positive natural numbers  $\mathbb{N}^* = \mathbb{N} - \{0\} = \{1, 2, 3, \dots\}$  .)

- a) ☐ This statement is true.
- b) ☒ This statement is false.

#### Question 5

Your answer is **INCORRECT**.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Undefined control sequence \square

- a) ☐ If you add up six consecutive integers, then the result is equivalent to 1 mod 6.
- b) ☒ If  $x \in \mathbb{Z}$  then  $\sum_{i=0}^5 x + i \not\equiv 0 \pmod{6}$ .
- c) ☐ The sum of 6 consecutive integers is never congruent to 0 mod 6.
- d) ☐ Technically no proposition was proven true since there is an algebraic mistake in Line (3).

#### Question 6

Your answer is **CORRECT**.

Use the Euclidean Algorithm to find the inverse of  $-25 \pmod{10}$  (if it exists).

- a) ☐  $-1/25$  is an inverse.

- b) ☐  $-5$  is an inverse.
- c) ☐  $-10/25$  is an inverse.
- d) ☐  $2$  is an inverse.
- e) ☒  $-25$  does not have an inverse mod  $10$  because  $\gcd(-25, 10) \neq 1$ .

#### Question 7

Your answer is CORRECT.

Of the options provided below, determine the one that best completes this sentence: "The modular equation  $16x \equiv 11 \pmod{17}$  \_\_\_\_\_"

- a) ☐ has no solutions.
- b) ☒ has exactly one solution.
- c) ☐ has multiple solutions.

#### Question 8

Your answer is CORRECT.

Which steps should one take when solving a congruence equation  $ax \equiv b \pmod{n}$ ? A helpful summary is presented below, only one step is missing:

Steps for solving  $ax \equiv b \pmod{n}$ .

Step 1.

Step 2. If  $\gcd(a, n) \mid b$ , then proceed to step 3, otherwise there are no solutions.

Step 3. Use work from Step 1 to calculate one solution  $x_0 \in \mathbb{Z}$ .

Step 4. Add  $\frac{n}{\gcd(a, n)}$  to  $x_0$  to create other solutions.

Of the following options, which could be used for the missing Step 1?

- a) ☐ Step 1. Use the Euclidean Algorithm to compute  $\gcd(b, n)$ .
- b) ☐ Step 1. Divide  $n$  by  $b$ .
- c) ☐ Step 1. Use the Euclidean Algorithm to compute  $\gcd(b, n)$ .
- d) ☐ Step 1. Multiply both sides by  $1/a$ .
- e) ☒ Step 1. Use the Euclidean Algorithm to compute  $\gcd(a, n)$ .

#### Question 9

**Your answer is CORRECT.**

Find a solution to the congruence equation  $13x \equiv -14 \pmod{5}$ .

- a) ☐  $x = 14/13$  is a solution.
- b) ☒  $x = 27$  is a solution.
- c) ☐  $x = 5$  is a solution.
- d) ☐  $x = 5$  is a solution.
- e) ☐  $x = 5/13$  is a solution.

**Question 10**

**Your answer is CORRECT.**

Find a solution to the congruence equation  $14x \equiv 18 \pmod{43}$ .

- a) ☒  $x = 720$  is a solution.
- b) ☐ There are no solutions.
- c) ☐  $x = 22$  is a solution.
- d) ☐  $x = 21$  is a solution.
- e) ☐  $x = 0$  is a solution.