In Search of a Model that Mimics Rayleigh-Benard Convection

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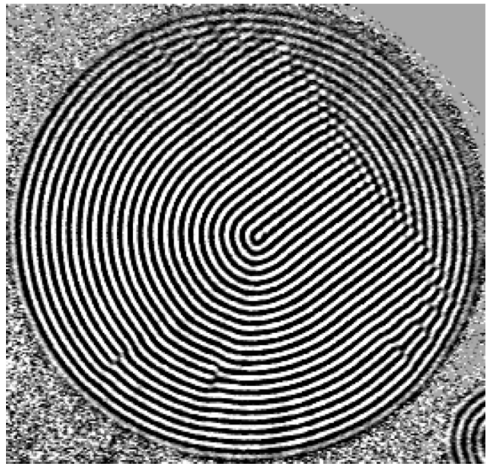
Pattern Forming Properties of Rayleigh-Benard Convection



- Experimentally, Rayleigh-Benard Convection is observed by trapping a thin section of fluid between two plates, and heating the bottom plate.
- There is a parameter, R, the Rayleigh number, with an associated critical value R_c. When $R > R_c$, the fluid is set in motion, and convection rolls emerge.
- Such convection rolls are known to form an array of patterns, as well as patten defects.

The Swift-Hohenberg Equation

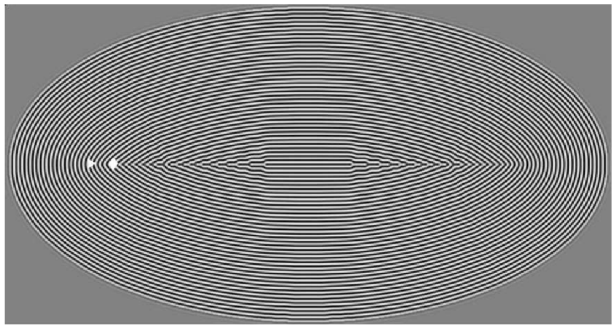


Figure 1: Numerical Simulation of SH

- $$w_t = -(1+\Delta)^2 w + Rw - w^3,$$

  where $w : \mathbb{R}^2 \to R$ represents advected temperature of *Boussinesq* equations.
- Swift-Hohenberg is known for its pattern forming behaviors, and replicates many of the patterns and pattern defects observed in experiments.
- We would like to have simpler models that capture the same pattern forming properties observed in experiments.

The Cross-Newell (and Regularized Cross-Newell) Equations

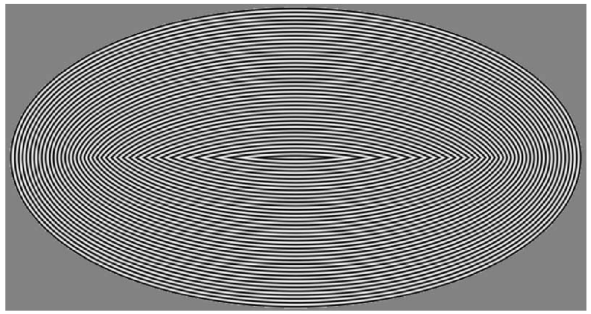


Figure 2: Numerical Simulation of CN

- Cross-Newell: $\Theta_T - kD_\perp(k)\nabla \cdot \vec{k} - D_{||}(k)\vec{k} \cdot \nabla k = 0.$
- Regularized Cross-Newell: $\tau(k)\Theta_T + \nabla \cdot \vec{k}B(k) + \eta\epsilon^2\nabla^4\Theta = 0$
- CN and RCN are derived assuming *the absence* of defects. (Or the absence of certain kinds of defects).
- The research goal is to find a way to modify CN/RCN in such a way that all defect types are possible.

Goal for the Semester

- Use PIML to find a model that captures the defects observed in Swift-Hohenberg simulations
- I will work on finding a way to numerically simulate Swift-Hohenberg. I know this is possible, since I have seen the simulations. My advisor knows where to find the method.
- It is my hope, that as long as there is data, PIML will output something useful.
- In parallel, I want to understand the CN/RCN derivations.