# PRINTABLE VERSION

## Quiz 11

## You scored 90 out of 100

**Question 1**

Your answer is CORRECT.

The congruence equation "$-60 \equiv -185 \bmod 25$" means

**a)** When $-60-(-185)$ is divided by $25$ the remainder is 0.

**b)** When $25-(-185)$ is divided by $-60$ the remainder is 0.

**c)** When $25$ is divided by $-60-(-185)$ the remainder is 0.

**d)** When $25-(-185)$ is divided by $-60$ the remainder is 0.

**Question 2**

Your answer is CORRECT.

The integers $-56$ and $3$ are congruent mod $n$ for which value of $n$?

**a)** $n = 60$

**b)** $n = 59$

**c)** There are no values of $n$ for which these two integers are congruent (except $n = 1$).

**d)** $n = 3$

**e)** $n = -56$

**Question 3**

Your answer is CORRECT.

Consider the following proposition:

$$\text{Proposition. If } a \equiv b \bmod n, \text{ then } a^2 \equiv b^2 \bmod n.$$

If you were writing a direct proof of this proposition, which of the following equations would be *most* helpful in your proof? (Hint: try to write a proof first!)

**a)** $a(a-b) = a^2 - ab$

b) $a^2 - b^2 = (a - b)(a + b)$

c) $ab = ba$

d) $a^2 - b^2 \geq a^2$

e) $a^2 - b^2 = (a - b)^2$

**Question 4**

Your answer is CORRECT.

Is the following statement true or false?

$\exists\, x, y, a, b \in \mathbb{Z},\ n \in \mathbb{N}^*,\ (x \equiv a \bmod n\ \wedge\ y \equiv b \bmod n)\ \wedge\ ((x + y) \not\equiv (a + b) \bmod n).$

(Note: for this problem $\mathbb{N}^*$ refers to the positive natural numbers $\mathbb{N}^* = \mathbb{N} - \{0\} = \{1, 2, 3, \ldots\}$ .)

a) This statement is false.

b) This statement is true.

**Question 5**

Your answer is INCORRECT.

A (direct) proof for a Proposition is presented below. Read through the proof and then determine which Proposition was proven.

Proposition.

Proof (Direct).

(1) Let $x \in \mathbb{Z}$ satisfy $x \not\equiv 0 \bmod 3$.

(2) By The Division Algorithm, there are only two cases to consider.

(3) When $x$ is divided by 3 either it has a remainder of 1 or of 2.

Case 1. $x \equiv 1 \bmod 3$

(4) It follows that $x^2 \equiv 1^2 \bmod 3 \equiv 1 \bmod 3$.

Case 2. $x \equiv 2 \bmod 3$

(5) It follows that $x^2 \equiv 2^2 \bmod 3 \equiv 4 \bmod 3 \equiv 1 \bmod 3$.

(6) Therefore, in all cases $x^2 \equiv 1 \bmod 3$.

a) $\forall\, x \in \mathbb{Z}, x \equiv 0 \bmod 3 \Rightarrow x^2 \not\equiv 1 \bmod 3$.

b) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 0 \bmod 3.$

c) ○ $\forall x \in Z, x \not\equiv 0 \bmod 3 \Rightarrow x^2 \equiv 1 \bmod 3.$

d) ◉ Technically no proposition was proven true since there is a mistake in Line (2); The Division Algorithm does *not* leave only two cases to consider.

**Question 6**

Your answer is CORRECT.

Use the Euclidean Algorithm to find the inverse of $-13 \bmod 28$ (if it exists).

a) ○ $28$ is an inverse.

b) ◉ $15$ is an inverse.

c) ○ $-1/13$ is an inverse.

d) ○ $-28/13$ is an inverse.

e) ○ $-13$ does not have an inverse mod $28$ because $\gcd(-13, 28) \neq 1$ .

**Question 7**

Your answer is CORRECT.

Of the options provided below, determine the one that best completes this sentence: "The modular equation $14x \equiv 27 \bmod 22$ ________________ "

a) ○ has multiple solutions.

b) ◉ has no solutions.

c) ○ has exactly one solution.

**Question 8**

Your answer is CORRECT.

Which steps should one take when solving a congruence equation $ax \equiv b \bmod n$ ? A helpful summary is presented below, only one step is missing:

Steps for solving $ax \equiv b \bmod n.$

Step 1. Use the Euclidean Algorithm to compute $\gcd(a, n)$.
Step 2.
Step 3. Use work from Step 1 to calculate one solution $x_0 \in Z$.
Step 4. Add $\dfrac{n}{\gcd(a, n)}$ to $x_0$ to create other solutions.

Of the following options, which could be used for the missing Step 2?

a) ○ Step 2. If $\gcd(a, n) \mid n$, then proceed to step 3, otherwise there are no solutions.

b) ○ Step 2. If $b \mid \gcd(a, n)$, then proceed to step 3, otherwise there are no solutions.

c) ○ Step 2. If $b \mid a$, then proceed to step 3, otherwise there are no solutions.

d) ◉ Step 2. If $\gcd(a, n) \mid b$, then proceed to step 3, otherwise there are no solutions.

e) ○ Step 2. If $\gcd(a, n) \mid a$, then proceed to step 3, otherwise there are no solutions.

**Question 9**

Your answer is CORRECT.

Find a solution to the congruence equation $23x \equiv 18 \bmod 17$.

a) ○ $x = 17/23$ is a solution.

b) ○ $x = 55$ is a solution.

c) ○ $x = 17$ is a solution.

d) ◉ $x = 54$ is a solution.

e) ○ $x = 18/23$ is a solution.

**Question 10**

Your answer is CORRECT.

Find a solution to the congruence equation $12x \equiv 25 \bmod 8$.

a) ○ $x = 5$ is a solution.

b) ○ $x = 2$ is a solution.

c) ○ $x = 0$ is a solution.

**d)** $x = 6$ is a solution.

**e)** There are no solutions.