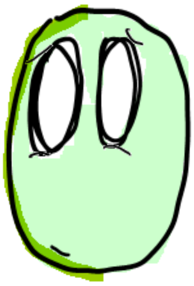
3336

OH 4

3:05 pm



unmute to ask questions!

8. Rewrite the following sentence using logical expressions.

If $\sin x \leq 0$, then it is not the case that $0 \leq x \leq \pi$.

read this as having hidden quantifiers

$$\forall x \in \mathbb{R}, \quad \sin x \leq 0 \Rightarrow \neg(0 \leq x \leq \pi)$$

Similar ex

If $x > 0$ then $2x > 0$.

$$\forall x \in \mathbb{R}, \quad x > 0 \Rightarrow 2x > 0$$

ex) if you are a dog, then you are not a cat.

$$D = \{\text{all dogs}\}, \quad C = \{\text{all cats}\}$$

$$\mathcal{U} = \{\text{all animals}\}$$

$$\forall x \in \mathcal{U}, \quad x \in D \Rightarrow x \notin C$$

ex) Differentiability implies Continuity

$$\boxed{f \text{ being diff'able} \Rightarrow f \text{ is continuous}}$$

$\forall x \qquad \exists x \qquad \exists! \, x$

$$\exists x, \, P(x)$$

Consider the following statement:

$$\exists\, x,\ \forall\, y,\ xy = 0.$$

"such that"

From the options provided below, which universal set U makes this statement true?

$x = 5$ $y = 5$ $5 \cdot 5 \neq 0$

a) $U = \{5\}$

b) $U = \{-1, 3\}$

c) $U = \{1/2, 1, 2\}$

d) $\boxed{U = \{-9, -6, -3, 0, 3, 6, 9\}}$ x

e) $U = \{2, 4, 6, 8, \ldots\}$

$x = -1$
$y = -1$ false
$y = 3$ false

$x = 3$
$y = -1$ false
$y = 3$ false

$$\underbrace{\exists x,\ \forall y,\ x \cdot y = 0}$$

there is an element x that when multiplied against anything $= 0$

OH!! $x = 0$

negating nested quantified statements

$$\neg\left(\forall x,\ \exists y,\ P(x,y)\right)$$

$$\exists x,\ \neg\left(\exists y,\ P(x,y)\right)$$

$$\exists x,\quad \forall y,\ \boxed{\neg P(x,y)}$$

$P(x,y)$: $x \cdot y > 0$

$\neg P(x,y)$: $x \cdot y \leq 0$

3. A new logical operator, ■, is partially defined by the following truth table information:

P	Q	P■Q	$\neg(P$■$Q) \wedge P$	$\neg(P$■$Q) \vee Q$
T	T	T		
T	F	T	F	
F	T	F		
F	F			T

Complete this truth table (no work need be included with this question).

$\neg(P$■$Q) \wedge P$ is False

$\wedge$ is False when at least one "part" is F

P is T so $\neg(P$■$Q)$ is F

P■Q is T

$\underbrace{\neg}_{\text{it is not the}} \underbrace{(P \Rightarrow Q)}_{\text{Both must have same truth value}} = \underbrace{P \oplus Q}_{\text{truth values must be different}}$

it is not that

Both must have same truth value

truth values must be different

For instance, if **T** is a tautology, **F** is a contradiction, and P, Q and R are abstract statements, then the following "equations" hold:

$$P \vee \mathbf{T} = \mathbf{T} \tag{24}$$

$$P \wedge \mathbf{T} = P \tag{25}$$

$$P \wedge \mathbf{F} = \mathbf{F} \tag{26}$$

$$\neg(\neg P \wedge Q) = P \vee \neg Q \tag{27}$$

$$\neg(\neg P \Rightarrow \neg Q) = P \wedge Q \tag{28}$$

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R) \tag{29}$$

De Morgans Laws

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

Quantified statements from Linear Algebra

Definition of "vector space"

1) $\underbrace{\exists!\, \vec{0} \in V, \quad \vec{0} + \vec{v} = \vec{v} = \vec{v} + \vec{0}}$

there is (only one) "zero vector"

2) $\forall\, \vec{v} \in V, \ \exists\, -\vec{v}, \quad \vec{v} + -\vec{v} = \vec{0}$

$$\exists!\, I_2 , \quad \forall M_2 , \quad I_2 \cdot M_2 = M_2$$

i.e. there is a unique identity matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

examples from before college

$$\exists x \in \mathbb{R}, \quad x^2 - x = 0$$

"Solve $x^2 - x = 0$"