Math 3336
Homework Assignment 2

Instructions

- Record your answers to the following 12 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (Click this link for instructions on how to do this).
- Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.

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1. Complete the following truth table (no work need be included with this question):

P	Q	$P \iff Q$	$\neg Q$	$(P \iff Q) \vee \neg Q$
T	T			
T	F			
F	T			
F	F			

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2. Use a truth table to show that $\neg(P \iff Q) = P \oplus Q$.

3. A new logical operator, $\blacksquare$, is partially defined by the following truth table information:

P	Q	$P \blacksquare Q$	$\neg(P \blacksquare Q) \wedge P$	$\neg(P \blacksquare Q) \vee Q$
T	T	T		
T	F		F	
F	T	F		
F	F			T

Complete this truth table (no work need be included with this question).

4. Consider the logical operator, $\blacksquare$, defined in the previous problem. Use a truth table to determine whether or not the following formula is correct:

$$(P \blacksquare Q) = \Big((\neg Q \wedge P) \vee P \Big).$$

5. Use De Morgan's Laws (and other logical equivalences) to explain why the following formula is correct:

$$\neg(\neg P \vee \neg Q) = (P \wedge Q).$$

6. Consider the following quantified statement:

$$\forall\, x \in U,\; x^2 + x = 0.$$

Write down a (non-empty) Universal Set, U, that makes this a true statement.

7. Carefully read the following statement:

$$\forall\, x \in \mathbb{R},\; \exists\, y \in \mathbb{R}, y^2 = x.$$

Is this statement true or false? If you think it is true, explain why. If you think it is false, then provide a counter-example.

8. Rewrite the following sentence using logical expressions.

If $\sin x \le 0$, then it is not the case that $0 \le x \le \pi$.

9. Consider the statement P below:

$$P: \quad \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y = 0.$$

(a) Briefly explain why P is true. (Remember: use some scratch paper to explore examples to convince yourself!)

(b) Write the statement $\neg P$ using logical expressions.

(c) Write the statement $\neg P$ using English words.

10. Consider the statement Q below:

$$Q: \quad \text{Every student at UH loves math.}$$

Write down an English-sentence version of $\neg Q$.

11. Consider the statement S below:

$$S: \;\; \exists\, a \in \mathbb{Z}, \forall\, b \in \mathbb{Z},\, a \cdot b = b.$$

Of the following options provided below, which correctly expresses $\neg S$? (No work need be included for this question.)

(a) $\neg S: \;\; \forall\, a \in \mathbb{Z}, \exists\, b \in \mathbb{Z},\, a \cdot b = b.$

(b) $\neg S: \;\; \exists\, a \in \mathbb{Z}, \forall\, b \in \mathbb{Z},\, a \cdot b \neq b.$

(c) $\neg S: \;\; \forall\, a \in \mathbb{Z}, \exists\, b \in \mathbb{Z},\, a \cdot b \neq b.$

(d) $\neg S: \;\; a = 1.$

12. What did you learn (or re-learn) by working through this assignment? Which questions, if any, were particularly helpful? Which ones, if any, were unhelpful?