[a] An experimenter is interested in the hypothesis testing problem with H_0: $\mu = 420$ versus H_a: $\mu < 420$, where μ is the average radiation level in a research laboratory. Suppose that a sample of 49 radiation level measurements is obtained and that the experimenter wishes to use $\sigma=10.0$ for the standard deviation of the radiation levels.

null hypothesis   alternative hypothesis

(a) What is the rejection region when $\alpha=0.01$?

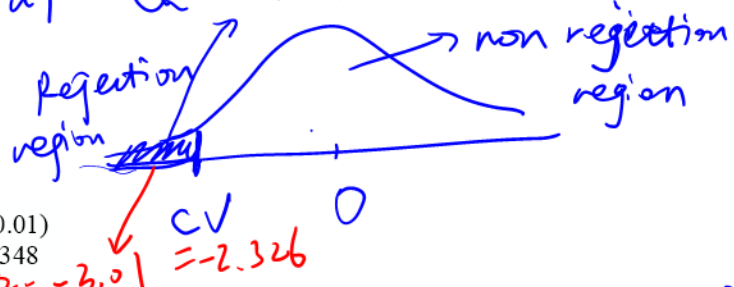
(b) Suppose that the sample mean is 415.7. What would be the experimenter's decision of the hypothesis test based on the rejection region?

(c) What is the p-value for this hypothesis test?

$H_0: \mu = 420$ $H_a: \mu < 420$ (left-tailed test)

$\sigma = 10.0 \Rightarrow$ z-test $= \dfrac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$

(a) $\alpha = 0.01$



CV: critical value(s) seperate rejection region and non-rejection region. it is determined by the type of the test and α.

> qnorm(0.01)
[1] -2.326348

CV = qnorm(0.01) = -2.326

Rejection region: reject H_0 if $z \le -2.326$.

(b) calculate test statistic :

> (415.7-420)/(10/sqrt(49))
[1] -3.01

$$z = \frac{\bar{x}-\mu}{(\sigma/\sqrt{n})} = \frac{(415.7-420)}{(10/\sqrt{49})} = -3.01$$

$\because$ -3.01 is in the rejection region

$\therefore$ We can reject H_0.

(c). p-value $= P(z < -3.01) =$ pnorm(-3.01) = 0.0013

left-tailed test

if p-value $< \alpha$ then we can reject H_0

> pnorm(-3.01)
[1] 0.001306238

Here p-value = 0.0013 $< \alpha = 0.01$ $\Rightarrow$ We can reject H_0.

. In a recent publication, it was reported that the average highway gas mileage of tested models of a new car was 33.5 mpg and approximately normally distributed. A consumer group conducts its own tests on a simple random sample of 12 cars of this model and finds that the mean gas mileage for their vehicles is 31.6 mpg with a standard deviation of 3.4 mpg.

a. Perform a test to determine if these data support the contention that the true mean gas mileage of this model of car is different from the published value.
b. Perform a test to determine if these data support the contention that the true mean gas mileage of this model of car is less than the published value.
c. Explain why the answers to part a and part b are different.

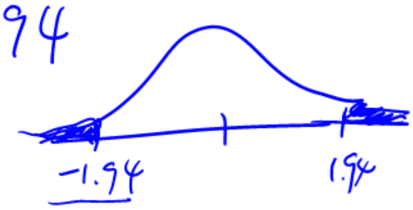
```
> (31.6-33.5)/(3.4/sqrt(12))
[1] -1.935821
```

$n = 12 \qquad \bar{x} = 31.6 \qquad s = 3.4$

(a) $H_0: \mu = 33.5 \qquad H_a: \mu \neq 33.5 \rightarrow$ two-tailed test

$\because \sigma$ is NOT given, $\therefore$ we have to use t-Test

$$t = \frac{\bar{x} - \mu}{(s/\sqrt{n})} = \frac{(31.6 - 33.5)}{(3.4/\sqrt{12})} = -1.94$$




```
> 2*pt(-1.94,11)
[1] 0.07844151
```

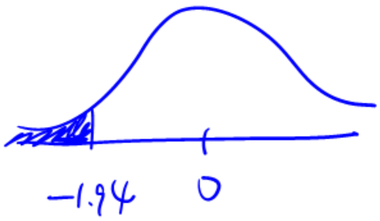
p-value $= 2 * P(t < -1.94)$

$= 2 * pt(-1.94, df = 12-1) = 0.078 > 5\%$

There is no/weak evidence that the true mean gas mileage of this model is different from published value.

(b) $H_0: \mu = 33.5 \qquad H_a: \mu < 33.5 \rightarrow$ left tailed test

$$t = \frac{\bar{x} - \mu}{(s/\sqrt{n})} = \frac{(31.6 - 33.5)}{(3.4/\sqrt{n})} = -1.94$$



p-value $= P(t < -1.94)$

```
> pt(-1.94,11)
[1] 0.03922076
```

$= pt(-1.94, df = 11) = 0.039 < 5\%$

There is evidence that the true mean gas milege of this model is less than the published value.

. For each of the following scenarios, state whether the null hypothesis should be rejected or not. State any assumptions that you make beyond the information that is given.

(a) H_0: $\mu = 6$, H_a: $\mu \neq 6$, n=25, $\bar{x}$=5.6, s=2.5, α=0.05

(b) H_0: $\mu = 6$, H_a: $\mu < 6$, n=25, $\bar{x}$=5.6, s=2.5, α=0.05

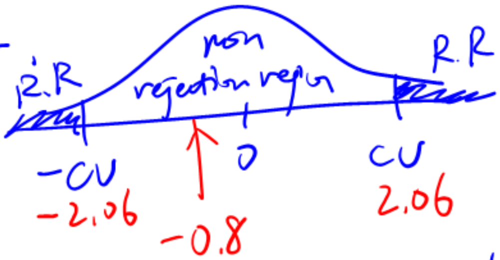
(c) H_0: $\mu = 25$, H_a: $\mu > 25$, n=81, $\bar{x}$=26.4, s=3.2, α=0.01

(d) H_0: $\mu = 50$, H_a: $\mu \neq 50$, n=55, p-value=0.053

```
> qt(0.025,24)
[1] -2.063899
```

$$t = \frac{\bar{x} - \mu}{(s/\sqrt{n})} = \frac{(5.6 - 6)}{(2.5/\sqrt{25})} = -0.8$$

(a) two-tailed test
$\alpha = 0.05$



$\frac{0.05}{2} = 0.025$

$$CV = \pm\, qt\left(\frac{\alpha}{2}, df = 25-1\right) = \pm\, qt(0.025, 24)$$
$$= \pm\, 2.06$$

-0.8 is in the Non-rejection region, we fail to reject H_0.

(b), left tailed test

$CV = qt(0.05, 24) = -1.71$

```
> qt(0.05,24)
[1] -1.710882
```

-0.8 is in the non rejection region, we fail to reject H_0

For each of the following scenarios, state whether the null h
any assumptions that you make beyond the information that

(a) H_0: $\mu = 6$, H_a: $\mu \neq 6$, n=25, $\bar{x}$=5.6, s=2.5, α=0.05

(b) H_0: $\mu = 6$, H_a: $\mu < 6$, n=25, $\bar{x}$=5.6, s=2.5, α=0.05

(c) H_0: $\mu = 25$, H_a: $\mu > 25$, n=81, $\bar{x}$=26.4, s=3.2, α=0.01

(d) H_0: $\mu = 50$, H_a: $\mu \neq 50$, n=55, p-value=0.053

```
> (26.4-25)/(3.2/sqrt(81))
[1] 3.9375
```

(c) $H_a: \mu > 25 \Rightarrow$ right-tailed test.

– use critical value method:

$$t = \frac{(\bar{X} - \mu)}{(s/\sqrt{n})} = \frac{(26.4 - 25)}{(3.2/\sqrt{81})} = 3.94$$

$\alpha = 0.01$ n=81

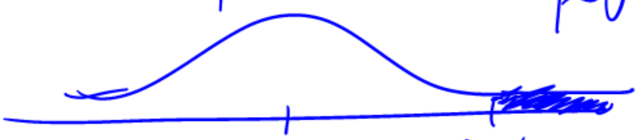
CV=2.37

```
> qt(1-.01,80)
[1] 2.373868
```

CV = qt(1 − 0.01, 80) = 2.37

3.94 is in the Reject region $\Rightarrow$ Reject H_0.

– p-value approach:



0 3.94

```
> 1-pt(3.94,80)
[1] 8.664187e-05
```

$$\text{p-value} = P(t > 3.94) = 1 - P(t < 3.94) = 1 - pt(3.94, df=80) \approx 0$$

p-value < α

Reject H_0. The data provide strong evidence that $\mu > 25$.