Discrete Math

Lecture 3

Conditional Statements

aka "if-then" statements

"implications"

notation: $P \Rightarrow Q$

read as: "if P then Q"

"P implies Q"

(note: one other way to notate this
most common $P \rightarrow Q$)

Example 3.1. *Use the truth table above to determine the truth value of* $P \Rightarrow Q$ *for each of the given statements.*

(1) $P : 4 \cdot (1/2) = 2,\ Q : 3 + 5 = 8$

(2) $P : \pi^2 < 10,\ Q :$ *some triangles have only two sides*

(3) $P : \pi^2 > 10,\ Q :$ *some triangles have only two sides*

(4) $P : 3 = 2,\ Q : 8^2 = 64$

$P: 4 \cdot (1/2) = 2$ is T

$Q: 3+5=8$ is T

$P \Rightarrow Q$

$\underbrace{\left(4 \cdot (1/2) = 2\right)}_{\text{T hyp.}} \Rightarrow \underbrace{\left(3+5=8\right)}_{\text{T concl.}}$ is T

Recall the meaning of $P \Rightarrow Q$

a kind of "promise"

whenever P is T, Q is also T

$\boxed{\begin{array}{l} P \Rightarrow Q \text{ is F when the promise is broken} \\ \text{is T otherwise} \end{array}}$

P	Q	P ⇒ Q
T	T	T ✓
T	F	F
F	T	T ✓
F	F	T ✓

"the promise was broken"

"vacuously true"

ex) if it rains, then I don't ride my bike.

$\underbrace{\text{it rains}}_{P}$, then $\underbrace{\text{I don't ride my bike.}}_{Q}$

it rains ⇒ I don't ride my bike.

$P \Rightarrow Q$

What happens when it's raining & you see me riding my bike?

$P \Rightarrow Q$ is false!

What happens when it's not raining & you see me ride my bike?

$P \Rightarrow Q$ is true (vacuously)

What happens when its not raining + I don't ride my bike?

$P \Rightarrow Q$ is T (vacuously)

ex) $P: e^0 = 1$

$Q: \pi > 5$

$$P \Rightarrow Q$$

$$\underset{T}{(e^0 = 1)} \Rightarrow \underset{F}{(\pi > 5)}$$

so $P \Rightarrow Q$ is F.

note: $(\pi > 5) \Rightarrow (e^0 = 1)$ is T

and so there's difference between

$P \Rightarrow Q$ and $Q \Rightarrow P$

<u>note</u> pretty much all Mathematical Results (Theorems) are if - then statements

<u>ex</u>) (lin. algebra) If $\det A = 0$, then A is not invertible

<u>ex</u>) (calculus) If a function is diff'able then it is continuous

<u>final note</u> open sentences

$P(x)$, $Q(x)$

$\neg P(x)$ open

$P(x) \wedge Q(x)$ open

$P(x) \vee Q(x)$ open

$P(x) \oplus Q(x)$ open

$\boxed{P(x) \Rightarrow Q(x) \text{ ~~open~~ statement}}$

the <u>real</u> explanation for how this works involves "quantifiers"

<u>ex</u> $P(x): \; x > 0$

$Q(x): \; x^3 > 0$

$P(x) \Rightarrow Q(x)$

$(x > 0) \Rightarrow (x^3 > 0)$

"if $x>0$, then $x^3 > 0$"

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