$r_2 + \frac{-4}{3} r_1 \to r_2$:

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 3 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 3 & 4 & 4 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_3 - r_1 \to r_3$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 3 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 0 & 2 & 5 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$r_3 + 3/4\, r_2 \to r_3$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3/4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 0 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 0 & 0 & 9 \end{bmatrix} = U \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3/4 & 1 \end{bmatrix}$$

$$L = (E_3 E_2 E_1)^{-1} = E_1^{-1} E_2^{-1} E_3^{-1}$$

$$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{3}{4} & 1 \end{bmatrix}$$

$$E_3^{-1} E_2^{-1} E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 1 & -\frac{3}{4} & 1 \end{bmatrix}$$

$$\to L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 1 & -\frac{3}{4} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -8/3 & 16/3 \\ 0 & 0 & 9 \end{bmatrix}$$

$L^{-1} = ?$ $\qquad L = (E_3 E_2 E_1)^{-1} \to L^{-1} = E_3 E_2 E_1$

$$\to L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3/4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4/3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ -2 & \frac{3}{4} & 1 \end{bmatrix}$$

$U^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & -8/3 & 16/3 & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftarrow r_2 \times \frac{-3}{8}}$$

$$\left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -\frac{3}{8} & 6 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftarrow \frac{2}{9} r_3 + r_2} \left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_1 \leftarrow r_1 + \frac{1}{9} r_3} \left[\begin{array}{ccc|ccc} 3 & 2 & 0 & 1 & 0 & \frac{1}{9} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftarrow -2r_2 + r_1} \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & \frac{3}{4} & \frac{-1}{3} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[r_3 \leftarrow r_3 \times \frac{1}{9}]{r_1 \leftarrow r_1 \times \frac{1}{3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{4} & \frac{-1}{9} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{array}\right] \Rightarrow U^{-1} = \left[\begin{array}{ccc} \frac{1}{3} & \frac{1}{4} & \frac{-1}{9} \\ 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{array}\right]$$

$A^{-1} = ?$ $\quad A^{-1} = U^{-1} L^{-1} = \left[\begin{array}{ccc} \frac{1}{3} & \frac{1}{4} & \frac{-1}{9} \\ 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ -2 & +\frac{3}{4} & 1 \end{array}\right]$

$$= \begin{bmatrix} 1 \times \frac{1}{3} + \frac{1}{4} \times \frac{4}{3} + \frac{-1}{9} \times 2 & \frac{1}{4} \times 1 + \frac{-1}{9} \times \frac{-3}{4} & \frac{-1}{9} \\ -\frac{3}{8} \times \frac{-4}{3} + \frac{2}{9} \times -2 & -\frac{3}{8} \times \frac{2}{9} \times \frac{3}{4} & \frac{2}{9} \\ +\frac{1}{9} \times -2 & \frac{1}{9} \times \frac{3}{4} & \frac{1}{9} \end{bmatrix}$$

$$= \left[\begin{array}{ccc} \frac{2}{9} & \frac{1}{6} & \frac{-1}{9} \\ \frac{1}{18} & \frac{-5}{24} & \frac{2}{9} \\ \frac{-2}{9} & \frac{1}{12} & \frac{1}{9} \end{array}\right] = A^{-1}$$