## EMSSL-syntax, rev2

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Angular momentum: \AngMom{derivative-frame}{reference-point}{point}

Example: \AngMom{0}{P}{A}

Output:

$$\bar{Q}\vec{h}_A$$

Acceleration: \AcclVec{derivative-frame}{point}{with-respect-to}

Example: \AcclVec{0}{A}{Q}

Output:

$$^{ar{O}}ec{a}_{^{A_{/_{Q}}}}$$

Angular Acceleration: \AngAccl{frame-1}{frame-2}

Example: \AngAccl{0}{B}

Output:

$$\bar{O}_{\vec{O}}\bar{B}$$

Angular Velocity: \AngAccl{frame-1}{frame-2}

Example: \AngAccl{0}{B}

Output:

$$\bar{O}_{\vec{Q}}\bar{B}$$

Cross two vectors: \Cross{vector-1}{vector-2}

Example: \Cross{A}{B}

Output:

$$A \times B$$

 $\label{local_coss_prod_frame} $$\operatorname{crossProd}_{a}_{a}_{b}_{c}_{d}_{e}_{f}$$$ 

Example: \CrossProd{frame}{a}{b}{c}{d}{e}{f}

Output:

$$\left| \begin{array}{ccc} \overrightarrow{i}_{\bar{O}} & \overrightarrow{j}_{\bar{O}} & \overrightarrow{k}_{\bar{O}} \\ a & b & c \\ d & e & f \end{array} \right|$$

Derivative: \Deriv[order] {with-respect-to-variable}

Example: \Deriv[]{x}

Output:

$$\frac{d}{dx}$$

Example: \Deriv[3]{t}

$$\frac{d^3}{dt^3}$$

Direction cosine matrix: \DirectCosMat

Example: \DirectCosMat

Output:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Sum of external forces: \Fext

Example: \Fext

Output:

$$\sum \vec{F}_{ext}$$

Frame: \Frame{vector}{frame}

Example: \Frame{A}{B}

Output:

 $\{A\}_{\bar{B}}$ 

Frame definition: \FrameDef{point}{frame}

Example: \FrameDef{Q}{0}

Output:

$$\overrightarrow{Q} = \{Q, \overrightarrow{i}_{\bar{O}}, \overrightarrow{j}_{\bar{O}}, \overrightarrow{k}_{\bar{O}}\}$$

Frame w/ derivative: \FrameDeriv[<order>]{<frame>}{<fun>} Inline version: \iFrameDeriv[<order>]{<frame>}{<fun>}

Example: \FrameDeriv[2]{A}{r}{t}

Output:

 $\frac{\bar{A}}{dt^2}r$ 

Inertia: \Inert{point}
Example: \Inert{A}

Output:

 $\tilde{I}_A$ 

Inertia w/ frame: \InertF{point}{frame}

Example: \InertF{A}{B}

Output:

 $[\tilde{I}_A]_{ar{B}}$ 

Inertia tensor: \InertMat
Example: \InertMat

Output:

 $\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$ 

Lagrangian: \Lagr Example: \Lagr

Output:

 $\mathcal{L}$ 

Omega matrix: \OmegaMat Example: \OmegaMat

Output:

$$\begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}$$

Partial derivative: \Partl[order] {with-respect-to-variable}

Example: \Partl[]{x}

Output:

 $\frac{\partial}{\partial x}$ 

Example: \Part1[3]{t}

Output:

$$\frac{\partial^3}{\partial t^3}$$

Partial framed derivative: \FramePartl[order]{frame}{with-respect-to-variable}

Example: \FramePart1[2]{A}{x}

Output:

$$\frac{\bar{A}}{\partial x^2} \frac{\partial^2}{\partial x^2}$$

Position vector: \PosVec{point}{with-respect-to-point}

Example: \PosVec{A}{Q}

Output:

$$\vec{r}_{A/_Q}$$

Quaternion angles: \QuatAngle

Example: \QuatAngle

Output:

$$\begin{array}{rcl} q_0 & = & \cos(\frac{\phi}{2}) \\ q_1 & = & a_x \sin(\frac{\phi}{2}) \\ q_2 & = & a_y \sin(\frac{\phi}{2}) \\ q_3 & = & a_z \sin(\frac{\phi}{2}) \end{array}$$

Quaternion BCO matrix: \QuatBCO

Example: \QuatBCO

Output:

$$\begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

Quaternion definition: \QuatDef{frame}

Example: \QuatDef{B}

Output:

$$q_0 + q_1 \overrightarrow{i}_{\bar{B}} + q_2 \overrightarrow{j}_{\bar{B}} + q_3 \overrightarrow{k}_{\bar{B}}$$

Quaternion derivative: \QuatDot

Example: \QuatDot

Output:

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{cases} q_0 \\ q_1 \\ q_2 \\ q_3 \end{cases}$$

Slanted fraction: \Rfrac{numerator}{denominator}

Example: \Rfrac{A}{B}

Output:

Rotation matrix form: \Rotate{angle}{axis}

Example: \Rotate{\phi}{X}

Output:

 $[R_X(\phi)]$ 

Rotation matrix: \RotateMat{frame-1}{frame-2}

Example: \RotateMat{B}{0}

Output:

 ${}^{\bar{B}}[C]^{\bar{O}}$ 

Rotation matrix derivative: \RotateMatDot{frame-1}{frame-2}

Example: \RotateMatDot{B}{0}

Output:

 ${}^{\bar{B}}[\dot{C}]^{\bar{O}}$ 

Rotation matrix about X: \RotateMatX{angle}

Example: \RotateMatX{\theta}

Output:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

Rotation matrix about Y: \RotateMatY{angle}

Example: \RotateMatY{\theta}

Output:

$$\begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Rotation matrix about Z: \RotateMatZ{angle}

Example: \RotateMatZ{\theta}

Output:

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sum of external torques: \Text[point]

Example: \Text

Output:

 $\sum \bar{\tau}$ 

Example: \Text[P]

Output:

 $\sum ec{ au}_P$ 

1st transport theorem: \TransOne{frame-1}{frame-2}{vector}

Inline version: \iTransOne{frame-1}{frame-2}{vector}

Example: \TransOne{0}{B}{A}

Output:

$$\frac{\bar{O}}{dt}A = \frac{\bar{B}}{dt}A + \bar{O}\vec{\omega}^{\bar{B}} \times A$$

2nd transport theorem: \TransTwo{frame-1}{frame-2}{vector}

Inline version: \iTransTwo{frame-1}{frame-2}{vector}

Example: \TransTwo{0}{B}{A}

Output:

$${}^{\bar{O}}\frac{d^2}{dt^2}A = {}^{\bar{B}}\frac{d^2}{dt^2}A + 2^{\bar{O}}\vec{\omega}^{\bar{B}}\times {}^{\bar{B}}\frac{d}{dt}A + {}^{\bar{O}}\vec{\alpha}^{\bar{B}}\times A + {}^{\bar{O}}\vec{\omega}^{\bar{B}}\times \left({}^{\bar{O}}\vec{\omega}^{\bar{B}}\times A\right)$$

Dyad: \UnitDyad{unit-vector}{unit-vector}{frame}

Example: \UnitDyad{i}{k}{B}

Output:

 $\vec{i}_{\bar{B}}\vec{k}_{\bar{B}}$ 

Unit vector: \UnitVec{vector}{frame}

Example: \UnitVec{i}{0}

Output:

 $\vec{i}_{\bar{O}}$ 

Left superscript frame: \UpRight{frame}{quantity}

Example: \UpRight{0}{A}

Output:

 $\bar{O}_A$ 

Horizontal vector: \VecExpressH{frame}{element-1}{element-2}{element-3}

Example: \VecExpressH{D}{a}{b}{c}

Output:

 $\{a\,\overrightarrow{i}_{\,\bar{D}}+b\,\overrightarrow{j}_{\,\bar{D}}+c\,\overrightarrow{k}_{\,\bar{D}}\}$ 

Vertical vector: \VecExpressV{element-1}{element-2}{element-3}

Example: \VecExpressV{a}{b}{c}

Output:

4-element vertical vector: \VecExpressVF{element-1}{element-2}{element-3}{element-4}

Example: \VecExpressVF{a}{b}{c}{d}

Output:

 $\begin{cases}
 a \\
 b
\end{cases}$   $\begin{bmatrix}
 c \\
 d
\end{bmatrix}$ 

Velocity vector \VelVec{derivative-frame}{point}{with-respect-to-point}

Example: \VelVec{0}{A}{Q}

Output:

 $\bar{O}_{\vec{V}_{A/_{O}}}$ 

Angular velocity definition, X \wX{frame-1}{frame-2}

Example:  $\wX\{B\}\{0\}$ 

Output:

 $\{0,0,1\}^{\bar{B}}[C]^{\bar{O}\bar{O}}[\dot{C}]^{\bar{B}} \begin{cases} 0\\1\\0 \end{cases}$ 

Angular velocity definition, Y \wY{frame-1}{frame-2}

Example:  $\wY\{B\}\{0\}$ 

Output:

 $\{1,0,0\}^{\bar{B}}[C]^{\bar{O}\bar{O}}[\dot{C}]^{\bar{B}} \left\{ \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \right\}$ 

Angular velocity definition, Z \wZ{frame-1}{frame-2}

Example:  $\wZ\{B\}\{0\}$ 

Output:

$$\{0,1,0\}^{\bar{B}}[C]^{\bar{O}\bar{O}}[\dot{C}]^{\bar{B}} \left\{ egin{matrix} 1 \\ 0 \\ 0 \end{matrix} \right\}$$

The velocity vector  ${}^{\bar{O}}\vec{v}_{^{A}/_{O}}$  should be fine.

The transport theorem, rev2, is  $\frac{\bar{O}}{dt}\vec{r}_{B/O} = \frac{\bar{A}}{dt}\vec{r}_{B/O} + \bar{O}\vec{\omega}^{\bar{A}} \times \vec{r}_{B/O}$ The transport theorem, rev3, is  $\frac{\bar{O}}{dt}\vec{r}_{B/O} = \frac{\bar{A}}{dt}\vec{r}_{B/O} + \bar{O}\vec{\omega}^{\bar{A}} \times \vec{r}_{B/O}$ 

The transport theorem, rev2, is  $\frac{\bar{O}_{d^2}}{dt^2}\vec{r}_{B/O} = \frac{\bar{A}_{d^2}}{dt^2}\vec{r}_{B/O} + 2^{\bar{O}}\vec{\omega}^{\bar{A}} \times \frac{\bar{A}_{d}}{dt}\vec{r}_{B/O} + ^{\bar{O}}\vec{\alpha}^{\bar{A}} \times \vec{r}_{B/O} + ^{\bar{O}}\vec{\omega}^{\bar{A}} \times \left(^{\bar{O}}\vec{\omega}^{\bar{A}} \times \vec{r}_{B/O}\right)$ The transport theorem, rev3, is  $\frac{\bar{O}_{d^2}}{dt^2}\vec{r}_{B/O} = \frac{\bar{A}_{d^2}}{dt^2}\vec{r}_{B/O} + 2^{\bar{O}}\vec{\omega}^{\bar{A}} \times \frac{\bar{A}_{d}}{dt}\vec{r}_{B/O} + ^{\bar{O}}\vec{\alpha}^{\bar{A}} \times \vec{r}_{B/O} + ^{\bar{O}}\vec{\omega}^{\bar{A}} \times \left(^{\bar{O}}\vec{\omega}^{\bar{A}} \times \vec{r}_{B/O}\right)$