PRINTABLE VERSION

Quiz 5

You scored 100 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

$$\begin{array}{|l|}\hline \text{Proposition. } P \Rightarrow Q \\ \\ \underline{\text{Proof}}.\ \text{Suppose } \neg Q. \\ \textit{Missing steps involving } \neg Q \textit{ and } \neg P \textit{ and any previously established facts} \\ \text{Therefore } \neg P.\ \square \\ \hline \end{array}$$

What type of proof was described in the outline?

a) A proof by contrapositive is described in this outline.

b) Wait a minute... The proof described in this outline isn't a valid proof technique!

c) A direct proof is described in this outline.

d) A proof by contradiction is described in this outline.

e) A proof by introspection is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P \wedge Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P \wedge Q$.

b) Suppose $\neg P$.

c) Suppose $\neg P \wedge \neg Q$.

d) Suppose $\neg Q$

e) Suppose $\neg P \vee \neg Q$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $A \subseteq B$ by...

a) First supposing $x \notin B$, and then showing $x \in A$.

b) First supposing $x \in A$, and then showing $x \in B$.

c) First supposing $x \in B$, and then showing $x \in A$.

d) First supposing $x \notin A$, and then showing $x \in B$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \notin B$, and then showing $x \in A$.

b) First supposing $x \notin A$, and then showing $x \in B$.

c) First supposing $x \notin A$, and then showing $x \notin B$.

d) First supposing $x \notin B$, and then showing $x \notin A$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If x^2 is an even integer, then x is even.

Proof. Suppose x^2 is even.
A previous result tells us that the if the product of two integers is even, then at least one of the factors is even.
When applied to $x^2 = x \cdot x$, this implies that x is even. $\square$

Determine the type of proof used.

a) A proof by contradiction was used.

b) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

c) A proof by indoctrination was used.

d) A direct proof was used.

e) A proof by contrapositive was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \subseteq X$ and $B \subseteq Y$ then $A \times B \subseteq X \times Y$.

Proof (Direct).
(1) Suppose $A \subseteq X \wedge B \subseteq Y$.
(2) In order to show $A \times B \subseteq X \times Y$ we will let $(a, b) \in A \times B$ and then conclude $(a, b) \in X \times Y$.
(3) $(a, b) \in A \times B$ means $a \in A \wedge b \in B$.
(4) Since $a \in A \subseteq X$ it follows that $a \in X$.
(5) Since $b \in B \subseteq Y$ it follows that $b \in Y$.
(6) By Definition of Cartesian Product $(a, b) \in X \times Y$. $\square$

Identify the mistake, if any, in this proof.

a) There is an error in Line (4) where the definition of subset is misused.

b) Hey, wait a second... this proof looks completely correct!

c) ○ There is an error in Line (6) where the definition of Cartesian Product is misused.

d) ○ There is an error in Line (3). The symbol $\wedge$ should be $\vee$.

e) ○ There is an error in Line (2). One is not allowed to just "let" $(a, b) \in A \times B$.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The set $S = \{x \in \mathbb{Z} : x^3 - x = 0\}$ has cardinality $|S| = 3$.

Proof. (Direct)
(1) We can rewrite the equation $x^3 - x = 0$ as $x(x-1)(x+1) = 0$.
(2) Because x is an integer, and because $\mathbb{Z} \subset \mathbb{R}$, it follows that x is a real number, too.
(3) The equation $x(x-1)(x+1) = 0$ only has solutions $x = 0, x = 1,$ and $x = -1$.
(4) The solutions $x = 1$ and $x = -1$ can be added to produce the other solution $x = 0$.
(5) It follows that $S = \{-1, 0, 1\}$.
(6) Therefore $|S| = 3$. $\square$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) ○ Only line (4) is not needed. All other lines are needed.

b) ○ All lines are needed.

c) ○ Only line (1) is not needed. All other lines are needed.

d) ◉ Only lines (2) and (4) are not needed. All other lines are needed.

e) ○ Only line (2) is not needed. All other lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a proof by contradiction of the proposition below:

$$\forall a, b, c \in [0, \infty), \ (ab = c) \Rightarrow (a \leq \sqrt{c} \vee b \leq \sqrt{c}).$$

Which of the following statements or properties do we need to use when composing this proof?

a) ○ Suppose $ab = c$ and that either $a > \sqrt{c}$ or $b > \sqrt{c}$.

b) ◉ Suppose $ab = c$ and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

c) ○ The fact that for real numbers x, y, if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

d) ○ The fact that for real numbers x, y, if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\exists n \in N, \ 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) ○ We need to show the claim is true for an abitrary natural, saying something like "Let $n \in N$."

b) We need only check that the claim is true for one, single natural number.

c) The proposition is a famous, unsolved problem. No one knows if it is true or false, and so it is not clear how to describe a proof for this.