

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = B$$

$$B = L_B U_B$$

$$L_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = U_B$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} A = L_B U_B \rightarrow A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} U_B$$

$$Ax = b \rightarrow L(U(x)) = b \quad L(y) = b$$

$$U(x) = y$$

$$b = \begin{bmatrix} 1 \\ \epsilon \\ -\epsilon \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \epsilon \\ -\epsilon \end{bmatrix}$$

$$\rightarrow \begin{cases} y_2 + y_3 = 1 \rightarrow y_3 = 0 \\ y_2 = \epsilon \\ y_1 = -\epsilon \end{cases} \quad y = \begin{bmatrix} -\epsilon \\ \epsilon \\ 0 \end{bmatrix}$$

$$Ux = y \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \epsilon \\ -\epsilon \\ 0 \end{bmatrix} \quad \begin{cases} x_1 + x_2 + x_3 = \epsilon \\ x_2 + x_3 = -\epsilon \\ \rightarrow x_2 = -\epsilon - x_3 \\ x_1 = \frac{\epsilon - x_2 + \epsilon x_3}{1} \end{cases}$$

$$\rightarrow x = \begin{bmatrix} \epsilon + x_3 \\ -\epsilon - x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} \epsilon \\ -\epsilon \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad Ly = b \rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{cases} y_2 + y_3 = 0 \rightarrow y_3 = -y_2 \\ y_2 = 1 \\ y_1 = 0 \end{cases}$$

$$y = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$Ux = y \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad \begin{cases} x_1 + x_2 + x_3 = 0 \\ x_2 + x_3 = 1 \\ 0 = -1 \quad \times \end{cases}$$

no solution