PRINTABLE VERSION

Quiz 5

You scored 100 out of 100

Question 1

Your answer is CORRECT.

An outline for a proof of an implication $P \Rightarrow Q$ is provided below:

Proposition. $P \Rightarrow Q$

Proof. Suppose $P \Rightarrow Q$ is false.
This means $\neg(P \Rightarrow Q) = P \wedge \neg Q$ is true.
Missing steps involving $P, \neg Q$, *and any previously established facts*
Therefore 0=1 (or some similarly weird conclusion) $\Rightarrow\Leftarrow$.

What type of proof was described in the outline?

a) Wait a minute... The proof described in this outline isn't a valid proof technique!

b) A proof by contradiction is described in this outline.

c) A direct proof is described in this outline.

d) A proof by contrapositive is described in this outline.

e) A proof by introspection is described in this outline.

Question 2

Your answer is CORRECT.

Suppose a mathematician wants to prove a statement of the form $P \wedge Q$. However, they wish to do so using a proof by contradiction. Of the following options which could be used as a first step in this proof?

a) Suppose $\neg P$.

b) Suppose $\neg P \wedge Q$.

c) Suppose $\neg Q$

d) Suppose $\neg P \wedge \neg Q$.

e) Suppose $\neg P \vee \neg Q$.

Question 3

Your answer is CORRECT.

Given two sets A and B one can prove $A \subseteq B$ by...

a) First supposing $x \notin A$, and then showing $x \in B$.

b) First supposing $x \in A$, and then showing $x \in B$.

c) First supposing $x \in A$, and then showing $x \notin B$.

d) First supposing $x \in B$, and then showing $x \in A$.

e) First supposing $x \notin B$, and then showing $x \in A$.

Question 4

Your answer is CORRECT.

Given two sets A and B one can prove $B \subseteq A$ by...

a) First supposing $x \notin B$, and then showing $x \notin A$.

b) First supposing $x \notin A$, and then showing $x \notin B$.

c) First supposing $x \notin A$, and then showing $x \in B$.

d) First supposing $x \notin B$, and then showing $x \in A$.

e) First supposing $x \in A$, and then showing $x \notin B$.

Question 5

Your answer is CORRECT.

A lovely little proof is presented below:

Proposition. If $2 + x$ is odd, then x is odd.

Proof. Suppose $2 + x$ is even. (We will show x is even.)
By definition of even this means $2 + x = 2m$ for some $m \in \mathbb{Z}$.
By subtracting 2 from both sides it follows that $x = 2m - 2 = 2(m - 1)$.
Because this expression is even the proof is complete. $\square$

Determine the type of proof used.

a) A proof by indoctrination was used.

b) A proof by contradiction was used.

c) A proof by contrapositive was used.

d) A direct proof was used.

e) Wait a minute... This so-called "proof" did not actually show the Proposition was true. Whoever wrote this deserves 0 points.

Question 6

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. If $A \subseteq X$ and $B \subseteq Y$ then $A \times B \subseteq X \times Y$.

Proof (Direct).
(1) Suppose $A \subseteq X \wedge B \subseteq Y$.
(2) In order to show $A \times B \subseteq X \times Y$ we will let $(a, b) \in A \times B$ and then conclude $(a, b) \in X \times Y$.
(3) $(a, b) \in A \times B$ means $a \in A \wedge b \in B$.
(4) Since $a \in A \subseteq X$ it follows that $a \in X$.
(5) Since $b \in B \subseteq Y$ it follows that $b \in Y$.
(6) By Definition of Cartesian Product $(a, b) \in X \times Y$. $\square$

Identify the mistake, if any, in this proof.

a) ◯ There is an error in Line (2). One is not allowed to just "let" $(a, b) \in A \times B$.

b) ◉ Hey, wait a second... this proof looks completely correct!

c) ◯ There is an error in Line (3). The symbol $\wedge$ should be $\vee$.

d) ◯ There is an error in Line (4) where the definition of subset is misused.

e) ◯ There is an error in Line (6) where the definition of Cartesian Product is misused.

Question 7

Your answer is CORRECT.

A proposition and an attempt at its proof are presented below.

Proposition. x is a multiple of 3 $\iff$ $(x+1)(x+2)-2$ is also a multiple of 3.

Proof. There are two parts or cases to prove.

($\Rightarrow$ Direct)
(1) Suppose $x = 3m$ for some $m \in \mathbb{Z}$.
(2) Since 3 is prime this means x is a multiple of a prime.
(3) It follows that $(x+1)(x+2)-2 = \left(x^2+3x+2\right)-2 = x^2+3x = (3m)^2+3\cdot 3m$
$= 9m^2+9m = 3\left(3m^2+3m\right)$, which is a multiple of 3.

($\Leftarrow$ By Contradiction)
(4) For a contradiction suppose $(x+1)(x+2)-2$ is multiple of 3, but that x is not a multiple of 3.
(5) Multiplying out this expression and combining like terms tells us $(x+1)(x+2)-2$
$= x^2+3x = 3b$ for some $b \in \mathbb{Z}$.
(6) From this equation we find $x^2 = 3b-3x = 3(b-x)$ and so x^2 is a multiple of 3.
(7) Since x is not a multiple of 3, it follows that x^2 is not a multiple of 3.
(8) Therefore x^2 is a multiple of 3 and x^2 is not a multiple of 3. $\Rightarrow\Leftarrow$

One or more lines in this proof are not needed -- the proof works perfectly well without them (in fact, it works better without them). Which lines are not needed?

a) ◯ Only line (3) is not needed. All other lines are needed.

b) ◯ Only line (5) is not needed. All other lines are needed.

c) ◯ Only line (4) is not needed. All other lines are needed.

d) ◯ All lines are needed.

e) ◉ Only line (2) is not needed. All other lines are needed.

Question 8

Your answer is CORRECT.

Suppose we want to write a proof by contradiction of the proposition below:

$$\forall a, b, c \in [0, \infty),\ (ab = c) \Rightarrow (a \le \sqrt{c} \vee b \le \sqrt{c}).$$

Which of the following statements or properties do we need to use when composing this proof?

a) Suppose $ab = c$ and that either $a > \sqrt{c}$ or $b > \sqrt{c}$.

b) Suppose $ab = c$ and that both $a > \sqrt{c}$ and $b > \sqrt{c}$.

c) The fact that for real numbers x, y, if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

d) The fact that for real numbers x, y, if $x > y$ then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

Question 9

Your answer is CORRECT.

A proposition is stated below. Take a few moments to carefully read it, and make sure you understand what, exactly, it claims to be true.

The recursively defined sequence $a_n = a_{n-1} - 1$ with initial conditions $a_0 = \pi$ has negative terms for all $n \geq 4$.

Of the following options, which one best describes what needs to be done in order to prove this claim?

a) This can only be proved by paying someone else to do it for me.

b) We need only check that the claim is true for one, single natural number.

c) We need to show the claim is true for an abitrary natural larger than 3, saying something like "Let $n \geq 4$. "