

Subject:

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 3 & 4 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & -1 & 0 & 0 & 0 \\ 0 & -8/3 & 16/3 & 1 & 0 & 0 \\ 0 & 2 & 5 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & -1 & 0 & 0 & 0 \\ 0 & -8/3 & 16/3 & 1 & 0 & 0 \\ 0 & 0 & 9 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4/3 & 8/3 & 0 \\ 1 & -2 & 9 \end{bmatrix} = L$$

$$L^{-1} \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 4/3 & 8/3 & 0 & 0 & 1 & 0 \\ 1 & -2 & 9 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 3/4 & 0 \\ 1 & -2 & 9 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 3/4 & 0 \\ 0 & -2 & 9 & -1 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 3/4 & 0 \\ 0 & 0 & 9 & -1 & 0 & 1 \end{array} \right] \rightarrow L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 3/8 & 0 \\ -2/9 & 1/12 & 1/9 \end{bmatrix}$$

$$U^{-1} \rightarrow \left[ \begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 3 & 2 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & 2 & -3 \\ 0 & -1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 2/3 & -1 \\ 0 & 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow U^{-1} = \begin{bmatrix} 1/3 & 2/3 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U^{-1} L^{-1} = \begin{bmatrix} 1/3 & 2/3 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 3/8 & 0 \\ -2/9 & 1/12 & 1/9 \end{bmatrix} = \begin{bmatrix} 2/9 & 1/6 & -1/9 \\ 1/18 & -5/24 & 2/9 \\ -2/9 & 1/12 & 1/9 \end{bmatrix} = A^{-1}$$