ارمان سرور ۹۵C۱۸۰۷

تجزیه LU : $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 4 \\ 8 & 4 & 4 \end{bmatrix}$ (۱

$U = \text{ref}(A)$

$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

$U = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 2 & 5 \end{bmatrix} \begin{matrix} r_2 - \frac{4}{3}r_1 \\ r_3 - r_1 \end{matrix} \Rightarrow l_{21} = \frac{4}{3}, \; l_{31} = 1$

$\Rightarrow \begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 0 & 9 \end{bmatrix} r_3 + \frac{3}{4}r_2 \Rightarrow l_{32} = -\frac{3}{4}$

$\Rightarrow A = LU = \begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 1 & -\frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 0 & 9 \end{bmatrix}$

(۲

$L^{-1} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 & 0 & 1 & 0 \\ 1 & -\frac{3}{4} & 1 & 0 & 0 & 1 \end{array}\right] \Rightarrow$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{4}{3} & 1 & 0 \\ 0 & -\frac{3}{4} & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} r_2 - \frac{4}{3}r_1 \\ r_3 - r_1 \end{matrix} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 & -1 & \frac{3}{4} & 1 \end{array}\right]$

$\underbrace{\qquad\qquad}_{L^{-1}}$

Subject:
Year: Month: Date:

$$U \Rightarrow \begin{bmatrix} 2 & 2 & -1 & 1 & 0 & 0 \\ 0 & -\frac{8}{4} & \frac{14}{4} & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{2}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{8}{4} & \frac{14}{4} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{bmatrix} \begin{matrix} r_1/3 \\ \\ 9/r_3 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{9} \\ 0 & -\frac{8}{4} & \frac{14}{4} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{bmatrix} r_1 + \frac{1}{4} r_2 - r_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{9} \\ 0 & 1 & 0 & 0 & -\frac{2}{8} & \frac{2}{9} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{bmatrix}$$

$$A^{-1} = U^{-1} L^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{9} \\ 0 & -\frac{2}{8} & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{4}{4} & 1 & 0 \\ -2 & \frac{2}{4} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{9} & \frac{1}{4} & -\frac{1}{9} \\ \frac{1}{18} & -\frac{5}{24} & \frac{2}{9} \\ -\frac{2}{9} & \frac{1}{12} & \frac{1}{9} \end{bmatrix}$$