PRINTABLE VERSION

Quiz 11

You scored 90 out of 100

Question 1

Your answer is CORRECT.

The congruence equation "$-79 \equiv -271 \bmod 48$ " means

a) -271 and 48 have the same remainder when they are divided by -79 .

b) -79 and -271 have the same remainder when they are divided by 48.

c) -79 and -271 have the same quotient when they are divided by 48.

d) -79 and 48 have the same remainder when they are divided by -271 .

Question 2

Your answer is CORRECT.

The integers -76 and -53 are congruent mod n for which value of n?

a) $n = 24$

b) $n = -53$

c) $n = 23$

d) There are no values of n for which these two integers are congruent (except $n = 1$).

e) $n = -76$

Question 3

Your answer is CORRECT.

Consider the following proposition:

$$\text{Proposition. If } a \equiv b \bmod n, \text{ then } a^4 \equiv b^4 \bmod n.$$

If you were writing a direct proof of this proposition, which of the following equations would be *most* helpful in your proof? (Hint: try to write a proof first!)

a) $a^3(a - b) = a^4 - a^3b$

b) ○ $a^4 - b^4 \geq b^4$

c) ○ $ab = ba$

d) ◉ $a^4 - b^4 = (a - b)(a^3 + a^2 b + ab^2 + b^3)$

e) ○ $a^4 - b^4 = (a - b)^4$

Question 4

Your answer is CORRECT.

Is the following statement true or false?

$\exists\, x, y, a, b \in \mathbb{Z},\ n \in \mathbb{N}^*,\ (x \equiv a \bmod n \ \wedge\ y \equiv b \bmod n) \ \wedge\ ((x + y) \not\equiv (a + b) \bmod n).$

(Note: for this problem $\mathbb{N}^*$ refers to the positive natural numbers $\mathbb{N}^* = \mathbb{N} - \{0\} = \{1, 2, 3, \ldots\}$.)

a) ◉ This statement is false.

b) ○ This statement is true.

Question 5

Your answer is INCORRECT.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Proposition.

Proof (Direct).

(1) Let $x \in \mathbb{Z}$ satisfy $x \not\equiv 0 \bmod 3$.

(2) By The Division Algorithm, there are only two cases to consider.

(3) When x is divided by 3 either it has a remainder of 1 or of 2.

Case 1. $x \equiv 1 \bmod 3$

(4) It follows that $x^2 \equiv 1^2 \bmod 3 \equiv 1 \bmod 3$.

Case 2. $x \equiv 2 \bmod 3$

(5) It follows that $x^2 \equiv 2^2 \bmod 3 \equiv 4 \bmod 3 \equiv 1 \bmod 3$.

(6) Therefore, in all cases $x^2 \equiv 1 \bmod 3$.

a) ◉ $\forall\, x \in \mathbb{Z},\ x \equiv 0 \bmod 3 \ \Rightarrow\ x^2 \not\equiv 1 \bmod 3$.

b) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 0 \bmod 3.$

c) ○ Technically no proposition was proven true since there is a mistake in Line (2); The Division Algorithm does *not* leave only two cases to consider.

d) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 1 \bmod 3.$

Question 6

Your answer is CORRECT.

Use the Euclidean Algorithm to find the inverse of -25 mod 10 (if it exists).

a) ○ $-1/25$ is an inverse.

b) ○ -5 is an inverse.

c) ○ $-10/25$ is an inverse.

d) ○ 2 is an inverse.

e) ◉ -25 does not have an inverse mod 10 because $\gcd(-25, 10) \neq 1$.

Question 7

Your answer is CORRECT.

Of the options provided below, determine the one that best completes this sentence: "The modular equation $-31x \equiv 32 \bmod 93$ ________________ "

a) ○ has exactly one solution.

b) ◉ has no solutions.

c) ○ has multiple solutions.

Question 8

Your answer is CORRECT.

Which steps should one take when solving a congruence equation $ax \equiv b \bmod n$? A helpful summary is presented below, only one step is missing:

Steps for solving $ax \equiv b \bmod n.$

Step 1. Use the Euclidean Algorithm to compute $\gcd(a, n)$.

Step 2. If $\gcd(a, n) \mid b$, then proceed to step 3, otherwise there are no solutions.

Step 3. Use work from Step 1 to calculate one solution $x_0 \in Z$.

Step 4.

Of the following options, which could be used for the missing Step 3?

a) Step 4. Add $\frac{a}{\gcd(a, n)}$ to x_0 to create other solutions.

b) Step 4. Add $\frac{b}{\gcd(a, n)}$ to x_0 to create other solutions.

c) Step 4. Add b to x_0 to create other solutions.

d) Step 4. Add $\frac{\gcd(a, n)}{b}$ to x_0 to create other solutions.

e) Step 4. Add $\frac{n}{\gcd(a, n)}$ to x_0 to create other solutions.

Question 9

Your answer is CORRECT.

Find a solution to the congruence equation $17x \equiv -15 \bmod 5$.

a) $x = 25$ is a solution.

b) $x = 5/17$ is a solution.

c) $x = 6$ is a soltuion.

d) $x = 15/17$ is a solution.

e) $x = 14$ is a solution.

Question 10

Your answer is CORRECT.

Find a solution to the congruence equation $1x \equiv 2 \bmod 14$.

a) $x = 3$ is a solution.

b) $x = 2$ is a solution.

c) $x = 0$ is a solution.

d) $x = 1$ is a solution.

e) There are no solutions.