« مسائل عددی »

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• تجزیه LU ماتریس زیر را بیابید.

$$A=\begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 3 & 4 & 4 \end{bmatrix} \qquad A=LU$$

$$\xrightarrow{row2-\frac{4}{3}row1} \begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{row3-row1} \begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 2 & 5 \end{bmatrix} \xrightarrow{row3+\frac{3}{4}row2}$$

$$U=\begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 0 & 9 \end{bmatrix} \Longrightarrow L=\begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 1 & -\frac{3}{4} & 1 \end{bmatrix}$$

$$A^{-1}=U^{-1}L^{-1}$$

• می دانیم $[B \mid I] \longrightarrow [I \mid B^{-1}]$.

محاسبه U^{-1}:

$$\left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & -\frac{8}{3} & \frac{16}{3} & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow{row1+\frac{3}{4}row2} \left[\begin{array}{ccc|ccc} 3 & 0 & 3 & 1 & \frac{3}{4} & 0 \\ 0 & -\frac{8}{3} & \frac{16}{3} & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{row2=-\frac{3}{8}row2 \\ row3=\frac{1}{9}row3}]{row1=\frac{row1}{3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{3} & \frac{1}{4} & 0 \\ 0 & 1 & -2 & 0 & -\frac{3}{8} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{array}\right] \xrightarrow[row2+2row3]{row1-row3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{4} & -\frac{1}{9} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{array}\right]$$

$$\Longrightarrow U^{-1}=\begin{bmatrix} \frac{1}{3} & \frac{1}{4} & -\frac{1}{9} \\ 0 & -\frac{3}{8} & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 & 0 & 1 & 0 \\ 1 & -\frac{3}{4} & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[row3-row1]{row2-\frac{4}{3}row1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{4}{3} & 1 & 0 \\ 0 & -\frac{3}{4} & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\text{row}_3 + \frac{3}{4}\text{row}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 & -2 & \frac{3}{4} & 1 \end{array}\right] \Longrightarrow L^{-1} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ -2 & \frac{3}{4} & 1 \end{array}\right]$$

$$\Longrightarrow A^{-1} = U^{-1} L^{-1} \Longrightarrow A^{-1} = \left[\begin{array}{ccc} \frac{1}{4} & \frac{1}{4} & -\frac{1}{6} \\ 0 & -\frac{3}{7} & \frac{2}{6} \\ 0 & 0 & \frac{1}{6} \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ -2 & 3/4 & 1 \end{array}\right]$$

$$\Longrightarrow A^{-1} = \left[\begin{array}{ccc} \frac{4}{6} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{14} & -\frac{5}{24} & \frac{2}{6} \\ -\frac{2}{6} & \frac{1}{12} & \frac{1}{6} \end{array}\right]$$