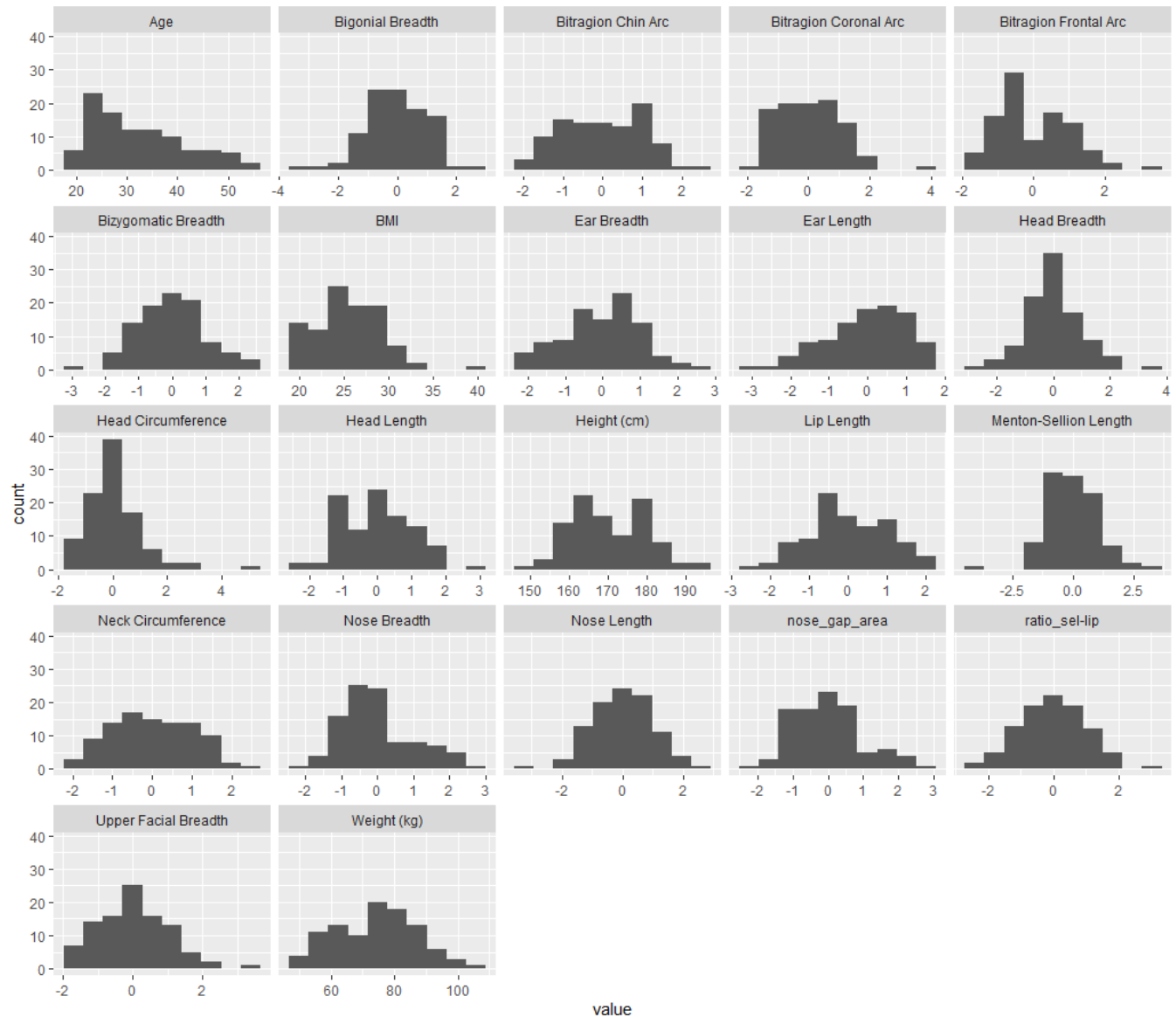


# Face Fit Exploratory Analysis and PCA

## Variable distribution



Most look normal.

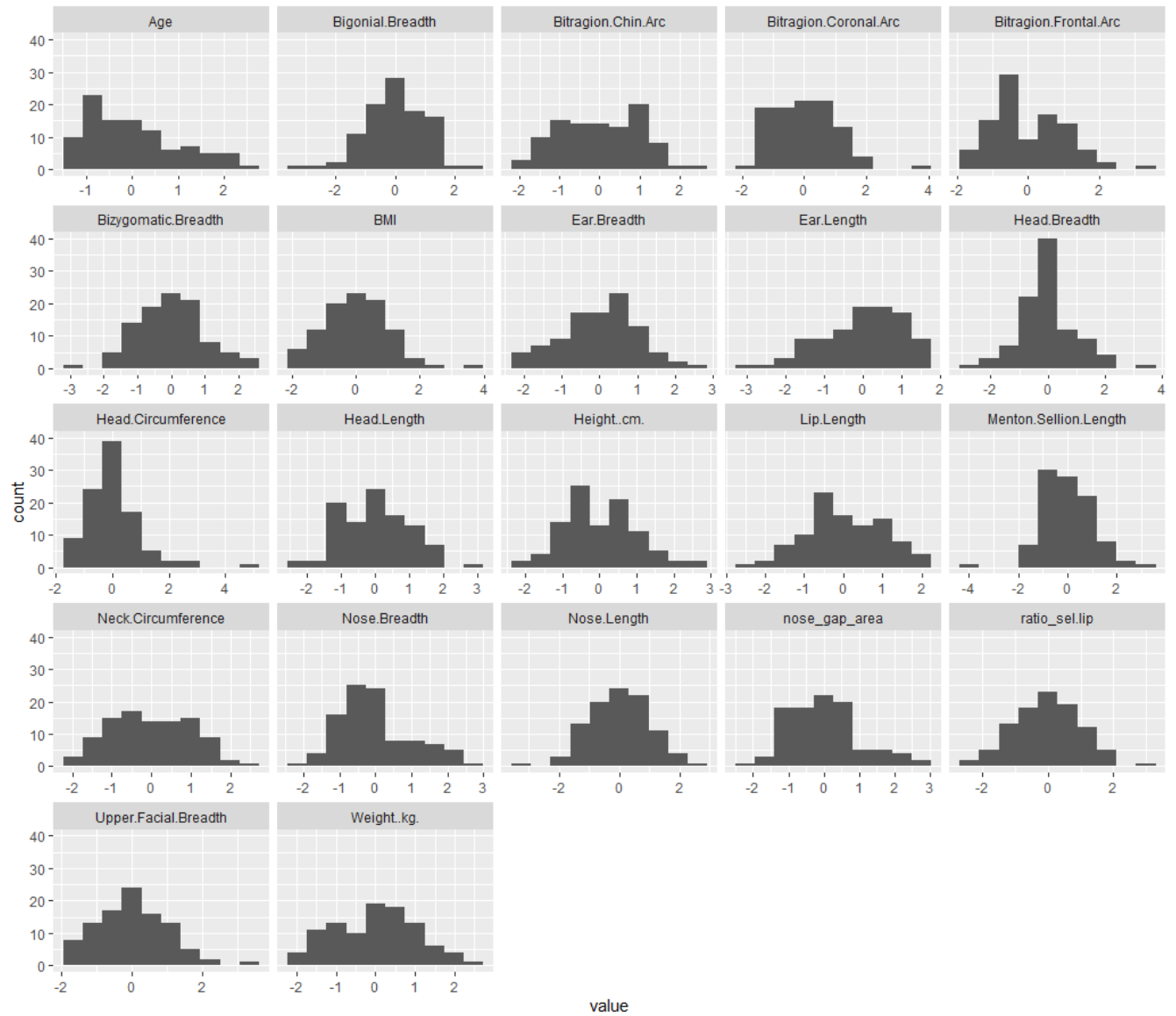
Right-skew for Age, head circumference and nose breadth. May need a transformation

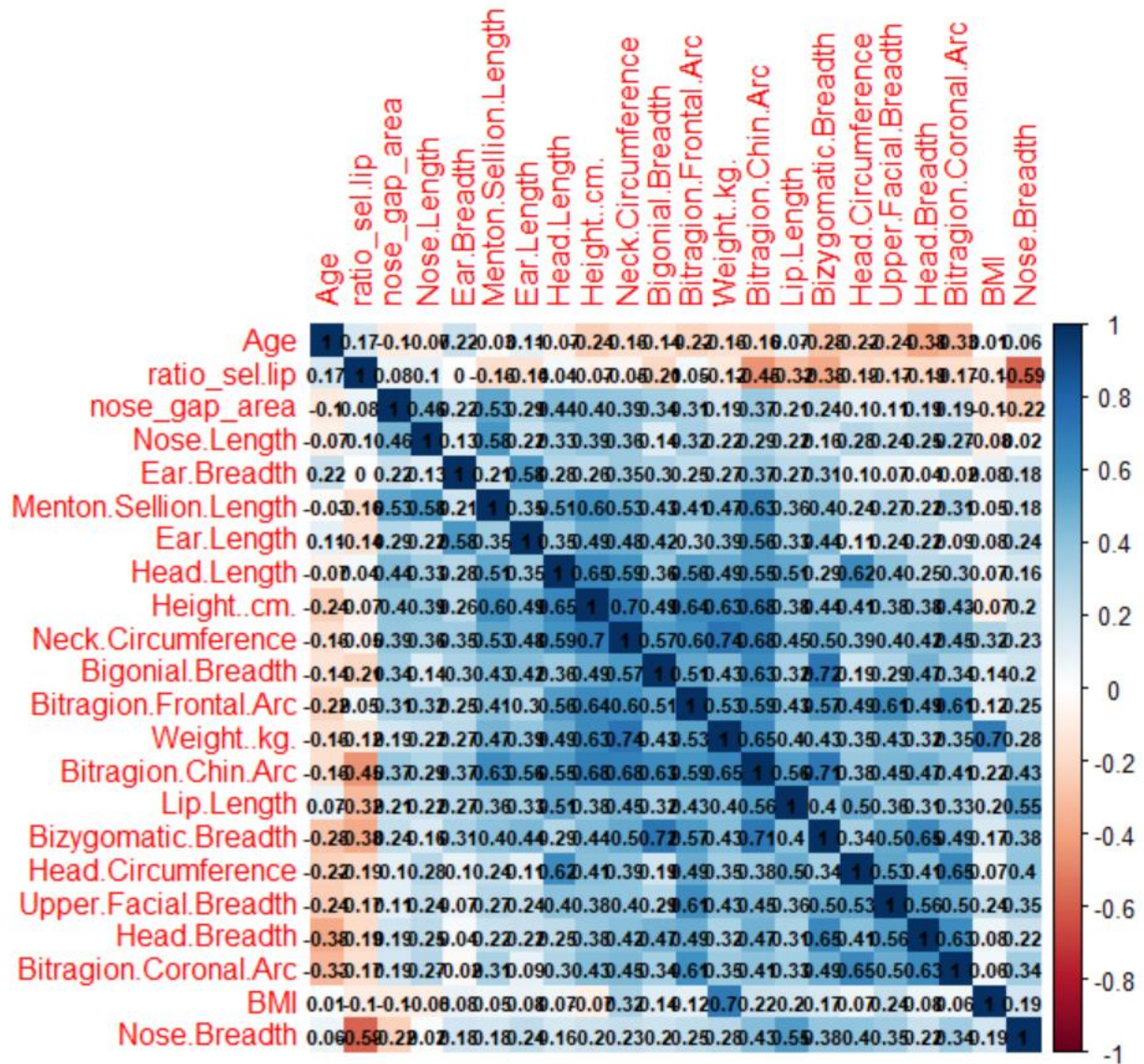
Left-skew for ear length

Uniform distribution for bitragion measurements and possibly neck circumference and nose\_gap\_area.

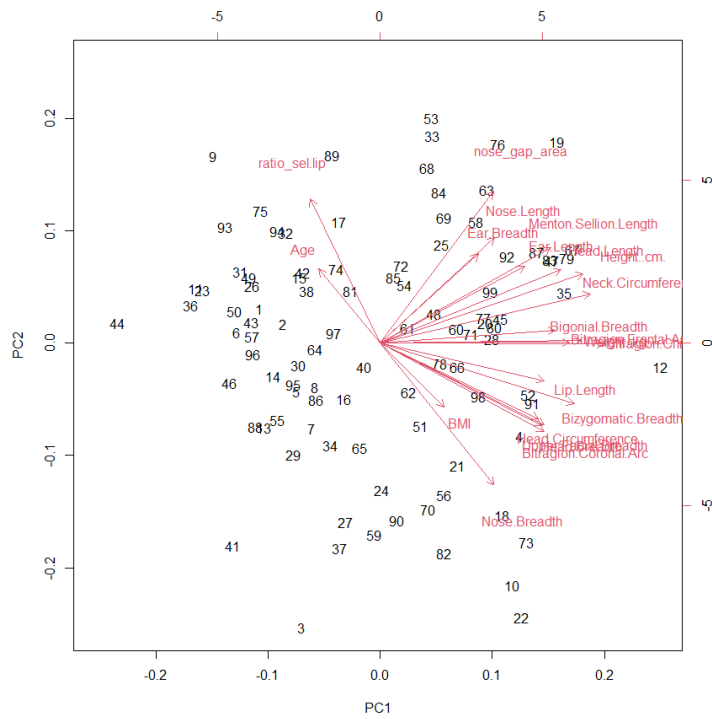
Weight, Age, BMI, and height need to be scaled and centered.

With scaling for all variables, include outliers



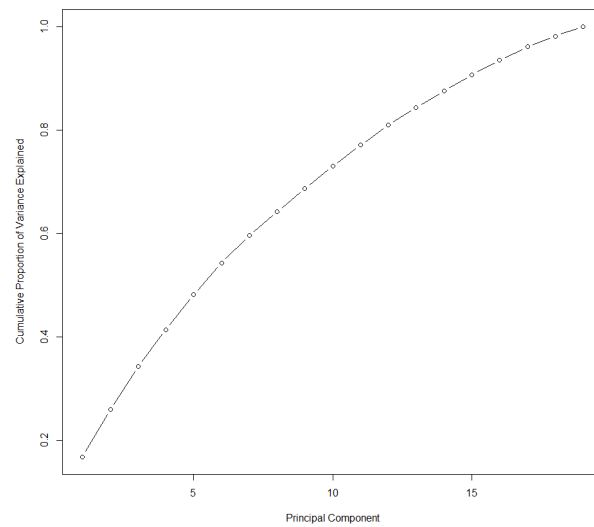
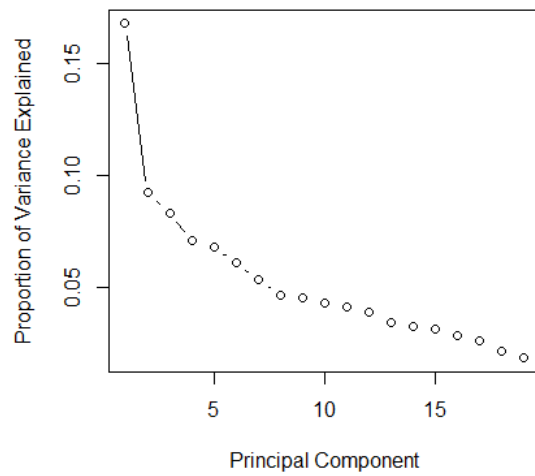


PCA (98 observations)



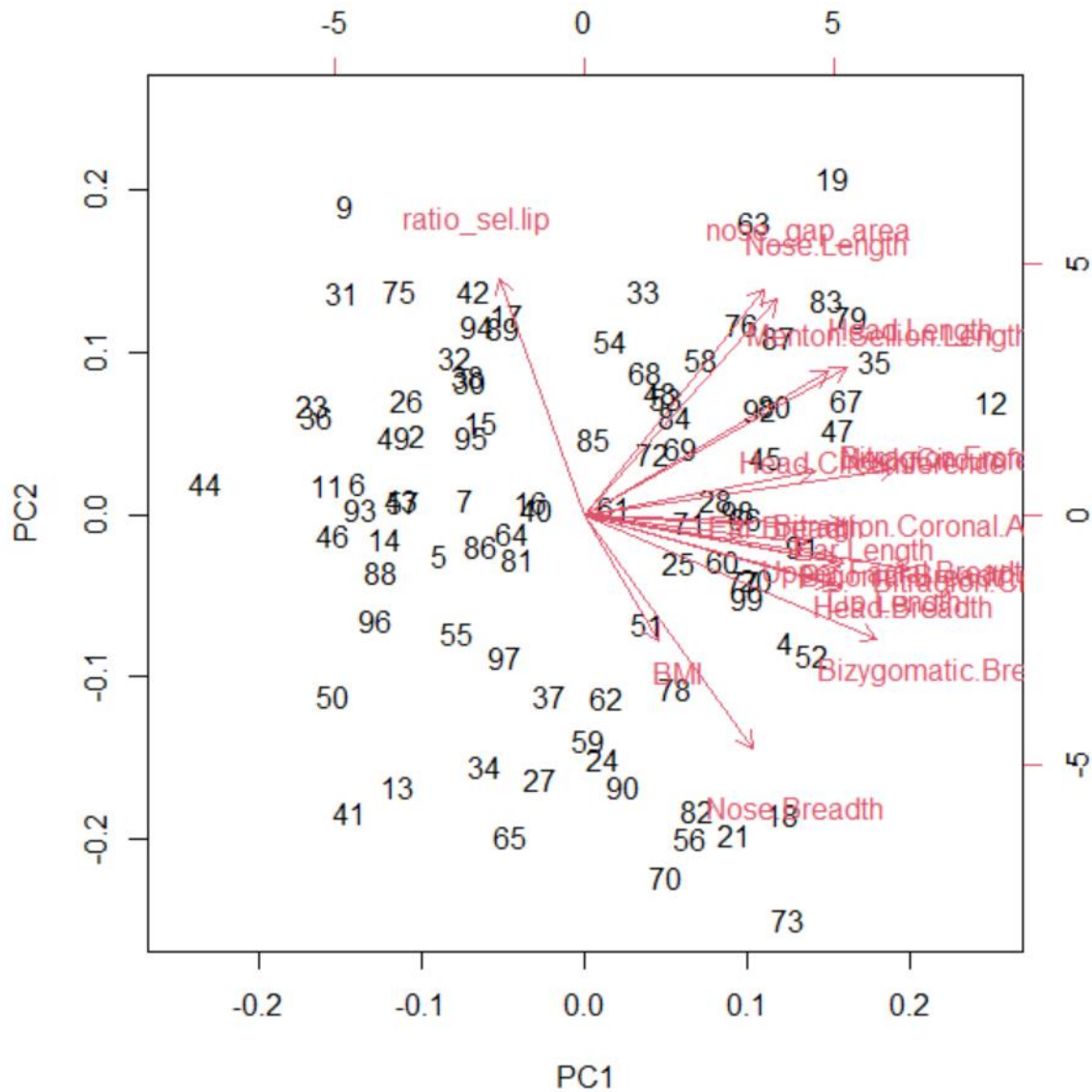
Remove some variables





```
> cumsum(pve)
[1] 0.1680199 0.2601842 0.3434930 0.4141386 0.4818818 0.5427806 0.595
9651 0.6422753 0.6873826 0.7300385 0.7710589 0.8098531 0.8436037 0.876
1122 0.9071149
[16] 0.9351058 0.9610018 0.9819793 1.0000000
```

PCA with outliers removed (91)

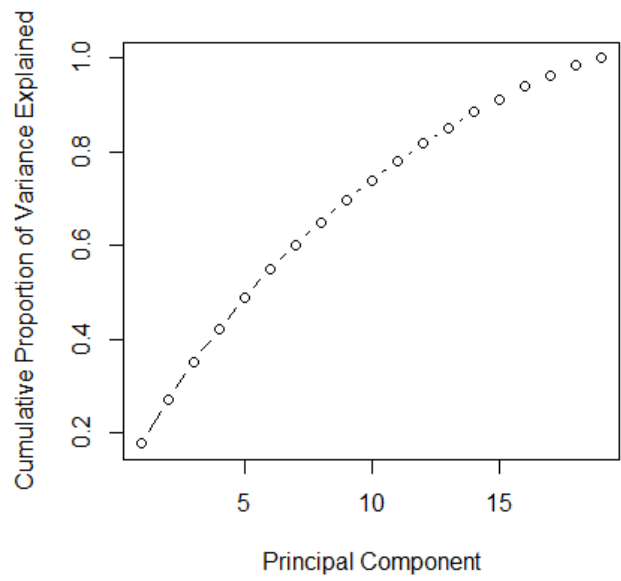
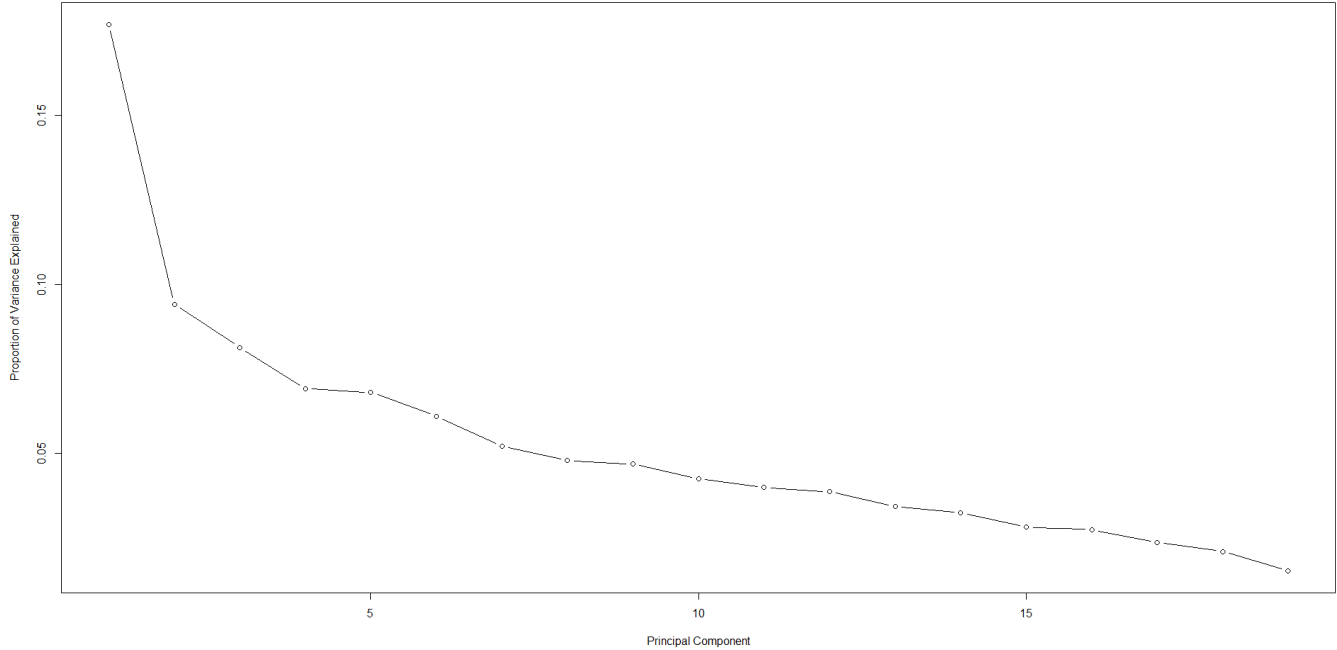


```
Standard deviations (1, ..., p=19):
[1] 2.7244968 1.4489630 1.2528949 1.0641833 1.0471490 0.9390362 0.8015775 0.7373125 0.7213920 0.6533415 0.6144736 0.5949551 0.52
73393 0.4998791 0.4332635
[16] 0.4215612 0.3637535 0.3221182 0.2343555

Rotation (n x k) = (19 x 19):
```

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
PC9	0.31298344	-0.09814171	0.163822697	-0.13376304	0.002499892	-0.122417142	7.354774e-02	-0.137403199
PC10	-0.096140953	0.25132128	-0.01721176	-0.343842168	-0.03552463	-0.009032435	0.063761370	5.779092e-02
Bitragion.Chin.Arc	0.29963269	0.08150416	-0.155855304	0.15240708	-0.146082882	0.223174780	-1.252609e-01	0.176005479
Bitragion.Coronal.Arc	0.22185036	0.07814445	-0.127505551	0.26480324	0.160997939	0.170956111	1.202745e-01	-0.042093005
834478	0.29914197	0.08337936	0.125790109	0.12499923	-0.212931175	-0.225228198	1.640220e-01	0.112746147
Bitragion.Frontal.Arc	0.15540211	-0.01691726	0.498247176	-0.01458845	-0.236258850	0.341399889	-2.370727e-01	0.200012750
813849	0.21438831	-0.05335268	0.409425027	-0.15744998	-0.100846989	0.130862205	-2.849413e-01	-0.335939315
Head.Circumference	0.23932516	-0.12648587	0.132688821	0.32938490	0.366224639	0.085575203	2.105844e-01	0.159313700
692537	0.24374687	-0.08384045	-0.287004230	0.17805184	-0.007852332	0.008378773	-4.833396e-01	-0.488829670
Neck.Circumference	0.23488541	0.25824145	0.117932894	-0.16604879	0.121415840	-0.299013377	-1.445071e-01	0.071441906
211530	0.16195952	-0.42250319	0.143837209	0.08477476	0.399697717	0.020606819	-1.109800e-01	0.279405687
Ear.Breadth	0.18583154	0.38943790	-0.035017875	-0.17136667	0.287865852	-0.326486241	-3.930305e-01	0.348260286
619706	-0.135467215							

Bigonial.Breadth	0.24899605	-0.08918309	0.006284172	-0.25560361	-0.319030321	0.017669679	3.662975e-01	0.197347343	0.29
761745	-0.016262919								
Bizygomatic.Breadth	0.28126158	-0.22498141	-0.120138955	-0.27203608	-0.223217535	0.096432085	-5.587082e-05	0.016847747	0.04
221360	0.038168034								
Head.Breadth	0.24684334	-0.13104936	-0.456439742	-0.17382455	-0.010960759	0.099478735	-2.678847e-02	-0.025811680	-0.37
788569	-0.366507251								
Head.Length	0.25250936	0.26748857	0.117740975	0.35852124	0.144874670	0.155685793	2.500804e-01	-0.274402274	0.24
028322	-0.178985099								
nose_gap_area	0.17296253	0.40678658	0.039242246	-0.21240039	0.043135070	-0.115586794	3.223231e-01	-0.292541342	-0.36
075726	0.541321179								
ratio_sel.lip	-0.08225021	0.42736952	-0.088348112	0.29283476	-0.345251054	0.266344946	-1.683478e-01	0.258162636	0.02
095823	-0.117900999								
BMI	0.07196002	-0.22789456	0.029397350	0.46215894	-0.394968942	-0.621001935	-3.868785e-02	0.006791015	-0.23
730589	0.083347097								



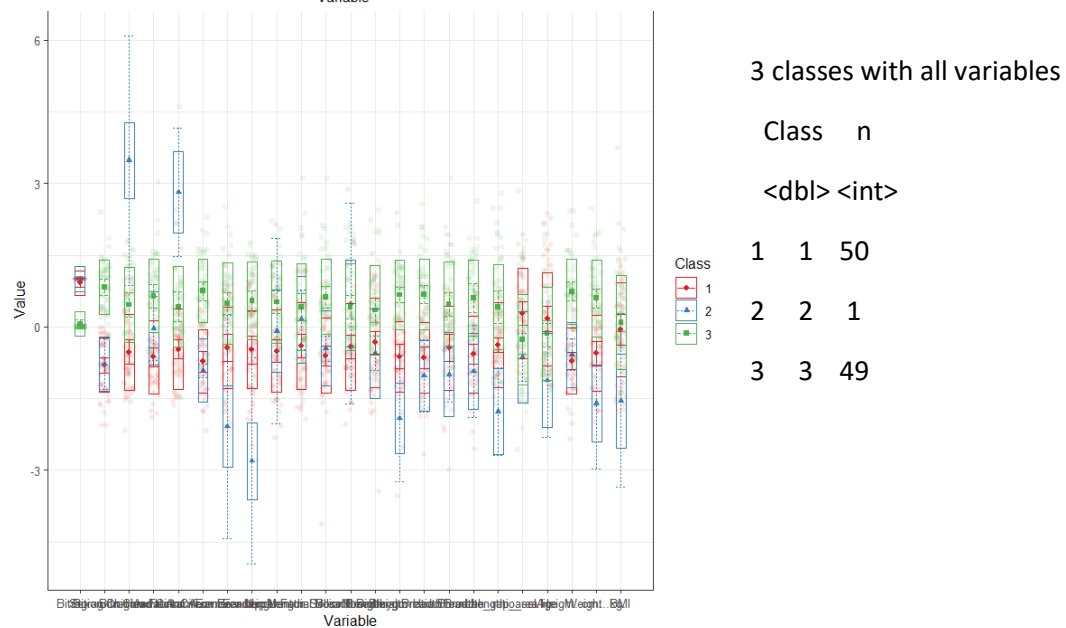
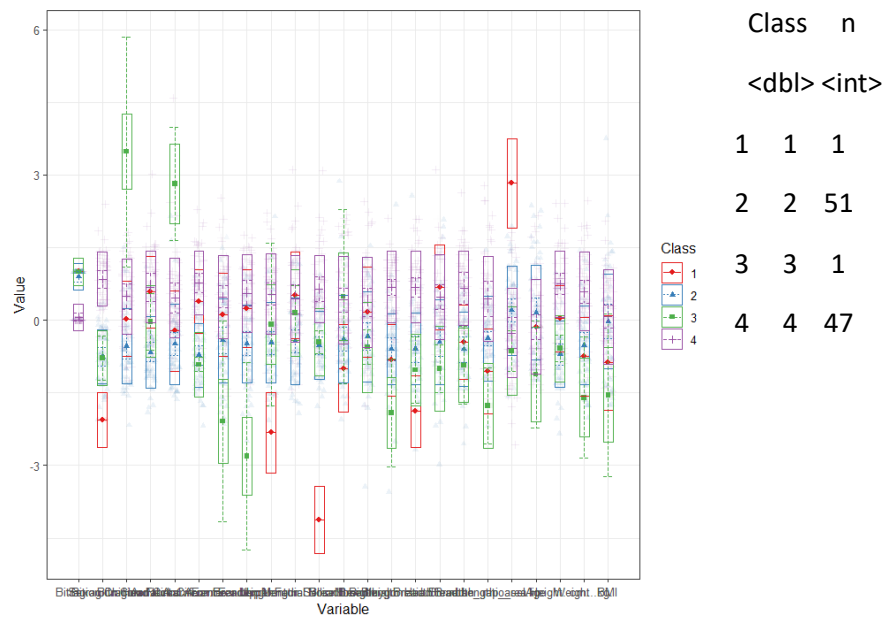


```
> cumsum(pve)
[1] 0.1768919 0.2709679 0.3523139 0.4214075 0.4893952 0.5503635 0.602
4070 0.6502781 0.6971155 0.7395347 0.7794303 0.8180586 0.8522968 0.884
7522 0.9128825
[16] 0.9402529 0.9638701 0.9847841 1.0000000
```

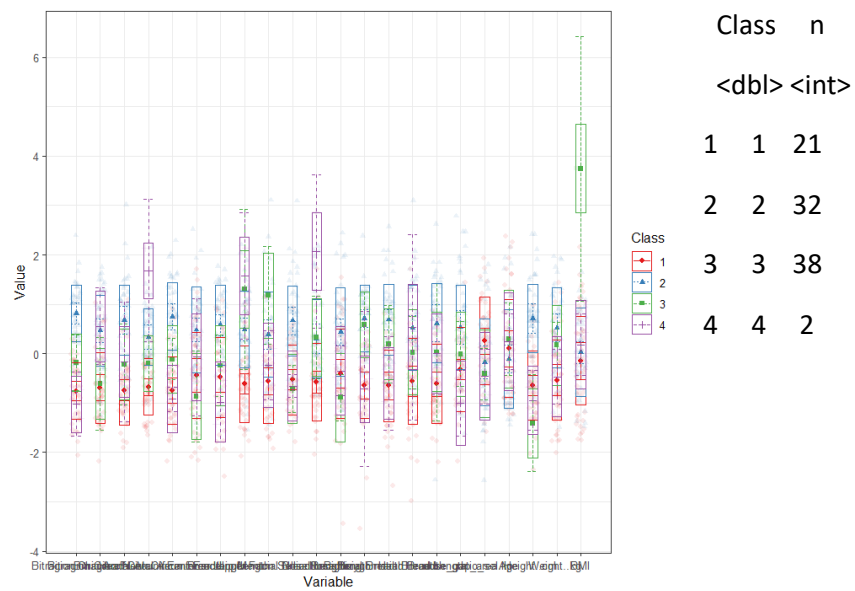
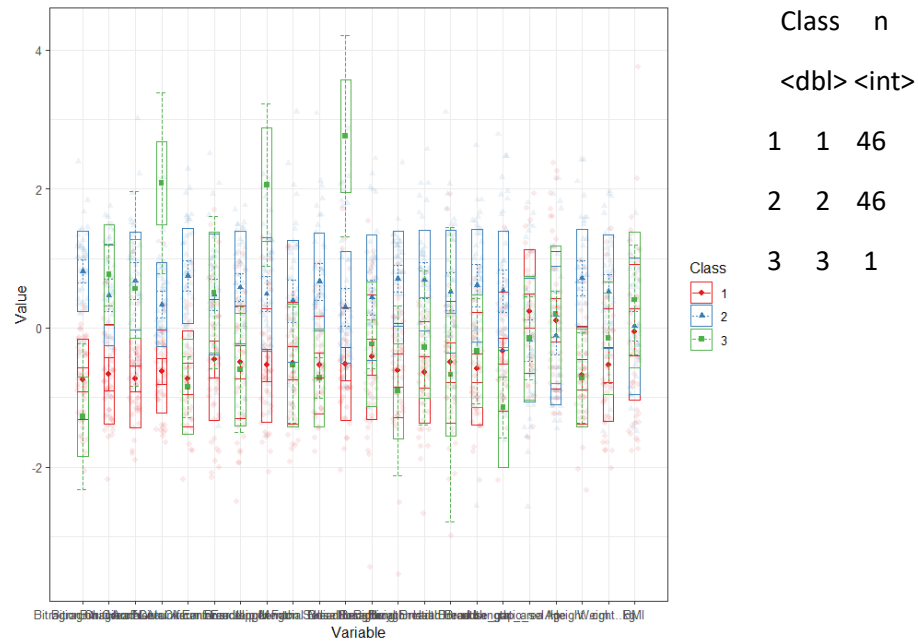
Factor Analysis, LCA, Bayesian profile regression, hierarchical clustering, k-means?

## LCA

Example output with 4 classes with all variables



Without 7 outliers and sex



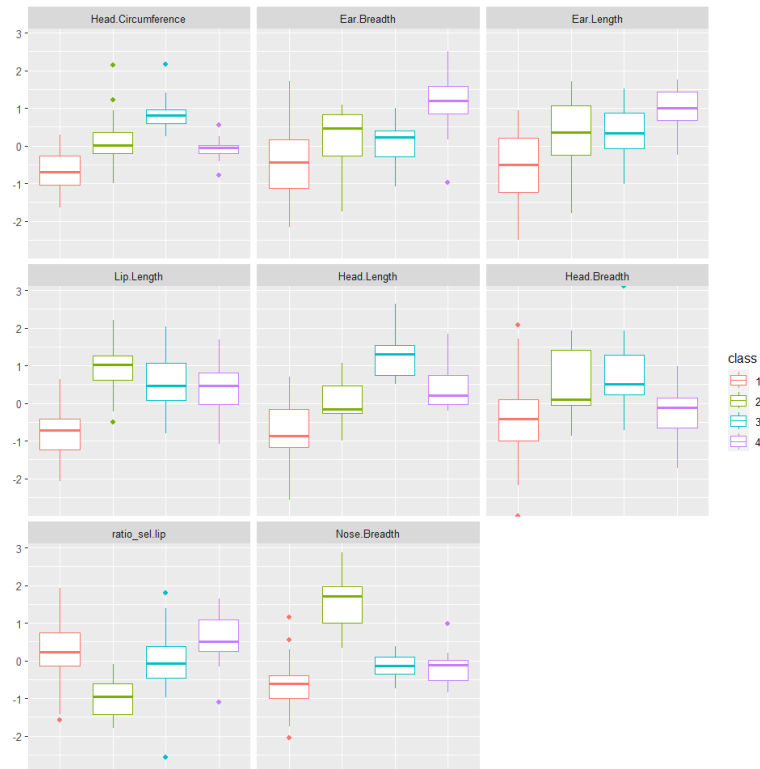
Compare tidyLPA solutions:

Model	Classes	AIC	BIC
1	2	5128.994	5298.678
1	3	5031.592	5259.526
1	4	5011.852	5298.036
1	5	5047.539	5391.973

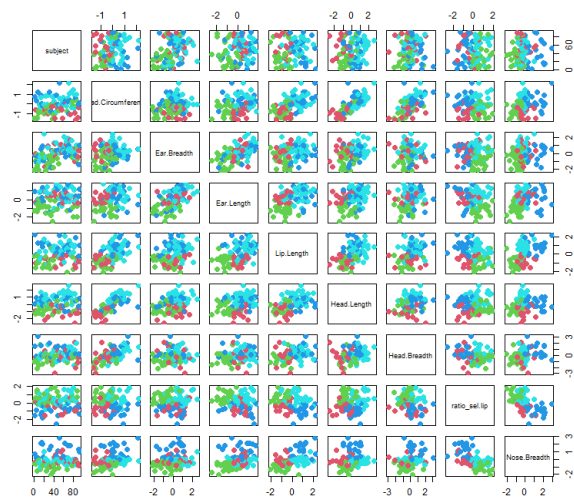
Best model according to AIC is Model 1 with 4 classes.

Best model according to BIC is Model 1 with 3 classes.

With smaller subset of variables



## Kmeans clustering



K = 4, nstart = 20

km.out\$tot.withinss = 332.5704 (within-cluster sum of squares)

Within cluster sum of squares by cluster:

[1] 62.21006 79.89359 89.98450 100.48230

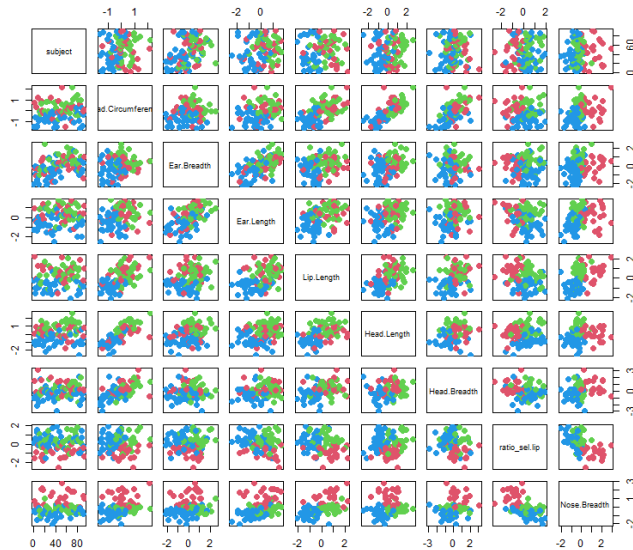
(between\_SS / total\_SS = 46.2 %)

(sum of squared distances of centroid mean of groups)/(sum of square distances of each data point to the global mean)

means clustering with 4 clusters of sizes 16, 26, 21, 29

## Cluster means:

	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
1	0.29172761	0.2648979	-0.7057184	-1.0131655	-0.1919569	-0.5646799	-0.1227434
2	-0.78499742	-0.9796451	-0.8448185	-0.5940373	-0.5632204	0.6882322	-0.9121198
3	-0.01488748	0.2722193	0.9067729	0.2267173	0.6520617	-0.9362417	1.2002521
4	0.64241935	0.6500054	0.4518716	0.9615764	0.1473800	0.4598564	-0.2573451



K=3, nstart = 20

km.out\$tot.withinss = 372.8572

Within cluster sum of squares by cluster:

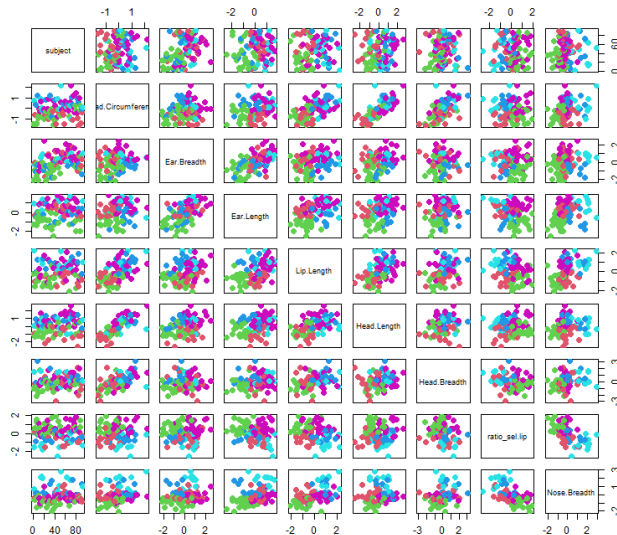
[1] 107.9379 104.9004 160.0188

(between\_SS / total\_SS = 39.7 %)

K-means clustering with 3 clusters of sizes 24, 30, 38

Cluster means:

	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
1	0.02711945	0.2649385	0.6818385	0.1172631	0.6193224	-0.9545004	1.1617096
2	0.65819053	0.6539821	0.4319330	0.9286597	0.1701395	0.4074541	-0.2656344
3	-0.46898289	-0.5958852	-0.8008522	-0.7811301	-0.5188328	0.3478502	-0.7328612



K= 5 , nstart = 20

km.out\$tot.withinss = 306.874

Within cluster sum of squares by cluster:

[1] 65.41579 79.89359 42.59014 35.53327 83.44119

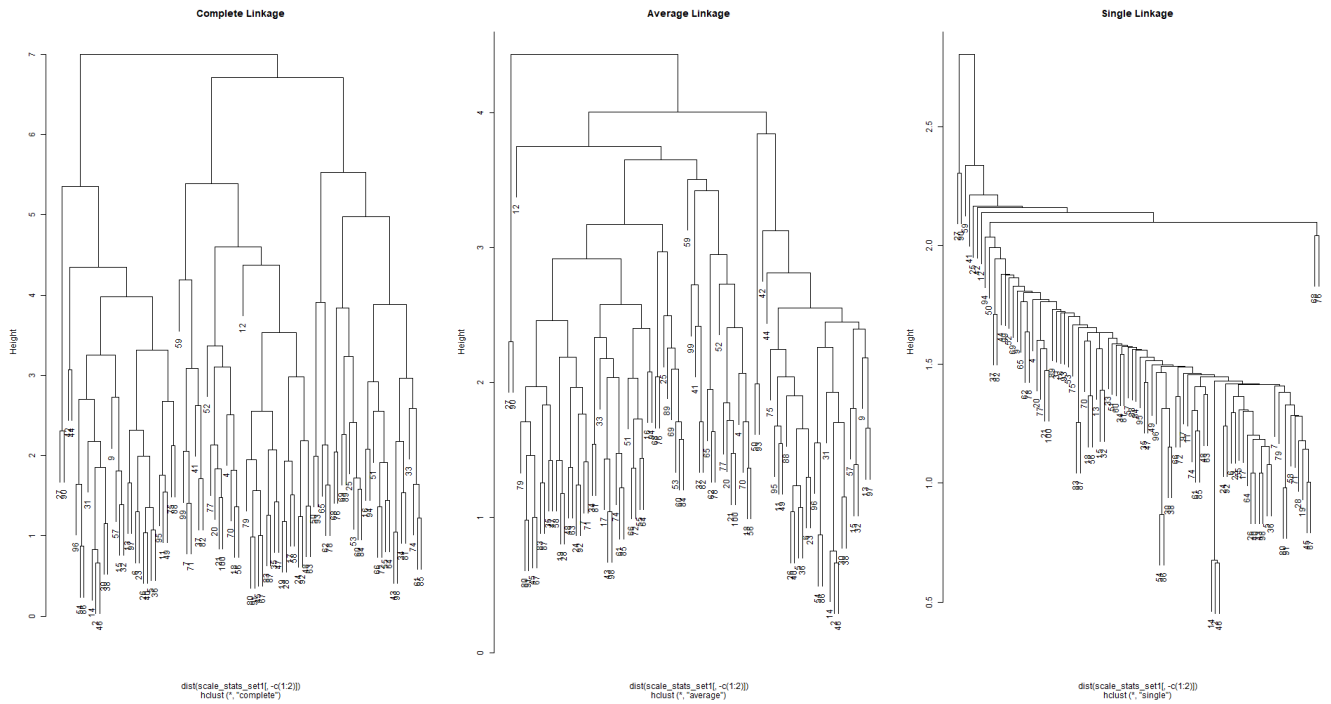
(between\_SS / total\_SS = 50.4 %)

K-means clustering with 5 clusters of sizes 17, 26, 14, 11, 24

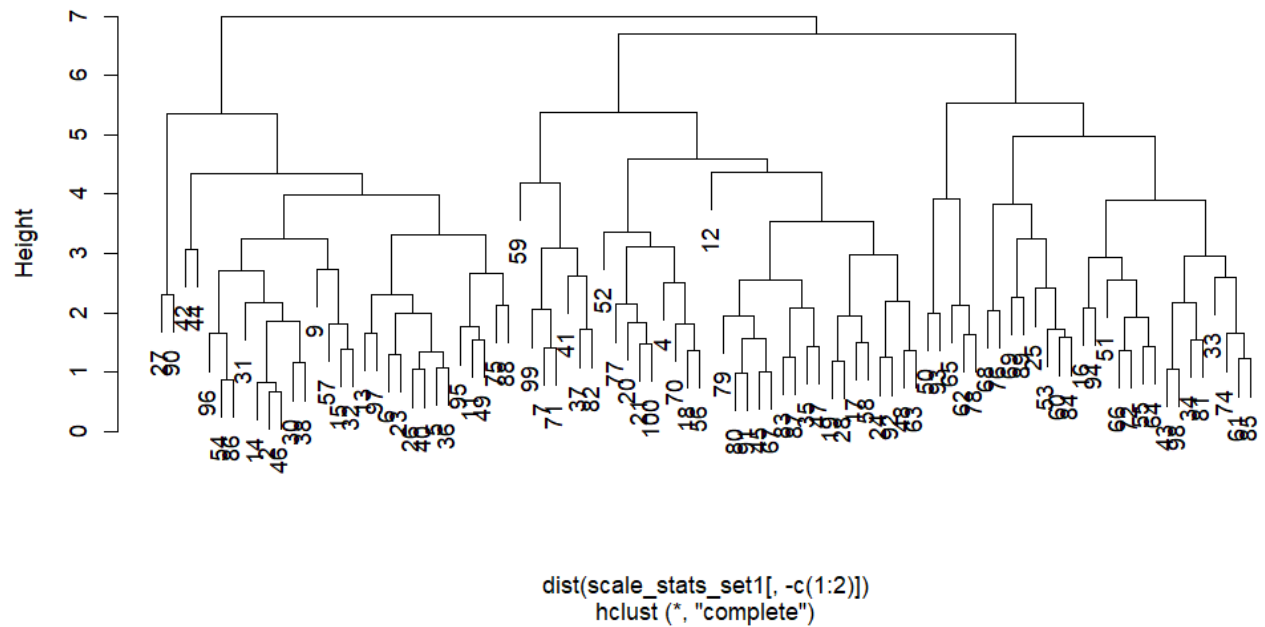
Cluster means:

	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
1	0.3017580	0.2695526	-0.6660923	-0.9692783	-0.10594893	-0.5666258	-0.09506646
2	-0.7849974	-0.9796451	-0.8448185	-0.5940373	-0.56322036	0.6882322	-0.91211979
3	-0.5130530	-0.3820276	0.7992453	0.6803242	0.58396397	-0.4475035	0.46103420
4	0.5717324	0.8774907	1.0607625	0.1549441	0.57015394	-1.1934750	1.63569140
5	0.7812068	0.8299467	0.3883651	0.9035390	0.09374783	0.5694082	-0.29385679

## Hierarchical Clustering



## Complete Linkage

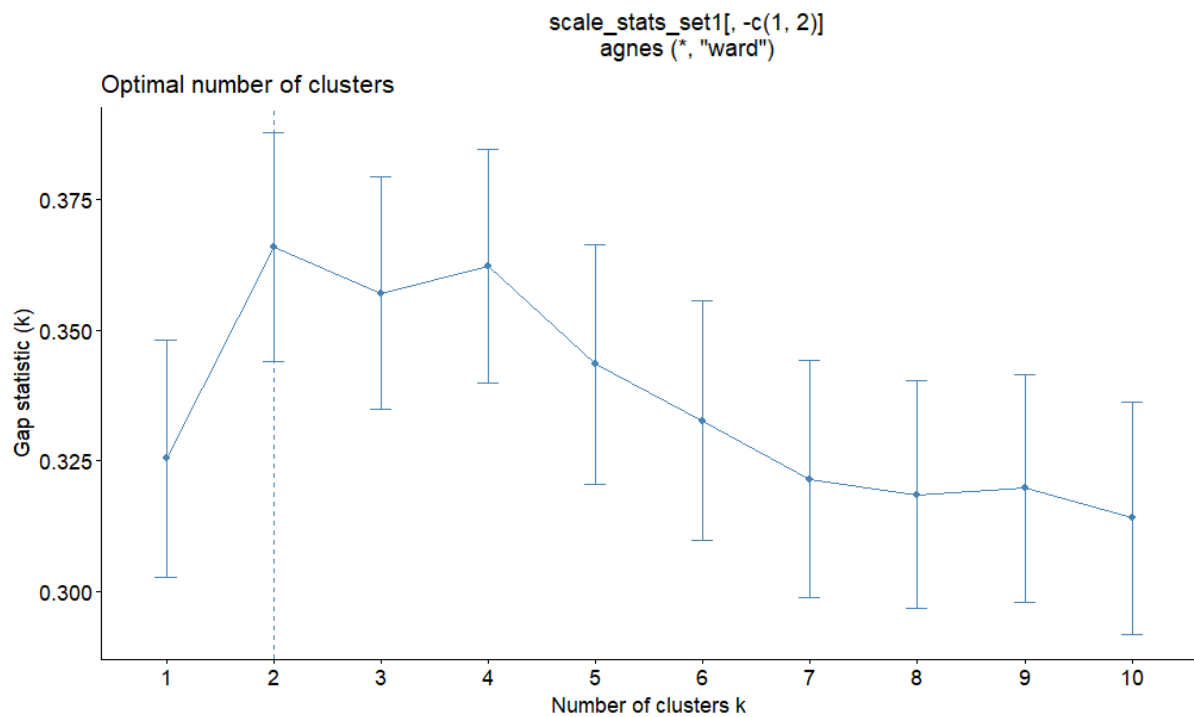
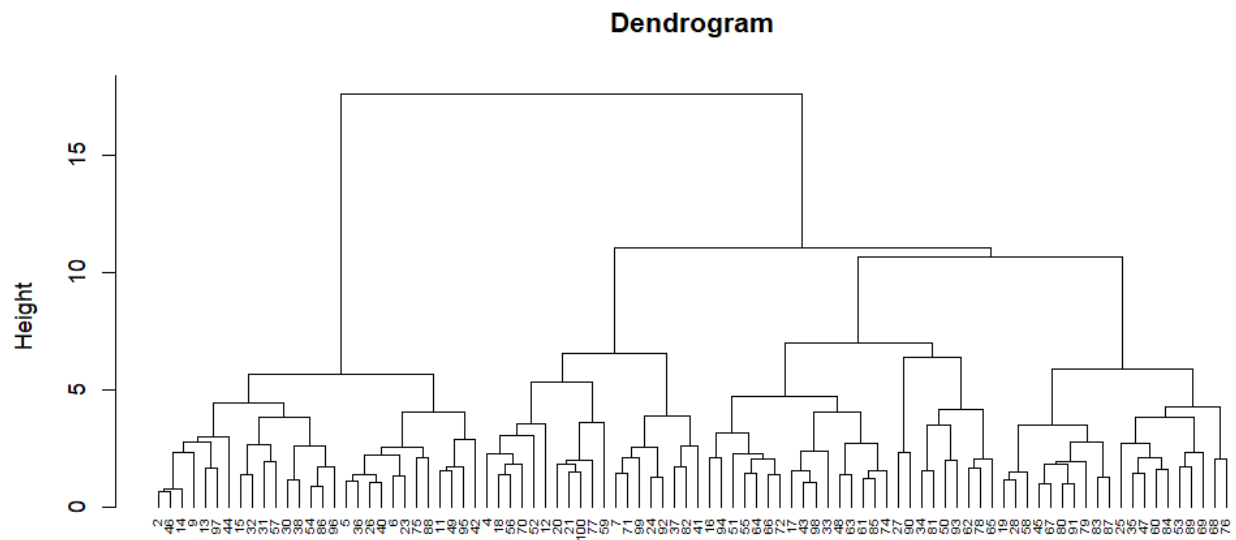


Cut at 6 clusters?

Another method

average single complete ward  
0.6319527 0.4735829 0.7583956 0.9063407

Since Ward's method is closest to one it does the best job of putting the data in strong clusters



Two clusters is the optimal, but 4 is also strong.

```
groups
1 2 3 4
28 19 25 20
```

cluster	Head.Circumference	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
1	-0.6848510	-0.76737710	-0.8861073	-0.8068998	-0.6008282	-0.64441377	0.6383809	-0.8463593
2	0.4122909	-0.09568445	0.2807804	0.9867346	0.3167739	0.66805222	-0.9530946	1.2119184
3	-0.4116520	0.45355312	0.2231551	-0.4336097	-0.4961828	0.07920286	-0.2374679	-0.1519401
4	0.5334525	0.72704842	0.8615826	0.6787605	1.2100108	0.18113988	0.4352375	-0.1733304

## Compare Methods

Are clusters/classes/groups meaningful?

## Factor Analysis



— Kaiser-Meyer-Olkin criterion (KMO)

! The overall KMO value for your data is mediocre.  
These data are probably suitable for factor analysis.

Overall: 0.656

For each variable:

Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
0.633	0.702	0.693	0.658	0.787	0.547	0.632

Call:

```
factanal(x = scale_stats_set1[, -c(1, 2)], factors = 3, n.obs = 7776, scores = "Bartlett")
```

Uniquenesses:

Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip	Nose.Breadth
0.556	0.287	0.005	0.623	0.872	0.005	0.484

Loadings:

	Factor1	Factor2	Factor3
Ear.Breadth		0.171	0.644
Ear.Length	0.221	0.181	0.795
Lip.Length	0.274	0.942	0.179
Head.Length		0.496	0.352
Head.Breadth		0.244	0.111
ratio_sel.lip	0.237		
Nose.Breadth	-0.997	0.591	0.379

	Factor1	Factor2	Factor3
SS loadings	1.530	1.401	1.237
Proportion Var	0.219	0.200	0.177
Cumulative Var	0.219	0.419	0.595

Test of the hypothesis that 3 factors are sufficient.  
The chi square statistic is 4.1 on 3 degrees of freedom.  
The p-value is 0.251

### 3 Factors

Uniquenesses:

Head.Circumference	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip
0.005	0.498	0.492	0.460	0.285	0.591	0.439
Nose.Breadth						
0.302						

Loadings:

	Factor1	Factor2	Factor3
Head.Circumference	0.977		0.195
Ear.Breadth			0.703
Ear.Length	0.147	0.201	0.668
Lip.Length	0.507	0.377	0.375
Head.Length	0.722	-0.144	0.415
Head.Breadth	0.591	0.235	
ratio_sel.lip		-0.749	
Nose.Breadth	0.223	0.767	0.245

	Factor1	Factor2	Factor3
SS loadings	2.161	1.410	1.356
Proportion Var	0.270	0.176	0.170
Cumulative Var	0.270	0.446	0.616

Test of the hypothesis that 3 factors are sufficient.

The chi square statistic is 25.56 on 7 degrees of freedom.

The p-value is 0.000604 (significant p-value indicates more factors are needed)

### 4 Factors

Uniquenesses:

Head.Circumference	Ear.Breadth	Ear.Length	Lip.Length	Head.Length	Head.Breadth	ratio_sel.lip
0.066	0.669	0.005	0.464	0.198	0.155	0.571
Nose.Breadth						
0.005						

Loadings:

	Factor1	Factor2	Factor3	Factor4
Head.Circumference	0.873		0.110	0.392
Ear.Breadth	0.207	0.101	0.525	
Ear.Length	0.130	0.155	0.968	0.133
Lip.Length	0.558	0.398	0.232	0.114
Head.Length	0.850		0.263	
Head.Breadth	0.254	0.165		0.868
ratio_sel.lip	0.110	-0.612	-0.110	-0.172
Nose.Breadth	0.242	0.959	0.130	

Negative numbers indicate a contrast between the variables

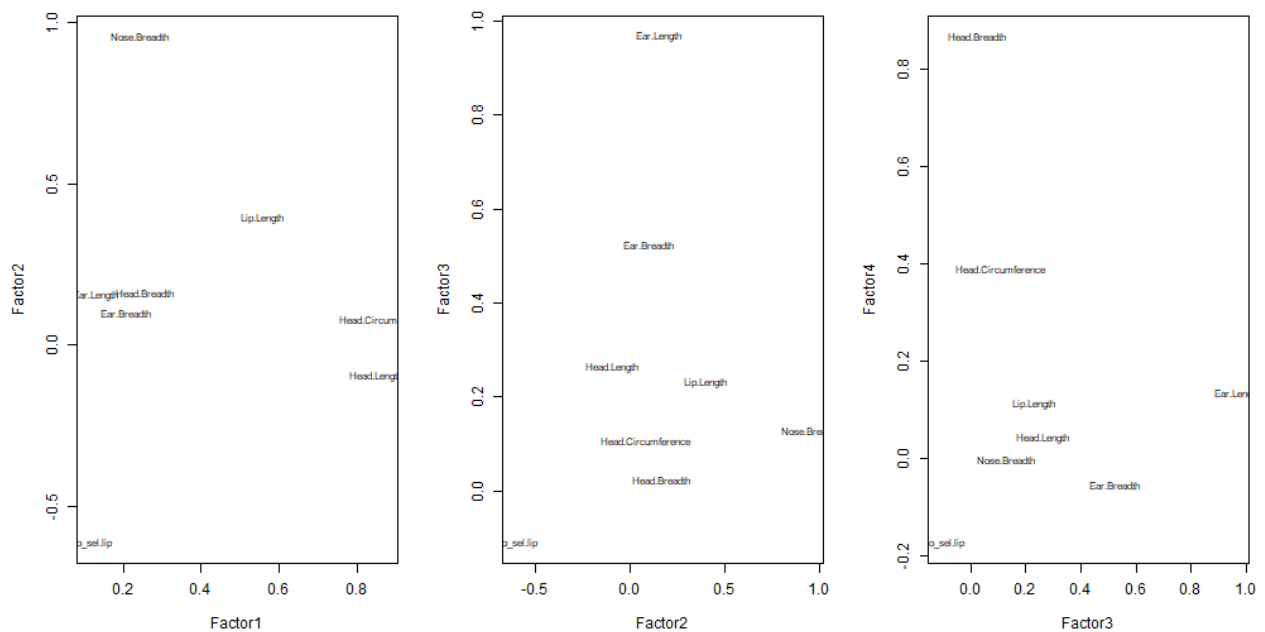
	Factor1	Factor2	Factor3	Factor4
SS loadings	1.990	1.530	1.376	0.971
Proportion Var	0.249	0.191	0.172	0.121
Cumulative Var	0.249	0.440	0.612	0.733

Test of the hypothesis that 4 factors are sufficient.

The chi square statistic is 2.56 on 2 degrees of freedom.

The p-value is 0.279





Factor loadings close to 1 or -1 indicate that the factor has a strong influence on the variable.  
Try different rotation methods to interpret factor loadings better. (varimax, equimax, quartimax)

## Compare Groupings

### Hierarchical/k-means

```
> table(scale_stats_cats$cluster,scale_stats_cats$k_clusters)
```

```
 1  2  3  4  5
1  0  1  2  5  2  0
2  9  0  0  0  1  0
3  2  6  1  1  5  1
4  3  1  7  0  0  0
```

### Hierarchical/LCA

```
> table(scale_stats_cats$cluster,scale_stats_cats$class)
```

```
 1  2  3  4
1  2  7  0  0  1
2  0  1  4  5  0
3  1  4  3  3  5
4  0  0  1  2  8
```

### LCA/k-means

```
> table(scale_stats_cats$class,scale_stats_cats$k_clusters)
```

```
 1  2  3  4  5
1  0  1  2  6  1  4  0
2  5  0  0  2  1  0
3  9  1  0  0  0  1
4  0  1  3  0  1  0
```

## Outcome Prediction

(cross validation,lowest RMSE)

**Also try supervised learning (xgboost, random forest, neural network, LDA, penalized regression to find important variables and better prediction)**

### *Hierarchical clustering*

```
glm.fit<-glm(KN95~cluster, data = complete_stats)
cv.err<-cv.glm(complete_stats, glm.fit)
cv.err$delta      #146.8020 146.7339
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-24.1474	-9.0115	0.3363	8.2062	24.8240

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	64.418	2.241	28.741	< 2e-16 ***
cluster2	8.430	3.525	2.391	0.01892 *
cluster3	2.558	3.263	0.784	0.43522
cluster4	11.362	3.472	3.272	0.00153 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 140.6621)

Null deviance: 14260 on 91 degrees of freedom  
Residual deviance: 12378 on 88 degrees of freedom  
AIC: 722.06

Number of Fisher Scoring iterations: 2

### *K-means Clustering*

```
cv.err$delta      #140.4521 140.3740
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-22.525	-8.152	-1.009	8.708	23.607

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	67.6929	3.0910	21.900	< 2e-16 ***
k_clusters2	4.6321	3.8895	1.191	0.23691
k_clusters3	-4.1775	3.8340	-1.090	0.27890
k_clusters4	0.1777	4.1741	0.043	0.96613
k_clusters5	13.1344	4.6599	2.819	0.00597 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 133.7633)

Null deviance: 14260 on 91 degrees of freedom  
Residual deviance: 11637 on 87 degrees of freedom  
AIC: 718.38

### *Factor Analysis*

```
cv.err$delta      #150.7898 150.6986
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-22.7096	-7.7240	-0.9772	8.9471	25.1268

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	69.3239	1.2441	55.724	<2e-16 ***
Factor1	2.4885	1.1970	2.079	0.0406 *
Factor2	3.1403	1.2440	2.524	0.0134 *
Factor3	1.9509	1.2458	1.566	0.1210
Factor4	0.7048	1.1400	0.618	0.5380

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 142.3876)

Null deviance: 14260 on 91 degrees of freedom  
Residual deviance: 12388 on 87 degrees of freedom  
AIC: 724.13

## Latent Class Analysis

cv.err\$delta #151.1825 151.1096

Deviance Residuals:

Min	1Q	Median	3Q	Max
-24.594	-7.605	0.000	9.218	22.000

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	64.800	1.877	34.530	<2e-16 ***
class2	8.494	3.466	2.450	0.0162 *
class3	7.130	3.277	2.175	0.0323 *
class4	9.229	3.720	2.481	0.0150 *

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 144.395)

Null deviance: 14260 on 91 degrees of freedom  
Residual deviance: 12707 on 88 degrees of freedom  
AIC: 724.47

## Supervised Learning

### Random Forest

Mean of squared residuals: 142.4891

% Var explained: 8.07

Number of trees: 500

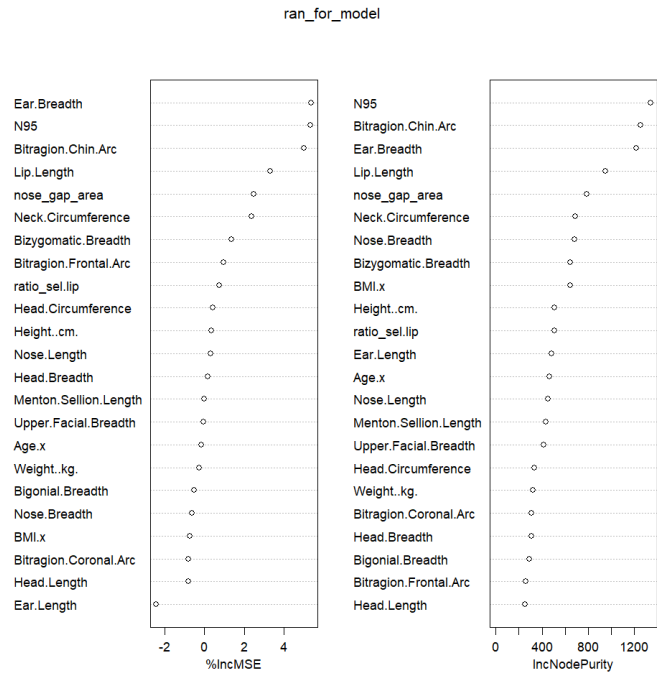
No. of variables tried at each split: 5

Mean of squared residuals: 139.0716

% Var explained: 10.27

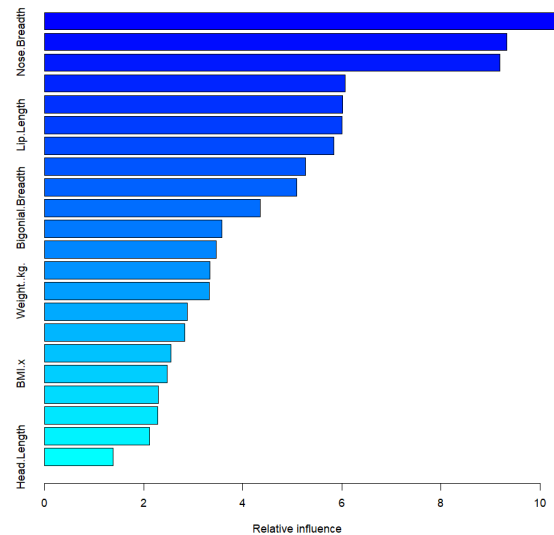
Importance indicates the total amount that the amount that the error is decreased due to splits in that variable. Larger % shows more importance because it is reducing the error (residual sum squares (RSS) or mean square error (MSE))

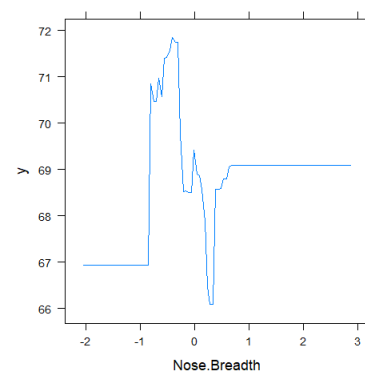
	%IncMSE	IncNodePurity
Bitracion.Chin.Arc	5.024717760	1256.9559
Bitracion.Coronal.Arc	-0.779033746	310.6126
Bitracion.Frontal.Arc	0.988387545	257.4253
Head.Circumference	0.417898491	332.1163
Neck.Circumference	2.362752430	687.5447
Ear.Breadth	5.385025178	1214.7591
Ear.Length	-2.407957275	480.8372
Lip.Length	3.304436818	951.2186
Upper.Facial.Breadth	-0.049421776	414.0601
Menton.Sellion.Length	0.005477826	433.7530
Nose.Breadth	-0.597461227	684.7406
Nose.Length	0.329356475	452.4049
Bigonial.Breadth	-0.491994626	293.6838
Bizygomatic.Breadth	1.380379078	646.9542
Head.Breadth	0.186922114	310.0315
Head.Length	-0.794316736	250.8811
nose_gap_area	2.496648011	789.2567
ratio_sel.lip	0.754019448	505.5186
Age.x	-0.135430154	462.7062
Height..cm.	0.372286170	507.9880
Weight..kg.	-0.263289913	318.6144
BMI.x	-0.736760054	642.6659
N95	5.319027997	1343.7329



## Boosting KN95

	var	rel.inf
nose_gap_area	nose_gap_area	10.305552
Nose.Breadth	Nose.Breadth	9.331326
Ear.Breadth	Ear.Breadth	9.196040
Ear.Length	Ear.Length	6.062168
Height..cm.	Height..cm.	6.015255
Lip.Length	Lip.Length	6.009020
Upper.Facial.Breadth	Upper.Facial.Breadth	5.845039
ratio_sel.lip	ratio_sel.lip	5.263364
Neck.Circumference	Neck.Circumference	5.092802
Bigonial.Breadth	Bigonial.Breadth	4.358403
Bitracion.Chin.Arc	Bitracion.Chin.Arc	3.574952
Nose.Length	Nose.Length	3.462573
Menton.Sellion.Length	Menton.Sellion.Length	3.342496
Weight..kg.	Weight..kg.	3.322764
Bizygomatic.Breadth	Bizygomatic.Breadth	2.882803
Bitracion.Coronal.Arc	Bitracion.Coronal.Arc	2.832282
Age.x	Age.x	2.545495
BMI.x	BMI.x	2.470395
Head.Breadth	Head.Breadth	2.300711
Bitracion.Frontal.Arc	Bitracion.Frontal.Arc	2.286070
Head.Circumference	Head.Circumference	2.117332
Head.Length	Head.Length	1.383157



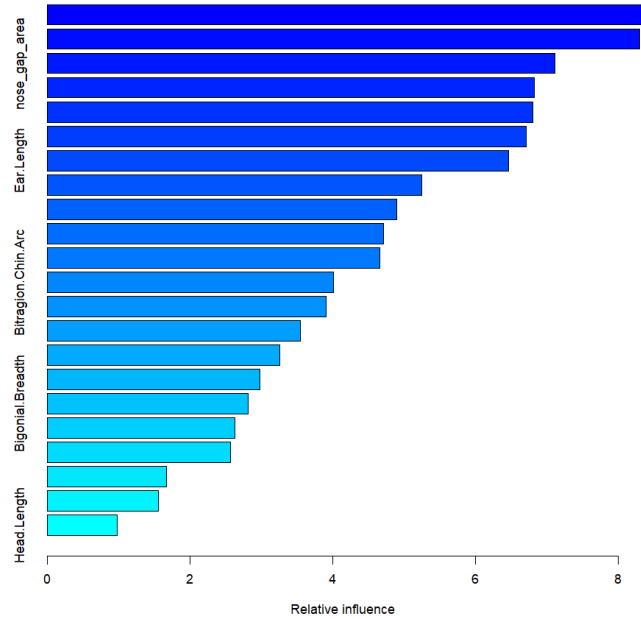


Feature	Red Bar	Green Bar	Blue Bar
Head Circumference	1.2	1.8	2.2
Ear Length	2.2	2.8	3.2
Nose Breadth	3.8	4.2	4.5
Bitrignon Coronal Arc	4.8	5.2	5.5
Bigonial Breadth	7.2	7.5	18.5

Feature	Relative influence
Head_Circumference	1.2
Bigonial Breadth	1.5
Lip_Length	2.0
Ear_Length	2.2
nose_gap_area	2.5
Head_Circumference	2.8
Bigonial Breadth	3.0
Lip_Length	3.8
Ear_Length	4.0
nose_gap_area	4.8
Head_Circumference	5.0
Bigonial Breadth	5.5
Lip_Length	6.0
Ear_Length	7.0
nose_gap_area	9.8
Head_Circumference	12.5

## MKF94

	var	rel.inf
Age.x	Age.x	8.3669942
Nose.Breadth	Nose.Breadth	8.2995780
nose_gap_area	nose_gap_area	7.1143605
Lip.Length	Lip.Length	6.8263855
BMI.x	BMI.x	6.7981118
Bizygomatic.Breadth	Bizygomatic.Breadth	6.7069722
Ear.Length	Ear.Length	6.4638645
Ear.Breadth	Ear.Breadth	5.2478557
Upper.Facial.Breadth	Upper.Facial.Breadth	4.9003659
ratio_sel.lip	ratio_sel.lip	4.7104439
Neck.Circumference	Neck.Circumference	4.6611234
Bitrignon.Chin.Arc	Bitrignon.Chin.Arc	4.0056550
Bitrignon.Frontal.Arc	Bitrignon.Frontal.Arc	3.9094330
Weight..kg.	Weight..kg.	3.5446295
Menton.Sellion.Length	Menton.Sellion.Length	3.2581455
Head.Breadth	Head.Breadth	2.9781748
Bigonial.Breadth	Bigonial.Breadth	2.8127678
Bitrignon.Coronal.Arc	Bitrignon.Coronal.Arc	2.6267851
Nose.Length	Nose.Length	2.5643238
Height..cm.	Height..cm.	1.6718965
Head.Circumference	Head.Circumference	1.5531240
Head.Length	Head.Length	0.9790094



KN95	Surgical	KF94	MKF94
nose_gap_area 10.305552	Bitrignon.Chin.Arc 12.780778	Bitrignon.Chin.Arc 17.0976217	Age.x 8.3669942
Nose.Breadth 9.331326	Bizygomatic.Breadth 10.926142	Bigonial.Breadth 7.3841740	Nose.Breadth 8.2995780
Ear.Breadth 9.196040	Nose.Breadth 9.841078	Menton.Sellion.Length 7.0697379	nose_gap_area 7.1143605
Ear.Length 6.062168	nose_gap_area 6.911546	Bizygomatic.Breadth 5.5107890	Lip.Length 6.8263855
Height..cm. 6.015255	ratio_sel.lip 5.750443	Age.x 5.4666848	BMI.x 6.7981118
Lip.Length 6.009020	Upper.Facial.Breadth 5.451953	Head.Length 5.3352446	Bizygomatic.Breadth 6.7069722
Upper.Facial.Breadth 5.845039	BMI.x 5.255903		Ear.Length 6.4638645
ratio_sel.lip 5.263364			Ear.Breadth 5.2478557
Neck.Circumference 5.092802			

1-Black

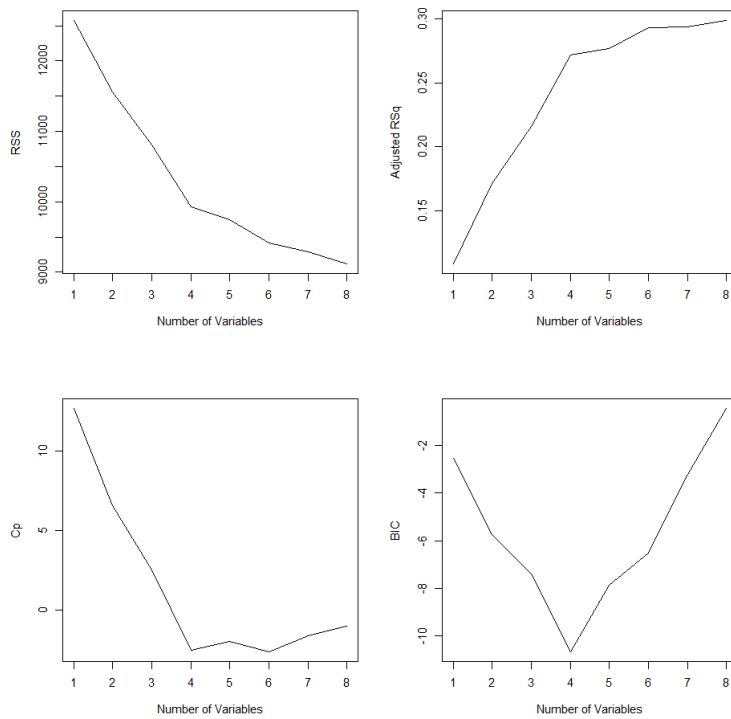
2-Green

3-purple

Shared with Elastic Net Regression Variables

## Best Subset Selection

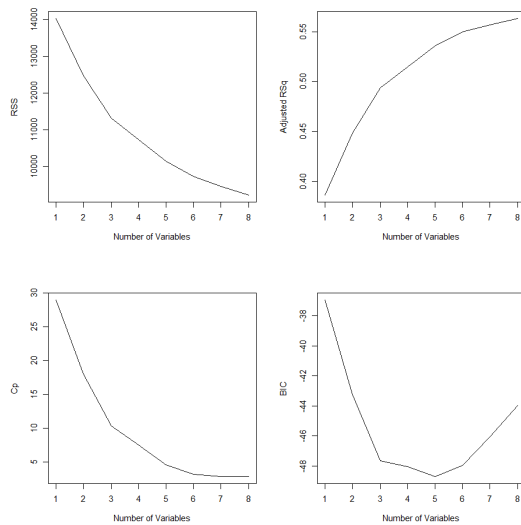
KN95



4 Variables  
 (Intercept)      Ear.Breadth      Lip.Length      Menton.Sellion.Length      nose\_gap\_area  
                     69.477436      3.354625      3.369148      4.244381      -5.218057

6 Variables  
 (Intercept)      Ear.Breadth      Lip.Length      Upper.Facial.Breadth      Menton.Sellion.Length      Bizygomatic.Breadth  
                     69.246286      2.702040      3.495693      -2.393592      4.279744      2.674251

nose\_gap\_area  
 -5.292972



KF94  
 (Intercept)      Neck.Circumference      Menton.Sellion.Length  
                     5.150786      5.546481      5.344939  
 Bizygomatic.Breadth      nose\_gap\_area      Age.x  
                     5.507095      -3.357483      4.726084

*Surgical*

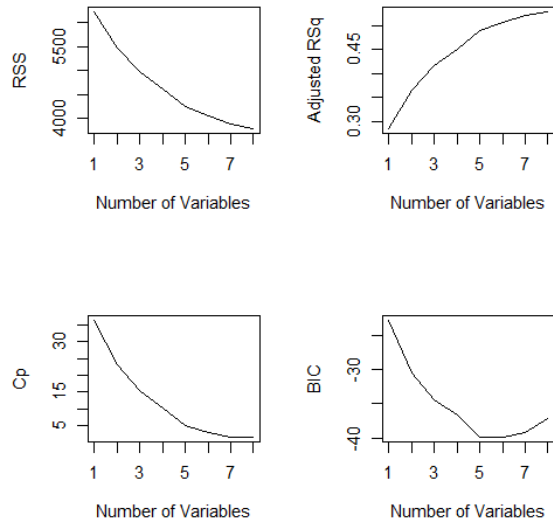
5 variables

(Intercept)	Bitragion.Chin.Arc	Ear.Breadth	Menton.Sellion.Length	Nose.Length	nose_gap_area
57.197332	4.751789	2.278044	4.246167	-3.941203	-3.222140

6 variables

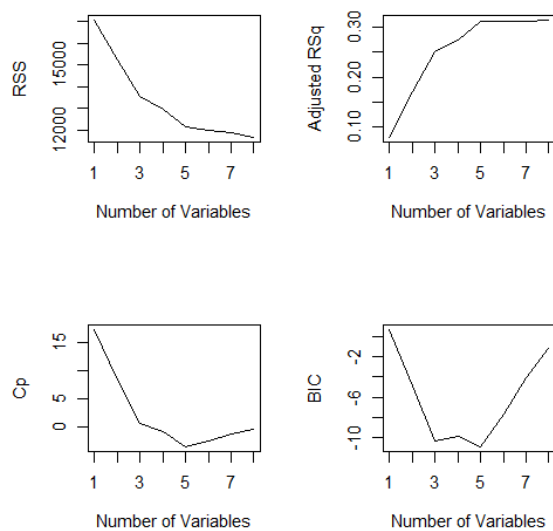
(Intercept)	Bitragion.Chin.Arc	Ear.Breadth	Menton.Sellion.Length	Nose.Breadth	Nose.Length
57.329008	3.786049	2.139793	4.153065	1.890864	-3.956852

nose\_gap\_area  
-2.469667



MKF94

(Intercept)	Bitragion.Chin.Arc	Neck.Circumference	Ear.Length	nose_gap_area	Age.x
66.819995	6.345920	4.740477	-6.230462	-3.426972	6.030945





### *Elastic Net Regression (nonzero coefficients)*

KN95                      s1  
(Intercept)        69.427926563  
Bitragion.Chin.Arc   0.868310870  
Head.Circumference   0.250662861  
Neck.Circumference   1.100548082  
Ear.Breadth        1.847755691  
Lip.Length         1.694117054  
Upper.Facial.Breadth -0.785949160  
Menton.Sellion.Length 1.015436809  
Nose.Breadth        0.562156594  
Nose.Length         0.673638269  
Bigonial.Breadth    0.430640694  
Bizygomatic.Breadth 0.934639906  
Head.Breadth        0.003943001  
nose\_gap\_area       -3.011890165  
Age.x               0.404811635  
BMI.x               -0.598810227

KF94                      s1  
(Intercept)        55.187267238  
Bitragion.Chin.Arc   2.286371160  
Bitragion.Coronal.Arc 0.974059176  
Bitragion.Frontal.Arc 0.881650356  
Head.Circumference   0.156788285  
Neck.Circumference   1.985462690  
Ear.Breadth        1.355486323  
Upper.Facial.Breadth -1.003675766  
Menton.Sellion.Length 2.801212790  
Nose.Breadth        0.512539091  
Nose.Length         -0.004712357  
Bigonial.Breadth    2.119372556  
Bizygomatic.Breadth 1.566417250  
Head.Length         0.482498608  
nose\_gap\_area       -1.808991830  
ratio\_sel.lip        -0.562179661  
Age.x               3.048380048  
Weight..kg.         1.431855205

Surgical                      s1  
(Intercept)        57.16858500  
Bitragion.Chin.Arc   2.52950592  
Bitragion.Coronal.Arc -0.54630017  
Head.Circumference   -0.98116649  
Neck.Circumference   0.86199879  
Ear.Breadth        1.83003310  
Ear.Length         -0.57717693  
Upper.Facial.Breadth -0.09360404  
Menton.Sellion.Length 2.95144412  
Nose.Breadth        2.02968682  
Nose.Length         -2.86874968  
Bizygomatic.Breadth 2.15101967  
nose\_gap\_area       -1.75495990  
Age.x               0.19971203

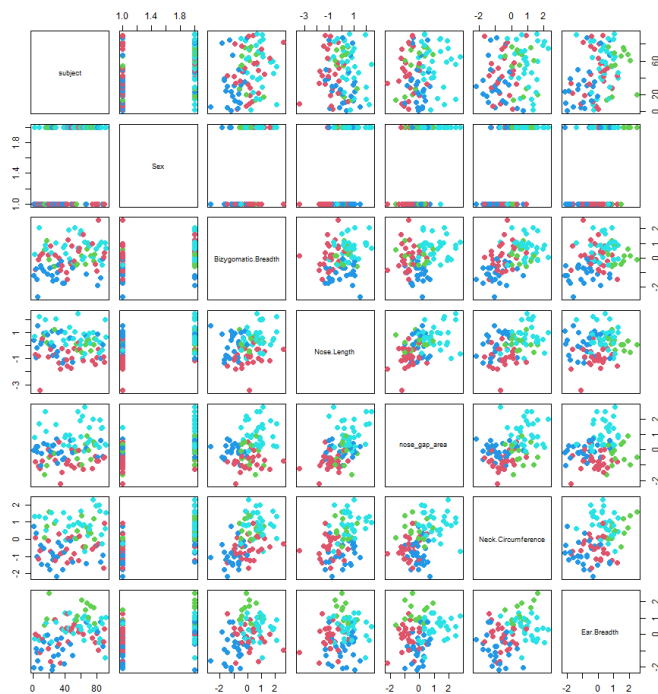
MKF94                      s1  
(Intercept)        66.6839526  
Bitragion.Chin.Arc   2.5236911  
Head.Circumference   -0.1976336  
Neck.Circumference   2.6639496

Ear.Length -3.4388664  
 Lip.Length 1.2209772  
 Menton.Sellion.Length 0.5617950  
 Nose.Breadth 0.5647308  
 Bigonial.Breadth 0.2579257  
 Bizygomatic.Breadth 0.8553541  
 nose\_gap\_area -1.8470165  
 Age.x 4.2235796  
 BMI.x 0.8374927

## Clustering with selected variables

(Bizygomatic Breadth, Nose Length, Nose Gap Area, Neck Circumference, Ear Breadth)

4 k-means clusters



K-means clustering with 4 clusters of sizes 25, 13, 24, 30

Cluster means:

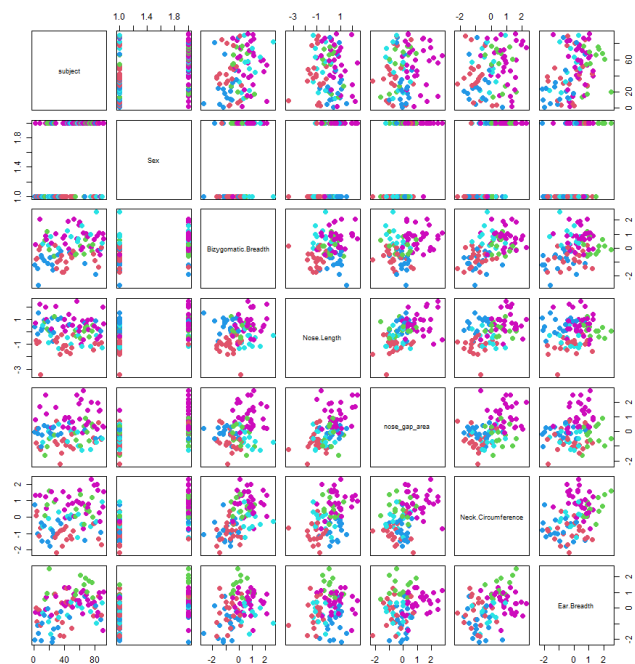
	Bizygomatic.Breadth	Nose.Length	nose_gap_area	Neck.Circumference	Ear.Breadth
1	0.1482384	-0.92412932	-0.75632430	-0.5393143	-0.1610692
2	0.1076945	0.19422189	-0.28171767	0.5775855	1.3355823
3	-1.0482425	0.06596069	-0.06428665	-0.8869017	-0.8833717
4	0.7493256	0.67971436	1.08701650	0.9289105	0.3480105

Within cluster sum of squares by cluster:

[1] 62.71906 21.03027 56.50618 78.93027

(between\_SS / total\_SS = 51.4 %)

5 Clusters



K-means clustering with 5 clusters of sizes 20, 13, 17, 12, 30

Cluster means:

	Bizygomatic.Breadth	Nose.Length	nose_gap_area	Neck.Circumference	Ear.Breadth
1	-0.7411918	-1.1274833	-0.67937915	-0.9549055	-0.2691778
2	0.1076945	0.1942219	-0.28171767	0.5775855	1.3355823
3	-0.9776698	0.4995841	-0.04710818	-0.7806054	-0.9439581
4	0.8326970	-0.6219534	-0.50521375	-0.2000081	-0.3164007
5	0.7493256	0.6797144	1.08701650	0.9289105	0.3480105

Within cluster sum of squares by cluster:

[1] 45.07786 21.03027 32.80232 20.57489 78.93027

(between\_SS / total\_SS = 56.0 %)

## Relative Influence

Add up to 100 or have the most important have 100. Most important is the variable that reduces the error the most when it is used for splitting. It has nothing to do with how it describes the variance. In `gbm()` the relative influence is averaged across all trees generated by the boosting algorithm. (empirical improvements by splitting on `x` at that point, best at training the model). Sex has low importance because it is the only binary variable. It can only be used once in a tree, while other variables can appear multiple times. Frequency gives it low importance even if it does a good job splitting the data and generating a prediction.

