$$D = \begin{bmatrix} A & B \\ C & I \end{bmatrix} \qquad DD^{-1} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & I \end{bmatrix} \begin{bmatrix} K & Z \\ y & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$$AK + By = I$$

$$AZ + BX = 0$$

$$CK + y = 0 \rightarrow y = -CK \quad (1)$$

$$CZ + X = I \rightarrow X = I - CZ \quad (2)$$

$$\Rightarrow AZ + BX = 0 \xrightarrow{(2)} AZ + B(I - CZ) = 0 \rightarrow AZ - BCZ + B = 0$$

$$\rightarrow (A - BC)Z = -B \xrightarrow{\text{در دو طرف } A-BC \text{ ضرب}} \boxed{Z = -(A-BC)^{-1}B}$$

$$\xRightarrow{(2)} \boxed{X = I + (A-BC)^{-1}B}$$

$$AK + By = I \xrightarrow{(1)} AK - BCK = I \rightarrow (A-BC)K = I$$

$$\xRightarrow{\text{در دو طرف ضرب}} \boxed{K = (A-BC)^{-1}} \qquad \xRightarrow{(1)} \boxed{y = -C(A-BC)^{-1}}$$

$$\Rightarrow D^{-1} = \begin{bmatrix} (A-BC)^{-1} & -(A-BC)^{-1}B \\ -C(A-BC)^{-1} & I + (A-BC)^{-1}B \end{bmatrix}$$