Math 3336 : Test 1 Review

Instructions

- This is not an assignment. Neither work nor answers are to be submitted.
- Also, **DON'T FORGET THAT** $0 \in \mathbb{N}$.

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1. Write down

(a) an example of a true statement.

5 is an odd integer.

(b) an example of a false statement.

5 is an even integer.

(c) an example of an open sentence.

x^2 is even

(d) an example of a non-statement (that is also *not* an open sentence).

Shut the door!

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2. suppose we have two finite sets, A and B, that satisfy

$$|A| = 5 \text{ and } |B| = 7$$

(a) Is it possible for $|A \cup B| = 5$?

no, $B \subseteq A \cup B$ so $|A \cup B| \geq |B| = 7$

(b) Is it possible for $|A \cup B| = 12$?

yes, if $A \cap B = \emptyset$, $|A \cup B| = 12$

(c) Is it possible for $|A \cup B| = 4$?

no (see (a))

(d) Compute $|\mathcal{P}(A)|$

$|\mathcal{P}(A)| = 2^5$

(e) Compute $|A \times B|$

$|A \times B| = 35$

(f) Compute $|\mathcal{P}(A) \times B|$

$|\mathcal{P}(A) \times B| = 2^5 \cdot 7$

(g) Is it possible for $|A \cap B| = 0$?

yes, if A & B are disjoint

3. Suppose P, Q and R are statements. Is it possible that the following is true?

$$(P \Rightarrow Q) \iff R \text{ is true}$$
$$P \vee R \text{ is false}$$

First one is true means $P \Rightarrow Q$ & R have same truth value. Second one means both P is F & R is F. Since P is F, $(P \Rightarrow Q)$ is true.

But this means $(P \Rightarrow Q)$ and R have diff. truth values

$\boxed{\text{impossible}}$

4. Fill in the following truth-tables:

P	Q	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \iff Q$	$P \oplus Q$
T	T	T	T	T	T	F
T	F	F	T	F	F	T
F	T	F	T	T	F	T
F	F	F	F	T	T	F

5. Is $\neg(P \wedge Q) \vee P = \neg P \vee (\neg Q \vee P)$? Explain your answer using a truth table.

P	Q	$\neg(P \wedge Q) \vee P$	$\neg P \vee (\neg Q \vee P)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	T	T

logically equiv.

both are tautologies!

6. Determine which, if any, of the following statements is true when using the universal set $U = \{2n : n \in \mathbb{Z}\}$.

X (a) $\exists!\, x \in U,\ x^4 = 16.$ ← multiple x's make this true

X (b) $\forall\, t \in U,\ 3t$ is odd. ← no t's make this true!

X (c) $\exists\, y \in U,\ y^3 = 27.$ ← no y's make this true

$\boxed{\text{(d) } \forall\, s \in U,\ \cos(\pi s) = 1.}$ ✓

(e) None of the other statements are true.

$\cos(\text{even} \cdot \pi) = 1$ ✓

7. Write down a non-empty universal set, U, that makes the following statement true:

$$\forall\, m \in U,\ 3|m \wedge 2|m.$$

one answer = {all multiples of 6} = $\{6a : a \in \mathbb{Z}\}$

other answers: $\{6\}$, $\{-6\}$, $\{0, 6\}$, etc. ✓

8. Consider the following claim:

$$\forall\, (x, y) \in \mathbb{R}^2,\ \exists\, (u, v) \in \mathbb{R}^2,\ (x, y) + (u, v) = (x, y).$$

If one wanted to prove this using the Contrapositive method, then what could be used as the first line of the proof? What could be used as the last line?

See OH notes from

T 9/27

9. A sequence of real numbers, $\{a_n\}$, satisfies the recurrence equation and initial condition

$$a_n = a_{n-1} + 5 \text{ and } a_0 = 3.$$

Write the first four terms of this sequence. Do *any* of the terms in the sequence equal 33? If so, which one(s)?

$a_0 = 3$

$a_1 = 3 + 5 = 8$

$a_2 = 8 + 5 = 13$

$a_3 = 13 + 5 = 18$

$a_4 = 18 + 5 = 23$

$a_5 = 23 + 5 = 28$

$\boxed{a_6 = 28 + 5 = 33}$ ✓

only $a_6 = 33$

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10. Consider the statement

$$P : \exists q \in \mathbb{Q}, \forall p \in \mathbb{Z}, q \cdot p = 1.$$

Use logical symbols to write $\neg P$. Which statement is true, P or $\neg P$? (You do not need to write a proof.)

$\neg P : \neg(\exists q \in \mathbb{Q}, \forall p \in \mathbb{Z}, qp = 1)$

$= \forall q \in \mathbb{Q}, \neg(\forall p \in \mathbb{Z}, qp = 1)$

$\boxed{= \forall q \in \mathbb{Q}, \exists p \in \mathbb{Z}, qp \neq 1}$

$\neg P$ is true (given any q, use $p = 0$ for ex.)

11. Consider the floor function $f(x) = \lfloor x \rfloor$ with domain and co-domain equal to $\mathbb{R}$.

(a) Where does f send the input $x = 5/3$?

$f(5/3) = \lfloor 5/3 \rfloor = 1$

(b) Is there an input $a \in \mathbb{R}$ that satisfies $f(a) = 5/3$? If so, identify the element(s) a that make this true. If not, explain why.

no, $f(x) = \lfloor x \rfloor$ only outputs integers.

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12. (HW 2 repeat) A new logical operator, ■, is partially defined by the following truth table information:

P	Q	$P■Q$	$\neg(P■Q) \wedge P$	$\neg(P■Q) \vee Q$
T	T	T	F	T
T	F	T	F	F
F	T	F	F	T
F	F	F	F	T

Complete this truth table. Is $P■Q$ a contradiction? A tautology? Neither?

not a tautology since its truth values are not all T

not a contradiction since its truth values are not all F

13. Write a proof of the following proposition:

Proposition. Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.

note: this talks about integers dividing integers

CP: if $\underbrace{\neg(a|c)}_{a \nmid c}$ then $\neg(a|b \wedge b|c)$

this is awkward

CP talks about not dividing

using CP method is probably a bad idea

Proposition. Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.

Proof (Direct)

Suppose $a|b$ and $b|c$. (We want to show $a|c$.)

This means $\exists\, m \in \mathbb{Z}$, $b = m \cdot a$ and $\exists\, n \in \mathbb{Z}$, $c = n \cdot b$.

It follows that $c = n \cdot b = n \cdot (ma) = (n \cdot m) \cdot a$.

This shows $a|c$ since $nm \in \mathbb{Z}$. $\square$