

# Outline: Unbalanced Supervised Problems (F. Chiaromonte)

Statistical Methods for Large, Complex Data

A reference article on subsampling in  
unbalanced classification problems:

*The Annals of Statistics*

2014, Vol. 42, No. 5, 1693–1724

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## LOCAL CASE-CONTROL SAMPLING: EFFICIENT SUBSAMPLING IN IMBALANCED DATA SETS

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For classification problems with significant class imbalance, subsampling can reduce computational costs at the price of inflated variance in estimating model parameters. We propose a method for subsampling efficiently for logistic regression by adjusting the class balance locally in feature space via an accept–reject scheme. Our method generalizes standard case-control sampling, using a pilot estimate to preferentially select examples whose responses are conditionally rare given their features. The biased subsampling is corrected by a post-hoc analytic adjustment to the parameters. The method is simple and requires one parallelizable scan over the full data set.

Standard case-control sampling is inconsistent under model misspecification for the population risk-minimizing coefficients  $\theta^*$ . By contrast, our estimator is consistent for  $\theta^*$  provided that the pilot estimate is. Moreover, under correct specification and with a consistent, independent pilot estimate, our estimator has exactly twice the asymptotic variance of the full-sample MLE—even if the selected subsample comprises a miniscule fraction of the full data set, as happens when the original data are severely imbalanced. The factor of two improves to  $1 + \frac{1}{c}$  if we multiply the baseline acceptance probabilities by  $c > 1$  (and weight points with acceptance probability greater than 1), taking roughly  $\frac{1+c}{2}$  times as many data points into the subsample. Experiments on simulated and real data show that our method can substantially outperform standard case-control subsampling.

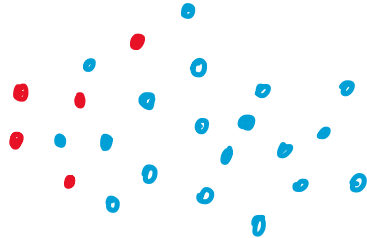
## Some more references

Dubey, Rashmi et al. "Analysis of sampling techniques for imbalanced data: An n = 648 ADNI study." *NeuroImage* vol. 87 (2014): 220-41. doi:10.1016/j.neuroimage.2013.10.005

Menardi, Torrelli (2010) "Training and assessing classification rules with unbalanced data". Working Paper 2-2010. Dipartimento B. De Finetti, Università' di Trieste.

Imbalanced Learning tools, MIT (includes SMOTE)  
<https://imbalanced-learn.org/stable/index.html>

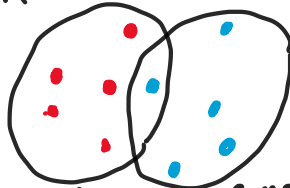
The data in feature space



"scarce" class  $n_r$

"abundant" class  $n_b$

① Reduce the abundant class



Keep these  $n_r$   
fixed

consider only  $n_r$   
of these at random  
... can "repeat"

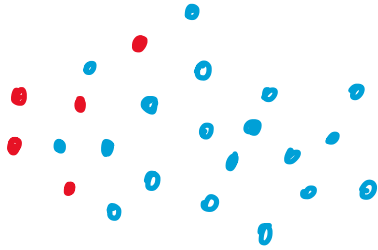
Form one, or several, datasets  
of size  $2n_r < n_r + n_b$ .  
The red points are always  
the same.

①B A variant

Bootstrap the red points  
(resampling with replacement)  
Sub-bootstrap the blue points  
(Select  $n_r$  blue points at  
random with replacement)

→ ... can "repeat" both  
Form one, or several, datasets  
of size  $2n_r < n_r + n_b$   
The red points are bootstrapped too.  
... **MAKES MORE SENSE STATISTICALLY**  
we simulate sampling from the  
two populations.

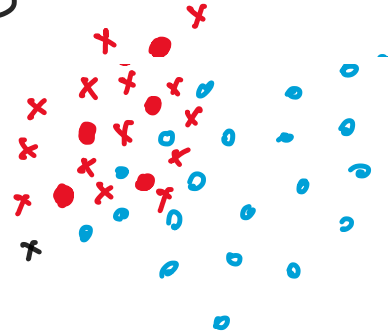
The data in feature space



"scarce" class  $n_R$

"abundant" class  $n_B$

② Augment the scarce class

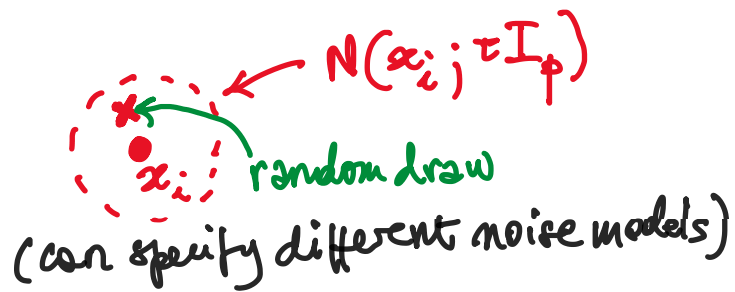


● = actual red points  
x = artificial red points

Form a dataset of size  
 $2n_B > n_R + n_B$

How do we create the artificial points? some options

- (i) Over-bootstrap the red points  
(select  $n_B$  red points at random with replacement)  
ADDING NOISE to each draw



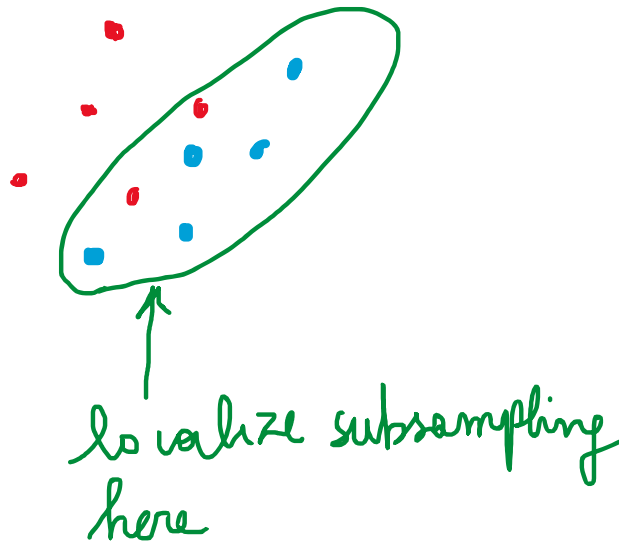
- (ii)  $n_B$  times over: select at random two red points and a point between them



(can "localize" the selection of the red points, e.g., first at random, second at random among its closest red neighbors)

## Localizing the reduction or the augmentation

preserve the abundant class  
where it is harder to discriminate,  
i.e., close to the red points



Focus the augmentation of the  
scarce class where it is harder  
to discriminate, i.e., close to  
the blue points.

... **MAKES MORE SENSE STATISTICALLY**

