In[13]:= `Grad[phase[r, alpha], {r, alpha}]`

Out[13]=
$\{phase^{(1,0)}[r, alpha], phase^{(0,1)}[r, alpha]\}$

In[14]:= `kvec = Grad[phase[x, y], {x, y}]`

Out[14]=
$\{phase^{(1,0)}[x, y], phase^{(0,1)}[x, y]\}$

In[15]:= `kvec[0]`

Out[15]=
$\{phase^{(1,0)}[x, y], phase^{(0,1)}[x, y]\}[0]$

In[16]:= `kvec`

Out[16]=
$\{phase^{(1,0)}[x, y], phase^{(0,1)}[x, y]\}$

In[17]:= `Part[kvec, 1]`

Out[17]=
$phase^{(1,0)}[x, y]$

`k = Sqrt[Part[kvec, 1]^2 + Part[kvec, 2]^2]`

Out[18]=
$\sqrt{phase^{(0,1)}[x, y]^2 + phase^{(1,0)}[x, y]^2}$

In[19]:= `B = 1 - k^2`

Out[19]=
$1 - phase^{(0,1)}[x, y]^2 - phase^{(1,0)}[x, y]^2$

In[20]:= `kvec * 2`

Out[20]=
$\{2\, phase^{(1,0)}[x, y], 2\, phase^{(0,1)}[x, y]\}$

In[21]:= `kvec * B`

Out[21]=
$\{phase^{(1,0)}[x, y]\left(1 - phase^{(0,1)}[x, y]^2 - phase^{(1,0)}[x, y]^2\right),$
$\quad phase^{(0,1)}[x, y]\left(1 - phase^{(0,1)}[x, y]^2 - phase^{(1,0)}[x, y]^2\right)\}$

In[22]:= `Div[kvec * B, {x, y}]`

Out[22]=
$phase^{(0,2)}[x, y]\left(1 - phase^{(0,1)}[x, y]^2 - phase^{(1,0)}[x, y]^2\right) +$
$\quad phase^{(0,1)}[x, y]\left(-2\, phase^{(0,1)}[x, y]\, phase^{(0,2)}[x, y] - 2\, phase^{(1,0)}[x, y]\, phase^{(1,1)}[x, y]\right) +$
$\quad \left(1 - phase^{(0,1)}[x, y]^2 - phase^{(1,0)}[x, y]^2\right) phase^{(2,0)}[x, y] +$
$\quad phase^{(1,0)}[x, y]\left(-2\, phase^{(0,1)}[x, y]\, phase^{(1,1)}[x, y] - 2\, phase^{(1,0)}[x, y]\, phase^{(2,0)}[x, y]\right)$

```
div = Div[kvec * B, {x, y}]
```

Out[23]=

$$\text{phase}^{(0,2)}[x, y]\left(1 - \text{phase}^{(0,1)}[x, y]^2 - \text{phase}^{(1,0)}[x, y]^2\right) + $$
$$\text{phase}^{(0,1)}[x, y]\left(-2\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(0,2)}[x, y] - 2\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(1,1)}[x, y]\right) + $$
$$\left(1 - \text{phase}^{(0,1)}[x, y]^2 - \text{phase}^{(1,0)}[x, y]^2\right)\text{phase}^{(2,0)}[x, y] + $$
$$\text{phase}^{(1,0)}[x, y]\left(-2\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(1,1)}[x, y] - 2\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(2,0)}[x, y]\right)$$

In[24]:=
```
lap = Laplacian[phase, {x, y}]
```

Out[24]=

$$0$$

In[25]:=
```
lap = Laplacian[phase[x, y], {x, y}]
```

Out[25]=

$$\text{phase}^{(0,2)}[x, y] + \text{phase}^{(2,0)}[x, y]$$

```
biharm = Laplacian[lap, {x, y}]
```

Out[26]=

$$\text{phase}^{(0,4)}[x, y] + 2\,\text{phase}^{(2,2)}[x, y] + \text{phase}^{(4,0)}[x, y]$$

In[27]:=
```
RHS = -2 * div - biharm
```

Out[27]=

$$-\text{phase}^{(0,4)}[x, y] - 2\left(\text{phase}^{(0,2)}[x, y]\left(1 - \text{phase}^{(0,1)}[x, y]^2 - \text{phase}^{(1,0)}[x, y]^2\right) + \right.$$
$$\text{phase}^{(0,1)}[x, y]\left(-2\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(0,2)}[x, y] - 2\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(1,1)}[x, y]\right) + $$
$$\left(1 - \text{phase}^{(0,1)}[x, y]^2 - \text{phase}^{(1,0)}[x, y]^2\right)\text{phase}^{(2,0)}[x, y] + $$
$$\left.\text{phase}^{(1,0)}[x, y]\left(-2\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(1,1)}[x, y] - 2\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(2,0)}[x, y]\right)\right) - $$
$$2\,\text{phase}^{(2,2)}[x, y] - \text{phase}^{(4,0)}[x, y]$$

In[28]:=
```
Simplify[RHS]
```

Out[28]=

$$-\text{phase}^{(0,4)}[x, y] + 2\,\text{phase}^{(0,2)}[x, y]\left(-1 + 3\,\text{phase}^{(0,1)}[x, y]^2 + \text{phase}^{(1,0)}[x, y]^2\right) + $$
$$8\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(1,1)}[x, y] - $$
$$2\,\text{phase}^{(2,0)}[x, y] + 2\,\text{phase}^{(0,1)}[x, y]^2\,\text{phase}^{(2,0)}[x, y] + $$
$$6\,\text{phase}^{(1,0)}[x, y]^2\,\text{phase}^{(2,0)}[x, y] - 2\,\text{phase}^{(2,2)}[x, y] - \text{phase}^{(4,0)}[x, y]$$

In[29]:=
```
Expand[%28]
```

Out[29]=

$$-2\,\text{phase}^{(0,2)}[x, y] + 6\,\text{phase}^{(0,1)}[x, y]^2\,\text{phase}^{(0,2)}[x, y] - \text{phase}^{(0,4)}[x, y] + $$
$$2\,\text{phase}^{(0,2)}[x, y]\,\text{phase}^{(1,0)}[x, y]^2 + 8\,\text{phase}^{(0,1)}[x, y]\,\text{phase}^{(1,0)}[x, y]\,\text{phase}^{(1,1)}[x, y] - $$
$$2\,\text{phase}^{(2,0)}[x, y] + 2\,\text{phase}^{(0,1)}[x, y]^2\,\text{phase}^{(2,0)}[x, y] + $$
$$6\,\text{phase}^{(1,0)}[x, y]^2\,\text{phase}^{(2,0)}[x, y] - 2\,\text{phase}^{(2,2)}[x, y] - \text{phase}^{(4,0)}[x, y]$$