Math 3336
Homework Assignment 6

Instructions

- Record your answers to the following 6 questions. Show your work when a question requires you to do so.
- Scan your work and save the file as a .pdf (make sure your work and answers are legible)
- Upload your scanned work to CASA CourseWare using the "Assignments" tab. (Click this link for instructions on how to do this).
- Homework submitted after 11:59pm on the indicated due date will be assigned a grade of 0.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.
- I understand that if any of the questions from this assignment (or future ones) are shared in ways that violate our Academic Honesty Policy, then the syllabus will change. Specificlaly, Homework and Quizzes will be worth zero points.

  Name:

  Signature:

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1. A proposition about the Fibonacci numbers and a proof (by induction) is presented below. Complete all of the missing spaces with appropriate terms or phrases. Note that for this problem we are defining the Fibonacci sequence starting at $n = 1$, i.e.

$$F_1 = F_2 = 1 \text{ and } F_n = F_{n-1} + F_{n-2}.$$

Proposition. For every natural number $n \geq 1$, if $3|n$ then F_n is even.

Proof (by induction).

Base Case. We will show the proposition is true when $n =$ _____. The recurrence equation that defines the Fibonacci numbers has $F_3 = F_2 + F_1 = 1 + 1 = 2$, and this is an even number.

Inductive Step. Suppose the proposition is true when $n = 3k$. (We wts that the proposition is true when $n =$ _______.) The recurrence equation that defines the Fibonacci numbers implies

$$\begin{aligned} F_{3k+3} &= F_{3k+2} + F_{3k+1} && (1)\\ &= F_{3k+1} + F_{3k} + F_{3k+1} && (2)\\ &= 2F_{3k+1} + F_{3k} && (3) \end{aligned}$$

where in line (2) we applied the recurrence equation a second time to the Fibonacci number F_{3k+2}. Since $2F_{3k+1}$ is even, and since our ____ tells us F_{3k} is even it follows that F_{3k+3} is even. $\square$

2. Recall (from our notes) the definition of a "path graph with n vertices," $\mathrm{PG}(n)$. Use induction to prove the following proposition:

Proposition. The graph $\mathrm{PG}(n)$ has $n-1$ edges.

Proof (By Induction).

Base Case.

Inductive Step.

3. Let $r \in \mathbb{R}$ be some fixed real number. Use induction to prove the following proposition.

$$\textbf{Proposition.} \quad \forall\, n \in \mathbb{N},\ \sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}$$

Proof (by induction)

Base Case

Inductive Step

4. Consider the recursively-defined sequence and initial condition

$$a_n = a_{n-1} + a_{n-2} + \cdots + a_1 + a_0$$
$$a_0 = 1$$

Write an inducion proof of the following proposition (and consider the hint that you may need to use the Proposition from problem 3 as part of your work):

Proposition. $\forall\, n \geq 1,\ a_n = 2^{n-1}$

Proof (by induction)

Base Case

Inductive Step

5. Use induction to prove the "extended De Morgan's law"

$$\text{For all } n \geq 1, \ \ \overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \cdots \cup \overline{A_n}$$

Here each A_i is *some* set, and there is *some* arbitrary universal set, U, that contains them all as subsets.

Proof (by induction)

Base Case

Inductive Step

6. Watch this video (from the amazing folks at Numberphile) about induction: Numberphile Induction Video. What, if anything, did you learn by watching this?