

$$\begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -20 \\ 0 & -8 & -16 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -20 \\ 0 & 0 & \frac{48}{7} \end{bmatrix}$$

$$\det A = 1 \times -7 \times \frac{48}{7} = \boxed{-48}$$

IDEA

$$\begin{bmatrix} 1+\lambda & 2+\lambda & 3+\lambda \\ 8+\lambda & 9+\lambda & 4+\lambda \\ 7+\lambda & 6+\lambda & 5+\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 1+\lambda & 2+\lambda & 3+\lambda \\ 1 & 3 & -1 \\ 6 & 4 & 2 \end{bmatrix} \rightarrow 2 \begin{vmatrix} 1+\lambda & 2+\lambda & 3+\lambda \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$2 \left( 3 \times (-2 - \lambda - 9 - 3\lambda) - 2(-1 - \lambda - 3 - \lambda) + (3 + 3\lambda - 2 - \lambda) \right)$$

$$2(-33 - 12\lambda + 8 + 4\lambda + 1 + 2\lambda) = -48 - 12\lambda$$

$$\begin{bmatrix} x^8 & x^2 & x^3 \\ x^8 & x^9 & x^4 \\ x^7 & x^6 & x^5 \\ x & x & x \end{bmatrix} \rightarrow \begin{bmatrix} x & x^2 & x^3 \\ 0 & 0 & x^4 - x^{10} \\ 0 & x^6 - x^8 & x^5 - x^9 \\ 0 & x - x & x - x \end{bmatrix}$$

$$\det = x(-x^4 - x^{10})(x^6 - x^8) = x(x^4(x^6 - 1) \cdot x^6(1 - x^2))$$

$$= x^{11}(x^6 - 1)(1 - x^2)$$