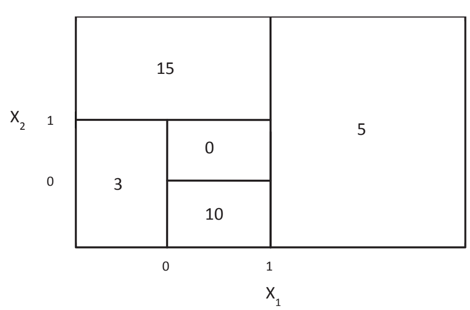
# Homework 5 Solutions - MATH 4322

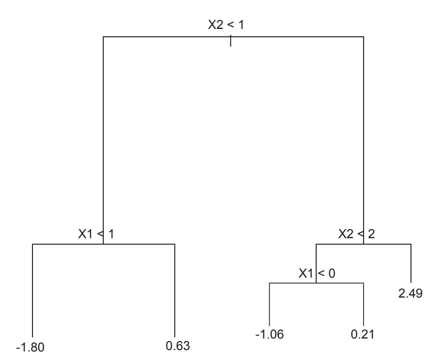
## Instructions

1. Due date: April 4, 2024
2. Answer the questions fully for full credit.
3. Scan or Type your answers and submit only one file. (If you submit several files only the recent one uploaded will be graded).
4. Preferably save your file as PDF before uploading.
5. Submit in Canvas.
6. These questions are from *An Introduction to Statistical Learning with Applications* in R by James, et. al., chapter 8.

## Problem 1

The questions relate to the following plots:

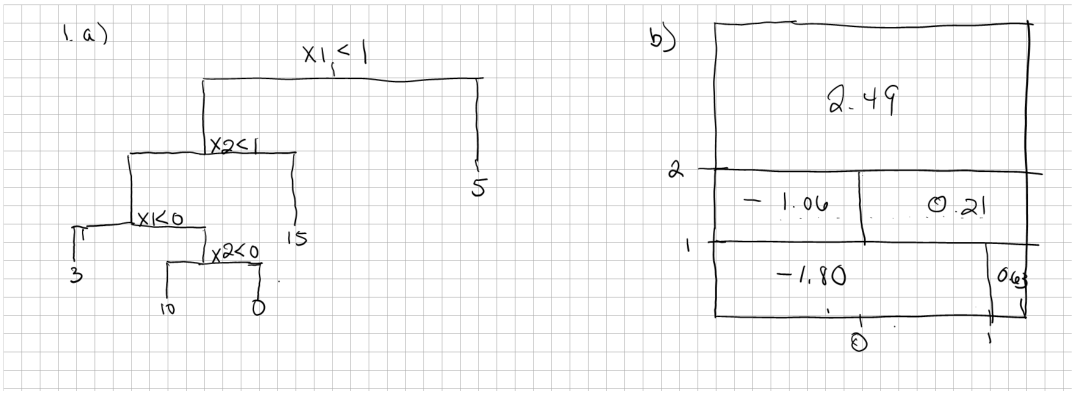



a) Sketch the tree corresponding to the partition of the predictor space illustrated on the left-hand plot. The numbers inside the boxes indicate the mean of $Y$ within each region.

b) Create a diagram similar to the left-hand plot using the tree illustrated in the right-hand plot. You should divide up the predictor space into the correct regions, and indicate the mean for each region.

**Answer**




## Problem 2

Suppose we produce ten bootstrapped samples from a data set containing red and green classes. We then apply a classification tree to each bootstrapped sample and, for a specific value of X, produce 10 estimates of $P(\text{Class is Red}|X)$:

$$0.1, 0.15, 0.2, 0.2, 0.55, 0.6, 0.6, 0.65, 0.7, \text{ and } 0.75.$$

There are two common ways to combine these results together into a single class prediction. One is the majority vote approach discussed in this chapter. The second approach is to classify based on the average probability. In this example, what is the final classification under each of these two approaches?

**Answer**

```r
px = c(0.1,0.15,0.2,0.2,0.55,0.6,0.6,0.65,0.7,0.75)
#Majority vote
vote = ifelse(px>= 0.5,1,0)
sum(vote)
```

```
[1] 6
```

```r
#Average
mean(px)
```

```
[1] 0.45
```

With the majority vote we get 6 out of 10 to be Red thus this approach would say that we have Red.
The average approach is at 0.45 which is less than 0.5, thus we would say with this approach we have Green.

## Problem 3

Provide a detailed explanation of the algorithm that is used to fit a regression tree.

**Answer**

1. Use recursive binary splitting to grow a large tree on the training data, stopping only when each terminal node has fewer than some minimum number of observations.
2. Apply cost complexity pruning to the large tree in order to obtain a sequence of best subtrees, as a function of $\alpha$.
3. Use K-fold cross-validation to choose $\alpha$. That is, divide the training observations into K folds. For each $k = 1, \ldots, K$:

(a) Repeat Steps 1 and 2 on all but the kth fold of the training data.
(b) Evaluate the mean squared prediction error on the data in the left-out $k$th fold, as a function of $\alpha$.

Average the results for each value of $\alpha$, and pick $\alpha$ to minimize the average error.

4. Return the subtree from Step 2 that corresponds to the chosen value of $\alpha$.

## Problem 4

We will use the `Carseats` data set that is in the `ISLR2` package to see to predict `Sales` using regression trees and related approaches.
a) Treating `Sales` as a quantitative variable, would we create regression or classification tree?

**Answer**: A regression tree.

b) Split the data set into a training set and a test set.

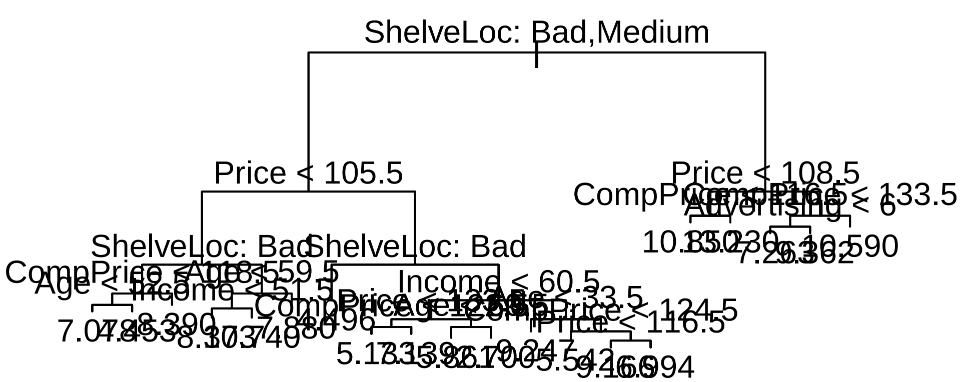
```
library(ISLR2)
set.seed(20)
index = sample(nrow(Carseats),round(0.7*nrow(Carseats)))
train = Carseats[index,]
test = Carseats[-index,]
```

c) Fit a tree to the training set. Plot the tree, and interpret the results. What test MSE do you obtain?

```r
library(tree)
tree.carseats = tree(Sales ~ ., train)
summary(tree.carseats)
```

```
Regression tree:
tree(formula = Sales ~ ., data = train)
Variables actually used in tree construction:
[1] "ShelveLoc"   "Price"       "CompPrice"   "Age"         "Income"
[6] "Advertising"
Number of terminal nodes:  20
Residual mean deviance:  2.363 = 614.4 / 260
Distribution of residuals:
    Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-4.18400 -0.88550 -0.08422  0.00000  0.95770  4.61000
```

```r
plot(tree.carseats)
text(tree.carseats,pretty = 0)
```



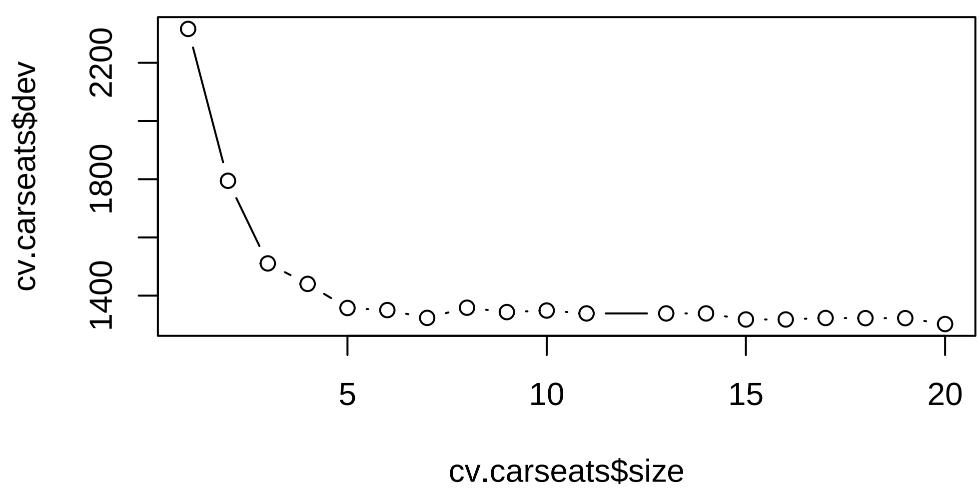
```
yhat = predict(tree.carseats,newdata = test)
#Test MSE
(test.mse = mean((yhat - test$Sales)^2))
```

```
[1] 5.157823
```

There are 20 nodes to this tree. The variables that are used is ShelveLoc, Price, CompPrice, Age, Income and Advertising. With 20 nodes this is very hard to interpret. The test MSE is 5.1578.

d) Use cross-validation in order to determine the optimal level of tree complexity. Does pruning the tree improve the test MSE?

```
cv.carseats = cv.tree(tree.carseats)
plot(cv.carseats$size,cv.carseats$dev,type = "b")
```




```
cv.carseats
```

```
$size
 [1] 20 19 18 17 16 15 14 13 11 10  9  8  7  6  5  4  3  2  1

$dev
 [1] 1302.356 1322.618 1322.618 1323.071 1318.090 1318.090 1338.809 1338.809
 [9] 1338.809 1348.762 1343.454 1358.499 1323.423 1350.313 1357.446 1440.282
[17] 1510.697 1794.510 2316.361

$k
 [1]      -Inf  24.80625  24.89140  25.46601  26.76843  26.80908  35.20185
 [8]  35.24160  35.43344  41.76738  43.47571  46.30820  56.97002  72.49995
[15]  92.24290 110.17098 135.94069 279.25998 531.52217

$method
[1] "deviance"

attr(,"class")
[1] "prune"         "tree.sequence"
```

It appears that pruning to 7 would be best.

```
prune.carseats = prune.tree(tree.carseats,best = 7)
prune.yhat = predict(prune.carseats,newdata = test)
#Test MSE
(mse.prune = mean((prune.yhat - test$Sales)^2))
```

```
[1] 4.679617
```

This does improve the test MSE.

e) Use the bagging approach in order to analyze this data. What test MSE do you obtain? Use the `importance()` function to determine which variables are most important.

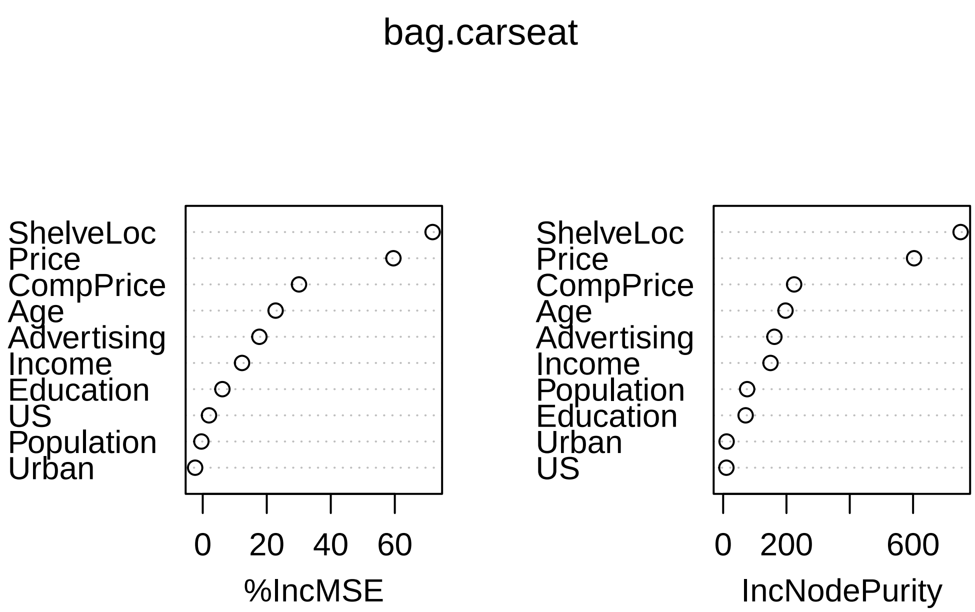
```
library(randomForest)
bag.carseat = randomForest(Sales ~., train, mtry = 10, importance = TRUE)
bag.yhat = predict(bag.carseat, newdata = test)
#Test MSE
(bag.mse = mean((bag.yhat - test$Sales)^2))
```

```
[1] 2.587705
```

```r
#Important Variables
importance(bag.carseat)
```

```
               %IncMSE IncNodePurity
CompPrice   30.0698643     224.29303
Income      12.2747407     149.63464
Advertising 17.6892477     161.90455
Population  -0.4244039      75.76704
Price       59.5642149     603.33628
ShelveLoc   71.8067989     750.23328
Age         22.7640834     196.99315
Education    6.1372741      71.01545
Urban       -2.3719424      10.91125
US           1.9533538      10.24566
```

```r
varImpPlot(bag.carseat)
```



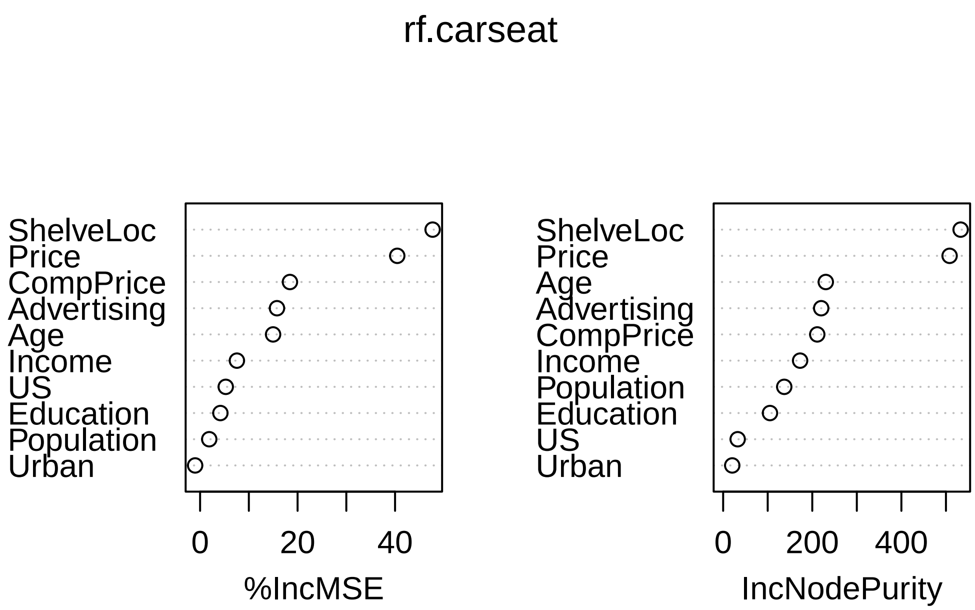

The two most important variables are `ShelveLoc` and `Price`.

f) Use random forests to analyze this data. What test MSE do you obtain? Use the `importance()` function to determine which variables are most important. Describe the effect of $m$, the number of variables considered at each split, on the error rate obtained.

```r
rf.carseat = randomForest(Sales ~., train, mtry = sqrt(10), importance = TRUE)
rf.yhat = predict(rf.carseat, newdata = test)
#Test MSE
(rf.mse = mean((rf.yhat - test$Sales)^2))
```

```
[1] 2.89114
```

```r
#Important Variables
varImpPlot(rf.carseat)
```




For my random samples, the random forests did not yield much of an improvement over the bagging.

## Problem 5

This problem involves the `OJ` data set which is part of the `ISLR2` package.

a) Create a training set containing a random sample of 800 observations, and a test set containing the remaining observations.

```r
library(ISLR2)
data(OJ)
set.seed(1000)
train = sample(nrow(OJ),800)
train.oj = OJ[train,]
test.oj = OJ[-train,]
```

b) Fit a tree to the training data, with `Purchase` as the response and the other variables as predictors. Use the `summary()` function to produce summary statistics about the tree, and describe the results obtained. What is the training error rate? How many terminal nodes does the tree have?

```r
library(tree)
tree.oj = tree(Purchase ~ ., OJ, subset = train)
summary(tree.oj)
```

```
Classification tree:
tree(formula = Purchase ~ ., data = OJ, subset = train)
Variables actually used in tree construction:
[1] "LoyalCH"     "PriceDiff"   "SalePriceMM"
Number of terminal nodes:  8
Residual mean deviance:  0.7486 = 592.9 / 792
Misclassification error rate: 0.16 = 128 / 800
```

Training error rate: 16% Number of terminal nodes: 8

c) Type in the name of the tree object in order to get a detailed text output. Pick one of the terminal nodes, and interpret the information displayed.

```r
tree.oj
```

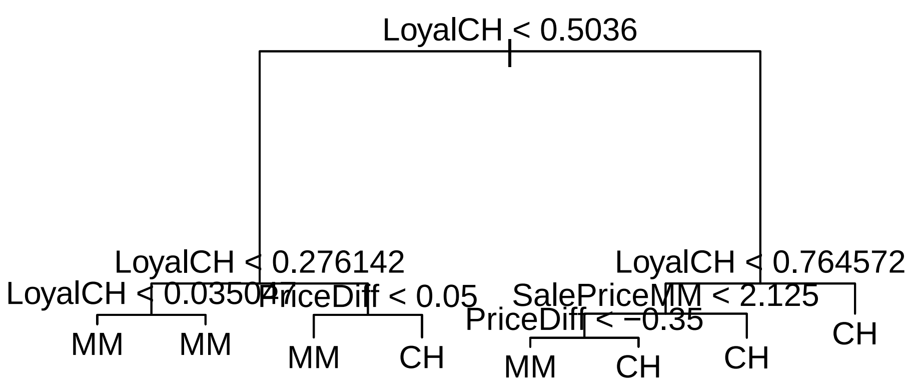
```
node), split, n, deviance, yval, (yprob)
      * denotes terminal node

 1) root 800 1066.00 CH ( 0.61500 0.38500 )
   2) LoyalCH < 0.5036 353  422.60 MM ( 0.28612 0.71388 )
```

```
    4) LoyalCH < 0.276142 170  131.00 MM ( 0.12941 0.87059 )
      8) LoyalCH < 0.035047 57   10.07 MM ( 0.01754 0.98246 ) *
      9) LoyalCH > 0.035047 113  108.50 MM ( 0.18584 0.81416 ) *
    5) LoyalCH > 0.276142 183  250.30 MM ( 0.43169 0.56831 )
     10) PriceDiff < 0.05 78   79.16 MM ( 0.20513 0.79487 ) *
     11) PriceDiff > 0.05 105  141.30 CH ( 0.60000 0.40000 ) *
  3) LoyalCH > 0.5036 447  337.30 CH ( 0.87472 0.12528 )
    6) LoyalCH < 0.764572 187  206.40 CH ( 0.75936 0.24064 )
     12) SalePriceMM < 2.125 120  156.60 CH ( 0.64167 0.35833 )
       24) PriceDiff < -0.35 16   17.99 MM ( 0.25000 0.75000 ) *
       25) PriceDiff > -0.35 104  126.70 CH ( 0.70192 0.29808 ) *
     13) SalePriceMM > 2.125 67   17.99 CH ( 0.97015 0.02985 ) *
    7) LoyalCH > 0.764572 260   91.11 CH ( 0.95769 0.04231 ) *
```

From my node 2): If LoyalCH $<$ 0.5036 there are 353 customers with this criteria the deviance is 422.6, the chance that the customer will by Minute Made is 71.388%.

d) Create a plot of the tree, and interpret the results.

```
plot(tree.oj)
text(tree.oj,pretty = 0)
```



The variables that appears to be used to predict if they will buy MM or CH is “LoyalCH”, “SalePriceMM”, and “PriceDiff”.

e) Predict the response on the test data, and produce a confusion matrix comparing the test labels to the predicted test labels. What is the test error rate?

```
tree.pred = predict(tree.oj,test.oj,type = "class")
(con.matrix = table(tree.pred,test.oj$Purchase))
```

```
tree.pred  CH  MM
       CH 150  38
       MM  11  71
```

```
#Test error rate
(con.matrix[1,2]+con.matrix[2,1])/sum(con.matrix)
```

```
[1] 0.1814815
```

f) Apply the `cv.tree()` function to the training set in order to determine the optimal tree size.

g) Produce a plot with tree size on the $x$-axis and cross-validated classification error rate on the $y$-axis.

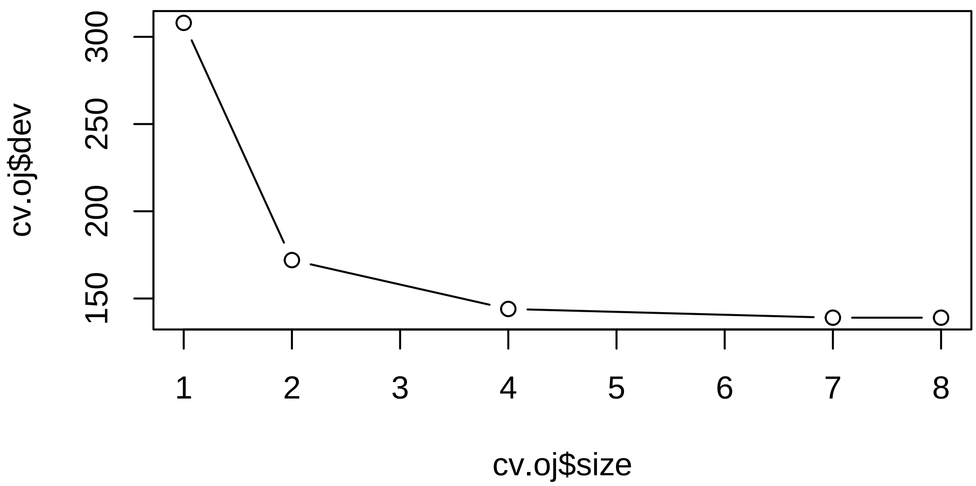
h) Which tree size corresponds to the lowest cross-validated classification error rate?

```
set.seed(2)
cv.oj = cv.tree(tree.oj,FUN = prune.misclass)
cv.oj
```

```
$size
[1] 8 7 4 2 1

$dev
[1] 139 139 144 172 308

$k
[1]       -Inf   0.000000   2.666667  10.500000 151.000000
```

```
$method
[1] "misclass"

attr(,"class")
[1] "prune"         "tree.sequence"
```

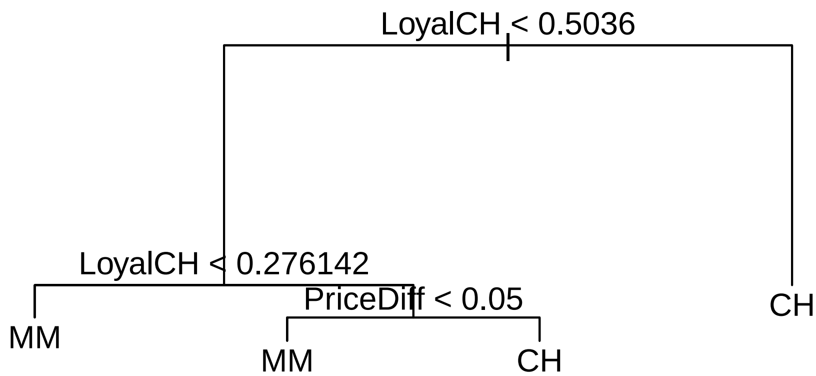
```r
plot(cv.oj$size,cv.oj$dev,type = "b")
```




It appears that the optimal tree size would be 4.

i) Produce a pruned tree corresponding to the optimal tree size obtained using cross-validation. If cross-validation does not lead to selection of a pruned tree, then create a pruned tree with five terminal nodes.

```r
prune.oj = prune.misclass(tree.oj,best = 4)
plot(prune.oj)
text(prune.oj,pretty = 0)
```




j) Compare the training error rates between the pruned and unpruned trees. Which is higher?

```
train.tree = predict(tree.oj,type = "class")
(train.matrix = table(train.tree,train.oj$Purchase))
```

```
train.tree  CH  MM
        CH 450  86
        MM  42 222
```

```
#Train error rate Un-pruned
(train.matrix[1,2]+train.matrix[2,1])/sum(train.matrix)
```

```
[1] 0.16
```

```
train.prune = predict(prune.oj,type = "class")
(prune.matrix = table(train.prune,train.oj$Purchase))
```

```
train.prune  CH  MM
         CH 454  98
         MM  38 210
```

```r
#Train error rate Pruned
(prune.matrix[1,2]+prune.matrix[2,1])/sum(prune.matrix)
```

```
[1] 0.17
```

The test error rate is slightly higher for the pruned trees.

k) Compare the test error rates between the pruned and unpruned trees. Which is higher?

```r
test.tree = predict(tree.oj,test.oj,type = "class")
(test.matrix = table(test.tree,test.oj$Purchase))
```

```
test.tree  CH  MM
       CH 150  38
       MM  11  71
```

```r
#Train error rate Un-pruned
(test.matrix[1,2]+test.matrix[2,1])/sum(test.matrix)
```

```
[1] 0.1814815
```

```r
test.prune = predict(prune.oj,test.oj,type = "class")
(prune.test.matrix = table(test.prune,test.oj$Purchase))
```

```
test.prune  CH  MM
        CH 150  44
        MM  11  65
```

```r
#Train error rate Pruned
(prune.test.matrix[1,2]+prune.test.matrix[2,1])/sum(prune.test.matrix)
```

```
[1] 0.2037037
```

Again slightly higher for the pruned tree.