PRINTABLE VERSION

Quiz 5

You scored 88.89 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $\neg P$.
Missing steps involving $\neg P$ *and* $\neg Q$ *and any previously established facts*
Therefore $\neg Q$. $\square$

What type of proof was described in the outline?

a) A proof by contradiction is described in this outline.

b) A direct proof is described in this outline.

c) A proof by contrapositive is described in this outline.

d) A proof by introspection is described in this outline.

e) Wait a minute... The proof described in this outline isn't a valid proof technique!

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P \lor Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P \lor \neg Q$.

b) Suppose $\neg P$.

c) Suppose $\neg P \land Q$.

d) Suppose $\neg Q$

e) Suppose $\neg P \land \neg Q$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \notin B$.

b) First supposing $x \notin B$, and then showing $x \in A$.

c) First supposing $x \in A$, and then showing $x \in B$.

d) First supposing $x \notin A$, and then showing $x \in B$.

e) ◉ First supposing $x \in B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) ○ First supposing $x \notin A$, and then showing $x \in B$.

b) ◉ First supposing $x \notin A$, and then showing $x \notin B$.

c) ○ First supposing $x \notin B$, and then showing $x \in A$.

d) ○ First supposing $x \notin B$, and then showing $x \notin A$.

e) ○ First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If $2+x$ is odd, then x is odd.

Proof. Suppose $2+x$ is even. (We will show x is even.)
By definition of even this means $2+x=2m$ for some $m \in \mathbb{Z}$.
By subtracting 2 from both sides it follows that $x=2m-2=2(m-1)$.
Because this expression is even the proof is complete. □

Determine the type of proof used.

a) ○ A proof by indoctrination was used.

b) ○ A proof by contradiction was used.

c) ○ A direct proof was used.

d) ○ A proof by contrapositive was used.

e) ◉ Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)
(1) Suppose $x, y \in \mathbb{Z}$ are integers.
(2) We can assume x is odd and that y is even.
(3) Since x is odd, it follows that $\exists\, y \in \mathbb{Z},\ x=2y+1$.
(4) Since y is even, it follows that $\exists\, m \in \mathbb{Z},\ y=2m$.
(5) We now have $x+y=(2y+1)+y=3y+1=3(2m)+1=2(3m)+1$.
(6) Because $x+y$ has the form of an odd number it is odd. □

Identify the mistake, if any, in this proof.

a) There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.

b) There is an error in Line (4) since where the definition of "even" is misapplied.

c) There is an error in Line (1) since we cannot simply assume $x, y \in Z$.

d) There is an algebraic mistake in Line (5).

e) There is an error in Line (2) since we do not know which integer is odd or even.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. $\forall n \in \mathbb{N}$, $n(n+1)$ is even.

Proof. (Direct)

(1) Let $n \in \mathbb{N}$. We will use cases to prove the proposition.

Case 1: n is even.

(2) In this case we have $n = 2m$ for some $m \in \mathbb{N}$.

(3) Because n is even, it follows that when n is divided by 2, there is no remainder.

(4) n being even implies $n+1$ is odd.

(5) It now follows that $n(n+1) = (2m)(2m+1) = 2 \cdot \left(m(2m+1)\right)$ which has the form of an even number.

(6) Therefore $n(n+1)$ is even, proving the proposition in this case.

Case 2: n is odd.

(7) In this case we have $n = 2\ell + 1$ for some $\ell \in \mathbb{Z}$.

(8) If n is not odd then it is even and Case 1 applies.

(9) It follows that $n(n+1) = (2\ell+1)(2\ell+1+1) = (2\ell+1)(2\ell+2) = 2(2\ell+1)(\ell+1)$

(10) Because the expression above has the form of an even number, $n(n+1)$ is even.

(11) If $n(n+1)$ is odd, then there is a contradiction.

(12) This completes the proof. □

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only lines (3) and (11) are not needed. All other lines are needed.

b) Only lines (4) and (11) are not needed. All other lines are needed.

c) Only line (8) is not needed. All other lines are needed.

d) Only lines (3),(4),(8), and (11) are not needed. All other lines are needed.

e) All lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a direct proof of the proposition below:

$$\forall\, a, b \in R,\ (a + b)^2 \leq 2\,(a^2 + b^2)\,.$$

Which of the following statements or properties do we need to use when composing this proof?

a) ◉ $2a^2 + 2b^2 - (a + b)^2 = a^2 - 2ab + b^2 = (a - b)^2 \geq 0.$

b) ○ $(a + b)^2 = a^2 + b^2$

c) ○ Let $a, b \in Q$.

d) ○ $a \cdot (a^2 + b^2) = a^3 + ab^2$.

Question 9

Your answer is INCORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\forall\, n \in N,\ 1 + 2 + \cdots + n = \frac{n(n + 1)}{2}.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) ◉ We need only check that the claim is true for one, single natural number.

b) ○ Nothing can describe an accurate proof strategy since this proposition is false.

c) ○ We need to show the claim is true for an abitrary natural, saying something like "Let $n \in N$."