Discrete Math

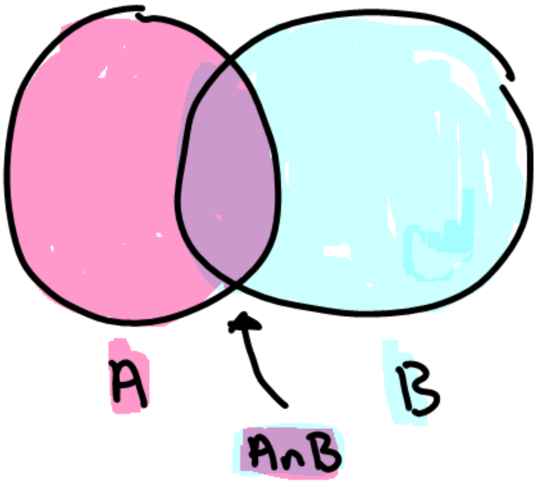
Lectures 28 & 29

Inclusion - Exclusion

&

The Pigeonhole Princ

Fact 3.6 Inclusion-Exclusion Formula
If A and B are finite sets, then $|A \cup B| = |A| + |B| - |A \cap B|$.



$$|A \cup B| = |A| + |B| - |A \cap B|$$

Note: this extends to more than two sets

2. How many 4-digit positive integers are there for which there are no repeated digits, or for which there may be repeated digits, but all digits are odd?

$\underline{9}\ \underline{9}\ \underline{8}\ \underline{7}$

$\underline{\ }\ \underline{5}\ \underline{5}\ \underline{5}$

1, 3, 5, 7, 9

5 options

$A = \{$ no repeated digits $\}$

$\boxed{9^2 \cdot 8 \cdot 7}$

$B = \{$ all odd digits $\}$

$\boxed{5^4}$

$9^2 \cdot 8 \cdot 7 + 5^4 - \boxed{\text{both}}$

$|A \cup B| = |A| + |B| - |A \cap B|$

we need to count this!

$A \cap B = \{$ all 4-digit nat. numbers that don't repeat AND have all odd digits $\}$

$|A \cap B| = 5 \cdot 4 \cdot 3 \cdot 2$

$\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }$

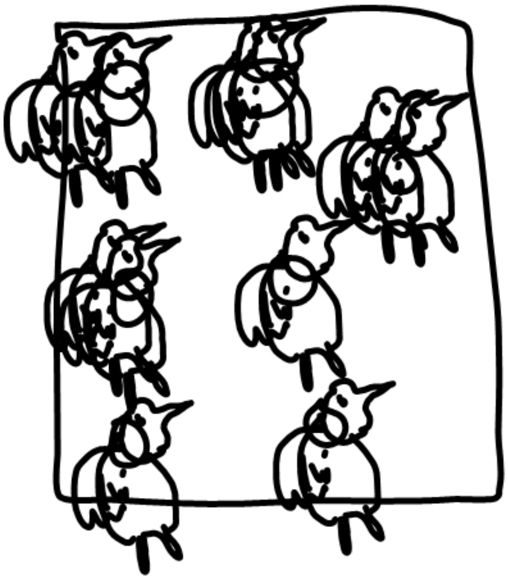
5 4 3 2

$$\boxed{9^2 \cdot 8 \cdot 7 + 5^4 - 5 \cdot 4 \cdot 3 \cdot 2}$$

$$4536 + 625 - 120$$

$$\boxed{= 5041}$$

The Pigeonhole Principle



if there are more pigeons than holes, at <u>least</u> one hole will have more than one pigeon.

7 pigeonholes
11 pigeons

$$\frac{11}{7} \approx 1.57\ldots$$

$$\left\lceil \frac{11}{7} \right\rceil = 2$$

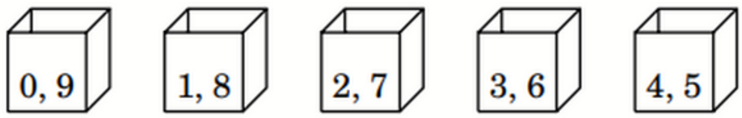
Extended P.H. Principle (Division princ.)

If we are trying to sort n objects into k boxes, then at least one box will contain $\left\lceil \frac{n}{k} \right\rceil$ objects distinct

Example 3.24 Pick six ~~integers~~ integers between 0 and 9 (inclusive). Show that two of them must add up to 9.

ex 0, 1, 5, 7, 6, 8

ex 2, 4, 7, 5, 1, 0



1 2 3 4 5

only 5 boxes

every time you pick an integer, place it in its box.

there will be at least one box that contains more than one integer

(since $5 < 6$, the pigeonhole princ. tells us this)