The random variable X has probability density function:

$$f(x) = \begin{cases} \frac{C}{4} - \frac{1}{2}x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Part a: Determine the value of C.

for a pdf the integral of the function should equal 1.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \longrightarrow \int_0^2 \frac{C}{4} - \frac{x}{2}\,dx = 1 \longrightarrow \left[\frac{Cx}{4} - \frac{x^2}{4}\right]\Bigg|_0^2 = 1$$

$$\longrightarrow \frac{2C}{4} - \frac{(2)^2}{4} = 1 \rightarrow \frac{C}{2} - 1 = 1 \rightarrow \frac{C}{2} = 2 \longrightarrow C = 4$$

Part b: Find $F(x)$, the cumulative distribution function of X.

the pdf after finding the value of C is: $f(x) = \begin{cases} 1 - \frac{x}{2} & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

to find cdf, you have to take the integral of the pdf.

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

this F(x) function will have three parts to it, when x < 0, when x is between 0 and 2, and when x is greater than 2.

when x is less than 0

$$\int_{-\infty}^{x} 0\,dt = 0$$

$$\boxed{F(x) = \begin{cases} 0 & x < 0 \\ x - \frac{x^2}{4} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}}$$

when x is between 0 and 2

$$\int_0^x 1 - \frac{t}{2}\,dt = \left[t - \frac{t^2}{4}\right]\Bigg|_0^x = x - \frac{x^2}{4}$$

when x is greater than 2

when x can be any value greater than 2, the integral will include the interval from 0 to 2 and beyond. We already know the function for 0 to 2, and anything beyond 2 we know is 0 from our pdf, so the integral will look like this:

$$\int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{0} 0\,dt + \int_0^2 1 - \frac{t}{2}\,dt + \int_0^x 0\,dt = 0 + \left[t - \frac{t^2}{4}\right]\Bigg|_0^2 + 0$$

$$= 2 - \frac{(2)^2}{4} = 1$$

Part c: Find $E[X]$.

$$E[X]=\int_{-\infty}^{\infty} x f(x)\,dx = \int_0^2 x\left(1-\frac{x}{2}\right)dx = \int_0^2 x-\frac{x^2}{2}\,dx = \left[\frac{x^2}{2}-\frac{x^3}{6}\right]\Bigg|_0^2 =$$

```
> (2^2)/2 - (2^3)/6
[1] 0.6666667
>
```

Part d: Find the variance and standard deviation of X.

$$Var[X]=E[x^2]-(E[x])^2 = \int_0^2 x^2\left(1-\frac{x}{2}\right)dx-(E[x])^2 = \int_0^2 x^2-\frac{x^3}{2}\,dx-(E[x])^2$$

$$= \left[\frac{x^3}{3}-\frac{x^4}{8}\right]\Bigg|_0^2 -(0.6666667)^2 =$$

```
> ((2^3)/3 - (2^4)/8) - (0.6666667)^2
[1] 0.2222222
>
```

<--- this is the variance

standard deviation is just the square root of the variance:

```
> sqrt(0.2222222)
[1] 0.4714045
>
```

Part e: Determine the third quartile of X.

third quartile is the 75th percentile

we want to solve: $F(x)=0.75 \rightarrow x-\frac{x^2}{4}=0.75$

you can use factoring or quadratic formula to solve this,
please try to see if you can factor first because quadratic formula is yucky.

am doing factoring, first gonna rewrite and rearrange the equation:

$x-\frac{x^2}{4}=\frac{3}{4}$ (3/4 is same thing as 0.75)

$4x-x^2=3$ (multiplied both sides by 4)

$-x^2+4x-3=0$ (rearranging to make it look like a quadratic equation)

$-(x^2-4x+3)=0$ (factored out the -1 because negatives are also yucky)

$-(x-1)(x-3)=0$

$x=1,\ x=3$

if we look at the cdf function, we only get values from between 0 and 2, so this is our answer.

(finding the percentile based on the cdf function is in the lecture 9 slides)