Discrete Math

Lecture 7

Translating

Example 7.1. *Consider the English and logical versions of statement P:*

P : *Every integer is either odd or even, but not both.*

P : $\forall x \in \mathbb{Z}$, $x = 2a$ *(for some* $a \in \mathbb{Z}$*)* $\oplus$ $x = 2b + 1$ *(for some* $b \in \mathbb{Z}$*).*

(note: isn't 0 neither even nor odd?
no! 0 is even)

Def. an integer x is <u>even</u> if

$\exists a \in \mathbb{Z}, \quad x = 2a$ ✓

an integer x is <u>odd</u> if

$\exists b \in \mathbb{Z}, \quad x = 2b + 1$ ✓

Note: there are <u>LOTS</u> of ways to define "even" + "odd"

Example 7.2. *Consider the English and logical versions of statement Q:*

Q: *Whenever a real number is squared the result is a non-negative real numer.*

Q: $x \in \mathbb{R} \Rightarrow x^2 \geq 0$.

note: $Q: \forall x \in \mathbb{R}, \ x^2 \geq 0$

Example 7.3. *Consider the statement*

$$P : \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y + x = 2.$$

This is an example of a statement with **nested quantifiers**, *and the order of these quantifiers matters (see the next example)! To see that P is true we carefully take apart the statement, step-by-step. We can also rewrite P in a natural language:*

P: *Given any real number, x, we can find another one, y, so that* $y + x = 2$.

tip for understanding complicated statements

try examples:

$P: \ \forall x \in \mathbb{R}, \ \exists y \in \mathbb{R}, \ y + x = 2$

pick an x

$x = \pi$

find a y so that

this is true

Scratch paper

$y + \pi = 2$

$-\pi \quad -\pi$

$y = 2 - \pi$

$x = \pi$

$y = 2 - \pi$ makes $y + x = 2$ T

$x = 13$

$y = -11$ makes $y + x = 2$ T

if I pick <u>any</u> x, I can always choose $y = \underline{2-x}$ so that $y+x=2$ is T

Example 7.4. *Consider the statement*

$$Q : \exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y + x = 2.$$

This complicated-looking sentence claims that there exists a special real number (named y) that, when added with any other real number *results in 2. Your familiarity with real numbers should tell you that Q is false.*

<u>try examples</u>

$\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, y + x = 2$

$x = 2 \longrightarrow y + 2 = 2 \longrightarrow y = 0$

$x = 3 \longrightarrow y + 3 = 2 \longrightarrow y = -1$

no single y can work!

slightly diff. version

$$\exists!\, y \in \mathbb{R},\ \forall x \in \mathbb{R},\ y + x = x$$

this statement is true!

Calc I ex.

$$\underbrace{\forall \varepsilon > 0,\ \exists\, \delta > 0,\quad 0 < |x - 2| < \delta \implies |x^2 - 4| < \varepsilon}_{\lim\limits_{x \to 2} x^2 = 4}$$