Lets Talk About Sets, Babeeeec.....

Math focuses on sets with additional structure

examples

sets that are vector spaces — Linear Algebra

sets that are "groups", "fields" — Abstract Algebra

$\mathbb{R} \leftarrow$ has additional structure; we can add, sub, multiply + divide elements!

ex) $\mathbb{N} = \{1, 2, 3, \ldots\}$ we can add elements, we can multiply them

we can also compare natural numbers; we can "order" them (using $\leq$)

Famous Property of $\mathbb{N}$ "The Well-Ordering Principle"

Every non-empty subset $S \subseteq \mathbb{N}$ has a smallest element.

ex $E = \{\text{even natural \#s}\} \subset \mathbb{N}$

smallest element of E is 2.

ex $P = \{\text{primes}\}$ smallest element is 2

ex $P_e = \{\text{perfect numbers}\} = \{n \in \mathbb{N} : n = \text{sum of its proper divisors}\}$

smallest element is $6 = 1 + 2 + 3$ ✓

non-ex $\mathbb{Z}$ is not 'well ordered'

$\mathbb{Z}$ itself has no smallest element!

↳ $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

useful for lots of "things" we will want to do including

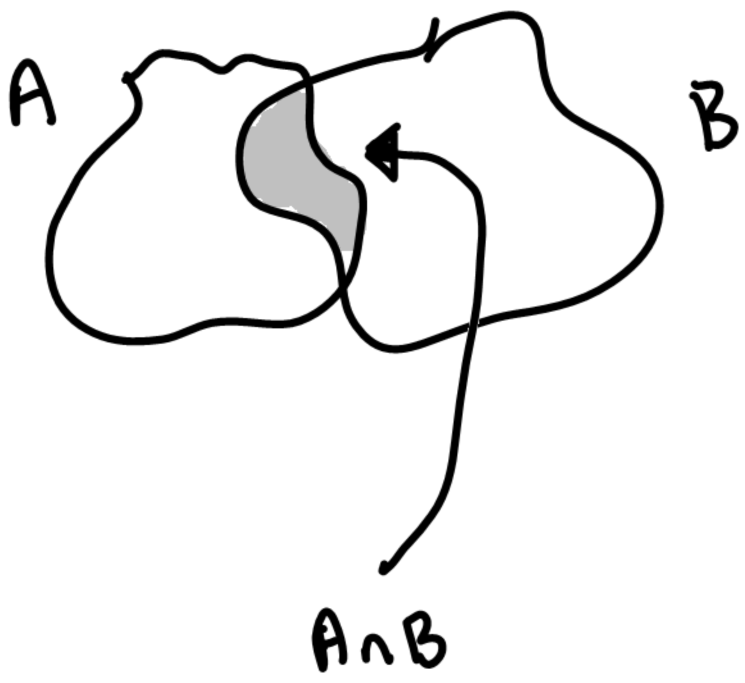
- proofs by induction
- division algorithm (1.9)

How do / should we think about sets and combo's of sets?

- for familiar sets like $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ thinking in terms of their algebraic properties is helpful !

- visualizing them
  - use Venn Diagrams (1.7) to visualize certain combo's of abstract sets

ex

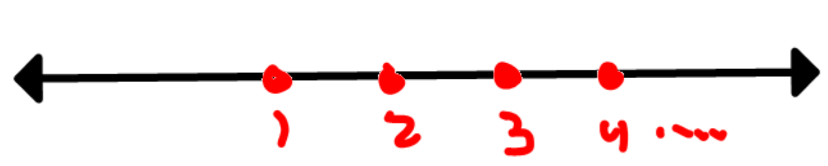


<u>note</u> Venn diagrams are not super helpful for

- $A \times B$
- $\mathcal{P}(A)$

for abstract sets, Cartesian products and power sets are often only or best understood in terms of set definitions.

ex) $\mathbb{N} = \{1, 2, 3, \ldots\}$



$\mathcal{P}(\mathbb{N})$ ← visualize this?

← how do we think about?

stuck w/ just the definition

$\mathcal{P}(\mathbb{N}) = \{\text{all subsets of } \mathbb{N}\}$

$\{1\} \in \mathcal{P}(\mathbb{N})$, $\mathbb{N} \in \mathcal{P}(\mathbb{N})$

$\{\text{evens}\}, \{\text{primes}\}, \{\text{perfect numbers}\} \in \mathcal{P}(\mathbb{N})$

ex) $S = \{(t, t^3) : t \in \mathbb{R}\}$

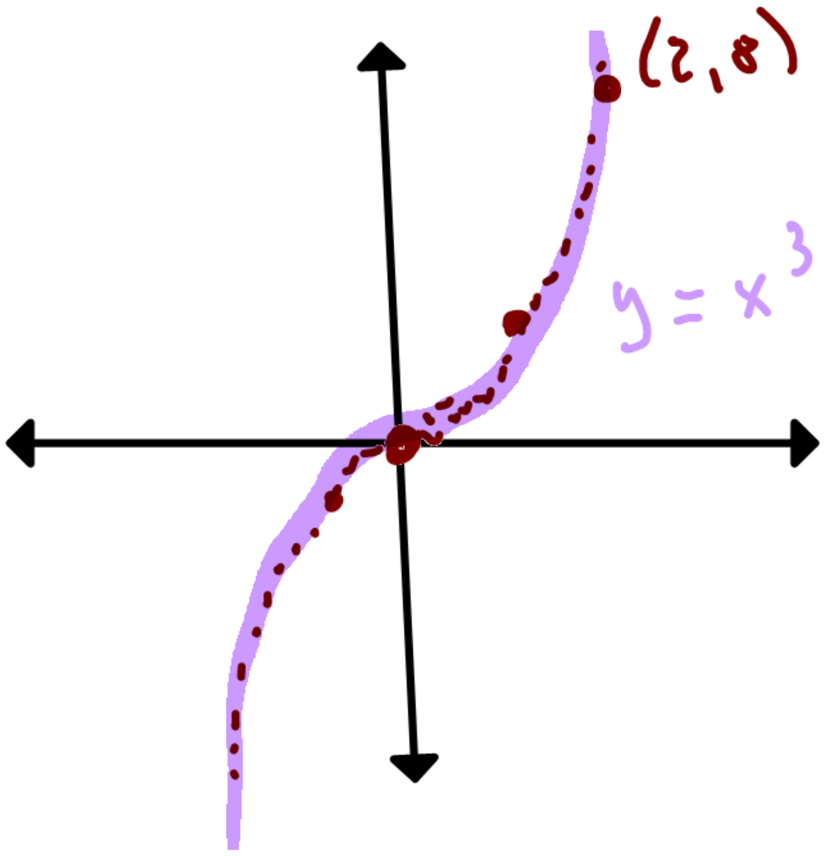
- a set of pairs of real numbers $\subseteq \mathbb{R}^2$
- $t = 0 : (0,0)$

  $t = 1 : (1,1)$

  $t = 2 : (2,8)$

  $t = \pi : (\pi, \pi^3)$

  $t = -\frac{1}{3} : (-1/3, -\frac{1}{27})$



we've discussed infinite unions & intersections like

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots = \bigcap_{i \in \mathbb{N}} A_i$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots = \bigcup_{i \in \mathbb{N}} A_i$$

these examples: $\mathbb{N}$ = "index set", it lets us keep track of each set we're combining.

we can use <u>other</u> index sets!

ex) $S_1 = \emptyset$

$S_2 = \mathbb{R}$

$S_3 = \{1, \mathbb{A}\}$

$S_4 = \{2\}$

$$\underbrace{S_1 \cap S_2 \cap S_3 \cap S_4}_{\bigcap_{i=1}^{4} S_i \,=\, \bigcap_{i \in \{1,2,3,4\}} S_i}$$

ex) the index can be big, like $\mathbb{R}$!

for each $t \in \mathbb{R}$ we might have a set B_t

we can notate the intersection / union of these!

$$\bigcap_{t \in \mathbb{R}} B_t \qquad \bigcup_{t \in \mathbb{R}} B_t$$

example: $B_t = [-t, t]$

$B_\pi = [-\pi, \pi]$

$$\bigcap_{t \in \mathbb{R}} B_t = \{0\} \qquad \bigcup_{t \in \mathbb{R}} B_t = \mathbb{R}$$

Problems With Sets

Read 1.10

Is the "set of all sets" a thing?

this would contain a strange "set"

$$A = \{ X : X \notin X \}$$

which sets aren't in A?

$S \in S$

$S = \{\{\{\cdots\}\}\}$

this is weird!

$\mathbb{Z} \in A \quad (\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \not\ni \mathbb{Z})$

$\boxed{\text{Is } A \in A\text{?}}$ → if yes, then no!

if no, then yes!

$\boxed{\text{oh sh** } A \text{ can't be a set!}}$