PRINTABLE VERSION

Quiz 8

You scored 100 out of 100

Question 1

Your answer is CORRECT.

A clockmaker assigns to each clock produced a serial number consisting of 4 capital letters of the English alphabet followed by 3 numerals (0 through 9). Here is one example of such a serial number:

$$MATH713$$

How many different serial numbers are possible if repetition of letters and digits is allowed?

a) $26^4 \cdot 9^3$

b) $26^3 \cdot 9^4$

c) $26^3 \cdot 10^4$

d) $4 \cdot 3$

e) $26^4 \cdot 10^3$

Question 2

Your answer is CORRECT.

Consider making lists from the symbols T, U, V, W, X, Y, Z. How many length-4 lists are possible if repition is allowed and the list does *not* contain a V ?

a) 4^7

b) $7 \cdot 6 \cdot 5 \cdot 4$

c) 7^4

d) 6^4

e) $4!$

Question 3

Your answer is CORRECT.

Of the options provided below, which one best completes the sentence "The notation $n!$ __________. "

a) ○ is very angry about natural numbers

b) ◉ $= n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$

c) ○ $= n^n$

d) ○ $= \dfrac{n!}{k!(n-k)!}$

e) ○ refers to the number of ways a non-repetitive length-k list may be formed using n symbols

Question 4

Your answer is CORRECT.

Suppose the set S has 8 elements. How many subsets of size 6 are there?

a) ○ 262144

b) ○ 40320

c) ◉ 28

d) ○ 94

Question 5

Your answer is CORRECT.

A (numerical) palindrome is a natural number that, when expressed in our standard digit system, reads the same forward as backward. For example, the number 12021 is a palindrome, as is 353. How many 12 digit palindromes are there?

a) ○ $9^2 \cdot 10^{10}$

b) ◉ $9 \cdot 10^5$

c) ○ 10^6

d) ○ 10^{12}

e) ○ $9 \cdot 10^{11}$

Question 6

Your answer is CORRECT.

This problem concerns lists of length 13 made from the (capital letters from the) English alphabet $A, B, C, \ldots, Y, Z$. How many lists will contain the word MATH?

a) $10 \cdot 26^9$

b) 26^9

c) 26^{12}

d) 9^{26}

e) 4^{12}

Question 7

Your answer is CORRECT.

Of the options provided below, which one best explains why the following formula is true?

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n} = 2^n$$

a) The sum on the left side adds up the different numbers of subsets of an n -element set, starting with the number of size-0 subsets, then the number of size-1 subsets, etc. The expression on the right equals the number of *all* possible subsets, and so these two values must be equal.

b) No explanation can be given becuase this equation is not true.

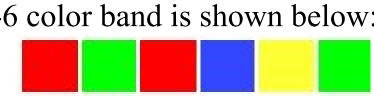
c) Both expressions involve n and subsets.

d) The sum on the left side adds up numbers from 1 to n . The expression on the right equals the number of *all* possible numbers between 1 and n , and so these two values must be equal.

Question 8

Your answer is CORRECT.

A length-n "color band" is a sequence of n squares arranged along a single row, where each square has been filled in with a particular color *and* the coloring obeys this one rule: *no two adjacent squares can have the same color.* An example of a length-6 color band is shown below:



How many length-3 color bands are possible when we are only allowed to pick from 6 colors?

a) $6 \cdot 5^2$

b) $\dfrac{6!}{(6-3)!} = 120$

c) $\dfrac{3!}{(3-6)!} = 0$

d) 6^3

e) $6^1 \cdot 5^2$

Question 9

Your answer is CORRECT.

How many 16-digit binary strings contain exactly 9 ones?

a) $\binom{9}{16} = 0$

b) $2^{16} - 2^9 = 65024$

c) $\binom{16}{9} = 11440$

d) 2^7

Question 10

Your answer is CORRECT.

Thank you for working hard on this quiz! As a token of your instructor's appreciation, take just a few moments to enjoy answering this question: Which of the following most accurately summarizes the content of this quiz?

a) Counting is super easy! We learned about it when I was, like, six years old.

b) The derivive of $\sin x$ is $\cos x$.

c) Counting cards isn't illegal, but it can get you banned from casinos.

d) Counting strings (and related objects) uses the Multiplication Principle and often involves expressions like $n!$ or $\dfrac{n!}{(n-k)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)$. Counting subsets (and related objects) is related to counting strings, but there are fewer subsets than strings since order doesn't matter; counting subsets (and related objects) often uses expressions like $\binom{n}{k} = \frac{n!}{n!(n-k)!}$.

e) None of the above.