The Complement of a Set

__idea__ given a set S, its __complement__ is notated by $\overline{S} = S^c$ and it means

$$\overline{S} = \{ x : x \notin S \}$$

this idea, as stated, falls apart!

__ex 1)__ $\overline{\mathbb{N}}$ $-5 \in \overline{\mathbb{N}}$ $-5 \notin \mathbb{N}$

my dog $\in \overline{\mathbb{N}}$

dream $\in \overline{\mathbb{N}}$

UH $\in \overline{\mathbb{N}}$

oh no! "$\overline{\mathbb{N}}$" is too "big"

there are too many things $\notin \mathbb{N}$!!

to properly define and use "set complement" we first need to specify a big set containing our starting set S

"universe of discourse"

$\mathcal{U}$

ex 2 $\overline{\mathbb{N}}$ using $\mathcal{U} = \mathbb{Z}$

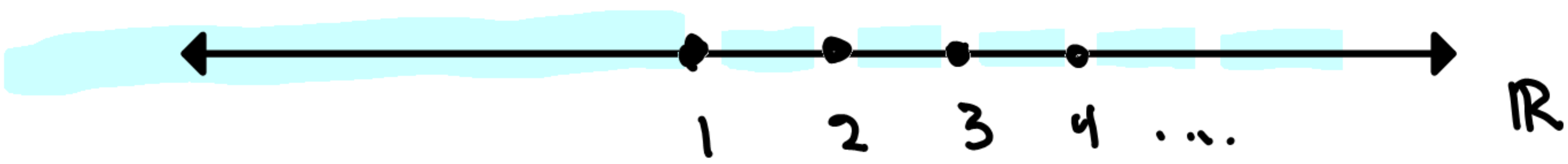
$$\overline{\mathbb{N}} = \{x \in \mathbb{Z} : x \in \mathbb{N}\}$$

$$= \mathcal{U} - \mathbb{N}$$

$$= \{\ldots, -5, -4, -3, -2, -1, 0\}$$

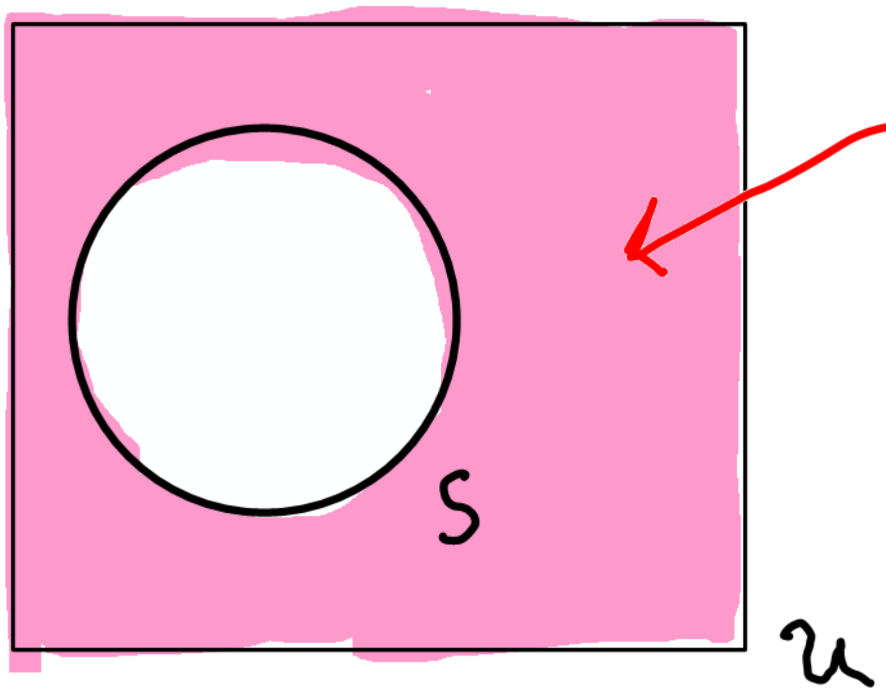
ex 3 $\overline{\mathbb{N}}$ with $\mathcal{U} = \mathbb{R}$

$$\overline{\mathbb{N}} = \mathbb{R} - \mathbb{N} = \{x \in \mathbb{R} : x \in \mathbb{N}\}$$



$$= (-\infty, 1) \cup (1, 2) \cup (2, 3) \cup \ldots$$

$$= (-\infty, 1) \cup \left(\bigcup_{i=1}^{\infty} (i, i+1) \right)$$



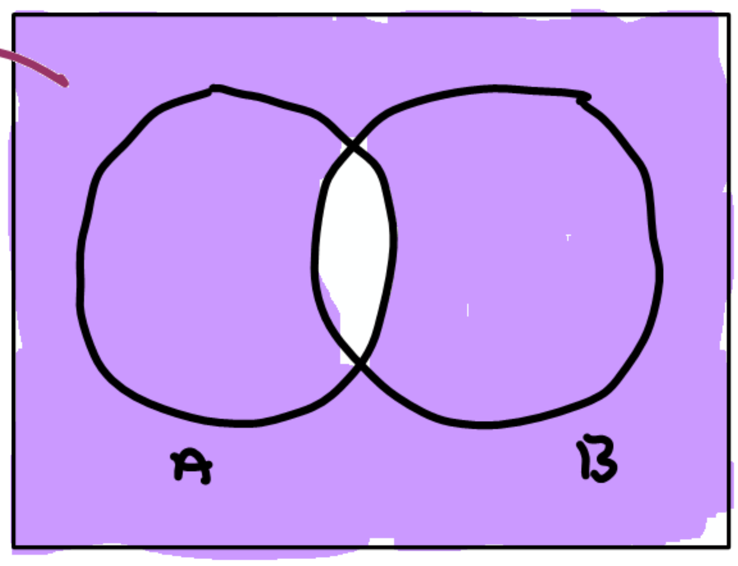
$\overline{S} = \mathcal{U} - S$

note: we don't always specify $\mathcal{U}$! Many times it is understood from context.

<u>note</u> $\cup$, $\cap$ and complement interact w/ one another in interesting + useful ways!

<u>ex</u>] $\overline{(A \cap B)} = ??$

$\overline{(A \cup B)} = ??$



draw + color

$\overline{A} \cup \overline{B}$ same?

<u>note</u> $\cap$, $\cup$ and complement can also be combined with

- Cartesian product
- power sets

$$\overline{(A \times B)} = ??$$