

$$\begin{array}{c}
 \begin{bmatrix} \textcircled{1} & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{\text{row } \tau = \text{row } \tau + \text{row } 1} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & \textcircled{2} & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{\text{row } \tau = \text{row } \tau + \frac{1}{2} \text{row } \tau} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \\
 \\
 \xrightarrow{\text{row } \epsilon = \text{row } \epsilon + \frac{2}{3} \text{row } \tau} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & \textcircled{\frac{5}{6}} & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{\text{row } \omega = \text{row } \omega + \frac{2}{5} \text{row } \epsilon} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & \frac{2}{5} & 1 \\ 0 & 0 & 0 & 0 & \frac{7}{5} \end{bmatrix}
 \end{array}$$

$$\det = 1 \times 2 \times \frac{3}{2} \times \frac{5}{6} \times \frac{7}{5} = \boxed{7}$$