Making New Sets from Old Ones : the Cartesian Product

Definition 1.1 An **ordered pair** is a list (x, y) of two things x and y, enclosed in parentheses and separated by a comma.

Definition 1.2 The **Cartesian product** of two sets A and B is another set, denoted as $A \times B$ and defined as $A \times B = \{(a,b) : a \in A, b \in B\}$.

$$A \times B = \{ (a,b) : a \in A, b \in B \}$$

ex 1) common example

$S = \{$names of students in this class$\}$

$I = \{$all student ID numbers$\}$

$$S \times I = \{ (\text{name}, ID) : \text{name} \in S, ID \in I \}$$

$(\text{Casey Douglas}, 0000001) \in S \times I$

$(\text{Casey Douglas}, 5671983) \in S \times I$

ex 2) a familiar math example :

$$\mathbb{R} \times \mathbb{R} = \{ (x,y) : x, y \in \mathbb{R} \}$$

$=$ the set of of pairs of real numbers

$(7, \pi) \in \mathbb{R} \times \mathbb{R}$ $\quad$ $(0,0) \in \mathbb{R} \times \mathbb{R}$

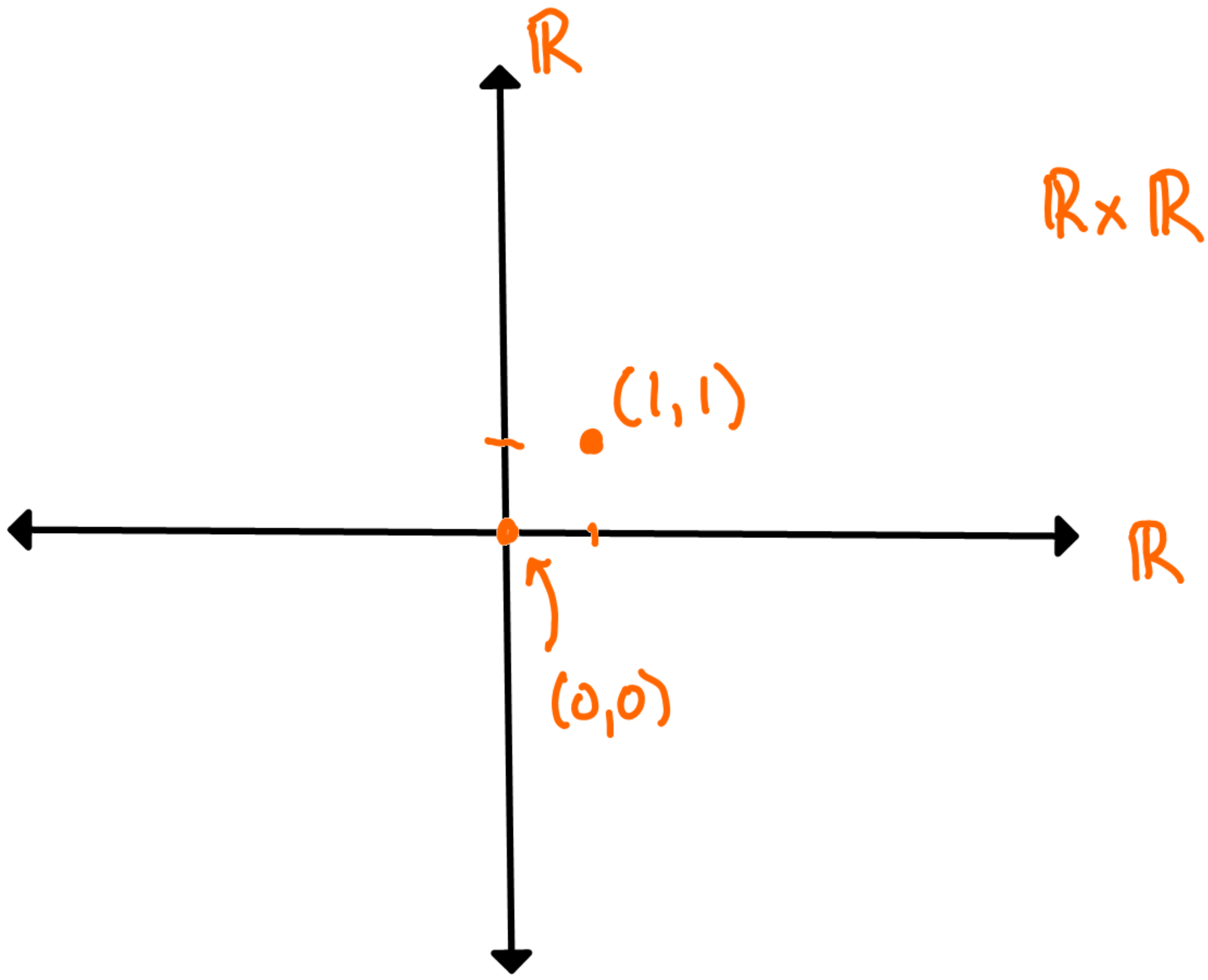
$(\underline{\quad}, -\frac{1}{2}) \notin \mathbb{R} \times \mathbb{R}$

no element here!

$(-e, \sqrt{-1}) \notin \mathbb{R} \times \mathbb{R}$

not a real number!

you've visualized + used $\mathbb{R} \times \mathbb{R}$ lots!



we can think of $\mathbb{R} \times \mathbb{R}$ as the plane $\mathbb{R}^2$

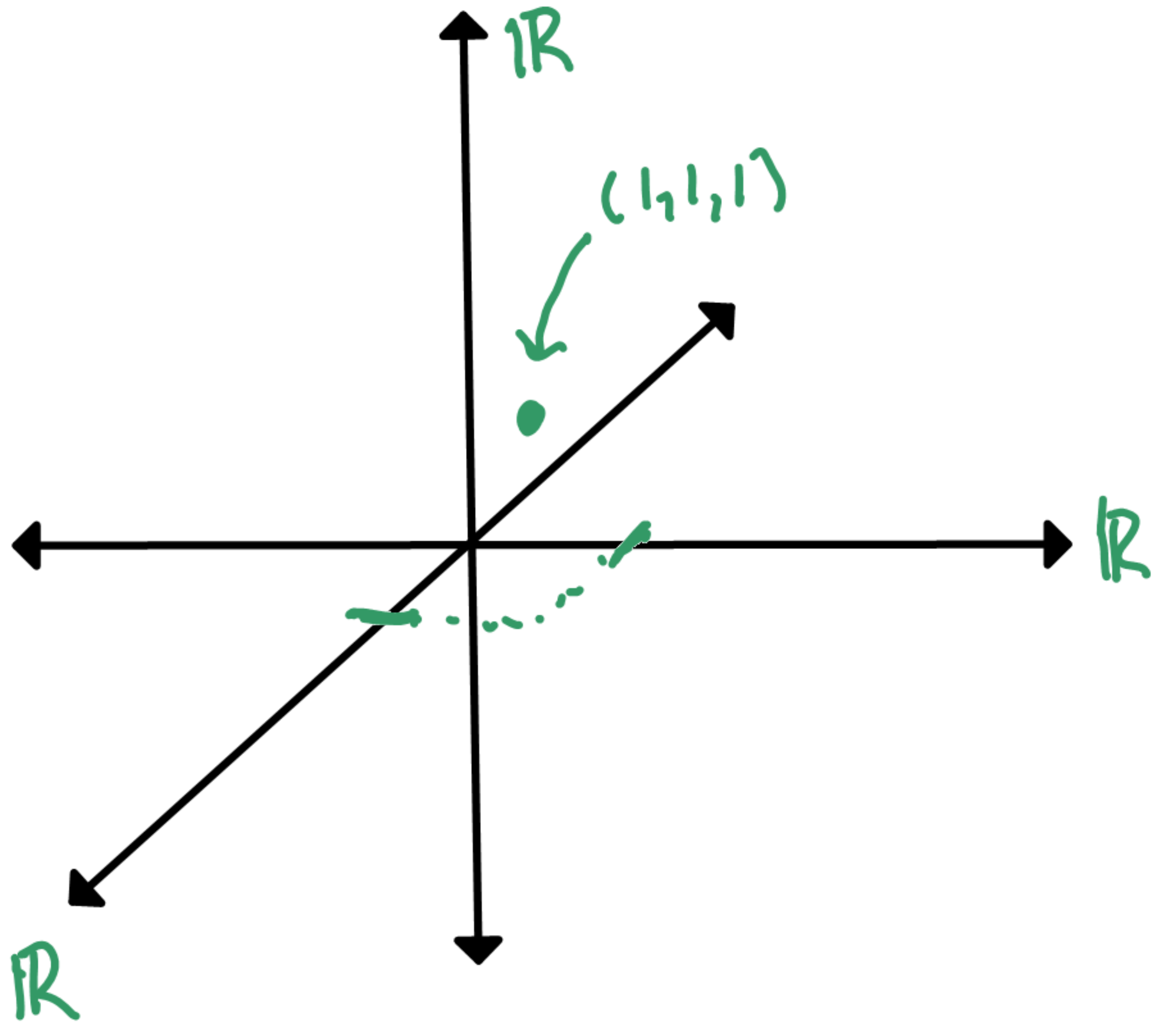
ex 3) what about $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$?

$$(\mathbb{R} \times \mathbb{R}) \times \mathbb{R} = \{ ((x,y), z) : x, y, z \in \mathbb{R} \}$$

$$\mathbb{R} \times (\mathbb{R} \times \mathbb{R}) = \{ (x, (y,z)) : x, y, z \in \mathbb{R} \}$$

we agree to write as $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{ (x, y, z) : x, y, z \in \mathbb{R} \}$

and we'll also write as $\mathbb{R}\times\mathbb{R}\times\mathbb{R} = \mathbb{R}^3$.



while we <u>can</u> visualize sets like $\mathbb{R}$, $\mathbb{R}^2$, + $\mathbb{R}^3$

we can't see the sets $\mathbb{R}^4, \mathbb{R}^5, \mathbb{R}^6, \dots, \mathbb{R}^n$

<u>but</u> we can still work with these as sets

<u>Def</u> given <u>any</u> set A, we can form the repeated product

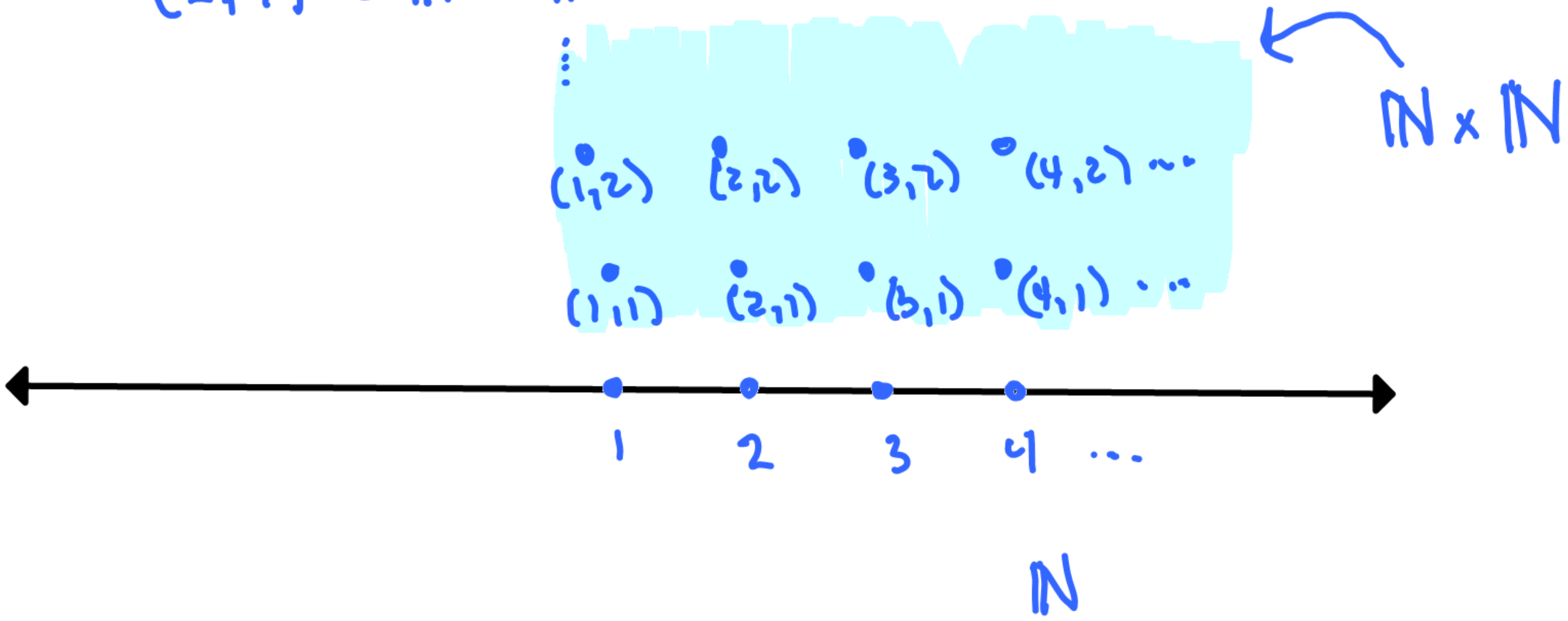
$$A^n = \underbrace{A\times A\times \cdots \times A}_{n \text{ times}}$$

$$= \{ (x_1, x_2, \dots, x_n) : \text{each } x_i \in A \}$$

ex 4) what about a set like

$$\mathbb{N} \times \mathbb{N} \ ?$$

$(1,2) \in \mathbb{N} \times \mathbb{N}$
$(2,1) \in \mathbb{N} \times \mathbb{N}$ $\Big\}$ note: $(1,2) \neq (2,1)$



Fact If $|A| = n$, $|B| = m$

then $|A \times B| = \underline{n \cdot m}$

ex 5) $A = \{1,2,3\}$ $B = \{s,t\}$

$$A \times B = \{(1,s), (1,t), (2,s), (2,t), (3,s), (3,t)\}$$

3×2 pairs!

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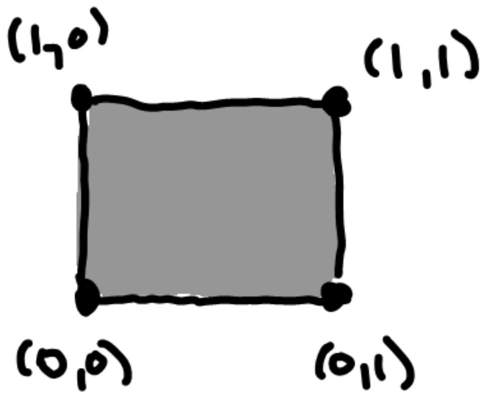
fun visual example

unit line segment

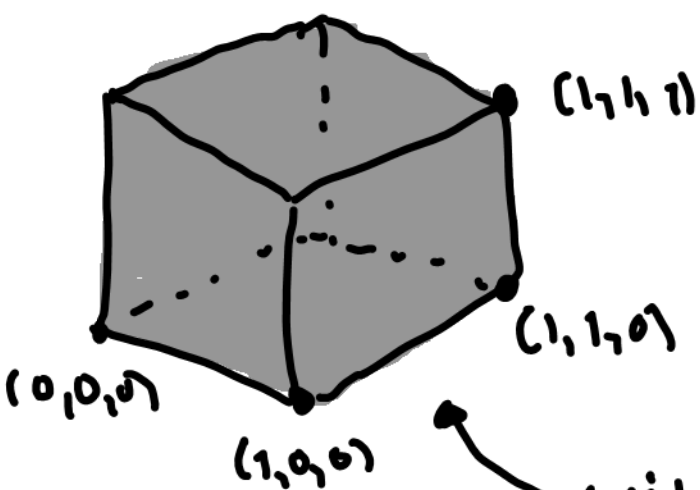
$$[0,1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$$

$$[0,1]^2 = [0,1] \times [0,1]$$
$$= \{(x,y) : x,y \in [0,1]\}$$



unit square!

$$[0,1]^3 = [0,1] \times [0,1] \times [0,1]$$
$$= \{(x,y,z) : x,y,z \in [0,1]\}$$



unit cube!

$$[0,1]^4 = [0,1] \times [0,1] \times [0,1] \times [0,1]$$

I need hallucinegenics to draw!