

Counting Subsets

$\binom{n}{k}$ = the # of size- k subsets
of a size- n set

= the # of ways one can
choose k elements from an
 n element set

= "n choose k"

ex] $n=4$ $X = \{\alpha, \beta, \delta, \gamma\}$

$\binom{4}{2}$ = # of ways to choose two elements from X

$\{\alpha, \beta\}, \{\alpha, \delta\}, \{\alpha, \gamma\}$ 3

$\{\beta, \delta\}, \{\beta, \gamma\}$ 2

$\{\delta, \gamma\}$ 1

6

$\binom{4}{1}$ = # of 1-element subsets

$\{\alpha\}, \{\beta\}, \{\delta\}, \{\gamma\}$

= 4

$$\binom{4}{3} = \# \text{ of 3-element subsets } = 4$$

$\{\beta, \delta, \gamma\}$, $\{\alpha, \delta, \gamma\}$, $\{\alpha, \beta, \gamma\}$,
 $\{\alpha, \beta, \delta\}$

$$\binom{4}{0} = \# \text{ of 0-element subset } = 1$$

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$$\binom{4}{4} = 1$$

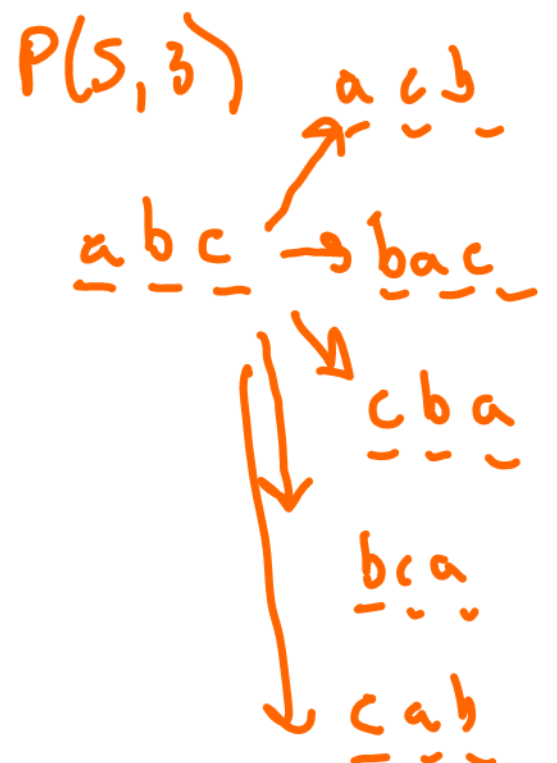
A Generic Formula for $\binom{n}{k}$

$$\binom{n}{k} = \frac{1}{k!} P(n, k)$$

ex $\binom{5}{3} = \# \text{ 3-element subsets from a 5-element set}$

$$X = \{a, b, c, d, e\}$$

$\{a, b, c\} \{a, b, d\} \{a, b, e\} \{a, c, d\} \{a, c, e\} \{a, d, e\} \{b, c, d\} \{b, c, e\} \{b, d, e\} \{c, d, e\}$




$$P(5,3) = 3! \cdot \binom{5}{3}$$

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$$\frac{5!}{(5-3)!} = 3! \binom{5}{3}$$

$$\boxed{\frac{5!}{3! \cdot (5-3)!} = \binom{5}{3}}$$

$$\binom{n}{k} = \frac{1}{k!} P(n, k) = \frac{1}{k!} \frac{n!}{(n-k)!}$$



$$\boxed{\binom{n}{k} = \frac{n!}{k! (n-k)!}}$$

Example 3.16 How many 7-digit binary strings (0010100, 1101011, etc.) have an odd number of 1's?

observation: there will be one, three, five or seven 1's

 $S_1 = \{ \text{7-digit binary strings w/ one 1} \}$ $\binom{7}{1} = 7$
 ----- \leftarrow where does the single 1 go?

$S_3 = \{ \text{" " w/ three 1's} \}$ $\binom{7}{3} = \frac{7!}{3!4!}$
 ----- \leftarrow where do the three 1's go?

$S_5 = \{ \text{" " five 1's} \}$ $\binom{7}{5} = \frac{7!}{5!2!}$

$S_7 = \{ \text{" " seven 1's} \}$ $\binom{7}{7} = 1$
 1 1 1 1 1 1 1