

$$\begin{vmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{vmatrix} \xrightarrow{r_1 \rightarrow r_1 + r_3} \begin{vmatrix} 8 & 8 & 8 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{vmatrix} \xrightarrow{\text{کردن سطر اول به صفر}} \begin{vmatrix} 8 & 8 & 7 \\ 8 & 9 & 6 \\ 8 & 4 & 5 \end{vmatrix} \xrightarrow{r_2 \rightarrow r_2 - r_1} \begin{vmatrix} 8 & 8 & 7 \\ 0 & +1 & -1 \\ 0 & -4 & -2 \end{vmatrix}$$

$$= 8(-2 - (-1) \times (-4)) = -48$$

$$\begin{vmatrix} 1+x & 1+x & 1+x \\ 1+x & 1+x & 1+x \\ 1+x & 1+x & 1+x \end{vmatrix} \xrightarrow{\substack{r_1 \rightarrow r_1 + r_3 \\ r_2 \rightarrow r_2 - r_1}} \begin{vmatrix} 1+x & 1+x & 1+x \\ 1 & 1 & -1 \\ 1+x & 1+x & 1+x \end{vmatrix} \xrightarrow{r_2 \rightarrow r_2 - \frac{r_1}{1+x}} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1+x & 1+x & 1+x \end{vmatrix}$$

$$= (1+x)(1-(-1) - (1-(-1))) + (1-1) = -4 - 4x$$

$$\begin{vmatrix} x^1 & x^2 & x^3 \\ x^4 & x^5 & x^6 \\ x^7 & x^8 & x^9 \end{vmatrix} = x^1 x^2 x^3 \begin{vmatrix} 1 & x & x^2 \\ x^3 & x^4 & 1 \\ x^7 & x^8 & 1 \end{vmatrix} = x^{10} (x^4 - x - x(x^8 - x^3) + x^7(x^4 - x^8))$$

$$= x^{10} (-x^9 + x^7 + x^3 - x) = -x^{19} + x^{17} + x^{13} - x^{11}$$