In[54]:=
```
phase = (2 / 3) * Power[r, 3 / 2] * Sin[(3 / 2) * alpha]
```
Out[54]=
$$\frac{2}{3}\, r^{3/2}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]$$

In[55]:=
```
kvec = Grad[phase, {r, alpha}, "Polar"]
```
Out[55]=
$$\left\{\sqrt{r}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right],\ \sqrt{r}\, \text{Cos}\left[\frac{3\,\text{alpha}}{2}\right]\right\}$$

In[56]:=
```
k = Sqrt[Part[kvec, 1]^2 + Part[kvec, 2]^2]
```
Out[56]=
$$\sqrt{r\, \text{Cos}\left[\frac{3\,\text{alpha}}{2}\right]^2 + r\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]^2}$$

In[57]:=
```
B = 1 - k^2
```
Out[57]=
$$1 - r\, \text{Cos}\left[\frac{3\,\text{alpha}}{2}\right]^2 - r\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]^2$$

In[58]:=
```
div = Div[kvec * B, {r, alpha}, "Polar"]
```
Out[58]=
$$\sqrt{r}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right] \left(-\text{Cos}\left[\frac{3\,\text{alpha}}{2}\right]^2 - \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]^2\right)$$

In[59]:=
```
lap = Laplacian[phase, {r, alpha}, "Polar"]
```
Out[59]=
$$0$$

In[60]:=
```
biharm = Laplacian[lap, {r, alpha}, "Polar"]
```
Out[60]=
$$0$$

In[61]:=
```
RHS = -2 * div - biharm
```
Out[61]=
$$-2\, \sqrt{r}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right] \left(-\text{Cos}\left[\frac{3\,\text{alpha}}{2}\right]^2 - \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]^2\right)$$

In[62]:=
```
Simplify[RHS]
```
Out[62]=
$$2\, \sqrt{r}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]$$

```
dfdr = D[phase, r]
```
Out[63]=
$$\sqrt{r}\, \text{Sin}\left[\frac{3\,\text{alpha}}{2}\right]$$

In[65]:=
```
dfda = D[phase, alpha]
```
Out[65]=

$$r^{3/2}\ \mathrm{Cos}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

```
d2fdr2 = D[phase, {r, 2}]
```
Out[66]=

$$\frac{\mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]}{2\sqrt{r}}$$

In[67]:=
```
d2fda2 = D[phase, {alpha, 2}]
```
Out[67]=

$$-\frac{3}{2}\ r^{3/2}\ \mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

```
d2fdadr = D[phase, {alpha, 1}, {r, 1}]
```
Out[68]=

$$\frac{3}{2}\sqrt{r}\ \mathrm{Cos}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

In[69]:=
```
d2fdrda = D[phase, {r, 1}, {alpha, 1}]
```
Out[69]=

$$\frac{3}{2}\sqrt{r}\ \mathrm{Cos}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

In[70]:=
```
d3fdr3 = D[phase, {r, 3}]
```
Out[70]=

$$-\frac{\mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]}{4\ r^{3/2}}$$

In[71]:=
```
d3fdr1da2 = D[phase, {r, 1}, {alpha, 2}]
```
Out[71]=

$$-\frac{9}{4}\sqrt{r}\ \mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

In[72]:=
```
d3fdr1da2 = D[phase, {alpha, 2}, {r, 1}]
```
Out[72]=

$$-\frac{9}{4}\sqrt{r}\ \mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]$$

In[73]:=
```
d4fdr4 = D[phase, {r, 4}]
```
Out[73]=

$$\frac{3\ \mathrm{Sin}\left[\frac{3\ \mathrm{alpha}}{2}\right]}{8\ r^{5/2}}$$

In[74]:=
```
d4fda4 = D[phase, {alpha, 4}]
```

Out[74]=

$$\frac{27}{8}\, r^{3/2}\, \mathrm{Sin}\left[\frac{3\,\mathrm{alpha}}{2}\right]$$

In[75]:=
```
d4fda2dr2 = D[phase, {alpha, 2}, {r, 2}]
```

Out[75]=

$$-\frac{9\,\mathrm{Sin}\left[\frac{3\,\mathrm{alpha}}{2}\right]}{8\,\sqrt{r}}$$

In[76]:=
```
d4fda2dr2 = D[phase, {r, 2}, {alpha, 2}]
```

Out[76]=

$$-\frac{9\,\mathrm{Sin}\left[\frac{3\,\mathrm{alpha}}{2}\right]}{8\,\sqrt{r}}$$