3336

OH 3

10:05

unmute to ask questions!

ex "Bob is funny"

is this a statement??

note 1) we can treat it as a statement

note 2) I don't think this is an actual statement

ex| Houston is in Texas. (True)

ex| Everyone is wearing a hat.

$\neg$(Everyone is wearing a hat)

= there is at least one person not wearing a hat

= some people are not wearing a hat

note: this green example uses "quantifiers"

## Quantifiers

$\forall$ = "every" or "for all"

$\exists$ = "some" or "there exists"

$\exists!$ = "there exists exactly one"

$\forall$ person, that person is wearing a hat

negation of "$\forall$" = "$\exists$"

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$P \Leftrightarrow Q$ = $\neg(P \oplus Q)$ ✓

| P | Q | $P \Leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

| P | Q | $P \oplus Q$ | $\neg(P \oplus Q)$ |
|---|---|---|---|
| T | T | F | T |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |

Statement 2? (pg. 25 - Casey's Notes)

$$\neg(\neg P \Rightarrow \neg Q) = P \wedge Q$$

| $P$ | $Q$ | $\neg P$ | $\neg Q$ | $\neg P \Rightarrow \neg Q$ | $\neg(\neg P \Rightarrow \neg Q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | T | T | F |
| F | T | T | F | F | T |
| F | F | T | T | T | F |

| $P$ | $Q$ | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

not the same

so $\neg(\neg P \Rightarrow \neg Q) \neq P \wedge Q$

| $P$ | $Q$ | $\neg P$ | $\neg P \wedge Q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | F | F |
| F | T | T | T |
| F | F | T | F |

$$\neg(\neg P \Rightarrow \neg Q) = \neg P \wedge Q$$

✓

<u>note</u> $\neg(A \Rightarrow B) = A \wedge \neg B$

$$\left( \text{above: } \neg(\neg P \Rightarrow \neg Q) = (\neg P) \wedge (\neg\neg Q) = \neg P \wedge Q \right)$$

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Quantifiers

we first talk about <u>sets</u>!

A <u>set</u> is a collection of objects

<u>ex</u> $\left\{ -1, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, ☺, \pi \right\} = S$

"is an element of"

$-1 \in S$

this set has 4 objects

no objects

<u>ex</u> $\emptyset$ = "the empty set" = $\{ \quad \}$

## familiar math sets

$\emptyset = \{ \}$

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ = "naturals"

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

"integers"

$\mathbb{Q}$ = "rationals" = { all possible fractions of integers }

such that

$\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$

$\mathbb{R}$ = { all real numbers }

ex) $T = \{\pi, 14, 0.23, e, \frac{1}{2}\}$

$14 \in T$ ✓

$3 \in T$ is false $\longrightarrow$ $3 \notin T$

ex) $\underbrace{\forall x \in \mathbb{R},\ x > 0}_{\text{false statement}}$

"every real number is positive"

ex) $\forall x \in T,\ x > 0$

true

ex) $\underbrace{\exists x \in \mathbb{R},\ x > 0}_{\text{true statement}}$

"there are real numbers that are positive"

> Quantified statements always refer to <u>some</u> set -- <u>"universal set"</u>

Mathematicians assume "if - then" statements are quantified (using $\forall$):

ex) if $\underbrace{x > 0 \text{ then } 2x > 0}_{T}$.

math: $\underbrace{\forall x \in \mathbb{R},\ x > 0 \Rightarrow 2x > 0}_{T}$