

# Day 1

Wednesday, March 21, 2018

10:58 PM

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$= \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ U_{12} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{matrix} = \rightarrow$$

$$\begin{aligned} \textcircled{1} U_{11} &= 3 & U_{11} &= 3 \\ \textcircled{2} U_{11} L_{21} &= 1 & L_{21} &= \frac{1}{3} \\ \textcircled{3} U_{11} L_{31} &= 2 & L_{31} &= 1 \end{aligned}$$

$\rightarrow U$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ 1 & L_{32} & 1 \end{bmatrix} \begin{bmatrix} 3 & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \rightarrow$$

$$L = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad u = \begin{bmatrix} u_{12} \\ u_{22} \\ u_{32} \end{bmatrix}$$

$$(1) u_{12} = 2$$

$$(2) u_{12} \frac{1}{2} + u_{22} = 0 \Rightarrow u_{22} = -\frac{1}{2}$$

$$(3) \underbrace{u_{12}}_2 + \underbrace{u_{22}}_{-\frac{1}{2}} L_{21} = 1 \Rightarrow L_{21} = -\frac{2}{1}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & u_{13} \\ 0 & -\frac{1}{2} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = )$$

$$(1) u_{13} = -1$$

$$(2) \frac{1}{2} u_{13} + u_{23} = 1 \Rightarrow u_{23} = \frac{3}{2}$$

$$(3) u_{13} - \frac{2}{1} u_{23} + u_{33} = 1 \Rightarrow u_{33} = 9$$

$$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & -\frac{1}{2} & 1 \end{bmatrix} \quad u = \begin{bmatrix} 2 & 2 & -1 \\ 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 9 \end{bmatrix}$$

$$u = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ -\frac{1}{\sqrt{2}} \\ 1 \end{bmatrix}$$

$$\begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 1 & -\frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 1 & 1 \end{vmatrix} = -2 \times 1 + 0 - 1 \times 1 \neq 0$$

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$$[L \ I] =$$

$$\begin{matrix} (1) \\ (2) \\ (3) \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 1 & 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} 0 \\ -\frac{1}{\sqrt{2}} \\ -1 \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 & -1 & 0 & 1 \end{bmatrix} + \frac{2}{3} \textcircled{R}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} + \frac{2}{3} & 1 & 1 \end{bmatrix}$$

$\underbrace{\hspace{10em}}_I \qquad \underbrace{\hspace{10em}}_{L^{-1}}$

$$\rightarrow [V \ I] =$$

$$\begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix} \begin{bmatrix} 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \times \\ \times \\ \times \end{matrix} \begin{matrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{9} \end{matrix}$$

$$\textcircled{1} \rightarrow \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{6} \end{bmatrix}$$

$$\textcircled{2} \rightarrow \begin{bmatrix} 1 & 0 & 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{6} \end{bmatrix} \quad \textcircled{2} \times 2 + \textcircled{3}$$

$$\textcircled{3} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{6} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{6} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{6} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} - \frac{1}{3} + \frac{2}{9} & \frac{1}{3} - \frac{1}{3} & -\frac{1}{9} \\ \frac{1}{3} - \frac{2}{9} & \frac{2}{3} + \frac{1}{9} & \frac{2}{9} \\ -\frac{2}{9} & -\frac{1}{3} & \frac{1}{9} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{9} & -\frac{1}{3} & \frac{1}{9} \\ \frac{1}{3} & \frac{2}{9} & -\frac{1}{9} \\ -\frac{1}{9} & \frac{1}{3} & \frac{2}{9} \end{bmatrix}$$

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