Homework 1 Solutions - MATH 4322 Spring 2024

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Instructions

1. Due date: January 30, 2024, 11:59 PM
2. Answer the questions fully for full credit.
3. Scan or Type your answers and submit only one file. (If you submit several files only the recent one uploaded will be graded).
4. Preferably save your file as PDF before uploading.
5. Submit in Canvas under Homework 1.
6. These questions are from *An Introduction to Statistical Learning*, second edition by James, et. al., chapter 2.
7. The information in the gray boxes are R code that you can use to answer the questions.

Problem 1

Explain whether each scenario is a classification or regression problem, and indicate whether we are most interested in inference or prediction. Finally, provide n and p.

a) We are interested in predicting the % change in the USD/Euro exchange rate in relation to the weekly changes in the world stock markets. Hence we collect weekly data for all of 2012. For each week we record the % change in the USD/Euro, the % change in the US market, the % change in the British market, and the % change in the German market.

Answer
Regression
Most interested in prediction
$n = 52$
$p = 4$

b) We are considering launching a new product and wish to know whether it will be a *success* or a *failure*. We collect data on 20 similar products that were previously launched. For each product we have recorded whether it was a success or failure, price charged for the product, marketing budget, competition price, and ten other variables.

Answer
Classification
Most interested in prediction
$n = 20$
$p = 14$

Problem 2

This is an exercises about bias, variance and MSE.

Suppose we have n independent Bernoulli trails with true success probability p. Consider two estimators of p: $\hat{p}_1 = \hat{p}$ where $\hat{p}$ is the sample proportion of successes and $\hat{p}_2 = \frac{1}{2}$, a fixed constant.

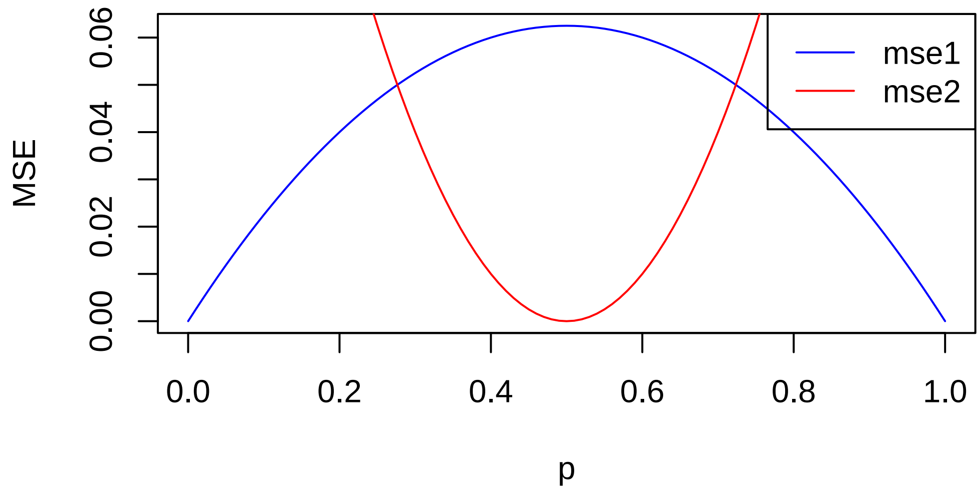
a) Find the expected value and bias of each estimator.

b) Find the variance of each estimator.

c) Find the MSE of each estimator and compare them by plotting against the true p. Use $n = 4$. Comment on the comparison.

Answer
a) $E(\hat{p}_1) = p$, $\text{Bias}(\hat{p}_1) = 0$, $E(\hat{p}_2) = \frac{1}{2}$, $\text{Bias}(\hat{p}_2) = \frac{1-2p}{2}$
b) $\text{Var}(\hat{p}_1) = \frac{p(1-p)}{n}$, $\text{Var}(\hat{p}_2) = 0$
c) $MSE(\hat{p}_1) = \text{Var}(\hat{p}_1) = \frac{p(1-p)}{n}$, $MSE(\hat{p}_2) = \text{Bias}(\hat{p}_2)^2 = \frac{(1-2p)^2}{4}$

```
p = seq(0,1,0.01)
mse.p1 = p*(1-p)/4
mse.p2 = (1-2*p)^2/4
plot(p,mse.p1,type = "l",col = "blue",ylab = "MSE")
lines(p,mse.p2,col="red")
legend("topright",legend = c("mse1","mse2"), col = c("blue","red"), lty = c(1,1))
```



Notice that the closer p gets to 0.5, the smaller the MSE for $\hat{p} = \frac{1}{2}$.

Problem 3

Describe the differences between a parametric and a non-parametric statistical learning approach. What are the advantages of a parametric approach to regression or classification (as opposed to a non-parametric approach)? What are its disadvantages?

Answer
Parametric methods involve a two-step model-based approach.
1. we make an assumption about the functional form, or shape, of f.
2. After a model has been selected, we need a procedure that uses the training data to *fit* or *train* the model.

Advantage: simplifies the problem of estimating f because generally it is much easier to estimate a set of parameters.
Disadvantage: the model we choose will usually not match the true unknown form of f.

Non-parametric methods do not make explicit assumptions about the functional form of f. Instead they seek an estimate of f that gets as close to the data points as possible without being too rough or wiggly.

Advantage: by avoiding the assumption of a particular functional form for f, they have the potential to accurately fit a wider range of possible shapes for f.

Disadvantage: since they do not reduce the problem of estimating f to a small number of parameters, a very large number of observations (far more than is typically needed for a parametric approach) is required in order to obtain an accurate estimate for f.

Problem 4

This exercise involves the `Auto` data set in `ISLR` package. Make sure that the missing values have been removed from the data.

```
library(ISLR)
Auto.new = na.omit(Auto)
```

(a) Which of the predictors are quantitative, and which are qualitative?

(b) What is the range of each quantitative predictor? You can answer this using the `summary()` function.

(c) What is the mean and standard deviation of each quantitative predictor?

(d) Now remove the 10th through 85th observations. What is the range, mean, and standard deviation of each predictor in the subset of the data that remains?

(e) Using the full data set, investigate the predictors graphically, using scatterplots or other tools of your choice. Create some plots highlighting the relationships among the predictors. Comment on your findings.

(f) Suppose that we wish to predict gas mileage (mpg) on the basis of the other variables. Do your plots suggest that any of the other variables might be useful in predicting mpg? Justify your answer.

Answer
a) Quantitative: mpg, cylinders, displacement, horsepower, weight, acceleration, and year. Categorical: origin and name.
b) Range

```
summary(Auto.new)
```

```
      mpg          cylinders      displacement     horsepower        weight    
 Min.   : 9.00   Min.   :3.000   Min.   : 68.0   Min.   : 46.0   Min.   :1613  
 1st Qu.:17.00   1st Qu.:4.000   1st Qu.:105.0   1st Qu.: 75.0   1st Qu.:2225  
 Median :22.75   Median :4.000   Median :151.0   Median : 93.5   Median :2804  
```

```
 Mean   :23.45   Mean   :5.472   Mean   :194.4   Mean   :104.5   Mean   :2978
 3rd Qu.:29.00   3rd Qu.:8.000   3rd Qu.:275.8   3rd Qu.:126.0   3rd Qu.:3615
 Max.   :46.60   Max.   :8.000   Max.   :455.0   Max.   :230.0   Max.   :5140

  acceleration        year           origin                      name
 Min.   : 8.00   Min.   :70.00   Min.   :1.000   amc matador       :  5
 1st Qu.:13.78   1st Qu.:73.00   1st Qu.:1.000   ford pinto        :  5
 Median :15.50   Median :76.00   Median :1.000   toyota corolla    :  5
 Mean   :15.54   Mean   :75.98   Mean   :1.577   amc gremlin       :  4
 3rd Qu.:17.02   3rd Qu.:79.00   3rd Qu.:2.000   amc hornet        :  4
 Max.   :24.80   Max.   :82.00   Max.   :3.000   chevrolet chevette:  4
                                                 (Other)           :365
```

mpg = 46.6 - 9 = 37.6
cylinders = 8 - 3 = 5
displacement = 455 - 68 = 387
horsepower = 230 - 46 = 184
weight = 5140 - 1613 = 3527
acceleration = 24.8 - 8 = 16.8
year = 82 - 70 = 12

c) Mean and standard deviation

Variable	Mean	Standard Deviation
mpg	23.45	7.81
cylinders	5.47	1.71
displacement	194.41	104.64
horsepower	104.47	38.49
weight	2977.58	849.4
acceleration	15.54	2.76
year	75.98	3.68

d) Remove 10 - 85

```r
Auto.new2 = rbind(Auto.new[1:9,],Auto.new[86:392,])
round(colMeans(Auto.new2[,1:7]),2) #means
```

```
         mpg    cylinders displacement   horsepower       weight acceleration
       24.40         5.37       187.24       100.72      2935.97        15.73
        year
       77.15
```

```
round(sqrt(diag(var(Auto.new2[,1:7]))),2) #standard deviations
```

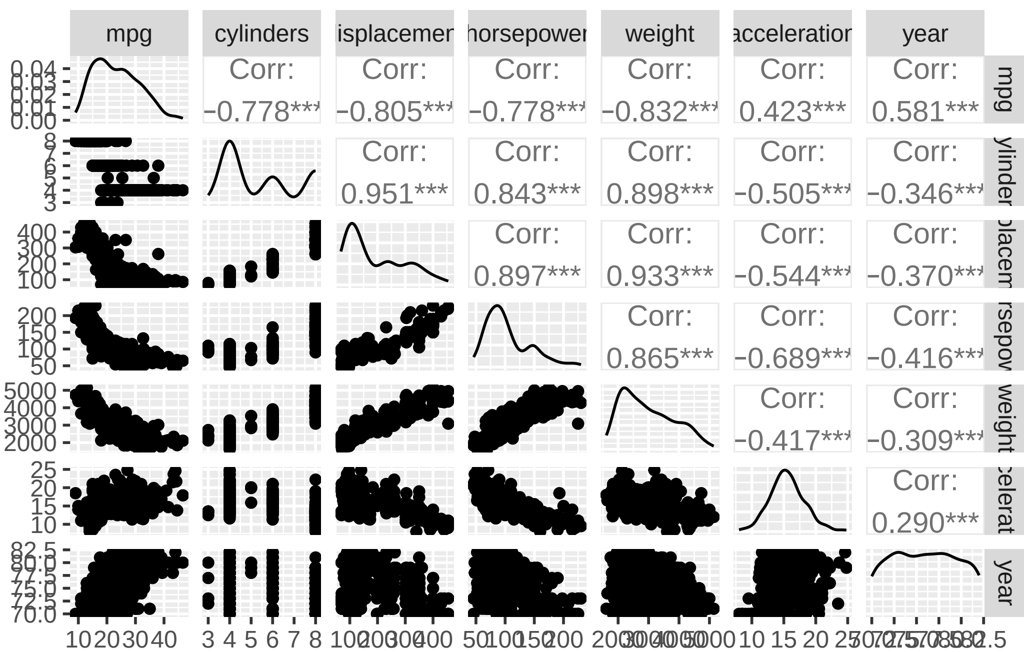
```
         mpg    cylinders displacement   horsepower       weight acceleration
        7.87         1.65        99.68        35.71       811.30         2.69
        year
        3.11
```

```
auto.range = sapply(Auto.new2[,1:7],range)
auto.range[2,] - auto.range[1,]
```

```
         mpg    cylinders displacement   horsepower       weight acceleration
        35.6          5.0        387.0        184.0       3348.0         16.3
        year
        12.0
```

e) Graphs of the predictors the graphs chosen can be up to you. I will graph the ggpairs.

```
library(ggplot2)
library(GGally)
ggpairs(Auto[,1:7])
```



Explanation is subjective.

f) Yes, `mpg` seems to be directly related to `displacement`, `horsepower`, and `weight`.

Problem 5

This exercise relates to the `College` data set, which can be found in the file `College.csv` attached to this homework set in Blackboard. It contains a number of variables for 777 different universities and colleges in the US. The variables are

- Private : Public/private indicator
- Apps : Number of applications received
- Accept : Number of applicants accepted
- Enroll : Number of new students enrolled
- Top10perc : New students from top 10% of high school class
- Top25perc : New students from top 25% of high school class
- F.Undergrad : Number of full-time undergraduates
- P.Undergrad : Number of part-time undergraduates
- Outstate : Out-of-state tuition
- Room.Board : Room and board costs
- Books : Estimated book costs
- Personal : Estimated personal spending
- PhD : Percent of faculty with Ph.D.'s
- Terminal : Percent of faculty with terminal degree
- S.F.Ratio : Student/faculty ratio
- perc.alumni : Percent of alumni who donate
- Expend : Instructional expenditure per student
- Grad.Rate : Graduation rate

Before reading the data into `R`, it can be viewed in Excel or a text editor.

a) Use the `read.csv()` function to read the data into `R`. Call the loaded data `college`. Make sure that you have the directory set to the correct location for the data. You can also import this data set into `RStudio` by using the **Import Dataset** → **From Text** drop down list in the Environment window.

b) Look at the data using the `View()` function. You should notice that the first column is just the name of each university. We will not use this column as a variable but it may be handy to have these names for later. Try the following commands in `R`:

```
rownames(college) <- college[,1]
college <- college[,-1]
View(college)
```

If you are getting an error make sure your data frame is named with a lowercase "c". Give a brief description of what you see in the data frame.

c) Use the `summary()` function to produce a numerical summary of the variables in the data set. Is there any variables that do not show a numerical summary?

```
summary(college)
```

```
   Private               Apps           Accept          Enroll    
 Length:777         Min.   :   81   Min.   :   72   Min.   :  35  
 Class :character   1st Qu.:  776   1st Qu.:  604   1st Qu.: 242  
 Mode  :character   Median : 1558   Median : 1110   Median : 434  
                    Mean   : 3002   Mean   : 2019   Mean   : 780  
                    3rd Qu.: 3624   3rd Qu.: 2424   3rd Qu.: 902  
                    Max.   :48094   Max.   :26330   Max.   :6392  
   Top10perc       Top25perc      F.Undergrad     P.Undergrad     
 Min.   : 1.00   Min.   :  9.0   Min.   :  139   Min.   :    1.0  
 1st Qu.:15.00   1st Qu.: 41.0   1st Qu.:  992   1st Qu.:   95.0  
 Median :23.00   Median : 54.0   Median : 1707   Median :  353.0  
 Mean   :27.56   Mean   : 55.8   Mean   : 3700   Mean   :  855.3  
 3rd Qu.:35.00   3rd Qu.: 69.0   3rd Qu.: 4005   3rd Qu.:  967.0  
 Max.   :96.00   Max.   :100.0   Max.   :31643   Max.   :21836.0  
    Outstate       Room.Board       Books           Personal   
 Min.   : 2340   Min.   :1780   Min.   :  96.0   Min.   : 250  
 1st Qu.: 7320   1st Qu.:3597   1st Qu.: 470.0   1st Qu.: 850  
 Median : 9990   Median :4200   Median : 500.0   Median :1200  
 Mean   :10441   Mean   :4358   Mean   : 549.4   Mean   :1341  
 3rd Qu.:12925   3rd Qu.:5050   3rd Qu.: 600.0   3rd Qu.:1700  
 Max.   :21700   Max.   :8124   Max.   :2340.0   Max.   :6800  
      PhD            Terminal       S.F.Ratio      perc.alumni   
 Min.   :  8.00   Min.   : 24.0   Min.   : 2.50   Min.   : 0.00  
 1st Qu.: 62.00   1st Qu.: 71.0   1st Qu.:11.50   1st Qu.:13.00  
 Median : 75.00   Median : 82.0   Median :13.60   Median :21.00  
 Mean   : 72.66   Mean   : 79.7   Mean   :14.09   Mean   :22.74  
 3rd Qu.: 85.00   3rd Qu.: 92.0   3rd Qu.:16.50   3rd Qu.:31.00  
 Max.   :103.00   Max.   :100.0   Max.   :39.80   Max.   :64.00  
     Expend        Grad.Rate     
 Min.   : 3186   Min.   : 10.00  
 1st Qu.: 6751   1st Qu.: 53.00  
 Median : 8377   Median : 65.00  
 Mean   : 9660   Mean   : 65.46  
 3rd Qu.:10830   3rd Qu.: 78.00  
```

```
 Max.   :56233   Max.   :118.00
```

Answer: The variable `Private` is listed as a character.

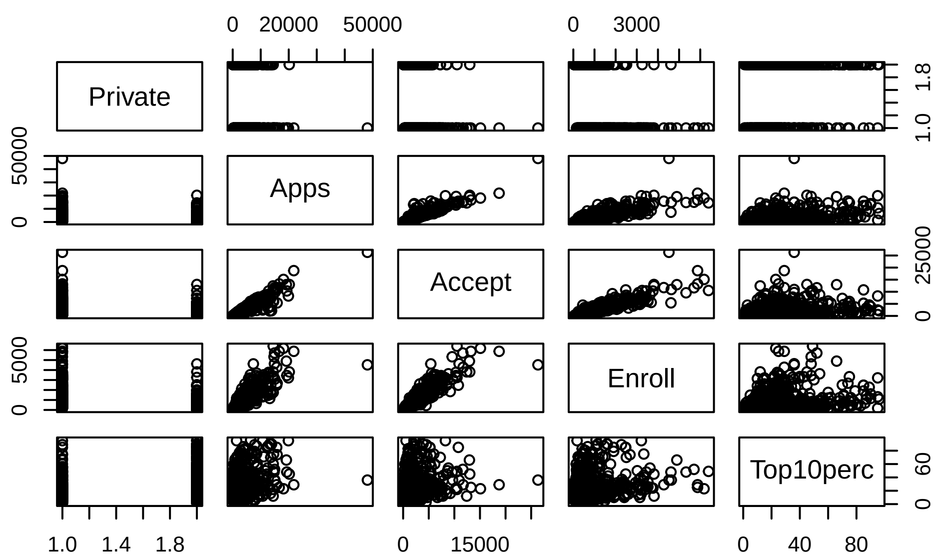
Type in the following in `R`:

```
college$Private <- as.factor(college$Private)
```

d) Use the `pairs()` function to produce a scatterplot matrix of the first five columns or variable of the dataset. Describe any relationships you see in these plots.

Answer

```
pairs(college[,1:5])
```

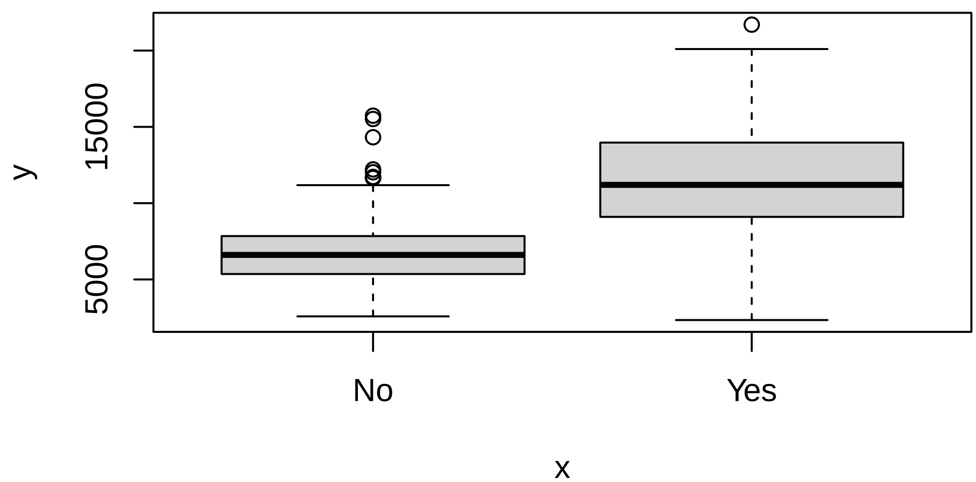


There seems to be a linear relationship between number of applications, number accepted, and number enrolled.

e) Use the `plot()` function to produce a plot of `Outstate` versus `Private`. What type of plot was produced? Give a description of the relationship. *Hint: 'Outstate is in the y-axis.*

Answer

```
plot(college$Private,college$Outstate)
```



This is a box plot. There seems to be a higher out state tution for private universities.

f) Create a new qualitative variable, called `Elite`, by *binning* the `Top10perc` variable. We are going to divide universities into two groups based on whether or not the proportion of students coming from the top 10% of their high school classes exceeds 50%. Type in the following in `R`:

```
Elite <- rep("No", nrow(college)) #this gives a column of No's for the same number of rows
Elite[college$Top10perc > 50] <- "Yes" #changes to Yes if top 10% is greater than 50
Elite <- as.factor(Elite)
college <- data.frame(college,Elite) #adds Elite as a column
```

Use the `summary()` function to see how many elite universities there are.

Answer

```
summary(Elite)
```

```
 No Yes
699  78
```

There are 78 so called elite schools.

Problem 6

This exercise involves the Boston housing data set.

(a) To begin, load in the Boston data set. The Boston data set is part of the ISLR2 library. You may have to install the ISLR2 library then call for this library.

```
library(ISLR2)
```

```
Attaching package: 'ISLR2'

The following objects are masked from 'package:ISLR':

    Auto, Credit
```

Now the data set is contained in the object Boston.

```
Boston
```

Read about the data set:

```
?Boston
```

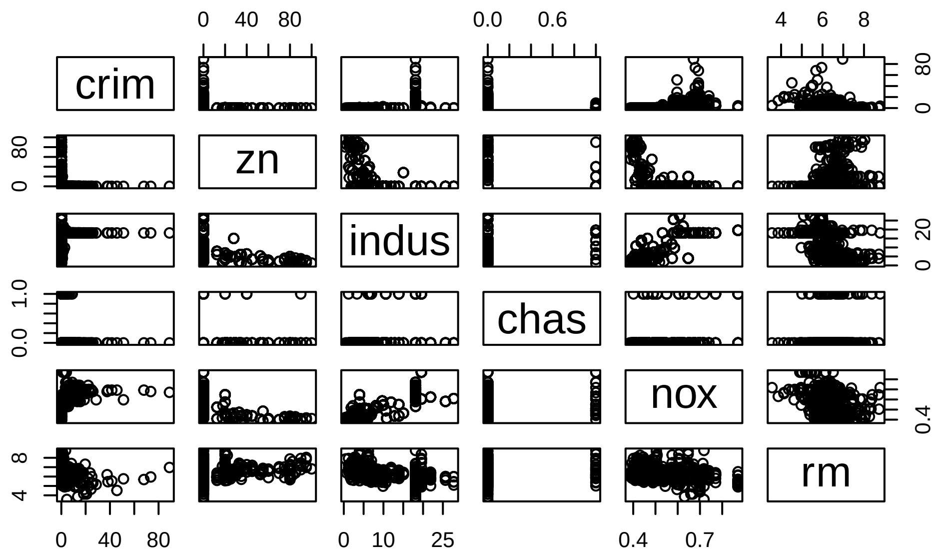
How many rows are in this data set? How many columns? What do the rows and columns represent?

Answer
There are 506 rows, this is the number of observations number of suburbs in Boston and 13 columns, this is the number of variables in the data set.

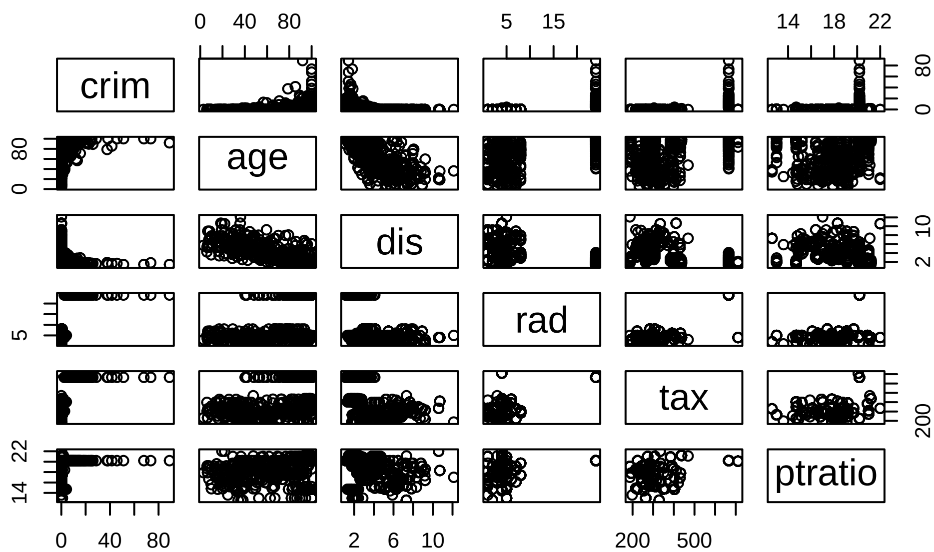
(b) Make some pairwise scatterplots of the predictors (columns) in this data set. Describe your findings.

Answer

```
pairs(Boston[1:6])
```




```r
pairs(Boston[c(1,7:11)])
```



(c) Are any of the predictors associated with per capita crime rate? If so, explain the relationship.

Answer

```
cor(Boston[,1],Boston[,2:11])
```

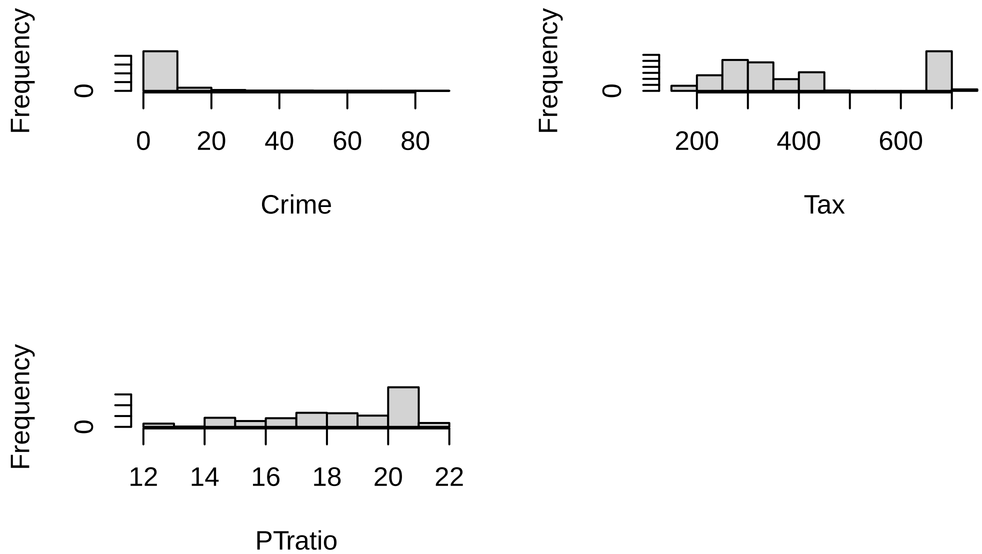
```
             zn     indus        chas       nox         rm       age        dis
[1,] -0.2004692 0.4065834 -0.05589158 0.4209717 -0.2192467 0.3527343 -0.3796701
           rad       tax   ptratio
[1,] 0.6255051 0.5827643 0.2899456
```

The highest correlation to the Crime rate appears to be `rad` index of accessibility to radial highways and `tax` full-value property-tax rate per $10,000. As these increase, the crime rate increases.

(d) Do any of the census tracts of Boston appear to have particularly high crime rates? Tax rates? Pupil-teacher ratios? Comment on the range of each predictor.

Answer

```
par(mfrow = c(2,2))
hist(Boston$crim,main = "",xlab = "Crime")
hist(Boston$tax, main = "",xlab = "Tax")
hist(Boston$ptratio, main = "",xlab = "PTratio")
```



The crime rate is skewed right. If you use the 1.5 IQR rule, there are some towns that are outliers.

Property tax seems to jump between 500 and 700. This shows bimodal.

Parent-teacher ratio is skewed left.

(e) How many of the census tracts in this data set bound the Charles river?

Answer

```r
sum(Boston$chas)
```

```
[1] 35
```

35 Suburbs bound the Charles River.

(f) What is the median pupil-teacher ratio among the towns in this data set?

Answer

```r
median(Boston$ptratio)
```

```
[1] 19.05
```

The median is 19.05%.

(g) Which census tract of Boston has lowest median value of owner occupied homes? What are the values of the other predictors for that census tract, and how do those values compare to the overall ranges for those predictors? Comment on your findings.
Answer

```r
which.min(Boston$medv) #This is the observation that has the lowest median value.
```

```
[1] 399
```

```r
Boston[which.min(Boston$medv),]
```

```
       crim zn indus chas   nox    rm age    dis rad tax ptratio lstat medv
399 38.3518  0  18.1    0 0.693 5.453 100 1.4896  24 666    20.2 30.59    5
```

(h) In this data set, how many of the census tracts average more than seven rooms per dwelling? More than eight rooms per dwelling? Comment on the census tracts that average more than eight rooms per dwelling.

Answer

```r
length(which(Boston$rm > 7)) #Number of suburbs average more than 7 rooms
```

```
[1] 64
```

```r
length(which(Boston$rm > 8)) #Number of suburbs average more than 8 rooms
```

```
[1] 13
```

```r
Boston[which(Boston$rm > 8),]
```

	crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	lstat	medv
98	0.12083	0	2.89	0	0.4450	8.069	76.0	3.4952	2	276	18.0	4.21	38.7
164	1.51902	0	19.58	1	0.6050	8.375	93.9	2.1620	5	403	14.7	3.32	50.0
205	0.02009	95	2.68	0	0.4161	8.034	31.9	5.1180	4	224	14.7	2.88	50.0
225	0.31533	0	6.20	0	0.5040	8.266	78.3	2.8944	8	307	17.4	4.14	44.8
226	0.52693	0	6.20	0	0.5040	8.725	83.0	2.8944	8	307	17.4	4.63	50.0
227	0.38214	0	6.20	0	0.5040	8.040	86.5	3.2157	8	307	17.4	3.13	37.6
233	0.57529	0	6.20	0	0.5070	8.337	73.3	3.8384	8	307	17.4	2.47	41.7
234	0.33147	0	6.20	0	0.5070	8.247	70.4	3.6519	8	307	17.4	3.95	48.3
254	0.36894	22	5.86	0	0.4310	8.259	8.4	8.9067	7	330	19.1	3.54	42.8
258	0.61154	20	3.97	0	0.6470	8.704	86.9	1.8010	5	264	13.0	5.12	50.0
263	0.52014	20	3.97	0	0.6470	8.398	91.5	2.2885	5	264	13.0	5.91	48.8
268	0.57834	20	3.97	0	0.5750	8.297	67.0	2.4216	5	264	13.0	7.44	50.0
365	3.47428	0	18.10	1	0.7180	8.780	82.9	1.9047	24	666	20.2	5.29	21.9