$$A = \begin{bmatrix} 0 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \longrightarrow PA = LU$$

$$P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \longrightarrow P_2 = P_1$$

$$U_0 = \begin{bmatrix} 0 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \rightarrow U_1 = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \rightarrow U_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = U$$

$$L_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow L_1 = L_0 \longrightarrow L_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \rightarrow L = L_2 + I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

a) $Ax = \begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$, b) $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

a) $Ax = b \rightarrow PAx = Pb \rightarrow PAx = \begin{bmatrix} -4 \\ 4 \\ 8 \end{bmatrix} \rightarrow LUx = Pb \rightarrow Ux = y$
$Ly = Pb$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 2 & 1 & 8 \end{array}\right] \rightarrow y = \begin{bmatrix} -4 \\ 4 \\ 0 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & -4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{ccc|c} 3 & 0 & -3 & -12 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow x = \begin{bmatrix} t-4 \\ 4-2t \\ t \end{bmatrix}$$

b) $Ax = b \rightarrow PAx = Pb \rightarrow LUx = Pb \rightarrow Ux = y$, $Ly = Pb$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right] \rightarrow \begin{cases} y_1 = 0 \\ y_2 = 1 \\ y_3 = -2y_2 = -2 \end{cases} \rightarrow y = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right] \rightarrow$$ بخاطر سطر آخر جواب نداره