PRINTABLE VERSION

Quiz 5

You scored 88.89 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $P \Rightarrow Q$ is false.
This means $\neg(P \Rightarrow Q) = P \wedge \neg Q$ is true.
Missing steps involving P, $\neg Q$, *and any previously established facts*
Therefore 0=1 (or some similarly weird conclusion) $\Rightarrow\Leftarrow$.

What type of proof was described in the outline?

a) Wait a minute... The proof described in this outline isn't a valid proof technique!

b) A proof by contrapositive is described in this outline.

c) A direct proof is described in this outline.

d) A proof by introspection is described in this outline.

e) A proof by contradiction is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P \wedge Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg Q$

b) Suppose $\neg P \vee \neg Q$.

c) Suppose $\neg P \wedge \neg Q$.

d) Suppose $\neg P$.

e) Suppose $\neg P \wedge Q$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \in A$, and then showing $x \notin B$.

b) First supposing $x \notin B$, and then showing $x \in A$.

c) First supposing $x \in A$, and then showing $x \in B$.

d) First supposing $x \notin A$, and then showing $x \in B$.

e) First supposing $x \in B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) ◯ First supposing $x \notin B$, and then showing $x \in A$.

b) ◯ First supposing $x \notin A$, and then showing $x \in B$.

c) ◉ First supposing $x \notin A$, and then showing $x \notin B$.

d) ◯ First supposing $x \notin B$, and then showing $x \notin A$.

e) ◯ First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If the product of two integers is even, then at least one of the integers is even.

Proof. Suppose $x, y \in \mathbb{Z}$ and neither x nor y is even. (We will show that xy is not even.)
This means x and y are both odd so that $x = 2n+1$ and $y = 2m+1$ for integers n, m.
It follows that $xy = (2n+1)(2m+1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$ which is odd since $(2nm + n + m)$
Therefore xy is not even.□

Determine the type of proof used.

a) ◯ A proof by indoctrination was used.

b) ◯ Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

c) ◯ A direct proof was used.

d) ◉ A proof by contrapositive was used.

e) ◯ A proof by contradiction was used.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. There do not exist integers a and b that satisfy $27a + 9b = 1$.

Proof. (By Contradiction)
(1) The proposition can be rewritten as $\forall\, a, b \in \mathbb{Z},\ 27a + 9b \neq 1$.
(2)For the sake of a contradiction we will assume that the negation of this proposition is trure; that is, we will assume $\forall$
(3) Dividing the equation above by 3 produces the equation $9a + 3b = \frac{1}{3}$.
(4) Since $a, b \in \mathbb{Z}$, it follows that $9a + 3b \in \mathbb{Z}$.
(5) However, $9a + 3b = \frac{1}{3} \notin \mathbb{Z}$.
(6) Therefore $9a + 3b$ is an integer, and $9a + 3b$ is not an integer. $\Rightarrow\Leftarrow$

Identify the mistake, if any, in this proof.

a) ◯ There is an algebraic mistake in Line (3).

b) ◉ There is a mistake in Line (2) since the negation of the proposition should use the quantifier ∃ , not ∀.

c) ◯ There is a mistake in Line (4) since Z is not closed under addition.

d) ◯ There is a mistake in Line (1); this is not a correct way to rewrite the proposition.

e) ◯ There is a mistake in Line (5) since 1/3 ∈ Z .

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \cup B = B$, then $A \subseteq B$.

Proof. (Direct)
(1) Suppose $A \cup B = B$. To prove $A \subseteq B$ we also let $x \in A$ and will end the proof by showing $x \in B$.
(2) Because B is a set $\emptyset \subseteq B$.
(3) Since $A \subseteq A \cup B$ and $x \in A$ it follows that $x \in A \cup B$.
(4) Since, by assumption $A \cup B = B$ it also follows that $x \in B$.
(5) Because $A \cup B = B$ a Venn diagram shows that $A \subseteq B$.
(6) If $x \notin B$, then there would be a contradiction. $\square$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) Only lines (2) and (5) are not needed. All other lines are needed.

b) Only ;ines (1) and (5) are not needed. All other lines are needed.

c) Only lines (1) and (2) are not needed. All other lines are needed.

d) All lines are needed.

e) Only lines (3) and (4) are not needed. All other lines are needed.

Question 8

Your answer is INCORRECT.

Suppose we want to write a direct proof of the proposition below:

$$\forall x \in Z,\ x^3 - x \text{ is a multiple of } 3.$$

Which of the following statements or properties do we need to use when composing this proof?

a) The fact that if x is a multiple of 3 then $7x$ is a multiple of 21.

b) Therefore $x^3 - x = 3m$

c) The definition of prime.

d) The fact that if x is a multiple of 3 then $5x$ is a multiple of 35.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

$$\exists n \in N,\ 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) We need to show the claim is true for an abitrary natural, saying something like "Let $n \in N$."

b) We need only check that the claim is true for one, single natural number.

c) The proposition is a famous, unsolved problem. No one knows if it is true or false, and so it is not clear how to describe a proof for this.