PRINTABLE VERSION

Quiz 11

You scored 90 out of 100

Question 1

Your answer is CORRECT.

The congruence equation "$-11 \equiv -119 \bmod 36$" means

a) ○ -11 and 36 have the same remainder when they are divided by -119.

b) ○ -11 and -119 have the same quotient when they are divided by 36.

c) ○ -119 and 36 have the same remainder when they are divided by -11.

d) ◉ -11 and -119 have the same remainder when they are divided by 36.

Question 2

Your answer is CORRECT.

The integers 14 and -36 are congruent mod n for which value of n?

a) ○ There are no values of n for which these two integers are congruent (except $n = 1$).

b) ◉ $n = 5$

c) ○ $n = 14$

d) ○ $n = 6$

e) ○ $n = -36$

Question 3

Your answer is CORRECT.

Consider the following proposition:

$$\text{Proposition. If } a \equiv b \bmod n, \text{ then } a^3 \equiv b^3 \bmod n.$$

If you were writing a direct proof of this proposition, which of the following statements could be used as your first line?

a) ○ Suppose n divides a and b.

b) ○ Suppose $a|n$ and $a|b$.

c) ○ Suppose $n|a$ and $b|a$.

d) ○ Suppose $(a-b)|n$.

e) ◉ Suppose $n|(a-b)$.

Question 4

Your answer is INCORRECT.

Is the following statement true or false?

$$\forall x \in Z,\ n \in N^*,\ \exists y \in Z,\ xy \equiv 1 \bmod n.$$

(Note: for this problem N^* refers to the positive natural numbers $N^* = N - \{0\} = \{1, 2, 3, \ldots\}$.)

a) ◉ This statement is true.

b) ○ This statement is false.

Question 5

Your answer is CORRECT.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Proposition.

Proof (Direct).

(1) Let $x \in Z$ satisfy $x \not\equiv 0 \bmod 3$.

(2) By The Division Algorithm, there are only two cases to consider.

(3) When x is divided by 3 either it has a remainder of 1 or of 2.

Case 1. $x \equiv 1 \bmod 3$

(4) It follows that $x^2 \equiv 1^2 \bmod 3 \equiv 1 \bmod 3$.

Case 2. $x \equiv 2 \bmod 3$

(5) It follows that $x^2 \equiv 2^2 \bmod 3 \equiv 4 \bmod 3 \equiv 1 \bmod 3$.

(6) Therefore, in all cases $x^2 \equiv 1 \bmod 3$.

a) ○ $\forall x \in Z, x \equiv 0 \bmod 3 \Rightarrow x^2 \not\equiv 1 \bmod 3$.

b) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 0 \bmod 3.$

c) ◉ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 1 \bmod 3.$

d) ○ Technically no proposition was proven true since there is a mistake in Line (2); The Division Algorithm does *not* leave only two cases to consider.

Question 6

Your answer is CORRECT.

Use the Euclidean Algorithm to find the inverse of 4 mod 5 (if it exists).

a) ○ 5 is an inverse.

b) ◉ 4 is an inverse.

c) ○ 4 does not have an inverse mod 5 because $\gcd(4, 5) \neq 1$.

d) ○ $1/4$ is an inverse.

e) ○ $5/4$ is an inverse.

Question 7

Your answer is CORRECT.

Of the options provided below, determine the one that best completes this sentence: "The modular equation $18x \equiv 31 \bmod 37$ ________________ "

a) ○ has multiple solutions.

b) ◉ has exactly one solution.

c) ○ has no solutions.

Question 8

Your answer is CORRECT.

Which steps should one take when solving a congruence equation $ax \equiv b \bmod n$? A helpful summary is presented below, only one step is missing:

Steps for solving $ax \equiv b \bmod n$.

Step 1. Use the Euclidean Algorithm to compute $\gcd(a, n)$.

Step 2. If $\gcd(a, n) \mid b$, then proceed to step 3, otherwise there are no solutions.

Step 3. Use work from Step 1 to calculate one solution $x_0 \in Z$.

Step 4.

Of the following options, which could be used for the missing Step 3?

a) Step 4. Add $\frac{n}{\gcd(a, n)}$ to x_0 to create other solutions.

b) Step 4. Add b to x_0 to create other solutions.

c) Step 4. Add $\frac{b}{\gcd(a, n)}$ to x_0 to create other solutions.

d) Step 4. Add $\frac{a}{\gcd(a, n)}$ to x_0 to create other solutions.

e) Step 4. Add $\frac{\gcd(a, n)}{b}$ to x_0 to create other solutions.

Question 9

Your answer is CORRECT.

Find a solution to the congruence equation $23x \equiv 19 \bmod 8$.

a) $x = 29$ is a solution.

b) $x = 19/23$ is a solution.

c) $x = 11$ is a solution.

d) $x = 8/23$ is a solution.

e) $x = 8$ is a solution.

Question 10

Your answer is CORRECT.

Find a solution to the congruence equation $-30x \equiv 31 \bmod 36$.

a) $x = 0$ is a solution.

b) $x = 1$ is a solution.

c) $x = 12$ is a solution.

d) There are no solutions.

e) ○ $x = 11$ is a solution.