

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad \begin{array}{l} \text{row } 2 \leftarrow \text{row } 2 + \text{row } 1 \\ \text{---} \end{array}$$

$$C_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\det(A) = \det(C_1) = 1 \cdot \det \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\text{row } 2 \leftarrow \frac{1}{r} \text{row } 1 + \text{row } 2$$

$$C_2 = \begin{bmatrix} r & 1 & 0 & 0 \\ 0 & 1/r & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \rightarrow \det(A) = \det(C_1) = \det(C_2)$$

$$= r \cdot \det \begin{bmatrix} 1/r & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\det \begin{bmatrix} 1/r & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \det \begin{bmatrix} 1/r & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -r & 0 \end{bmatrix}$$

$$= (-1)^{r,r} \begin{vmatrix} 1/r & 1 \\ 1 & -r \end{vmatrix} = -1 \cdot r - r \cdot \frac{r}{r} = -r$$

$$\Rightarrow \boxed{\det(A) = r}$$