Thin Boundary Layer Equations wall model in Trio_U VDF

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23rd July 2004

1 Introduction

This document is meant to expose the implementation of the TBLE wall model in the Trio_U code. This wall model consist in embedding a one-dimensional fine grid between the wall and the first computation node. It was introduced in 1996 by Balaras [1] and has been increasingly investigated by other researchers (Cabot [3, 4, 5], Diurno [6], Wang and Moin [8]...).

First, the solving method of the TBLE will be explained. Then, results of LES with TBLE wall model will be discussed. Finally, various keywords and the implementation will be exposed.

2 Numerical solving method in the TBLE fine grid

2.1 Equations in the fine mesh

In the TBLE (*Thin Boundary Layer Equation*) wall-model, the wall shear stress is evaluated thanks to boundary layer equations applied in a one-dimensional fine grid in the near-wall region.

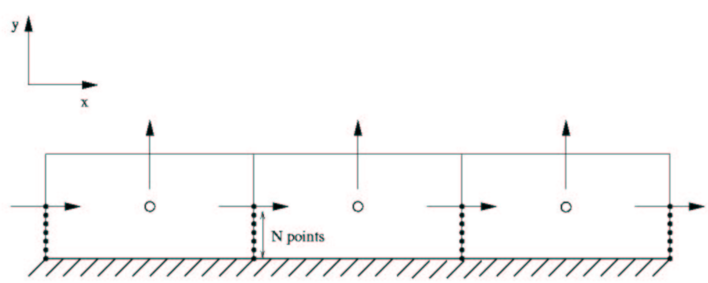

Figure 1: TBLE fine grid near the wall

Equations to solve for the velocity components U and W (to simplify, we will concentrate on the first one), respectively in the x-direction (streamwise direction) and the z-direction (spanwise direction):

$$\frac{\partial}{\partial y}\left(a\frac{\partial U}{\partial y}\right)+F=\frac{\partial U}{\partial t} \tag{1}$$

$$F=Q-\frac{\partial P}{\partial x} \tag{2}$$

$$a=\nu+\nu_t=\nu+D(\kappa y_w)^2\left(\left(\frac{\partial U}{\partial y}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2\right)^{1/2} \tag{3}$$

$$D=1-exp\left(-\left(\frac{y_w^+}{25}\right)^3\right) \tag{4}$$

(2) is the pressure gradient expression in the bi-periodical channel flow case. Q is the source term ensuring the global mass conservation in the channel and P represents the pressure.

(3) is the total viscosity composed by the molecular viscosity ν and the turbulent viscosity ν_t which is evaluated with a mixing length model in the fine mesh.

D is the Van Driest damping function.

2.2 Numerical method

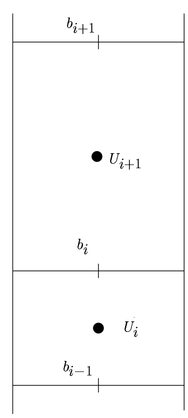

Figure 2: i-Node in the fine mesh

Now, we will describe the numerical method used to solve equation (1) for the first velocity component. Equation (1) is solved iteratively (iteration number is noted p) in the fine grid (i is the node number in the one-dimensional grid).

$$\frac{2}{y_{i+1}-y_{i-1}}\left(a^p_{i+1/2}\frac{U^{p+1}_{i+1}-U^{p+1}_{i}}{y_{i+1}-y_i}-a^p_{i-1/2}\frac{U^{p+1}_{i}-U^{p+1}_{i-1}}{y_{i}-y_{i-1}}\right)+F=\frac{U^{p+1}_i-U^n_i}{\Delta t_{LES}} \tag{5}$$

U^n_i is the velocity in the fine grid at the previous LES time step and Δt_{LES} is the LES time step.

$$a^p_{i+1/2}=\nu+D(\kappa y_w)^2\left(\left(\frac{U^p_{i+1}-U^p_i}{y_{i+1}-y_i}\right)^2+\left(\frac{W^p_{i+1}-W^p_i}{y_{i+1}-y_i}\right)^2\right)^{1/2} \tag{6}$$

Now, the coefficients b_i, A_i and C_i will be defined :

$$b^p_i=\frac{2\,a^p_{i+1/2}}{(y_{i+1}-y_i)\,(y_{i+1}-y_{i-1})} \tag{7}$$

Equation (5) can be written as :

$$\left(b^p_i+b^p_{i-1}+\frac{1}{\Delta t_{LES}}\right)U^{p+1}_i=b^p_iU^{p+1}_{i-1}+b^p_{i-1}U^{p+1}_{i-1}+F+\frac{U^n_i}{\Delta t_{LES}} \tag{8}$$

We introduce A_i and C_i such as :

$$\boxed{C^p_iU^{p+1}_i=b^p_{i-1}U^{p+1}_{i-1}+A^p_i\quad\text{for}\quad 2\le i\le N-2 \qquad (9)}$$

C^p_i and A^p_i can be obtained applying (9) to U^{p+1}_{i+1} and replacing its new value in (8) :

$$\boxed{\begin{aligned} C^p_i &= b^p_i+b^p_{i-1}-\frac{b^p_i\times b^p_i}{C^p_{i+1}}+\frac{1}{\Delta t_{LES}} && (10)\\ A^p_i &= F+b^p_i\frac{A^p_{i+1}}{C^p_{i+1}}+\frac{U^n_i}{\Delta t_{LES}} && (11)\end{aligned}}$$

2.3 Boundary conditions

▶ **i=N-1**

In the TBLE wall-model, the top velocity boundary condition U_N of the fine mesh is equal to the velocity at the first cell of the LES-mesh (graphic 3). Then, (5) and (7) applied to the top boundary node leads to :

$$(2b_{N-1}+b_{N-2})U_{N-1}=2b_{N-1}U_N+b_{N-2}U_{N-2}+F \tag{12}$$

Thus C^p_{N-1} and A^p_{N-1} are :

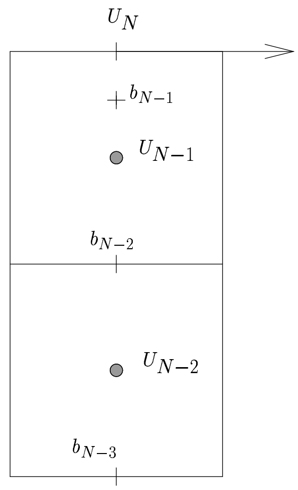

Figure 3: Top boundary condition in the fine mesh

$$\boxed{\begin{aligned} C^p_{N-1} &= 2b_{N-1} + b_{N-2} + \frac{1}{\Delta t_{LES}} && (13) \\ A^p_{N-1} &= F + 2b_{N-1}U_N + \frac{U^n_{N-1}}{\Delta t_{LES}} && (14) \end{aligned}}$$

▶ **i=1**

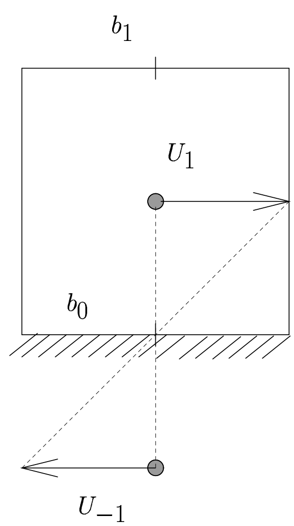

Figure 4: Bottom boundary condition in the fine mesh

To force the no-slip condition, a virtual node $i = -1$ is introduced so that $U_{-1} = -U_1$ (graphic 4). Accordingly to (9) :

$$C^p_1 U_1 = b_0 U^p_{-1} + A^p_1 \tag{15}$$

And therefore,

$$C_1^p U_1^p = -b_0 U_1^p + A_1^p \tag{16}$$

Hence : $$\boxed{U_1 = \frac{A_1}{C_1 + \frac{\nu}{(y_1 - y_w)^2}} \quad \text{assuming that} \quad b_0 = \frac{\nu}{(y_1 - y_w)^2} \qquad (17)}$$

2.4 Numerical procedure

▶ First, the TBLE boundary conditions are set : $U_N^n(LES) = U_N^n(TBLE)$ and $U_0^n(LES) = U_0^n(TBLE)$. $U_N^n(LES)$ is the first LES node.

▶ Then, the TBLE mesh is computed. The iteration procedure goes on while the friction velocity has not converged. While the convergence criterion is not satisfied the following instructions are performed :

- b_i^p coefficients are computed from U_i^p
- Set of A_{N-1}^p and C_{N-1}^p then calculation of A_i^p and C_i^p
- Calculate U_1^{p+1}
- Calculate U_i^{p+1}
- Calculate wall shear stress τ_{wj}^{p+1} in the streamwise ($j = 1$) and spanwise ($j = 3$) directions
- Friction velocity convergence check $\left(\frac{\|u_\tau^p - u_\tau^{p+1}\|}{u_\tau^{p+1}} < \epsilon\right)$ with ϵ very small (10^{-5} for example).

▶ Finally, we obtain, from the TBLE mesh, the wall shear stress τ_w^n to compute the LES velocity at n+1 instant.

3 LES with TBLE model : results

TBLE wall model is tested at relatively low Reynolds number ($Re_\tau = 590$) and the results are compared with Moser DNS ones [7] (the results and the numerical method have also been described in [2]). To study the behavior of the TBLE inner mesh, we added the inner mesh profiles to the outer one (except for the normal component since it is not computed in the inner mesh).

Regarding the TBLE results the mean velocity profiles (figure 5) are in great agreement (for both inner and outer mesh) with the DNS. Moreover, we can notice that the continuity of the mean velocity at the interface between the inner and the outer mesh is ensured.

However, the continuity of the fluctuations (figures 6 and 8) is not obtained. Given that a RANS eddy viscosity is implemented to simulate the flow in the inner mesh, it could seem surprising to look for fluctuations in the near wall region. Nonetheless, the upper boundary condition is evolving like the LES of the outer mesh and we can observe a linear fluctuation profile developing from the wall up to the outer LES fluctuation. As for the other fluctuation profiles (figures 7 and 8), they are underestimated comparing with the DNS because the meshes are too coarse to capture all the turbulent scales.

4 TBLE user guide using TRIO_U code

4.1 Required TBLE keywords

N : number of nodes in the TBLE grid
In fact, it is the only keyword required.

4.2 Optional TBLE keywords

facteur : stretching ratio for the TBLE grid (to refine the TBLE **facteur** must be greater than 1).
modele_visco : file name containing the description of the eddy viscosity model
Nb_comp : number of component to solve in the fine grid (1 if 2D simulation (2D not available yet), 2 if 3D simulation)
stats : statistics of the TBLE velocity and turbulent viscosity profiles. 4 values are required : the starting time of velocity averaging, the starting time of the RMS fluctuations, the ending time of the statistics computation and finally the print time period for the statistics.
Here you can find an example of the TBLE command line in a data file :
Turbulence_paroi Paroi_TBLE { N 6 modele_visco Diffu_lm facteur 1.33 stats 9. 10. 20. 2. }

5 TBLE implementation in TRIO_U code

Files needed to modify the TBLE wall model :
../ThHyd/Turbulence/Diffu_totale_base.h and *.cpp
../ThHyd/Turbulence/Diffu_totale.h and *.cpp
../ThHyd/Turbulence/Diffu_lm.h and *.cpp
../ThHyd/Turbulence/Eq_couch_lim.h and *.cpp
../ThHyd/Turbulence/Vect_Eq_couch_lim.h and *.cpp
../VDF/Turbulence/ParoiVDF_TBLE.h and *.cpp

A few explanations can be provided about the implementation of the TBLE wall model. To perform the computation, an *Eq_couch_lim* object, in which TBL Equations are solved, is affected at each wall cell. Hence, we have vectors of *Eq_couch_lim* objects (see file **Vect_Eq_couch_lim.h** and ***.cpp**). The boundary conditions of TBLE are set in the file **ParoiVDF_TBLE.cpp** and the wall shear stress provided to the LES is retrieved in this same file. The TBL Equations are solved and the wall shear stress is computed in the *"aller_au_temps"* method of Eq_couch_lim objects (see file **Eq_couch_lim.h** and ***.cpp**).

The TBLE turbulent viscosity model is defined in the file **Diffu_lm.h**. It should be recalled that the TBLE user can develop his own turbulent viscosity model in specific files **My_Diffu_lm.h** and ***.cpp** and then used for the TBLE computation using the following keywords in the data file : *modele_visco My_Diffu_lm*.

6 Conclusion and future work

A brand new wall model has been implemented in the Trio_U code. Its numerical solving method and its implementation have been entirely described in this document. All the future work on TBLE will first focus on taking into account the convective terms in the TBLE equation. Their effects on the LES results should be analyzed and discussed. Then, this wall model should be implemented for unstructured mesh (Trio_U VEF) and tested in flow configurations with adverse pressure gradients.

References

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List of Figures

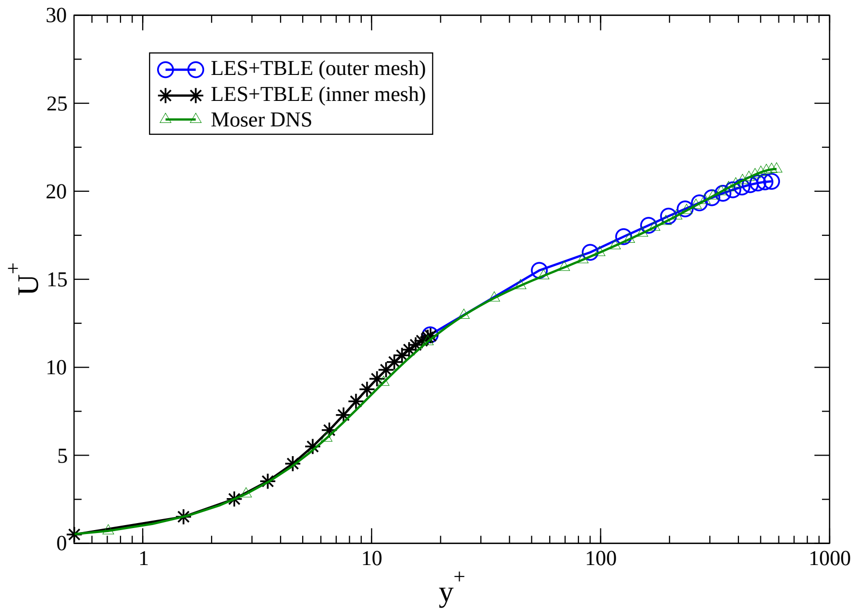

Figure 5: Reduced mean velocity profiles ($Re_\tau = 590$).

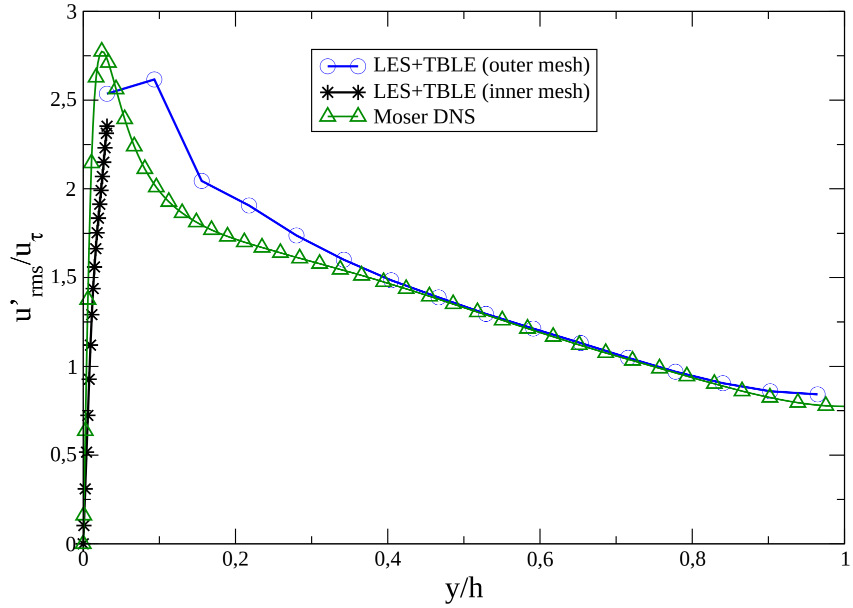

Figure 6: Streamwise RMS velocity fluctuations profiles ($Re_\tau = 590$).

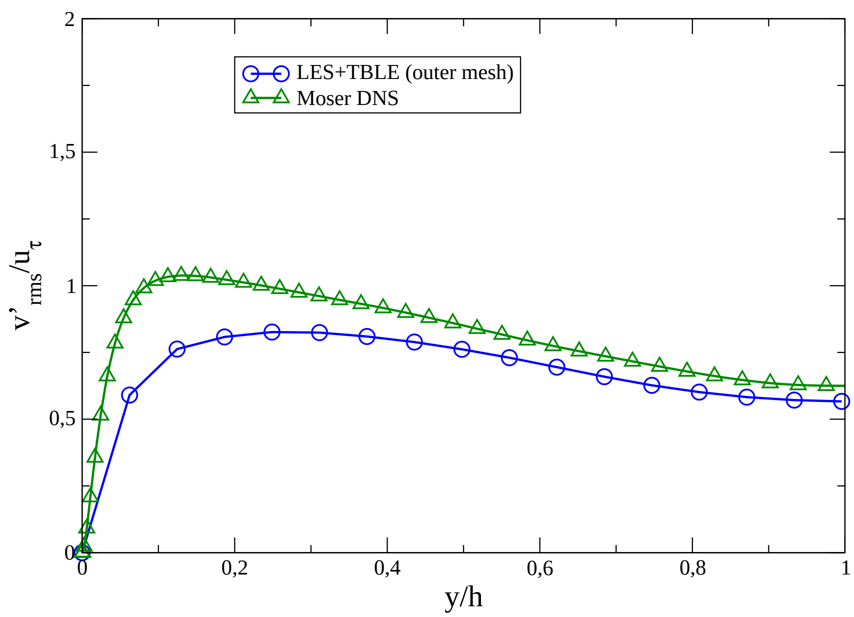

Figure 7: Normal RMS velocity fluctuations profiles ($Re_\tau = 590$).

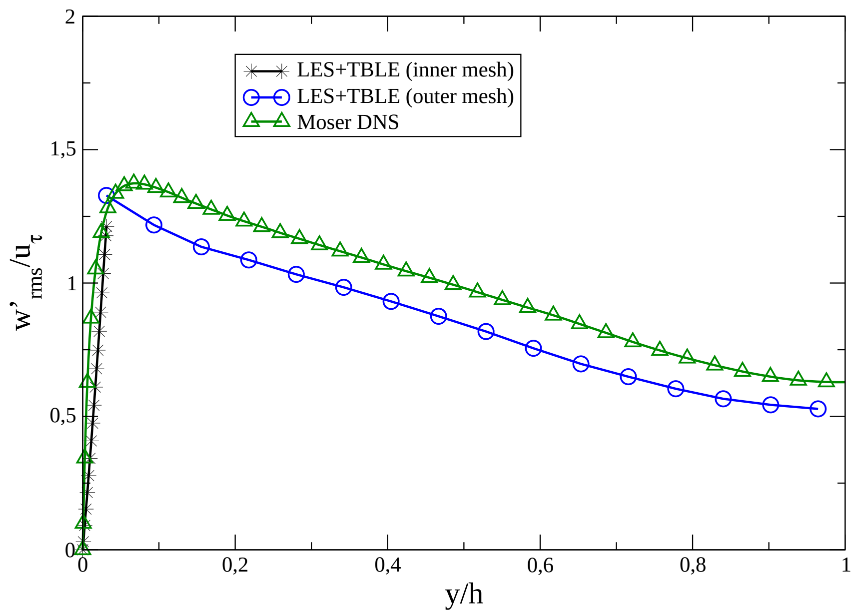

Figure 8: Spanwise RMS velocity fluctuations profiles ($Re_\tau = 590$).

Contents