$A \xrightarrow{\text{ابعاد}} n\times n \quad v \longrightarrow n\times 1 \quad u \longrightarrow n\times 1$

برای اثبات سوال کافی است نشان دهیم:

$$① \quad (A+uv^t)\left(A^{-1}-\frac{A^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}\right)=I_n$$

و

$$② \quad \left(A^{-1}-\frac{A^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}\right)(A+uv^t)=I_n$$

ابتدا به اثبات اولی می‌پردازیم:

$$(A+uv^t)\left(A^{-1}-\frac{A^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}\right)$$

$$=\underbrace{AA^{-1}}_{I_n}-\frac{\overbrace{AA^{-1}}^{I_n}uv^tA^{-1}}{1+v^tA^{-1}u}+uv^tA^{-1}-\frac{uv^tA^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}$$

دقت شود که: $v^t_{1\times n}\times A^{-1}_{n\times n}\times u_{n\times 1}=C_{1\times 1}\longrightarrow$ اسکالر

ادامه: $=I_n+uv^tA^{-1}-\frac{uv^tA^{-1}+uv^tA^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}$

$$=I_n+uv^tA^{-1}-\frac{(u+uv^tA^{-1}u)v^tA^{-1}}{1+v^tA^{-1}u}$$

$$=I_n+uv^tA^{-1}-\frac{u\cancel{(1+v^tA^{-1}u)}v^tA^{-1}}{\cancel{1+v^tA^{-1}u}}$$

$$=I_n+\cancel{uv^tA^{-1}-uv^tA^{-1}}=I_n \quad ①$$

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حال به اثبات دوم می پردازیم:

$$\left(A^{-1} - \frac{A^{-1}uv^tA^{-1}}{1+v^tA^{-1}u}\right)(A+uv^t)$$

$$= \underbrace{A^{-1}A}_{I_n} + A^{-1}uv^t - \frac{A^{-1}uv^t\overbrace{A^{-1}A}^{I_n}}{1+v^tA^{-1}u} - \frac{A^{-1}uv^tA^{-1}uv^t}{1+v^tA^{-1}u}$$

$$= I_n + A^{-1}uv^t - \frac{A^{-1}uv^t + A^{-1}uv^tA^{-1}uv^t}{1+v^tA^{-1}u}$$

$$= I_n + A^{-1}uv^t - \frac{A^{-1}u(v^t + v^tA^{-1}uv^t)}{1+v^tA^{-1}u}$$

$$= I_n + A^{-1}uv^t - \frac{A^{-1}u\cancel{(1+v^tA^{-1}u)}v^t}{\cancel{1+v^tA^{-1}u}}$$

$$= I_n + \cancel{A^{-1}uv^t - A^{-1}uv^t} = I_n \quad (2)$$

$(1), (2) \longrightarrow$ حکم