Homework 6 - MATH 4322

Instructions

1. Due date: November 28, 2023
2. Answer the questions fully for full credit.
3. Scan or Type your answers and submit only one file. (If you submit several files only the recent one uploaded will be graded).
4. Preferably save your file as PDF before uploading.
5. Submit in Canvas.
6. These questions are based on the Neural Networks lectures.
7. The information in the gray boxes are R code that you can use to answer the questions.

Problem 1

You are given:

- n data samples $\mathbf{x}_i = (x_{1,i}, \cdots, x_{p,i}), i = 1, \dots, n$
- n corresponding to true responses (or labels) $y_i, i = 1, \dots, n$.

and asked to train a single linear neuron "network" to approximate function $f(.)$ such that $f(x_i) = y_i, i = 1, \dots, n$. Provided the train steps for your "network" by answering the following questions.

a) What is the formula to calculate an output $\hat{y}_i$ from an input $\mathbf{x}_i$? What are the model parameters in that formula?

   ANS:

$$\hat{y}_i = \sum_{i=1}^{p} w_i x_i + b,$$

   The model parameters are: Weight of inputs: w1,w2,....,wp and Bias: *b*

b) What criteria do we need to optimize in order to estimate the model parameters?

ANS:
For the sum of squared residuals over all samples:

$$\frac{1}{2}\sum_{i=1}^{p}(\hat{y}_i - y_i)^2$$

If the data samples are fed one at a time, for the ith sample:

$$\frac{1}{2}(\hat{y}_i - y_i)^2$$

c) What is the name of the method used to optimize this criteria in case you do not have access to an analytically solution?

ANS: Gradient Decent

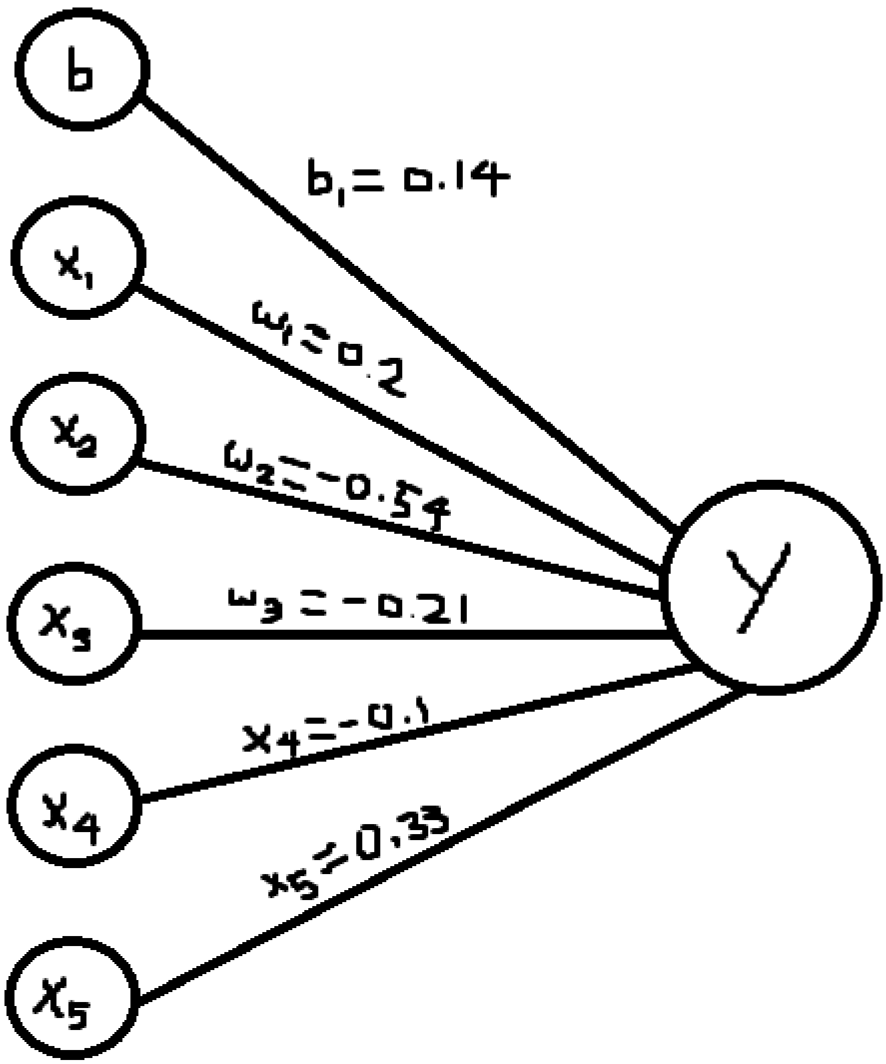
Problem 2

Presume that for a single linear neuron model with input variables $x_1, \ldots, x_5$, you are given the following parameter values:

- weights: $w_1 = 0.2, w_2 = -0.54, w_3 = -0.21, w_4 = -0.1, w_5 = 0.33,$
- bias: $b = 0.14.$

a) Draw a mathematical model of this linear neuron that takes an arbitrary input vector $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$.

ANS:



b) Calculate the linear neuron output for the case of $x_1 = 4, x_2 = -3, x_3 = 7, x_4 = 5, x_5 = -1$. Show your work.

ANS:

$$\hat{y} = 0.14 + 0.2 \cdot (4) - 0.54 \cdot (-3) - 0.21 \cdot (7) - 0.1 \cdot (5) + 0.33 \cdot (-1) = 0.26$$

Problem 3

You are given an artificial neural network (ANN) of linear neurons with

- Input layer of two neurons: x_1, x_2
- Fully-connected hidden layer of three neurons: h_1, h_2, h_3
- One output neuron, y.

The following weight matrices are provided:

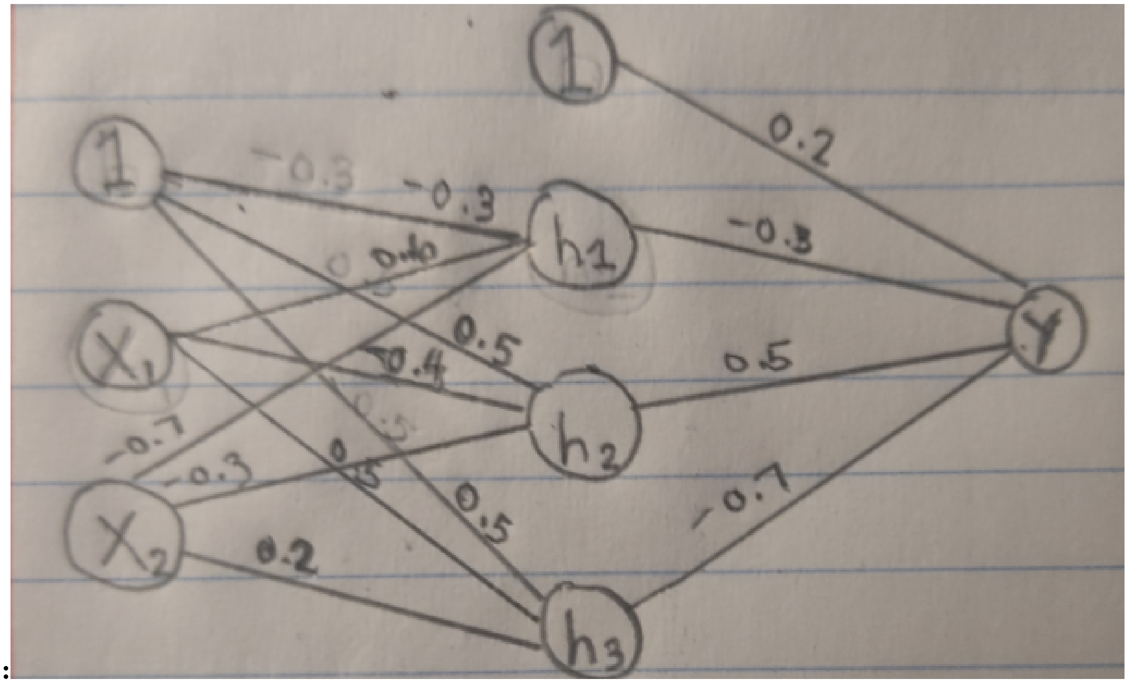
1) Between input & hidden layer:

		Hidden h_1	h_2	h_3
	1 (bias)	-0.3	0.5	0.5
Input	x_1	0.6	-0.4	0.5
	x_2	-0.7	-0.3	0.2

2) Between hidden & output layer:

		Output y
	1 (bias)	0.2
Hidden	h_1	-0.3
	h_2	0.5
	h_3	-0.7

a) Draw this ANN as was done in lecture slides.

ANS: 

b) Calculate the output of this ANN for the case of $x_1 = 10, x_2 - -5$. Show work.

ANS:

$$
\begin{aligned}
\hat{h}_1 &= -0.3 + 0.6 \cdot (10) - 0.7 \cdot (-5) = 9.2 && (1)\\
\hat{h}_2 &= 0.5 - 0.4 \cdot (10) - 0.3 \cdot (-5) = -2 && (2)\\
\hat{h}_3 &= 0.5 + 0.5 \cdot (10) + 0.2 \cdot (-5) = 4.5 && (3)\\
\hat{y} &= 0.2 - 0.3 \cdot (9.2) + 0.5 \cdot (-2) - 0.7 \cdot (4.5) = -6.71 && (4)\\
& && (5)
\end{aligned}
$$

Problem 4

- We want to predict the 'medv' value based on the input of the other thirteen variables.
- We will run a regression neural network for the `Boston` data set.
- We will split the data into training/testing by a 70/30 split.

a) Type and run the following in `R`.

```r
library(neuralnet)
library(MASS)

data = Boston #renaming the Boston data set to "data"
summary(data)
```

What is the mean of `age`? What is the mean of `ptratio`?

ANS: The mean of age = 68.57. The mean of ptratio is 19.05.

b) Normalizing data

- It is recommended to **normalize** (or scale, or standardize, either works) features in order for all the variables to be on the same scale.

- With normalization, data units are eliminated, allowing you to easily compare data from different locations.
- This avoids unnecessary results or difficult training processes resulting in algorithm convergence problems.
- There are different methods for scaling the data.
- The **z-normalization**
$$x_{scale} = \frac{x - \bar{x}}{s}$$
- The **min-max scale**
$$x_{scale} = \frac{x - x_{min}}{x_{max} - x_{min}}$$
- And so forth
- The function in `R` is scale(x,center = ,scale =)
- For this example we will use the min-max method to get all the scaled data in the range [0, 1].
- In order to scale we need to find the minimum and maximum value for each of the columns in the data set. To do this we use the apply function.
- The `apply` function returns a vector or an array or a list of values obtained by **applying** a function to margins of an array or matrix.

- Type and run the following:

```r
max_data = apply(data,2,max)
#2=columns, we are getting the maximum value from each column
min_data = apply(data,2,min)
data_scaled = scale(data, center = min_data, scale = max_data - min_data)
head(data_scaled)
```

What is the scaled value of the first observation for `medv`?

ANS: the scaled value of the first observation for medv is 0.42222.

c) Now we can split the data into training and testing data sets. We will use the 70/30 split

```
set.seed(10)
index = sample(1:nrow(data), round(0.7*nrow(data)))
train_data = as.data.frame(data_scaled[index,])
test_data = as.data.frame(data_scaled[-index,])
dim(train_data)
```

How many observations do we have in the training data set?

ANS: There are 354 observations in the training data set

d) Type and run the following

```
set.seed(1)
net_data = neuralnet(medv ~ ., data = train_data,
                     hidden = 10, linear.output = TRUE)

plot(net_data)
```

Apply the test data set to determine the MSE

```
predict_net = predict(net_data,test_data)
predict_net_start = predict_net*(max(data$medv) - min(data$medv)) + min(data$medv)
test_data_start = test_data$medv*(max(data$medv) - min(data$medv)) + min(data$medv)
sum((predict_net_start - test_data_start)^2)/nrow(test_data)
```

What is the test MSE for this model?

ANS: The Test MSE is 15.37819.

e) Let us compare this test MSE to the linear regression model. Type and run the following:

```
lm.boston = lm(medv ~ ., data = data, subset = index )
summary(lm.boston)
test = data[-index,]
predict_lm = predict(lm.boston,test)
sum((predict_lm - test$medv)^2)/nrow(test)
```

What is the training MSE for the linear model?

ANS: The training MSE for the linear model is 17.77379.