18.369 Problem Set 5

Due Wednesday, 3 April 2024.

Problem 1: A triangular metal box

Consider the two-dimensional solutions in a *triangular* perfect-metal box with side L. Don't try to solve this analytically; instead, you will use symmetry to sketch out what the possible solutions will look like for both E_z and H_z polarizations.

(a) List the symmetry operations in the space group (choose the origin at the center of the triangle so that the space group is symmorphic), and break them into conjugacy classes. (This group is traditionally called $C_{3\text{v}}$). Verify that the group is closed under composition (i.e. that the composition of two operations always gives another operation in the group) by giving the "multiplication table" of the group (whose rows and columns are group members and whose entries give their composition).

(b) Find the character table of $C_{3\text{v}}$, using the rules from the representation-theory handout.

(c) Give unitary representation matrices D for each irreducible representation of $C_{3\text{v}}$.

(d) Sketch possible $\omega \neq 0$ E_z and H_z solutions that would transform as these representations. What representation should the lowest-ω mode (excluding $\omega = 0$) of each polarization correspond to?

(e) If there are any (non-accidental) degenerate modes, show how given one of the eigenfunction we can compute the other orthogonal eigenfunction(s). (For example, in the square case we could get one from the $90°$ rotation of the other for a degenerate pair, but the triangular structure is not symmetric under $90°$ rotations.) Hint: use your representation matrices.