Subsets & Power Sets

Links: Math 3336

Lecture Video 11: Subsets, Power Sets; Textbook Section 1.3, 1.4

Subsets

A set *B* is a **subset** of another set *A* if every $b \in B$ satisfies $b \in A$ (i.e. if every element of *B* is also an element of *A*)
"every element in the second set is in the first set"

> **notation**
> subset:
> $B \subseteq A$
> *allows for the possibility that* $B = A$
>
> proper subset:
> $B \subset A$
> "*there are elements in A that are not in B*"

example
$\mathbb{N} \subseteq \mathbb{Z}$ (can also be written as the proper subset $\mathbb{N} \subset \mathbb{Z}$)
all together:
$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$
"naturals are a subset of integers which are a subset of rationals which are a subest of reals"
(remember the familiar sets talked about in lecture video 9)

extended further (chain of familiar sets)
$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$
"naturals are a *proper* subset of integers, which are a proper subset of rationals, which are a proper subset of reals, which are a proper subset of complex numbers" (with $\mathbb{C}$ being complex numbers)

The empty set, ϕ, is a subset of **all** sets!
Given any set *S*, $\phi \subseteq S$ (this is vaccuously true)

Power Set

The power set of a given set, *S*, is $\mathscr{P}(S) = \{all\, subsets\, of\, S\}$
$\mathscr{P} = \{x :\, x \subseteq S\}$

The power set is a *set of sets*, for instance, $\phi \in \mathscr{P}(S)$
its also true that $\phi \subseteq \mathscr{P}(S)$, as well as $S \in \mathscr{P}(S)$

summary:
$x \in \mathscr{P}(S) \Leftrightarrow x \subseteq S$
"x is an element power set of some set S, is the same thing as saying that x is a subset of S"

if $|S| = n$, then $|\mathscr{P}(S)| = 2^n$
also $|S| = n = log_2(|\mathscr{P}(S)|)$