PRINTABLE VERSION

Quiz 11

You scored 100 out of 100

Question 1

Your answer is CORRECT.

The congruence equation "$-78 \equiv -166 \bmod 22$ " means

a) -166 and 22 have the same remainder when they are divided by -78 .

b) -78 and 22 have the same remainder when they are divided by -166 .

c) -78 and -166 have the same quotient when they are divided by 22.

d) -78 and -166 have the same remainder when they are divided by 22.

Question 2

Your answer is CORRECT.

The integers 94 and -29 are congruent mod n for which value of n?

a) $n = -29$

b) There are no values of n for which these two integers are congruent (except $n = 1$).

c) $n = 41$

d) $n = 94$

e) $n = 42$

Question 3

Your answer is CORRECT.

Consider the following proposition:

$$\text{Proposition. If } a \equiv b \bmod n, \text{ then } a^2 \equiv b^2 \bmod n.$$

If you were writing a direct proof of this proposition, which of the following statements could be used as your first line?

a) Suppose $(a - b) | n$.

b) ◉ Suppose $n|(a-b)$.

c) ○ Suppose $a|n$ and $a|b$.

d) ○ Suppose n divides a and b.

e) ○ Suppose $n|a$ and $b|a$.

Question 4

Your answer is CORRECT.

Is the following statement true or false?

$$\forall x, y, a, b \in Z,\ n \in N^*,\ (x \equiv a \bmod n\ \wedge\ y \equiv b \bmod n) \Rightarrow xy \equiv ab \bmod n.$$

(Note: for this problem N^* refers to the positive natural numbers $N^* = N - \{0\} = \{1, 2, 3, \ldots\}$.)

a) ○ This statement is false.

b) ◉ This statement is true.

Question 5

Your answer is CORRECT.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Proposition.

Proof (Direct).

(1) Let $x \in Z$ satisfy $x \not\equiv 0 \bmod 3$.

(2) By The Division Algorithm, there are only two cases to consider.

(3) When x is divided by 3 either it has a remainder of 1 or of 2.

Case 1. $x \equiv 1 \bmod 3$

(4) It follows that $x^2 \equiv 1^2 \bmod 3 \equiv 1 \bmod 3$.

Case 2. $x \equiv 2 \bmod 3$

(5) It follows that $x^2 \equiv 2^2 \bmod 3 \equiv 4 \bmod 3 \equiv 1 \bmod 3$.

(6) Therefore, in all cases $x^2 \equiv 1 \bmod 3$.

a) ○ $\forall x \in Z, x \equiv 0 \bmod 3 \Rightarrow x^2 \not\equiv 1 \bmod 3$.

b) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 0 \bmod 3.$

c) ◉ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 1 \bmod 3.$

d) ○ Technically no proposition was proven true since there is a mistake in Line (2); The Division Algorithm does *not* leave only two cases to consider.

Question 6

Your answer is CORRECT.

Use the Euclidean Algorithm to find the inverse of 40 mod 19 (if it exists).

a) ○ 40 does not have an inverse mod 19 because $\gcd(40, 19) \neq 1$.

b) ○ 19 is an inverse.

c) ○ $1/40$ is an inverse.

d) ○ $19/40$ is an inverse.

e) ◉ 10 is an inverse.

Question 7

Your answer is CORRECT.

Of the options provided below, determine the one that best completes this sentence: "The modular equation $-17x \equiv -50 \bmod 51$ ________________ "

a) ○ has multiple solutions.

b) ○ has exactly one solution.

c) ◉ has no solutions.

Question 8

Your answer is CORRECT.

Which steps should one take when solving a congruence equation $ax \equiv b \bmod n$? A helpful summary is presented below, only one step is missing:

Steps for solving $ax \equiv b \bmod n.$

Step 1. Use the Euclidean Algorithm to compute $\gcd(a, n)$.

Step 2. If $\gcd(a, n) \mid b$, then proceed to step 3, otherwise there are no solutions.

Step 3. Use work from Step 1 to calculate one solution $x_0 \in Z$.

Step 4.

Of the following options, which could be used for the missing Step 3?

a) Step 4. Add $\dfrac{n}{\gcd(a, n)}$ to x_0 to create other solutions.

b) Step 4. Add b to x_0 to create other solutions.

c) Step 4. Add $\dfrac{b}{\gcd(a, n)}$ to x_0 to create other solutions.

d) Step 4. Add $\dfrac{a}{\gcd(a, n)}$ to x_0 to create other solutions.

e) Step 4. Add $\dfrac{\gcd(a, n)}{b}$ to x_0 to create other solutions.

Question 9

Your answer is CORRECT.

Find a solution to the congruence equation $17x \equiv 9 \bmod 5$.

a) $x = 13$ is a solution.

b) $x = 5/17$ is a solution.

c) $x = 9/17$ is a solution.

d) $x = 5$ is a solution.

e) $x = 12$ is a solution.

Question 10

Your answer is CORRECT.

Find a solution to the congruence equation $-25x \equiv 2 \bmod 8$.

a) $x = 14$ is a solution.

b) $x = 0$ is a solution.

c) $x = 15$ is a solution.

d) There are no solutions.

e) $x = 13$ is a solution.