MATH 3336

Homework Assignment 3

Instructions

- Record your answers to the following 9 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (<u>Click this link</u> for instructions on how to do this).
- ullet Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.
- 1. Write out the resulting set in "set-builder notaiton" (that is, list out the elements in between squiggly braces $\{\ \}$ If the set is infinite, you can use "..." after listing out enough of the elements). No work need be included for this question.

(a)
$$\mathbb{Q} \cap \{0, -1, -2, 1/2, 1/3, 1/4, \pi\} =$$

(b)
$$(\mathbb{N} \cup \mathbb{Z}) \cap \{-4, -3, -2, -1, 0, 1, 2, \sqrt{2}, \pi\} =$$

(c)
$$\mathbb{Z} - \mathbb{N} =$$

(d)
$$\mathbb{Q} \cap (\mathbb{R} - \mathbb{Q}) =$$

- **2.** What does it mean for two sets A and B to satisfy $A \cap B = \emptyset$?
 - (a) $A = \emptyset$
 - (b) $B = \emptyset$
 - (c) This means the two sets have no elements in common; they are **disjoint**.
 - (d) $A \subseteq B$.
 - (e) $B \subseteq A$.
- 3. Is it possible for a set A to satisfy $|\mathcal{P}(A)| = 9$? If it is, give an example. If it is not possible, explain why.

4. Sketch a picture of the Cartesian product $\{-1,0,1\} \times \mathbb{R}$ as a subset of the usual "xy-plane \mathbb{R}^2 ."

5. Consider the set $A = \{-2, \heartsuit, 1/2\}$. Suppose we are told that

$$|A \times \mathcal{P}(B)| = 96$$

where B is some other finite set. What is the cardinality of set B? How do you know your answer is correct?

6. Suppose we have two sets S and T, each one defined by specifying conditions for its elements:

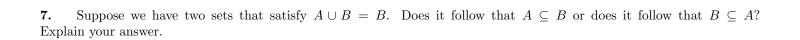
$$S = \left\{ \forall \, x \in U : P(x) \right\} \ \text{ and } \ T = \left\{ \forall \, x \in U : Q(x) \right\}.$$

Which of the following statements tells us S = T?

- (a) $\forall x \in U, P(x) \Rightarrow Q(x)$
- (b) $\exists x \in U, P(x) \iff Q(x)$.
- (c) $\forall x \in U, P(x) \iff Q(x)$.
- (d) $\exists x \in U, P(x) \Rightarrow Q(x)$.

Which of the following statements tells us $S \subseteq T$?

- (a) $\forall x \in U, P(x) \Rightarrow Q(x)$
- (b) $\exists x \in U, P(x) \iff Q(x)$.
- (c) $\forall x \in U, P(x) \iff Q(x)$.
- (d) $\exists x \in U, P(x) \Rightarrow Q(x)$.



8. One familiar set is missing from the "chain of inclusion" below. Fill it in! (No work need be included for this question.)

$$\emptyset \subseteq \{0\} \subseteq \mathbb{N} \subseteq \underline{\qquad} \subseteq \mathbb{Q} \subseteq \mathbb{R}.$$

^{9.} What did you learn (or re-learn) by working through this assignment? Which questions, if any, were particularly helpful? Which ones, if any, were unhelpful?