Discrete Math

Lecture 8

Negation

P, negation "$\neg P$"

$=$ "it is not true that P"

$$\neg\left(\forall x \in \mathcal{U},\ P(x)\right)$$

$$\exists x \in \mathcal{U},\ \neg P(x)$$

a counter-example

ex $\forall n \in \mathbb{N},\ n^2 > 0$ is F

is this T or F?

$0 \in \mathbb{N}$ $0^2 > 0$ is F

0 is a counter-example

$\neg(\forall n \in \mathbb{N},\ n^2 > 0)$

$\exists n \in \mathbb{N},\ \neg(n^2 > 0)$

$\exists n \in \mathbb{N},\ n^2 \leq 0$ is T

since we can use $n = 0$.

$\neg(\exists x \in \mathcal{U},\ P(x))$

$\forall x \in \mathcal{U},\ \neg P(x)$

"$\neg\forall = \exists$"　"$\neg\exists = \forall$"

ex)

$$\neg\left(\forall x \in \mathcal{U},\ \exists y \in \mathcal{U},\ P(x,y) \wedge Q(x,y)\right)$$

$$= \exists x \in \mathcal{U},\ \neg\left(\exists y \in \mathcal{U},\ P(x,y) \wedge Q(x,y)\right)$$

$$= \exists x \in \mathcal{U},\ \forall y \in \mathcal{U},\ \neg\left(P(x,y) \wedge Q(x,y)\right)$$

De Morgan's

$$= \exists x \in \mathcal{U},\ \forall y \in \mathcal{U},\ \neg P(x,y) \vee \neg Q(x,y)$$