PRINTABLE VERSION

Quiz 8

You scored 90 out of 100

Question 1

Your answer is CORRECT.

A clockmaker assigns to each clock produced a serial number consisting of 2 capital letters of the English alphabet followed by 5 numerals (0 through 9). Here is one example of such a serial number:

AC27182

How many different serial numbers are possible if repetition of letters and digits is allowed?

a) ◉ $26^2 \cdot 10^5$

b) ○ $2 \cdot 5$

c) ○ $26^2 \cdot 9^5$

d) ○ $26^5 \cdot 9^2$

e) ○ $26^5 \cdot 10^2$

Question 2

Your answer is INCORRECT.

Consider making lists from the symbols $\mathrm{T}, \mathrm{U}, \mathrm{V}, \mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$. How many length-4 lists are possible if repetition is not allowed and the list must contain a W (in any position)?

a) ○ $4!$

b) ○ $4 \cdot (6 \cdot 5 \cdot 4)$

c) ◉ $7 \cdot 6 \cdot 5 \cdot 4$

d) ○ $7^4 - 6^4 = 1105$

e) ○ 7^4

Question 3

Your answer is CORRECT.

Of the options provided below, which one best completes the sentence "The notation $n!$ _____. "

a) ○ is very angry about natural numbers

b) ◉ $= n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$

c) ○ $= n^n$

d) ○ $= \dfrac{n!}{k!(n-k)!}$

e) ○ refers to the number of ways a non-repetitive length-k list may be formed using n symbols

Question 4

Your answer is CORRECT.

Suppose the set S has 4 elements. How many subsets of size 2 are there?

a) ◉ 6

b) ○ 24

c) ○ 16

d) ○ 12

Question 5

Your answer is CORRECT.

A (numerical) palindrome is a natural number that, when expressed in our standard digit system, reads the same forward as backward. For example, the number 12021 is a palindrome, as is 353. How many 8 digit palindromes are there?

a) ○ $9 \cdot 10^7$

b) ○ 10^8

c) ○ 10^4

d) ◉ $9 \cdot 10^3$

e) ○ $9^2 \cdot 10^6$

Question 6

Your answer is CORRECT.

This problem concerns lists of length 11 made from the (capital letters from the) English alphabet $A, B, C, \ldots, Y, Z$. How many lists will contain the word MATH?

a) 4^{10}

b) 26^7

c) $8 \cdot 26^7$

d) 26^{10}

e) 7^{26}

Question 7

Your answer is CORRECT.

Of the options provided below, which one best explains why the following formula is true?

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

a) The expression on the left counts the number of size-$(k+1)$ subsets from the set $\{1, 2, 3, \cdots, n, n+1\}$. The expression on the right counts the number of size $(k+1)$ subsets from an n-element set as well as the number of size k subsets from an n-element set and then adds these together.

b) The expression on the left counts the number of size-$(k+1)$ subsets from the set $\{1, 2, 3, \cdots, n, n+1\}$, and these can be split into two types. Subsets that do not contain the element $n+1$ can be thought of as size-$(k+1)$ subsets of $\{1, 2, 3, \cdots, n\}$, and there are $\binom{n}{k+1}$ of these. Subsets that do contain the element $n+1$ are obtained by taking k-element subsets of $\{1, 2, 3, \cdots, n\}$ and inserting the element $n+1$ into them, which means there are $\binom{n}{k}$ of these.

c) No explanation can be given becuase this equation is not true.

d) This equation is true because $\binom{n+1}{k+1} - \binom{n}{k+1} - \binom{n}{k}$ is a number and the empty set has size $|\emptyset| = 0$.

Question 8

Your answer is CORRECT.

A length-n "color band" is a sequence of n squares arranged along a single row, where each square has been filled in with a particular color *and* the coloring obeys this one rule: *no two adjacent squares can have the same color.* An example of a length-6 color band is shown below:

How many length-4 color bands are possible when we are only allowed to pick from 7 colors?

a) $7 \cdot 6^3$

b) $\frac{7!}{(7-4)!} = 840$

c) $7^2 \cdot 6^2$

d) 7^4

e) $\frac{4!}{(4-7)!} = 0$

Question 9

Your answer is CORRECT.

How many 14-digit binary strings contain exactly 7 zeroes?

a) $\binom{7}{14} = 0$

b) 2^7

c) $\binom{14}{7} = 3432$

d) $2^{14} - 2^7 = 16256$

Question 10

Your answer is CORRECT.

Thank you for working hard on this quiz! As a token of your instructor's appreciation, take just a few moments to enjoy answering this question: Which of the following most accurately summarizes the content of this quiz?

a) Counting strings (and related objects) uses the Multiplication Principle and often involves expressions like $n!$ or $\frac{n!}{(n-k)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-k+1)$. Counting subsets (and related objects) is related to counting strings, but there are fewer subsets than strings since order doesn't matter; counting subsets (and related objects) often uses expressions like $\binom{n}{k} = \frac{n!}{n!(n-k)!}$.

b) Counting cards isn't illegal, but it can get you banned from casinos.

c) Counting is super easy! We learned about it when I was, like, six years old.

d) The derivive of e^x is e^x.

e) None of the above.