The Division & Pigeonhole Principles

ex) consider the fact that a discrete math course has 119 students.

each student will be assigned a grade

| A | A- | B+ | B | B- | C+ | C | C- | D | D- | F |

119 students to place in or share among 11 boxes

• most uniform distribution : $\frac{119}{11} = 10.\overline{81}$

we can't do this !!

tells us : $\frac{119}{11}$ rounds up to 11

at least one box will have 11 students.

the above example demonstrates "the division principle"

math / computation note: rounding a real number up to the nearest integer = taking the "ceiling" of that real number

ex: "ceiling of $\frac{119}{11} = 10.\overline{81}$" = 11

$$\left\lceil \frac{119}{11} \right\rceil = \lceil 10.8181\ldots \rceil = 11$$

($\lceil x \rceil$ = the smallest integer $\geq x$)

ex: $\lceil \pi \rceil = 4$ $\qquad$ $\lceil 14 \rceil = 14$

$\lceil -\pi \rceil = -3$

(note: there's a thing called "the floor", $\lfloor \pi \rfloor = 3$)

Division Principle

If we have n objects sorted into k "boxes"

and if $n > k$, then there is a box that

contains at least $\left\lceil \frac{n}{k} \right\rceil$ objects.

Pigeonhole Principle

If n objects are to be sorted into k boxes

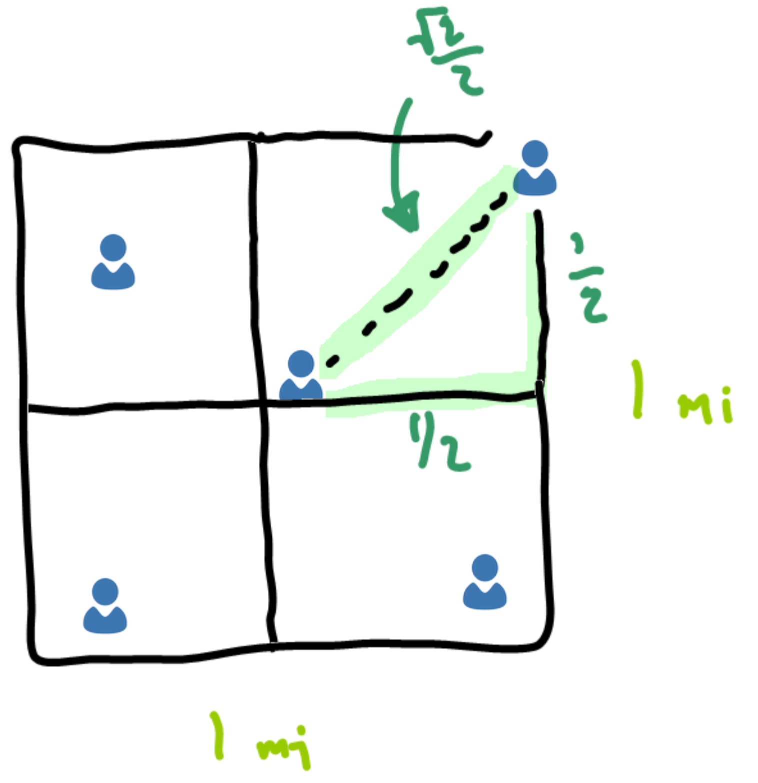
and

$n > K$, then at least one box contains more than one object

$n < K$, then at least one box contains no objects

ex) consider a large square park, 1 mi × 1 mi

and 5 people are at the park



How close do two people HAVE to be?

can argue that 2 people will be within $\frac{\sqrt{2}}{2}$ miles of each other.

dividing into 4 similar $1/2$ mi × $\frac{1}{2}$ mi regions, $5 > 4$

Pigeonhole Principle $\Rightarrow$ one of the 4 regions contains 2 (or more) people!

Pyth Thm tells us those 2 people are at most $\frac{\sqrt{2}}{2}$ miles apart!