Union, Intersection & Difference

Links: Math 3336

Lecture Video 12: Union, Intersection, Difference; Textbook Section 1.5

(more ways to create new sets from two given sets)

Definitions

Union
Given two sets *A* and *B*: $A \cup B$ = "A union B" = $\{x : x \in A \ or \ x \in B\}$

> **example**
> $A = \{1, 2, \$\}, \ \ B = \{0, 1\}$
> $A \cup B = \{1, 2, \$, 0\}$

(needs to be in at least one set)

Intersection
Given two sets *A* and *B*: $A \cap B$ = "A intersection B" = $\{x : x \in A \ and \ x \in B\}$

> **example**
> $A = \{1, 2, \$\}, \ \ B = \{0, 1\}$
> $A \cap B = \{1\}$

(needs to be in both sets)

Difference
the Difference of two sets:
$A - B$ = "A takeaway B or A difference B" = $\{x : x \in A \ and \ x \notin B\}$
(*all elements x, such that x is an element of A but not an element of B*)

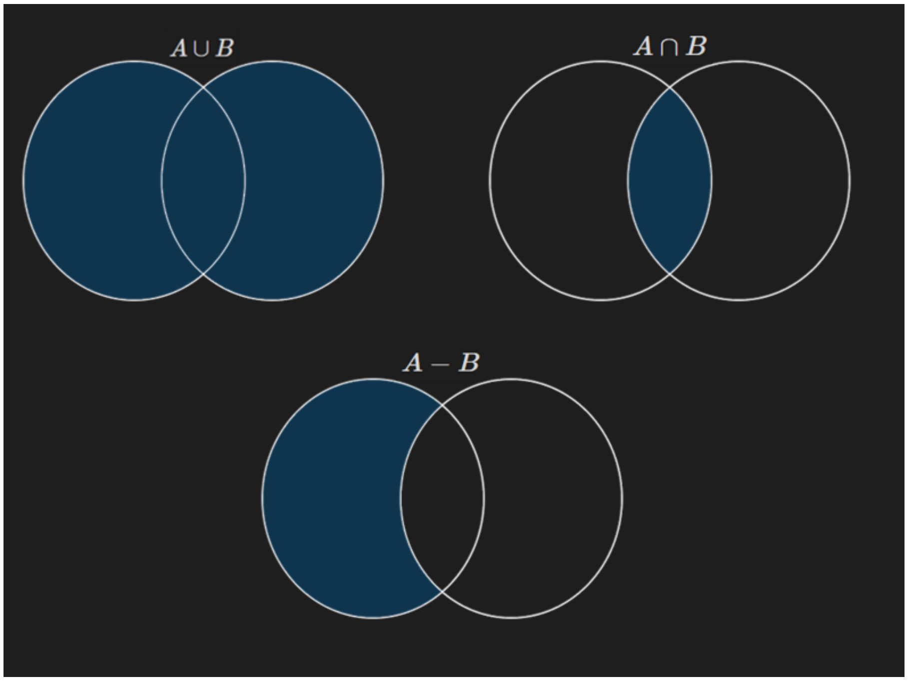
> **example**
> $A = \{1, 2, \$\}, \ \ B = \{0, 1\}$
> $A - B = \{2, \$\}$

(throwing out things in A that are in B)

Disjoint

Two sets are **disjoint** if their intersection is empty, i.e.: "A and B are disjoint if $A \cap B = \phi$"

Union, Intersection, and Difference visualized as Venn Diagrams:



Operations on more than two sets

These operations can be expanded to more than two sets, for instance:
$A \cup B \cup C = \{x : x \in A \ or \ x \in B \ or \ x \in C\}$

union of inifinite amounts of sets
$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \ldots = \{x : x \in A_i \ \ for\, some\ i\}$

intersection of infinite amounts of sets
$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \ldots = \{x : x \in A_i \ \ for\, every\ i\}$

example

> $\mathbb{N} \cap \mathbb{Z} = \mathbb{N}$
> (*remember that the naturals are a [subset]() of the integers*)
>
> $\mathbb{N} \cup \mathbb{Z} \cup \mathbb{Q} = \mathbb{Q}$
> *(answer is the biggest one since all the others are subsets of it)*

example 2

$S_i = [0, i] \quad i \in \mathbb{N}$

$S_1 = [0,1],\ \ S_2 = [0,2],\ \ S_3 = [0,3], \ldots$
only $[0,1]$ is in all of them, thus:
$\bigcap_{i=1}^{\infty} S_i = [0,1]$

however all the intervals will be in the intervals of all the other sets as the index increases, thus:
$\bigcup_{i=1}^{\infty} S_i = \{x : 0 \leq x\}$ (or in calculus notation: $[0,\infty)$)