3336

Office Hours

12:00

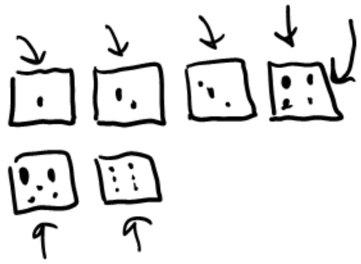
Use the Pigeonhole Principle to answer the following question. What is the fewest number of times needed to roll a 6-sided dice so that 8 or more of the rolls result in the same number? (Obviously you'll need to roll the dice at *least* 8 times, but that may not be enough to guarantee the desired outcome... or will it!?)

Simpler version

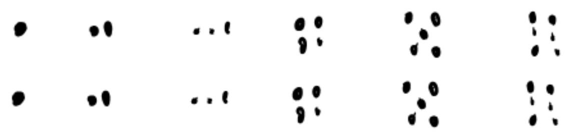
2

how many rolls will produce $\boxed{\text{two}}$ of the same number?

1 roll not enough | 2 rolls not enough | 3 rolls not enough | 4 rolls " " | 5 rolls " " | 6 rolls " " | $\boxed{\text{7 rolls}}$



how many rolls will produce $\boxed{3}$ of the same number?



← 6 rolls

← 6 more rolls

← 1 more roll

13 rolls ensure repeating 3 outcomes

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1. (Part A) Write a proof of the following proposition (make certain to label the style of proof you are using):

Proposition. $\forall\, a, b \in \mathbb{Z}$, $(a+b)^3 \equiv (a^3+b^3) \mod 3$

$\boxed{X \equiv Y \text{ mod } 3}$ → want to show $X - Y = 3m$, $m \in \mathbb{Z}$

$a=1$ $b=1$

LHS

$2^3 = 8$

RHS

$1^3 + 1^3 = 1+1 = 2$

$8 \equiv 2 \mod 3$ ✓

i $2x \equiv 7 \bmod 5$

multiply both sides by 2^{-1}

$$x \equiv 2^{-1} \cdot 7 \bmod 5 = 3 \cdot 7 \bmod 5 = 21 \bmod 5 = 1 \bmod 5$$

work in para b: $2^{-1} \equiv \underline{3} \bmod 5$

$\boxed{x=1}$

$2x \equiv 7 \bmod 5$

you need the inverse of 2

one way	another way
Euclid. alg. to get $\gcd(2,5)=1$	fill out a multi. table for $\mathbb{Z}_5$ and see that $2^{-1} \equiv 3 \bmod 5$
Bezout's Id to get 2^{-1}	

$$\underbrace{2 \cdot 3 = 6 \equiv 1 \bmod 5}$$

$2 \cdot 3 = 1$ in $\mathbb{Z}_5$

$$2^{-1} 2x \equiv 2^{-1} 7 \bmod 5$$

$$x \equiv 2^{-1} \cdot 7 \bmod 5$$

$$\equiv 3 \cdot 7 \bmod 5$$

$$\equiv 21 \bmod 5$$

$\times_5$

$\times_5$	0	1	2	3	4
0					
1					
2	0	2	4	1	3
3	0	3	1	4	2
4					

$\underbrace{\boxed{2\cdot 3 = 1}}$ in $\mathbb{Z}_5$

tells you $3 = 2^{-1}$

$\underbrace{\boxed{3\cdot 2 = 1}}$

tells you $\boxed{3^{-1} = 2}$ in $\mathbb{Z}_5$

5. It is well known that the equation $x^2 + 1 = 0$ has no solutions in the set $\mathbb{R}$.

Are there solutions if we instead use the set $\mathbb{Z}_{10}$? That is, does the analogous equation $x^2+1 \equiv 0 \bmod 10$ have any solutions? If there are solutions, find them and express them mod-10 (i.e. as integers from the set $\{0, 1, \cdots 9\}$) and neatly show your work.

$$x^2+1=0$$

$x=0$: $0^2+1=1$ ✗

$x=1$: $1^2+1=2$ ✗

$x=2$: $2^2+1=5$ ✗

$x=3$: $3^2+1 = 9+1 = 10 \equiv 0 \bmod 10$ ✓

9. How many prime numbers are there?

(a) There are 10 prime numbers.

(b) There are 17 prime numbers.

(c) The are infinitely many prime numbers.

right!!

(d) There are finitely many prime numbers, but the exact amount is not currently known.

(e) There are zero prime numbers.

wrong!!