Addition & Subtraction Principles

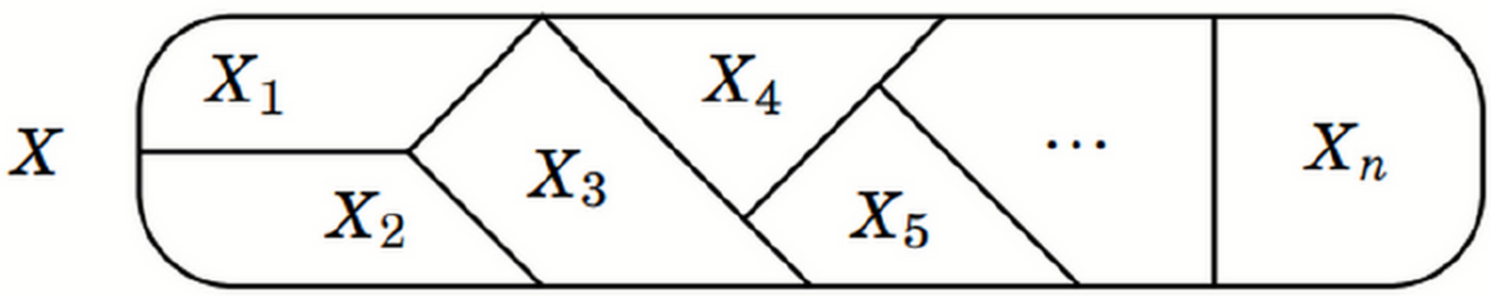
these are intuitive ideas which we describe in terms of sets

Addition Principle

$$|X| = |X_1| + |X_2| + \cdots + |X_n|$$

provided:

1) X is a finite set
2) each $X_i \subseteq X$
3) the subsets are disjoint : $X_i \cap X_j = \emptyset$



very similar to the notion a partition of a set S

a partition of S is a collection subsets that are pairwise disjoint and none of them are empty!

for partitions, the big set, S, and the subsets can be infinite!

example (from the book)

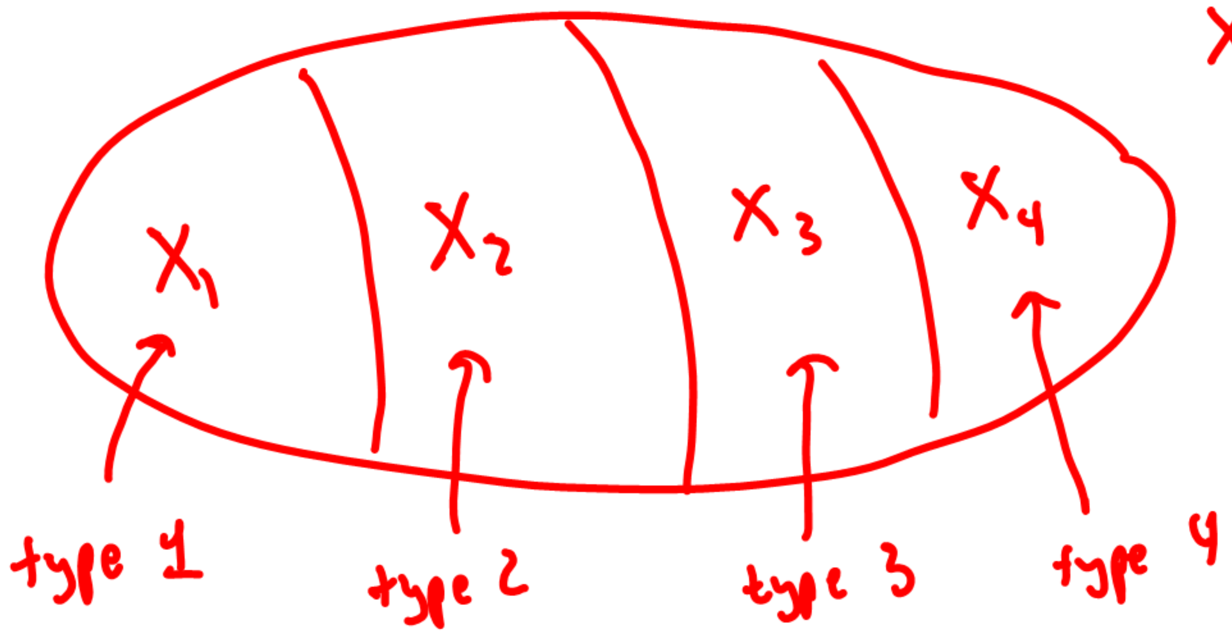
Example 3.5 How many length-4 non-repetitive lists can be made from the symbols A, B, C, D, E, F, G, if the list must contain an E?

In Example 3.3 (c) our approach was to divide these lists into four types, depending on whether the E is in the first, second, third or fourth position.

Type 1	Type 2	Type 3	Type 4
E □ □ □	□ E □ □	□ □ E □	□ □ □ E
6 5 4	6 5 4	6 5 4	6 5 4
$6 \cdot 5 \cdot 4 = 120$ type 1 lists	120 these	120 these	120 these

there are a total $4 \cdot 120 = 480$ lists meeting these conditions!

$X = \{$ lists meeting conditions $\}$



$$|X| = |X_1| + |X_2| + |X_3| + |X_4|$$

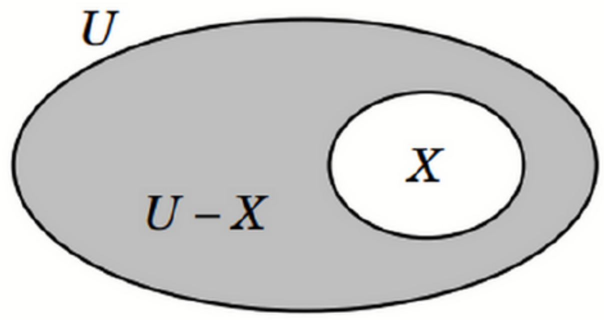
$$= 120 + 120 + 120 + 120$$

$$= 4 \cdot 120 = 480 \checkmark$$

The subtraction principle

If $X \subseteq \mathcal{U}$ ($\mathcal{U}$ is finite)

then $|\overline{X}| = |\mathcal{U}| - |X|$



$\left(\mathcal{U} = X \cup \overline{X} \xrightarrow[\text{add. princ.}]{} |\mathcal{U}| = |X| + |\overline{X}| \right)$

5. How many integers between 1 and 9999 have no repeated digits? How many have at least one repeated digit? (base 10)

ex) $\mathcal{U} = [1, 9999] \cap \mathbb{N}$ = all numbers between 1 + 9999

$X = \{ x \in \mathcal{U} : x \text{ has no repeated digit} \}$

$X_1 = \{ 1 \text{ digit numbers in } X \}$ (1, 2, 3, 4, 5, 6, 7, 8, 9) $\quad |X_1| = 9$

$X_2 = \{ 2 \text{ digit } '' \quad '' \}$ (10, ~~11~~, 12, 13, 14, 15, 16, 17, 18, 19) $\quad |X_2| = 9 \cdot 9$ (9, $10 - 1 = 9$)

$X_3 = \{ 3 \text{ digit } '' \quad '' \} \rightarrow |X_3| = 9 \cdot 9 \cdot 8$ (9 9 8)

$X_4 = \{ 4 \text{ digit } '' \quad '' \}$

9 9 8 7

$|X_4| = 9 \cdot 9 \cdot 8 \cdot 7$

$X = X_1 \cup X_2 \cup X_3 \cup X_4$

$|X| = |X_1| + |X_2| + |X_3| + |X_4| = 9 + 9 \cdot 9 + 9 \cdot 9 \cdot 8 + 9 \cdot 9 \cdot 8 \cdot 7$

$$|X| = 9 + 81 + 648 + 4536$$
$$= 5274$$

ex) How many have at least one repeated digit?

i.e. How many have one or more repeated digits?

$$X = \{x \in \mathcal{U} : x \text{ has no repeated digits}\}$$

this question is asking for $|\overline{X}|$

$$\overline{X} = \{x \in \mathcal{U} : \sim(x \text{ has no repeated digits})\}$$

subtraction principle:

$$|\overline{X}| = |\mathcal{U}| - |X|$$

we calculated this already

to calculate this....

$\mathcal{U} = [1, 9999] \cap \mathbb{N}$ $\quad |\mathcal{U}| = 9999$

$$|\overline{X}| = 9999 - 5274$$
$$= 4725$$