PRINTABLE VERSION

Quiz 5

You scored 44.44 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose P.
Missing steps involving P and Q and any previously established facts
Therefore Q. $\square$

What type of proof was described in the outline?

a) A proof by introspection is described in this outline.

b) Wait a minute... The proof described in this outline isn't a valid proof technique!

c) A proof by contrapositive is described in this outline.

d) A direct proof is described in this outline.

e) A proof by contradiction is described in this outline.

Question 2

Your answer is INCORRECT.

Suppose a mathematician wants to prove a statement of the form P. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P \vee \neg Q$.

b) Suppose $\neg P \wedge \neg Q$.

c) Suppose $\neg Q$

d) Suppose $\neg P$.

e) Suppose $\neg P \wedge Q$.

Question 3

Your answer is INCORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \in B$.

b) First supposing $x \in B$, and then showing $x \in A$.

c) First supposing $x \notin A$, and then showing $x \in B$.

d) First supposing $x \in A$, and then showing $x \notin B$.

e) ○ First supposing $x \notin B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $A \subseteq B$ by...

a) ○ First supposing $x \notin A$, and then showing $x \notin B$.

b) ○ First supposing $x \notin A$, and then showing $x \in B$.

c) ○ First supposing $x \in A$, and then showing $x \notin B$.

d) ◉ First supposing $x \notin B$, and then showing $x \notin A$.

e) ○ First supposing $x \notin B$, and then showing $x \in A$.

Question 5

Your answer is INCORRECT.

A lovely little proof is presented below:

Proposition. If $2+x$ is odd, then x is odd.

Proof. Suppose $2+x$ is even. (We will show x is even.)
By definition of even this means $2+x=2m$ for some $m \in \mathbb{Z}$.
By subtracting 2 from both sides it follows that $x=2m-2=2(m-1)$.
Because this expression is even the proof is complete. □

Determine the type of proof used.

a) ○ A proof by indoctrination was used.

b) ○ A proof by contradiction was used.

c) ○ A direct proof was used.

d) ◉ A proof by contrapositive was used.

e) ○ Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. The sum of an odd integer and an even integer is odd.

Proof. (Direct)
(1) Suppose $x, y \in \mathbb{Z}$ are integers.
(2) We can assume x is odd and that y is even.
(3) Since x is odd, it follows that $\exists\, y \in \mathbb{Z},\ x=2y+1$.
(4) Since y is even, it follows that $\exists\, m \in \mathbb{Z},\ y=2m$.
(5) We now have $x+y=(2y+1)+y=3y+1=3(2m)+1=2(3m)+1$.
(6) Because $x+y$ has the form of an odd number it is odd. □

Identify the mistake, if any, in this proof.

a) There is an error in Line (4) since where the definition of "even" is misapplied.

b) There is an error in Line (3) where the variable name "y" is used when this name already refers to a different variable.

c) There is an error in Line (2) since we do not know which integer is odd or even.

d) There is an error in Line (1) since we cannot simply assume $x, y \in \mathbb{Z}$.

e) There is an algebraic mistake in Line (5).

Question 7

Your answer is INCORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. x is a multiple of 3 $\iff$ $(x+1)(x+2)-2$ is also a multiple of 3.

Proof. There are two parts or cases to prove.

($\Rightarrow$ Direct)
(1) Suppose $x = 3m$ for some $m \in \mathbb{Z}$.
(2) Since 3 is prime this means x is a multiple of a prime.
(3) It follows that $(x+1)(x+2)-2 = \left(x^2+3x+2\right)-2 = x^2+3x = (3m)^2+3\cdot 3m$
$= 9m^2+9m = 3\left(3m^2+3m\right)$, which is a multiple of 3.

($\Leftarrow$ By Contradiction)
(4) For a contradiction suppose $(x+1)(x+2)-2$ is multiple of 3, but that x is not a multiple of 3.
(5) Multiplying out this expression and combining like terms tells us $(x+1)(x+2)-2$
$= x^2+3x = 3b$ for some $b \in \mathbb{Z}$.
(6) From this equation we find $x^2 = 3b-3x = 3(b-x)$ and so x^2 is a multiple of 3.
(7) Since x is not a multiple of 3, it follows that x^2 is not a multiple of 3.
(8) Therefore x^2 is a multiple of 3 and x^2 is not a multiple of 3. $\Rightarrow\Leftarrow$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only line (4) is not needed. All other lines are needed.

b) Only line (2) is not needed. All other lines are needed.

c) Only line (5) is not needed. All other lines are needed.

d) Only line (3) is not needed. All other lines are needed.

e) All lines are needed.

Question 8

Your answer is INCORRECT.

Suppose we want to write a proof by contradiction of the proposition below:

$$\forall a, b, c \in [0, \infty),\ (ab = c) \Rightarrow (a \leq \sqrt{c} \lor b \leq \sqrt{c}).$$

Which of the following statements or properties do we need to use when composing this proof?

a) The fact that for real numbers x, y , if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

b) The fact that for real numbers x, y $\sqrt{x} < \sqrt{y}$.

c) Suppose $ab = c$ and that both $a < \sqrt{c}$ and $b < \sqrt{c}$.

d) Suppose $ab = c$ and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\exists\, n \in N,\ 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) We need to show the claim is true for an abitrary natural, saying something like "Let $n \in N$. "

b) We need only check that the claim is true for one, single natural number.

c) The proposition is a famous, unsolved problem. No one knows if it is true or false, and so it is not clear how to describe a proof for this.